Laboratory 8: Using Thermocouples for Temperature and Heat Transfer Measurements

- ***Lab Objectives:***

-- Understanding Type J, K Thermocouples and the Seebeck Effect

-- Thermocouple Signal Output Levels

-- Amplifying the Thermocouple Output

-- Measurement Curve Fitting and Confidence Level Analysis

-- Reference Temperatures Software Compensation

-- End-to-End Temperature Calculations

-- Lumped-Mass Capacitance Time Constant

J, K Thermocouples (1)

Type J ***Positive Terminal: Iron …. Negative Terminal: Constantan***

The Type J may be used, exposed or unexposed, where there is a deficiency of free oxygen. For cleanliness and longer life, a protecting tube is recommended. Since JP (iron) wire will oxidize rapidly at temperatures over 540°C (1000°F), it is recommended that larger gauge wires be used to compensate. Maximum recommended operating temperature is 760°C (1400°F).

Type K ***Positive Terminal: Chromel …. Negative Terminal: Alumel***

Due to its reliability and accuracy, Type K is used extensively at temperatures up to 1260°C (2300°F). It's good practice to protect this type of thermocouple with a suitable metal or ceramic protecting tube, especially in reducing atmospheres. In oxidizing atmospheres, such as electric furnaces, tube protection is not always necessary when other conditions are suitable; however, it is recommended for cleanliness and general mechanical protection. Type K will generally outlast Type J because the JP (iron) wire rapidly oxidizes, especially at higher temperatures.

Constantan is a copper-nickel alloy usually consisting of 55% Copper and 45% Nickel.

Chromel is an alloy made of approximately 90 percent nickel and 10 percent chromium

Alumel is an alloy consisting of 95% nickel, 2% manganese, 2% aluminium and 1% silicon.

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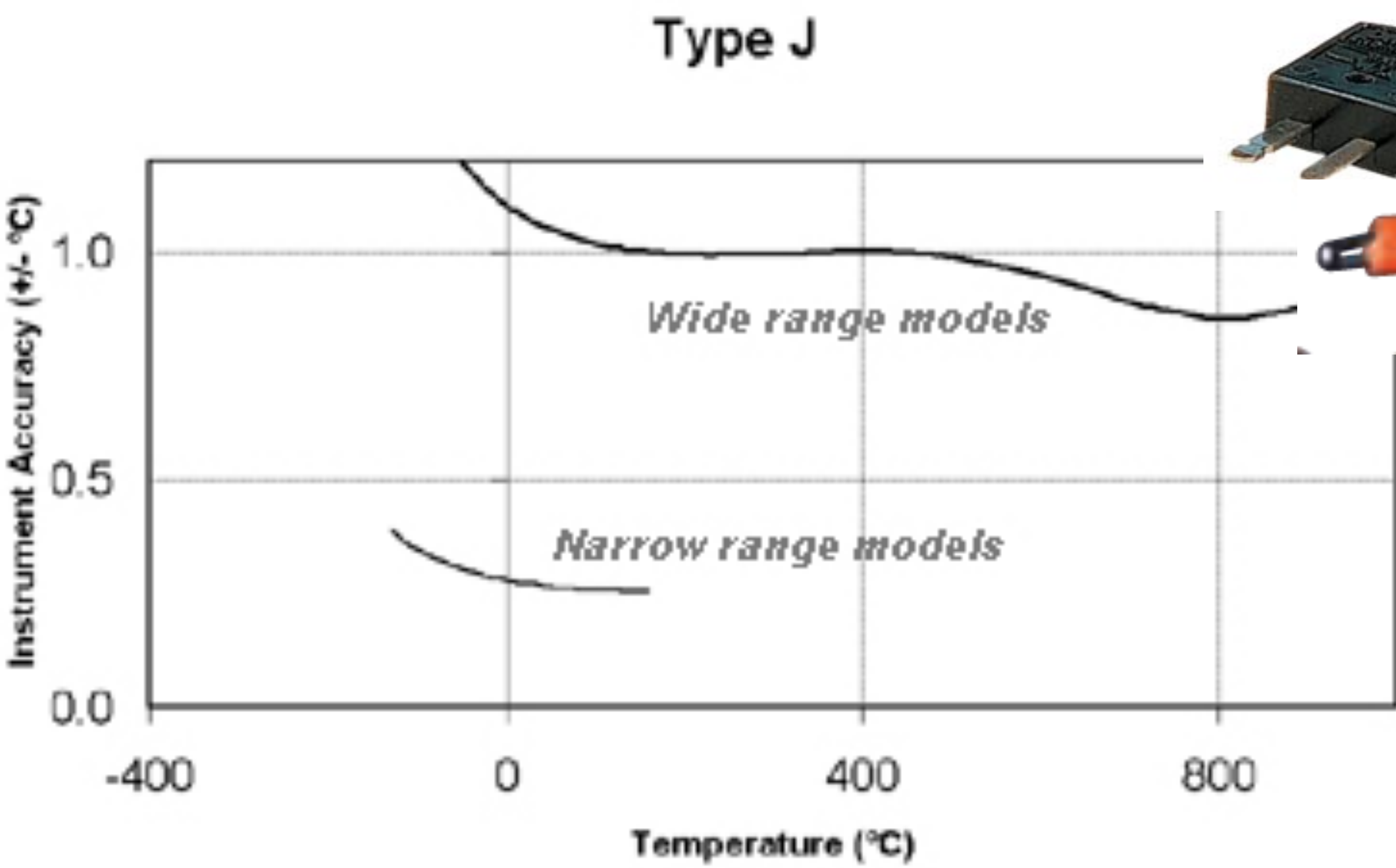
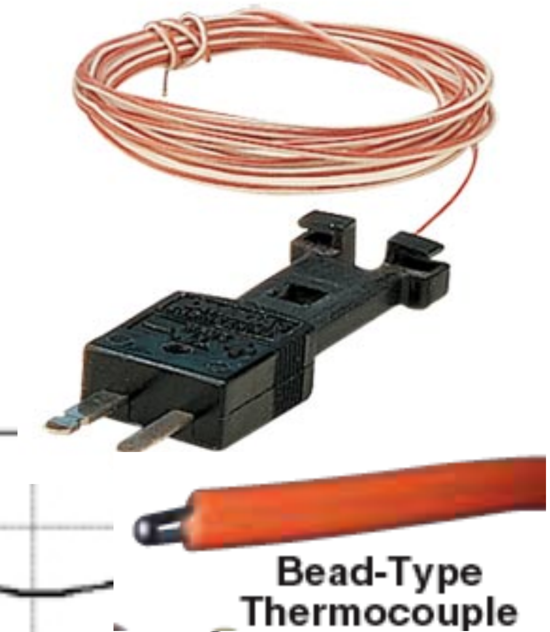
J, K Thermocouples (2)

- **Graphical Display of Accuracy**





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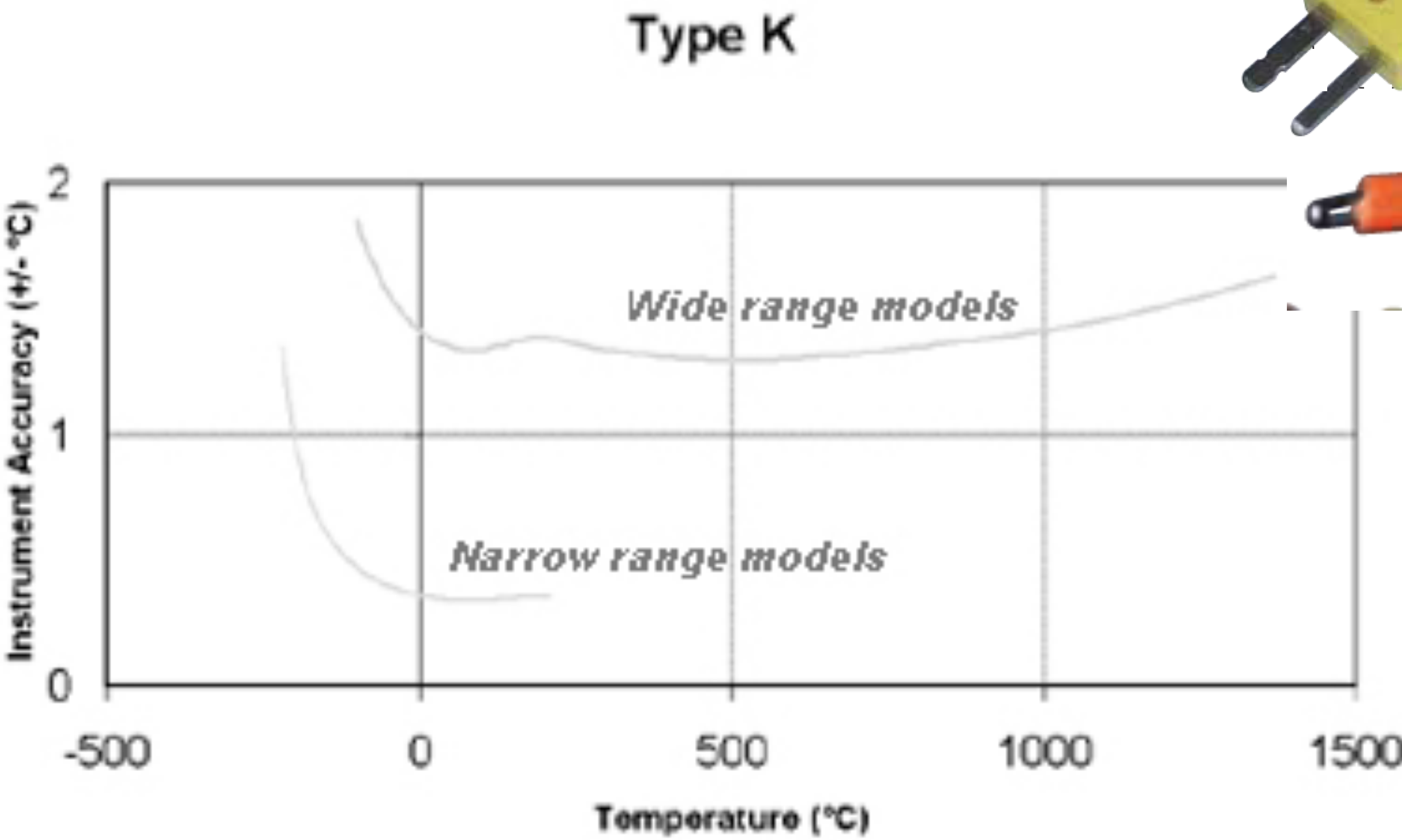
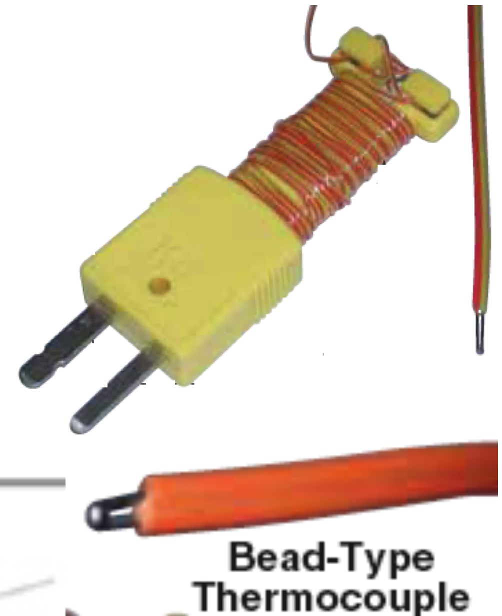
J, K Thermocouples (3)

- **Graphical Display of Accuracy**





Type J Calibration

Type J Temperature to Voltage Calibration Data

	-210 to 760 C	760 C to 1200 C
0	0	296.456257
0	0.0503811878	-1.49761278
	3.04758369E-5	0.00317871039
	-8.56810657E-8	-3.18476867E-6
	1.32281953E-10	1.5720819E-9
	-1.70529583E-1	-3.06913691E-1
	2.09480907E-16	0
	-1.25383953E-1	0
	1.56317257E-23	0

$$V(T) = \sum_{0}^{n} a_n T^n$$

Type J Voltage to Temperature Calibration Data 2

	-8.095 to 0 mV	0 to 42.919 mV	42.919 to 69.553 mV
0	0	0	-3113.58187
0	19.528268	19.78425	300.543684
	-1.2286185	-0.2001204	-9.9477323
	-1.0752178	0.01036969	0.17027663
	-0.59086933	-0.0002549687	-0.00143033468
	-0.17256713	3.585153E-6	4.73886084E-6
	-0.028131513	-5.344285E-8	0
	-0.002396337	5.09989E-10	0
	-8.3823321E-5	0	0

$$T(V) = \sum_{0}^{n} b_n V^n$$

Type K Calibration

Type K Temperature to Voltage Calibration Data

-210 to 0 C	1372 C	exponentiaL COEFFS
0	-0.0176004137	0.1185976
0.039450128	0.038921205	-0.0001183432
2.36223736E-5	1.855877E-5	126.9686
-3.28589068E-7	-9.94575929E-8	0
-4.99048288E-9	3.18409457E-10	0
-6.75090592E-11	-5.60728449E-13	0
-5.74103274E-13	5.60750591E-16	0
-3.10888729E-15	-3.202072E-19	0
-1.04516094E-17	9.71511472E-23	0
-1.98892669E-20	-1.21047213E-26	0
-1.63226975E-23	0	0
0	0	0

Type K Voltage to Temperature Calibration Data

-5.891 to 0 mV	0 to 20.644 mV	20.644 to 54.886 mV
0	0	-131.8058
25.173462	25.08355	48.30222
-1.1662878	0.07860106	-1.646031
-1.0833638	-0.2503131	0.05464731
-0.8977354	0.0831527	-0.0009650715
-0.37342377	-0.01228034	8.802193E-6
-0.086632643	0.0009804036	-3.11081E-8
-0.010450598	-4.41303E-5	0
-0.0005192057	1.057734E-6	0
0	-1.052755E-8	0

$$V(T) = \sum_{0}^{n} a_n T^n + a_{e_0} e^{a_{e_1} \cdot (T - a_{e_2})^2}$$

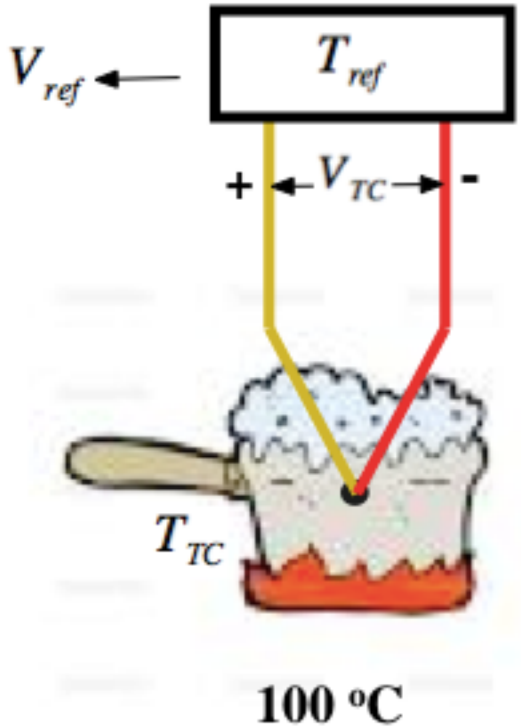
$$T(V) = \sum_{0}^{n} b_n V^n$$

Type J Thermocouple Output Levels

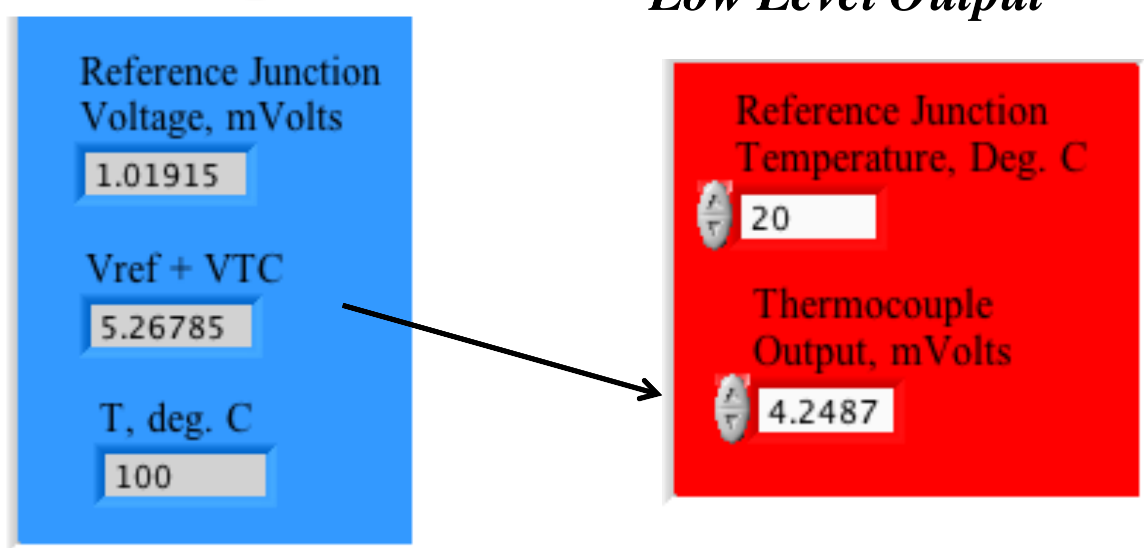
$$T_{TC} \rightarrow V(T) = \sum_{0}^{n} a_n T^n \quad = 4.2487\ mV$$

$$T_{ref} \rightarrow V(T) = \sum_{0}^{n} a_n T^n \quad = 0.8136\ mV$$

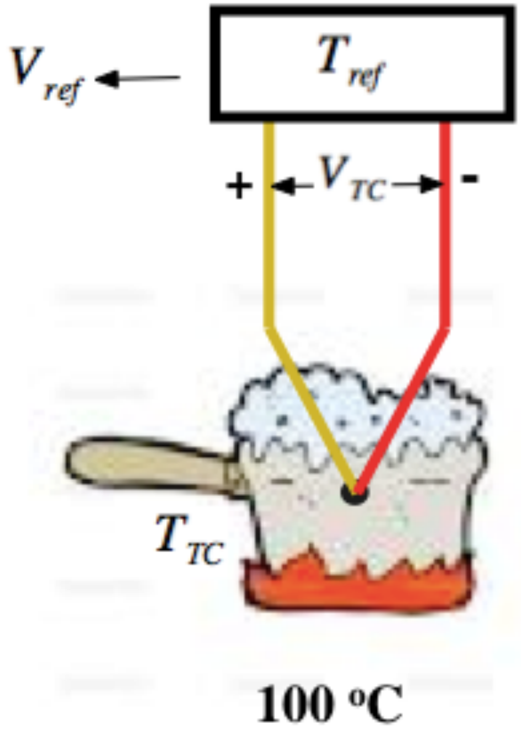
$$T(V) = \sum_{0}^{n} b_n V^n \quad = 100\ ^oC$$



Low Level Output



Type K Thermocouple Output Levels

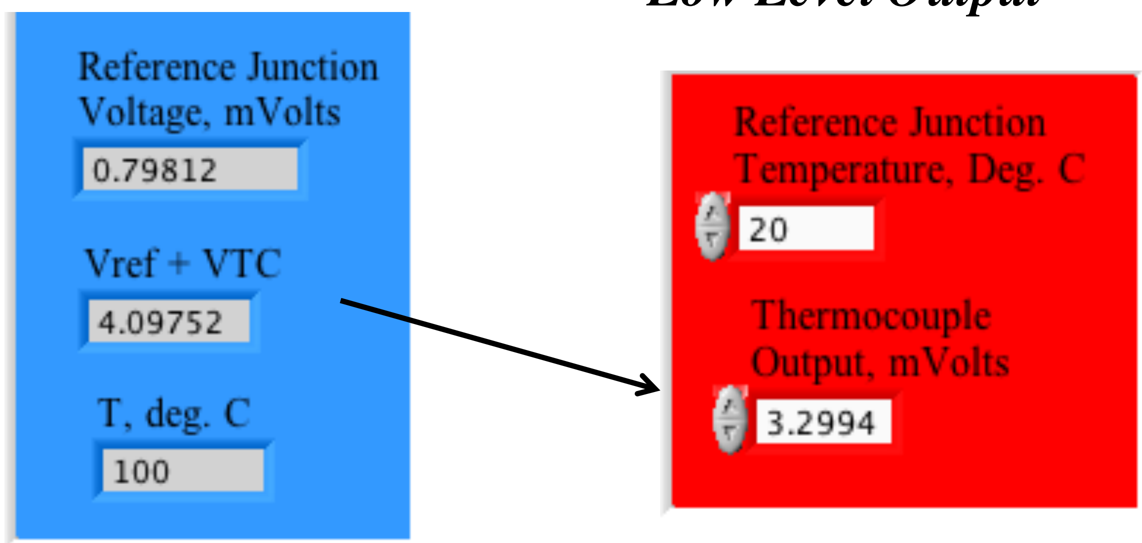


$$T_{TC} \rightarrow V(T) = \sum^{n} a_n T^n$$ **= 4.2487 mV**

$$T_{ref} \rightarrow V(T) = \sum_{0}^{n} a_n T^n$$ **= 0.8136 mV**

$$T(V) = \sum_{0}^{n} b_n V^n$$ = 100 °C

Low Level Output



MyDAQ Analog Input

Analog Input

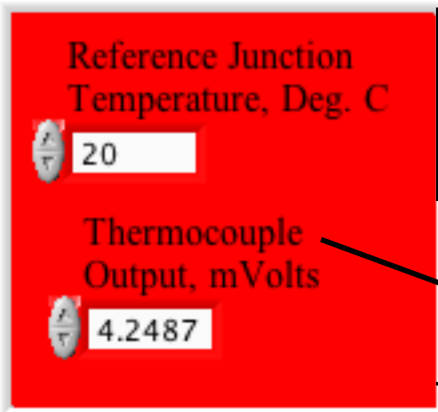
Number of channels .... 2 differential or 1 stereo audio input

ADC resolution .... 16 bits

Range

Analog input .... ±10 V

Input data

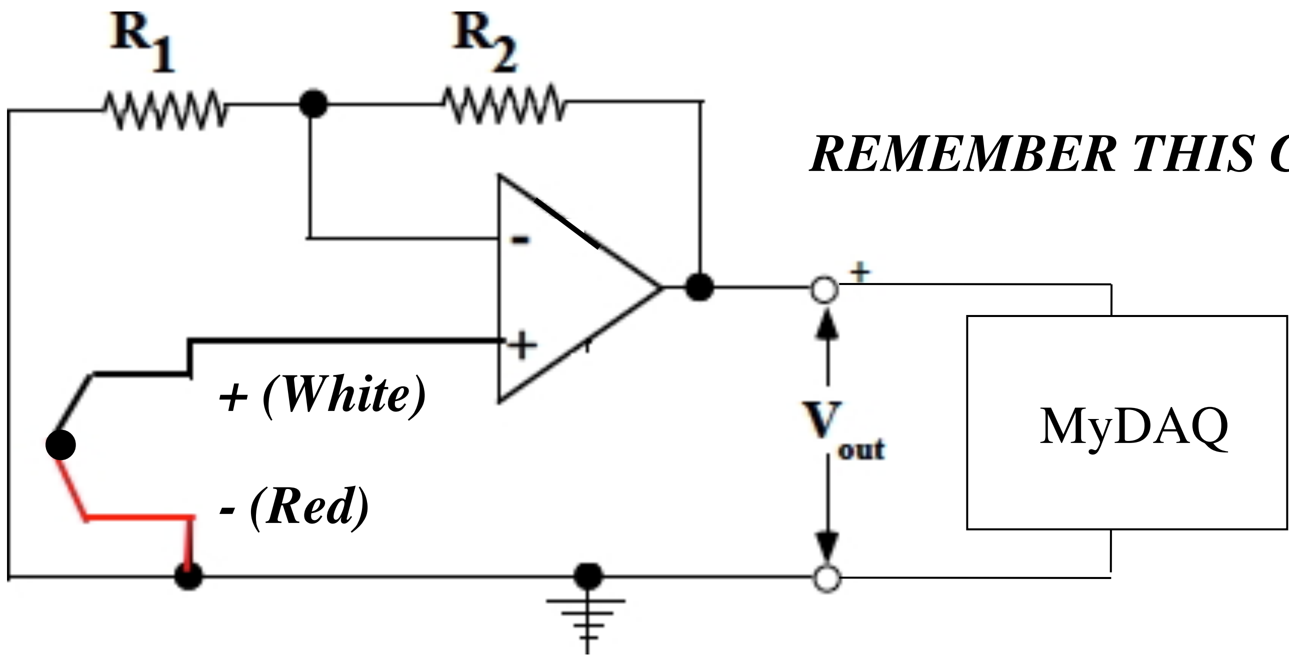


$$lsb \rightarrow \frac{+10_V - (-10_V)}{2^{16}} \times 1000_{\frac{mV}{V}} = \pm 0.3052_{\frac{mV}{bit}}$$

$$\frac{0.3052_{\frac{mV}{bit}}}{4.2487_{mV @ 100^o C}} = \pm 7.18_{\%error} \rightarrow$$

≈ Much Bigger than Sensor Precision Error ± 0.75%

Let's Amplify the TC Output to Achieve Better Resolution



REMEMBER THIS CIRCUIT?!

$$V_o = V_{TC} \cdot \left(\frac{R_1 + R_2}{R_1} \right)$$

$R_1 = 100$ Ohms
$R_2 = 2000$ Ohms → Gain = 20

See Appendix III for Derivation

Let's Amplify the TC Output to Achieve Better Resolution (2)

-- @ 100 °C → $V_{TC} = 4.2487\ mV$

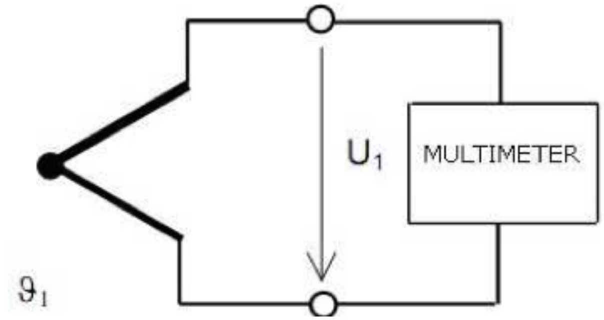
-- Amplified Signal = *20 x 4.2487 mV = 84.9748 mV*

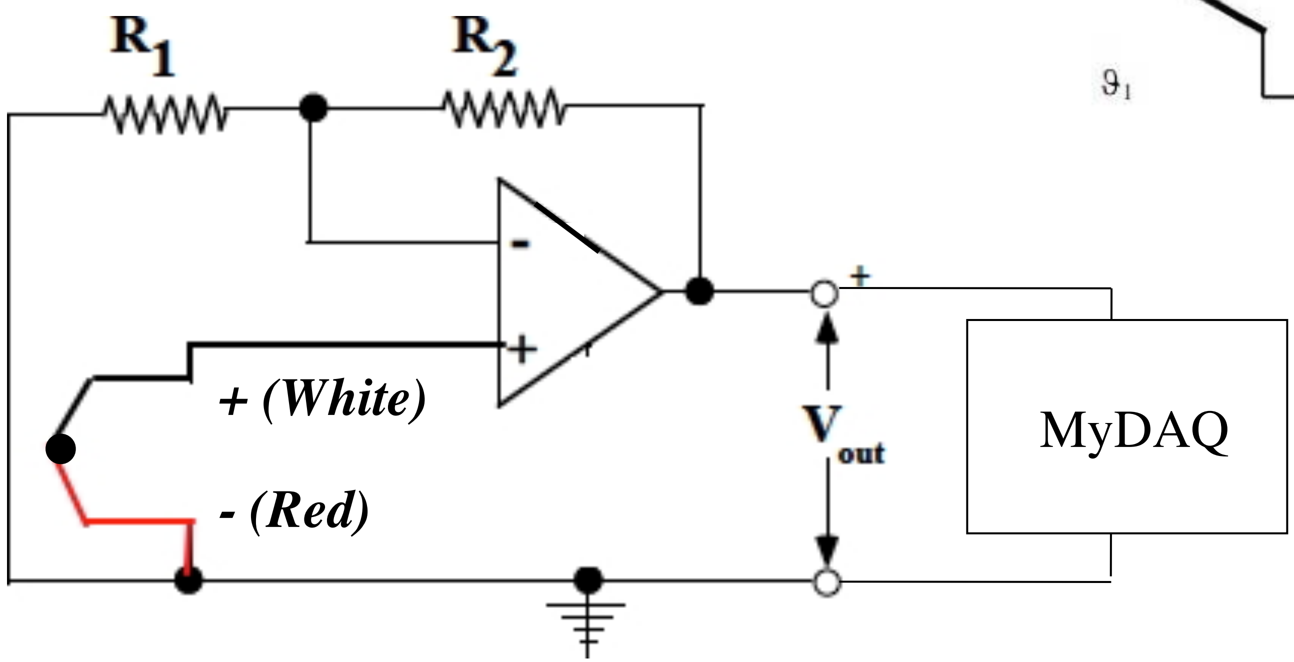
-- *Resolution error now has been significantly reduced*

$$lsb \rightarrow \frac{+20_V}{2^{16}} \times 1000_{\frac{mV}{V}} = 0.3052_{\frac{mV}{bit}}$$

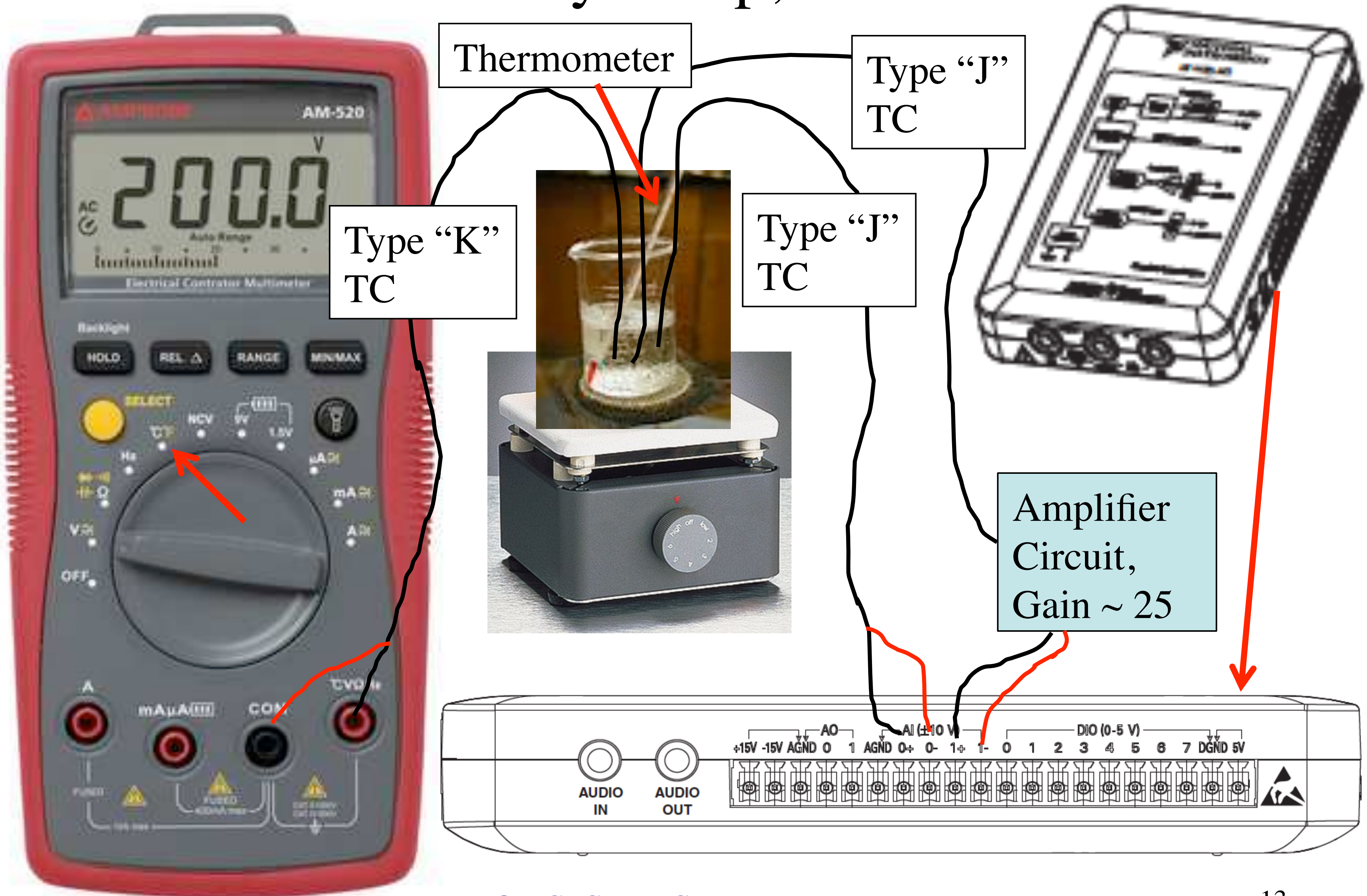
$$\frac{0.3052_{\frac{mV}{bit}}}{84.974_{mV@100^o C}} = 0.359_{\%error}$$

Part 1, Thermocouple Systems Calibration





Laboratory Setup, Part 1

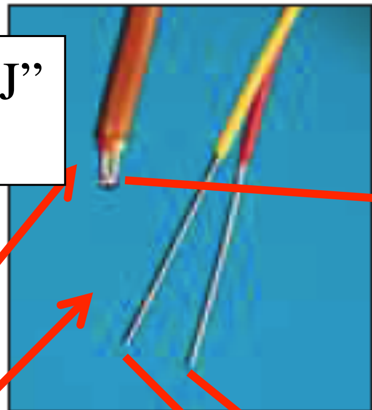


Laboratory Setup (2)

Type "J" TC

Sensing bead

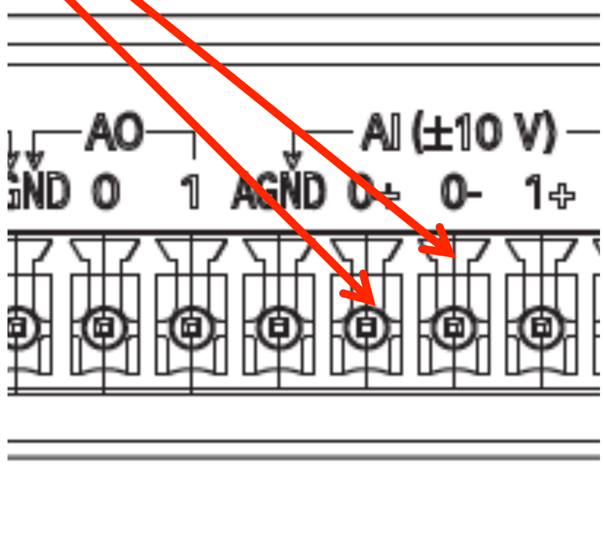
Matched TC Metal Wire Pairs

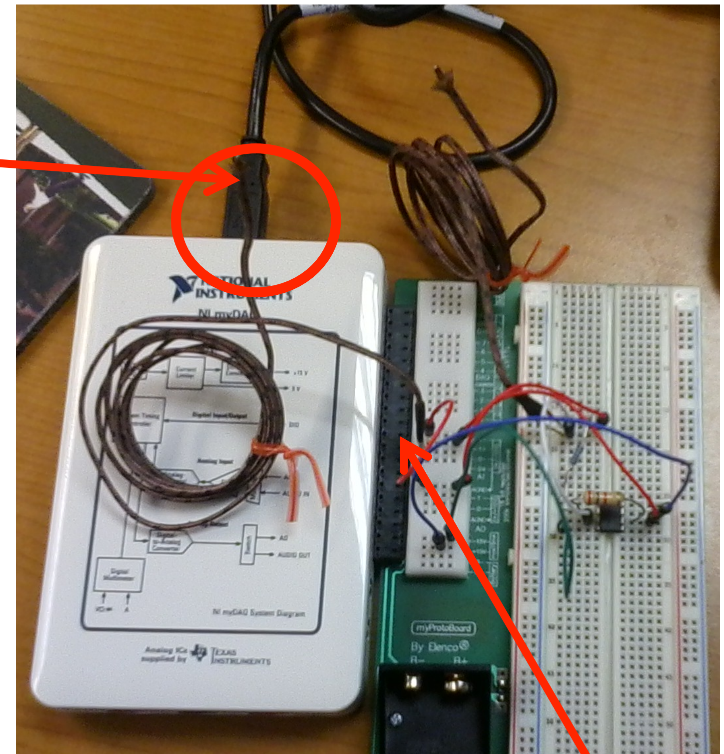


+ (White)

- (Red)

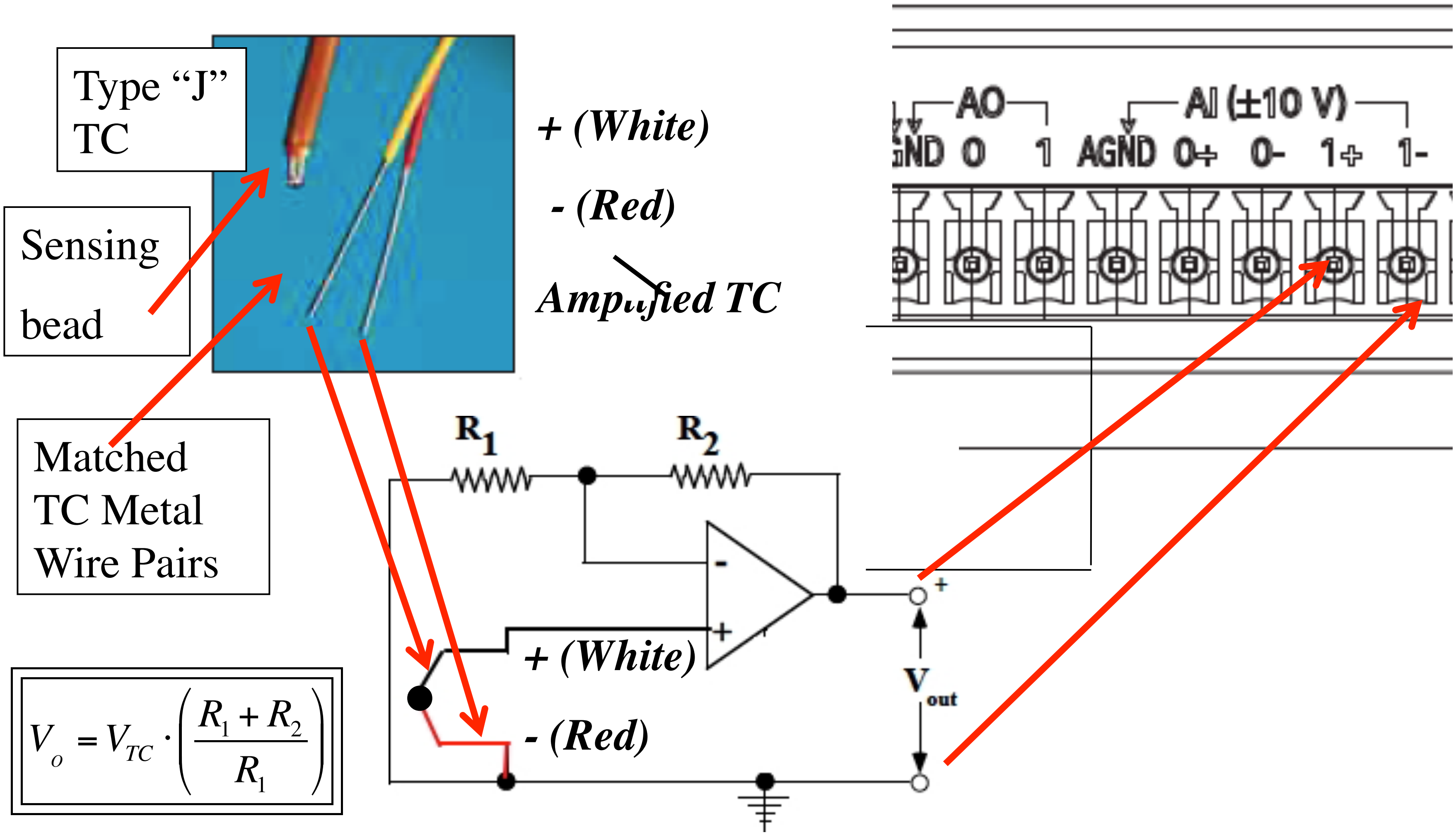
Unamplified TC





WIRE DIRECTLY TO DAQ AI0

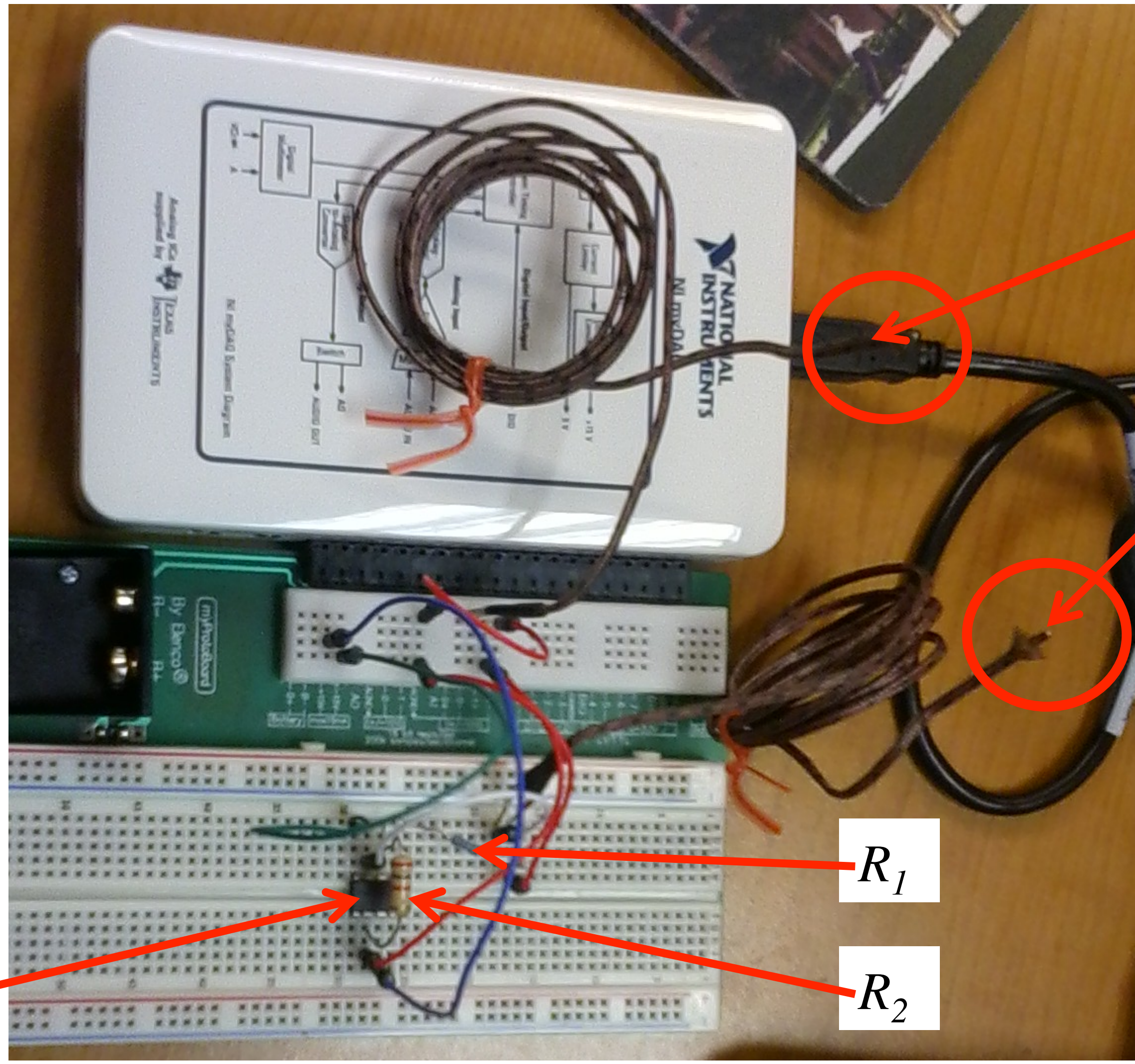
Laboratory Setup (3)



$$V_o = V_{TC} \cdot \left(\frac{R_1 + R_2}{R_1} \right)$$

Don't forget the ± 15V Power to Amplifier

Laboratory Setup (4)



TC (1) BEAD

TC (2) BEAD

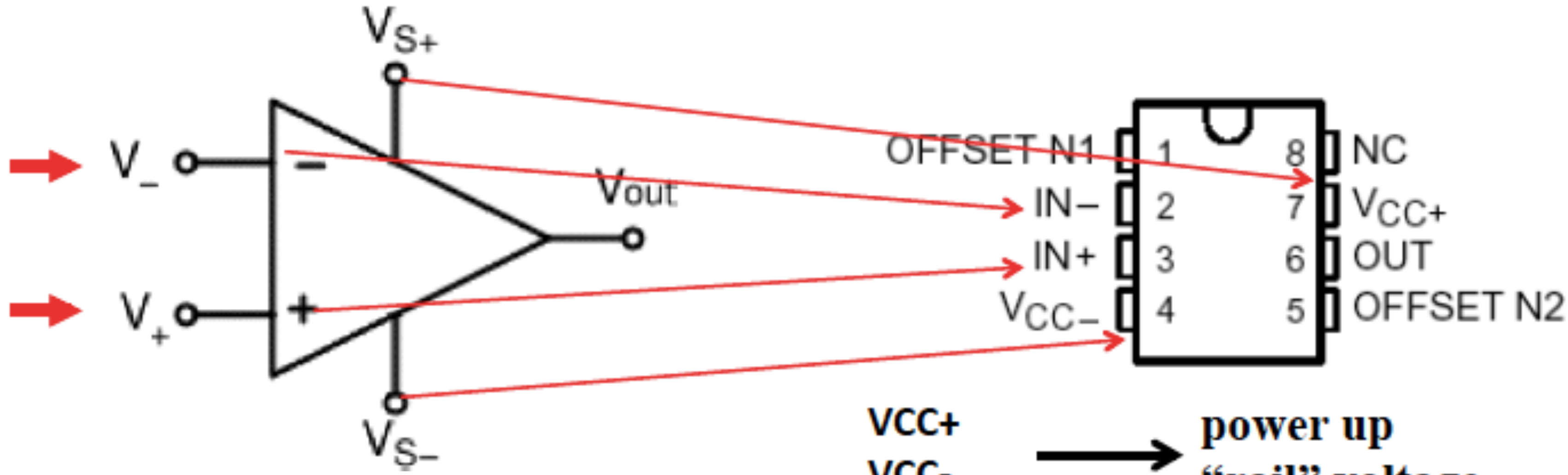
Pick R_1, R_2 to give amplifier gain near 20-25.

R_1

R_2

OP AMP

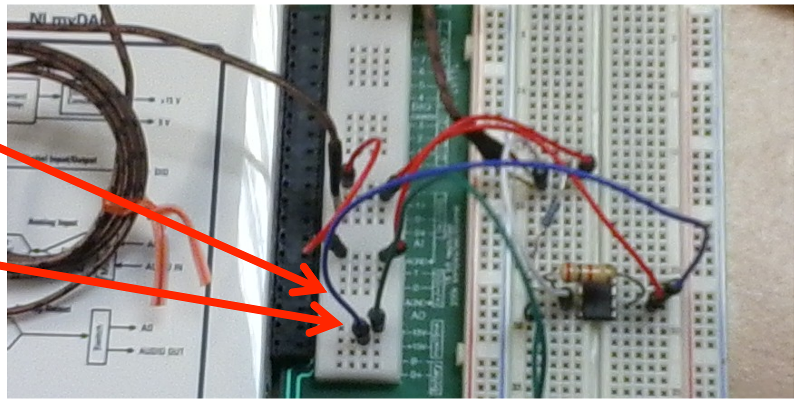
Laboratory Setup (5)



VCC+
VCC- → **power up "rail" voltage**

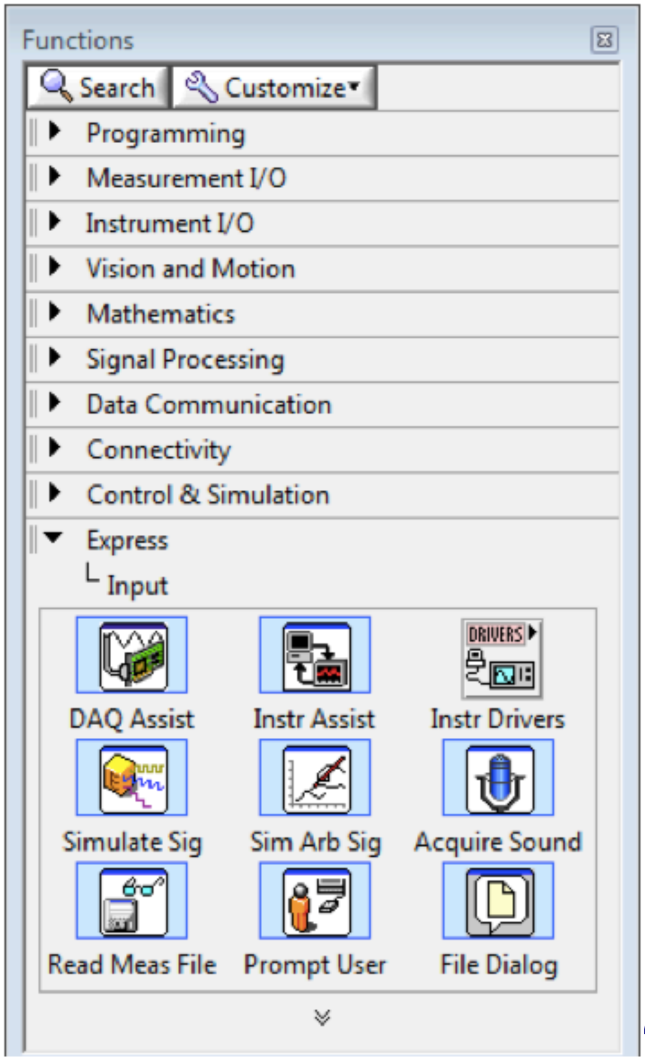
+ 15 VDC

- 15 VDC



VI Programming Procedure (1)

- Modify VI from labs 2 & 6 to now acquire TWO CHANNELS from the MyDAQ
- Easiest method is to use the DAQ assistant



- The DAQ Assistant Express VI can be found on the Functions palette by going to Express » Input.

- Place the DAQ Assistant on the block diagram by dragging and dropping it from the Functions palette.

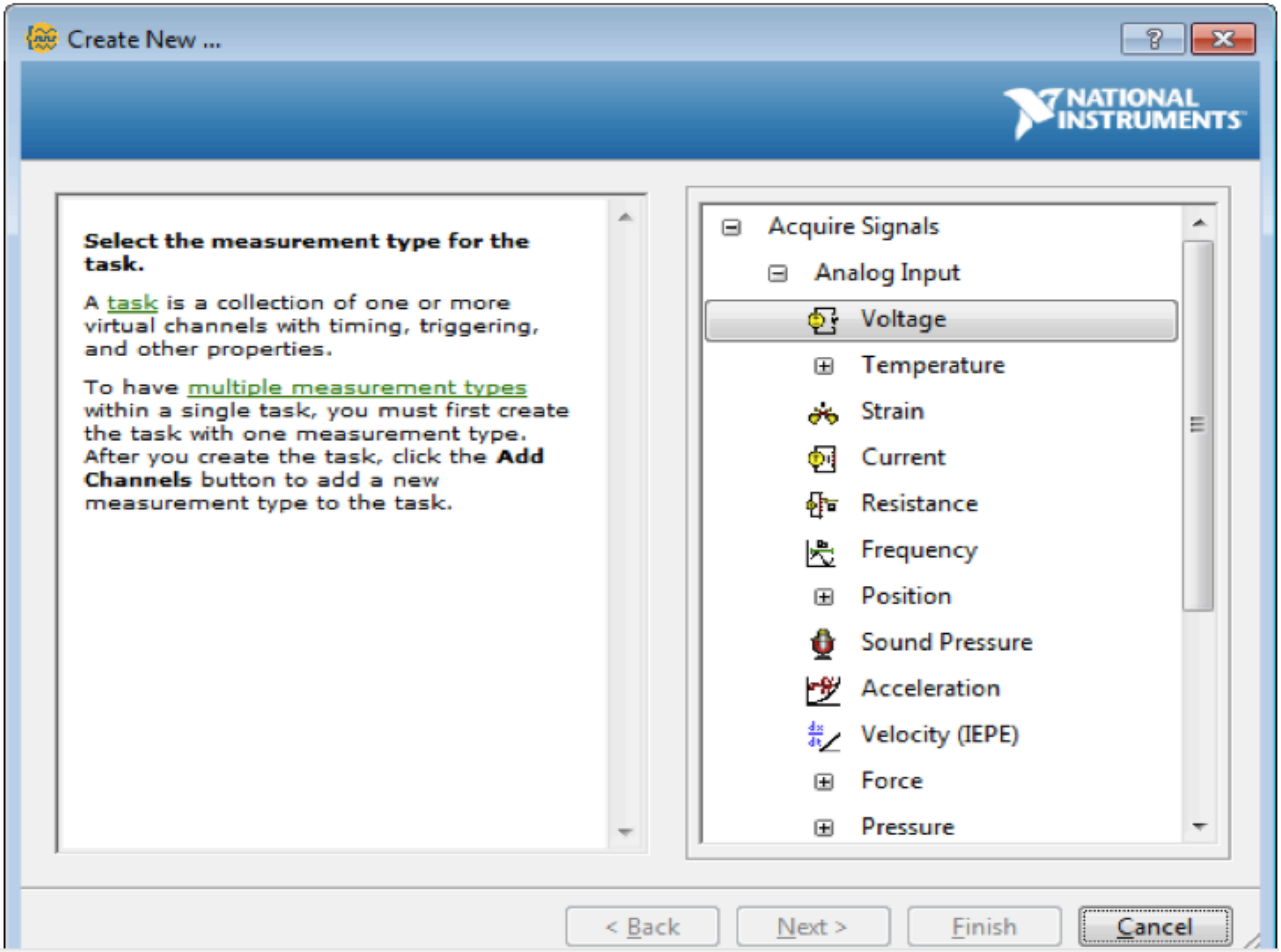
- The Assistant should automatically launch when you drop the VI on the diagram.

- **Create the Task**

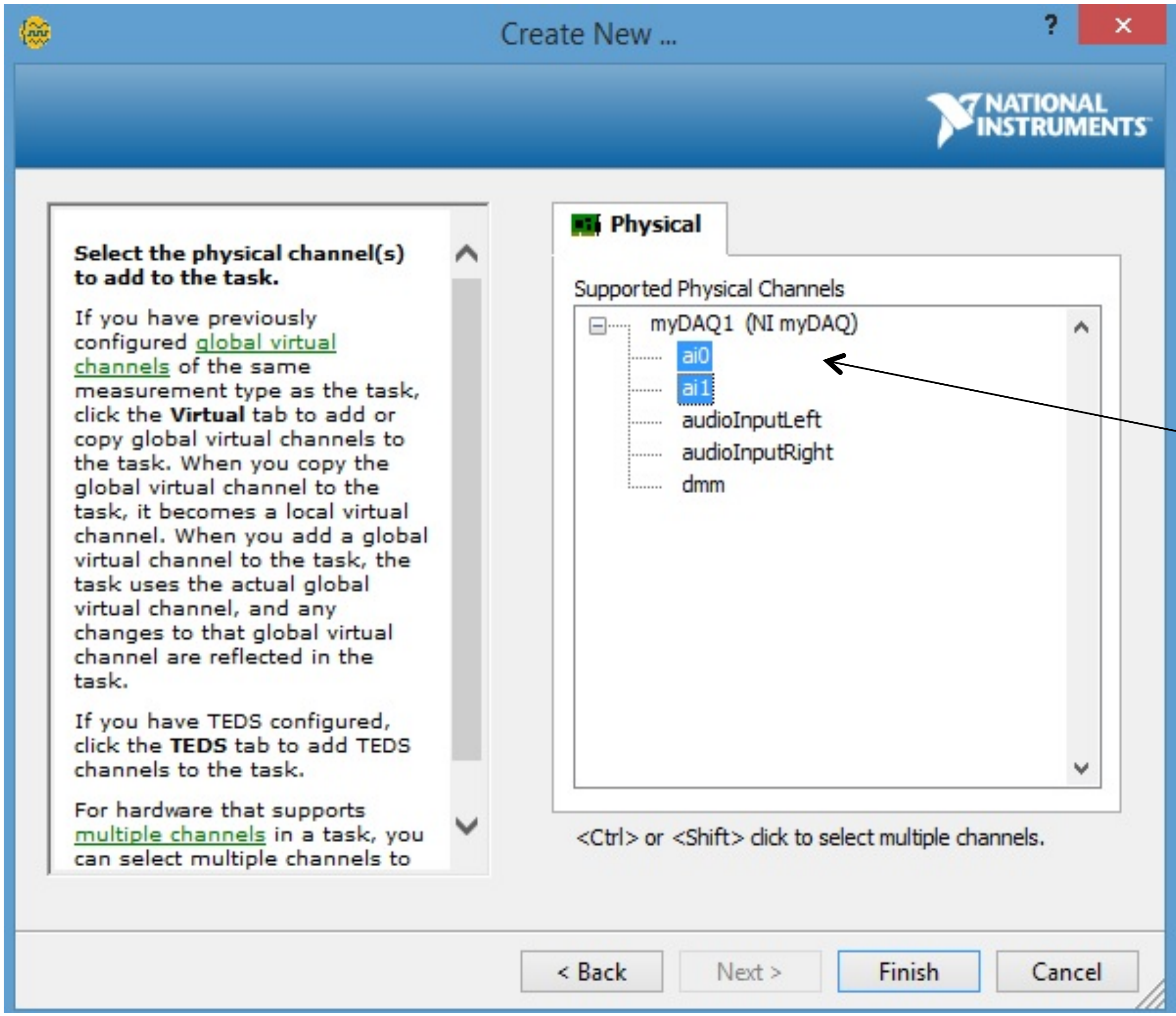
  -- Select **Acquire Signals** and then **Analog Input** for your Measurement Type.

  -- Next, select **Voltage.**

VI Programming Procedure (2)



VI Programming Procedure (3)



- Will show "MyDAQ"
- and available channels

Select both a0 and a1 (shift drag)

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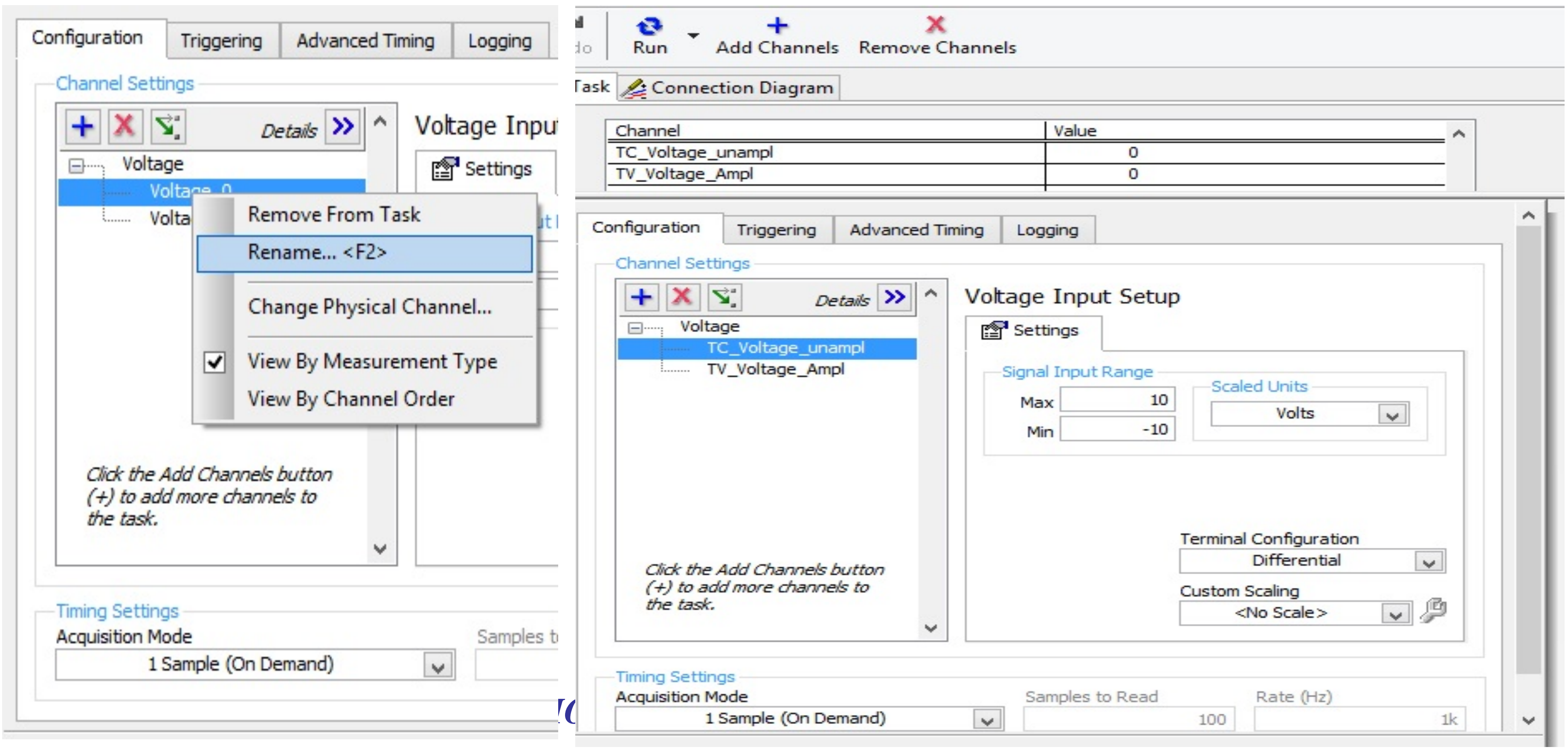
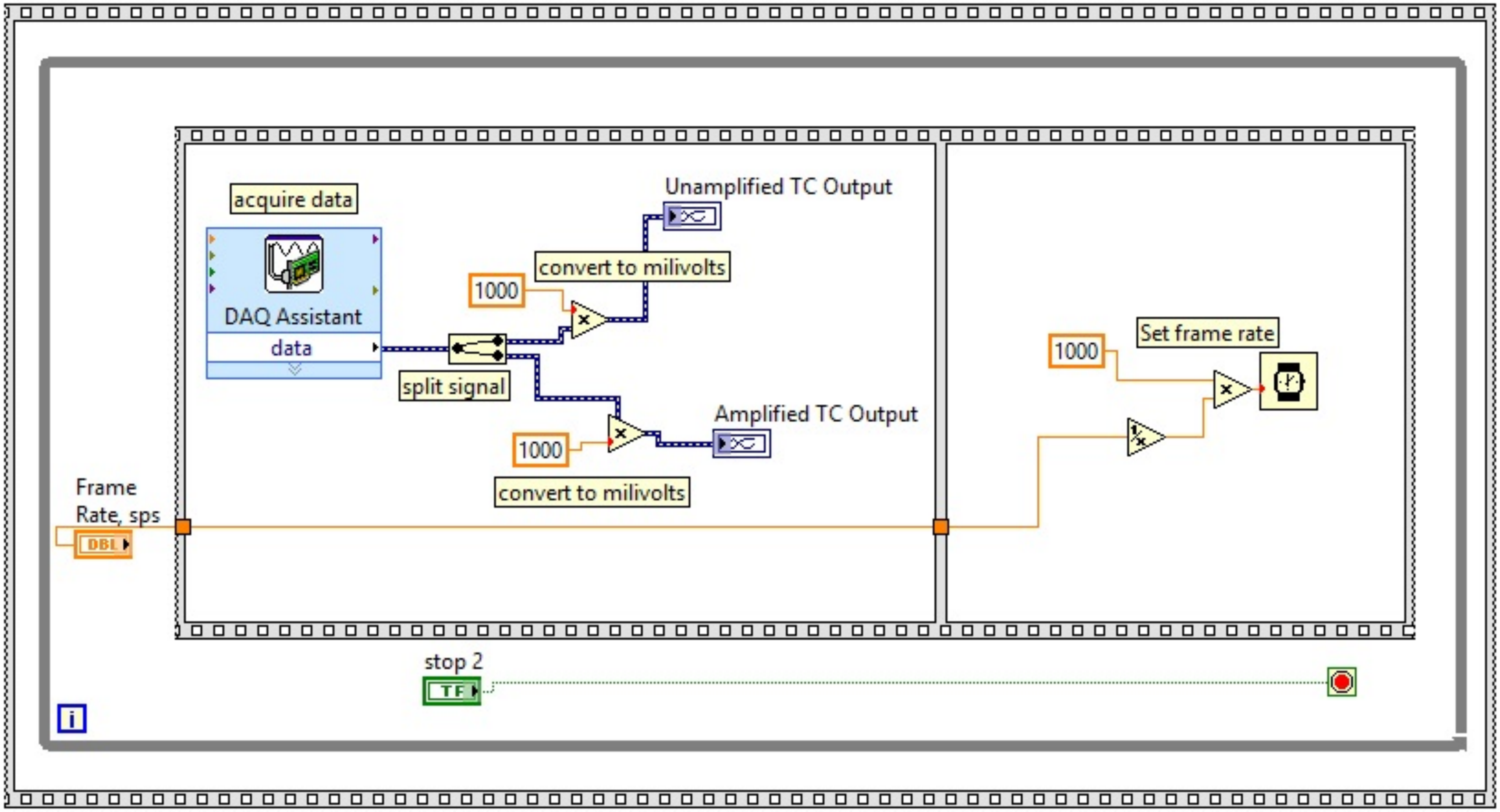
VI Programming Procedure (4)

- ***Rename Channels (right click on channel)***
  - ACH0 → Unamplified TC Voltage
  - ACH1 → Amplified TC Voltage
- ***Set Acquisition Mode: 1 Sample (On Demand)***
  - 1-sample, On demand
- **Set Voltage Range for Both Channels:** +- 10 Volts (Default for MyDAQ)



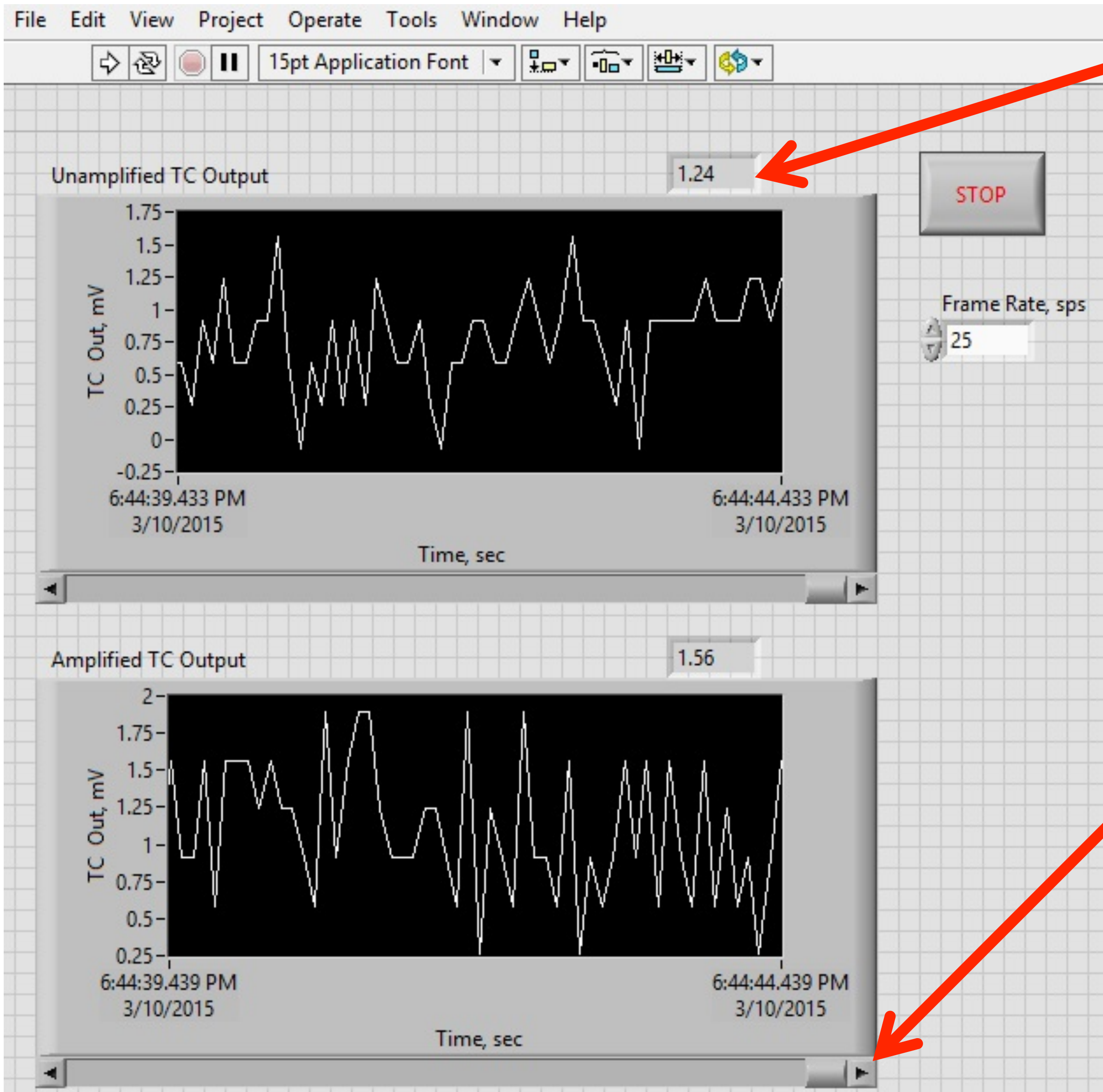
VI Programming Procedure (6)



• Split Output from DaQ assistant Bock .. And wire to two separate digital displays and strip charts

Functions >> Express >> Signal manipulation >> Split Signals

VI Programming Procedure (7)

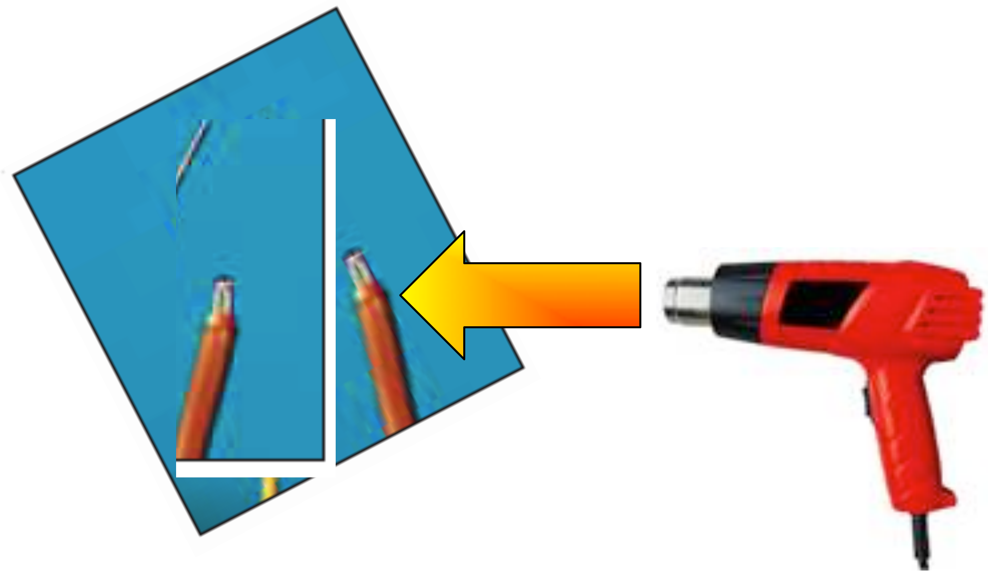


"Right click" for Digital Display

"Right click" for Time-Axis Scroll Bar, Drag to desired size.

Lab Procedure, Part 1 (1)

- Power up your Op-Amp, then run the VI
- Use Heat gun to heat the beads of both TC's simultaneously
- What do you notice of the comparative responses? Noise levels?



Lab Procedure, Part 1 (2)

• Fill Beaker with water .. Insert all three TC's

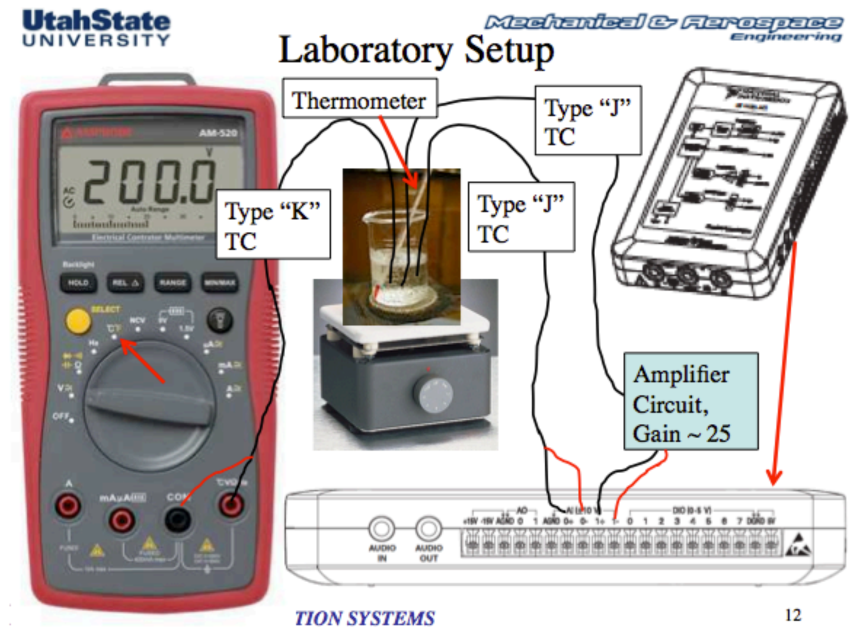
- *"K" from multimeter*
- *"J" unamplified*
- *"J" Amplified*

Manual thermometer

• Place beaker onto Corning lab Heater, turn set point to "high"

• Run VI and as water heats up, Read Multimeter Temperature Output and Thermometer Value, and mVolt outputs from TC'sa connected to MyDAQ

• Log at least 10 data points as water warms from tap temperature to just below boiling.



Lab Procedure, Part 1 (3)

- Populate Spreadsheet below

Data Point Number	Thermometer Temperature ^oC	MultiMeter "K" Temperature ^oC	Unamplified Type "J" mVolts	Amplified Type "J" mVolts
1				
2				
3				
…				
9				
10				

Lab Procedure, Part 1 (4)

- **Plot and Curve Fit Data .. Produce 4 graphs**

1) Thermometer Temperature vs Unamplified TC mVolts
2) Type “K” Temperature (DMM) vs Unamplified TC mVolts

3) Thermometer Temperature vs Amplified TC mVolts
4) Type “K” Temperature (DMM) vs Amplified TC mVolts

- **Compare First and Second order Curve Fits for each, Use RMSE fit error and Student –t distribution to assess curve confidence interval to 95% level of confidence**

- **Attach Plots for each to lab report**

- **Use VI developed for Homework 5 to make this task EASY!**

Lab Procedure, Part 1 (5)

- **Select Coefficients for "Best fit" for both type "J" TC's**

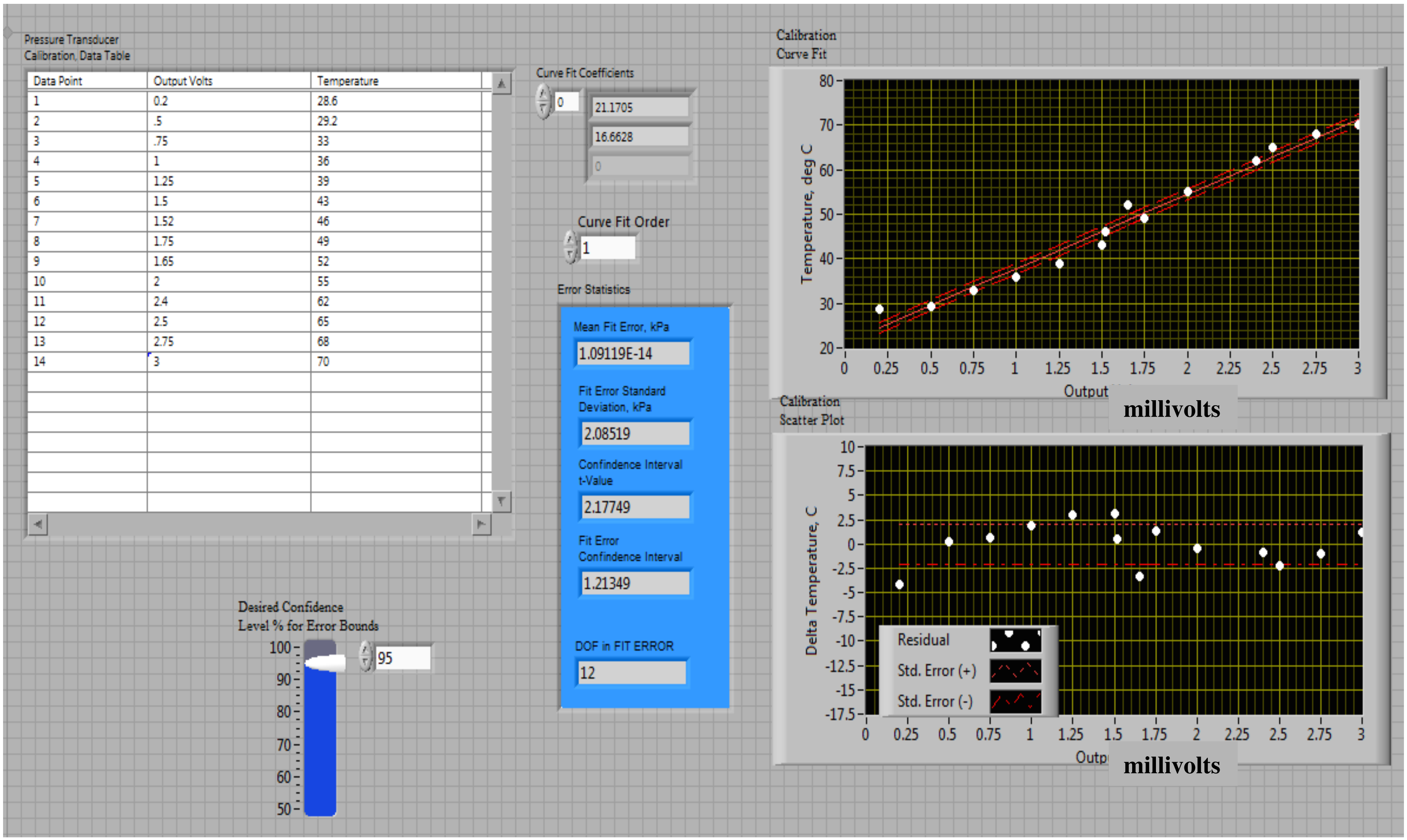
1) These will be your calibrations for the "Installed" TC systems

2) Use Coefficients to Scale mVolts display on VI of front panel to calculate temperature in deg. C

3) Add additional stripchart graph and display both TC outputs in T deg. C on the same chart.

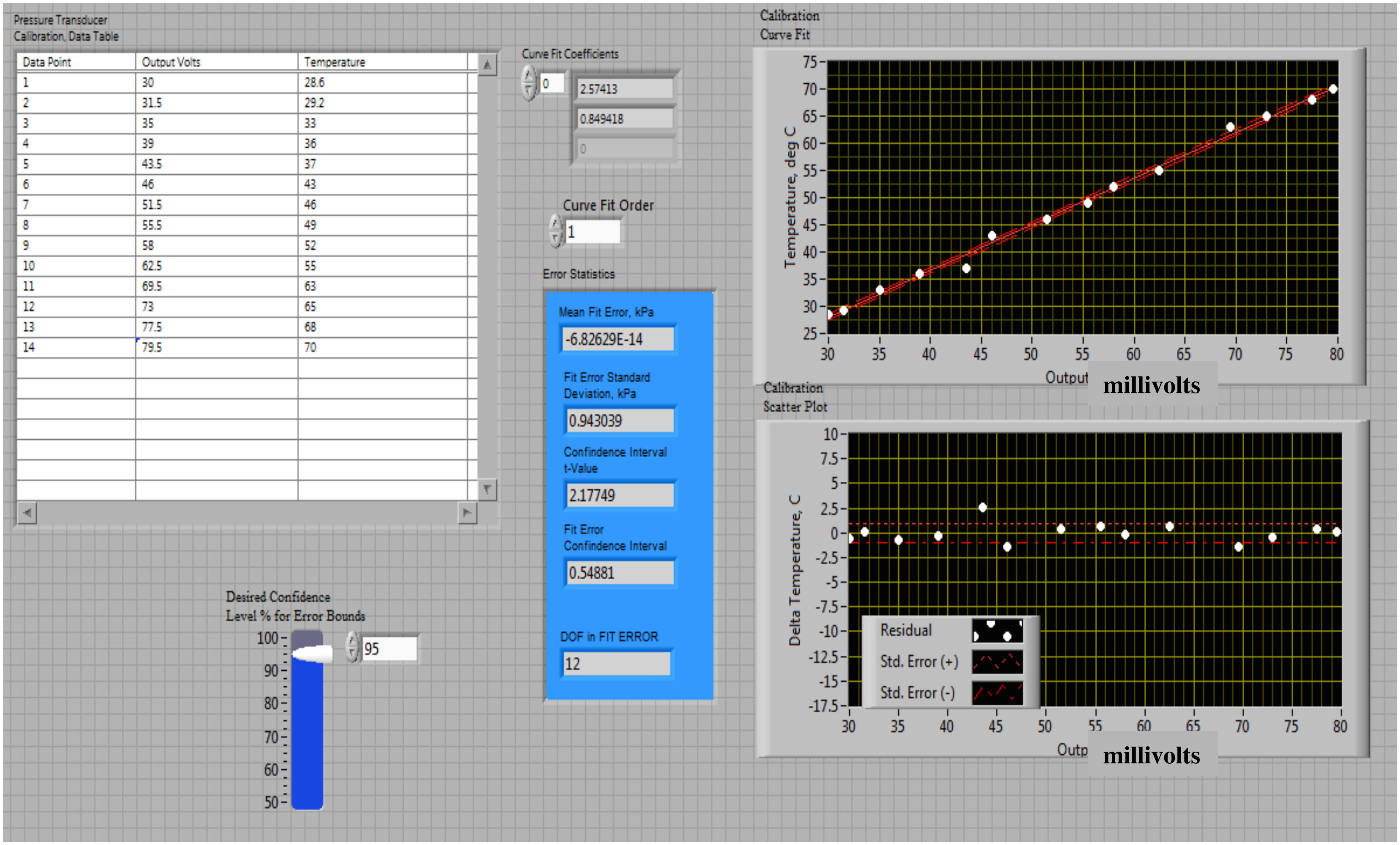
Lab Procedure, Part 1 (5)

- **Example Plots and Analysis, Unamplified TC**



Lab Procedure, Part 1 (6)

- **Example Plots and Analysis, Amplified TC**



Data Point	Output Volts	Temperature
1	30	28.6
2	31.5	29.2
3	35	33
4	39	36
5	43.5	37
6	46	43
7	51.5	46
8	55.5	49
9	58	52
10	62.5	55
11	69.5	63
12	73	65
13	77.5	68
14	79.5	70

VI Programming Procedure, Part 2

• Use curve Fit coefficients from calibration to calculate temperature directly from the output from TC (1) and TC (2)

• Select the Curve Fit Array from Each Calibration VI and Paste Onto Front Panel of TC-Read VI. → Right Click Indicator and Select *"Change to Control"*

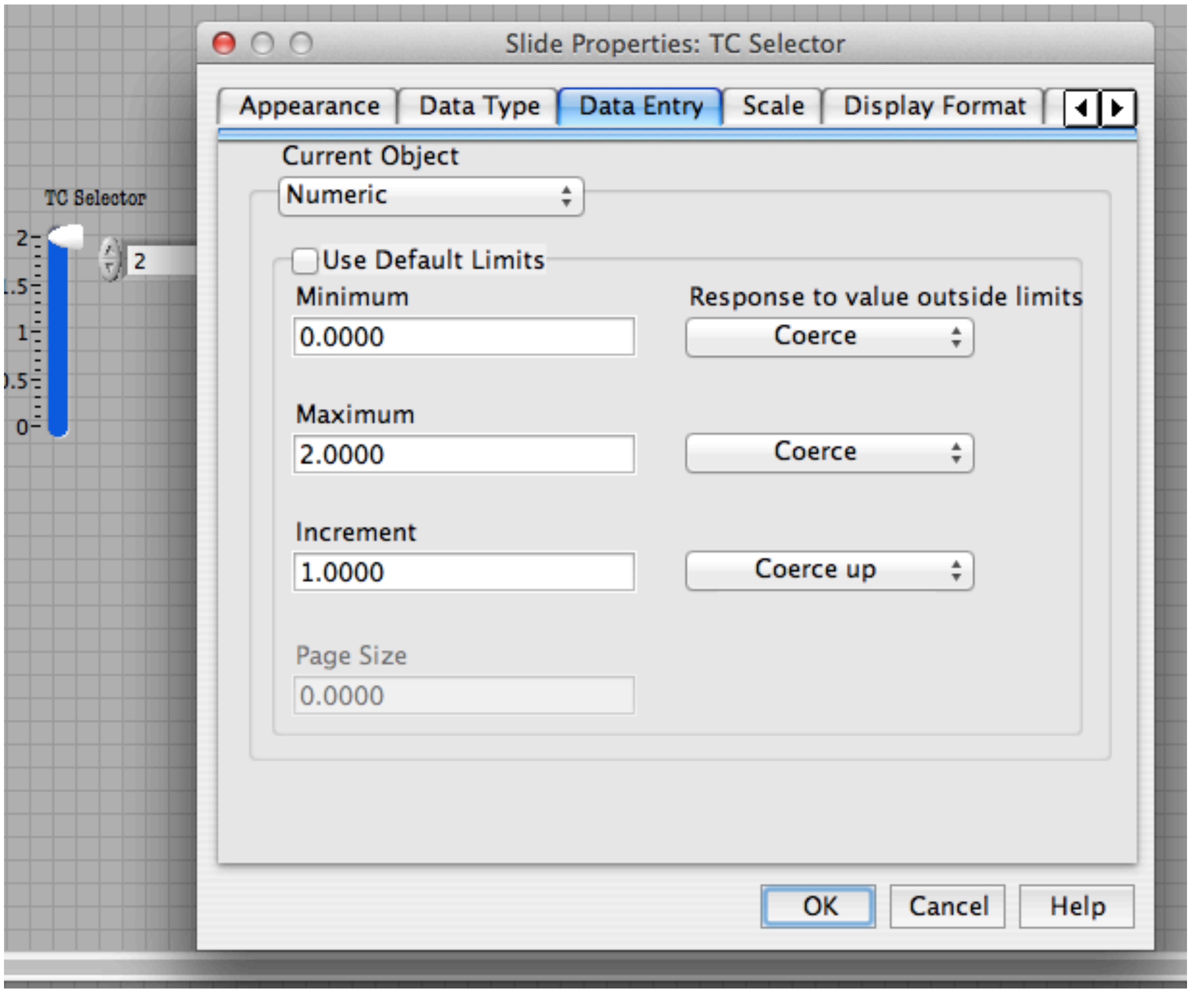
• Right Click again on Control and Select *"Data Operations >> Make Current Value Default"*

• Make a New Strip chart Display on Front Panel and Title that Indicator *"TC Output Display"*. Y-axis units are now temperature in degrees C.

• Now Grab a *"Vertical Pointer Slide"* control from the Numeric Control Menu and Paste onto front panel, Label This Bar *"TC Selector"*

VI Programming Procedure, Part 2 (2)

• *Right Click on Control and Select data Entry .. Set minimum value to zero, maximum value to 2*



• Set Upper Limit on Indicator to 2

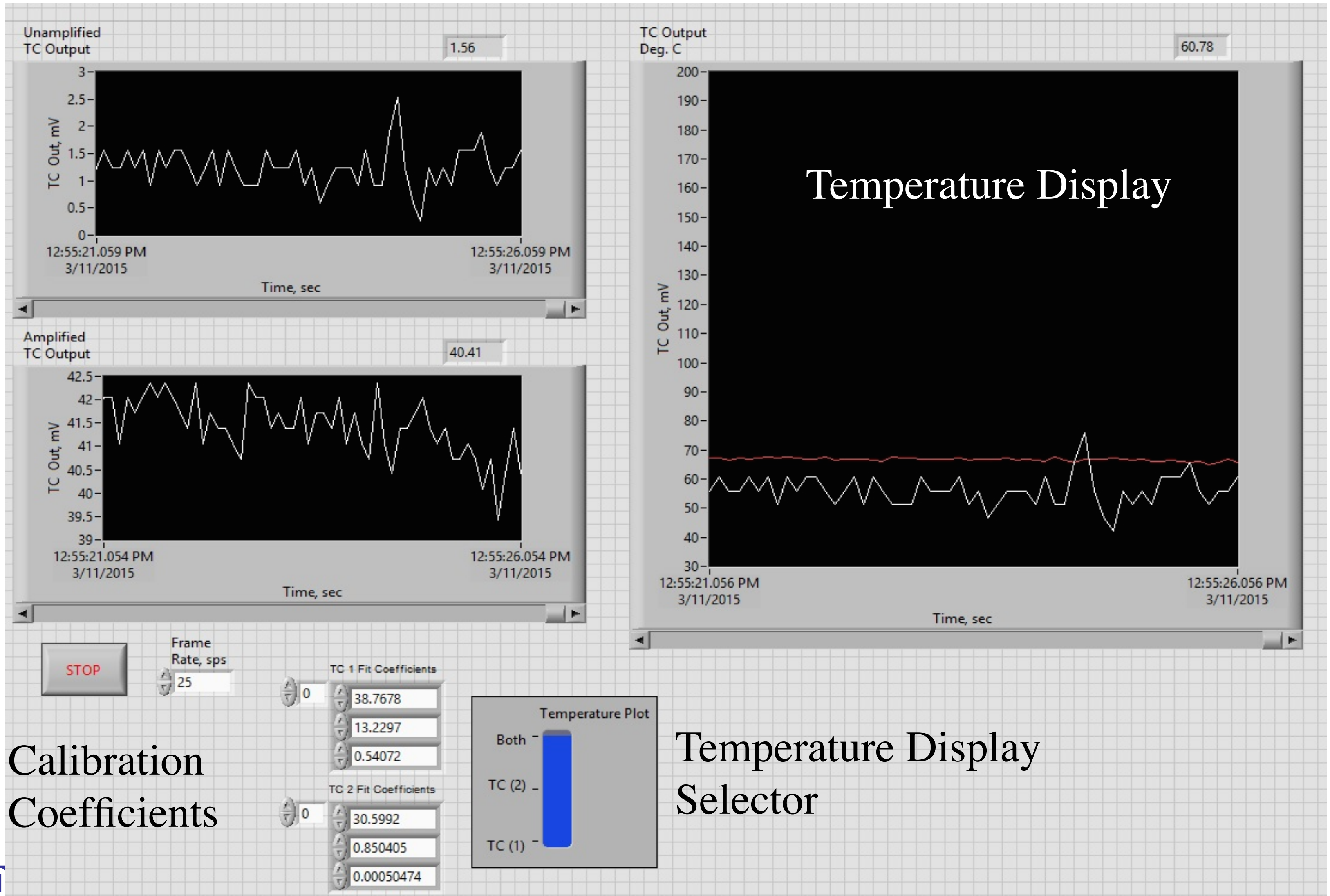
• Right click on Indicator and Select *"Representation>>I32"* Sets this control to an integer input

• Right Click on Indicator and select *"Visible Items >> Digital Display"*

This control will allow you to select either of both TC for temperature display

VI Programming Procedure, Part 2 (3)

- *Your revised front panel should now look approximately like*



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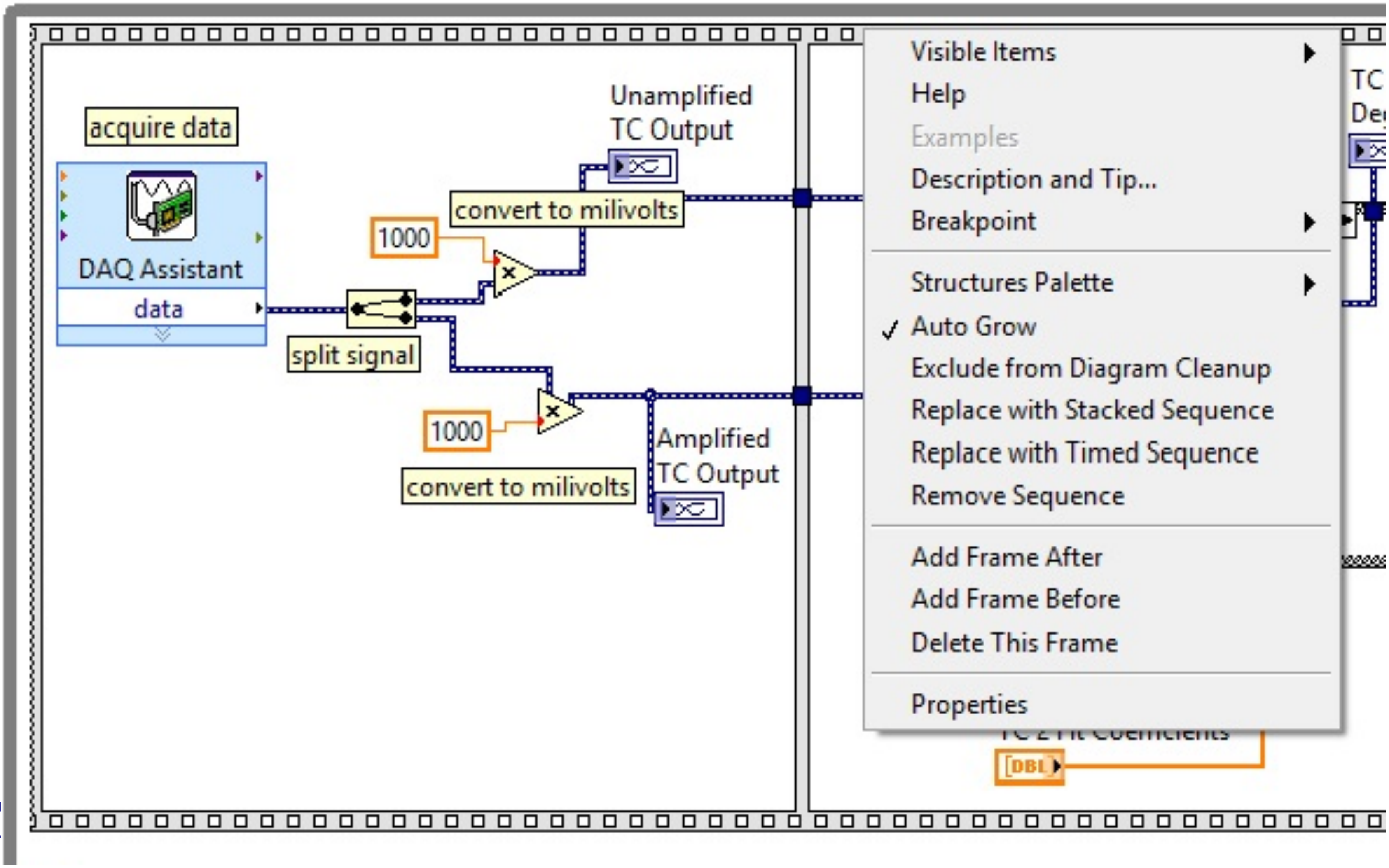
VI Programming Procedure, Part 2 (4)

• *Add another frame to your Block diagram for the Volts to temperature conversion … right click on first frame .. Select add frame after*



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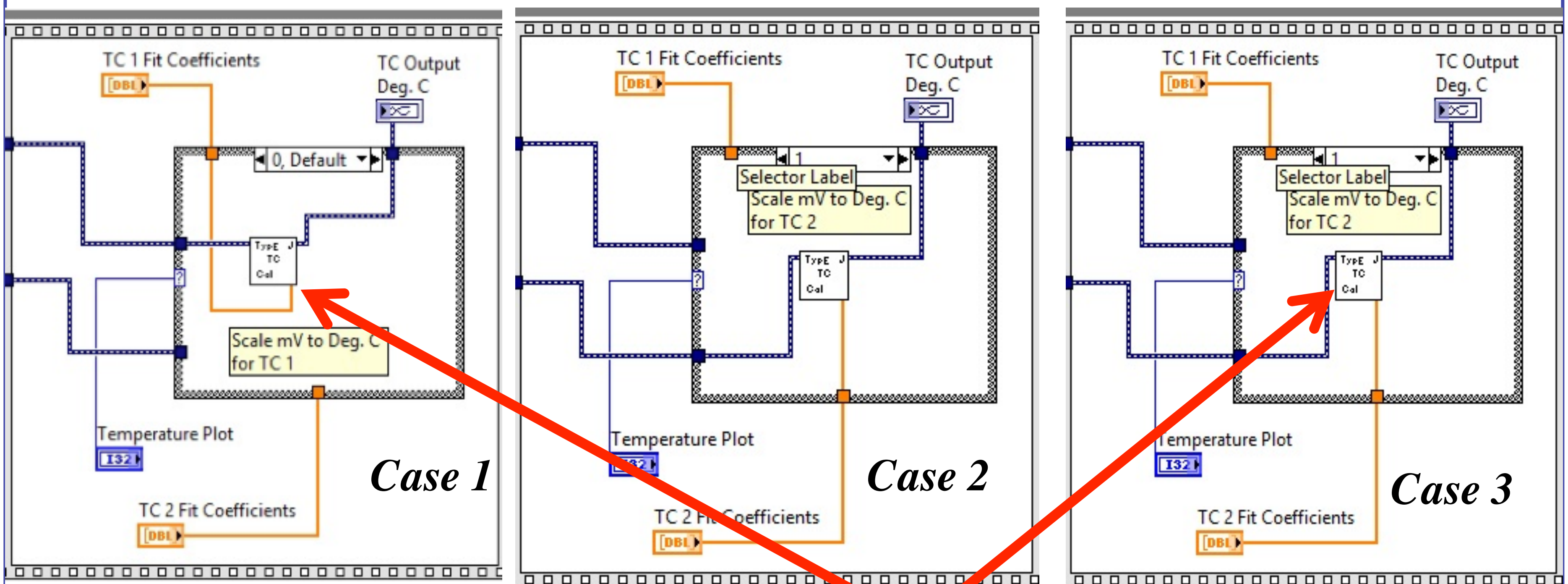
VI Programming Procedure, Part 2 (5)

- Add a case structure to allow you to select the display options → *"Functions >> Programming >> Structures >> Case Structures "*
- Wire your selector to this case structure, select case *"1"*, right click, and select *"add frame after"*

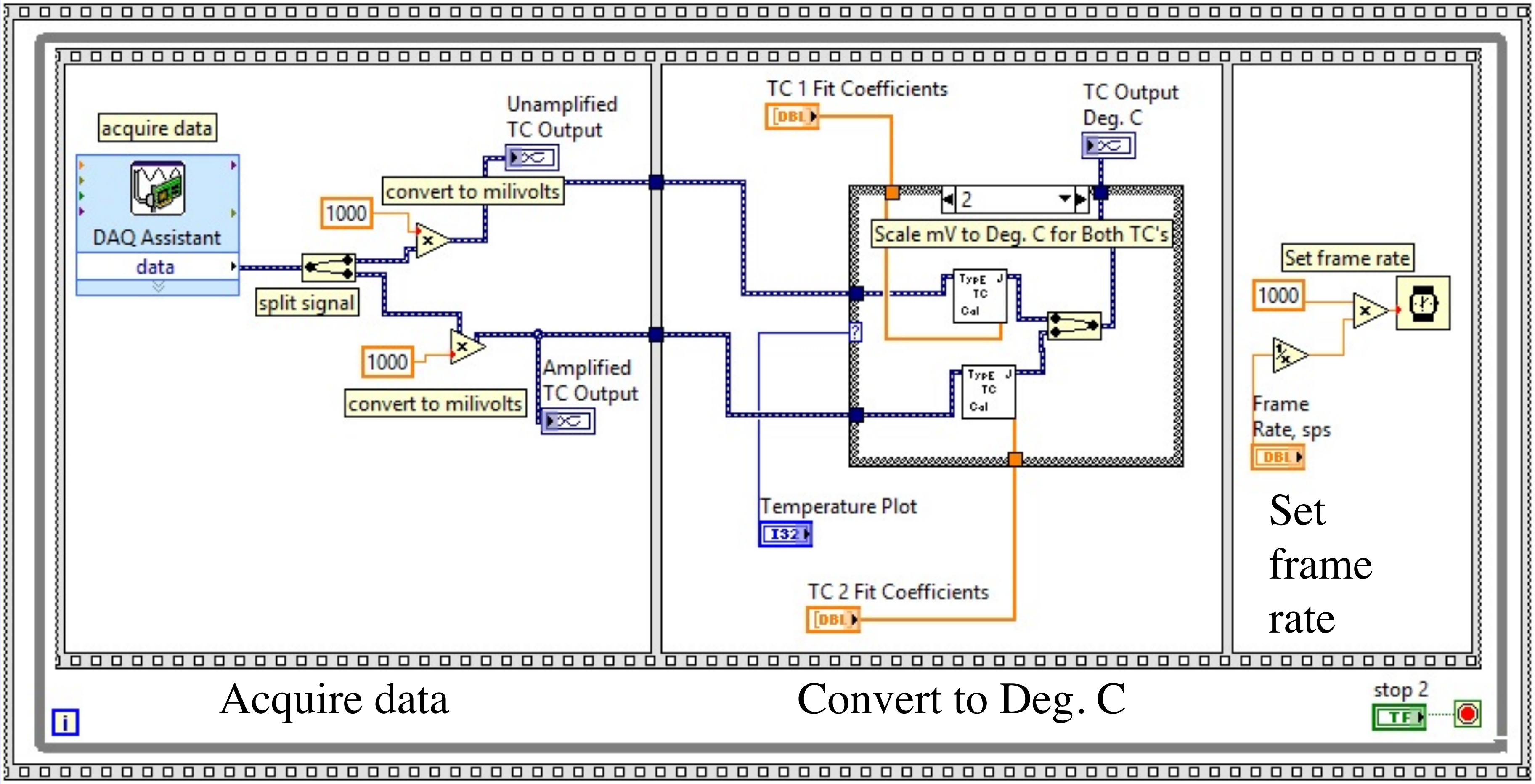
→ Populate the three cases in the middle frame as shown below



… The VI ***"eval_poly_waveform.vi"*** is found on the lab 8 web page

VI Programming Procedure, Part 2 (6)

- The Final Modified Block Diagram should Look something like



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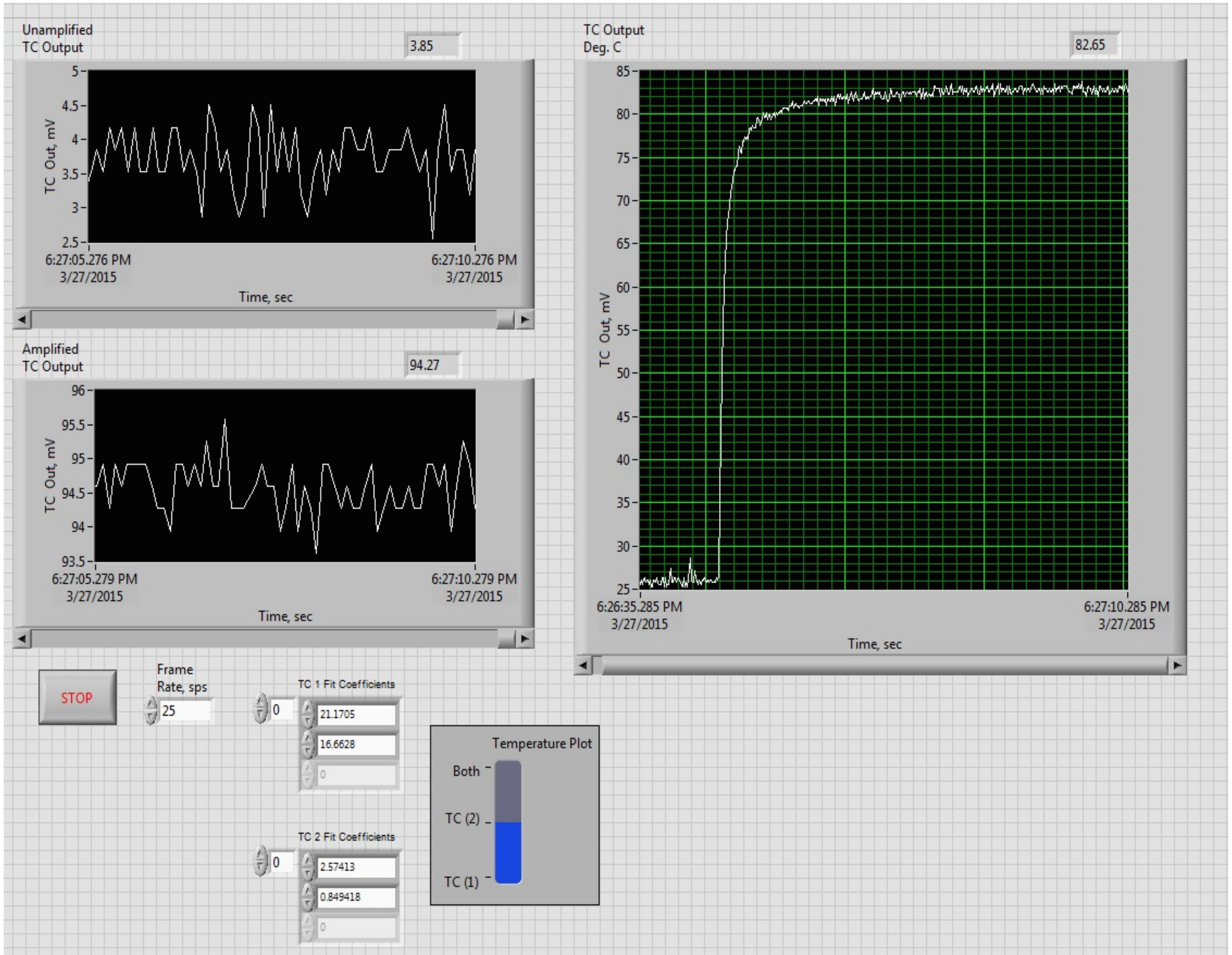
VI Programming Procedure, Part 2 (7)



- Run your VI and test for Functionality
  - Heat both Type "J" TC's with heat gun and compare amplified and unamplified TC temperature outputs
  - Which signal would you rather use?

Lab Procedure Part 2 (1)

• Solder your Amplified TC – tip to a Penny. You might have to clean the penny And sand the surface first to get good contact.

• Once you have a good connection run your VI to check functionality .. Then stop your VI, right click on temperature display graph and select *"Data Operations >> clear chart>> "*.

• Heat water in beaker to just before boil ~ 90-90 ^{o}C. Immediately immerse penny on bottom of beaker and watch the time response.

• Run the VI until the temperature stabilizes, then stop VI and right click on Temperature display … *select "Data Operations >> Make Current Value Default" .. Save your VI*

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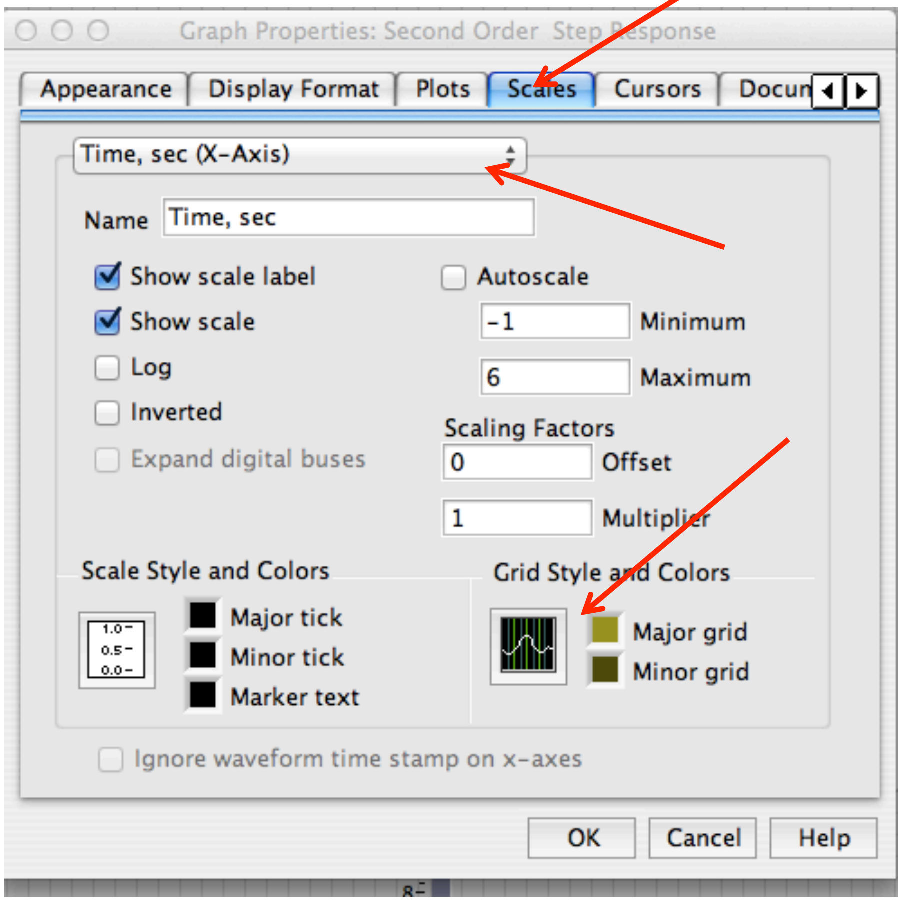
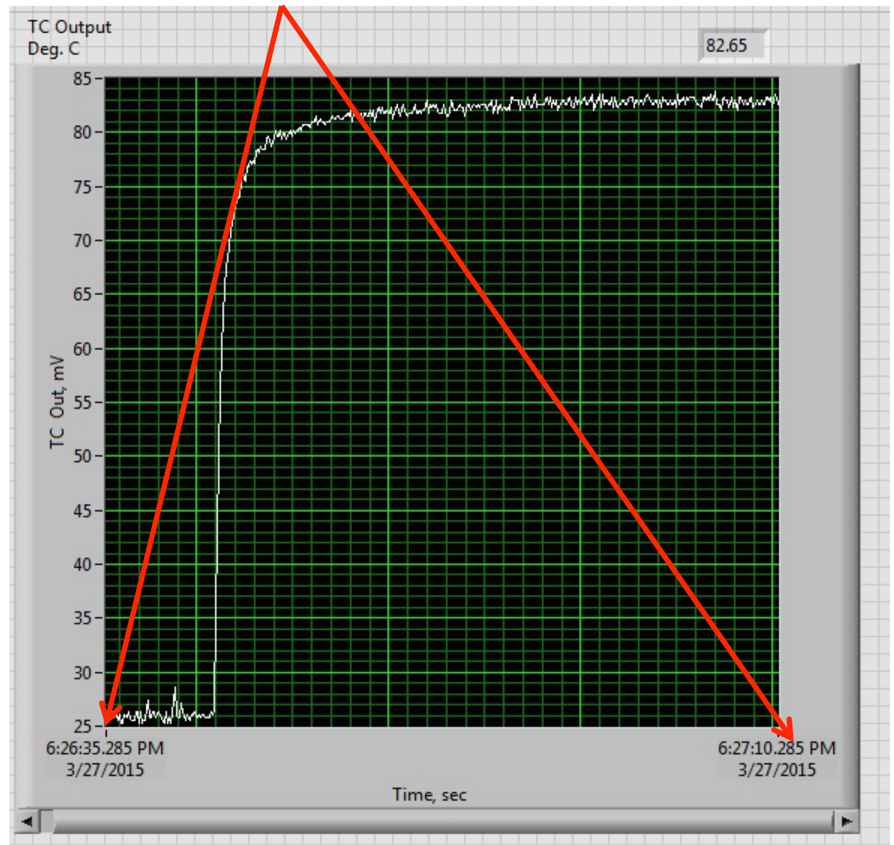
Lab Procedure Part 2 (2)

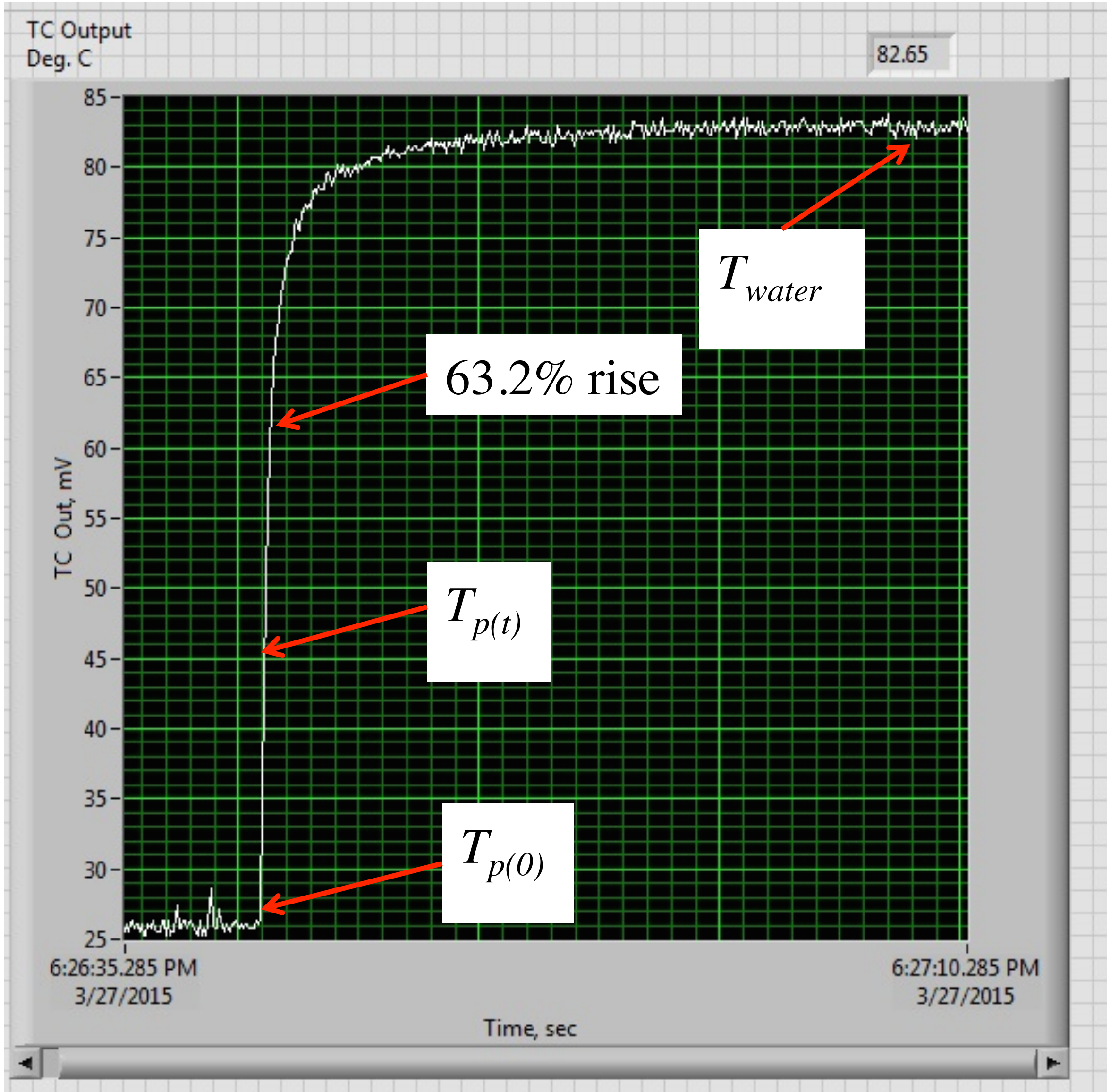
- Right click on your your temperature display graph, and select *"Formatting"*
- *In the pop-up menu select the "Scales" tab .. And then for both x and y scales Select major and minor grids* .. This action will add grids that will help you to read your chart accurately

Manually change the graph limits to display the entire temperature response curve





Lab Procedure Part 2 (3)



- Attach this figure to your lab report

- Read T_o and $T_{final} = T_{water}$ from graph + sufficient intermediate points to plot a log-graph as per slide 56 of Appendix

$$-\ln\left(\frac{T_{water} - T_p(t)}{T_{water} - T_{p(0)}}\right)$$

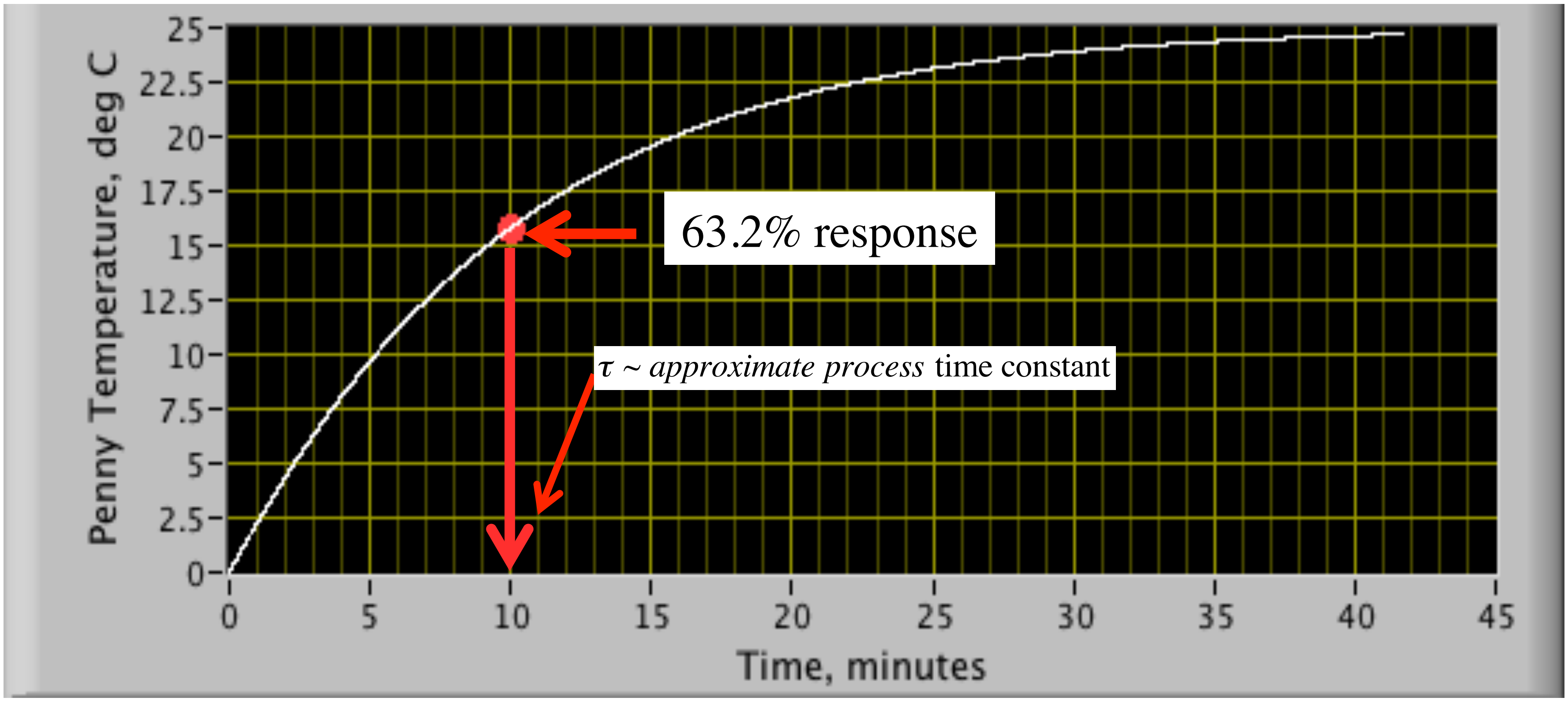
-- Plotted versus time

The slope of this curve is =

$$\frac{1}{\tau} = \frac{h \cdot A_{surf}}{m \cdot c_p}$$

Lab Procedure Part 2 (4)

Verify this result by reading the $1-\frac{1}{e}=63.2\%...$ rise time from your linear time history graph *(as per slide 55 of Appendix)*



Lab Procedure Part 2 (4)

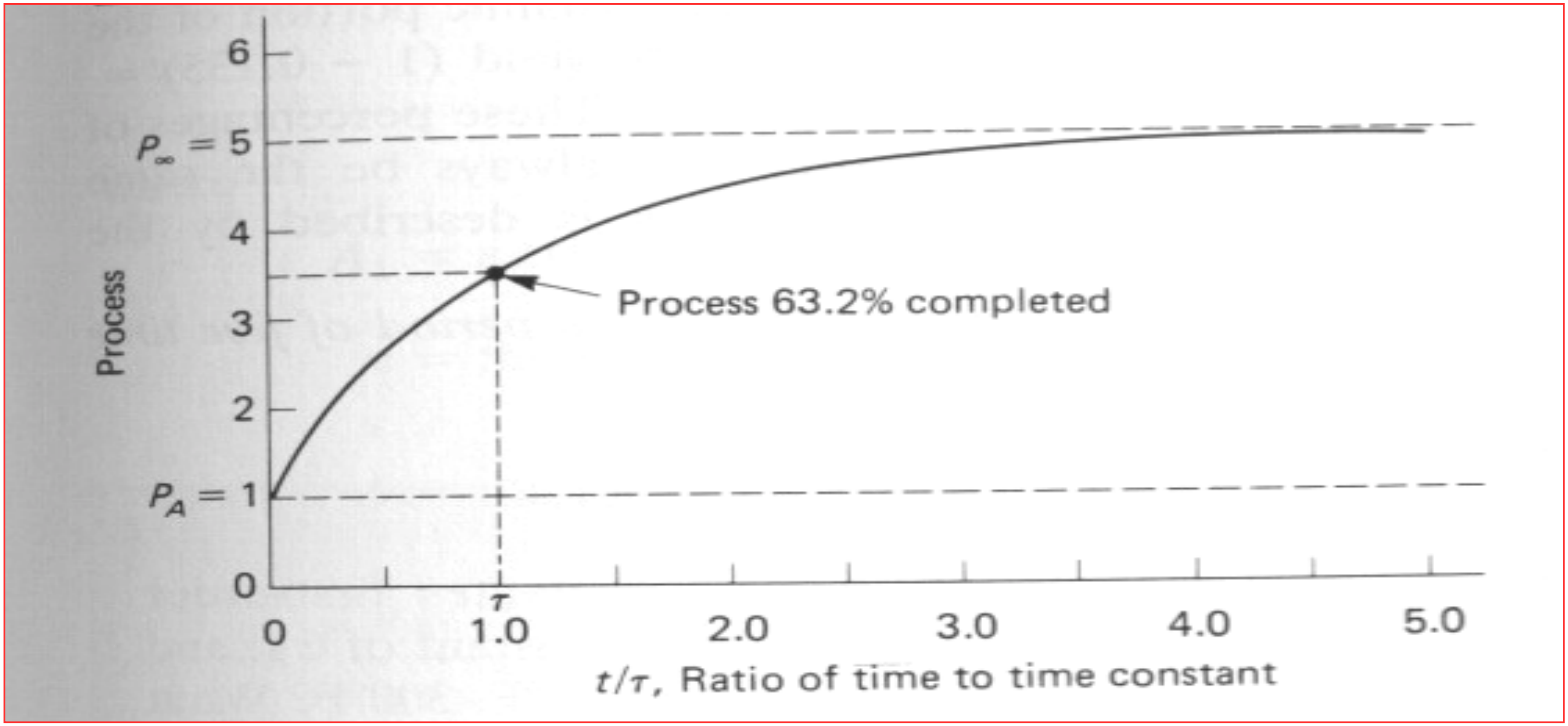
- Using the Previously described material, thermodynamic, and transport parameters for the Penny ..calculate the convective heat transfer coefficient and the Biot Number ***(see appendix I)***

- What can you conclude about the applicability of the "lumped-mass" assumption to this configuration?

$$\tau_c = \frac{m \cdot c_p}{h \cdot A_{surf}} \rightarrow h = \frac{m \cdot c_p}{\tau \cdot A_{surf}}$$

$$Bi = \frac{h \cdot L_c}{k} = \frac{h \cdot V}{k \cdot A_{surf}} \rightarrow Bi = \frac{m \cdot c_p}{\tau \cdot A_{surf}} \cdot \frac{V}{k \cdot A_{surf}} = \frac{m \cdot c_p \cdot V}{\tau \cdot k \cdot A^2{}_{surf}}$$

Appendix I, Heat Transfer Measurements



Lumped Capacitance Method

Temperature Versus Heat (1)

• Often the concepts of heat and temperature are thought to be the same, but they are not.

• Temperature is a number that is related to the average kinetic energy of the molecules of a substance. If temperature is measured in Kelvin degrees, then this number is directly proportional to the average kinetic energy of the molecules.

• Heat is a measurement of the total energy in a substance. That total energy is made up of not only of the *kinetic energies* of the molecules of the substance, but total energy is also made up of the *potential energies* of the molecules.

Temperature Versus Heat (2)

• When heat, (i. e., energy), goes into a substance one of two things can happen:

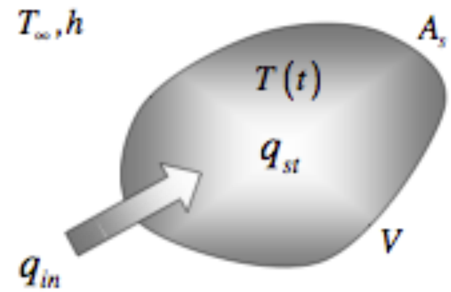
1. The substance can experience a raise in temperature. That is, the heat can be used to speed up the molecules of the substance.

2. The substance can change state. For example, if the substance is ice, it can melt into water. This change does not cause a raise in temperature. The moment before melting the average kinetic energy of the ice molecules is the same as the average kinetic energy of the water molecules a moment after melting. Although heat is absorbed by this change of state, the absorbed energy is not used to speed up the molecules. The energy is used to change the bonding between the molecules.

Transient Heat Conduction
Lumped Capacitance Method

The temperature of the solid is assumed **spatially uniform**

The temperature is a function of time only $T(t)$



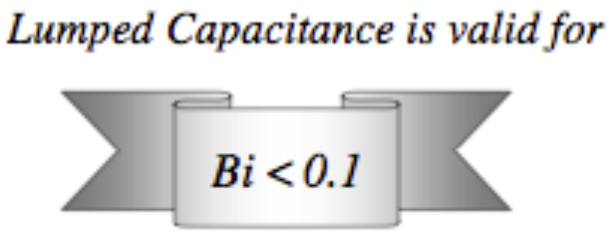
$$q_{in} = q_{st}$$

Energy Balance :

$$\begin{bmatrix} \textit{net rate of heat transfer} \\ \textit{into the solid through} \\ \textit{its boundaries} \end{bmatrix} = \begin{bmatrix} \textit{rate of increase of} \\ \textit{the internal energy} \\ \textit{of the solid} \end{bmatrix}$$

Initial condition:
$$T(0) = T_0$$

Transient Heat Conduction
Lumped Capacitance Method (2)



- Biot number (Bi) -- dimensionless quantity used in heat transfer calculations.
- Named after the French physicist Jean-Baptiste Biot (1774–1862
- Ratio of the heat transfer resistances inside of and at the surface of a body.
- Determines temperatures inside a body will vary significantly in space, as body heats or cools over time from thermal gradient at surface.
- Problems involving small Biot numbers are thermally simple, due to uniform temperature fields inside the body.

Transient Heat Conduction
Lumped Capacitance Method (3)

Lumped Capacitance is valid for

$Bi < 0.1$

$$Bi = \frac{hL_c}{k} = \frac{\text{conduction thermal resistance}}{\text{convection thermal resistance}}$$

Characteristic length

$$L_c = \frac{V}{A_s}$$

$L_c \rightarrow$ *Characteristic Length of Solid*

$V \rightarrow$ *Volume of Solid*

$A_{surf} \rightarrow$ *Surface Area* of *Solid*

$k \rightarrow$ *Thermal Conductivity of Material*

$h \rightarrow$ *Convective Heat Transfer Coefficient*

Transient Heat Conduction
Lumped Capacitance Method (4)

- Consider a room temperature penny immersed into a hot water bath

- The penny is 19.0 mm in diameter and is 1.25 mm thick.
- Material properties .. Penny is 97.5% zinc with copper coating … use zinc material properties.
-

$$c_p = 385 \frac{J}{kg^\circ K} \qquad \rho = 7140 \frac{kg}{m^3} \qquad k = 112 \frac{W}{m-K} \rightarrow$$

$$V = 350.14_{mm^3} \qquad A_s = 641.67_{mm^2} \qquad L_c = 0.5467_{mm} \qquad m = \rho \cdot V = 2.500_g$$

$$L_c = \frac{V}{A_s} = \frac{\frac{\pi \cdot D^2}{4} \cdot t}{2 \cdot \frac{\pi \cdot D^2}{4} + t \cdot \pi \cdot D}$$

Let's evaluate heat transfer coefficient and calculate Biot Number

Lumped Capacitance Method (5)

- Relationship between temperature and heat transfer

…. Heat flows from "cold to hot"

$$m \cdot c_p \frac{\partial T}{dt} = \frac{\partial q}{dt} \rightarrow$$

$$m \rightarrow mass\ of\ object_{\sim kg}$$

$$c_p \rightarrow specific\ heat\ of\ object_{\sim \frac{J}{kg^\circ K}}$$

$$T \rightarrow temperature\ of\ object_{\sim^\circ K}$$

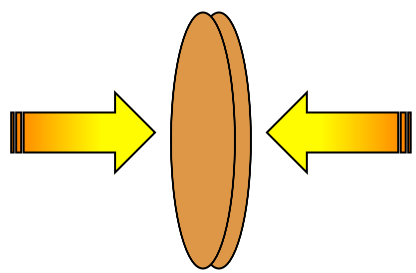
$$\frac{\partial q}{dt} \rightarrow rate\ of\ heat\ transfer_{\sim \frac{J}{\sec}\text{...watts}}$$

$m \cdot c_p$ → "heat capacity" ~ $J/^oK$

Transient Heat Conduction
Lumped Capacitance Method (6)

Let's evaluate heat transfer coefficient and calculate Biot Number

$$\dot{Q}_{in} = h \cdot A_{surf}\left(T_{water} - T_{surf}\right)$$



Equating Rate of Heat Input Versus Rate of Internal Energy Stored by Lumped Capacitance

$$m \cdot c_p \cdot \dot{T}_p = h \cdot A_{surf}\left(T_{surf} - T_{water}\right)$$

Assume Low Biot Number and $T_p = T_{surf}$

T_{surf} = *Surface Temperature of Penny*
T_p = *Mean Internal Temperature of Penney*

Transient Heat Conduction
Lumped Capacitance Method (7)

Solve for T_p

$$\dot{T}_p + \frac{h \cdot A_{surf}}{m \cdot c_p} \cdot T_p = \frac{h \cdot A_{surf}}{m \cdot c_p} \cdot T_{water}$$

Assume .. $\frac{h \cdot A_{surf}}{m \cdot c_p}$ .. Approximately Constant

multiply both sides by $\rightarrow e^{\left(\frac{h \cdot A_{surf}}{m \cdot c_p}\right) \cdot t}$

$$e^{\left(\frac{h \cdot A_{surf}}{m \cdot c_p}\right) \cdot t} \cdot \left(\dot{T}_p + \frac{h \cdot A_{surf}}{m \cdot c_p} \cdot T_p\right) = e^{\left(\frac{h \cdot A_{surf}}{m \cdot c_p}\right) \cdot t} \cdot \frac{h \cdot A_{surf}}{m \cdot c_p} \cdot T_{water}$$

Transient Heat Conduction
Lumped Capacitance Method (8)

$$e^{\left(\frac{h\cdot A_{surf}}{m\cdot c_p}\right)\cdot t}\cdot\left(\dot{T}_p+\frac{h\cdot A_{surf}}{m\cdot c_p}\cdot T_p\right)=\frac{d}{dt}\left[e^{\left(\frac{h\cdot A_{surf}}{m\cdot c_p}\right)\cdot t}\cdot T_p\right]$$

Integrate right hand side $\rightarrow$

$$\int_0^t\frac{d}{dt}\left[e^{\left(\frac{h\cdot A_{surf}}{m\cdot c_p}\right)\cdot t}\cdot T_p\right]\cdot dt=e^{\left(\frac{h\cdot A_{surf}}{m\cdot c_p}\right)\cdot t}\cdot T_{p_{(t)}}-e^{\left(\frac{h\cdot A_{surf}}{m\cdot c_p}\right)\cdot\cdot 0}\cdot T_{p_{(0)}}=$$

$$e^{\left(\frac{h\cdot A_{surf}}{m\cdot c_p}\right)\cdot t}\cdot\left(T_{p_{(t)}}-e^{-\left(\frac{h\cdot A_{surf}}{m\cdot c_p}\right)\cdot t}T_{p_{(0)}}\right)$$

Transient Heat Conduction
Lumped Capacitance Method (9)

Integrate for T_p

Integrate left hand side $\rightarrow$

$$e^{\left(\frac{h\cdot A_{surf}}{m\cdot c_p}\right)\cdot t}\cdot\left[T_{p\,(t)}-e^{-\left(\frac{h\cdot A_{surf}}{m\cdot c_p}\right)\cdot t}\cdot T_{p\,(0)}\right]=\int_0^t e^{\left(\frac{h\cdot A_{surf}}{m\cdot c_p}\right)\cdot\tau}\cdot\frac{h\cdot A_{surf}}{m\cdot c_p}\cdot T_{water}\cdot d\tau=$$

$$\frac{h\cdot A_{surf}}{m\cdot c_p}\cdot T_{water}\cdot\frac{m\cdot c_p}{h\cdot A_{surf}}\left(e^{\left(\frac{h\cdot A_{surf}}{m\cdot c_p}\right)\cdot t}-1\right)=T_{water}\cdot\left(e^{\left(\frac{h\cdot A_{surf}}{m\cdot c_p}\right)\cdot t}-1\right)$$

$$\textit{Divide by...}\ e^{\left(\frac{h\cdot A_{surf}}{m\cdot c_p}\right)\cdot t}\rightarrow\left[T_{p\,(t)}-e^{-\left(\frac{h\cdot A_{surf}}{m\cdot c_p}\right)\cdot t}\cdot T_{p\,(0)}\right]=$$

$$e^{-\left(\frac{h\cdot A_{surf}}{m\cdot c_p}\right)\cdot t}T_{water}\cdot\left(e^{\left(\frac{h\cdot A_{surf}}{m\cdot c_p}\right)\cdot t}-1\right)=T_{water}\cdot\left(1-e^{-\left(\frac{h\cdot A_{surf}}{m\cdot c_p}\right)\cdot t}\right)$$

Transient Heat Conduction

Lumped Capacitance Method (10)

Integrate for T_p

$$\text{Solve for} \rightarrow T_p(t)$$

$$T_{p(t)} = e^{-\left(\frac{h \cdot A_{surf}}{m \cdot c_p}\right) \cdot t} \cdot T_{p(0)} + T_{water} \cdot \left(1 - e^{-\left(\frac{h \cdot A_{surf}}{m \cdot c_p}\right) \cdot t}\right) =$$

$$e^{-\left(\frac{h \cdot A_{surf}}{m \cdot c_p}\right) \cdot t} \cdot \left(T_{p(0)} - T_{water}\right) + T_{water}$$

T_{water} = final temperature of penny

Transient Heat Conduction
Lumped Capacitance Method (11)

Simplify and normalize

$$T_p(t) = e^{-\left(\frac{h \cdot A_{surf}}{m \cdot c_p}\right) \cdot t} \cdot \left(T_{p(0)} - T_{water}\right) + T_{water} \rightarrow$$

$$T_p(t) - T_{p(0)} = e^{-\left(\frac{h \cdot A_{surf}}{m \cdot c_p}\right) \cdot t} \cdot \left(T_{p(0)} - T_{water}\right) + T_{water} - T_{p(0)} =$$

$$\left(T_{water} - T_{p(0)}\right) \cdot \left(1 - e^{-\left(\frac{h \cdot A_{surf}}{m \cdot c_p}\right) \cdot t}\right) \rightarrow \boxed{\frac{T_p(t) - T_{p(0)}}{T_{water} - T_{p(0)}} = 1 - e^{-\left(\frac{h \cdot A_{surf}}{m \cdot c_p}\right) \cdot t}}$$

$$\rightarrow Let\ldots \tau_c = \frac{m \cdot c_p}{h \cdot A_{surf}} \rightarrow \frac{T_p(t) - T_{p(0)}}{T_{water} - T_{p(0)}} = 1 - e^{-t/\tau_c} \rightarrow \tau_c \equiv \text{"time constant"}$$

$$\boxed{when.... \ \tau \equiv t \rightarrow \frac{T_p(t) - T_{p(0)}}{T_{water} - T_{p(0)}} = 1 - e^{-1} = 1 - \frac{1}{e} \approx 0.63.2\%}$$

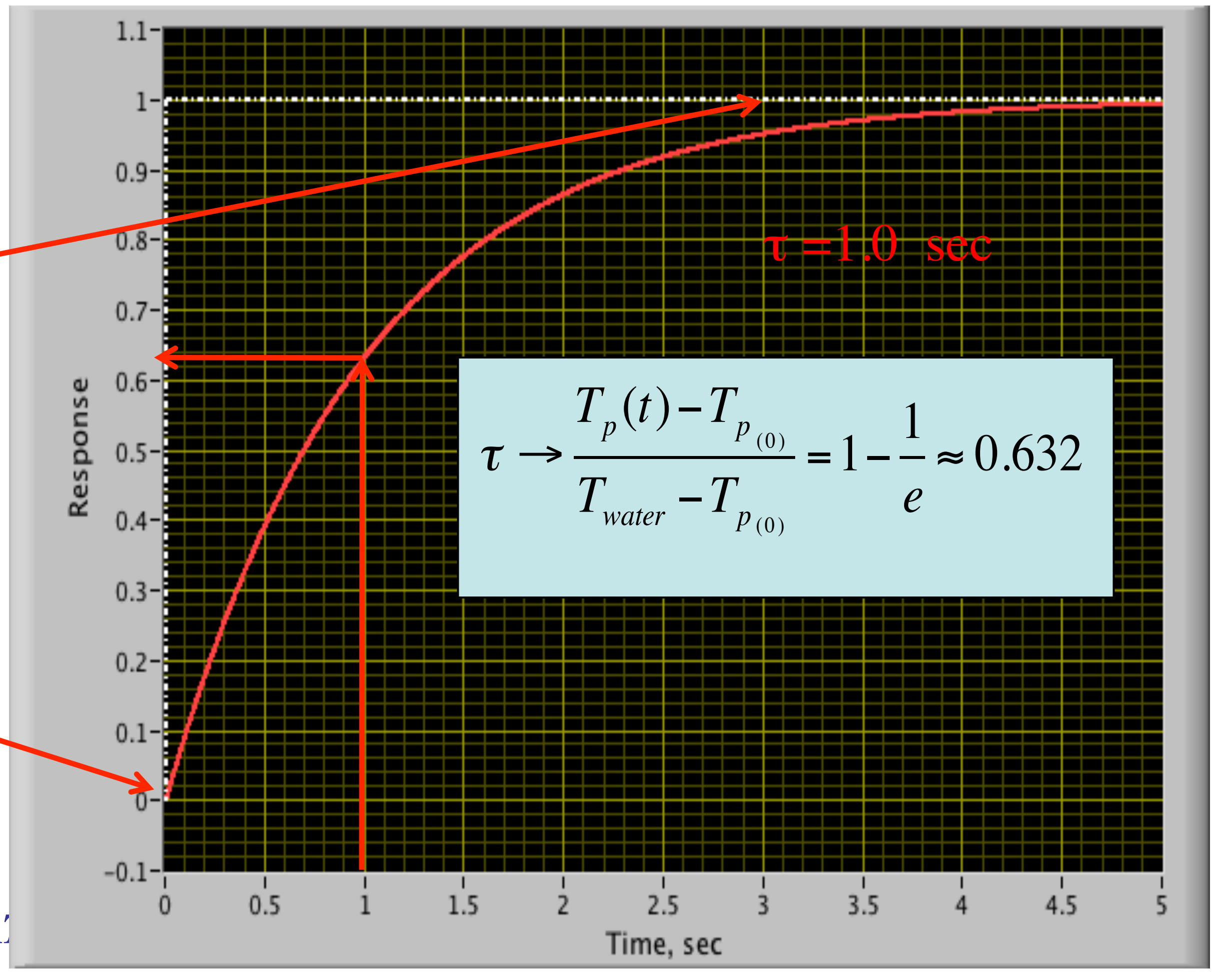
Transient Heat Conduction
Lumped Capacitance Method (12)

$$\frac{T_p(t)-T_{p_{(0)}}}{T_{water}-T_{p_{(0)}}}=1-e^{-\left(\frac{h\cdot A_{surf}}{m\cdot c_p}\right)\cdot t}=1-e^{-t/\tau_c}\rightarrow \tau_c\equiv\text{"time constant"}$$

$$\tau\rightarrow\frac{T_p(t)-T_{p_{(0)}}}{T_{water}-T_{p_{(0)}}}=1-\frac{1}{e}\approx 0.63.2\%$$

$$T_p(t)=T_{water}$$

$$T_p(t)=T_p(0)$$



Transient Heat Conduction
Lumped Capacitance Method (13)

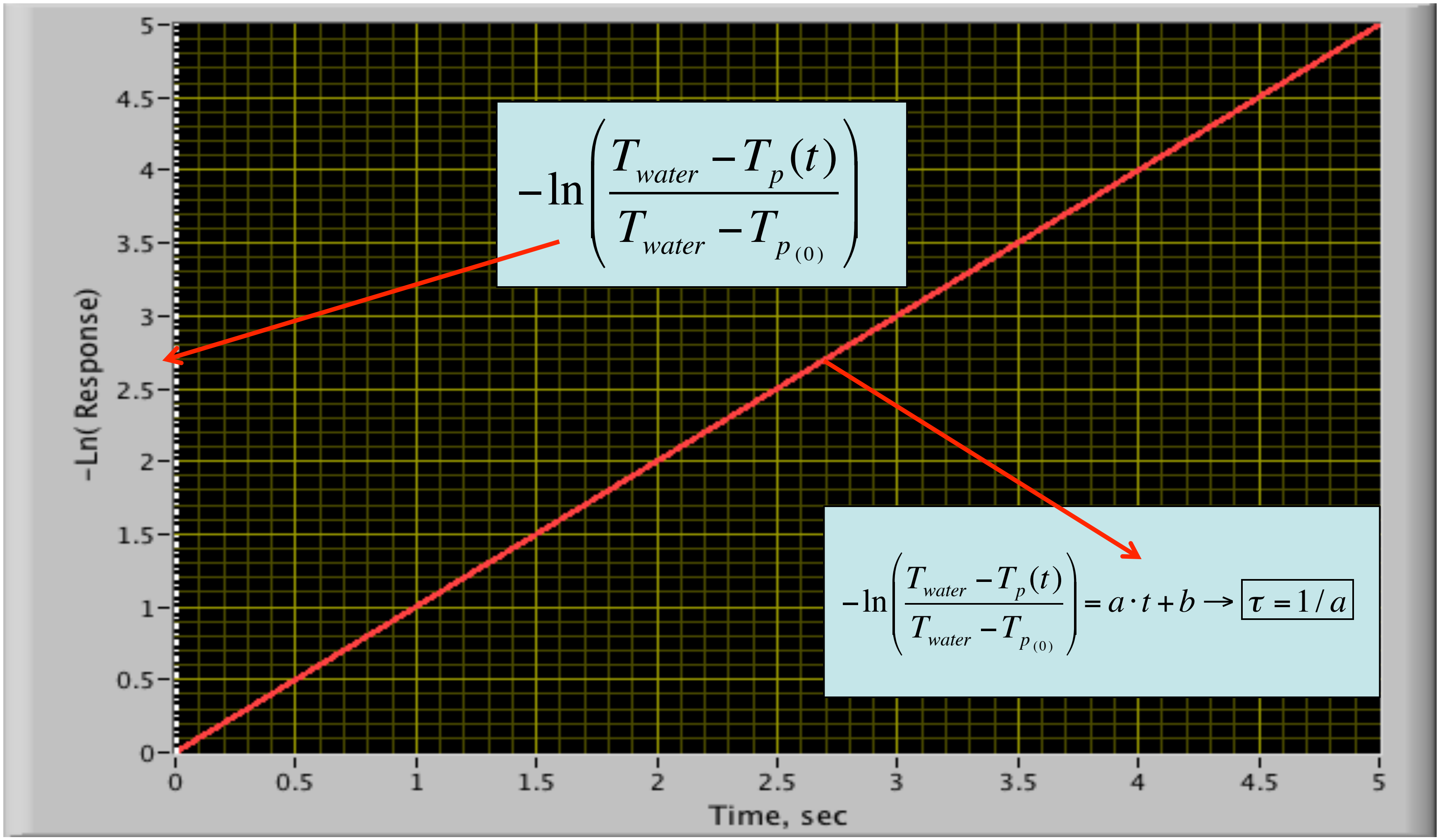
$$\frac{T_p(t)-T_{p_{(0)}}}{T_{water}-T_{p_{(0)}}}=1-e^{-t/\tau_c}\rightarrow solve\ for\ e^{-t/\tau_c}$$

$$\rightarrow e^{-t/\tau_c}=1-\frac{T_p(t)-T_{p_{(0)}}}{T_{water}-T_{p_{(0)}}}-1=\frac{\left(T_{water}-T_{p_{(0)}}\right)-\left(T_p(t)-T_{p_{(0)}}\right)}{T_{water}-T_{p_{(0)}}}=\frac{T_{water}-T_p(t)}{T_{water}-T_{p_{(0)}}}\rightarrow$$

$$take\ natural\ log\rightarrow \boxed{t/\tau_c=-\ln\left(\frac{T_{water}-T_p(t)}{T_{water}-T_{p_{(0)}}}\right)}$$

$$\tau_c=1/slope..on\log\ plot$$

Transient Heat Conduction
Lumped Capacitance Method (14)



Appendix II Thermocouple Types

Thermocouple Types

Thermocouple Type	Useful/General Application Range	Notes
B	1370-1700°C (2500-3100°F)	Easily contaminated, require protection.
C*	1650-2315°C (3000-4200°F)	No oxidation resistance. Vacuum, hydrogen or inert atmospheres.
E**	95-900°C (200-1650°F)	Highest output of base metal thermocouples. Not subject to corrosion at cryogenic temperatures.
J	95-760°C (200-1400°F)	Reducing atmosphere recommended. Iron leg subject to oxidation at elevated temperatures--use larger gauge to compensate.
K**	95-1260°C (200-2300°F)	Well suited for oxidizing atmospheres.
N	650-1260°C (1200-2300°F)	For general use, better resistance to [illegible] Type K.
R	870-1450°C (1600-2640°F)	Oxidizing atmosphere recommended. Easily contaminated, require protection.
S	980-1450°C (1800-2640°F)	Laboratory standard, highly reproducible. Easily contaminated, require protection.
T**	-200-350°C (-330-660°F)	Most stable at cryogenic temperatures ranges. Excellent in oxidizing and reducing atmospheres within temperature range.

What we are using in this lab

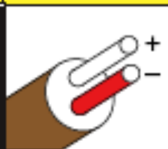
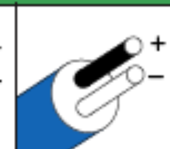
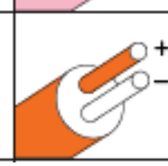
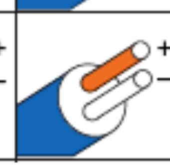
UtahState UNIVERSITY

Mechanical & Aerospace Engineering

Thermocouple Types (2)

- *Color Coded Connectors*

Connectors | Connectors

ANSI Code	ANSI MC 96.1 Color Coding: Thermocouple Grade	ANSI MC 96.1 Color Coding: Extension Grade	Alloy Combination: + Lead	Alloy Combination: – Lead	Comments Environment Bare Wire	Maximum T/C Grade Temp. Range	EMF (mV) Over Max. Temp. Range	IEC 584-3 Color Coding: Thermocouple Grade	IEC 584-3 Color Coding: Intrinsically Safe	IEC Code
J			IRON Fe (magnetic)	CONSTANTAN COPPER-NICKEL Cu-Ni	Reducing, Vacuum, Inert. Limited Use in Oxidizing at High Temperatures. Not Recommended for Low Temperatures.	–210 to 1200°C –346 to 2193°F	–8.095 to 69.553			J
K			CHROMEGA® NICKEL-CHROMIUM Ni-Cr	ALOMEGA® NICKEL-ALUMINUM Ni-Al (magnetic)	Clean Oxidizing and Inert. Limited Use in Vacuum or Reducing. Wide Temperature Range, Most Popular Calibration	–270 to 1372°C –454 to 2501°F	–6.458 to 54.886			K
T			COPPER Cu	CONSTANTAN COPPER-NICKEL Cu-Ni	Mild Oxidizing, Reducing Vacuum or Inert. Good Where Moisture Is Present. Low Temperature & Cryogenic Applications	–270 to 400°C –454 to 752°F	–6.258 to 20.872			T
E			CHROMEGA® NICKEL-CHROMIUM Ni-Cr	CONSTANTAN COPPER-NICKEL Cu-Ni	Oxidizing or Inert. Limited Use in Vacuum or Reducing. Highest EMF Change Per Degree	–270 to 1000°C –454 to 1832°F	–9.835 to 76.373			E
N			OMEGA-P® NICROSIL Ni-Cr-Si	OMEGA-N® NISIL Ni-Si-Mg	Alternative to Type K. More Stable at High Temps	–270 to 1300°C –450 to 2372°F	–4.345 to 47.513			N
R	NONE ESTABLISHED		PLATINUM-13% RHODIUM Pt-13% Rh	PLATINUM Pt	Oxidizing or Inert. Do Not Insert in Metal Tubes. Beware of Contamination. High Temperature	–50 to 1768°C –58 to 3214°F	–0.226 to 21.101			R

• *Color Codes Connectors*

Thermocouple Types (3)

Connectors | Connectors

ANSI Code	ANSI MC 96.1 Color Coding: Thermocouple Grade	ANSI MC 96.1 Color Coding: Extension Grade	Alloy Combination: + Lead	Alloy Combination: – Lead	Comments Environment Bare Wire	Maximum T/C Grade Temp. Range	EMF (mV) Over Max. Temp. Range	IEC 584-3 Color Coding: Thermocouple Grade	IEC 584-3 Color Coding: Intrinsically Safe	IEC Code
S	NONE ESTABLISHED		PLATINUM-10% RHODIUM Pt-10% Rh	PLATINUM Pt	Oxidizing or Inert. Do Not Insert in Metal Tubes. Beware of Contamination. High Temperature	–50 to 1768°C –58 to 3214°F	–0.236 to 18.693			S
U	NONE ESTABLISHED		COPPER Cu	COPPER-LOW NICKEL Cu-Ni	Extension Grade Connecting Wire for R & S Thermocouples, Also Known as RX & SX Extension Wire.					U
B	NONE ESTABLISHED		PLATINUM-30% RHODIUM Pt-30% Rh	PLATINUM-6% RHODIUM Pt-6% Rh	Oxidizing or Inert. Do Not Insert in Metal Tubes. Beware of Contamination. High Temp. Common Use in Glass Industry	0 to 1820°C 32 to 3308°F	0 to 13.820			B
G* (W)	NONE ESTABLISHED		TUNGSTEN W	TUNGSTEN-26% RHENIUM W-26% Re	Vacuum, Inert, Hydrogen. Beware of Embrittlement. Not Practical Below 399°C (750°F). Not for Oxidizing Atmosphere	0 to 2320°C 32 to 4208°F	0 to 38.564	NO STANDARD USE ANSI COLOR CODE		G (W)
C* (W5)	NONE ESTABLISHED		TUNGSTEN-5% RHENIUM W-5% Re	TUNGSTEN-26% RHENIUM W-26% Re	Vacuum, Inert, Hydrogen. Beware of Embrittlement. Not Practical Below 399°C (750°F) Not for Oxidizing Atmosphere	0 to 2320°C 32 to 4208°F	0 to 37.066	NO STANDARD USE ANSI COLOR CODE		C (W5)
D* (W3)	NONE ESTABLISHED		TUNGSTEN-3% RHENIUM W-3% Re	TUNGSTEN-25% RHENIUM W-25% Re	Vacuum, Inert, Hydrogen. Beware of Embrittlement. Not Practical Below 399°C (750°F)–Not for Oxidizing Atmosphere	0 to 2320°C 32 to 4208°F	0 to 39.506	NO STANDARD USE ANSI COLOR CODE		D (W3)

* *Not official symbol or standard designation*

† *JIS color code also available.*

UtahState UNIVERSITY

Mechanical & Aerospace Engineering

Thermocouple Tolerances

(Reference Junction at 0°C)

- No sampling and reference sensor error

American Limits of Error ASTM E230-ANSI MC 96.1

ANSI Code		Standard Limits†		Special Limits†	
J	Temp Range	>0 to 750°C	>32 to 1382°F	0 to 750°C	32 to 1382°F
	Tolerance Value	2.2°C or 0.75%	4.0°F or 0.75%	1.1°C or 0.4%	2.0°F or 0.4%
K	Temp Range	>0 to 1250°C	>32 to 2282°F	0 to 1250°C	32 to 2282°F
	Tolerance Value	2.2°C or 0.75%	4.0°F or 0.75%	1.1°C or 0.4%	2.0°F or 0.4%
	Temp. Range*	-200 to 0°C	-328 to 32°F		
	Tolerance Value	2.2°C or 2.0%	4.0°F or 2.0%		
T	Temp Range	>0 to 350°C	>32 to 662°F	0 to 350°C	32 to 662°F
	Tolerance Value	1.0°C or 0.75%	1.8°F or 0.75%	0.5°C or 0.4%	1°F or 0.4%
	Temp. Range*	-200 to 0°C	-328 to 32°F		
	Tolerance Value	1.0°C or 1.5%	1.8°F or 1.5%		
E	Temp Range	>0 to 900°C	>32 to 1652	0 to 900°C	32 to 1652°F
	Tolerance Value	1.7°C or 0.5%	3°F or 0.5%	1.0°C or 0.4%	1.8°F or 0.4%
	Temp. Range*	-200 to 0°C	-328 to 32°F		
	Tolerance Value	1.7°C or 1.0%	3°F or 1.0%		
N	Temp Range	>0 to 1300°C	>32 to 2372°F	0 to 1300°C	32 to 2372°F
	Tolerance Value	2.2°C or 0.75%	4.0°F or 0.75%	1.1°C or 0.4%	2.0°F or 0.4%
	Temp. Range*	-270 to 0°C	-454 to 32°F		
	Tolerance Value	2.2°C or 2.0%	4.0°F or 2.0%		
R S	Temp Range	0 to 1450°C	32 to 2642°F	0 to 1450°C	32 to 2642°F
	Tolerance Value	1.5°C or 0.25%	2.7°F or 0.25%	0.6°C or 0.1%	1°F or 0.1%
B	Temp Range	800 to 1700°C	1472 to 3092°F	Not Established	
	Tolerance Value	0.5%	0.9°F		
G*C*D*	Temp Range	0 to 2320°C	32 to 4208°F	Not Established	
	Tolerance Value	4.5°C or 1.0%	0.9°F		

** Not official symbol or standard designation* *† Whichever value is greater.*

Note: *Material is normally selected to meet tolerances above 0°C. If thermocouples are needed to meet tolerances below 0°C, the purchaser shall state this as selection of material is usually required.*

Thermocouple Accuracies (2)

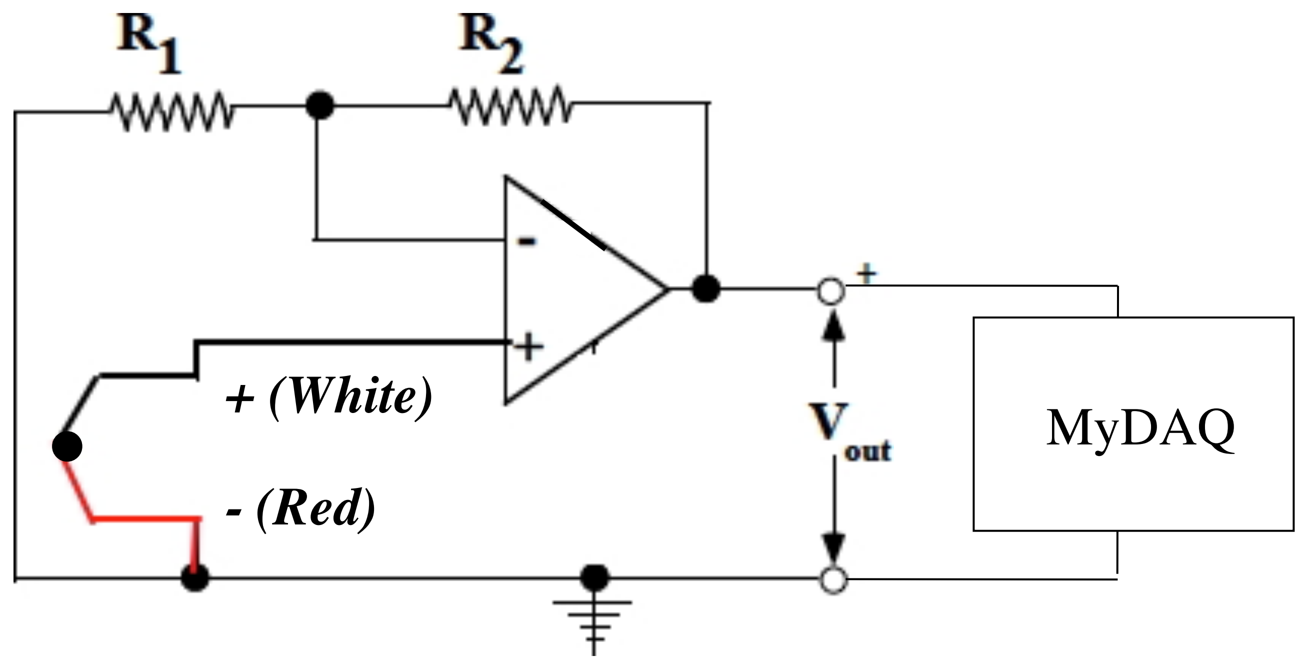
• **No sampling and reference sensor error**

IEC Tolerance Class EN 60584-2; JIS C 1602

IEC Code		Class 1	Class 2	Class 3†
J	Temp Range Tolerance Value Temp. Range Tolerance Value	-40 to 375°C ±1.5°C 375 to 750°C ±0.4% Reading	-40 to 333°C ±2.5°C 333 to 750°C ±0.75% Reading	Not Established
K N	Temp Range Tolerance Value Temp. Range Tolerance Value	-40 to 375°C ±1.5°C 375 to 1000°C ±0.4%	-40 to 333°C ±2.5°C 333 to 1200°C ±0.75% Reading	-167 to 40°C ±2.5°C -200 to -167°C ±1.5% Reading
T	Temp Range Tolerance Value Temp. Range Tolerance Value	-40 to 125°C ±0.5°C 125 to 350°C ±0.4% Reading	-40 to 133°C ±1°C 133 to 350°C ±0.75% Reading	-67 to 40°C ±1°C -200 to -67°C ±1.5% Reading
E	Temp Range Tolerance Value Temp. Range Tolerance Value	-40 to 375°C ±1.5°C 375 to 800°C ±0.4% Reading	-40 to 333°C ±2.5°C 333 to 900°C ±0.75% Reading	-167 to 40°C ±2.5°C -200 to -167°C ±1.5% Reading
R S	Temp Range Tolerance Value Temp. Range Tolerance Value	0 to 1100°C ±1°C 1100 to 1600°C ±[1 + 0.3% x (Rdg-1100)]°C	0 to 600°C ±1.5°C 600 to 1600°C ±0.25% Reading	Not Established
B	Temp Range Tolerance Value Temp. Range Tolerance Value	Not Established	600 to 1700°C ±0.25% Reading	600 to 800°C +4°C 800 to 1700°C ±0.5% Reading

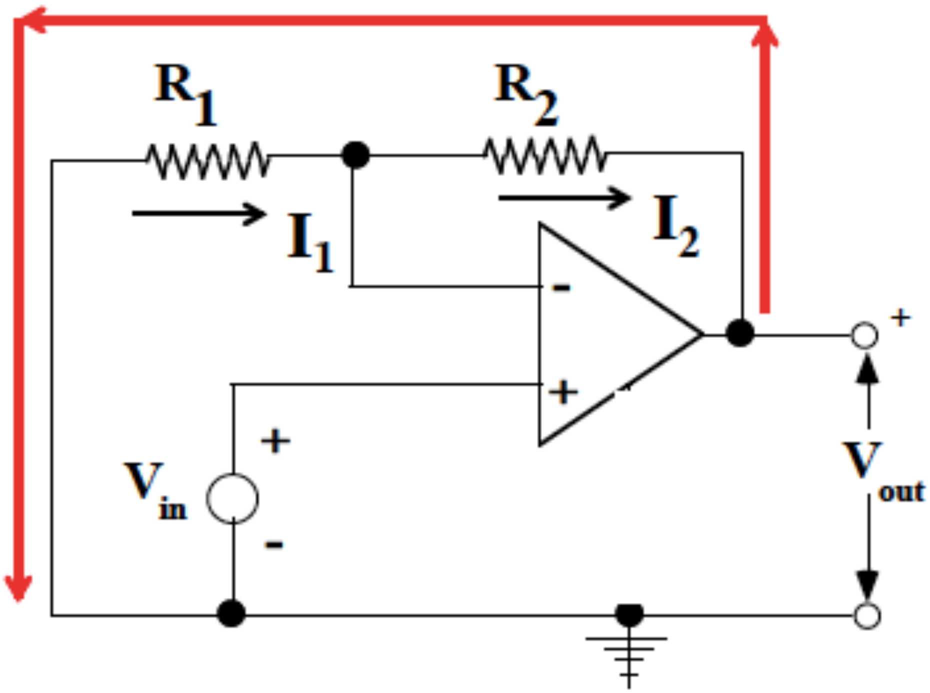
† Material is normally selected to meet tolerances above -40°C. If thermocouples are needed to meet limits of Class 3, as well as those of Class 1 or 2, the purchaser shall state this, as selection of material is usually required.

Appendix III, TC Amplifier Circuit Analysis



Appendix III, TC Amplifier Circuit Analysis

- **Now Look at the Op Amp Configuration**



- **Calculating the Voltage Drop Around the Loop**

$$V_{out} = I_2 R_2 + I_1 R_1$$

- **Ideal Op Amp draws no Current (infinite impedance)**

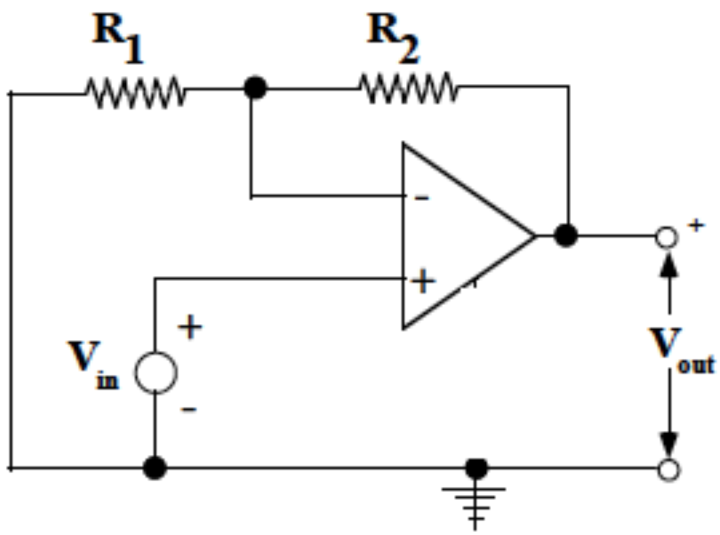
$$\rightarrow I_2 = I_1$$

- **Also Since Ideal Op Amp has Infinite Gain**

$$V_{out} = A \cdot \left\{ V_{in} - \left[V_{out} - I_2 R_2 \right] \right\} \rightarrow$$

$$A = \infty \rightarrow \boxed{V_{out} = V_{in} + I_2 R_2}$$

Appendix III, TC Amplifier Circuit Analysis



- **Collecting terms**

$$V_{out} = I_2 R_2 + I_1 R_1 \rightarrow I_2 = I_1 \rightarrow V_{out} = I_2 \left[R_2 + R_1 \right]$$

$$V_{in} = V_{out} - I_2 R_2 = I_2 \left[R_2 + R_1 \right] - I_2 R_2 = I_2 R_1$$

$$\boxed{\frac{V_{out}}{V_{in}} = \frac{I_2 \left[R_2 + R_1 \right]}{I_2 R_1} = 1 + \frac{R_2}{R_1}}$$

Amplified Output!