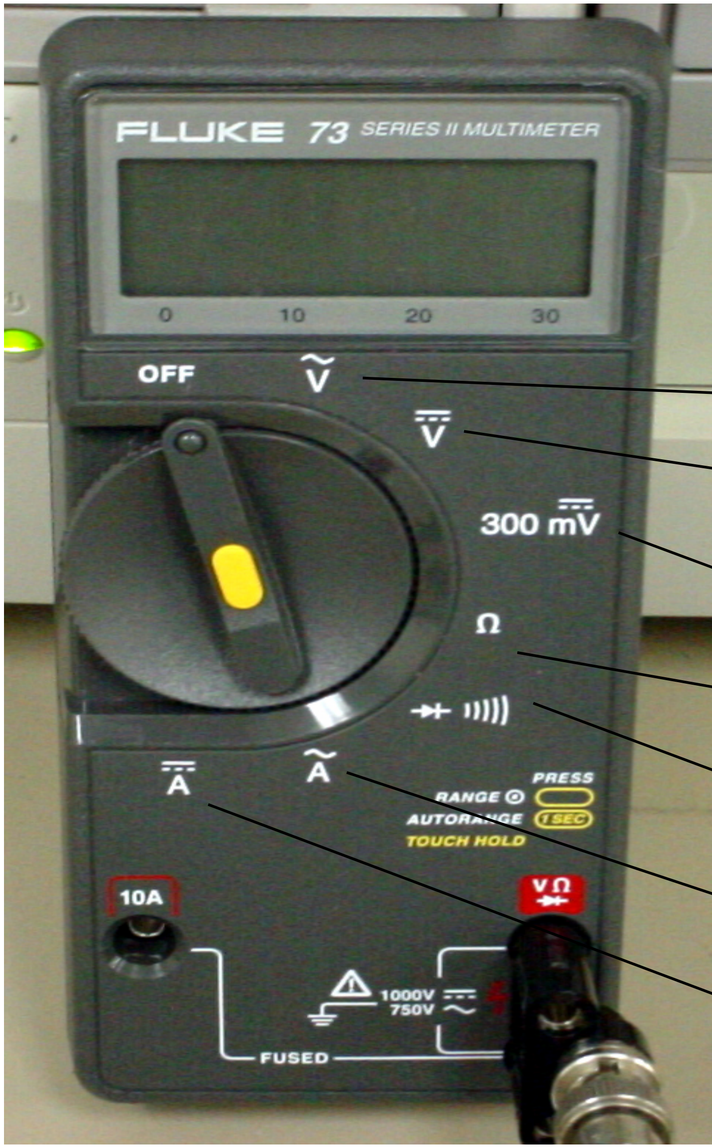
Homework 3 :



3.17. In order to determine the power dissipated across a resistor, the current flow and resistance values are measured separately. If $I = 3.2$ A and $R = 1000\ \Omega$ are measured values, determine the uncertainty if the following instruments are used:

Instrument	Resolution	Uncertainty (% of reading)
Voltmeter	1.0 mV	0.5%
Ohmmeter	1.0 Ω	0.1%
Ammeter	0.1 A	0.5%

$Assume..V_{nom} = I \cdot R = 3200V$

AC voltage
DC voltage
DC milivolts
Resistance
Diode Test
AC current
DC current

$$P = I^2 \quad R$$

Ammeter
Ohmmeter

$$P = I \quad V$$

Ammeter
Voltmeter

- ***Which method is more Accurate?***
- ***What is largest contributing Error source?***

Hints:

$$P = I^2 \quad R \quad \left| \begin{array}{l} Ammeter \\ Ohmmeter \end{array} \right.$$

$$Assume..V_{nom} = I \cdot R = 3200V$$

$$P = I \quad V \quad \left| \begin{array}{l} Ammeter \\ Voltmeter \end{array} \right.$$

Use Error Propagation Formula:

$$z = f(x, y, u, v)$$

$$U_z^2 \cong U_x^2 \left(\frac{\partial f}{\partial x} \right)^2 + U_y^2 \left(\frac{\partial f}{\partial y} \right)^2 + U_u^2 \left(\frac{\partial f}{\partial u} \right)^2 + U_v^2 \left(\frac{\partial f}{\partial v} \right)^2 + \cdots$$

$$\rightarrow \left| \begin{array}{l} U_x^2 = \left(U_{x\,-resolution} \right)^2 + \left(U_{x\,-uncertainty} \right)^2 \\ U_x^2 = \left(U_{y\,-resolution} \right)^2 + \left(U_{y\,-uncertainty} \right)^2 \\ \ldots \end{array} \right.$$

HW 3.17 (2)

- ***Ammeter, Ohmmeter Method First***

$$P = I^2 \cdot R \rightarrow \left| \begin{array}{c} Ammeter \\ Ohmmeter \end{array} \right.$$

$$\rightarrow U_P = \sqrt{\left(\frac{\partial P}{\partial I}\right)^2 \cdot U_I^{\ 2} + \left(\frac{\partial P}{\partial R}\right)^2 \cdot U_R^{\ 2}} \rightarrow \left. \begin{array}{c} \frac{\partial P}{\partial I} = 2 \cdot I \cdot R \\ \frac{\partial P}{\partial R} = I^2 \end{array} \right| \rightarrow$$

$$\frac{U_P}{P_{nom}} = \sqrt{\frac{(2 \cdot I \cdot R)^2 \cdot U_I^{\ 2} + \left(I^2\right)^2 \cdot U_R^{\ 2}}{\left(I^2 \cdot R\right)^2}} = \sqrt{4 \cdot \left(\frac{U_I}{I}\right)^2 + \left(\frac{U_R}{R}\right)^2}$$

HW 3.17 (3)

- ***Ammeter, Ohmmeter Method***

$$\frac{U_P}{P_{nom}} = \sqrt{4 \cdot \left(\frac{U_I}{I}\right)^2 + \left(\frac{U_R}{R}\right)^2}$$

Ammeter Uncertainty and *Resolution* are Independent Error Sources .. *RSS*

- Current Measurement Error …

$$\left(\frac{U_I}{I_{nom}}\right) = \sqrt{\left(\frac{\delta I}{I}_{Uncertainty}\right)^2 + \left(\frac{\delta I}{I}_{Resolution}\right)^2} =$$

$$\frac{\delta I}{I}_{Uncertainty} = \frac{0.5\%}{100} = .005$$

$$\frac{\delta I}{I}_{Resolution} = \frac{0.1_{Amp}}{3.2_{Amp}} = .03125$$

$$\left(\frac{UI}{I}_{nom}\right) = \sqrt{\left(\frac{0.5\%}{100}\right)^2 + \left(\frac{0.1_{Amp}}{3.2_{Amp}}\right)^2} = 0.03125$$

HW 3.17 (4)

- ***Ammeter, Ohmmeter Method***

$$\frac{U_P}{P_{nom}} = \sqrt{4 \cdot \left(\frac{U_I}{I}\right)^2 + \left(\frac{U_R}{R}\right)^2}$$

Ohmmeter Uncertainty and *Resolution* are Independent Error Sources .. *RSS*

- Resistance Measurement Error …

$$\left(\frac{U_R}{R}\right) = \sqrt{\left(\frac{\delta R}{R}_{Uncertainty}\right)^2 + \left(\frac{\delta R}{R}_{Resolution}\right)^2} =$$

$$\frac{\delta R}{R}_{Uncertainty} = \frac{0.1\%}{100} = .001$$

$$\frac{\delta R}{R}_{Resolution} = \frac{1.0_\Omega}{1000_\Omega} = .001$$

$$\left(\frac{U_R}{R}\right) = \sqrt{\left(\frac{0.1\%}{100}\right)^2 + \left(\frac{1.0_\Omega}{1000_\Omega}\right)^2} = 0.001414$$

HW 3.17 (5)

- ***Ammeter, Ohmmeter Method***

$$\frac{U_P}{P_{nom}} = \sqrt{4 \cdot \left(\frac{U_I}{I}\right)^2 + \left(\frac{U_R}{R}\right)^2}$$

$$\frac{I}{I}^{2} = \frac{0.5\%}{100}^{2} + \frac{0.1_{Amp}}{3.2_{Amp}}^{2} = 0.0010$$

$$\frac{R}{R}^{2} = \frac{0.1\%}{100}^{2} + \frac{1.0}{1000}^{2} = 2 \;\; 10^{6}$$

$$(4 \;\; 0.001 + 2 \cdot \;\; 10^{6})^{0.5} = 0.0633 \; (6.33\%)$$

HW 3.17 (6)

- ***Ammeter, Ohmmeter Method***

$$\frac{U_P}{P_{nom}} = \sqrt{4 \cdot \left(\frac{U_I}{I}\right)^2 + \left(\frac{U_R}{R}\right)^2} = \left(4\ 0.001 + 2 \cdot\ 10^{\ 6}\right)^{0.5}$$

$$= 0.0633\ (6.33\%)$$

$$\begin{bmatrix} 2 \cdot \dfrac{\delta I}{I}\ _{Uncertainty} \\ 2 \cdot \dfrac{\delta I}{I}\ _{Re\,soultion} \\ \dfrac{\delta R}{R}\ _{Uncertaint\,y} \\ \dfrac{\delta R}{R}\ _{Resolution} \end{bmatrix} = \begin{bmatrix} \left(\dfrac{2 \cdot 0.5}{100}\right) \\ \left(\dfrac{2 \cdot 0.1}{3.2}\right) \\ \left(\dfrac{0.1}{100}\right) \\ \left(\dfrac{1.0}{1000}\right) \end{bmatrix} = \begin{bmatrix} 0.01 \\ 0.0625 \\ 0.001 \\ 0.001 \end{bmatrix} \rightarrow \text{Ammeter resolution error is greatest contributor}$$

HW 3.17 (7)

Ammeter, Voltmeter Method Next

$$P = I \cdot V \rightarrow \left. \begin{array}{c} Ammeter \\ Voltmeter \end{array} \right| \rightarrow$$

$$\rightarrow U_P = \sqrt{\left(\frac{\partial P}{\partial I}\right)^2 \cdot U_I^{\,2} + \left(\frac{\partial P}{\partial V}\right)^2 \cdot U_V^{\,2}} \rightarrow \left. \begin{array}{c} \dfrac{\partial P}{\partial I} = V \\ \dfrac{\partial P}{\partial V} = I \end{array} \right| \rightarrow$$

$$\frac{U_P}{P_{nom}} = \sqrt{\frac{(V)^2 \cdot U_I^{\,2} + (I)^2 \cdot U_V^{\,2}}{(I \cdot V)^2}} = \sqrt{\left(\frac{U_I}{I}\right)^2 + \left(\frac{U_V}{V}\right)^2}$$

HW 3.17 (8)

- ***Ammeter, Voltmeter Method***

$$\frac{U_P}{P_{nom}} = \sqrt{\left(\frac{U_I}{I}\right)^2 + \left(\frac{U_V}{V}\right)^2}$$

Ammeter Uncertainty and *Resolution* are Independent Error Sources .. *RSS*

- Current Measurement Error …

$$\left(\frac{U_I}{I}\right) = \sqrt{\left(\frac{\delta I}{I}_{Uncertainty}\right)^2 + \left(\frac{\delta I}{I}_{Resolution}\right)^2} =$$

$$\frac{\delta I}{I}_{Uncertainty} = \frac{0.5\%}{100} = .005$$

$$\frac{\delta I}{I}_{Resolution} = \frac{0.1_{Amp}}{3.2_{Amp}} = .03125$$

$$\left(\frac{U_I}{I}\right) = \sqrt{\left(\frac{0.5\%}{100}\right)^2 + \left(\frac{0.1_{Amp}}{3.2_{Amp}}\right)^2} = 0.03125$$

Same Calculation as earlier

HW 3.17 (9)

- ***Ammeter, Voltmeter Method***

$$\frac{U_P}{P_{nom}} = \sqrt{\left(\frac{U_I}{I}\right)^2 + \left(\frac{U_V}{V}\right)^2}$$

Voltmeter Uncertainty and *Resolution* are Independent Error Sources .. *RSS*

- Voltage Measurement Error …

$$\left(\frac{U_V}{V}\right) = \sqrt{\left(\frac{\delta V}{V}_{Uncertainty}\right)^2 + \left(\frac{\delta V}{V}_{Resolution}\right)^2} =$$

$$\frac{\delta V}{V}_{Uncertainty} = \frac{0.5\%}{100} = .005$$

$$\frac{\delta V}{V}_{Resolution} = \frac{1 \cdot 10^{-3}{}_{Volts}}{\left(3.2_{Amps} \cdot 1000_{\Omega}\right)} = 3.125 \cdot 10^{-7}$$

$$\left(\frac{U_V}{V}\right) = \sqrt{\left(\frac{0.5\%}{100}\right)^2 + \left(\frac{1 \cdot 10^{-3}{}_{Volts}}{\left(3.2_{Amps} \cdot 1000_{\Omega}\right)}\right)^2} = 0.005$$

HW 3.17 (10)

- ***Ammeter, Voltmeter Method***

$$\frac{U_P}{P_{nom}} = \sqrt{\left(\frac{U_I}{I}\right)^2 + \left(\frac{U_V}{V}\right)^2} \rightarrow$$

$$\frac{I}{I}^{2} = \frac{0.5\%}{100}^{2} + \frac{0.1_{Amp}}{3.2_{Amp}}^{2} = 0.0010$$

$$\frac{V}{V}^{2} = \frac{0.5\%}{100}^{2} + \frac{1 \quad 10^{3}{}_{Volts}}{\left(3.2_{Amps} \quad 1000\right)}^{2} = 2.5 \quad 10^{5}$$

$$\frac{U_P}{P_{nom}} = \sqrt{\left(\frac{U_I}{I}\right)^2 + \left(\frac{U_V}{V}\right)^2} = \left(0.001 + 2.5 \cdot 10^{5}\right)^{0.5} = 0.0321 \; (3.21\%)$$

HW 3.17 (6)

- ***Ammeter, Voltmeter Method***

$$\frac{U_P}{P_{nom}} = \sqrt{\left(\frac{U_I}{I}\right)^2 + \left(\frac{U_V}{V}\right)^2} \rightarrow \left(0.001 + 2.5\cdot 10^{5}\right)^{0.5} = 0.0321\ (3.21\%)$$

$$\begin{bmatrix} \frac{\delta I}{I}_{Uncertainty} \\ \frac{\delta I}{I}_{Resoultion} \\ \frac{\delta V}{V}_{Uncertainty} \\ \frac{\delta V}{V}_{Resolution} \end{bmatrix} = \begin{bmatrix} \left(\frac{0.5}{100}\right) \\ \left(\frac{2\cdot 0.1}{3.2}\right) \\ \left(\frac{0.5}{100}\right) \\ \left(\frac{10^{-3}}{3.2\cdot 1000}\right) \end{bmatrix} = \begin{bmatrix} 0.005 \\ 0.03125 \\ 0.005 \\ 3.125\times 10^{-7} \end{bmatrix} \rightarrow \text{Ammeter resolution error is greatest contributor}$$

HW 3.17 (9)

- ***Voltmeter- Ammeter*** method is most accurate

$$(0.001 + 2.5 \cdot 10^{5})^{0.5} = 0.0321\ (3.21\%)$$

$$(4\ 0.001 + 2 \cdot 10^{6})^{0.5} = 0.0633\ (6.33\%)$$

- ***Ammeter, Ohmmeter Method***

Ammeter resolution
error is greatest contributor