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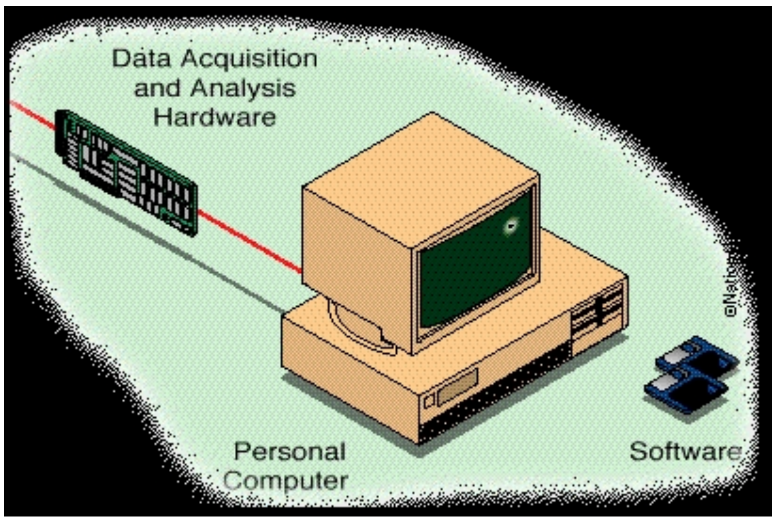
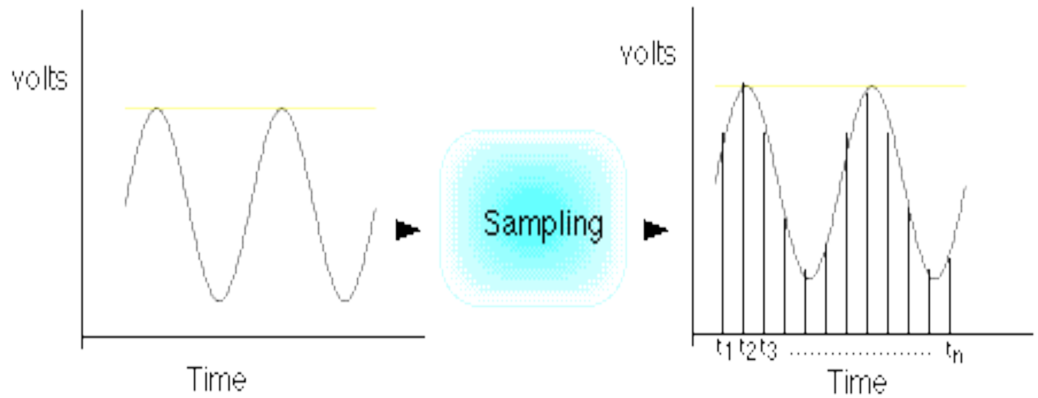
Mechanical & Aerospace Engineering

Section 2.3: Sampling, Nyquist Frequency, and Signal Aliasing

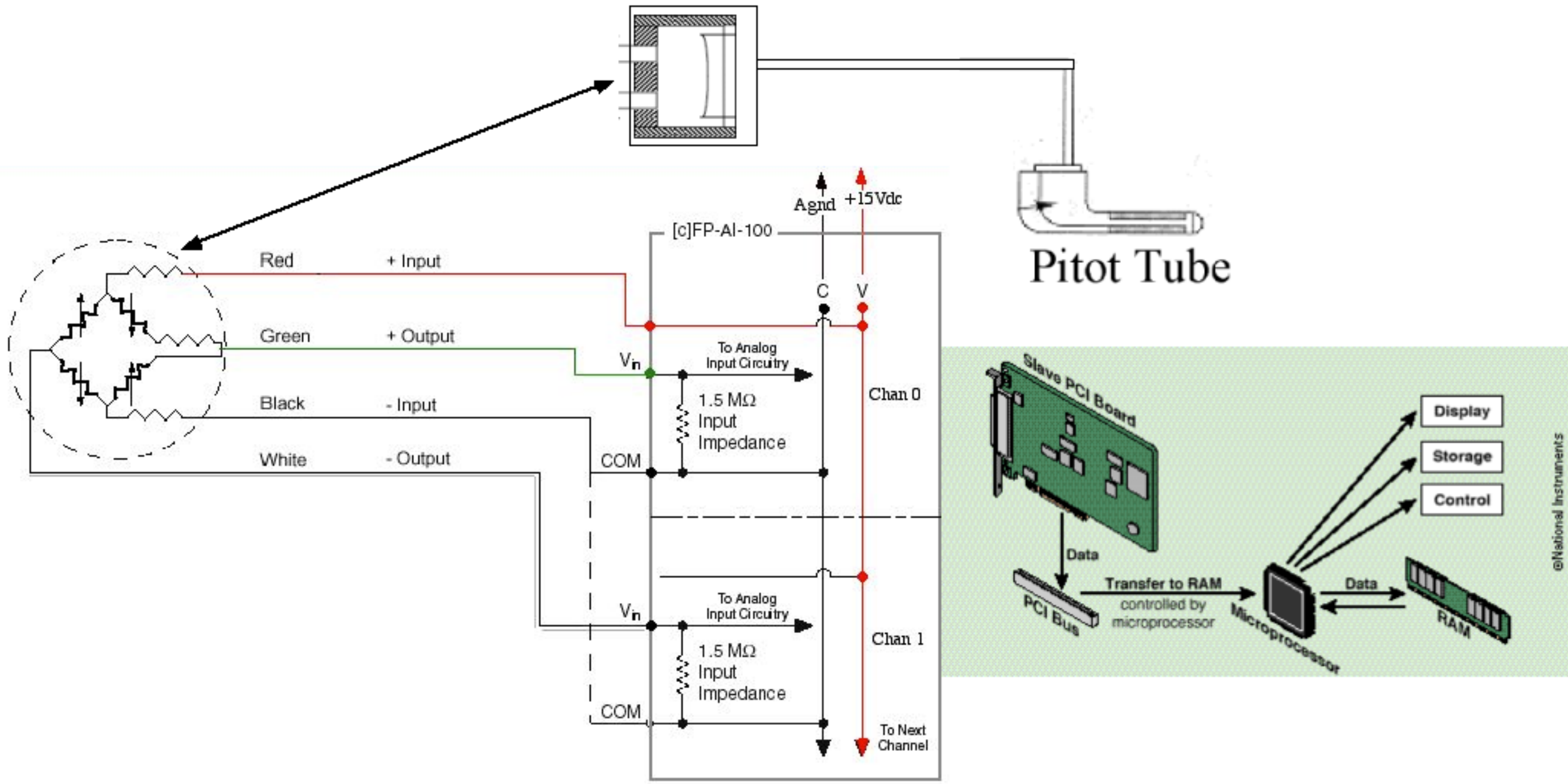
Beckwith Chapter 8-section 8.11, Chapter 7, Figliola and Beasley





Measurement and Data Sampling

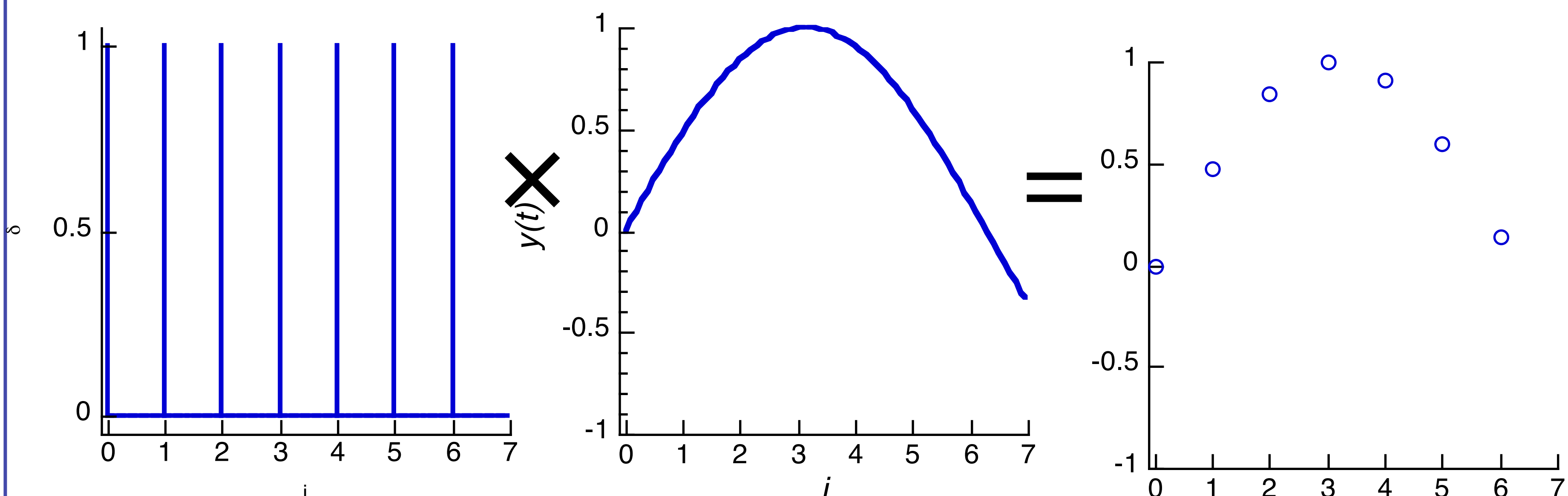
• When perform a measurement … a *transducer* converts the *measurand* into an electrical signal … and this signal is *"sampled"* using a digital computer



Discrete Signals

More times than not, modern measurements are made using digital data acquisition equipment (DAQ). Most real processes are analog in nature. As such, we need to be able to move freely between these two ways of thinking.

Both Signal Amplitude ***and*** Time Quantization



Discrete Signals (2)

• When sampling or ***discretizing*** a signal, it is important to consider both accuracy and acquisition speed.

• Signal must be acquired fast enough that salient information is not lost while the analog signal is being sampled.

• Condition that stipulates this speed is known as the *Nyquist Sampling Theorem*. *(more on this later)*

• Speech analysis, telecommunication, and earthquake analysis are examples of common applications where the frequency of the signal must be known.

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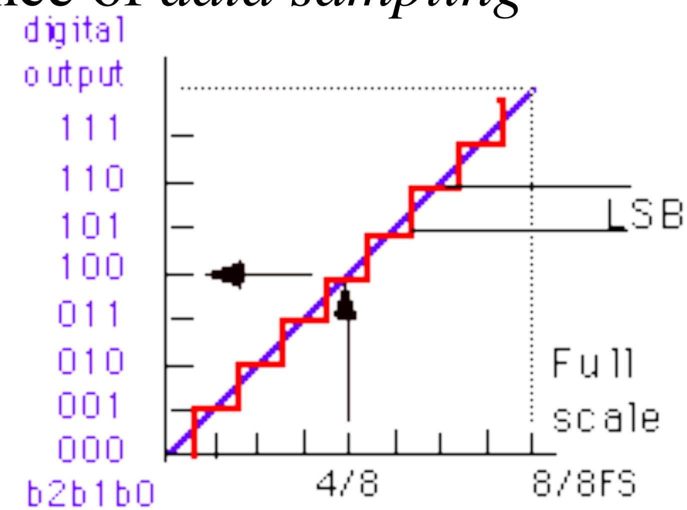
We have Already Discussed Signal Magnitude Quantization

• **Resolution** determines the ability to see fine details in the **measurement**.

• Defined as the smallest incremental value that can be Discerned by a system

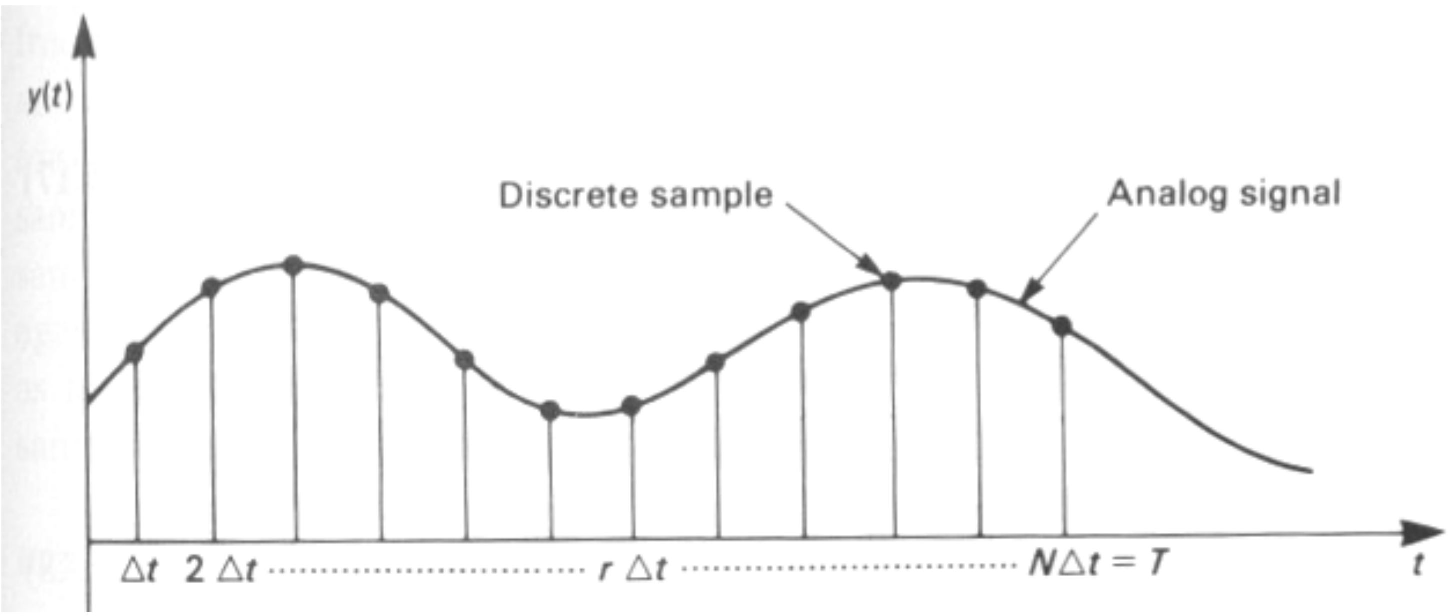
• Typically a consequence of *data sampling*



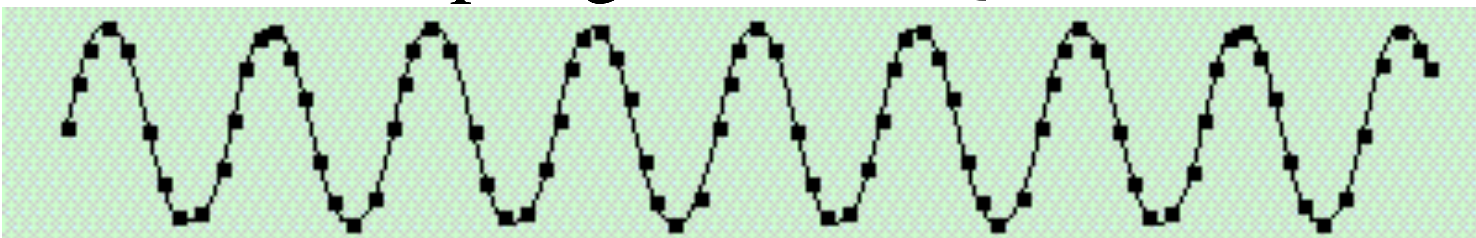
Data Sampling – Time Quantization

When we sample a signal … we do not capture the entire original signal

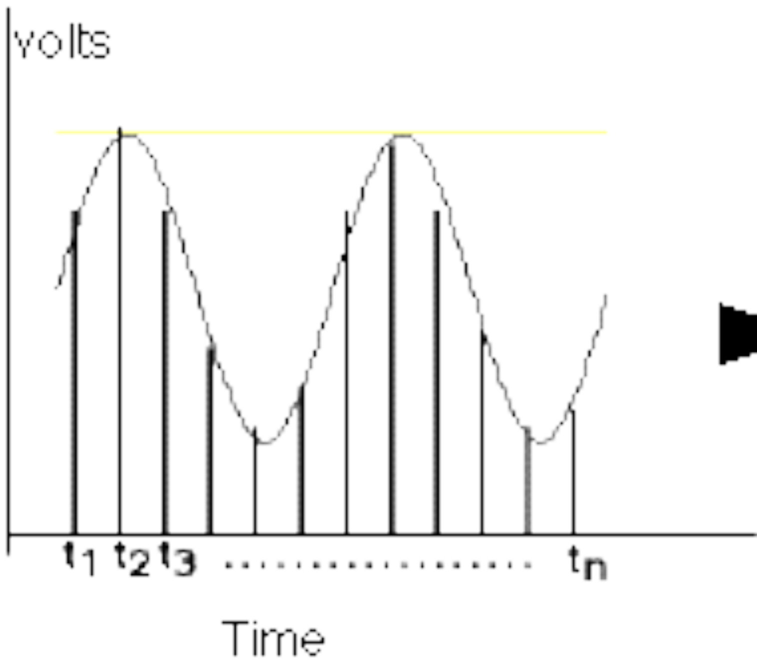
… but only a *subset of the signal* … so if would stand to reason that we do not capture all of the frequencies imbedded in the signal

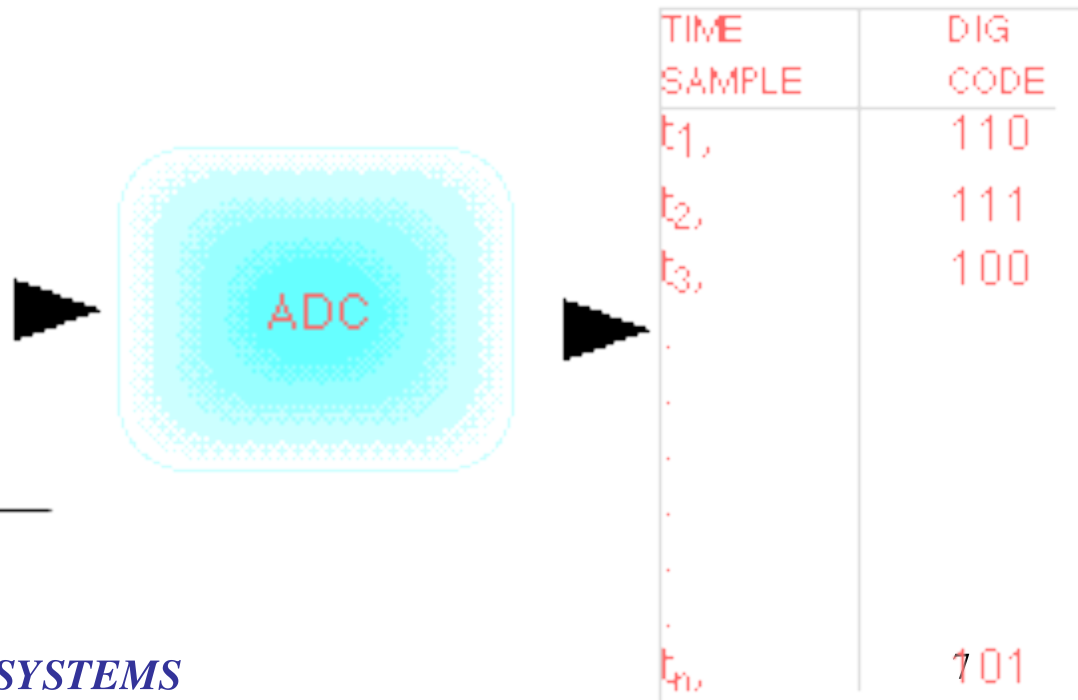


Data Sampling -- Time Quantization (2)



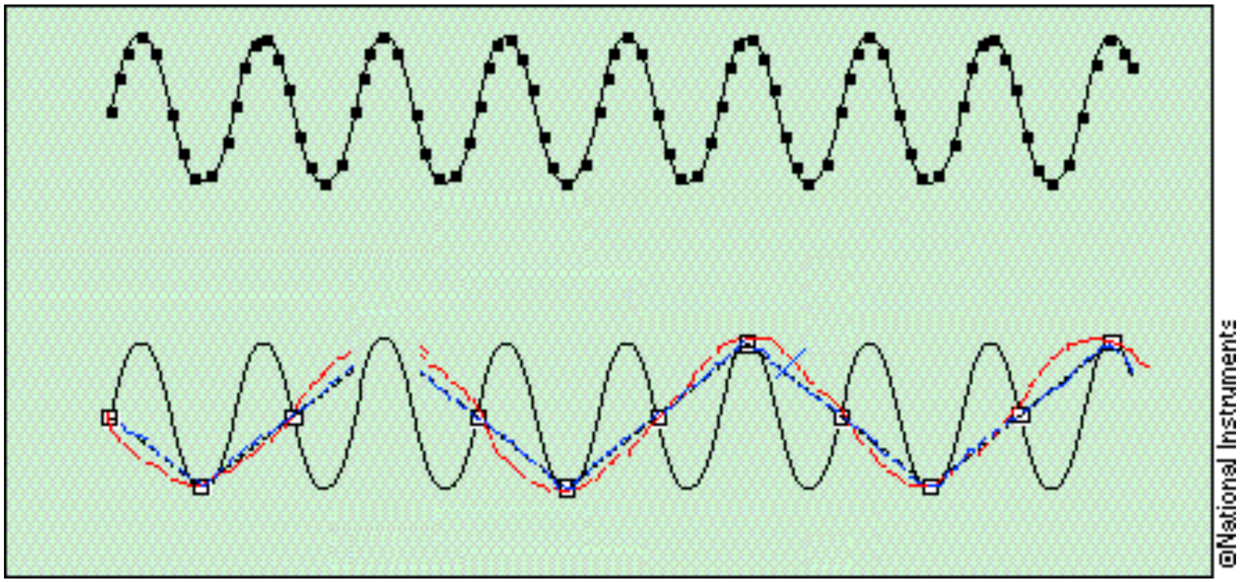
- Waveform is "grabbed" at intervals (regular or aperiodic) and each "sample" is represented as a number on the computer





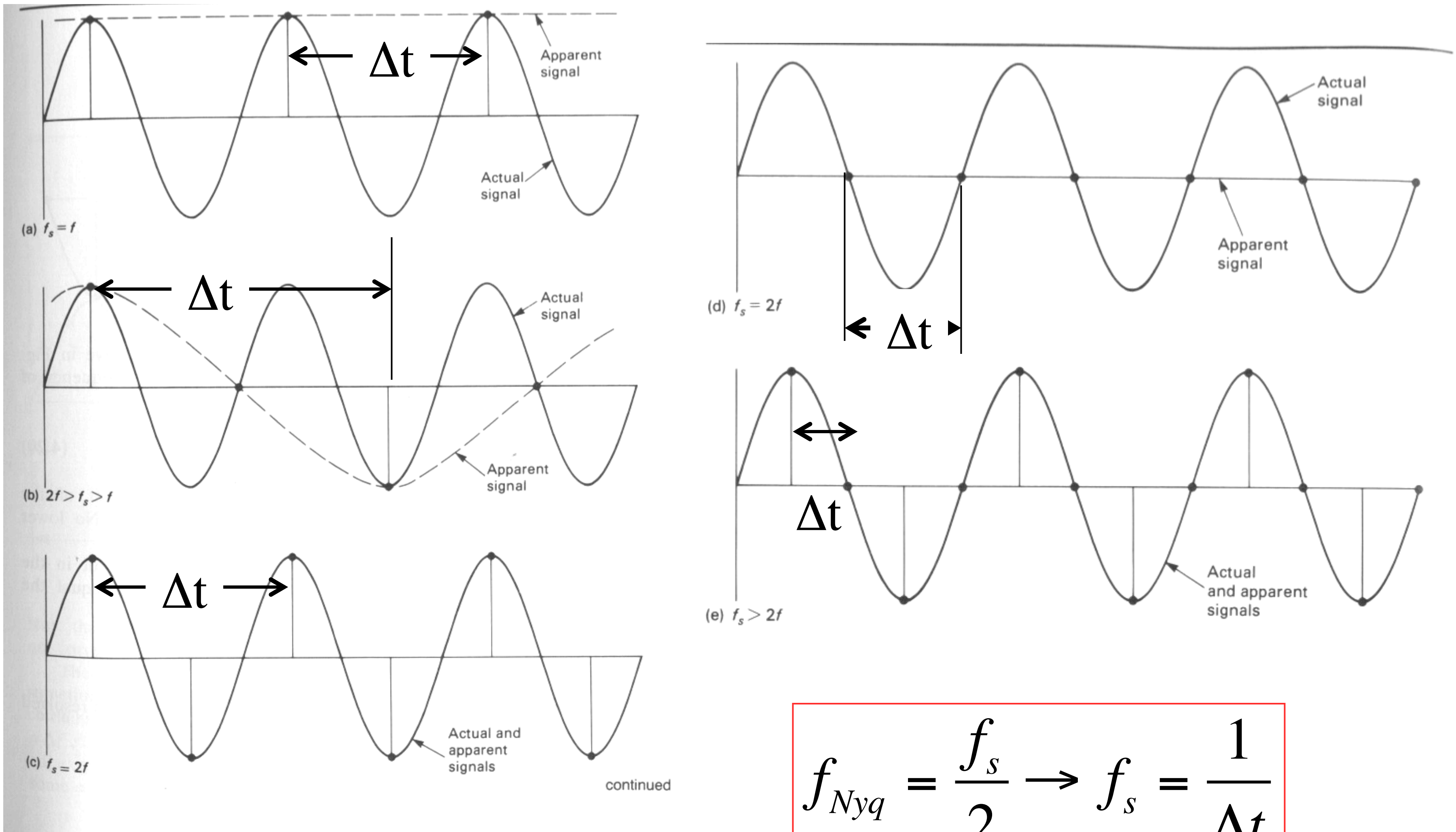
Data Sampling -- Time Quantization (2)

- Frame rate … how fast did you sample?



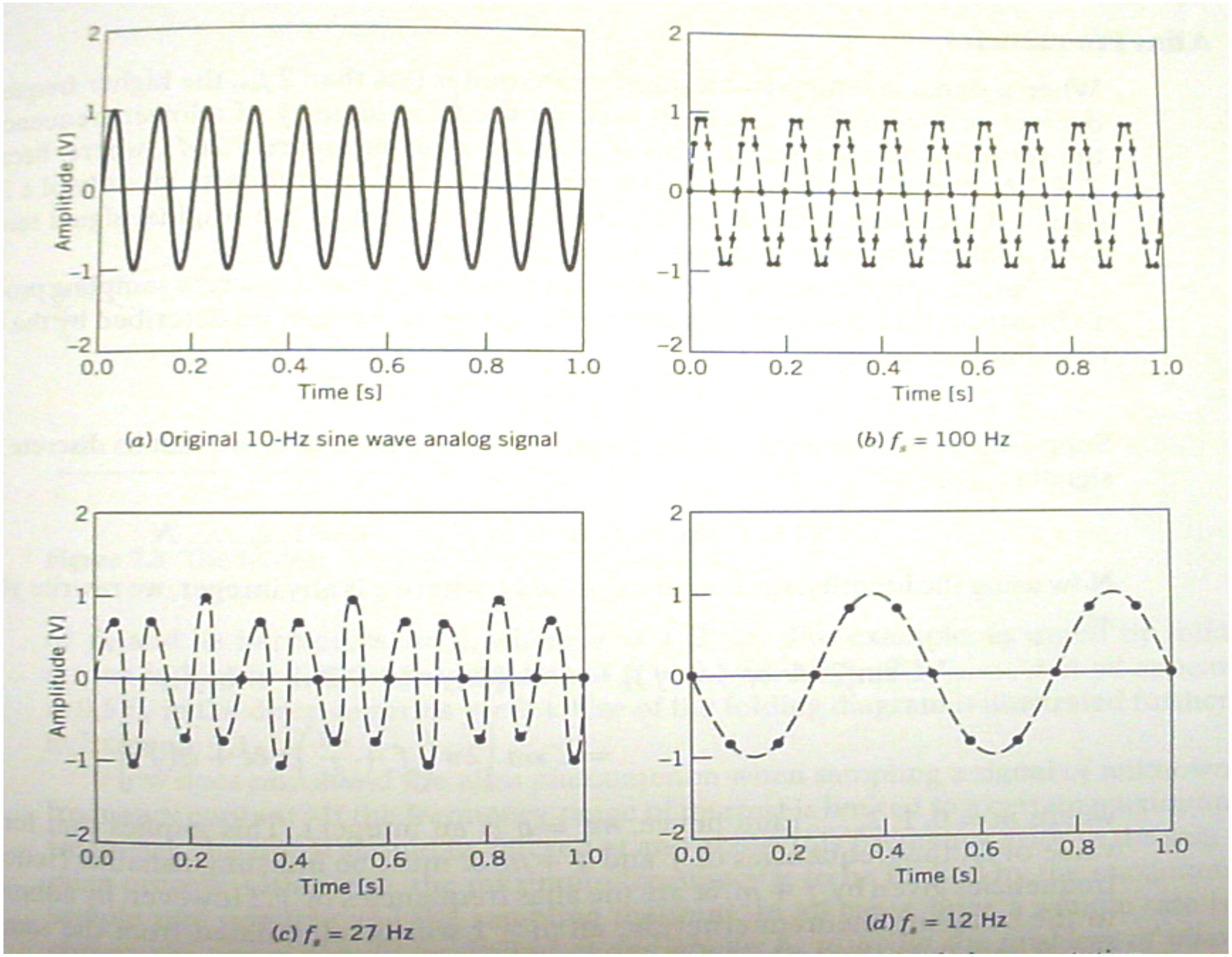
- How Accurately Can you Resolve frequencies within the Sampled Signal?

Sampling and the Nyquist Frequency



$$f_{Nyq} = \frac{f_s}{2} \rightarrow f_s = \frac{1}{\Delta t}$$

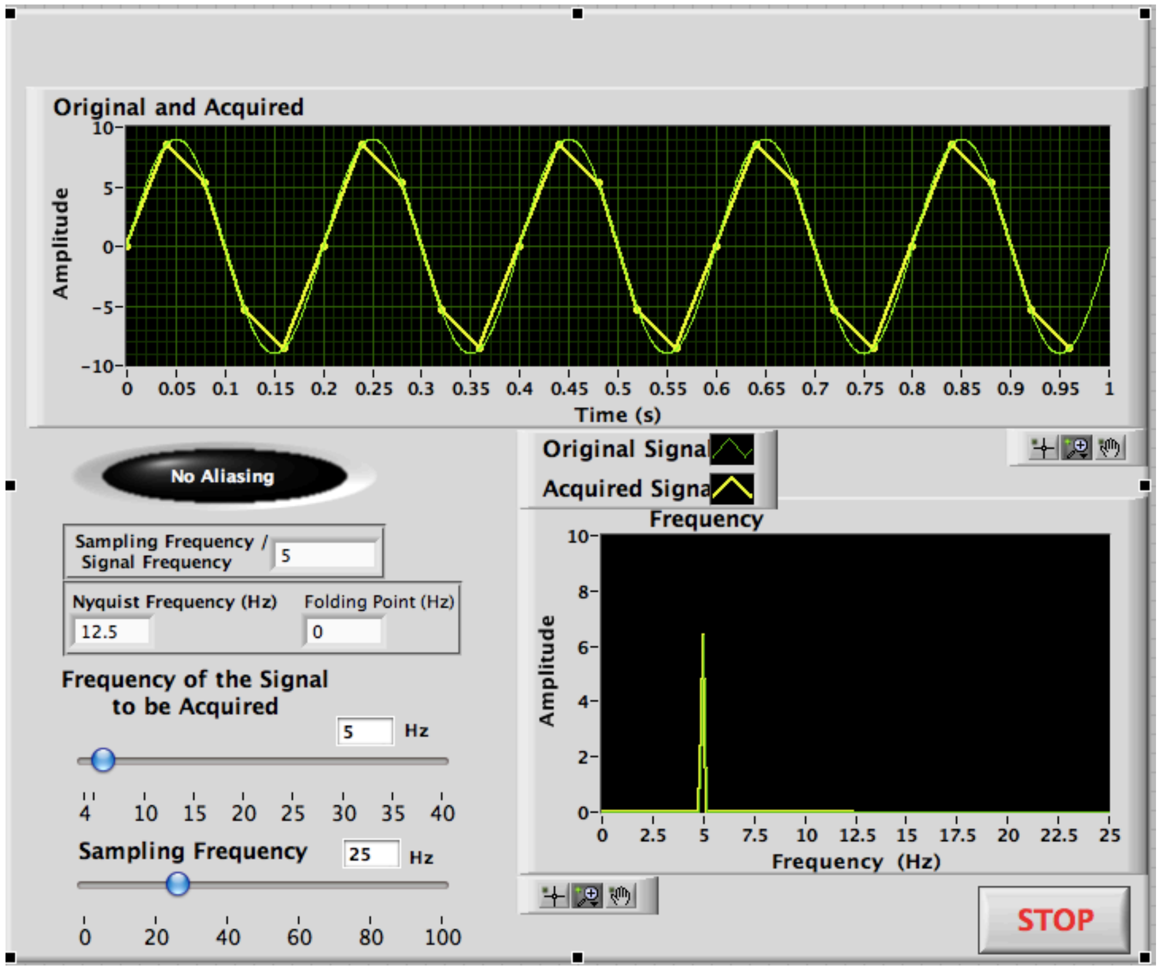
Aliasing Example



Frequencies above Nyquist Frequency will "Alias" to Lower frequencies

Aliasing Examples (4)

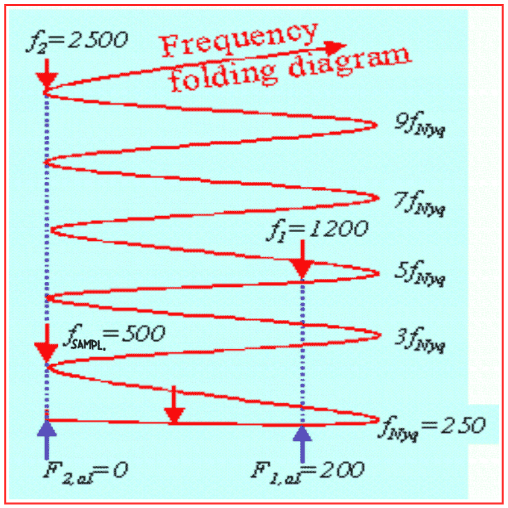
- Aliasing Example VI

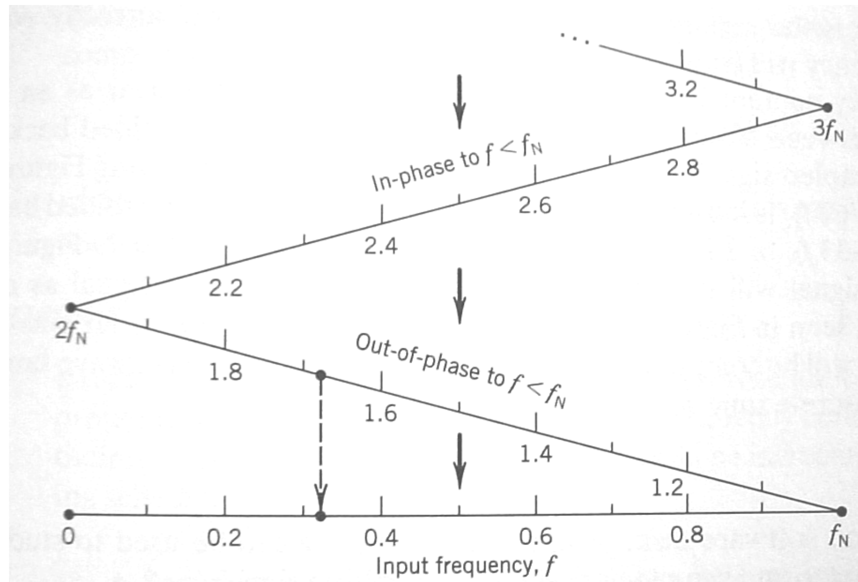


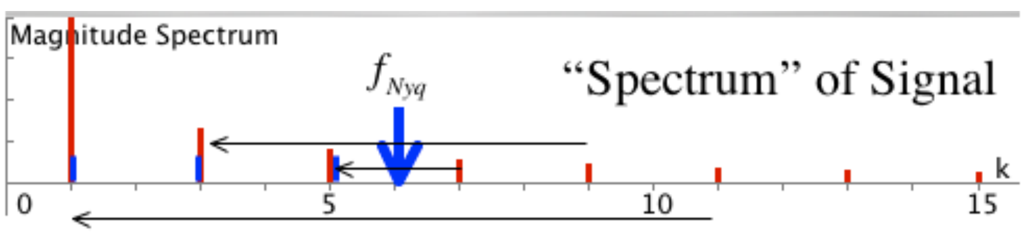
Nyquist Frequency and Aliasing

If we sample too slowly, frequencies in our input signal can appear as lower frequencies. This phenomena is known as aliasing.

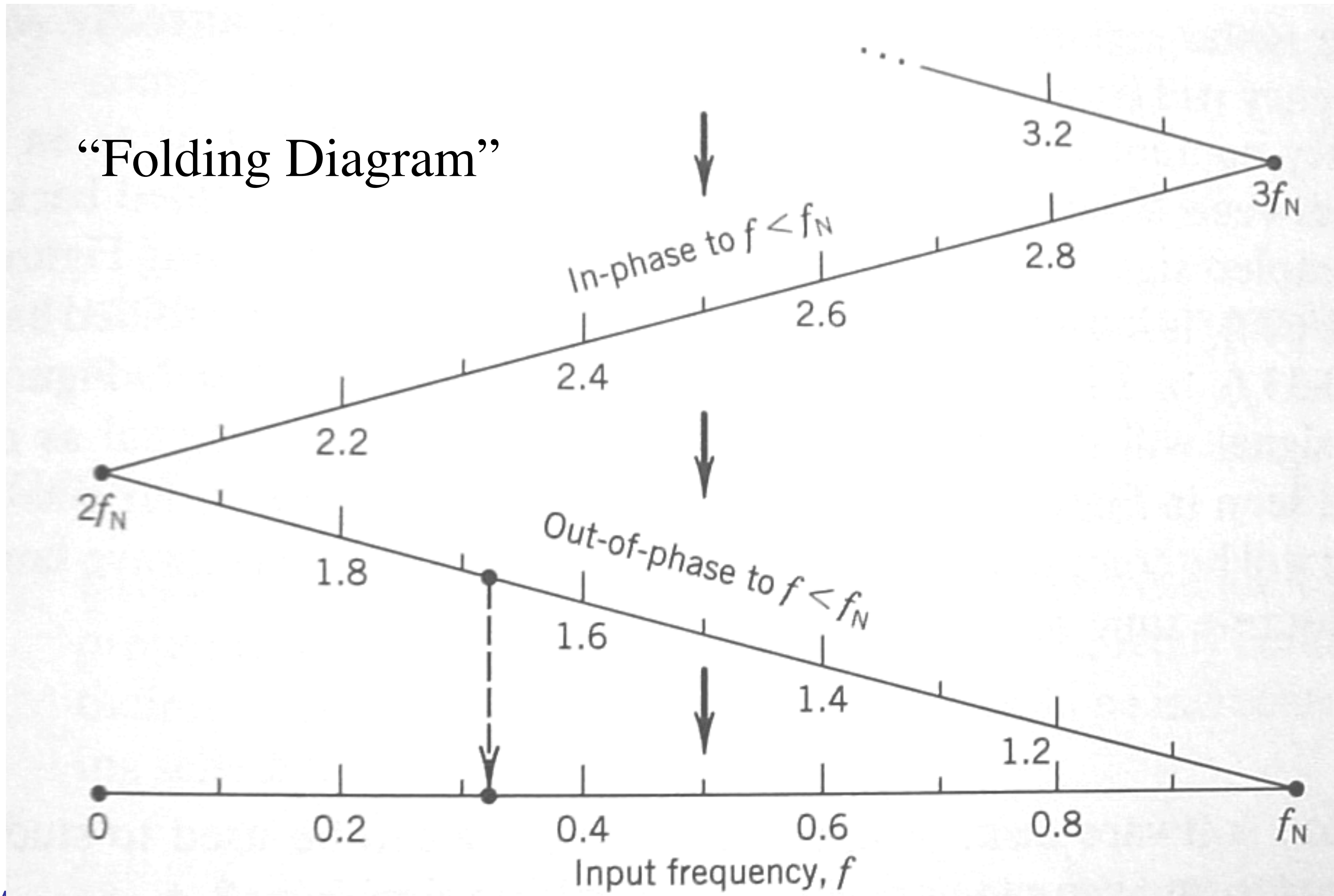
$$f_{Nyq} = \frac{f_s}{2} \rightarrow f_s = \frac{1}{\Delta t}$$





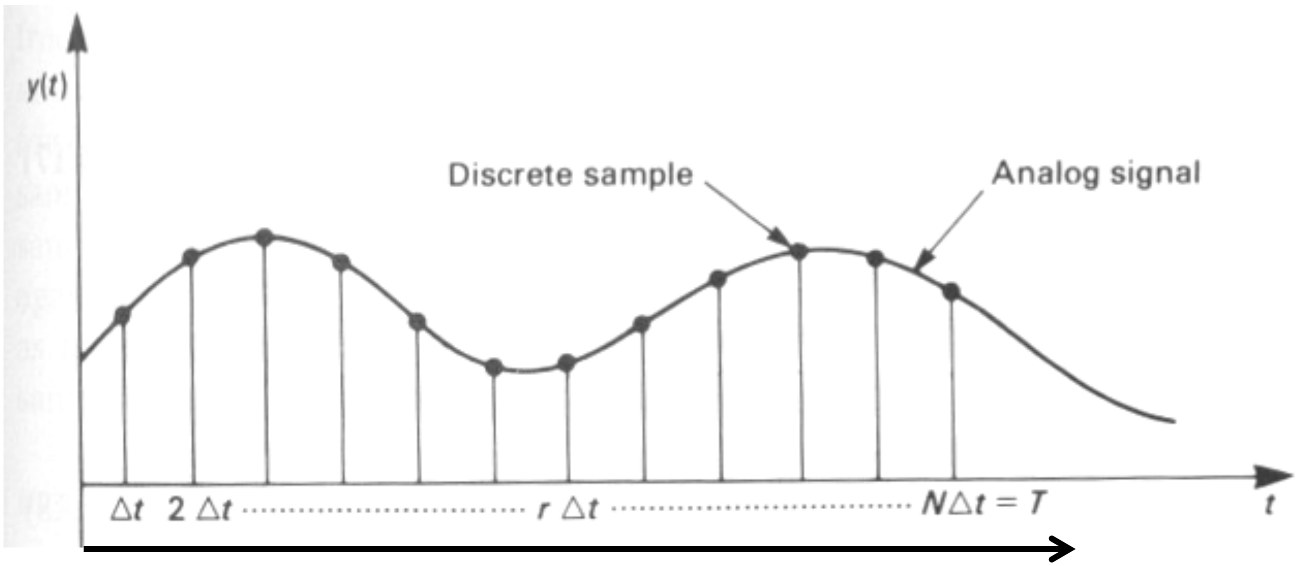


Nyquist Frequency and Aliasing (2)



Frequency Resolution

- Duration of the sample $T = N\,\Delta t = N/f_s$ *[sec]*



- Minimum frequency that can be resolved is function of Sample length …. "bandwidth resolution"

- Must capture a *full period* of the frequency

$$\Delta f = \frac{1}{N\Delta t} = \frac{f_s}{N}$$

Nyquist Frequency, Resolution Bandwidth. and Aliasing (2)

Definitions:

Sampling time, T: Total measuring time of a signal.

Sampling interval Δt: Time between two samples

Sample Rate : $1/\Delta t$, the number of samples per second.

Nyquist Frequency, F_{nyq}: maximum frequency that can be captured by a sample interval, ΔT

$$f_{Nyq} = \frac{f_s}{2} \rightarrow f_s = \frac{1}{\Delta t}$$

Resolution Bandwith: minimum frequency that can be represented by a sample

$$\Delta f = \frac{1}{N \Delta t} = \frac{f_s}{N}$$

- **Faster you sample, the higher frequency you can represent**
- **The longer you sample, the smaller the frequency represented**