

Pulse Oximeter Prototype Design Project

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A comprehensive analysis of pulse oximeter fabrication, integration, and testing has been conducted. The complete design, testing, and analysis process is henceforth described. Two prototypes were fabricated and tested. The first, employing a photo-sensitive resistor as the light detector, uses a Wheatstone bridge configuration to monitor change in resistance. Although similar in construction the second employs a photo-diode with a transimpedance amplifier to output voltage. LabVIEW software in conjunction with an NI MyDAQ allowed processing of output voltages. Both circuits utilize Light Emitting Diodes as light sources, producing wavelengths matching each light detector respectively. Team member's resting heart rates were measured, however, due to inconsistent results from the photo-resistor the team proceeded with the photo-diode. Further results were obtained as each member's heart rate was measured and recorded, first after vigorous activity and again after one minute of recovery. As confidence intervals were obtained and Student-t tests performed the team determined that the differences in found data and the national data were statistically significant with a 95% confidence interval. Using this data as a model for USU's entire Engineering population suggests that the resting heart rate of an engineer at USU is higher than that of their fellow average American.

Nomenclature

R	= resistance
V_{ex}	= excitation voltage
P	= power
I	= current
\bar{x}	= mean
S_x	= standard deviation
x_i	= individual data point
n	= number of samples
t	= student-t value
μ_x	= true mean
ν	= degrees of freedom

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I. Introduction

Pulse Oximetry finds its advantage in that it provides a non-invasive process for obtaining information regarding a subjects heart rate and oxygen levels in the blood. This method is photo based and the concept is relatively simple by nature. The technology is widely used for two purposes. First, to monitor the oxygenation status is the blood through its light absorptive characteristics. Second, to determine a heart rate through the pulsating nature of the heart beat and the slight change in the volume of blood flowing through the veins with each pulse.

The overarching principle for the design of the pulse-oximeter is to use light absorption of blood to create an electrical signal, more specifically a potential difference that can be measured. Bloods absorption of light varies with the amount of oxygen that it contains. Arteries carry a bright red, oxygen rich blood while veins carry a darker red blood lacking oxygen. The figure below presents the molar extinction coefficient (light absorption) of blood in Arteries (HbO₂) and Veins (Hb) as a function of wavelength. As the figure shows, there is a significant difference between the two in the 650 to 700 nm wavelengths (red light) as well as the 900 to 1000 nm wavelengths (infrared light). True pulse oximeters operate by careful measurements of light absorption at both wavelengths. By comparing the ratios of light absorbed at these wavelengths blood oxygen content can be determined as well as a heart beat.

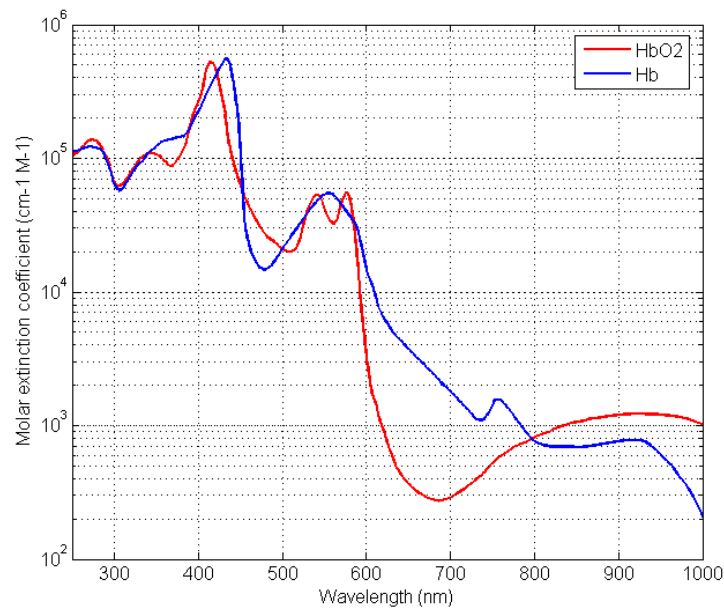


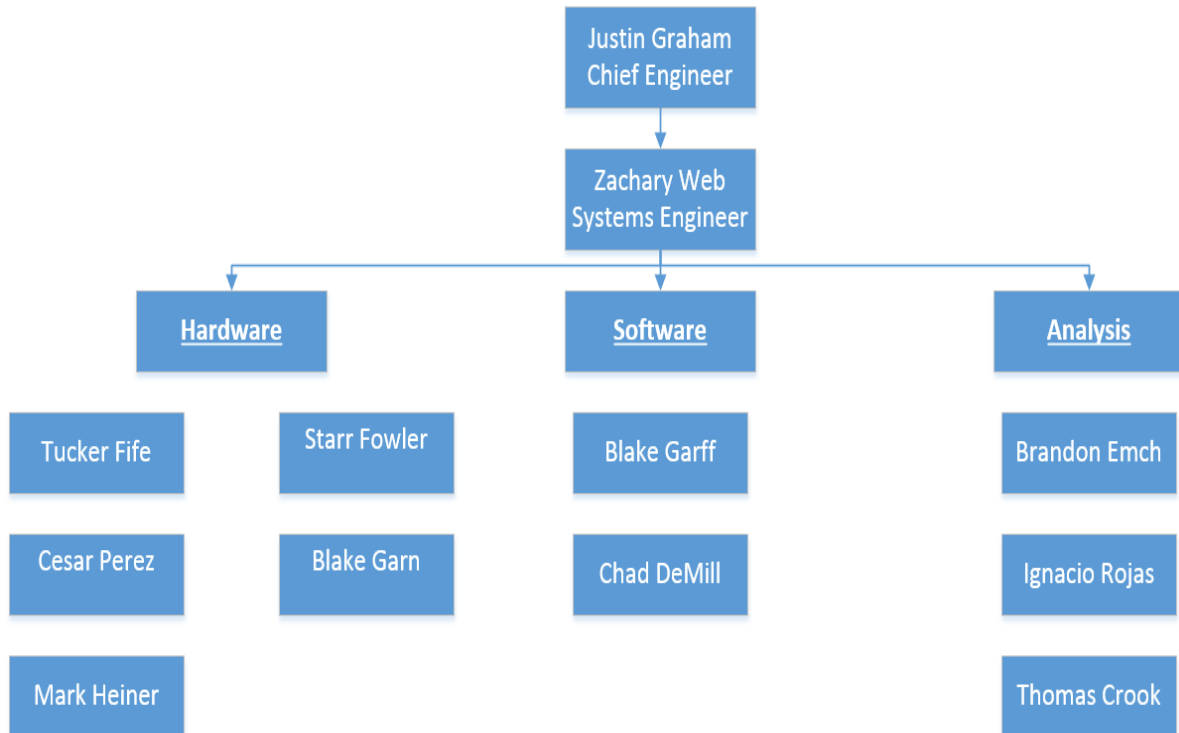
Figure 1. Light absorption levels of Blood as a function of wavelength
© 2012 Patrick Dear, Mark Bunney, BrainMap¹

Our project while much more simplistic in design uses the same idea as the pulse oximeter in that we will use light absorption to measure heart rate. Our design does not include gathering data at both the red wavelength and infrared wavelength and thus we will not be able to determine oxygen content but we hope to be able to accurately determine heart rate.

In order for the concept to work, there must be a light source and a light detector placed opposite from one another with the semi-transparent extremity passing between. Fingers, toes and ear lobes are most commonly used and provide accurate results as the tissues found therein are semi-transparent and allow transmission of light. Two different prototype concepts involving different light detectors and conditioning circuits were to be considered. Each of the possible options were considered and are thoroughly described in the Theory and Requirements section.

Once the fabrication process was complete, data on individual team members heart rates were recorded and measured using the NI MyDAQ and LabVIEW software. The compiled data could then be statistically analyzed and compared to data on the National average.

II. Team Members and Roles



III. Theory and Requirements

Two different prototype concepts involving different light detectors and conditioning circuits were to be considered:

A. Photo-Resistive Sensing Circuit:

The first prototype employs the use of a photo-sensitive resistor. Cadmium Sulfide (CdS) sensors are most commonly used however other sensors, including Cadmium Selenide and Cadmium Sulfide Selenide sensors, are available. A typical photo-resistor can be seen in Fig. 2 below.

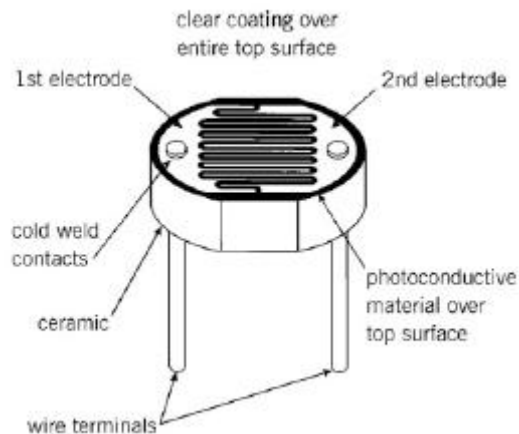


Figure 2. Typical Photo-Resistor. Courtesy of Final Project Description

For the photo-sensitive resistor two possible circuit conditioning configurations were considered, including the voltage divider, and Wheatstone bridge. The conditioning circuits respond to the change in resistance provided by the photo-sensitive resistor and output a proportional voltage to be processed by a NI MyDAQ and LabVIEW software.

B. Photo-Generation Sensing Circuit:

The second prototype employs the use of a photo-diode and essentially acts as a solar cell, outputting a small current that is proportional to the light received by the sensor. In order to interpret the data provided by the photo-diode a conditioning circuit must be used to convert the provided current into a voltage that is readable by the NI MyDAQ and LabVIEW software. A transimpedance amplifier² as seen in Fig. 3 provides a sensible opportunity for this conversion.

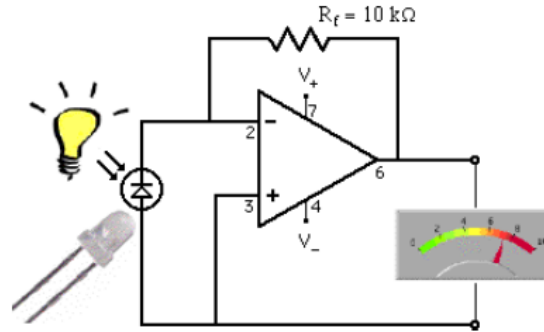


Figure 3. Transimpedance Amplifier for Photo-Diode. Courtesy of Final Project Description

As the illuminated light reaches the photo-diode a current is generated and flows through R_f to the operational amplifier's output terminal. Because no current flows into the op-amp, the voltage drop across the photo-diode is zero, and the output voltage is directly proportional to the light incident upon the sensor.

For both prototypes Light Emitting Diodes (LEDs) are most commonly used as the light source. Multiple colors and wavelengths are available however in order to ensure more accurate results and maximum sensitivity, its important that the wavelength of the LED match the response of the light detector.

IV. Design Schematic and Analysis

A. Photo Resistive Circuit:

One design approach to the pulse oximeter was a photo-resistive circuit. The idea is to have a resistor that is sensitive to the light that will be absorbed by the finger and will be sensitive enough to determine when there is a higher concentration of blood due to pulsation from the heart. If this is the case we can plot the data over time to see the periodic behavior of the blood absorption and thus extract a heartbeat frequency. This idea seems simple enough but as it turns out the circuit can be so sensitive to light that it can pick up frequencies from the light in the room and produce several hundred frequencies that will make it difficult. To help overcome this problem we process the data received by eliminating frequencies that are well beyond what we are looking for. This is done through a digital filter.

Our particular design consisted of a wheatstone bridge with one arm as the photo-resistor which was positioned to receive light from an LED after passing through a finger. We focused our efforts on using a red light, as it could most easily pass through the finger without being absorbed. The wheatstone bridge allowed us to get a large potential difference for the signal. As the photo-resistor's resistance is not constant we chose to use a variable resistor to balance the circuit as best we could before gathering data. The figure below shows a schematic of the circuit described.

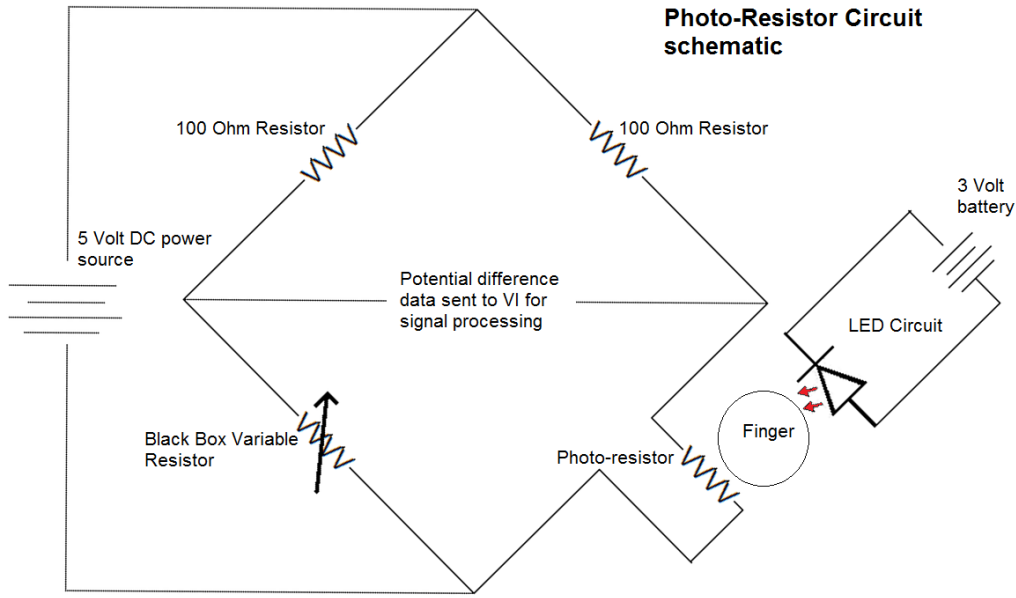


Figure 4. Photo-Resistor Circuit Schematic. Note that the LED is powered by a separate source to ensure that no additional noise was introduced to the sensor circuit.

On the photo resistor, the voltage output was maximized by using a balanced Wheatstone bridge. The circuit we constructed is shown below:

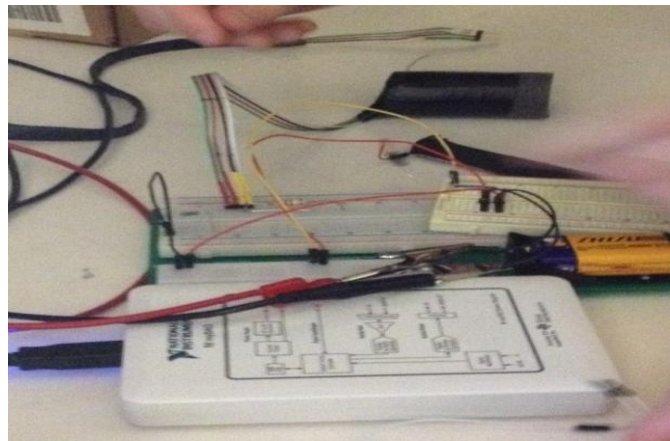


Figure 5. Wheatstone Bridge Circuit

B. Photo Diode Circuit:

For the photo diode circuit, this was accomplished by using an op amp and changing the resistance in the feedback loop of the amplifier. The higher the resistance, the higher the gain of the op amp and the more sensitive the reading of the pulse. The limiting factor for the resistor was voltage saturation. The op amp has a maximum output of about 15 V. The maximum current output of the photo diode was about 50 μA (45 to 75 μA). The maximum resistance was obtained by Ohms law, $R = V/I$, ($15 \text{ V} / 50 \mu\text{A} = 300 \text{ k}\Omega$). After the circuit was implemented, going above the calculated resistance was the optimal choice for a constant reading. The following is a picture of the photo diode circuit created:

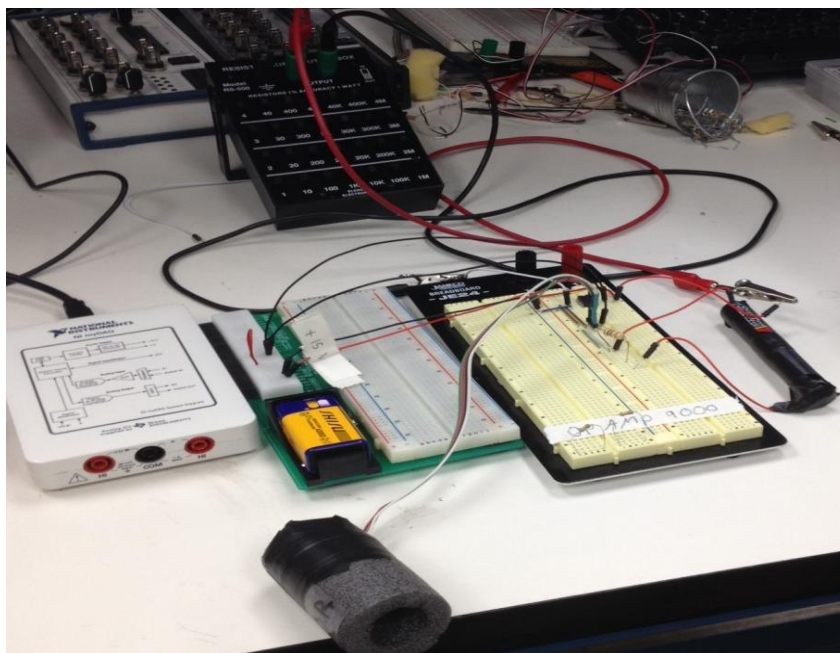


Figure 6. Photo Diode Circuit

The op amp circuit used was based off of a circuit presented in pulsesensor.com³, shown below. The placement of the finger is included.

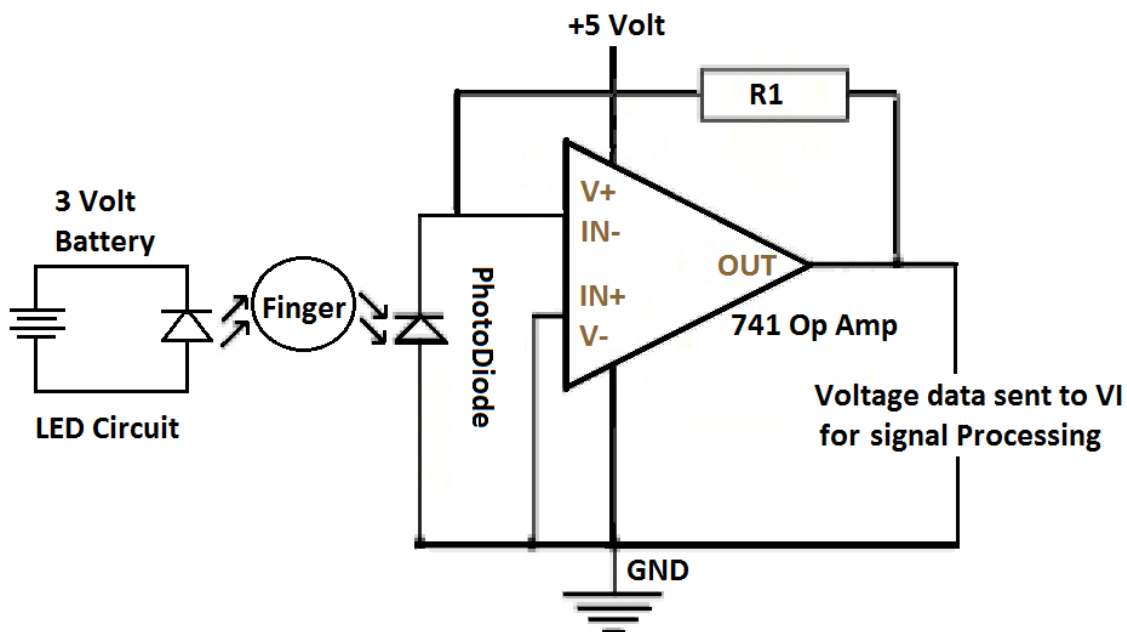


Figure 7. Op Amp Circuit

V. Parts and Components

In order to select proper components different wave lengths of light produced by the most common LED's available were considered and an attempt was made to match those wave lengths to an available photo resistor and a photo diode. Two CdS photo cells were located, from Spark fun and Adafruit, with a useable spectrum ranging from approximately 400 – 800 nm with the optimal wavelength at approximately 500 nm. A blue LED (450-495nm) and a red LED (620 - 740nm) were evaluated from each company. A search for an appropriate photodiode that would work with these light spectrums proved to be a more difficult task. While not optimal, a photodiode from Sparkfun was chosen despite not being able to sense wavelengths at its optimal wavelength. Although Adafruit had a cheaper photo resistor, they would only sell LED's in packages of 25 and they did not have a photodiode that fit our design. If we went with Adafruit for the photo resistor we would have to pay shipping and handling from different companies and also have more LED's than we needed. Ultimately, Sparkfun was chosen so that we could buy everything we needed in one place and have the proper amount of hardware. All of the component specifications provided by each company can be seen in Appendix A.

VI. Assembly Process & Images

Step 1: Using a 4 cord wire, we chose one end of the wire to be the Sensor end to be connected to the LED and photo resistor while the opposite end will be connected to the header. On the sensor end we split all 4 wires from one another about 3 inches from the end so we could strip and expose the metal from the plastic in order to twist the ends together.

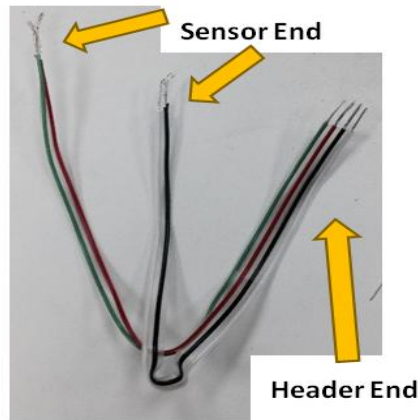


Figure 8. Sensor Wiring

Step 2: Choosing the header end, slip a small piece of heat shrink over each wire. Next, solder individually exposed wires to the Header pin. Then carefully shrink the heat shrink over each solder joint by using a heat gun.

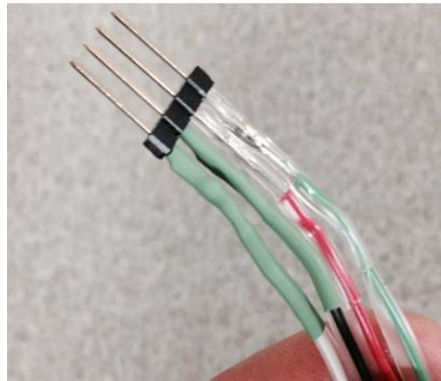


Figure 9. Header Pin

Step 3: Next, slip a small piece of heat shrink over each wire of the sensor end. Then, solder the black and white wires to the LED bulb. Then solder the green and red wires to the photo resistor. Be careful to note that the white and red wires remain the negative for each assembly while the black and green wires are positive. Then carefully shrink the heat shrink over each solder joint by using a soldering iron.



Figure 10. Heat Shrink

Step 4: To ensure our sensors get the best reading we bought a black Styrofoam tube casting from Home Depot and using the soldering iron burnt two very small holes for our photo resistor and LED.



Figure 11. Styrofoam Tube Casting

Step 5: Next, place the photo resistor and LED in each hole of the Styrofoam tube casting.

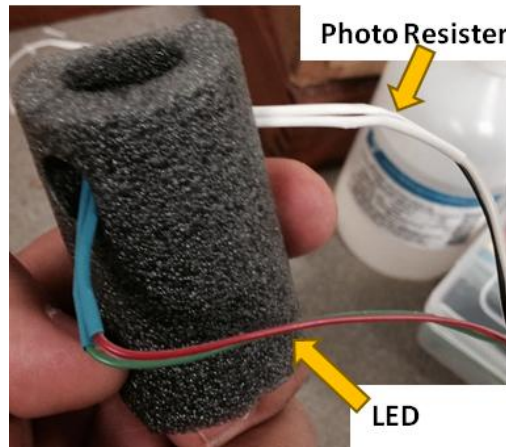


Figure 12. Placement

Step 6: We then wrapped multiple layers of black electrical tape around the Styrofoam casting, photo resistor and LED wires. Then wrapped an aluminum foil around the complete assembly which acted a shield against the fluorescent light radiation. Lastly, to secure the aluminum foil to the assembly we added one more layer of black electrical tape.



Figure 13. Completed Prototype

VII. Test and Analysis

A. Circuit Analysis

With this experiment, uncertainty wasn't really an issue. An accurate voltage reading came secondary to differentiating between blood pulses. The accuracy of this measurement was improved by optimizing the voltage output.

In order to ensure that the wattage drawn by each component was within specified limits an analysis on individual components was considered using the equations provided within section 3.1 slide 24 of class notes.⁴ The equations can be seen in Fig 14 as follows.

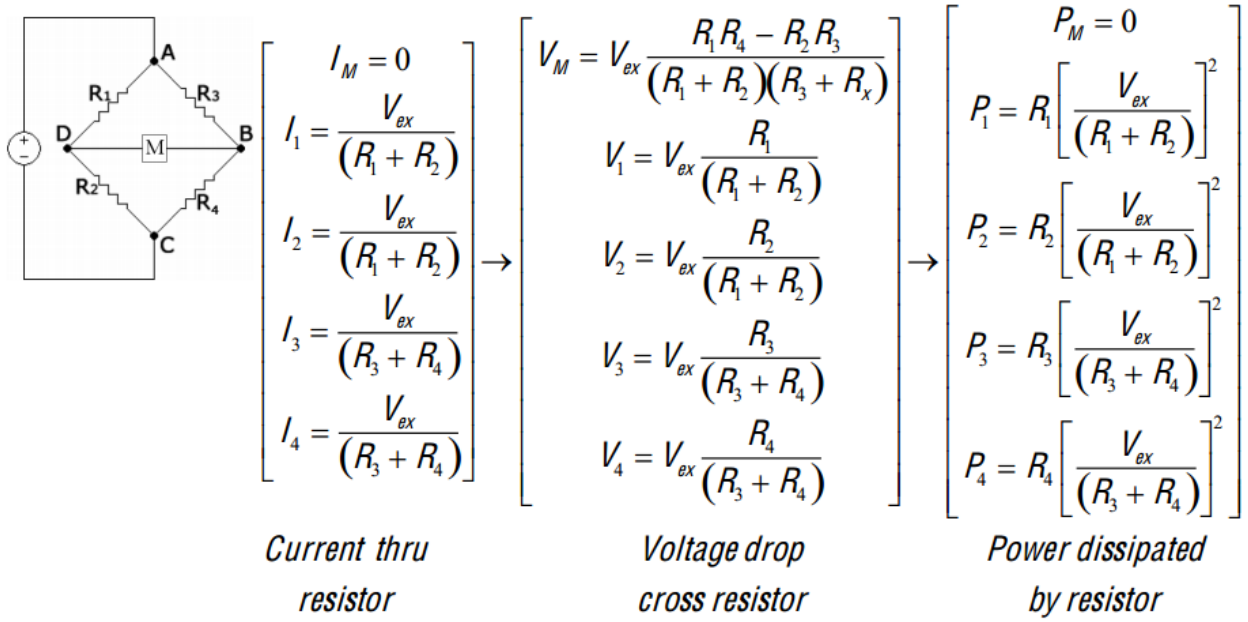


Figure 14. Equations for Power Calculations

Wheatstone Bridge:

$$R_1 := 100\text{ohm} = 100\Omega$$

$$R_2 := 5000\text{ohm} = 5 \times 10^3 \Omega$$

$$R_3 := 100\text{ohm} = 100\Omega$$

$$R_{4a} := 8000\text{ohm} = 8 \times 10^3 \Omega \quad (\text{light})$$

$$R_{4b} := 1000000\text{ohm} = 1 \times 10^6 \Omega \quad (\text{dark})$$

$$V_{ex} := 5\text{V}$$

In full light at the photo resistor:

$$P_{4a} := R_{4a} \left[\frac{V_{ex}}{(R_3 + R_{4a})} \right]^2 = 3.048 \times 10^{-3} \text{W}$$

In total darkness at the phot resistor:

$$P_{4b} := R_{4b} \left[\frac{V_{ex}}{(R_3 + R_{4b})} \right]^2 = 2.5 \times 10^{-5} \text{W}$$

Photodiode Circuit:

$$R_1 := 300000 \text{ ohm} = 3 \times 10^5 \Omega$$

$$I := .00005 \text{ A} = 5 \times 10^{-5} \text{ A}$$

$$P := I^2 \cdot R_1 = 7.5 \times 10^{-4} \text{ W}$$

LED Circuit:

$$V_{\text{ex}} := 3 \text{ V}$$

$$R_1 := 360 \text{ ohm}$$

$$I := \frac{V_{\text{ex}}}{R_1} = 8.333 \times 10^{-3} \text{ A}$$

$$P_1 := I^2 \cdot R_1 = 0.025 \text{ W}$$

This is the power dissipated by the resistor, to find the power dissipated through the led its resistance value would need to be known.

B. Results and Discussion

A small sample of resting and recovery heart rates was taken using our monitors to represent the entire body of engineering students at USU. Both monitors were used to take the resting pulse however only the photo diode monitor was used to take the maximum and recovered heart rate due to inconsistent “finicky” readings from the other. A limited number of maximum and recovered heart rates were taken because consistent readings weren’t obtained before the heart rate was close to resting rate again. All data is displayed in Table 1 as seen below.

Table 1: Measured Heart Rates

Student	Resting Resistor	Resting Diode	Maximum	Recovered	Recovery
1	72	76	114	84	30
2	65	89	115	97	18
3	70	81	105	91	14
4	74	79	168	129	39
5	88	84	88	82	6
6	65	71	88	79	9
7	74	82	147	87	60
8	63	65	97	77	20
9	65	75	172	108	64
10	66	85	141	93	48
11	73	75			

The following equations were used to find the mean, standard deviation, and degrees of freedom for these data sets. All values shown are for resting diode data.

$$\bar{x} = \frac{\sum_{i=1}^n x_i}{n} = 78.364 \quad \text{Eq. (1)}$$

$$S_x = \sqrt{\sum_{i=1}^n \frac{(x_i - \bar{x})^2}{n-1}} = 6.860 \quad \text{Eq. (2)}$$

$$DOF = n - 1 = 10 \quad \text{Eq. (3)}$$

A “t” value was then found from tables⁴ for a 95% confidence interval and the calculated degrees of freedom.
 $t = 2.228$

A true mean confidence interval was then calculated.

$$\bar{x} - t \times \frac{S_x}{\sqrt{n}} \leq \mu_x \leq \bar{x} + t \times \frac{S_x}{\sqrt{n}}$$

$$73.756 \leq \mu_x \leq 82.971$$

This process was then repeated to obtain confidence intervals for other data sets. Results are tabulated in table (2). Student T-tests were then performed on each set to determine whether the differences in our data and known national averages.

The effective degrees of freedom was found with subscripts one and two representing our data and the national averages respectively and then used to find the maximum t value for which our data would be statistically equivalent to the national date. (Standard deviation calculations for the national average were calculated by Professor Whitmore):

$$v = \frac{[(S_1^2/n) + (S_2^2/n)]^2}{\frac{(S_1^2/n)^2}{n_1 - 1} + \frac{(S_2^2/n)^2}{n_2 - 1}} = 10 \quad \text{Eq. (4)}$$

$$t_{max} = 2.228$$

A t value comparing the data was found to be:

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{(S_1^2/n) + (S_2^2/n)}} = 2.431 \quad \text{Eq. (5)}$$

Since $t > t_{max}$ the differences in our data and the national data are statistically significant for a 95% confidence interval. Taking this data as a model for the entire engineering population of USU would mean that the average heart rate of engineers at USU are higher than the average American. This was repeated for other data sets and the results are displayed in Table 2 below.

Table 2: Calculated Statistics

Value	Resting Resistor	Resting Diode	Recovery
Mean	70.455	78.364	30.80
Standard Deviation	7.451	6.86	19.93
Degrees of Freedom	10	10	9
t	2.228	2.228	2.262
Confidence Interval	5.005	4.608	15.026
v	10	10	9
t_{max}	2.228	2.228	2.262
t	-0.154	2.431	0.183

C. "Lessons Learned"

During the fabrication process attention to detail is very important. Any mistake made during this step can lead to headaches down the road, for instance if care is not taken to make sure that every solder joint provides a good connection it can greatly extend the troubleshooting process. Another important aspect is the location of the LED and photo-resistor/diode. Care should be taken to ensure that they line up concentrically. During the testing stages the accuracy of the readings seemed to be heavily dependent on the location of ones finger. It would be a good idea to design a housing for the components that incorporated a "fixture" to ensure that each person's finger was located to the same position every time. An example of this may be a back stop.

It is a good idea to have at least one person who was part of the fabrication process also be apart of the integration. This helps to minimize questions like "Where does this wire go?" When it comes to building the circuits the cleaner looking the better, as it can often be confusing to come back to the circuit after a while and remember what you had done. Labeling each portion of the circuit is worth the extra time.

As long as the fabrication and integration between hardware and software are done correctly the testing is a very straightforward process. Every problem we had during the testing stage seemed to trace back to a mistake we had made earlier on in the design process. The first stages of testing could be more accurately named "debugging."

VIII. Conclusion

Pulse oximetry is an incredibly valuable tool in the medical field. The ability to measure blood oxygen content and heart rate by simply attaching a device to the end of a finger is so easy these devices are used extensively in and out of hospitals everywhere. The device we built is capable of measuring heart rate.

The results from our photo diode device indicate that statistically, lab group 506 has a higher resting heart rate than the national average and that there is no significant statistical difference between the recovery rates of the two groups. These results were based on a small sample size. A larger sample size would give more accurate results. Additionally there is a 10-15 second delay before a reading can be taken, this may have exaggerated the recovery beats per minute data. Taking into account the stress that a junior in the USU engineering program experiences during finals week it makes sense that our resting heart rates are slightly above average.

Appendix

The specifications provided by each company for individual components are listed as follows:

LED				
Company:	Spark Fun	Spark Fun	Adafruit	Adafruit
Name:	LED - Basic Red 5mm	LED - Basic Blue 5mm	LED - Red	Led - Blue
Part # :	COM-09590	COM-11372	297	301
Max Current:	20 mA	30 mA	20 mA	20 mA
Wave Length:	620 -740 nm	450 - 495 nm	640 nm	465 nm
Price:	\$0.35	\$0.35	\$8.00	\$8.00
Quantity:	1	1	25	25

Photo Resistor		
Company:	Spark Fun	Adafruit
Name:	Mini - Photocell	Photo cell
Part #:	SEN-09088	161
Type:	CdS	CdS
Light resistance :	8-20 kOhm	5 - 10 k ohm
Dark resistance :	~10k Ohm	200 k ohm
Max power:	100 mW	100 mW
Max Voltage:	150 V	Na
Wave Length:	540 nm	520 nm
Price:	\$1.50	0.95
Quantity:	4	1

Photodiode		
Company:	Spark Fun	AdaFruit
Name:	Miniature Solar Cell	NA
Part #:	BPW34	NA
Wave Length:	430 - 1100 nm	NA
Price:	1.5	NA
Quantity:	4	NA
Shipping:	3.93	NA

CdS PHOTOCONDUCTIVE CELLS

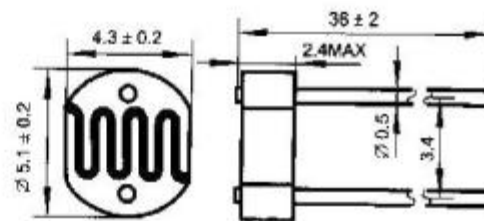
GL5528



- ▲ Epoxy encapsulated
- ▲ Quick response
- ▲ Small size
- ▲ High sensitivity
- ▲ Reliable performance
- ▲ Good characteristic of spectrum

Light Resistance at 10Lux (at 25°C)	8~20KΩ
Dark Resistance at 0 Lux	1.0MΩ(min)
Gamma value at 100-10Lux	0.7
Power Dissipation(at 25°C)	100mW
Max Voltage (at 25°C)	150V
Spectral Response peak (at 25°C)	540nm
Ambient Temperature Range:	- 30~+70°C

Outline



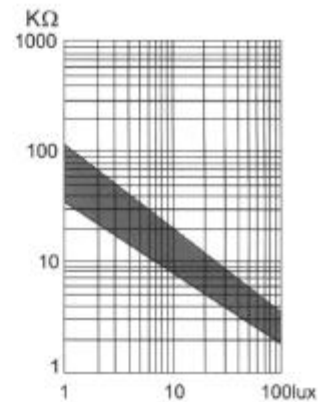
Measuring Conditions

1. Light Resistance:
measured at 10 lux with standard light A (2854k color temperature) and 2h pre-illumination at 400-600 lux prior to testing.
2. Dark Resistance:
measured 10 seconds after pulsed 10 lux.
3. Gamma Characteristic:
between 10 lux and 100 lux and given by

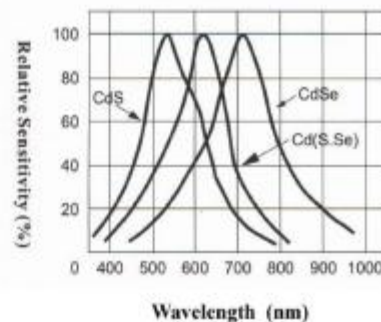
$$T = \frac{\log(R_{10}/R_{100})}{\log(100/10)} = \log(R_{10}/R_{100})$$

R₁₀, R₁₀₀ cell resistance at 10 lux and 100 lux.
The error of T is +0.1.
4. P_{max}:
Max. power dissipation at ambient temperature of 25°C.
5. V_{max}:
Max. voltage in darkness that may be applied to the cell continuously.

Illuminance Vs. Photo Resistance



Spectral Response



BPW34, BPW34S

Vishay Semiconductors



Silicon PIN Photodiode, RoHS Compliant



FEATURES

- Package type: leaded
- Package form: top view
- Dimensions (L x W x H in mm): 5.4 x 4.3 x 3.2
- Radiant sensitive area (in mm²): 7.5
- High photo sensitivity
- High radiant sensitivity
- Suitable for visible and near infrared radiation
- Fast response times
- Angle of half sensitivity: $\varphi = \pm 65^\circ$
- Lead (Pb)-free component in accordance with RoHS 2002/95/EC and WEEE 2002/96/EC



RoHS
COMPLIANT

DESCRIPTION

BPW34 is a PIN photodiode with high speed and high radiant sensitivity in miniature, flat, top view, clear plastic package. It is sensitive to visible and near infrared radiation. BPW34S is packed in tubes, specifications like BPW34.

APPLICATIONS

- High speed photo detector

PRODUCT SUMMARY

COMPONENT	I_{rs} (μA)	φ (deg)	$\lambda_{0.1}$ (nm)
BPW34	50	± 65	430 to 1100
BPW34S	50	± 65	430 to 1100

Note

Test condition see table "Basic Characteristics"

ORDERING INFORMATION

ORDERING CODE	PACKAGING	REMARKS	PACKAGE FORM
BPW34	Bulk	MOQ: 3000 pcs, 3000 pcs/bulk	Top view
BPW34S	Tube	MOQ: 1800 pcs, 45 pcs/tube	Top view

Note

MOQ: minimum order quantity

ABSOLUTE MAXIMUM RATINGS

PARAMETER	TEST CONDITION	SYMBOL	VALUE	UNIT
Reverse voltage		V_R	60	V
Power dissipation	$T_{amb} \leq 25^\circ\text{C}$	P_V	215	mW
Junction temperature		T_J	100	$^\circ\text{C}$
Operating temperature range		T_{amb}	- 40 to + 100	$^\circ\text{C}$
Storage temperature range		T_{stg}	- 40 to + 100	$^\circ\text{C}$
Soldering temperature	$t \leq 3$ s	T_{sd}	260	$^\circ\text{C}$
Thermal resistance junction/ambient	Connected with Cu wire, 0.14 mm ²	R_{thJA}	350	K/W

Note

$T_{amb} = 25^\circ\text{C}$, unless otherwise specified

www.vishay.com
386

For technical questions, contact: detectorsupport@vishay.com

Document Number: 81521
Rev. 2.0, 08-Sep-08

References

- ¹Dear, Patrick & Bunny, Mark Jr. "High Level Design Final Project." Brain Map
- ²D'Donnell, W. "Things You Should Know about LEDs and Photodiodes." *UNLV Physics*. n.d. Web. 30 April 2014.
http://www.physics.unlv.edu/~bill/PHYS483/LED_PIN.pdf
- ³Murphy, Joel. "Anatomy of the DIY Heart Rate Monitor." *Pulse Sensor*. 1 Aug 2011. Web. 24 April 2014.
<<http://pulsesensor.com/2011/08/01/anatomy-of-the-diy-heart-rate-monitor/>>
- ⁴Whitmore, Stephen. "Section 3.1 Introduction to Wheatstone Bridge." *Class Notes*. n.d.. Web. 30 April 2014.
<http://www.neng.usu.edu/classes/mae/3340/Section_3/section3.1.pdf>
- ⁵Figliola, Richard S. *Theory and Design for Mechanical Measurements*. New Jersey: John Wiley and Sons, 2011