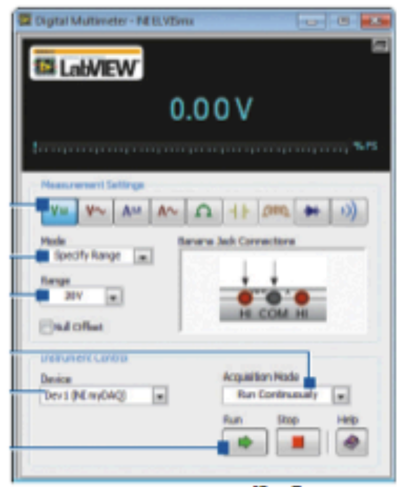
Uncertainty Analysis

• Given the relationship between the known resistors and the unknown resistors, find the uncertainty in your measurement that is due to the uncertainty in the values of R_1 and R_2 ($100\Omega, \pm 1\%$).

• **Assume that the precision error in decade resistor/Elenco Box is very small.** … However their setting resolution is only *1 Ohm*. ***Account for this value in the total error budget.***

• How do you account for the fact that the "Elvis" voltmeter only has two digits of precision in voltage?…



• Compare this calculated value to the standard

deviation you computed

based on your 20 resistors

based on all 80 resistors

• Comment on the manufacturer's claim as to the accuracy of these resistors.

Uncertainty Analysis (2)

- Precision Uncertainty *… Due to Bridge Resistors …*

$$R_x = f(R_1, R_2, R_3) = \frac{R_2 R_3}{R_1}$$

$$U_{R_x} = \sqrt{\left[\frac{\partial}{\partial R_1}\left(\frac{R_2 \cdot R_3}{R_1}\right)\right]^2 \cdot U_{R_1}^2 + \left[\frac{\partial}{\partial R_2}\left(\frac{R_2 \cdot R_3}{R_1}\right)\right]^2 \cdot U_{R_2}^2 + \left[\frac{\partial}{\partial R_3}\left(\frac{R_2 \cdot R_3}{R_1}\right)\right]^2 \cdot U_{R_3}^2}$$

$$\left[\begin{array}{c} \dfrac{\partial f}{\partial R_1} = -\dfrac{R_2 R_3}{R_1^2} \\ \dfrac{\partial f}{\partial R_2} = \dfrac{R_3}{R_1} \\ \dfrac{\partial f}{\partial R_3} = \dfrac{R_2}{R_1} \end{array}\right] \rightarrow U_{R_x} = \sqrt{\left[\frac{R_2 \cdot R_3}{R_1^{\,2}}\right]^2 \cdot U_{R_1}^2 + \left[\frac{R_3}{R_1}\right]^2 \cdot U_{R_2}^2 + \left[\frac{R_2}{R_1}\right]^2 \cdot U_{R_3}^2}$$

Uncertainty Analysis (3)

- Precision Uncertainty *… Due to Bridge Resistors …*

$$R_x = f(R_1, R_2, R_3) = \frac{R_2 R_3}{R_1}$$

$$Normalize \rightarrow \frac{U_{R_x}}{R_x} = \sqrt{\left(\frac{R_1}{R_2 \cdot R_3}\right)^2 \left(\left[\frac{R_2 \cdot R_3}{R_1^2}\right]^2 \cdot U_{R_1}^2 + \left[\frac{R_3}{R_1}\right]^2 \cdot U_{R_2}^2 + \left[\frac{R_2}{R_1}\right]^2 \cdot U_{R_3}^2\right)} = \sqrt{\left(\frac{U_{R_1}}{R_1}\right)^2 + \left(\frac{U_{R_2}}{R_2}\right)^2 + \left(\frac{U_{R_3}}{R_3}\right)^2}$$

Assume Precision Uncertainty of Elenco Box ~ 0

Elenco Resolution Error $\rightarrow$

$$R_x = \frac{R_2 \cdot R_3}{R_1} \rightarrow R_1 \approx R_2 \rightarrow \frac{1}{n}\sum_{i-1}^{n} R_x \approx \frac{1}{n}\sum_{i-1}^{n} R_3 \rightarrow \boxed{\left(\frac{U_{R_3}}{R_3}\right) \approx \frac{1_\Omega}{\bar{R}_x}}$$

$$\bar{R}_x \equiv \frac{1}{n}\sum_{i-1}^{n} R_x$$

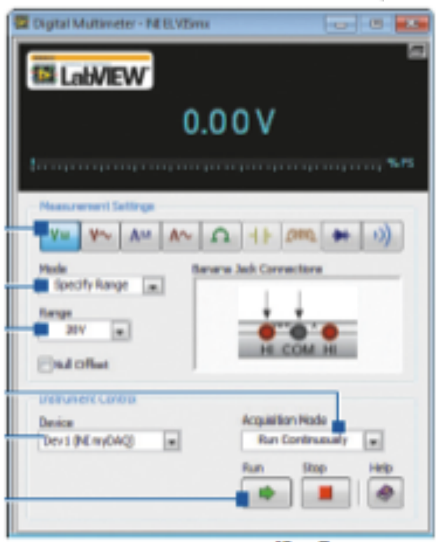
$$R_x = \left(\frac{R_2 R_3}{R_1}\right) \rightarrow \frac{R_3}{R_x} = \frac{R_1}{R_2} \approx \frac{100\Omega}{100\Omega} \sim 1$$

Uncertainty Analysis (4)

… Uncertainty Due to Bridge Resistors …

$$\frac{U_{R_x}}{R_x} = \sqrt{\left(\frac{U_{R_1}}{R_1}\right)^2 + \left(\frac{U_{R_2}}{R_2}\right)^2 + \left(\frac{U_{R_3}}{R_3}\right)^2} = \sqrt{\left(\frac{U_{R_1}}{R_1}\right)^2 + \left(\frac{U_{R_2}}{R_2}\right)^2 + \left(1_\Omega \Big/ \frac{1}{n}\sum_{i=1}^{n} R_x\right)^2}$$

- How do you account for the fact that the meter only has two digits of precision



Voltmeter Resolution Error $\rightarrow$

$$U_{R_x V_M} \approx \frac{U_{V_M}}{\partial V_M / \partial R_x} \rightarrow V_M = \frac{R_1 \cdot R_x - R_2 \cdot R_3}{(R_1 + R_2)\cdot(R_x + R_3)} \text{Vex}$$

$U_{V_M} \rightarrow$ *Resolution Error in Voltmeter Reading*

$U_{R_x V_M} \rightarrow$ *Error in* R_x *Due to Voltmeter Resolution Error*

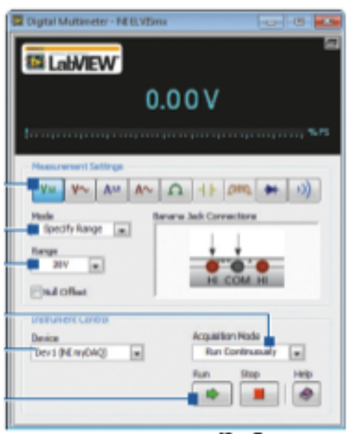
Uncertainty Analysis (5)

… Uncertainty Due to Voltmeter Resolution…

$$V_M = V_{ex}\frac{R_1R_x - R_2R_3}{(R_1+R_2)(R_3+R_x)} \rightarrow$$

$$\frac{\partial V_M}{\partial R_x} = V_{ex}\left[\frac{R_1}{(R_1+R_2)(R_3+R_x)} - \frac{R_1R_x - R_2R_3}{(R_1+R_2)(R_3+R_x)^2}\right] = V_{ex}\left[\frac{R_1(R_3+R_x) - R_1R_x + R_2R_3}{(R_1+R_2)(R_3+R_x)^2}\right] =$$

$$V_{ex}\left[\frac{R_1R_3 + R_2R_3}{(R_1+R_2)(R_3+R_x)^2}\right] = V_{ex}\left[\frac{(R_1+R_2)\cdot R_3}{(R_1+R_2)(R_3+R_x)^2}\right] = V_{ex}\left[\frac{R_3}{(R_3+R_x)^2}\right]$$



$U_{V_M} \rightarrow$ *Resolution Error in Voltmeter Reading*

$U_{R_xV_M} \rightarrow$ *Error in R_x Due to Voltmeter Resolution Error*

Uncertainty Analysis (6)

Voltmeter Resolution Error →

$$U_{R_xV_M} \approx \frac{U_{V_M}}{\partial V_M / \partial R_x} = \frac{U_{V_M}}{V_{ex}\dfrac{R_3}{\left(R_x + R_3\right)^2}} = \frac{\left(R_x + R_3\right)^2}{V_{ex} \cdot R_3} U_{V_M}$$

Normlize Error →

$$\frac{U_{R_xV_M}}{R_x} = \frac{1}{R_x}\frac{\left(R_x + R_3\right)^2}{R_3}\frac{U_{V_M}}{V_{ex}} = \frac{1}{\dfrac{R_2 \cdot R_3}{R_1}}\frac{\left(R_x + R_3\right)^2}{R_3}\frac{U_{V_M}}{V_{ex}} = \frac{R_1}{R_3}\frac{\left(R_x + R_3\right)^2}{R_2 \cdot R_3}\frac{U_{V_M}}{V_{ex}}$$

Since $\rightarrow R_1 \approx R_2 \rightarrow R_x \approx R_3 \rightarrow \dfrac{U_{R_xV_M}}{R_x} = \dfrac{\left(2 \cdot R_x\right)^2}{R_3 \cdot R_3}\dfrac{U_{V_M}}{V_{ex}} = 4 \cdot \dfrac{U_{V_M}}{V_{ex}}$ ***For Balanced Bridge***

U_{V_M} → *Resolution Error in Voltmeter Reading*

$U_{R_xV_M}$ → *Error in* R_x *Due to Voltmeter Resolution Error*

Uncertainty Analysis (7)

$$\frac{U_{R_x}}{R_x} = \sqrt{\left(\frac{U_{R_1}}{R_1}\right)^2 + \left(\frac{U_{R_2}}{R_2}\right)^2 + \left(1_\Omega \Big/ \frac{1}{n}\sum_{i-1}^{n} R_x\right)^2 + \left(\frac{U_{R_x V_M}}{R_x}\right)^2} =$$

$$\sqrt{\underbrace{\left(\frac{U_{R_1}}{R_1}\right)^2 + \left(\frac{U_{R_2}}{R_2}\right)^2}_{\text{Resistor Precision Error}} + \underbrace{\left(1_\Omega \Big/ \frac{1}{n}\sum_{i-1}^{n} R_x\right)^2}_{\text{Elenco Resolution Error}} + \underbrace{\left(4 \cdot \frac{U_{V_M}}{V_{ex}}\right)^2}_{\text{Voltmeter Resolution Error}}}$$

Resistor Precision Error **Elenco Resolution Error** ***Voltmeter Resolution Error***