HW 4.1

PROBLEM 2.3

KNOWN:

$$y(t) = 25 + 10\sin 6\pi t$$

FIND: $\bar{y}$ and y_{rms} for the time periods t_1 to t_2 listed below

a) 0 to 0.1 s
b) 0.4 to 0.5 s
c) 0 to 1/3 s
d) 0 to 20 s

SOLUTION:

For the function $y(t)$

$$\bar{y} = \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} y(t)\,dt$$

and

$$y_{rms} = \sqrt{\frac{1}{t_2 - t_1}\int_{t_1}^{t_2} \left[y(t)\right]^2 dt}$$

Thus in general,

$$\begin{aligned}
\bar{y} &= \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} \left(25 + 10\sin 6\pi t\right) dt \\
&= \frac{1}{t_2 - t_1}\left[25t - \frac{10}{6\pi}\cos 6\pi t \Big|_{t_1}^{t_2} \right] \\
&= \frac{1}{t_2 - t_1}\left[25\left(t_2 - t_1\right) - \frac{10}{6\pi}\left(\cos 6\pi t_2 - \cos 6\pi t_1\right) \right]
\end{aligned}$$

and

$$\begin{aligned}
y_{rms} &= \left\{ \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} \left(25 + 10\sin 6\pi t\right)^2 dt \right\}^{1/2} \\
&= \left\{ \frac{1}{t_2 - t_1}\left[625t - \frac{500}{6\pi}\cos 6\pi t + 100\left(\frac{-1}{12\pi}\sin 6\pi t\cos 6\pi t + \frac{1}{2}t \right) \right]\Bigg|_{t_1}^{t_2} \right\}^{1/2}
\end{aligned}$$

The resulting values are

a) $\bar{y} = 25.69 \qquad y_{rms} = 31.85$

b) $\bar{y} = 25.69 \qquad y_{rms} = 31.85$

c) $\bar{y} = 25 \qquad y_{rms} = 25.98$

d) $\bar{y} = 25 \qquad y_{rms} = 25.98$

COMMENT: The average and rms values for the time period 0 to 20 seconds represents the long-term average behavior of the signal. The result in parts a) and b) are accurate over the specified time periods and for a measured signal may have specific significance. The period 0 to 1/3 represents one complete cycle of the simple periodic signal and results in average and rms values which accurately represent the long-term behavior of the signal.

HW 4.2

PROBLEM 2.9

KNOWN: Functions:

a) $\sin \frac{10\pi t}{5}$

b) $8\cos 8t$

c) $\sin 5n\pi t$ for n=1 to ∞

FIND: The period, frequency in Hz, and circular frequency in rad/s are found from

$$\omega = 2\pi f = \frac{2\pi}{T}$$

SOLUTION:

a) $\omega = 2\pi$ rad/s $\quad f = 1$ Hz $\quad T = 1$ s

b) $\omega = 8$ rad/s $\quad f = 4/\pi$ Hz $\quad T = \pi/4$ s

c) $\omega = 5n\pi$ rad/s $\quad f = 5n/2$ Hz $\quad T = 2/(5n)$ s

HW 4.3

PROBLEM 2.20

KNOWN:
At time zero ($t = 0$)

$$x = 0$$

$$\frac{dx}{dt} = 5 \text{ cm / s} \quad f = 1 \text{ Hz}$$

FIND:

a) period, T
b) amplitude, A
c) displacement as a function of time, $x(t)$
d) maximum speed

SOLUTION:

The position of the particle as a function of time may be expressed

$$x(t) = A \sin 2\pi t$$

so that

$$\frac{dx}{dt} = 2A\pi \cos 2\pi t$$

Thus, at t = 0 $\frac{dx}{dt} = 5$

From these expressions we find

a) $T = 1$ s

b) amplitude, $A = 5/2\pi$

c) $x(t) = (5/2\pi)\sin 2\pi t$

d) maximum speed $= 5$ cm/s

HW 4.4

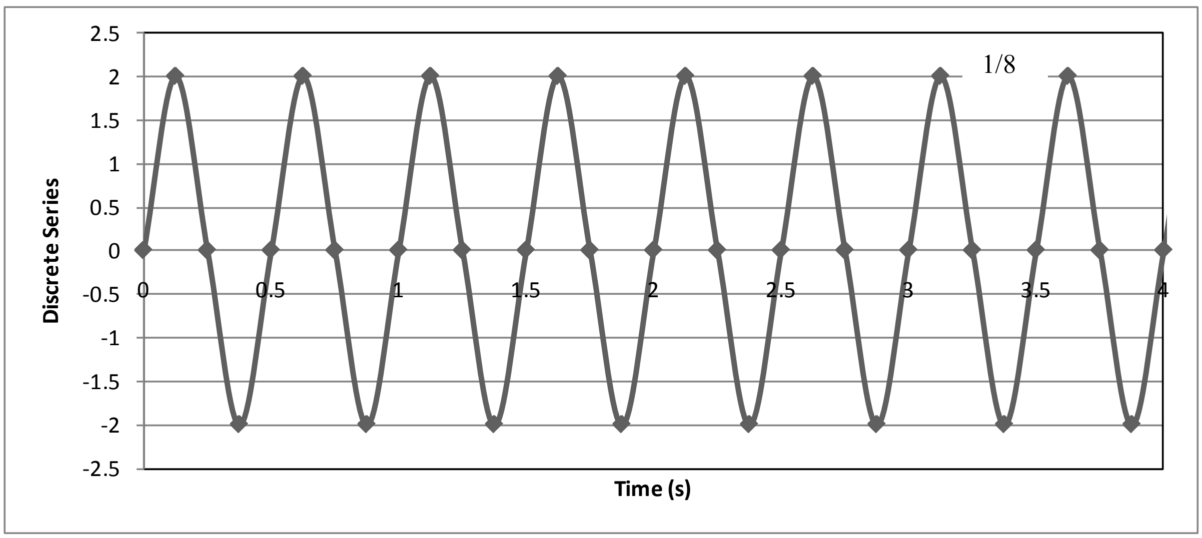
PROBLEM 7.1

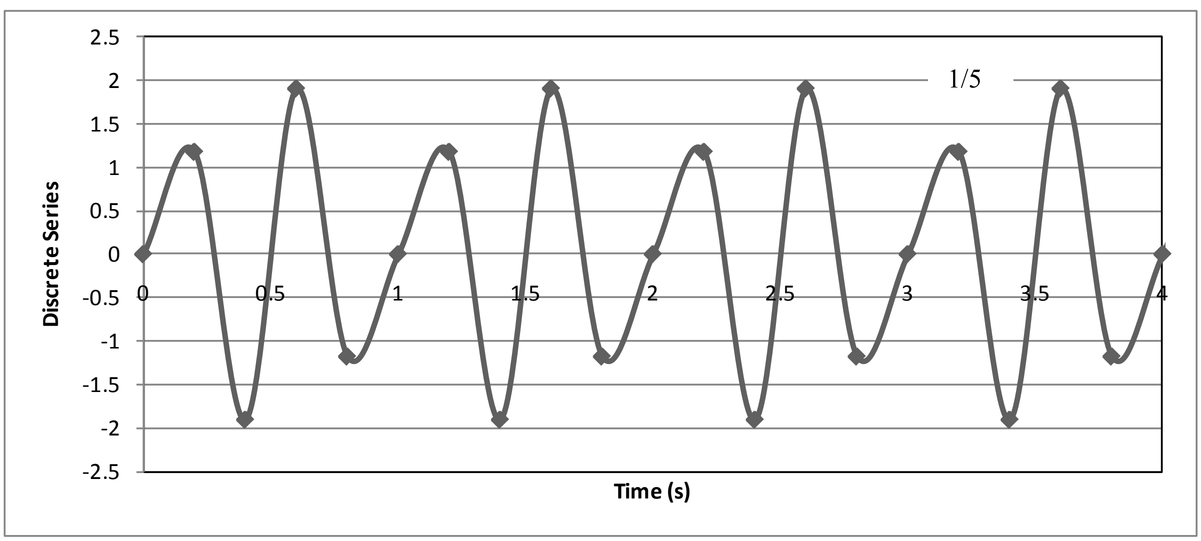
KNOWN: $E(t) = 2\sin 4\pi t$ mV

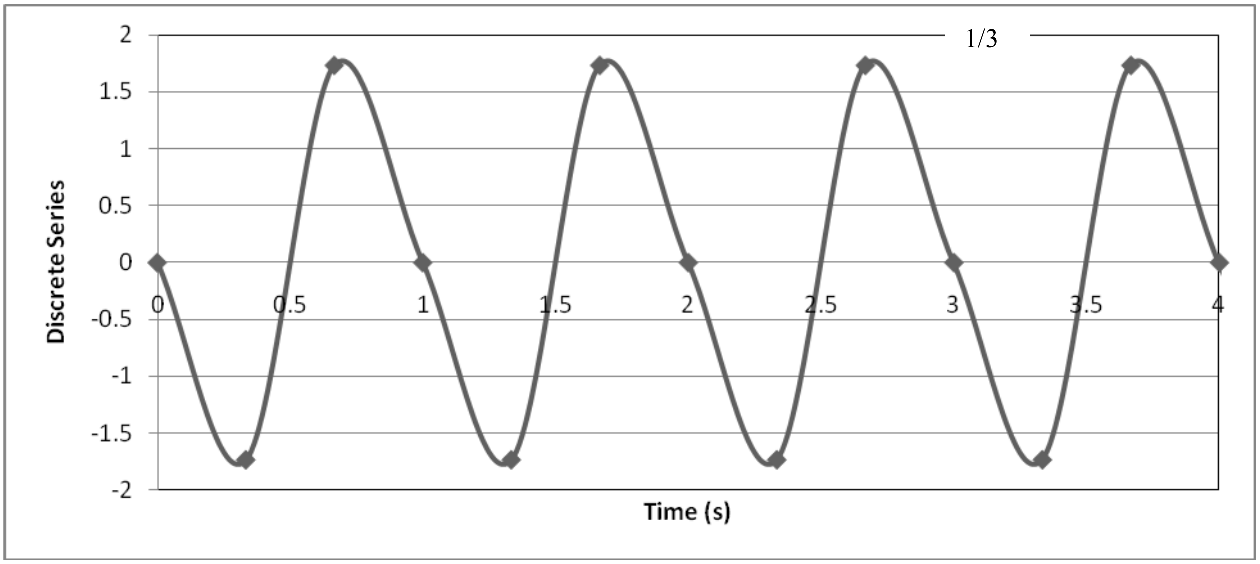
FIND: Convert to a discrete time series and plot

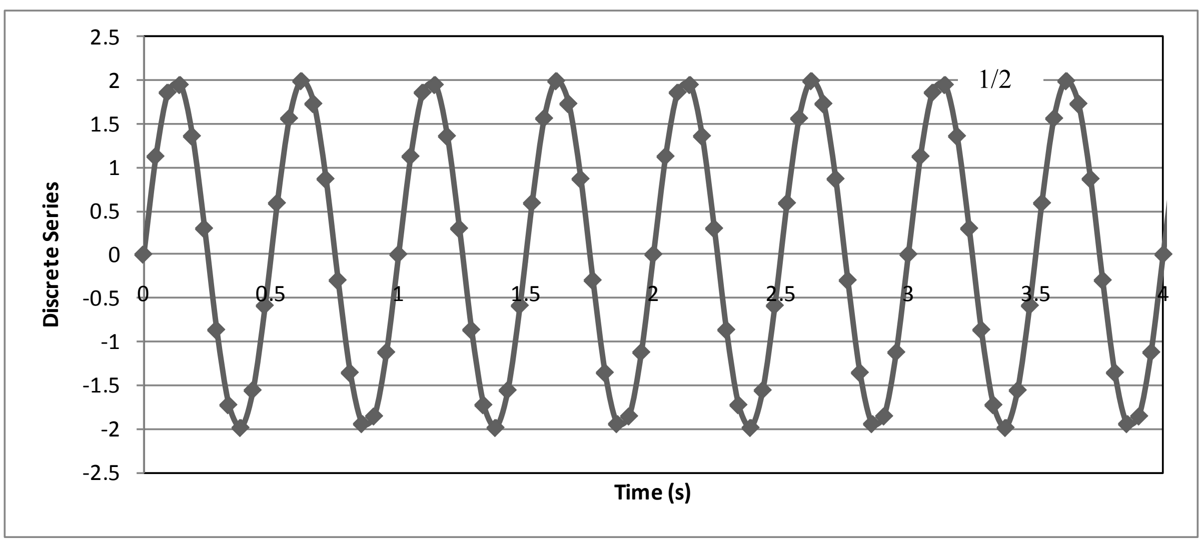
SOLUTION

The signal is converted to a discrete time series for using $N = 128$ and sample time increments of 1/8, 1/5, 1/3, 1/21 s and plotted below. The series created with a time increment of 1/3 s fails the Sampling Theorem criterion and portrays a signal with a different frequency content; this frequency is the alias frequency. We plot 4 s of each signal:









At $f_s = 8$ Hz, the series is an exact replica. At $f_s = 5$ Hz, frequency content is clear but there is amplitude distortion. At $f_s = 3$ Hz, there is aliasing to $f_a = 1$ Hz with amplitude distortion. At 21 Hz (more than 10 times the highest frequency) the reconstructed signal becomes nearly exact.