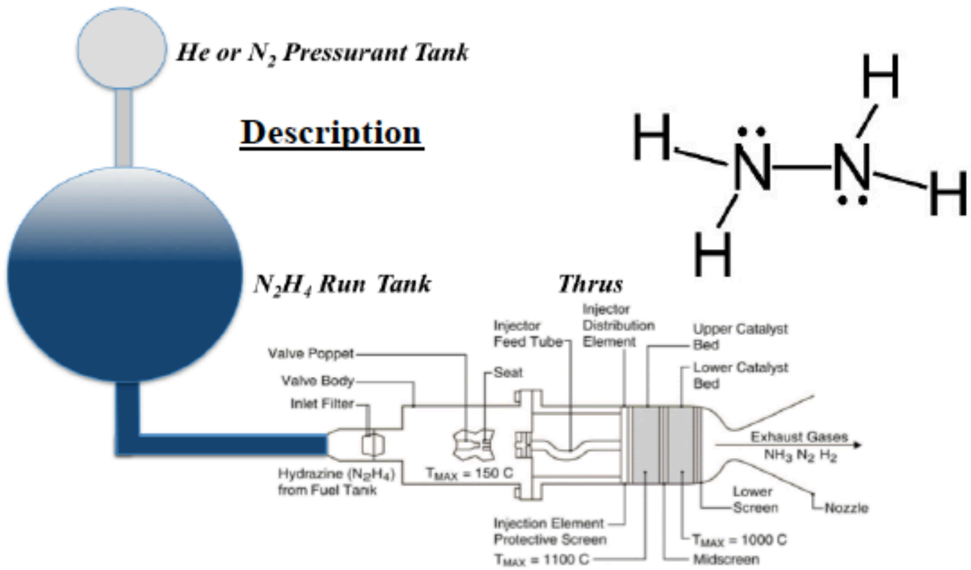
# Examples of Using the t-Test to Assess Statistical Significance Between Multiple Data Sets

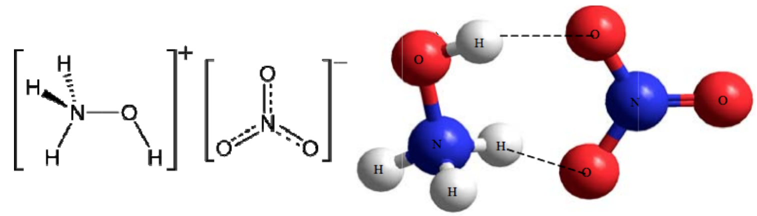
$$t = \frac{\bar{x} - \mu}{S_x / \sqrt{n}} \longrightarrow t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\left(S_1^2 / n_1\right) + \left(S_2^2 / n_2\right)}}$$

# Problem 1: "Green Mono-Propellants"

# Example 1 .. “Green Propellants” Comparison

**Description**



Monopropellant thrusters using hydrazine ($N_2H_4$) are commonly used on a variety of spacecraft and satellites. However, hydrazine is a highly toxic and dangerously unstable substance. Although procedures are in place to allow hydrazine to be managed safely on tightly controlled military and NASA-owned flight experiments; the toxicity and explosion potential of hydrazine requires extreme handling precautions. Increasingly, with a growing regulatory burden, infrastructure requirements associated with hydrazine transport, storage, servicing, and clean up of accidental releases are becoming cost prohibitive. The use of hydrazine as a propellant has been compromised by stringent laws to protect personnel who have to work with substances which are highly toxic and carcinogenic.

Recently, much more benign, low toxicity (“green”) storable liquid propellants have attracted significant attention as possible replacements for hydrazine.

**Hydroxylammonium nitrate** (HAN)

Because of the high-energy and relatively benign environmental characteristics (compared to hydrazine), multiple development activities have sought to adapt HAN-based monopropellants for rocketry applications.

# "Green Propellant" Comparisons

• We've Been asked to compare the performance of two HAN-based propellant formulations

1. ... The well known Army-Developed HAN *LP 1846 HAN-Based* Monopropellant for Artillery and Ordnance Applications that consists of the mass proportions shown by the table below

| LP 1846 | | | |
|---|---|---|---|
| | HAN | TEAN | Water |
| Molecular Structure | $H-N(H)(H)-O-H^+NO_3^-$ | $(HO-CH_2-CH_2)_3-N-H^+NO_3^-$ | $H-O-H$ |
| Wt. % | 60.8 | 19.2 | 20.0 |

| HAN269MEO15 | | | | |
|---|---|---|---|---|
| | HAN | AN (Ammonium Nitrate) | Methanol | Water |
| Molecular Structure | $H-N(H)(H)-O-H^+NO_3^-$ | $H-N(H)(H)-H^+NO_3^-$ | $H-C(H)(H)-O-H$ | $H-O-H$ |
| Wt. % | 69.7 | 0.6 | 14.79 | 14.91 |

2. .. The lesser-known known Aerojet Formulation HAN269MEO15 which consists of the mass proportions shown by the table above

# "Green Propellants" (1)

The following results were collected for 8 HAN LP 1846 thruster tests

Test Results, HAN LP 1846

| Test # | P0, kPa | F, N | Mdot , g/s | Isp, s |
|---|---|---|---|---|
| 1 | 3085.72 | 37.4 | 29.61 | 151.54 |
| 2 | 2994.65 | 36.12 | 28.64 | 151.77 |
| 3 | 3150.54 | 38.31 | 29.94 | 153.45 |
| 4 | 3021.96 | 36.51 | 29.01 | 151.54 |
| 5 | 2962.97 | 35.69 | 28.71 | 149.83 |
| 6 | 2946.36 | 35.47 | 27.8 | 154.26 |
| 7 | 2947.05 | 35.49 | 28.1 | 153.12 |
| 8 | 2857.94 | 34.12 | 29.19 | 139.75 |
| Mean | 2995.9 | 36.1396 | 28.8768 | 150.659 |
| Std Dev | 90.7273 | 1.28611 | 0.718911 | 4.618 |

# "Green Propellants" (2)

The following test results were collected for HAN 269MEO15:

Test Results, HAN 269MEO15

| Test # | P0, kPa | F, N | Mdot , g/s | Isp, s |
|---|---|---|---|---|
| 1 | 3131.72 | 38.08 | 29.61 | 154.72 |
| 2 | 3066.18 | 37.19 | 28.63 | 156.67 |
| 3 | 3204.74 | 39.13 | 29.97 | 157.16 |
| 4 | 3065.72 | 37.16 | 29.01 | 154.55 |

Mean Values

| | | | |
|---|---|---|---|
| 3117.09 | 37.8904 | 29.303 | 155.774 |

Std Dev. Values

| | | | |
|---|---|---|---|
| 66.1503 | 0.92865 | 0.599395 | 1.33276 |

• ***Only 4 HAN 269MEO tests were performed due to limited propellant availability***

# "Green Propellants" (3)

- Based on these test results, **Aerojet claims HAN269MEO15 generates superior performance when compared to LP 1846 … let's evaluate that claim!**
- Use the *"t-test"* and the appropriate degrees of freedom to assess whether the observed differences in Specific Impulse are statistically significant. **(25 pts)**
- Assume a 95% confidence interval for this assessment.
- Use Specific Impulse as the Figure of Merit

$$I_{sp} = \frac{1}{g_0}\frac{\mathrm{I}_{mpulse}}{M_{\substack{propellant\\consumed}}} \equiv \frac{\int_0^{t_{burn}} F_{(t)} \cdot dt}{W_{\substack{propellant\\consumed}}}$$

# "Green Propellants" (solution)

- *Calculate t-variable .. Use specific impulse as the figure of Merit*

$$t_{meas} = \frac{\bar{x}_{269\,MEO} - \bar{x}_{LP1846}}{\sqrt{\frac{S^2_{269\,MEO}}{n_{269\,MEO}} + \frac{S^2_{LP1846}}{n_{LP1846}}}} = \frac{155.774 - 150.659}{\left(\left(\frac{1.33276^2}{4}\right) + \left(\frac{4.618^2}{8}\right)\right)^{0.5}} = 2.90054$$

sec

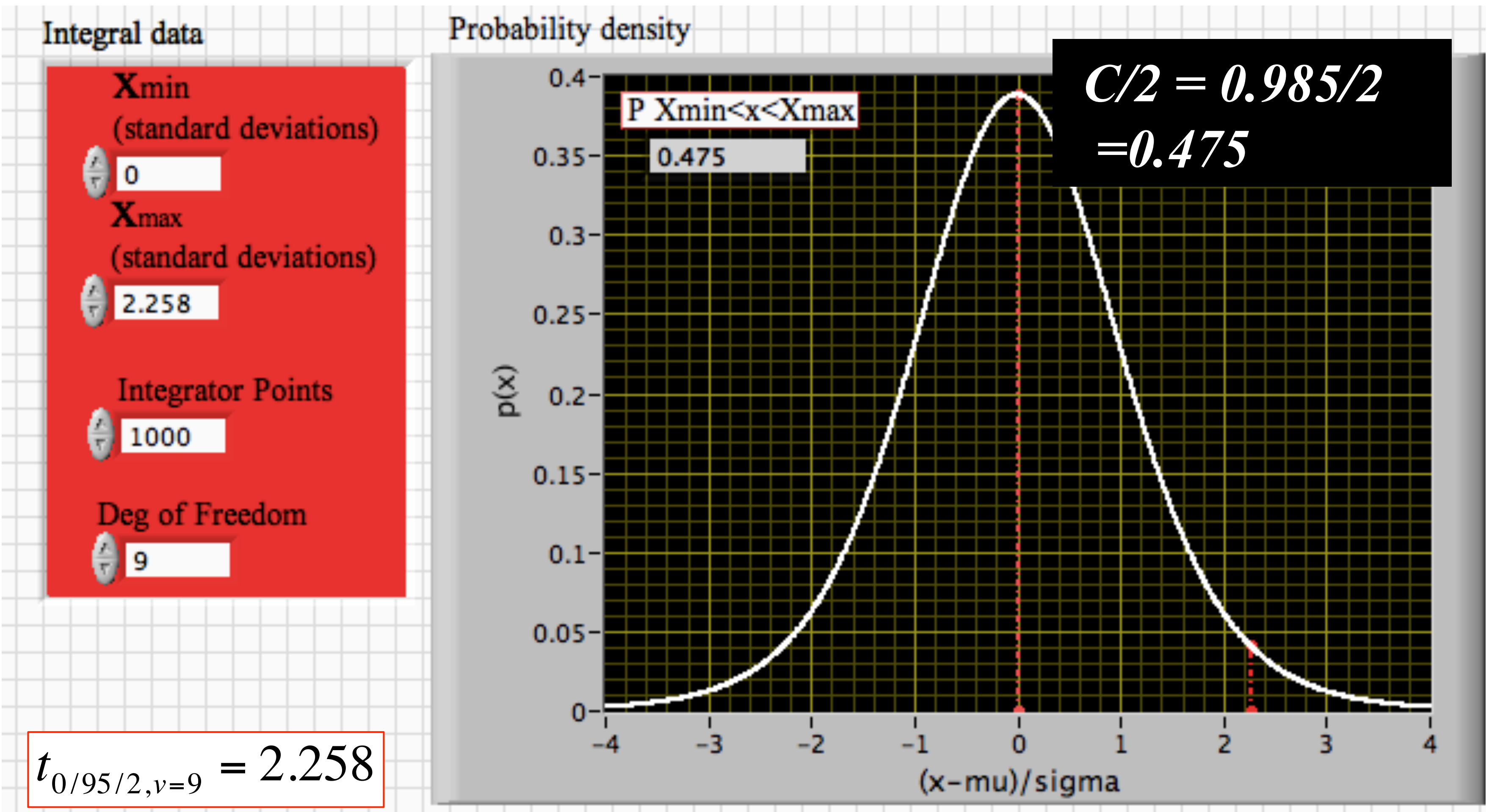
- *Calculate effective DOF*

$$v_{avg} = \frac{\left[\frac{S^2_{269\,MEO}}{n_{269\,MEO}} + \frac{S^2_{LP1846}}{n_{LP1846}}\right]^2}{\left[\frac{\left(\frac{S^2_{269\,MEO}}{n_{269\,MEO}}\right)^2}{n_{269\,MEO}-1} + \frac{\left(\frac{S^2_{LP1846}}{n_{LP1846}}\right)^2}{n_{LP1846}-1}\right]} = \frac{\left(\left(\frac{1.33276^2}{4}\right) + \left(\frac{4.618^2}{8}\right)\right)^2}{\frac{\left(\frac{1.33276^2}{4}\right)^2}{4-1} + \frac{\left(\frac{4.618^2}{8}\right)^2}{8-1}} = 8.94707$$

***Round to 9 DOF***

# "Green Propellants" (solution)

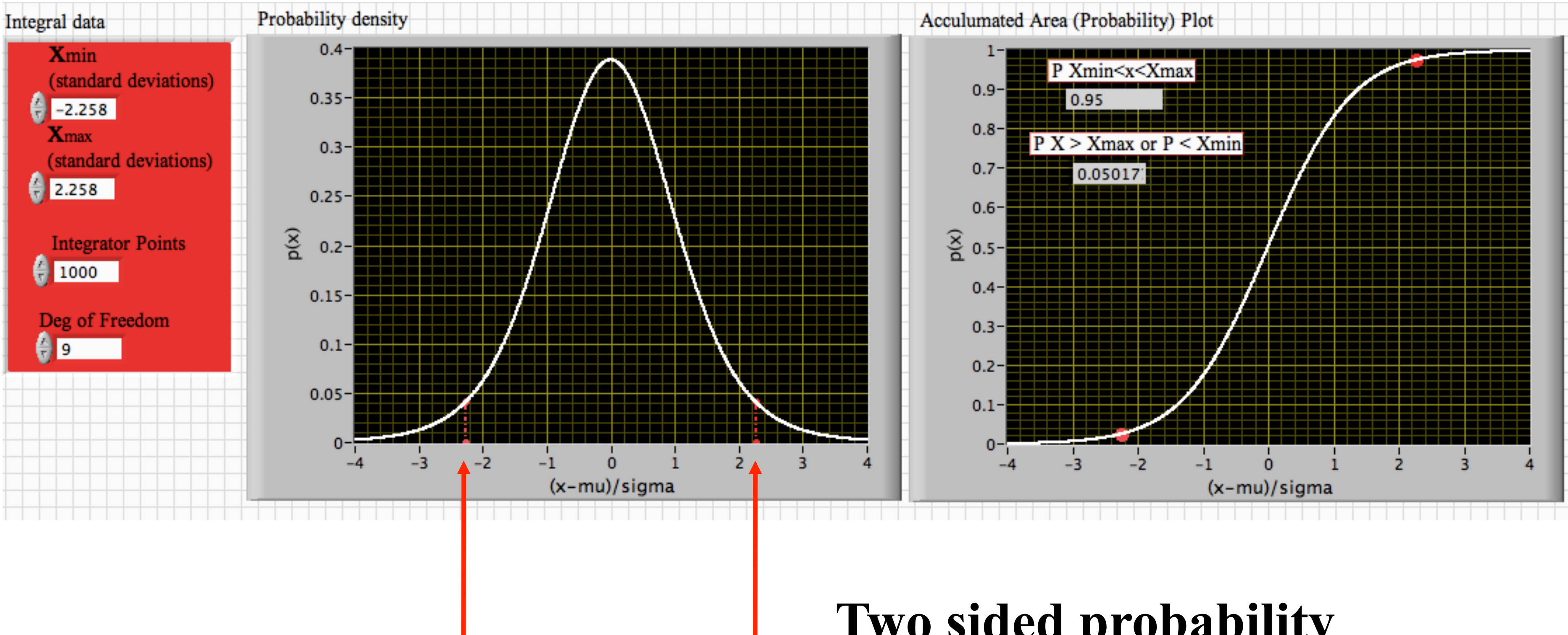
- *Find t-value from student-t curve corresponding to 95% confidence level*




$$t_{0/95/2,\nu=9} = 2.258$$

# "Green Propellants" (solution)

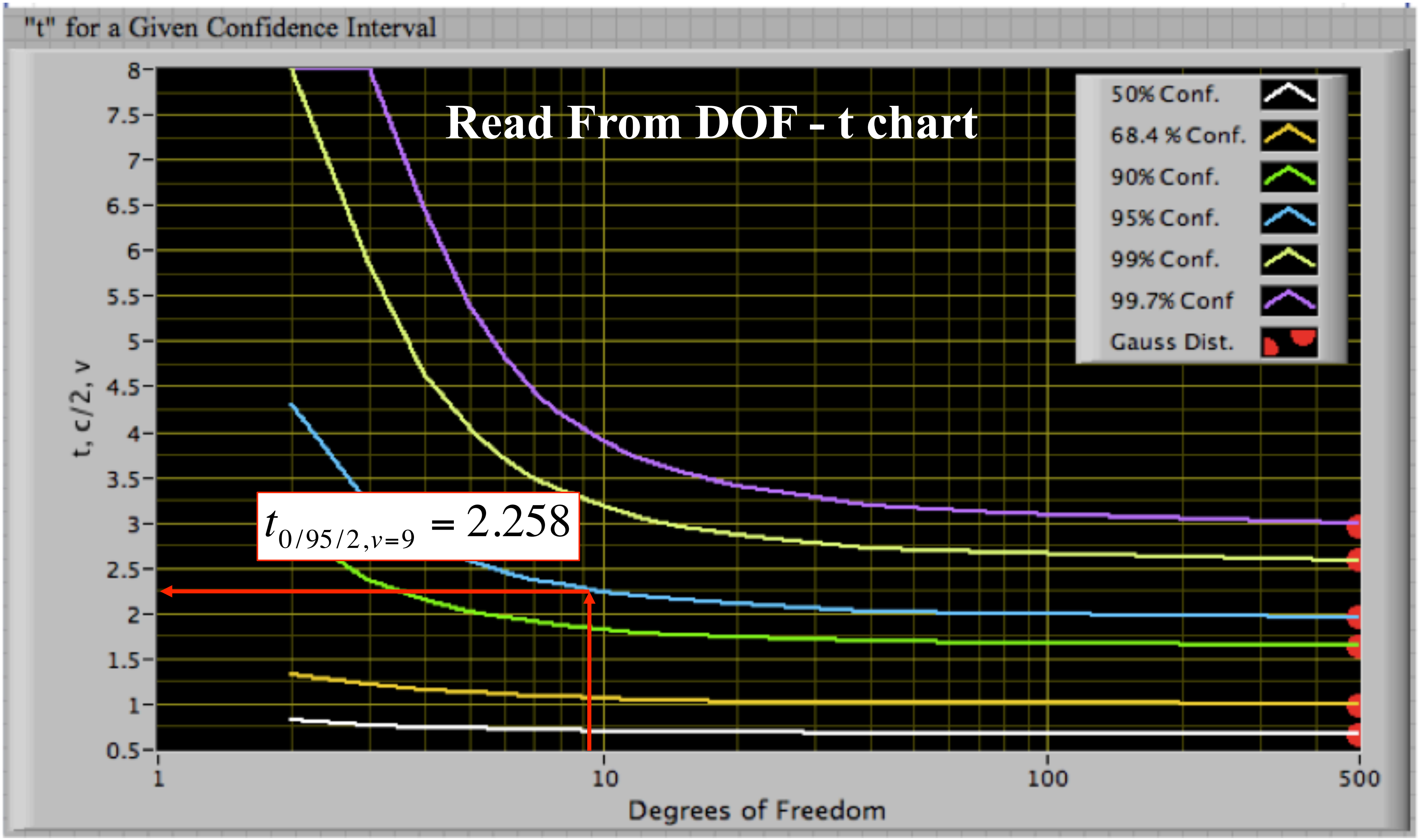
- *Find t-value from student-t curve corresponding to 95% confidence level*




**Two sided probability**

**Gives same solution**

# "Green Propellants" (solution)



# “Green Propellants” (solution)

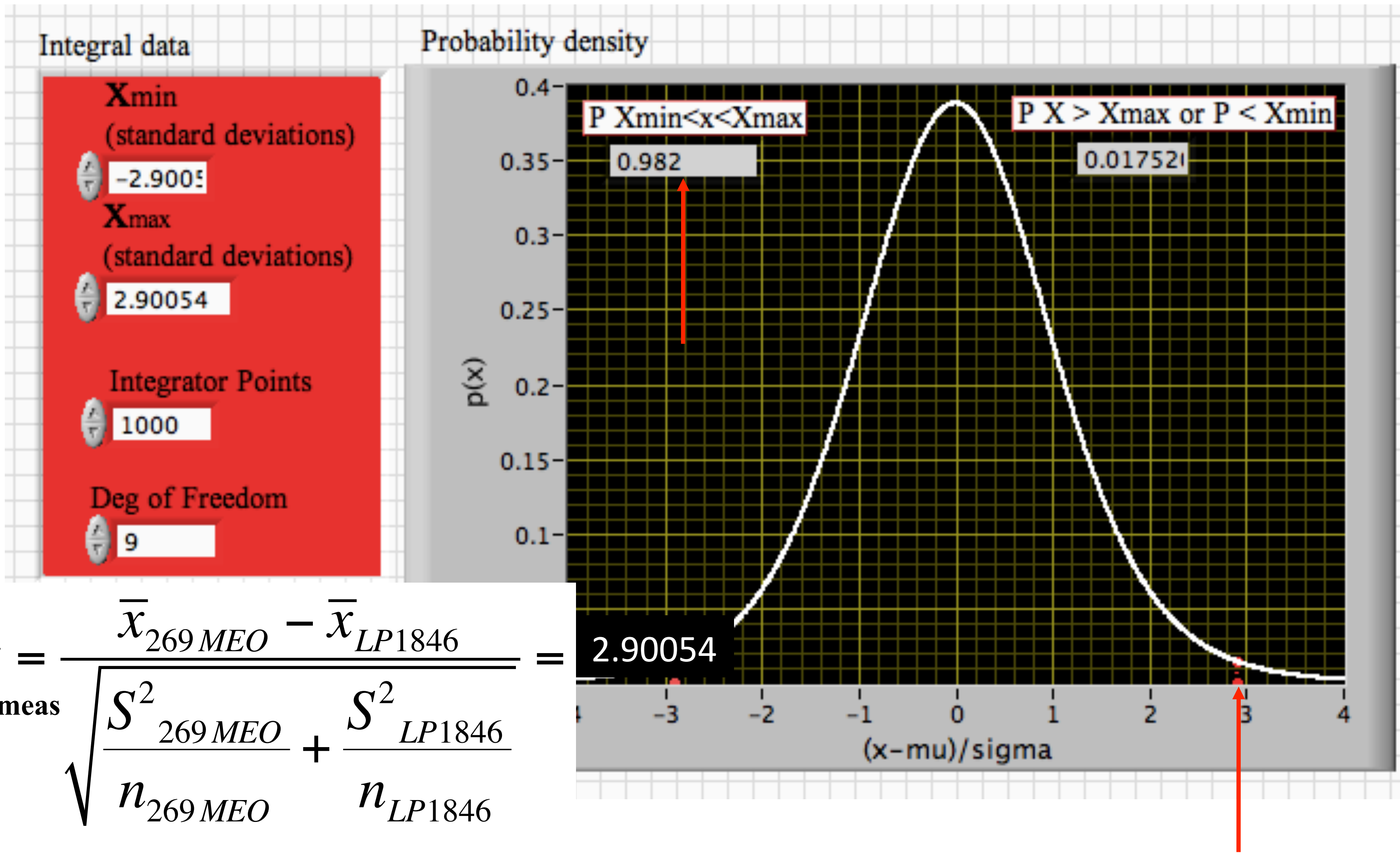
*Compare t-values*

$$t_{\text{meas}} = \frac{\bar{x}_{269\,MEO} - \bar{x}_{LP1846}}{\sqrt{\frac{S^2_{269\,MEO}}{n_{269\,MEO}} + \frac{S^2_{LP1846}}{n_{LP1846}}}} = 2.90054 > t_{c/2,\,v} = 2.258$$

…. Performance of HAN 269MEO15 is statistically better than HAN LP1846 at a 95% confidence level

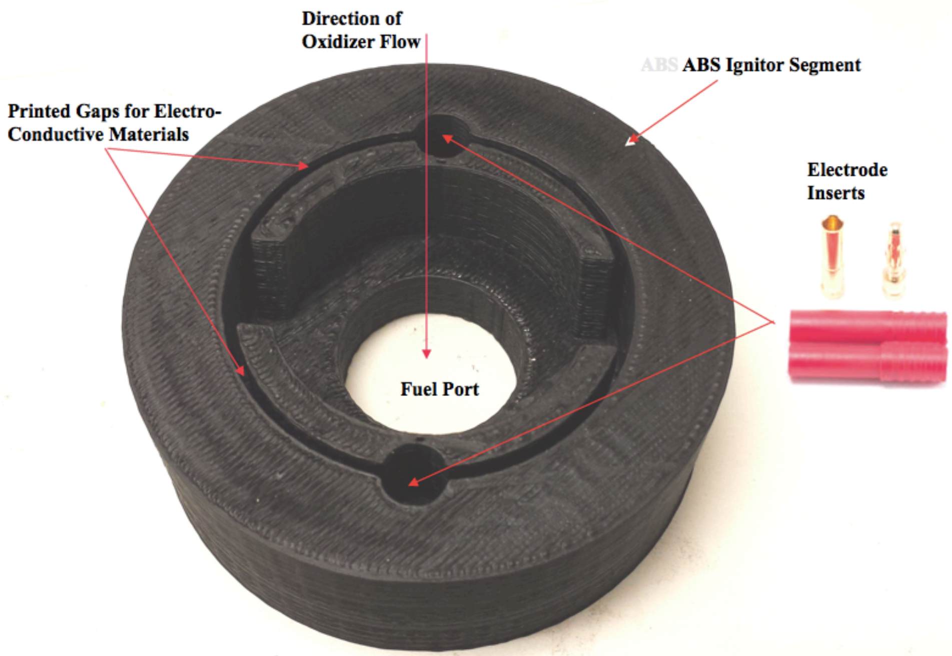
…. In fact the statistical difference is valid up to a 98.2% confidence level!

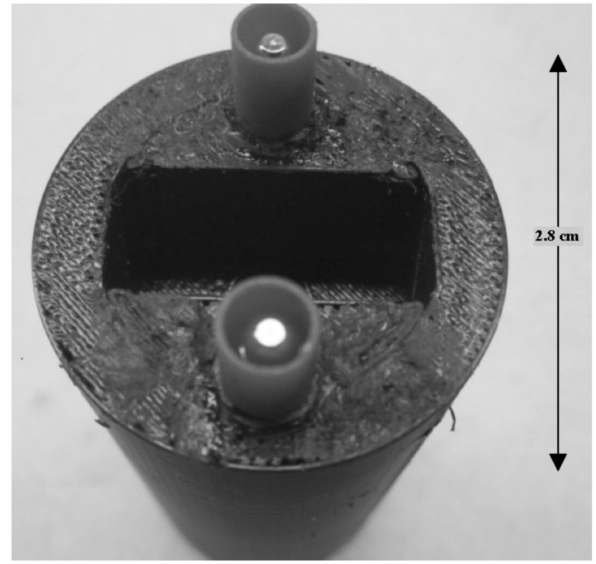
# "Green Propellants" (solution)

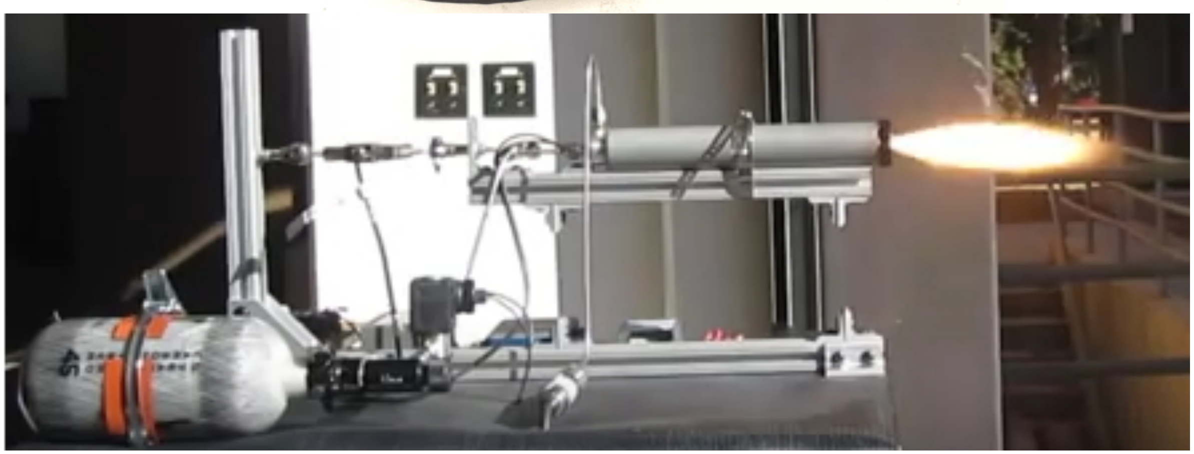


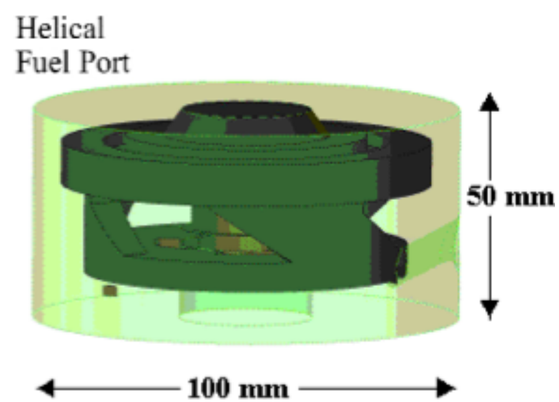

$$t_{meas} = \frac{\bar{x}_{269\,MEO} - \bar{x}_{LP1846}}{\sqrt{\frac{S^2_{269\,MEO}}{n_{269\,MEO}} + \frac{S^2_{LP1846}}{n_{LP1846}}}} = 2.90054$$

# Problem 2: "3-D Printed Fuel Grains"





# "3-D Printed Fuel Grains"

• We've Been asked to Measure the enthalpy of vaporization of a candidate thermo-plastic hybrid rocket fuel made from acrylonitrile butadiene styrene (ABS).

• The enthalpy of vaporization $h_v$ is critical to ensure proper fuel ignitability and performance.

• A series of pyrolysis tests are performed using three different sample sets:

1. Sample set 1 is made from extruded ABS
2. Sample set 2 is made from ABS, but additively printed using a 3-D FDM printer set at the highest print density, $\rho_1$.
3. Sample set 3 is made from printed ABS, but printed with approximately 15% lower density, $\rho_2$.

**Table 1 Summarizes the test results**

# Table 1: Pyrolysis Experiment Results

| Sample # | Extruded (1) $h_v$, MJ/kg | Extruded $\rho$, g/cm³ | Printed(2) $h_v$, MJ/kg | Printed $\rho$, g/cm³ | Printed(3) $h_v$, MJ/kg | Printed $\rho$, g/cm³ |
|---|---|---|---|---|---|---|
| 1 | 3.102 | 1.0324 | 3.005 | 0.9573 | 2.832 | 0.8064 |
| 2 | 3.050 | 1.0267 | 3.081 | 0.9726 | 2.657 | 0.8156 |
| 3 | 3.430 | 1.0502 | 2.965 | 0.9782 | 3.124 | 0.8157 |
| 4 | 3.177 | 1.0163 | 3.062 | 0.9905 | 3.002 | 0.8019 |
| 5 | 3.031 | 1.0354 | 3.134 | 0.9695 | 3.300 | 0.8167 |
| 6 | 3.371 | 1.0517 | 3.071 | 0.9523 | 2.912 | 0.8671 |
| 7 | 3.334 | 0.9947 | 3.132 | 0.9788 | 2.680 | 0.8505 |
| 8 | 3.213 | 1.0276 | 3.111 | 1.0004 | 2.744 | 0.8332 |
| $\overline{x}$ | ***3.2135*** | ***1.0294*** | ***3.07013*** | ***0.9749*** | ***2.9064*** | ***0.8259*** |
| $S_x$ | ***0.1512*** | ***0.01839*** | ***0.0598*** | ***0.01593*** | ***0.2558*** | ***0.0227*** |

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# Pyrolysis Experiment Results Assessment

## • Confidence Intervals on Mean Values

(i) Based on a 95% confidence level and assuming a student-t probability distribution calculate, the confidence intervals for the mean estimates of $h_v$ and $\rho$ for each of the three sample sets.

ii) Sketch a plot with the mean density ( ρ )of each sample set plot on the x-axis and the mean enthalpy of vaporization plotted on the y axis --- Show error bars on the data point based on the calculated intervals calculated in part i)

## • t-Test to Assess Effects of "Extrusion vs. Printing"

i) Assess the effect of extrusion versus additive manufacturing on the resulting enthalpy vaporization by performing a t-test comparing the mean enthalpies of vaporization of the extruded sample 1 versus the high density printed sample (2)

ii) What can you conclude about the two sample sets?

• ***Assume a 95% confidence level***

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# Pyrolysis Experiment Results Assessment (2)

- **t-test to Assess Effects of Material Density**

i) Assess the effect of material density on the resulting enthalpy vaporization by performing a t-test comparing the mean enthalpies of vaporization of the high density printed sample (2) versus the low density printed sample (3)

ii) Can you statistically conclude anything about the effects of print density on the enthalpy of vaporization?

- ***Assume a 95% confidence level***

# Solution

**95% Confidence Interval Calculation, Enthalpy of Vaporization, $h_v$, Material Density ρ.**

| | Sample set 1 Extruded $h_v$ | Sample set 1 Extruded ρ | Sample set 2 Printed $h_v$ | Sample set 2 Printed ρ | Sample set 3 Printed $h_v$ | Sample set 3 Printed ρ |
|---|---|---|---|---|---|---|
| Z c/2 | 2.36286 | 2.36286 | 2.36286 | 2.36286 | 2.36286 | 2.36286 |
| Mean, Low | 3.0872 | 1.01403 | 3.02014 | 0.961648 | 2.71777 | 0.806924 |
| Mean, High | 3.3398 | 1.04472 | 3.12011 | 0.988252 | 3.09498 | 0.844851 |
| Confidence Interval | 0.126301 | 0.0153428 | 0.0499812 | 0.0133015 | 0.188608 | 0.0189632 |

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# Solution (2)

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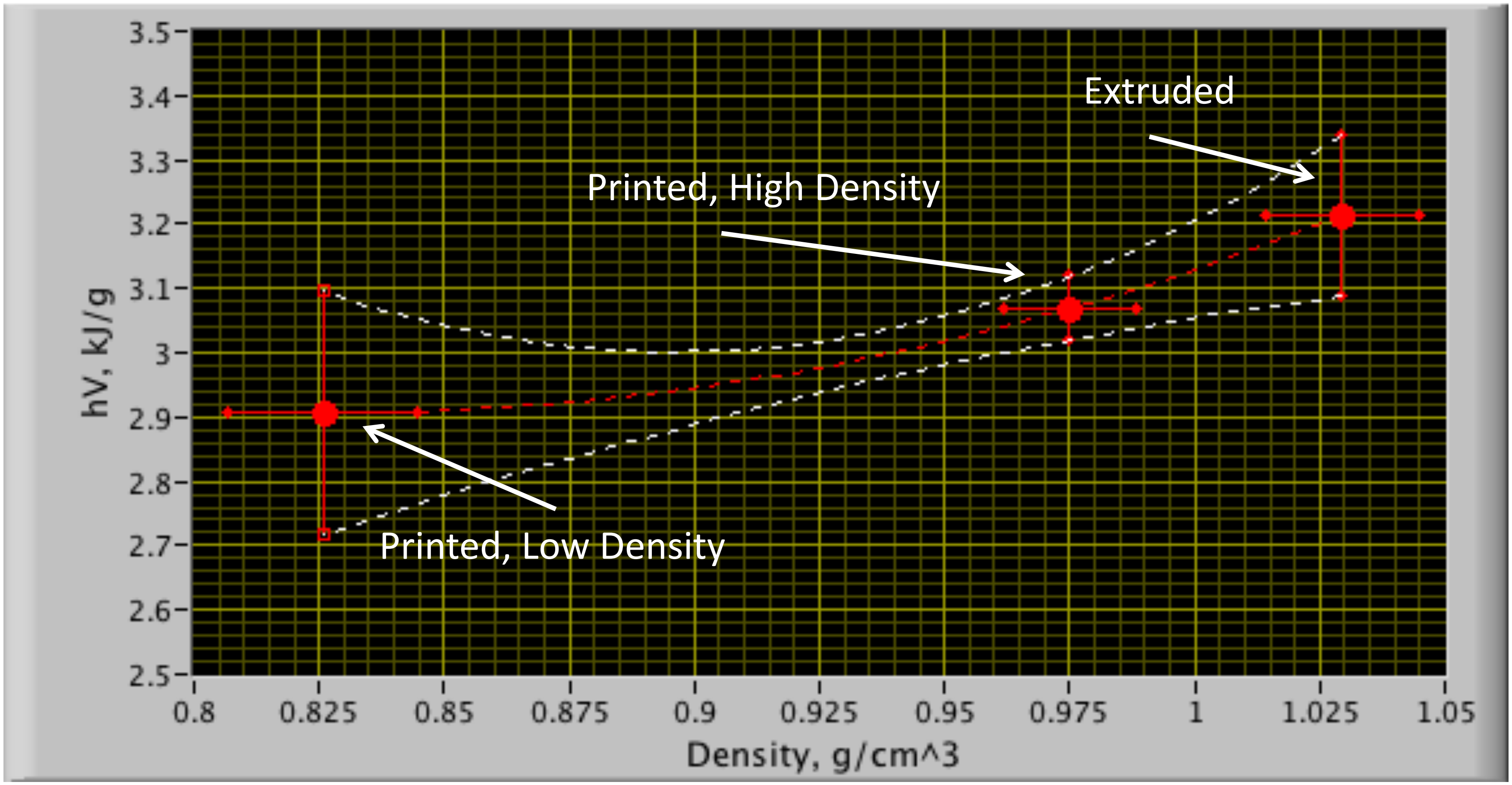
hv Plots




**Data Plot with 95% Error boundaries**

**→ is the red line statistically significant?**

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# Effect of Printing vs Extrusion

| Sample # | Extruded (1) $h_V$, MJ/kg | Printed(2) $h_V$, MJ/kg |
|---|---|---|
| 1 | 3.102 | 3.005 |
| 2 | 3.050 | 3.081 |
| 3 | 3.430 | 2.965 |
| 4 | 3.177 | 3.062 |
| 5 | 3.031 | 3.134 |
| 6 | 3.371 | 3.071 |
| 7 | 3.334 | 3.132 |
| 8 | 3.213 | 3.111 |
| $\overline{x}$ | ***3.2135*** | ***3.0713*** |
| $S_x$ | ***0.1512*** | ***0.0598*** |

t-test 1 comparison of Samples 1 and 2

$$t_{meas} = \frac{\left|h_{v_{(1)}} - h_{v_{(2)}}\right|}{\sqrt{\frac{S_{x_{(1)}}^2}{n_{(1)}} + \frac{S_{x_{(2)}}^2}{n_{(2)}}}} = \frac{3.2135 - 3.07013}{\left(\frac{0.151187^2}{8} + \frac{0.0598294^2}{8}\right)^{0.5}}$$

**=2.494**

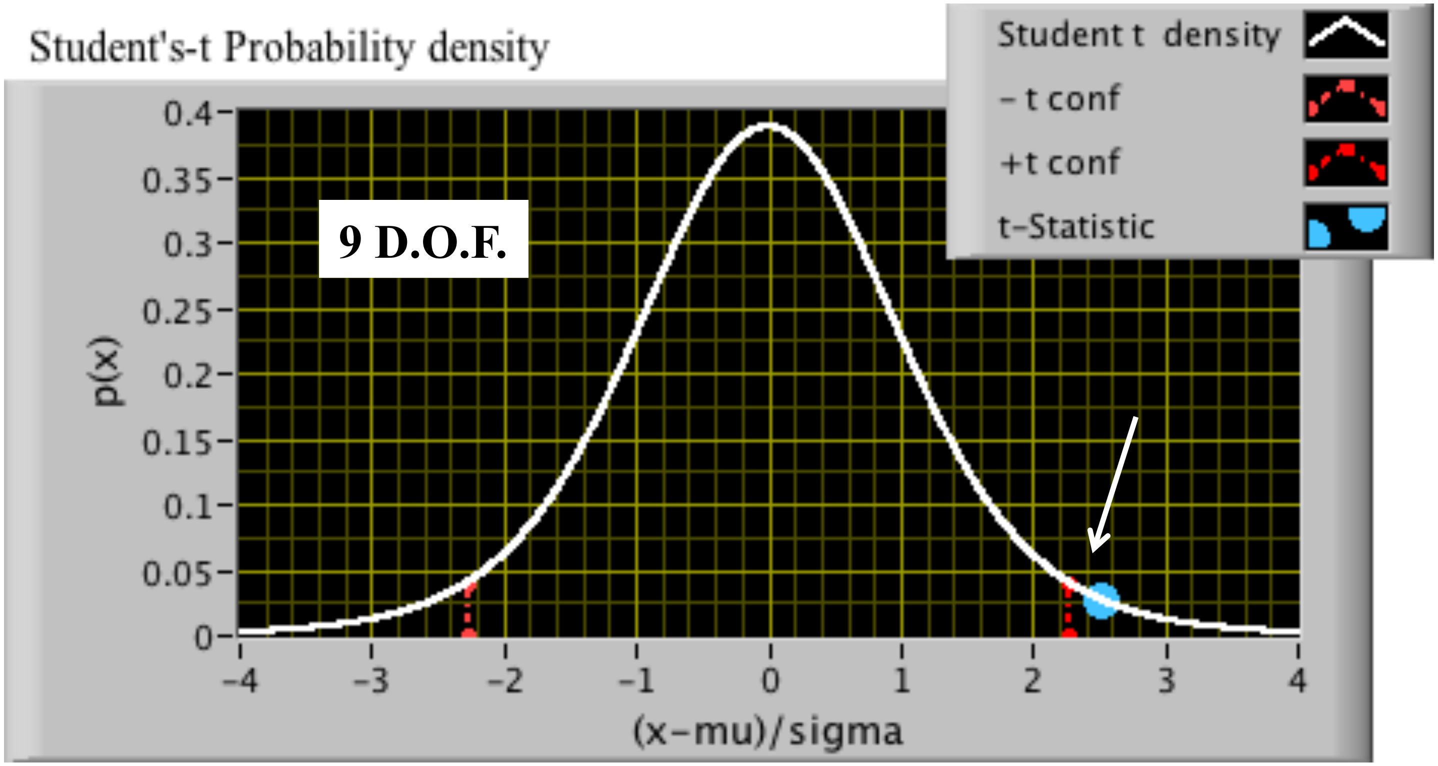
Compute Effective degrees of freedom

$$\nu_{DOF} = \frac{\left[\left(\frac{S_{x_{(1)}}^2}{n_{(1)}}\right) + \left(\frac{S_{x_{(2)}}^2}{n_{(2)}}\right)\right]^2}{\frac{\left(\frac{S_{x_{(1)}}^2}{n_{(1)}}\right)^2}{n_{(1)} - 1} + \frac{\left(\frac{S_{x_{(2)}}^2}{n_{(2)}}\right)^2}{n_{(2)} - 1}} = \frac{\left(\frac{0.1512^2}{8} + \frac{0.0598294^2}{8}\right)^2}{\frac{\left(\frac{0.1512^2}{8}\right)^2}{8 - 1} + \frac{\left(\frac{0.0598294^2}{8}\right)^2}{8 - 1}}$$

**= 9.1396 ~ 9 D.O.F**

# Effect of Printing vs Extrusion (2)

Student's-t Probability density




t-test statistics

| DOF | S2 xbar 1 | S2 xbar 2 |
|---|---|---|
| 9 | 0.00285718 | 0.00044744 |

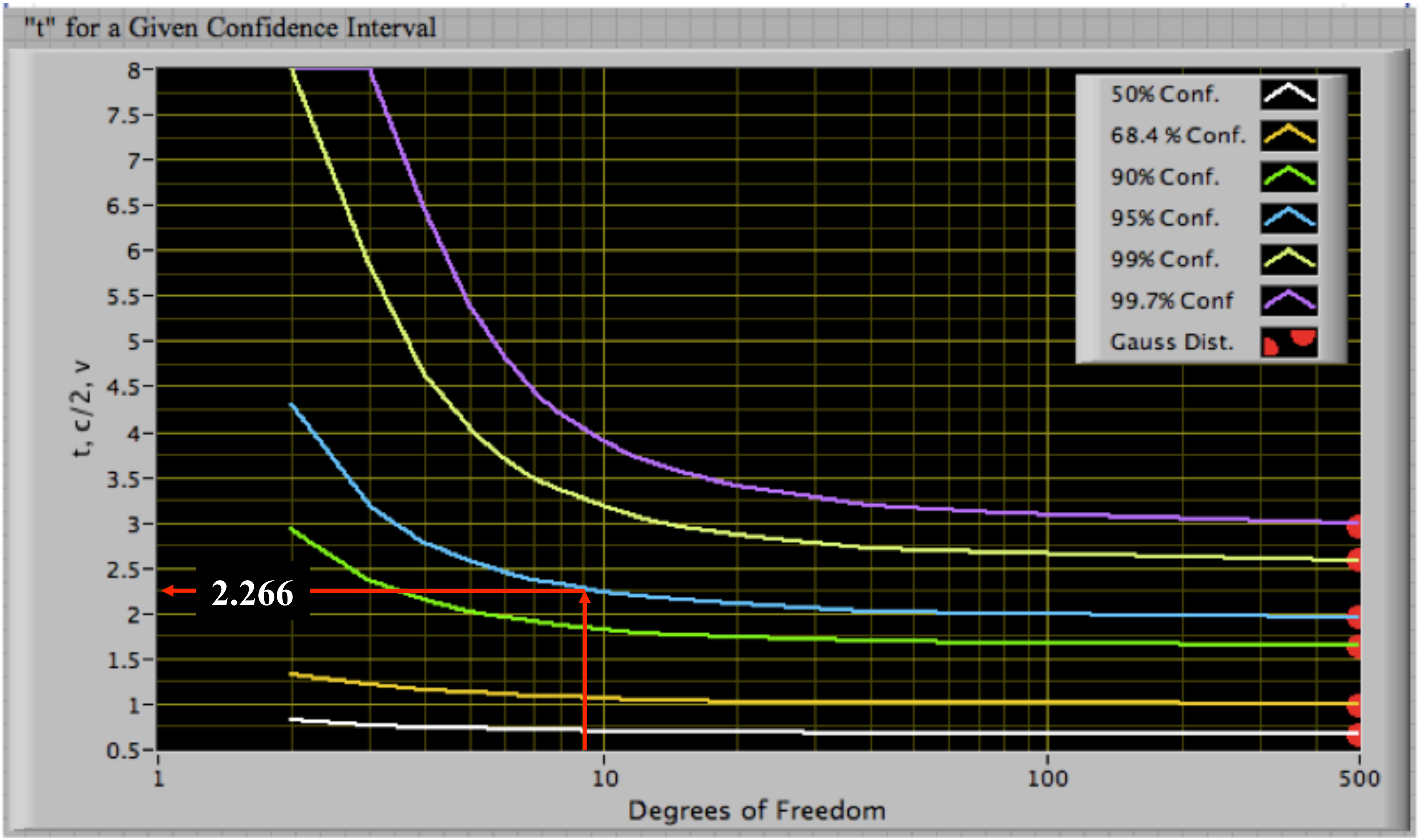
| RMS 1, 2 | t-Statistics | 95% Conf t-Value |
|---|---|---|
| 0.0574858 | 2.49409 | 2.26604 |

**DOF = 9 → $t_{95\%\ Confidence} = 2.266$**

**$t_{meas} = 2.393 > t_{95\%\ Confidence} = 2.266$**

**Printing Fuel Material Lowers $h_v$.**

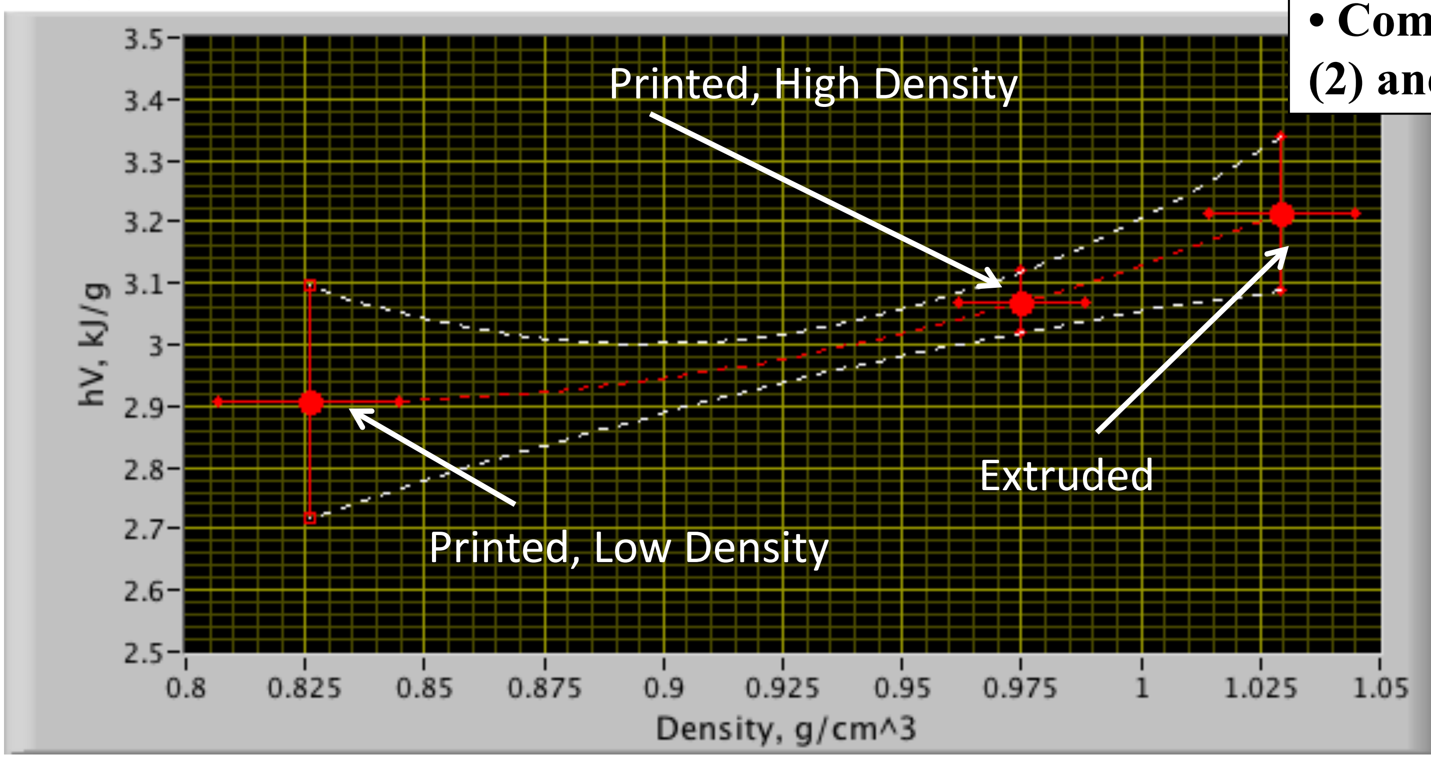
# Effect of Printing vs Extrusion (3)



# Effect of Material Density

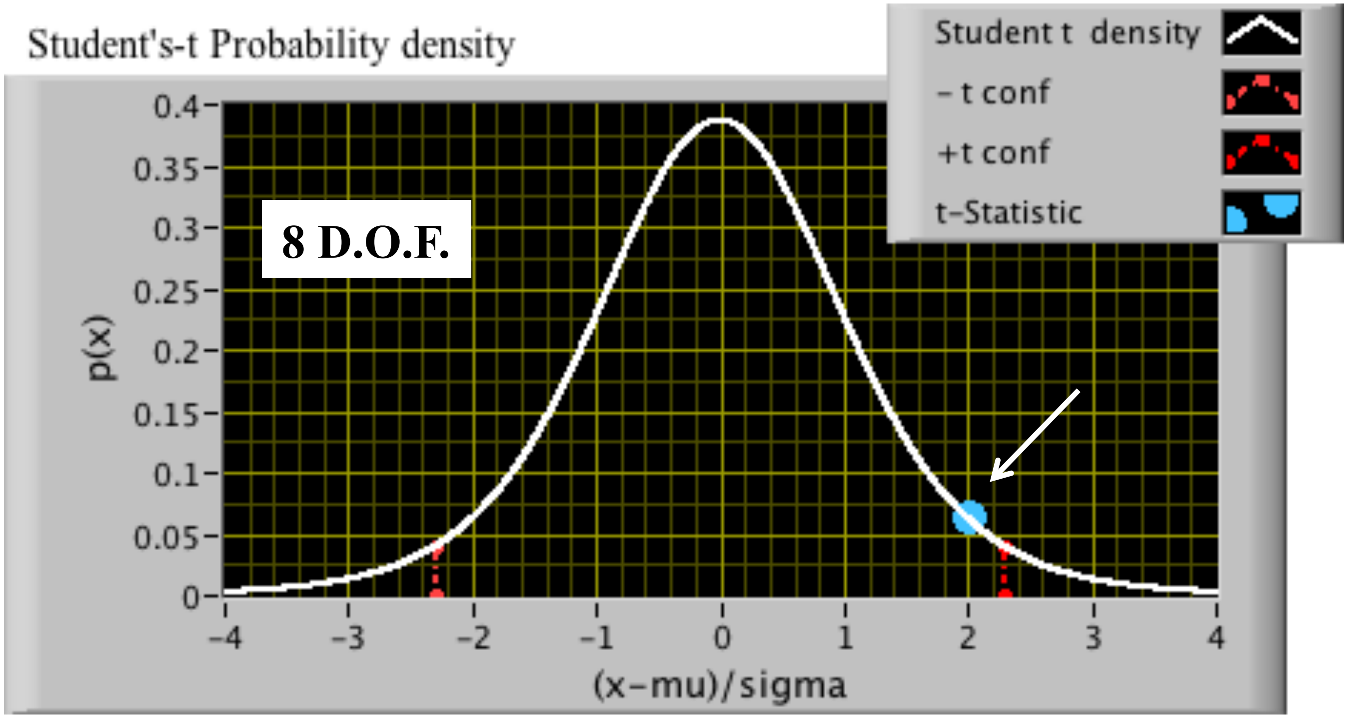
| Sample # | Extruded (1) $h_V$, MJ/kg | Extruded $\rho$, g/cm$^3$ | Printed(2) $h_V$, MJ/kg | Printed $\rho$, g/cm$^3$ | Printed(3) $h_V$, MJ/kg | Printed $\rho$, g/cm$^3$ |
|---|---|---|---|---|---|---|
| $\overline{x}$ | ***3.2135*** | ***1.0294*** | ***3.07013*** | ***0.9749*** | ***2.9064*** | ***0.8259*** |
| $S_x$ | ***0.1512*** | ***0.01839*** | ***0.0598*** | ***0.01593*** | ***0.2558*** | ***0.0227*** |

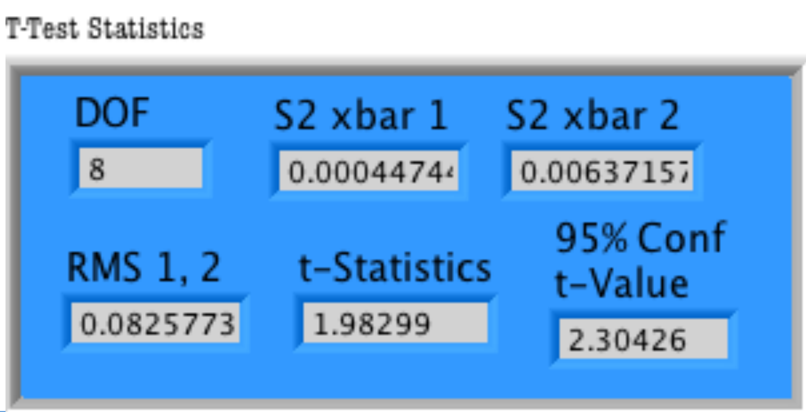
hv Plots




- **Compare Samples (2) and (3)**

# Effect of Material Density(2)



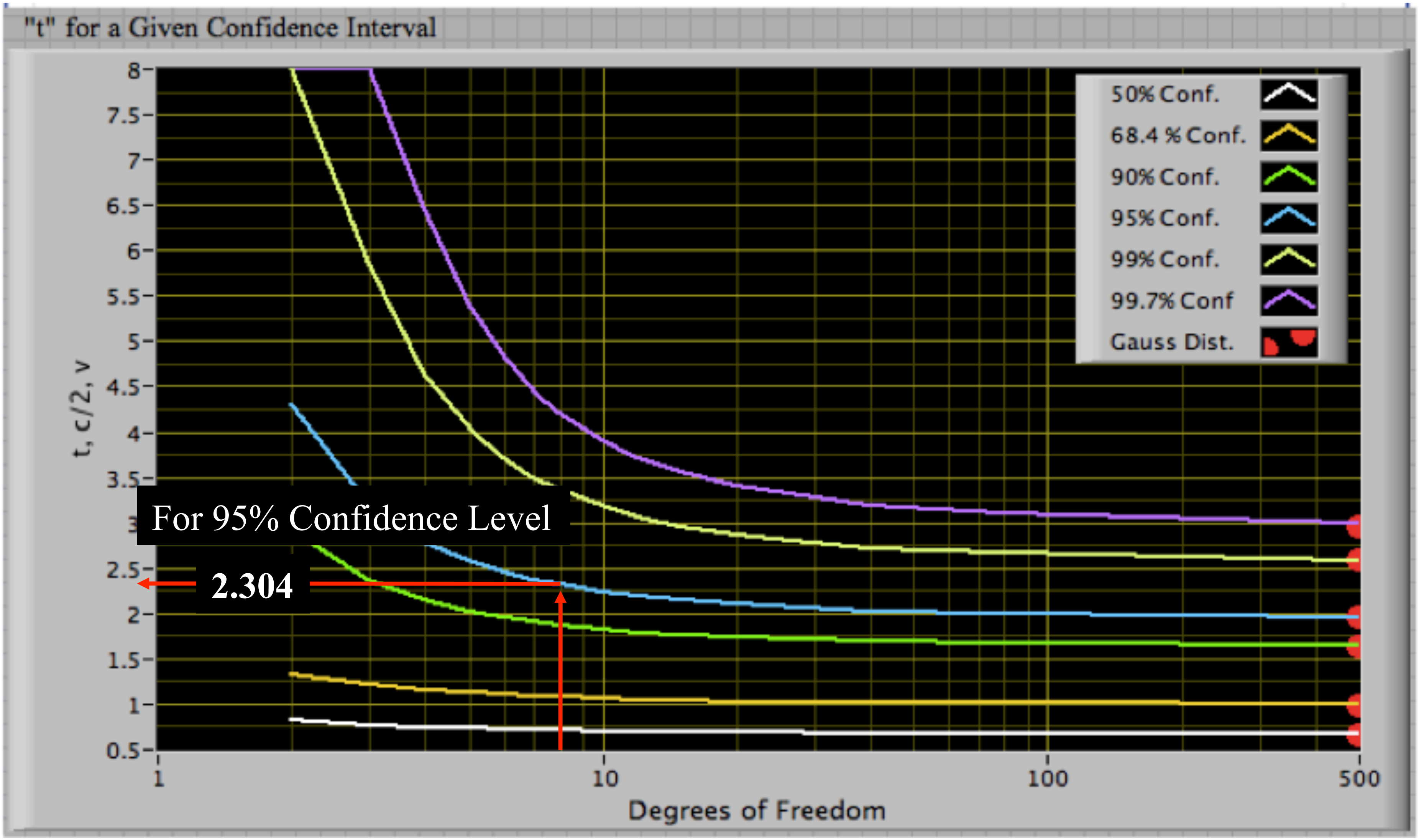




$\text{DOF} = 8 \rightarrow t_{95\%\ \text{Confidence}} = 2.304$

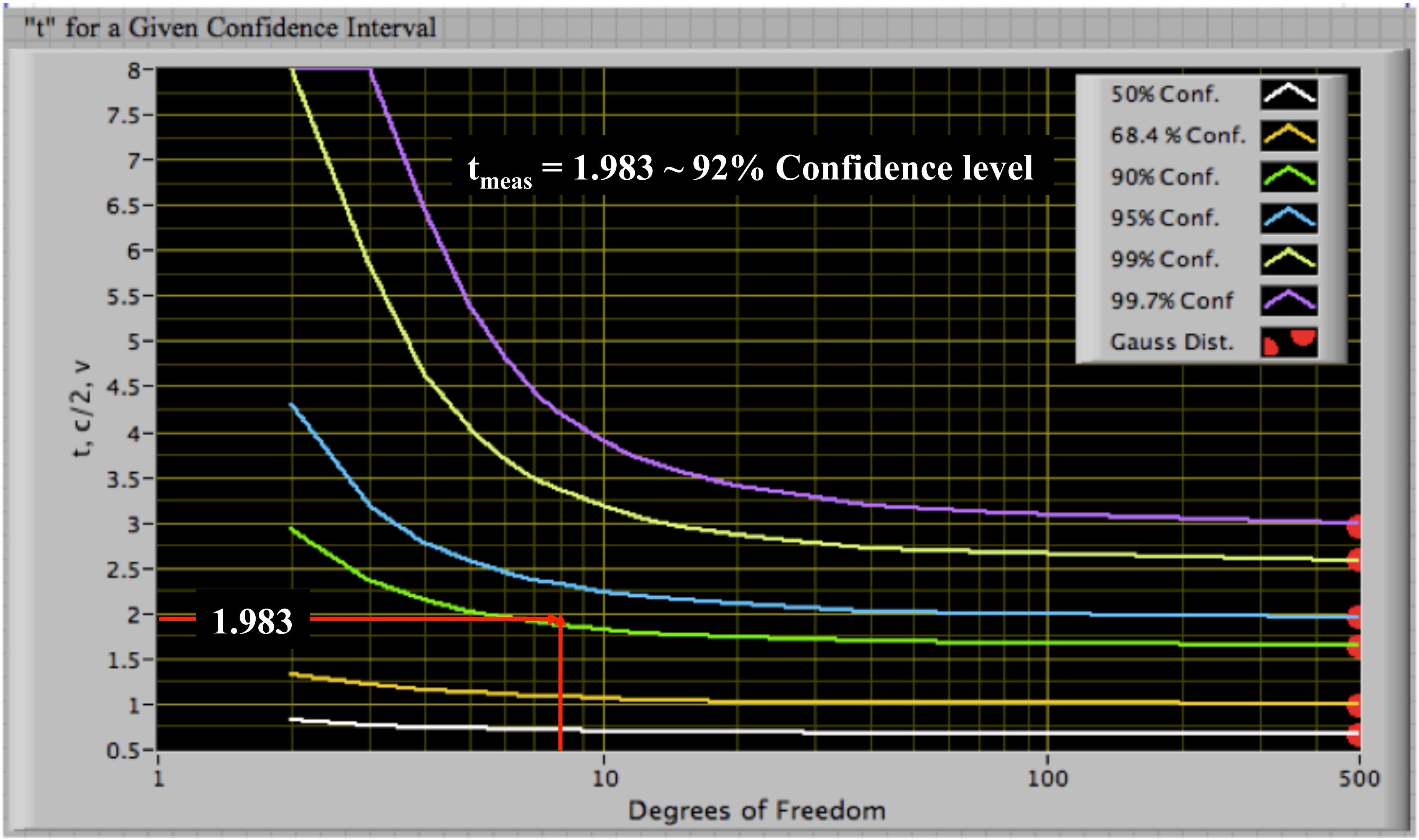
$t_{meas} = 1.983 > t_{95\%\ \text{Confidence}} = 2.3046$

**Fuel Density Not factor on $h_v$ (@95%)**

# Effect of Material Density(3)

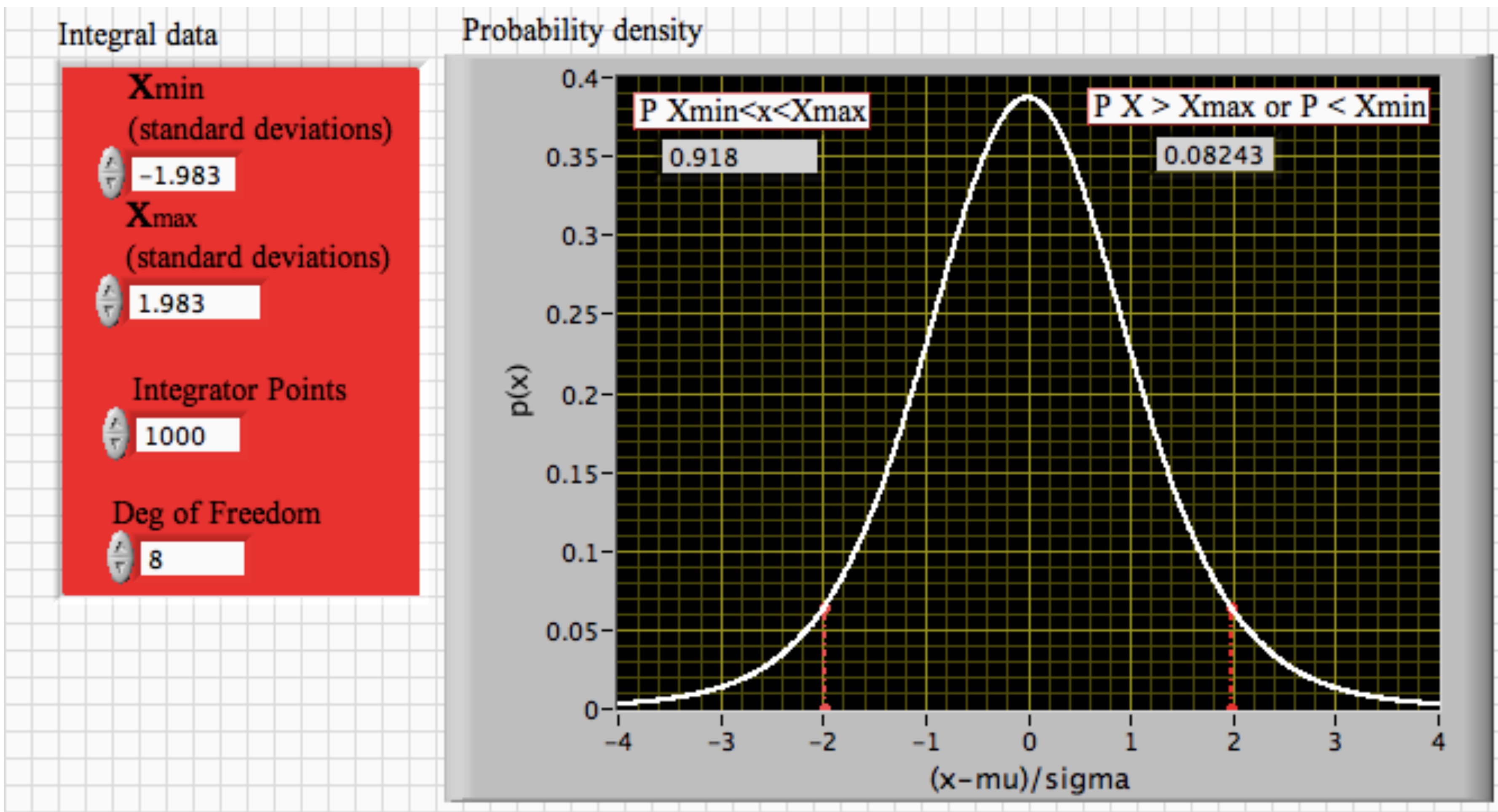


# Effect of Material Density(4)



# Effect of Material Density (5)

**$t_{meas}$ = 1.983 ~ 92% Confidence Level**




- *So there is a likely material density effect .. But data only show a 92% confidence level*