UtahState UNIVERSITY

Mechanical & Aerospace Engineering

Laboratory 5: Wheastone Bridge and Measurement Uncertainty

- ***Lab Objectives:***

--Understand Wheatstone Bridge Properties

--Power Dissipation in Resistor

--Reading the Resistor Labeling Code

--Balancing a Resistance Bridge

--Using a resistance bridge to measure and unknown resistance

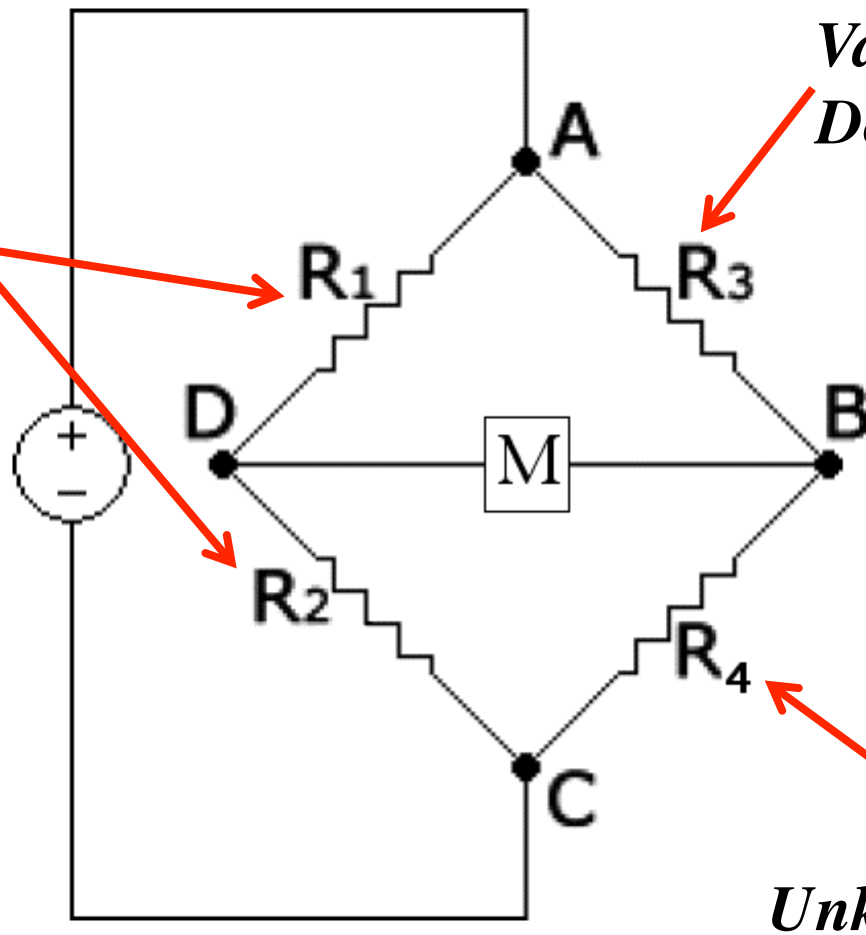
--Understand how to Calculate statistics levels for a random population

Sir Charles Wheatstone

Chapters 7, 3, Beckwith

You will build this bridge …

Known Precision 100 Ω resistors



Variable Decade Resistor

Unknown Resistor

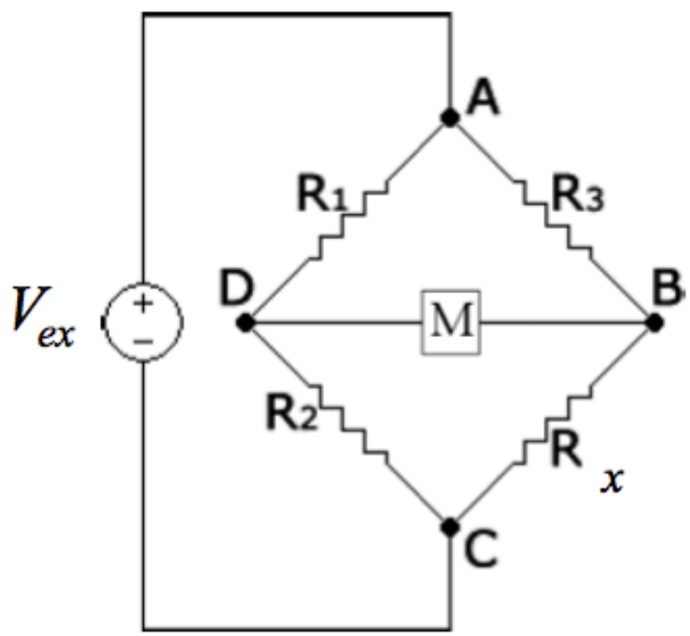
- You will Balance this bridge … Using R_3 To determine R_4

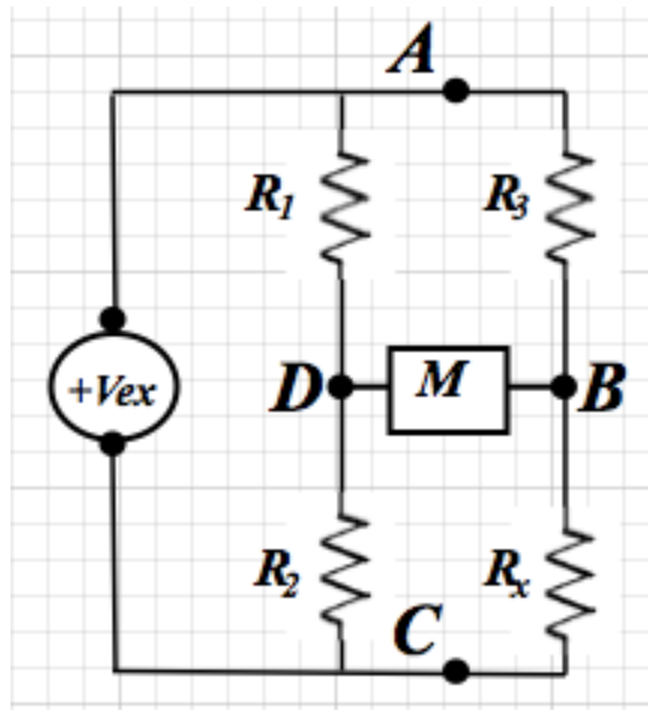
Wheatstone Bridge (1)

- A Circuit that amplifies small changes in an unknown resistance R_x

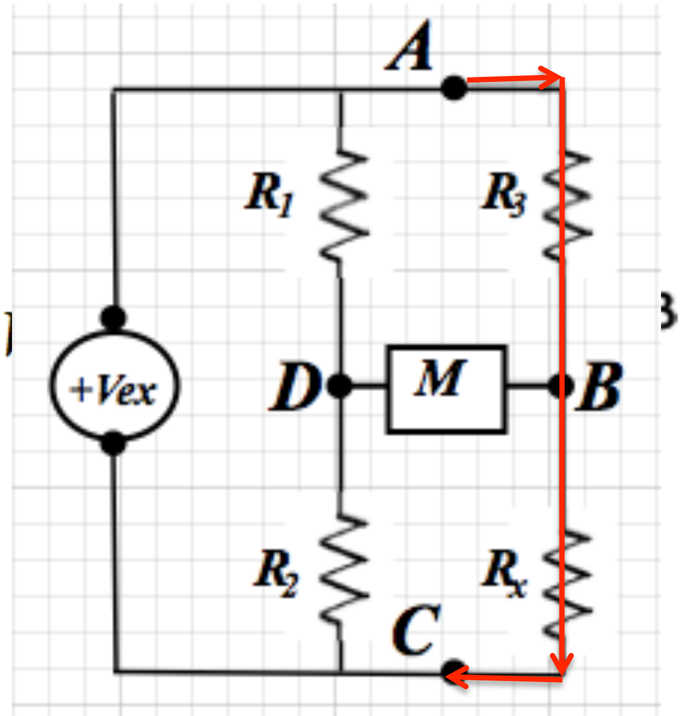
REDRAW CIRCUIT AS PAIR OF VOLTAGE DIVIDERS

- Consider the circuit



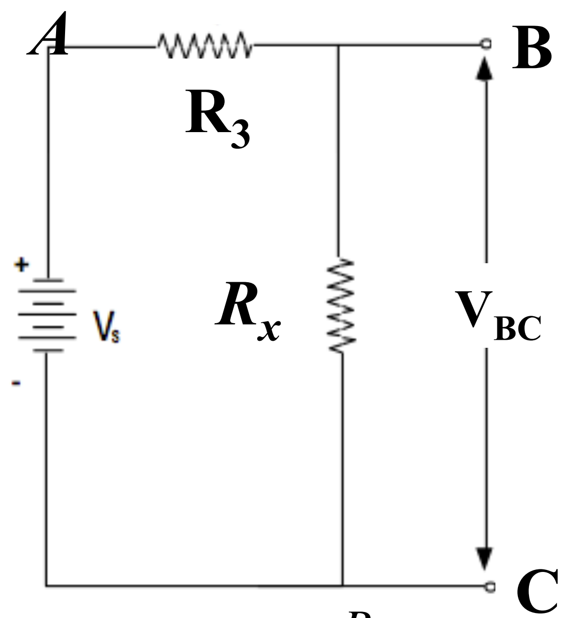


Wheatstone Bridge (2)



- What is Voltage Drop from *B to C*?

… *Looks like Voltage Divide Circuit* …



$$V_{BC} = V_{ex} \cdot \frac{R_x}{R_3 + R_x}$$

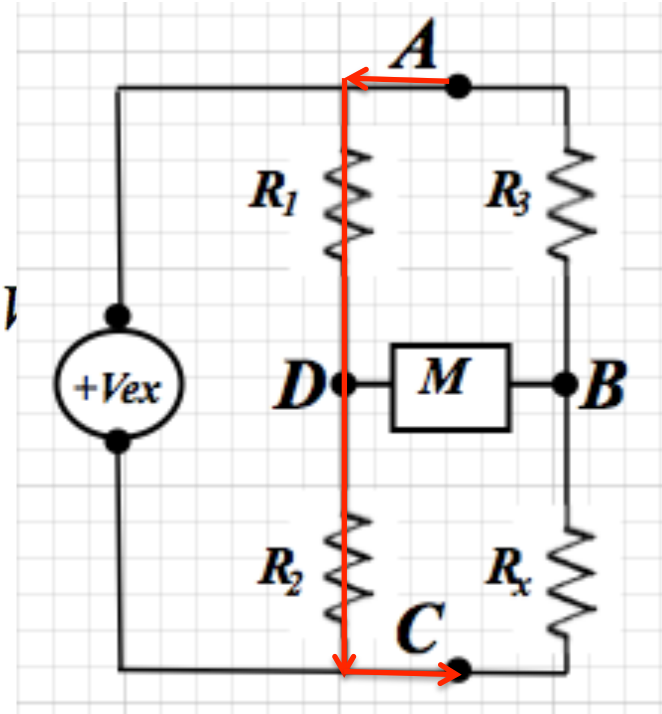
$$V_{BC} = V_{ex} \frac{R_x}{R_3 + R_x}$$

$$I_3 = \frac{V_{ex}}{(R_3 + R_x)}$$

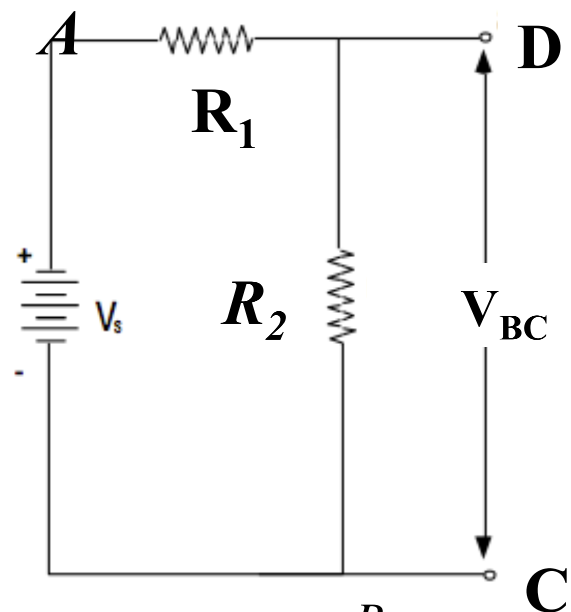
$$I_x = \frac{V_{ex}}{(R_3 + R_x)}$$

Wheatstone Bridge (3)

- What is Voltage Drop from *D to C*?



… *Looks like Voltage Divide Circuit* …



$$V_{DC} = V_{ex}\frac{R_2}{R_1 + R_2}$$

$$I_1 = \frac{V_{ex}}{(R_1 + R_2)}$$

$$I_2 = \frac{V_{ex}}{(R_1 + R_2)}$$

$$V_{DC} = V_{ex} \cdot \frac{R_2}{R_1 + R_2}$$

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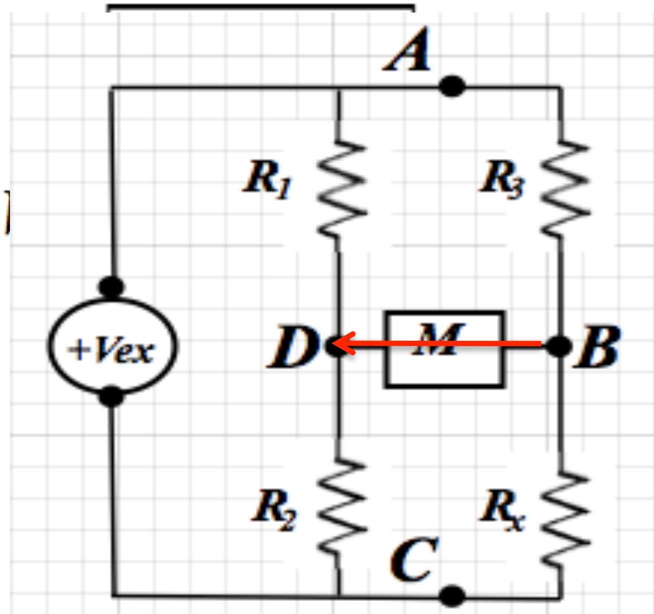
Wheatstone Bridge (4)



- What is Voltage Drop from *B to D*? (across meter)
  - *Assume Impedance of M is Very Large > MΩ, thus negligible current flows across meter*

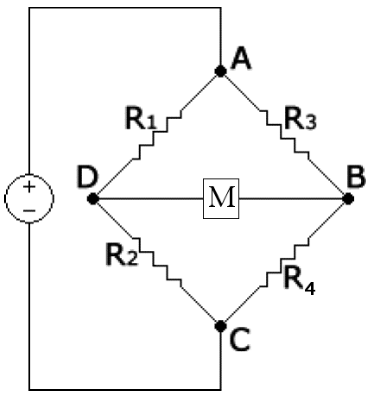
$$V_{BD} = V_{BC} + V_{CD} = V_{BC} - V_{DC} =$$

$$V_{ex}\left(\frac{R_x}{R_3 + R_x} - \frac{R_2}{R_1 + R_2}\right) = V_{ex}\left(\frac{R_x(R_1 + R_2) - R_2(R_3 + R_x)}{(R_1 + R_2)(R_3 + R_x)}\right) =$$

$$V_{ex}\frac{R_1 R_x - R_2 R_3}{(R_1 + R_2)(R_3 + R_x)}$$

Resistance Bridge Summary (see Appendix I)

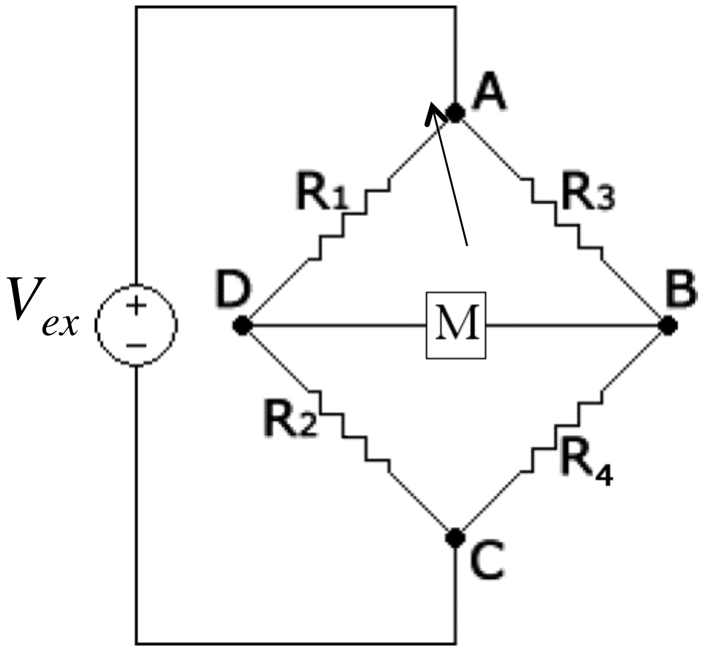
- *.. Infinite Meter Impedance*



$$\left[\begin{array}{c} I_M = 0 \\ I_1 = \frac{V_{ex}}{(R_1 + R_2)} \\ I_2 = \frac{V_{ex}}{(R_1 + R_2)} \\ I_3 = \frac{V_{ex}}{(R_3 + R_4)} \\ I_4 = \frac{V_{ex}}{(R_3 + R_4)} \end{array}\right] \rightarrow \left[\begin{array}{c} V_M = V_{ex}\frac{R_1 R_4 - R_2 R_3}{(R_1 + R_2)(R_3 + R_x)} \\ V_1 = V_{ex}\frac{R_1}{(R_1 + R_2)} \\ V_2 = V_{ex}\frac{R_2}{(R_1 + R_2)} \\ V_3 = V_{ex}\frac{R_3}{(R_3 + R_4)} \\ V_4 = V_{ex}\frac{R_4}{(R_3 + R_4)} \end{array}\right] \rightarrow \left[\begin{array}{c} P_M = 0 \\ P_1 = R_1\left[\frac{V_{ex}}{(R_1 + R_2)}\right]^2 \\ P_2 = R_2\left[\frac{V_{ex}}{(R_1 + R_2)}\right]^2 \\ P_3 = R_3\left[\frac{V_{ex}}{(R_3 + R_4)}\right]^2 \\ P_4 = R_4\left[\frac{V_{ex}}{(R_3 + R_4)}\right]^2 \end{array}\right]$$

Current thru resistor | *Voltage drop cross resistor* | *Power dissipated by resistor*

"Balanced" Bridge



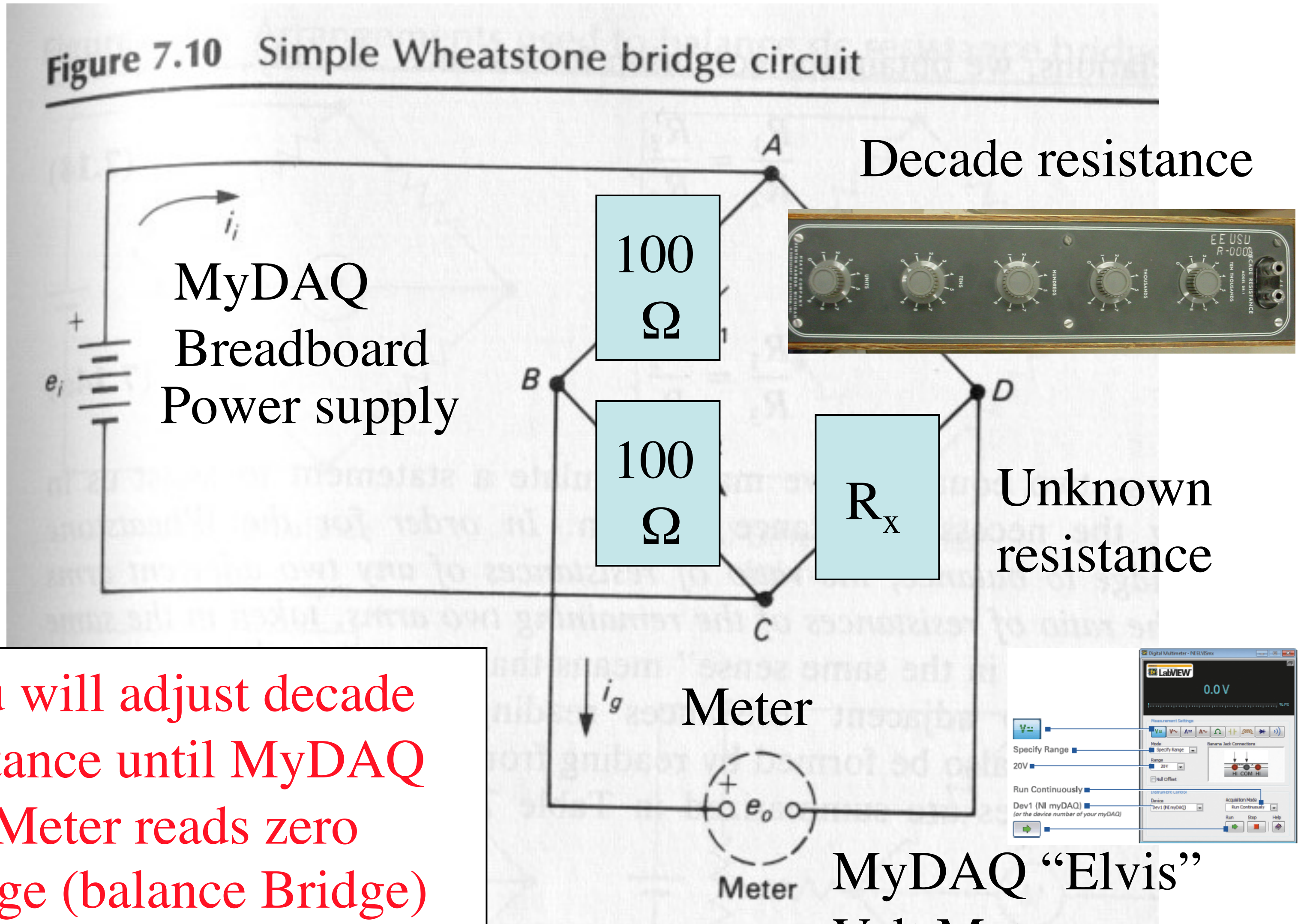
$$V_{Meter} = V_{ex} \frac{R_1 \cdot R_x - R_2 \cdot R_3}{(R_1 + R_2) \cdot (R_3 + R_x)}$$

- $R_1 \cdot R_x = R_2 \cdot R_3 \rightarrow V_{Meter} = 0$
- Condition for Balance, $V_M = 0$

→ $$V_{meter} = 0 \rightarrow R_x = R_3 \frac{R_2}{R_1}$$

- Good way to sense *unknown resistance*
- Adjust R_3 until $V_M = 0$

Your Lab Circuit



MyDAQ Breadboard Power supply

100 Ω

100 Ω

R_x

Decade resistance

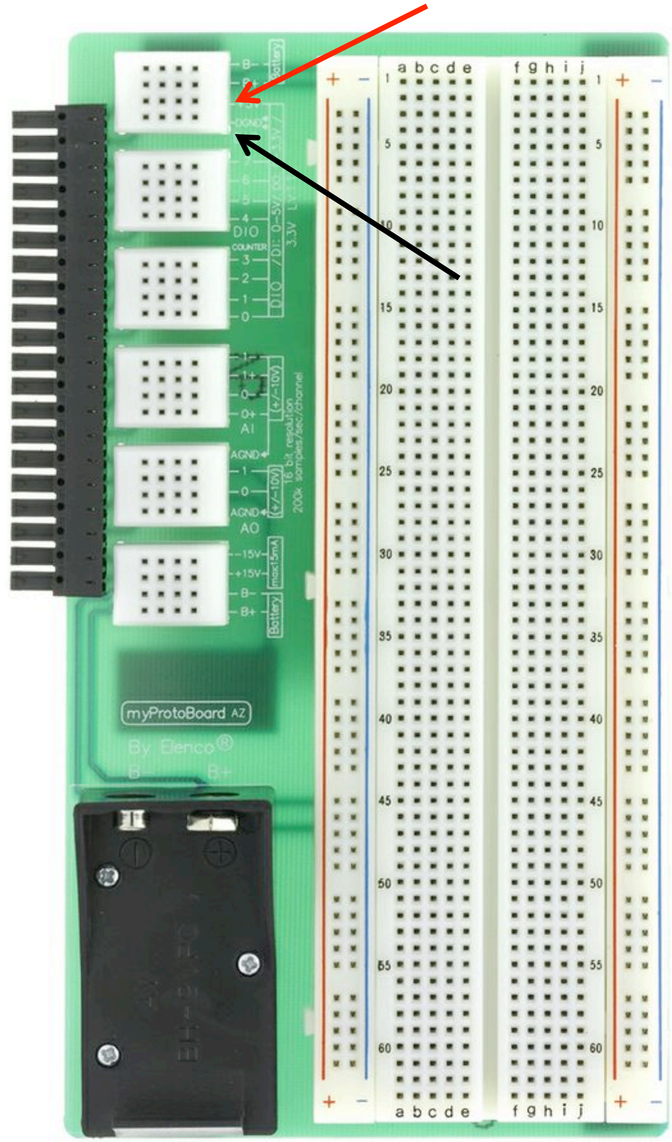
Unknown resistance

Meter

MyDAQ "Elvis" Volt Meter

- You will adjust decade resistance until MyDAQ Volt Meter reads zero voltage (balance Bridge)

+ 5V Power Supply Using MyDAQ



Decade Variable Resistor Box

There are 3 decade resistors that measure higher resistances, and 2 that measure lower resistances. They will be marked “High Resistance” or “Low Resistance”. These can create a resistance anywhere between 0.01 and 900 ohms in 0.01-ohm increments (Low Resistance), or 1 to 99,000 Ohms in 1-Ohm increments (High Resistance).

Use the “High Resistance” Boxes for this Lab
1-Ohm resolution

Elenco 1-Watt Resistor Substitution Box

You will also be using Elenco 1-Watt Boxes for half of your resistors instead of decade resistors

1-Ohm resolution

Elenco 1-Watt Resistor Substitution Box (2)

INTRODUCTION

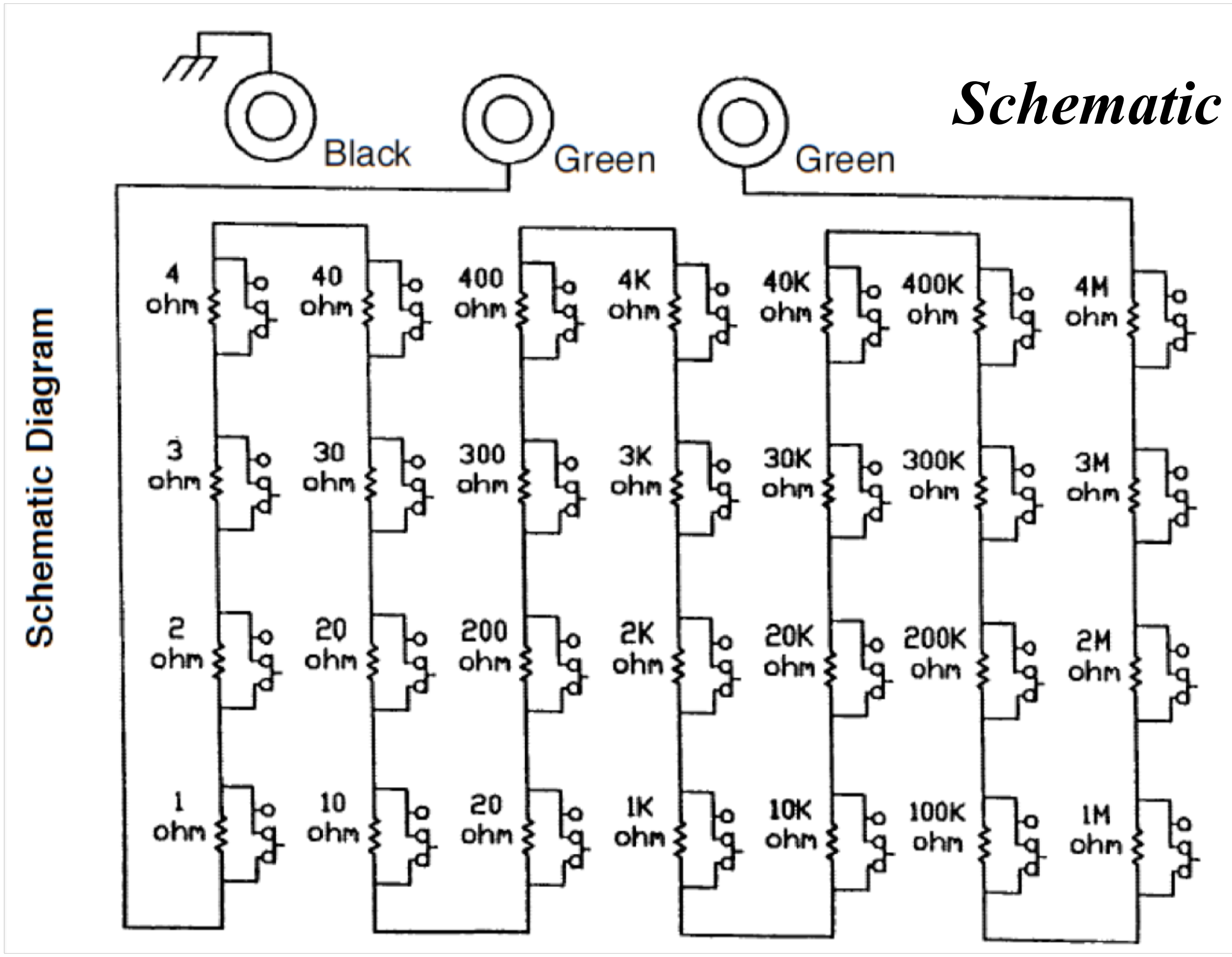
The RS-500 Resistance Substitution Box is a convenient instrument for determining the desired resistance values in circuits under design or test. The resistance obtainable is form 0Ω to 11,111,110Ω in 1 ohm steps. All resistors are precision 1% resistors.

OPERATION

All of the resistors are wired in series. A switch on the front panel shorts each resistor. Putting the switch in the ON (up) position removes the short and places the resistor across the output terminals. The two green binding posts serve as the output terminals. The black binding post is tied to the front panel. To place any resistance across the output terminals turn ON those switches that add up to the desired resistance. For example, for 5,071Ω, turn on the 3kΩ, 2kΩ, 40Ω, 30Ω and 1Ω switches.

Elenco 1-Watt Resistor Substitution Box (3)

Schematic Diagram

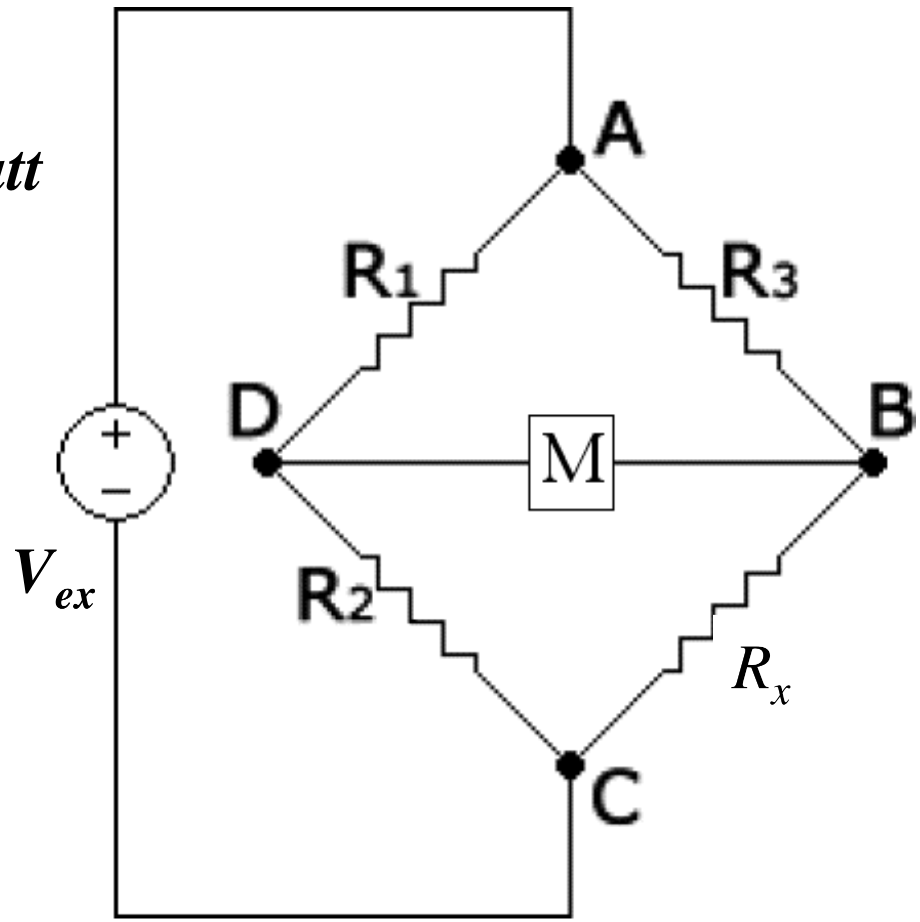


Excitation Voltage …

The Precision 100 Ω resistors (R_1, R_2) (±1%) Have a 1/4th watt capability

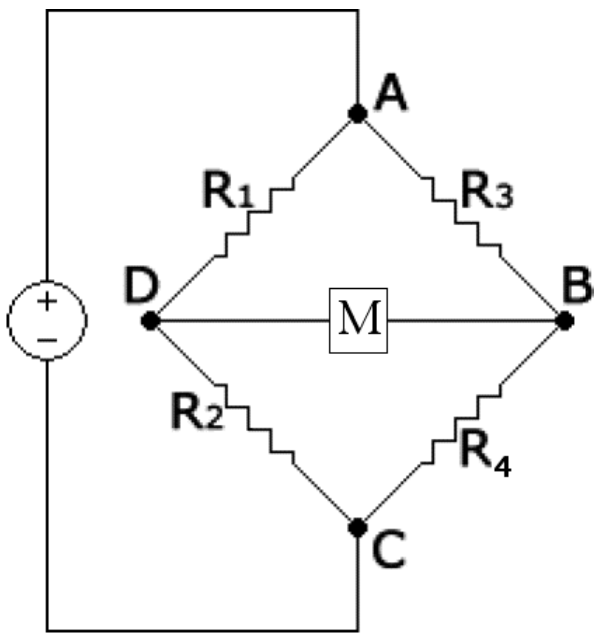
… but to be safe … we want
To limit power dissipated
*To **1/16th watt safety factor 4***

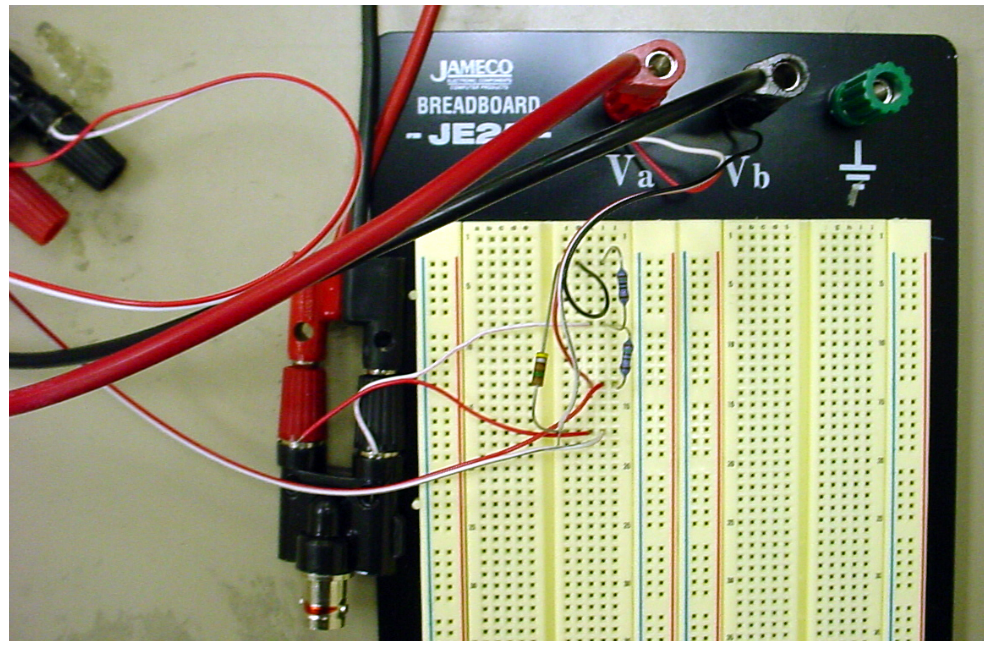
… Verify that V_{ex} = 5VDC
*Will dissipate **1/16th watt in R_1, R_2***



Do the analysis before lab, and show it to the lab instructor before lab, staple it to this report.

Example Wheatstone Bridge on Breadboard





Resistance Table

20 resistors selected at random …

You will fill out this table

.. Also populate .xlsx spreadsheet *(download example from Lab 5 section of Web page)*

Table I: Resistance Values

Resistor Number	Nominal resistance (Ohms) (color code)	Spec Accuracy (%) (color code)	Actual resistance (Ohms)	R_i/R_{nom}

***Also save data in Text.CSV* format (“comma, separated, values”)**

Sample Statistics

- Sample Mean, Standard Deviation Estimates

"sample mean"

$$\bar{x} = \sum_{i=1}^{n} \frac{\left(\frac{R_i}{R_{nom}}\right)_i}{n}$$

R_{nom} → Value from Resistor Color Code

- Sample Standard Deviation Estimates

"sample Std. Dev."

$$S_x = \sqrt{\sum_{i=1}^{n} \frac{\left(\frac{R_{x_i}}{R_{nom}} - \bar{x}\right)^2}{n-1}}$$

$$\{x_n\} = \begin{Bmatrix} \left(\frac{R}{R_{nom}}\right)_1 \\ \left(\frac{R}{R_{nom}}\right)_2 \\ \left(\frac{R}{R_{nom}}\right)_3 \\ \cdot \\ \cdot \\ \cdot \\ \left(\frac{R}{R_{nom}}\right)_{n-1} \\ \left(\frac{R}{R_{nom}}\right)_n \end{Bmatrix}$$

Lab Report Summary (1)

• When you are finished, get together with the other 4 groups in your section and make a text file containing all 80 samples. . …

• Using all the data, compute an estimate of the normalized mean and the standard deviation for *your 20 resistors, and the* the *pool of 80 resistors from your lab section*.

• Exchange spreadsheets and cut/paste to get all 80 resistors.

• Using only your 20 resistors, estimate the normalized mean, standard deviation, and the precision uncertainty of the mean to 95% confidence, ***assuming student-t distribution***

• Repeat Calculations using all 80 resistors for your section (95% confidence, ***assume Gauss distribution)?***

• Which set would you expect to have the smallest confidence interval??

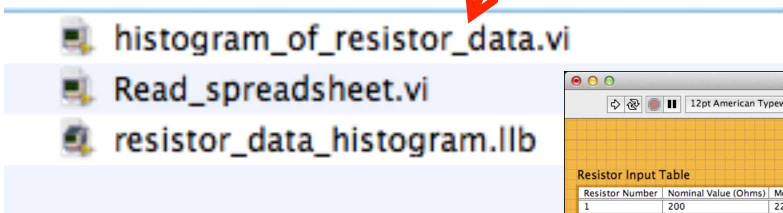
Lab Report Summary (2)

- Down Load and Open Histogram Code … See Link on Section 4 of web page

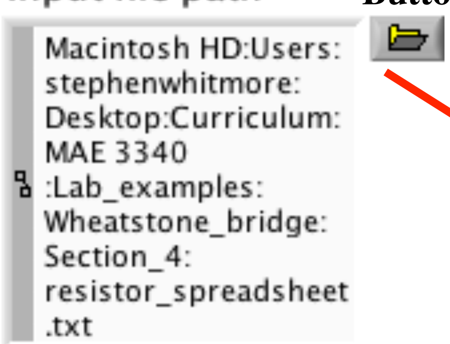
  [Lab 5 Histogram, Statistical Analysis VI]() **… http://www.neng.usu.edu/classes/mae/3340/**

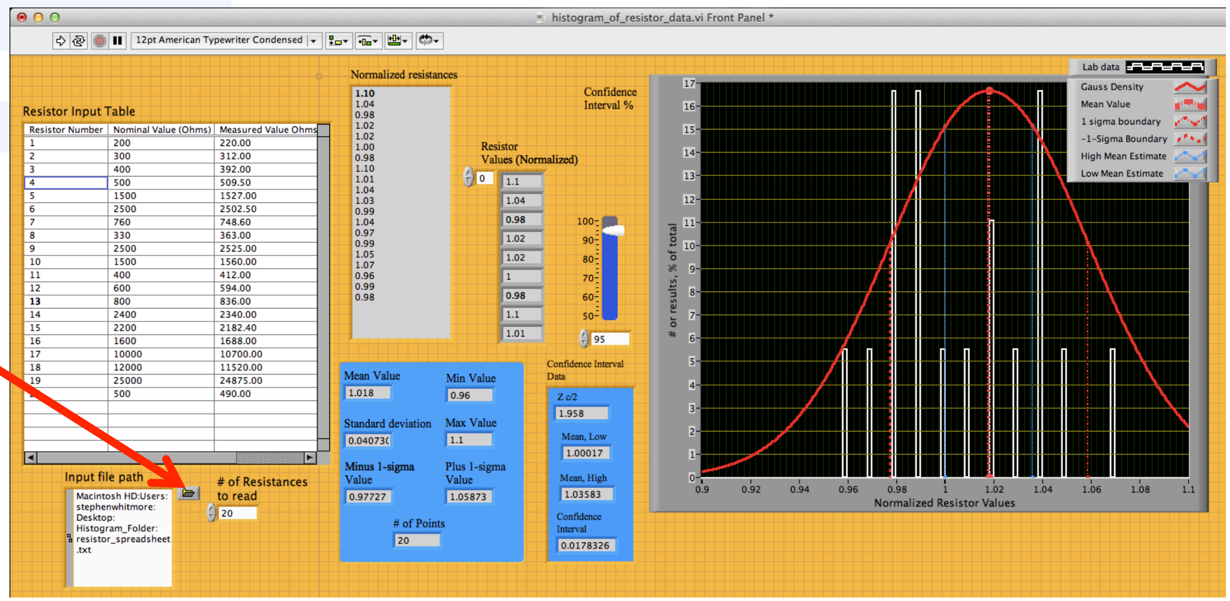
  Section_4/Histogram_Folder.zip

This VI will read your spreadsheet file … if saved on *Text.CSV* format



"Browse Button"

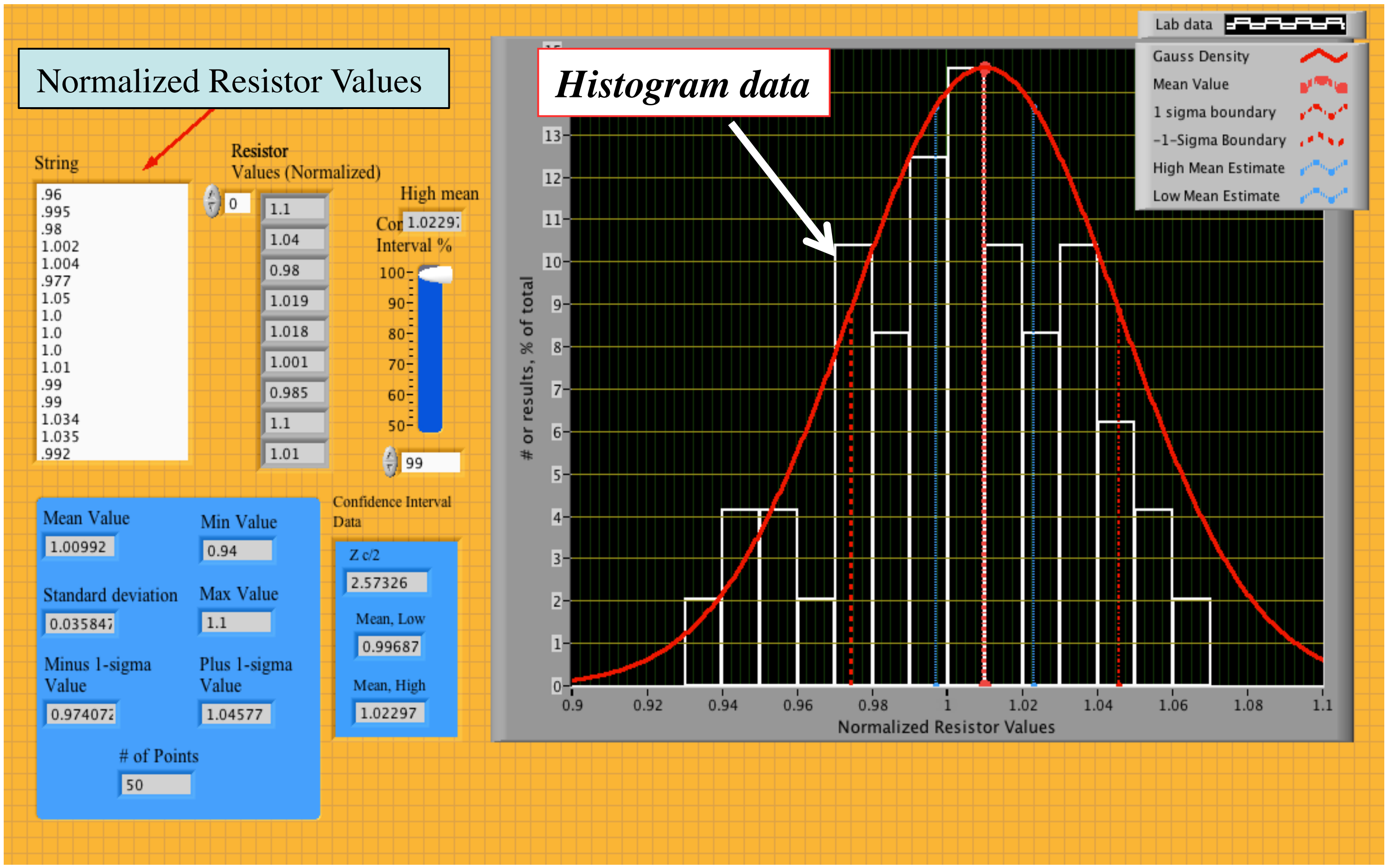




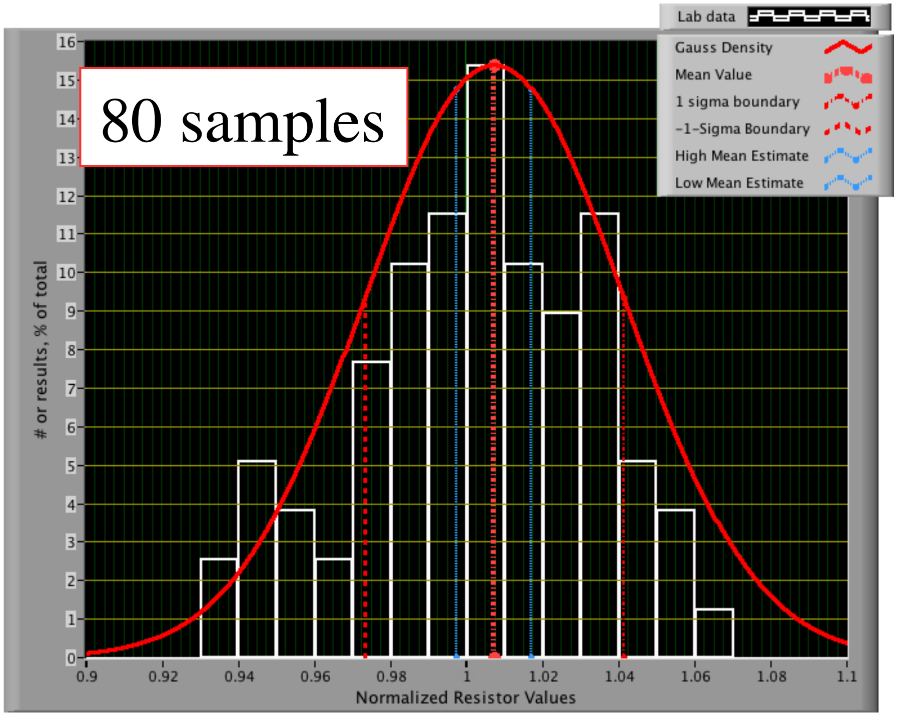
- *Make a histogram of the data with all your 20 samples and 80 samples* for you're a section and and attach it to this lab (screenshot)

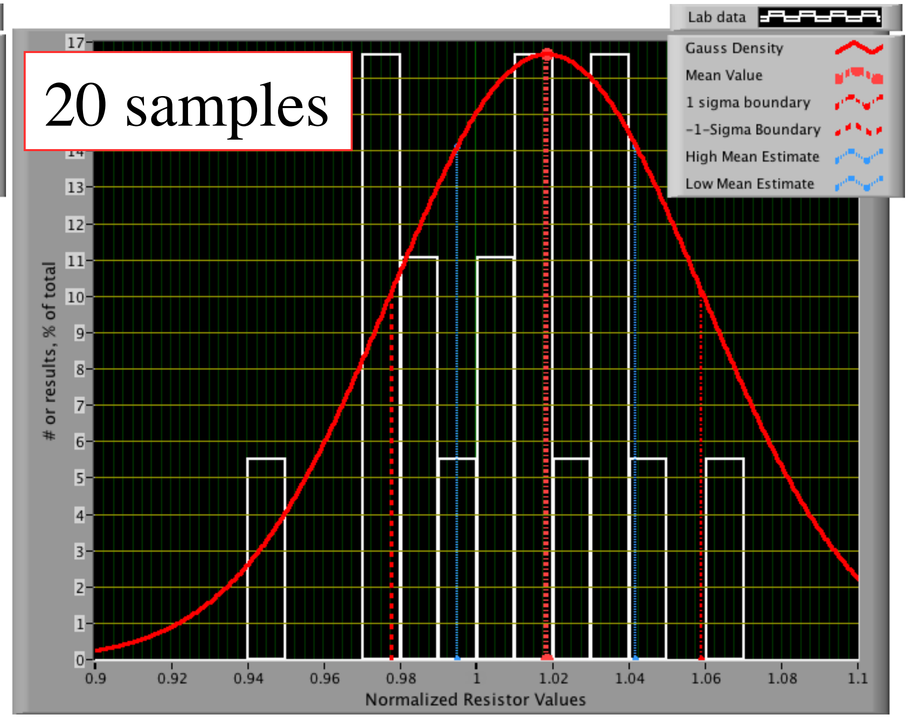
Lab Report Summary (3)

• *A histogram divides the range of possible values up Into "bins" and plots the % of occurrences of values within that bin*



Example Histograms



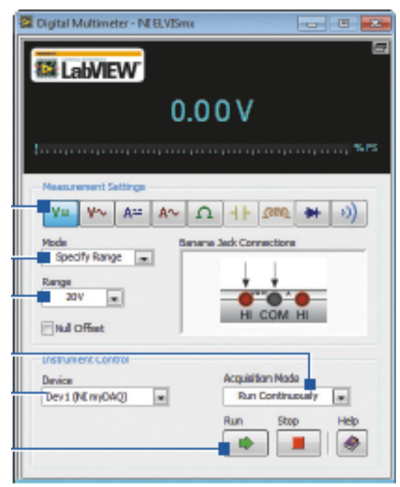


Error Analysis (1)

- Given the relationship between the known resistors and the unknown resistors, find the uncertainty in your measurement that is due to the uncertainty in the values of R_1 and R_2 (100Ω, $\pm$1%).

- **Assume that the precision error in decade resistor/Elenco Box is very small.** … However their setting resolution is only *1 Ohm*. ***Account for this value in the total error budget.***

- How do you account for the fact that the "Elvis" voltmeter only has two digits of precision in voltage?… ***see next page***



- Compare this calculated value to the standard

  deviation you computed

  based on your 20 resistors

  based on all 80 resistors

- Comment on the manufacturer's claim as to the accuracy of these resistors.

Uncertainty Analysis (6)

- How do you account for the fact that the meter only has two digits of precision

$$\delta V_M \approx \frac{\partial V_M}{\partial R_x} \cdot \delta R_x \rightarrow \boxed{\delta R_x \approx \frac{\delta V_M}{\partial V_M / \partial R_x}}$$

$$V_M = V_{ex} \frac{R_1 R_x - R_2 R_3}{(R_1 + R_2)(R_3 + R_x)} \rightarrow$$

$$\frac{\partial V_M}{\partial R_x} = V_{ex}\left[\frac{R_1}{(R_1 + R_2)(R_3 + R_x)} - \frac{R_1 R_x - R_2 R_3}{(R_1 + R_2)(R_3 + R_x)^2}\right] = V_{ex}\left[\frac{R_1(R_3 + R_x) - R_1 R_x + R_2 R_3}{(R_1 + R_2)(R_3 + R_x)^2}\right] =$$

$$V_{ex}\left[\frac{R_1 R_3 + R_2 R_3}{(R_1 + R_2)(R_3 + R_x)^2}\right] = V_{ex}\left[\frac{(R_1 + R_2) \cdot R_3}{(R_1 + R_2)(R_3 + R_x)^2}\right] = V_{ex}\left[\frac{R_3}{(R_3 + R_x)^2}\right]$$

Uncertainty Analysis (7)

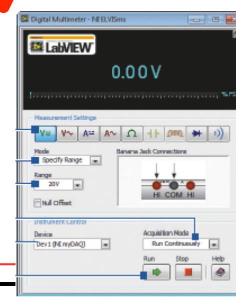
- How do you account for the fact that the meter only has two digits of precision

$$\delta R_x \approx \frac{\delta V_M}{\partial V_M / \partial R_x} \rightarrow \frac{\partial V_M}{\partial R_x} = V_{ex}\left[\frac{R_3}{\left(R_3 + R_x\right)^2}\right] \rightarrow R_x \approx \frac{\delta V_M}{V_{ex}\left[\frac{R_3}{\left(R_3 + R_x\right)^2}\right]} =$$

$$\frac{\left(R_3 + R_x\right)^2}{R_3}\left[\frac{\delta V_M}{V_{ex}}\right] = R_3 \cdot \left(1 + \frac{R_x}{R_3}\right)^2 \left[\frac{\delta V_M}{V_{ex}}\right] \rightarrow \boxed{\left[\frac{\delta R_x}{R_x}\right]_{Meter} = \left(\frac{R_3}{R_x}\right) \cdot \left(1 + \frac{R_x}{R_3}\right)^2 \left[\frac{\delta V_M}{V_{ex}}\right]}$$

- But …. For a balanced bridge

$$R_x = \left(\frac{R_2 R_3}{R_1}\right) \rightarrow \frac{R_3}{R_x} = \frac{R_1}{R_2} \approx \frac{100\Omega}{100\Omega} = 1$$



$$\boxed{\left[\frac{\delta R_x}{R_x}\right]_{Meter} \approx 4\frac{\delta V_M}{V_{ex}}}$$

Uncertainty Analysis (8)

- Precision Uncertainty

$$R_x = f(R_1, R_2, R_3) = \frac{R_2 R_3}{R_1}$$

$$\delta R_x = \sqrt{\left(\frac{\partial f}{\partial R_1}\right)^2 \cdot \delta R_1^2 + \left(\frac{\partial f}{\partial R_2}\right)^2 \cdot \delta R_2^2 + \left(\frac{\partial f}{\partial R_3}\right)^2 \cdot \delta R_3^2}$$

$$\left[\begin{array}{l} \dfrac{\partial f}{\partial R_1} = -\dfrac{R_2 R_3}{R_1^2} \\[2ex] \dfrac{\partial f}{\partial R_2} = \dfrac{R_3}{R_1} \\[2ex] \dfrac{\partial f}{\partial R_3} = \dfrac{R_2}{R_1} \end{array}\right] \rightarrow \delta R_x = \sqrt{\left(\frac{R_2 R_3}{R_1^2}\right)^2 \cdot \delta R_1^2 + \left(\frac{R_3}{R_1}\right)^2 \cdot \delta R_2^2 + \left(\frac{R_2}{R_1}\right)^2 \cdot \delta R_3^2}$$

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Uncertainty Analysis (8)

- Precision Uncertainty

$$Normalize \rightarrow \frac{\delta R_x}{R_x} = \frac{1}{\frac{R_2 R_3}{R_1}} \cdot \sqrt{\left(\frac{R_2 R_3}{R_1^2}\right)^2 \cdot \delta R_1^2 + \left(\frac{R_3}{R_1}\right)^2 \cdot \delta R_2^2 + \left(\frac{R_2}{R_1}\right)^2 \cdot \delta R_3^2} = \sqrt{\left(\frac{\delta R_1}{R_1}\right)^2 + \left(\frac{\delta R_2}{R_2}\right)^2 + \left(\frac{\delta R_3}{R_3}\right)^2}$$

- **Total Uncertainty** … Root Sum Square Precision and Resolution Uncertainty

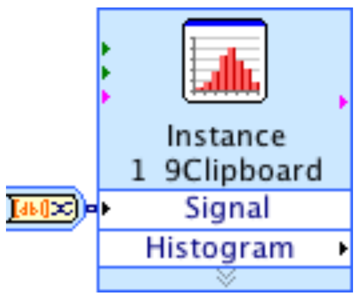
$$\rightarrow \frac{\delta R_x}{R_x} = \sqrt{\underbrace{\left(\frac{\delta R_1}{R_1}\right)^2 + \left(\frac{\delta R_2}{R_2}\right)^2 + \left(\frac{\delta R_3}{R_3}\right)^2}_{\text{Precision Uncertainty}} + \underbrace{\left(\frac{\delta R_x}{R_x}\right)^2}_{\text{Meter Resolution}}}$$

Meter Resolution

Precision Uncertainty

How do you account for Resolution Of only 1 Ohm on Decade Resistor/Elenco Box?

Appendix
Histogram Plots Using Labview



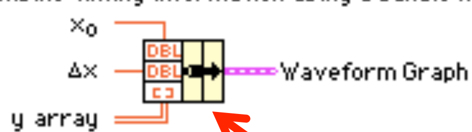
Using Labview for Histogram Plot (2)

Waveform Graphs:

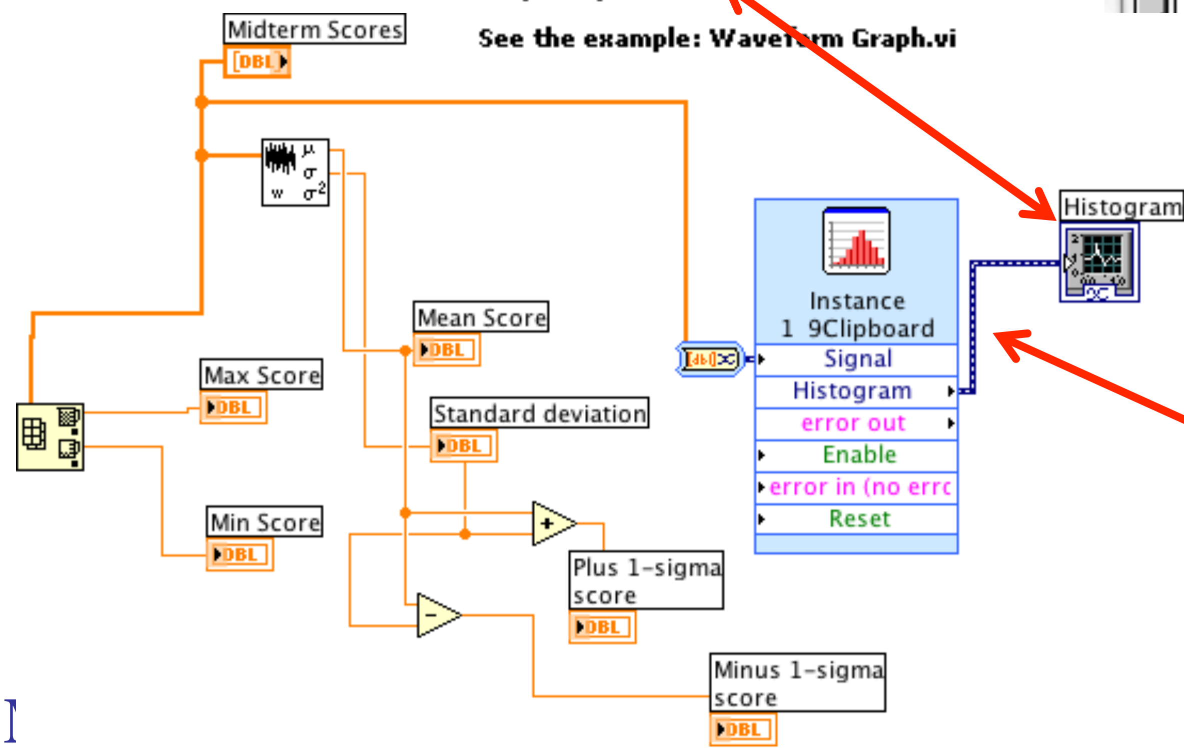
Wire data directly to waveform graph:

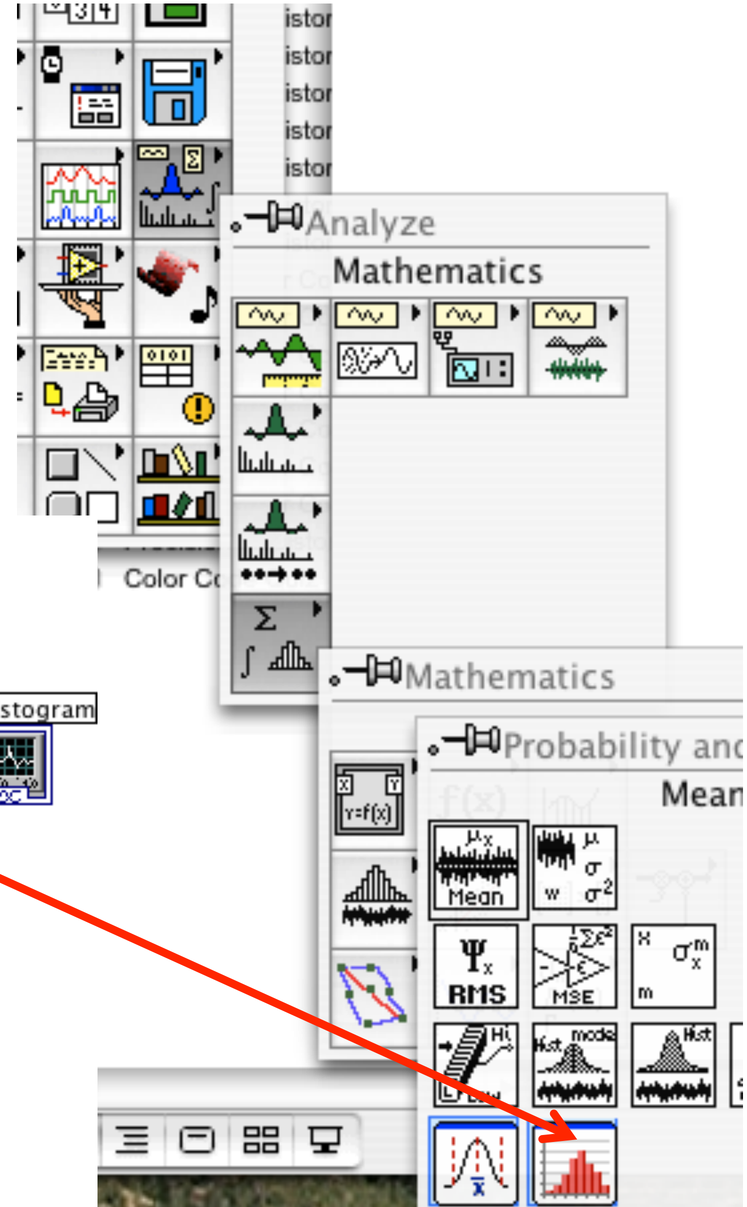
Y Array	Resulting Graph
1D	Single Plot
WDT	Single Plot
2D	Multiplot

WDT (Waveform Data Type) includes timing info. Others default to 0 for x_0 and 1 for Δx.
Combine timing information using a bundle node:



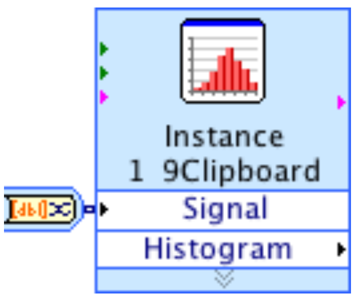
See the example: Waveform Graph.vi



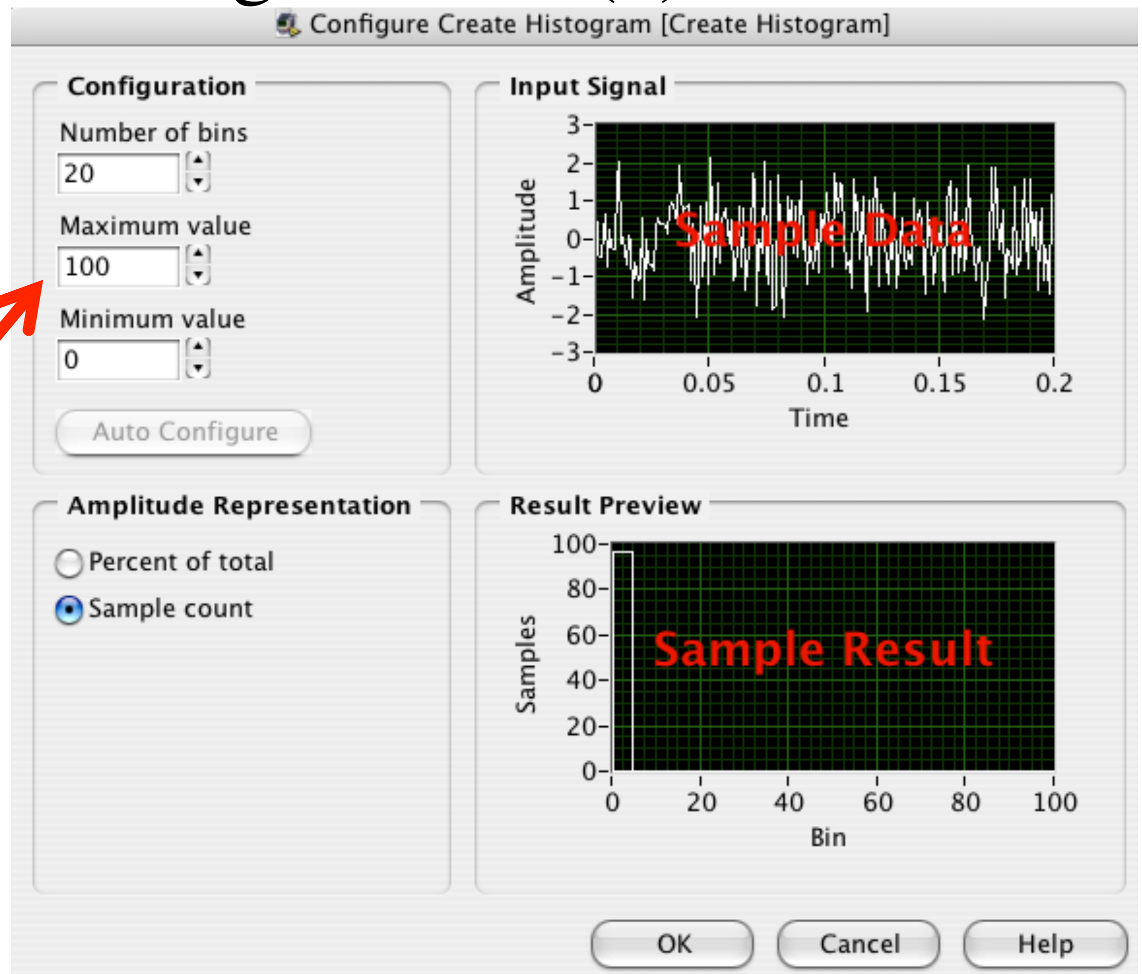


Using Labview for Histogram Plot (3)

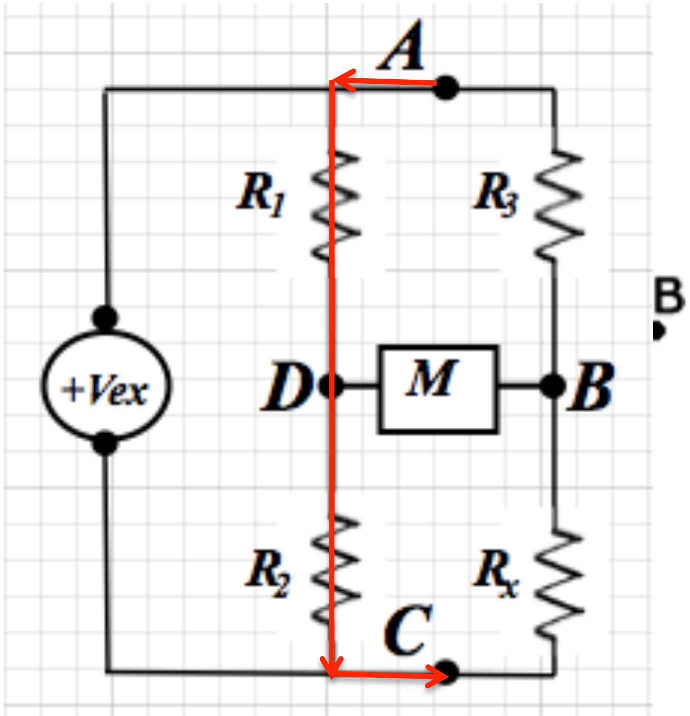
Set bin intervals
And min/max
On front panel



"double click"



Wheatstone Bridge (3)



$$V_{DC} = V_{ex}\frac{R_2}{R_1 + R_2}$$

Watts Dissipated in First Leg of Bridge

$$I_{R_1,R_2} = \frac{V_{ex}}{R_1 + R_2} = \frac{5_V}{200_\Omega} = 0.025_{Amps}$$

$$P_{R1} = \left(I_{R_1,R_2}\right)^2 \cdot R_1 = \left(0.025\,Amps\right)^2 \cdot 100_\Omega = 0.0625_{Watts}$$

$$P_{R_2} = \left(I_{R_1,R_2}\right)^2 \cdot R_2 = \left(0.025\,Amps\right)^2 \cdot 100_\Omega = 0.0625_{Watts}$$

Precision Uncertainty Analysis (2)

Standard Error of the Sample Mean

$$S_{\bar{x}} = \frac{S_x}{\sqrt{n}}$$

• Confidence Interval for Sample Mean Estimate

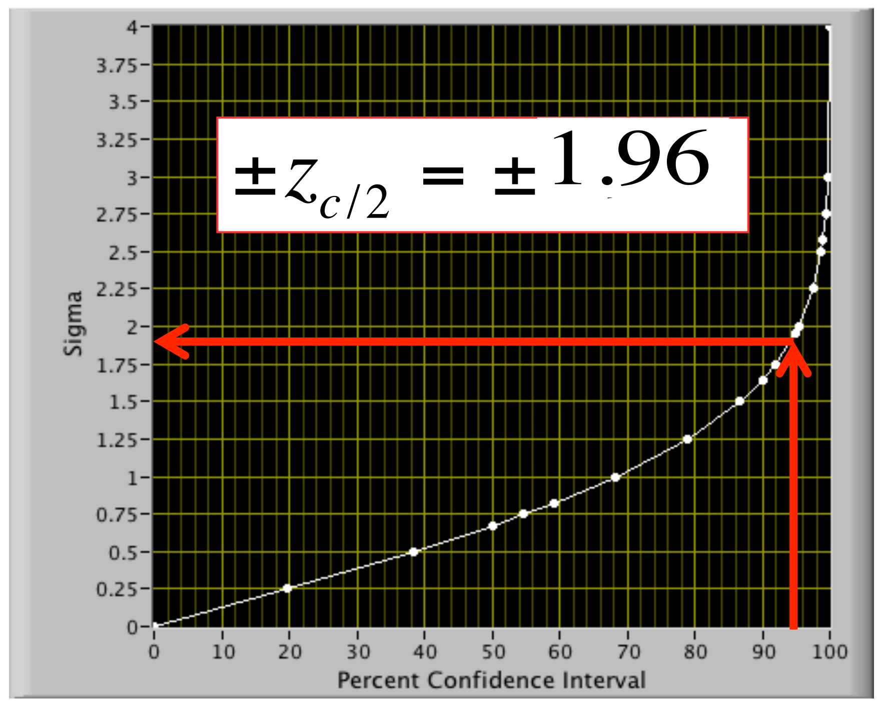
$$\bar{x} - z_{c/2}\frac{S_x}{\sqrt{n}} < \mu < \bar{x} + z_{c/2}\frac{S_x}{\sqrt{n}}$$

95% Confidence Interval --->

$$0.95 = P_{(\bar{x})} = \frac{1}{\sqrt{2\pi}\cdot\sigma_{\bar{x}}}\int_{z=-\frac{(\bar{x}-\mu_{\bar{x}})^2}{\sigma_{\bar{x}}^2}}^{z=\frac{(\bar{x}-\mu_{\bar{x}})^2}{\sigma_{\bar{x}}^2}}\left(e^{-\frac{(x-\mu)^2}{2\sigma^2}}\right)dx$$

"Gauss Distribution"

Confidence Interval



Precision Uncertainty Analysis (3)

$$\rightarrow let \ldots z = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} \rightarrow$$

$$0.95 = P_{(x)} = \frac{1}{\sqrt{2\pi} \cdot \sigma_{\bar{x}}} \cdot \int_{z_{\cdot c/2}}^{z_{\cdot c/2}} \left(e^{-z/2}\right) dz$$

Integral data

Xmin (standard deviations)

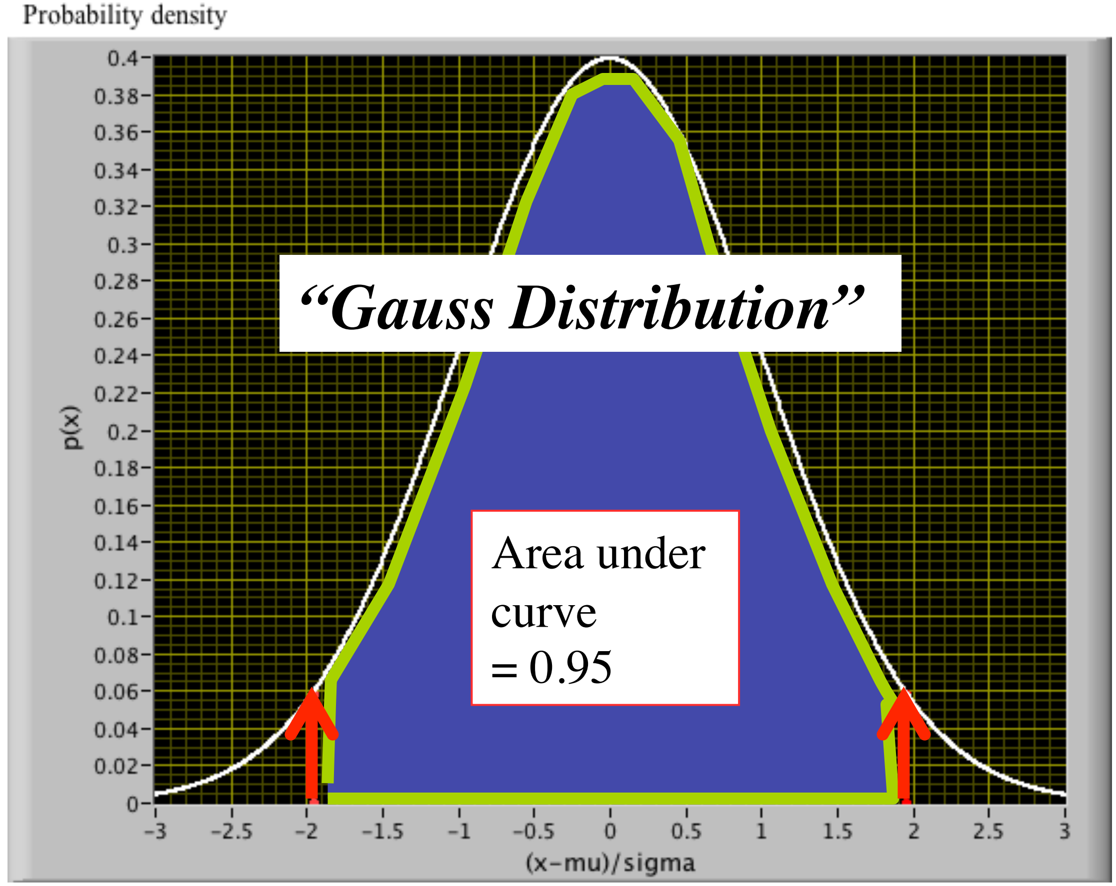
-1.958

Xmax (standard deviations)

1.958

Points in Integrator

1000



$$\pm z_{c/2} = \pm 1.96$$

Precision Uncertainty Analysis (4)

$$\pm z_{c/2} = \pm 1.96$$

$$\bar{x} - \left(1.96 \cdot \frac{S_x}{\sqrt{n}}\right) < \mu_{\bar{x}} < \bar{x} + \left(1.96 \cdot \frac{S_x}{\sqrt{n}}\right) ....$$

$$or ... \boxed{\mu_{\bar{x}} = \bar{x} \pm \left(1.96 \frac{S_x}{\sqrt{n}}\right)}$$

- precision uncertainty

….in mean estimate

Assuming Gauss Distribution