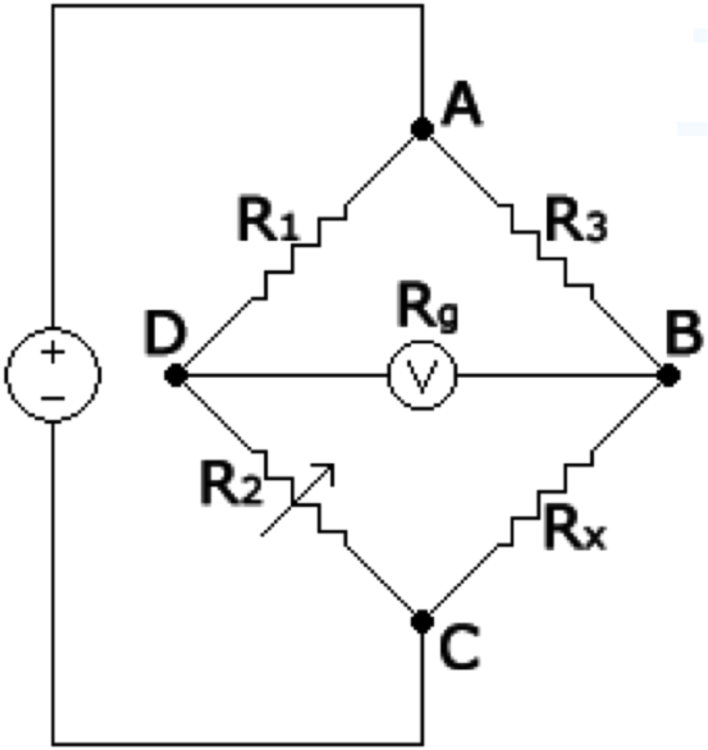
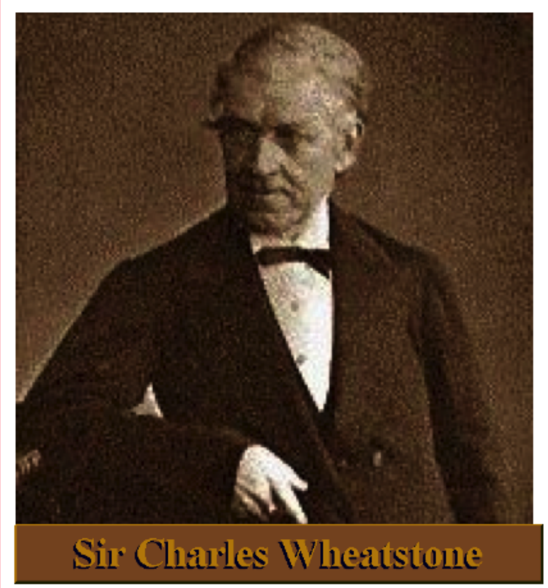
UtahState UNIVERSITY

Mechanical & Aerospace Engineering

Section 4.1 : Introduction to the Fundamental Tool of Modern Measurement, The Resistance Bridge



Beckwith Chapter 7
Sections 7.7-7.9



Resistance and Resistivity (1)

• ***Electrical resistivity*** is a measure of how strongly a material opposes the flow of electric current.

• A low resistivity indicates a material that readily allows the movement of electrical charge.

• The SI unit of electrical resistivity is the ohm meter.

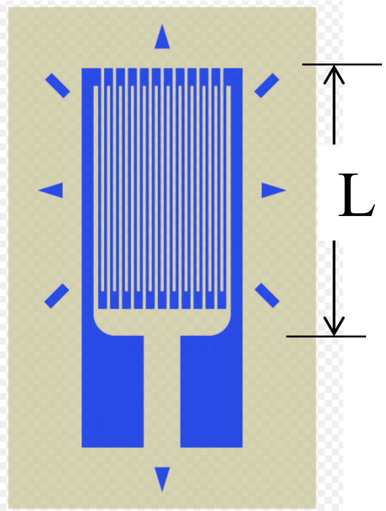
• The ***Resistance*** of Specimen is calculated by

$$R = \rho \cdot \frac{L}{A}$$

ρ = resistivity of material *(Ω-m)*
L = length of specimen (m)
A = cross sectional area of specimen (m^2)

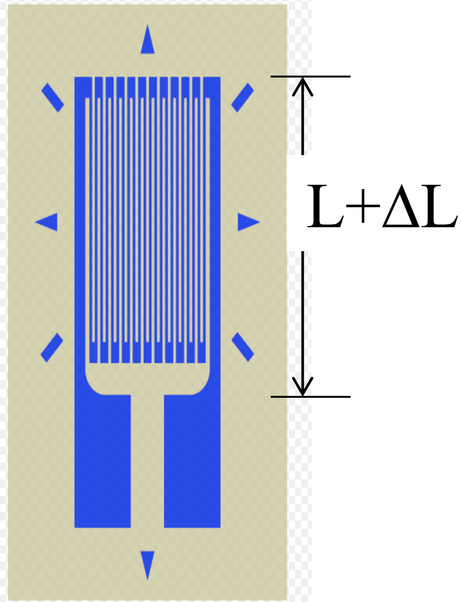
Resistance and Resistivity (2)

• ***Now consider a device made of a material with a known Resistivity …. And the design is far more sensitive to strain in the vertical direction than in the horizontal direction.***



• Now stretch the device ….

• Cross section does not change Much … but length changes significantly



$$R = \rho \cdot \frac{L}{A}$$

$$R + \Delta R = \rho \cdot \left(\frac{L + \Delta L}{A - \Delta A} \right)$$

Resistance and Resistivity (3)

- Normalize by R and collect terms

$$\frac{R+\Delta R}{R}=\left(\frac{L+\Delta L}{A-\Delta A}\right)\frac{A}{L}=\left(\frac{L+\Delta L}{L}\right)\frac{A}{A-\Delta A}\rightarrow$$

$$1+\frac{\Delta R}{R}=\left(1+\frac{\Delta L}{L}\right)\left(\frac{A}{A-\Delta A}\right)\rightarrow$$

$$\frac{\Delta R}{R}=1-\left(\frac{A}{A-\Delta A}\right)+\left(\frac{\Delta L}{L}\right)\left(\frac{A}{A-\Delta A}\right)$$

$$1-\left(\frac{A}{A-\Delta A}\right)=\left(\frac{\Delta A}{A-\Delta A}\right)$$ For small deflections ~ 0

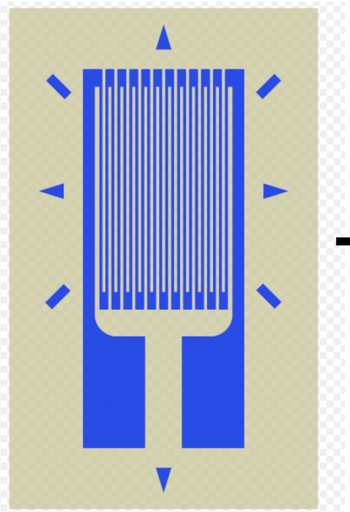
Resistance and Resistivity (4)

- Solving for area ratio

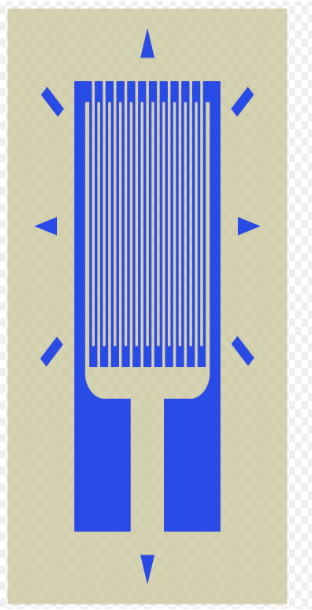
$$\frac{\Delta R}{R} = \left(\frac{\Delta L}{L}\right)\left(\frac{A}{A-\Delta A}\right) \rightarrow \left(\frac{A}{A-\Delta A}\right) \equiv G_F = \frac{\Delta R}{R} \Big/ \varepsilon$$

G_F ---> Gauge factor

ε ---> linear strain



$$\Delta R = G_F \varepsilon R_g$$



- ***Strain Gauge … change in resistance is proportional To the strain on the sensor***

Strain Gauge (1)

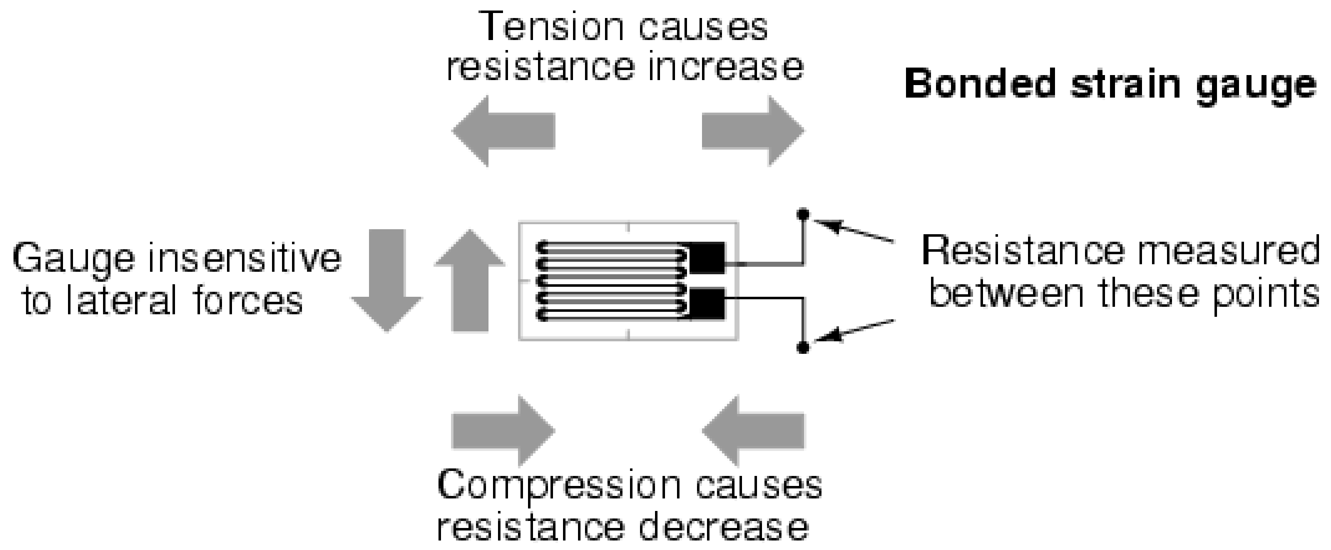
- A **strain gauge** is a sensor used to measure deformation of an object.

- Most common type of strain gauge consists of an insulating flexible backing which supports a metallic foil pattern.

- Gauge is attached to the object by a suitable adhesive.

- As object being tested is deformed, the foil is deformed, causing its electrical resistance to change …. Which should be sensible As a change in current thru sensor …

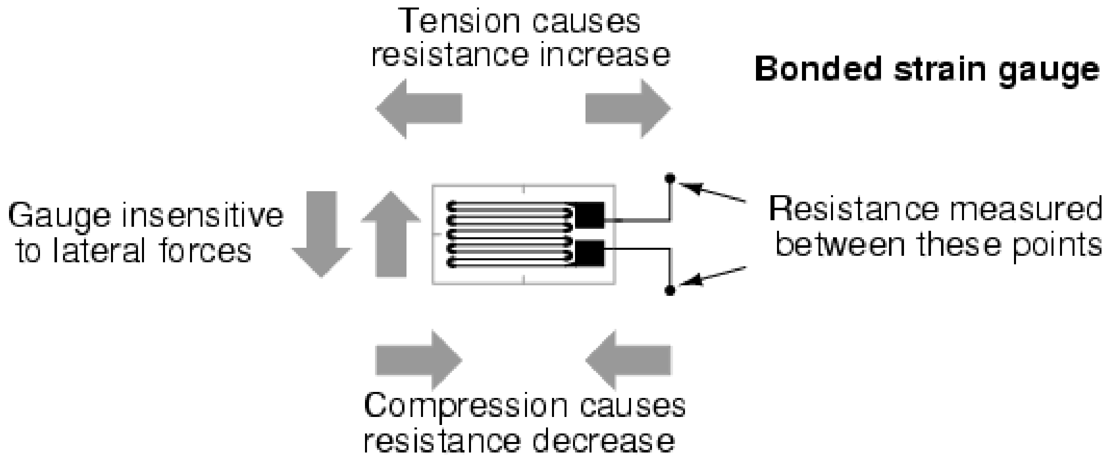
Strain Gauge (2)



- After being strained, gauge is designed to “bounce-back” to original shape

- Given limits imposed by the elastic limits of the gauge material and test specimen, Resistance changes only a small fraction of a percent (~0.1%)for full force range of the gauge.

Strain Gauge (3)



- Forces great enough to induce greater resistance changes permanently deform the test specimen and/or gauge.

- In order to use the strain gauge as a practical instrument, must measure very small changes in resistance with high accuracy.

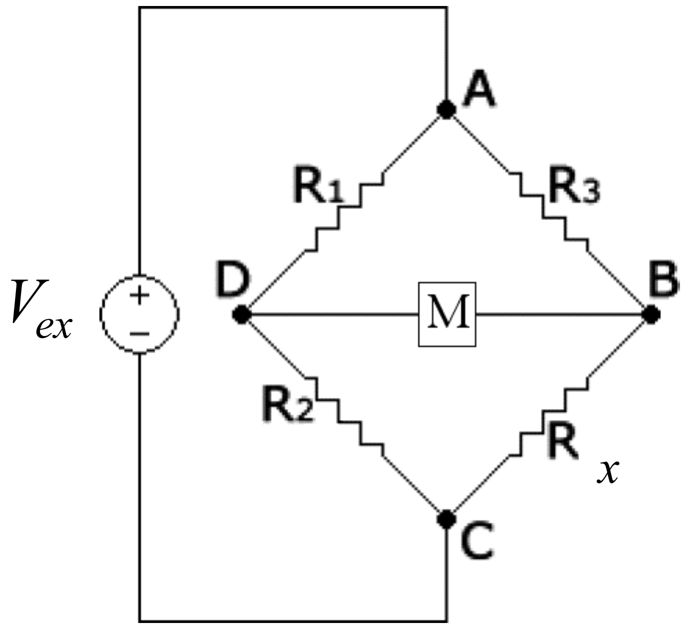
- Typical strain gauge resistances range from 30 Ω to 100 Ω (unstressed).

- ***ΔR ~ 0.03 to 0.1 Ω*** … a very difficult task for a multimeter

Wheatstone Bridge (1)

- Need a device that amplifies the changes in resistance

- Consider the circuit



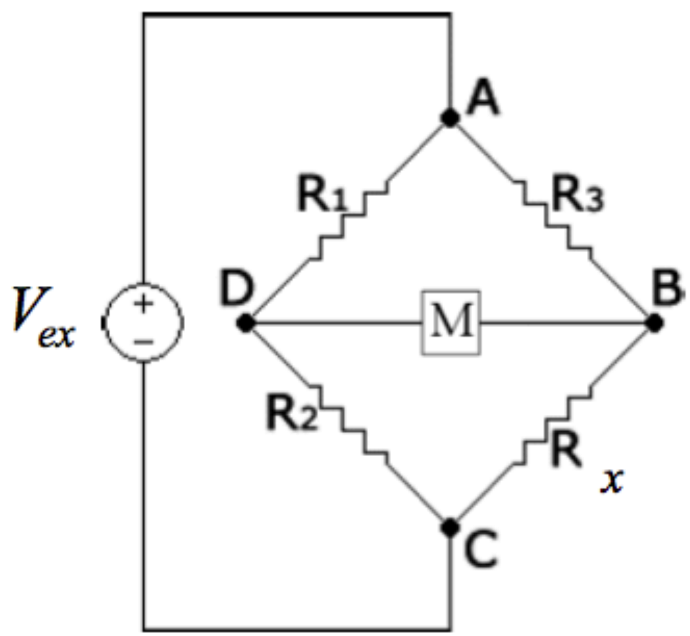
- What will voltage Across the volt Meter terminals {*D*,*B*} read?

… Oh Boy …
… this will be fun …

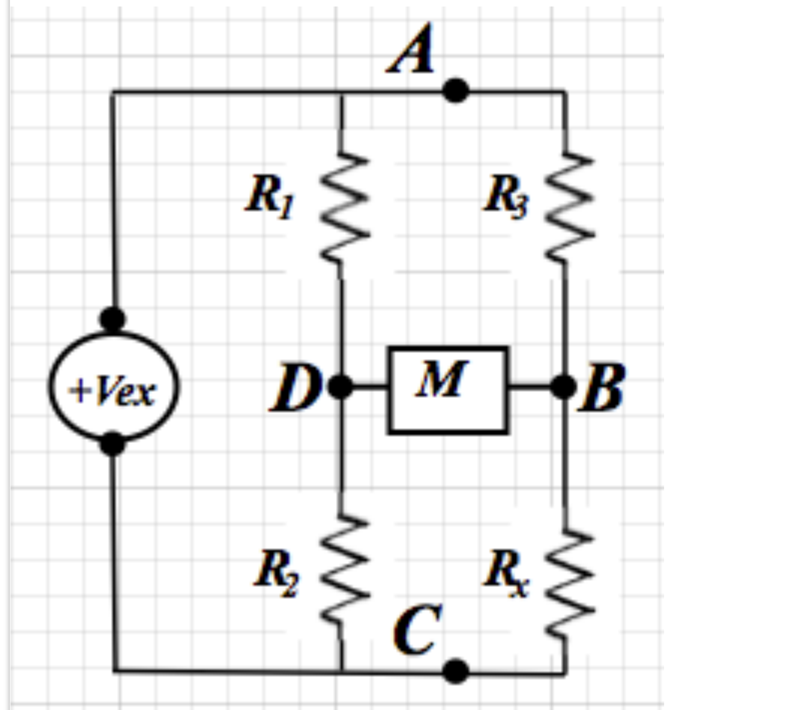
Wheatstone Bridge (1)

- Need a device that amplifies the changes in resistance

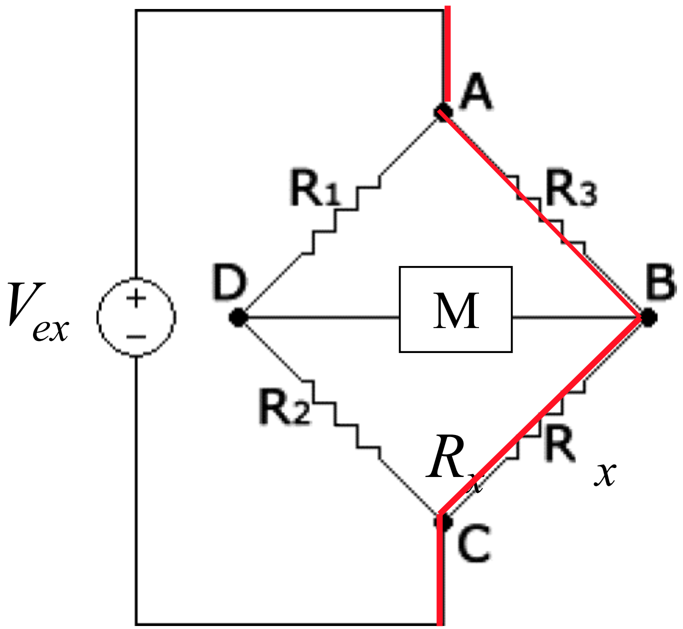
  - Consider the circuit



REDRAW CIRCUIT AS PAIR OF VOLTAGE DIVIDERS

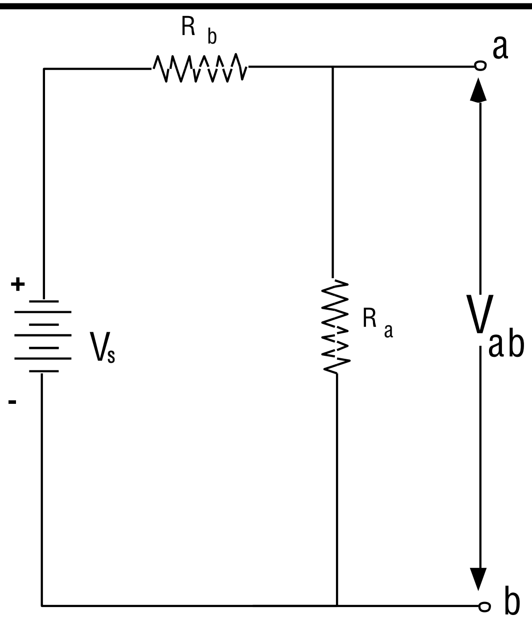


Wheatstone Bridge (2)



- What is Voltage Drop from *B to C*?

…
Looks like Voltage Divide Circuit
…

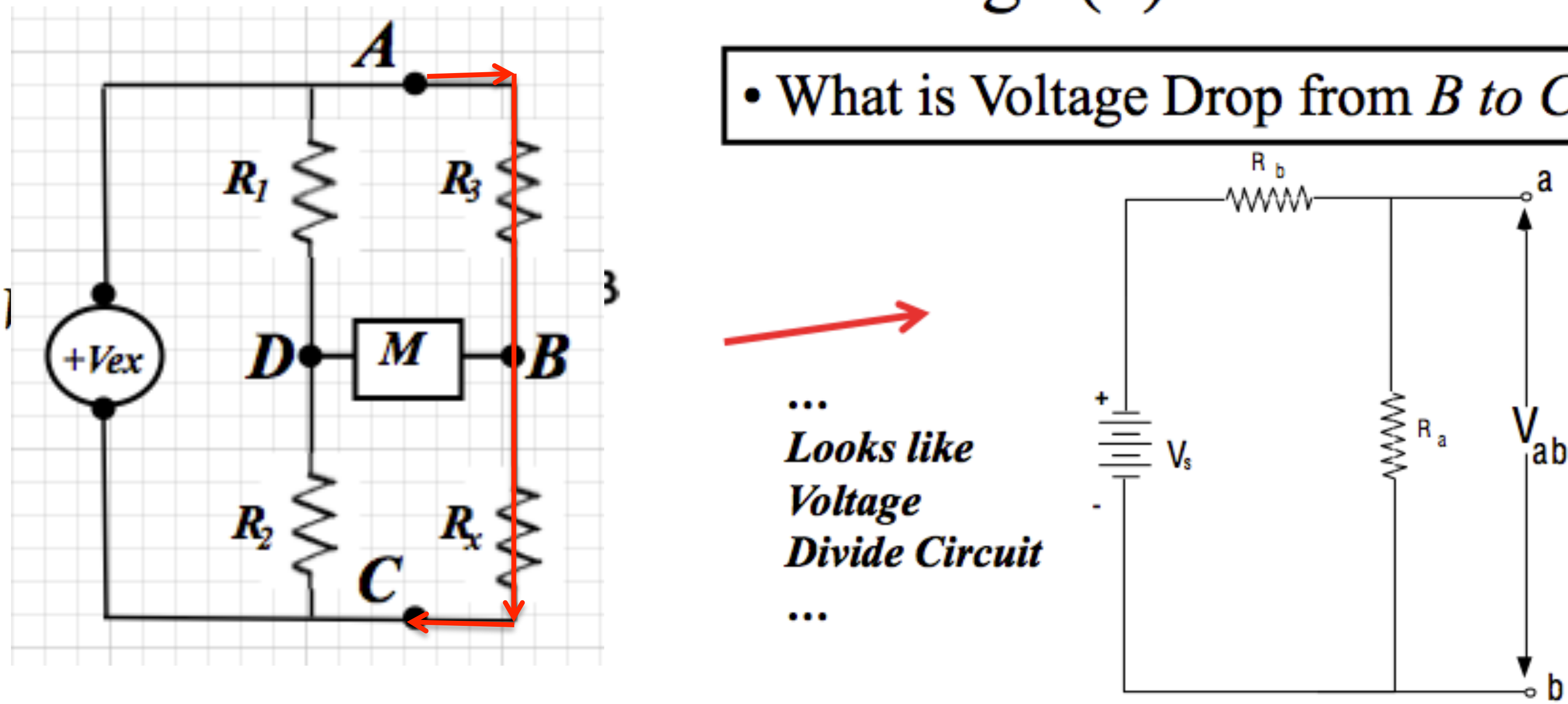


$$V_{ab} = V \cdot \frac{R_a}{(R_a + R_b)}$$

$$V_{BC} = V_{ex} \frac{R_x}{R_3 + R_x}$$

Wheatstone Bridge (2)

- What is Voltage Drop from *B to C*?

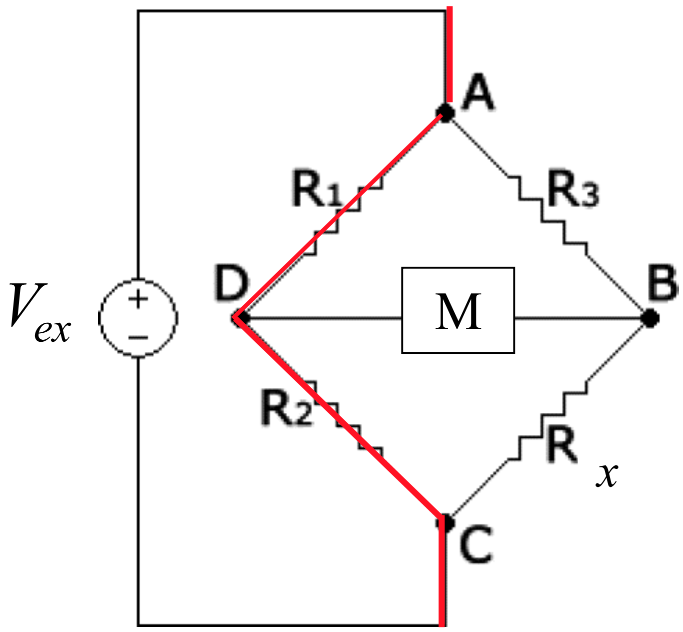


… *Looks like Voltage Divide Circuit* …

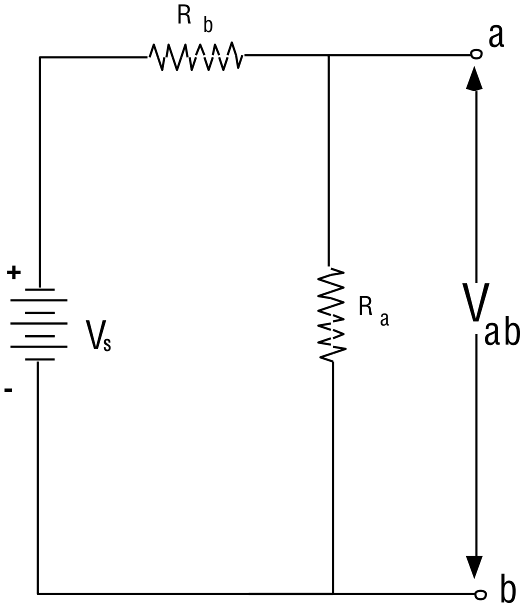
$$V_{BC} = V_{ex}\frac{R_x}{R_3 + R_x}$$

$$V_{ab} = V \cdot \frac{R_a}{(R_a + R_b)}$$

Wheatstone Bridge (3)



- What is Voltage Drop from *D to C*?

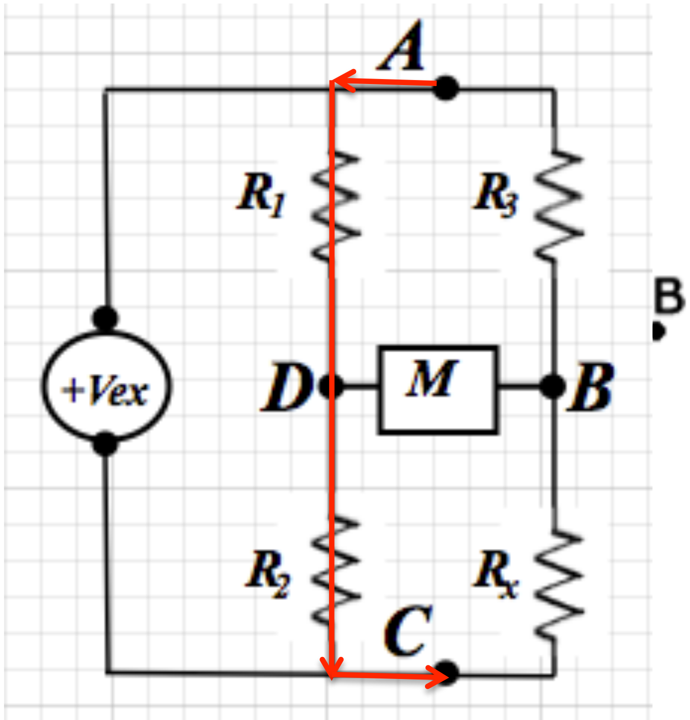


… Looks like Voltage Divide Circuit …

$$V_{DC} = V_{ex}\frac{R_2}{R_1 + R_2}$$

$$V_{ab} = V \cdot \frac{R_a}{(R_a + R_b)}$$

Wheatstone Bridge (3)



- What is Voltage Drop from *D to C*?

$$V_{DC} = V_{ex}\frac{R_2}{R_1 + R_2}$$

...
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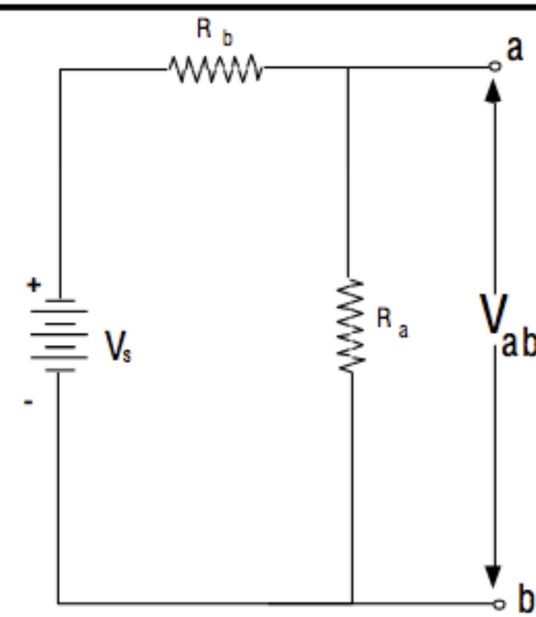
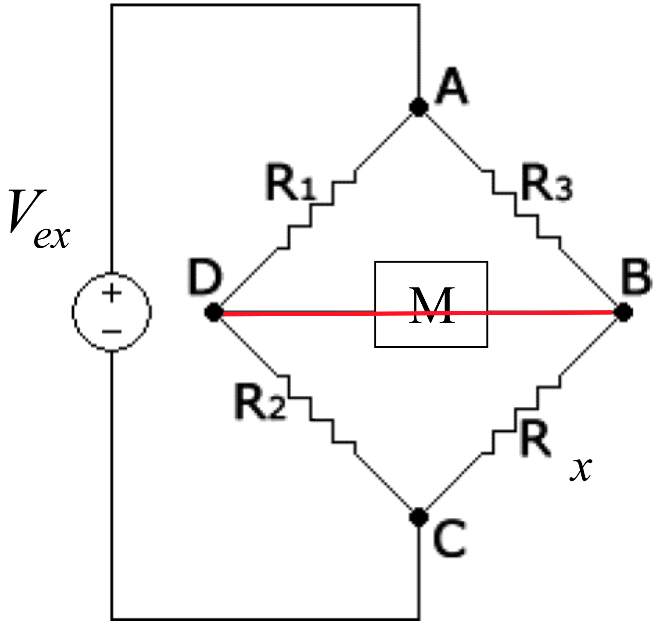


$$V_{ab} = V \cdot \frac{R_a}{(R_a + R_b)}$$

Wheatstone Bridge (4)



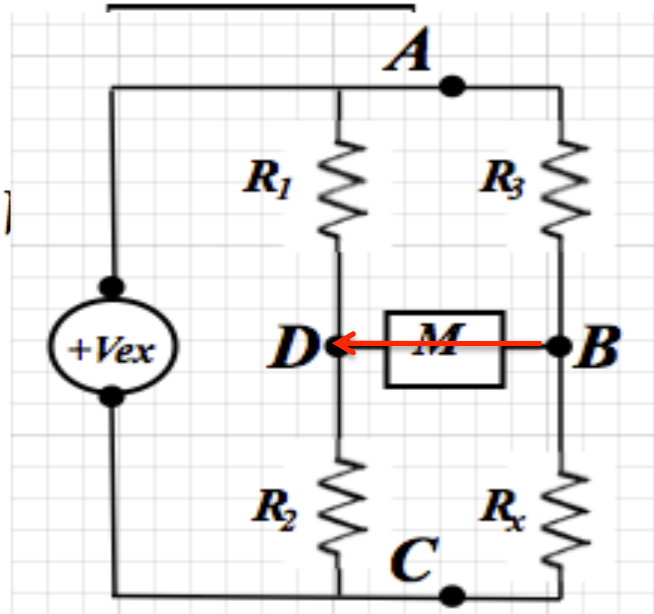
- What is Voltage Drop from *B to D*? (across meter)
  - *Assume Impedance of M is Very Large > MΩ, thus negligible current flows across meter*

$$V_{BD} = V_{BC} + V_{CD} = V_{BC} - V_{DC} =$$

$$V_{ex}\left(\frac{R_x}{R_3 + R_x} - \frac{R_2}{R_1 + R_2}\right) = V_{ex}\left(\frac{R_x\left(R_1 + R_2\right) - R_2\left(R_3 + R_x\right)}{\left(R_1 + R_2\right)\left(R_3 + R_x\right)}\right) =$$

$$V_{ex}\frac{R_1 R_x - R_2 R_3}{\left(R_1 + R_2\right)\left(R_3 + R_x\right)}$$

Wheatstone Bridge (4)



- What is Voltage Drop from *B to D*? (across meter)
  - *Assume Impedance of M is Very Large > MΩ, thus negligible current flows across meter*

$$V_{BD} = V_{BC} + V_{CD} = V_{BC} - V_{DC} =$$

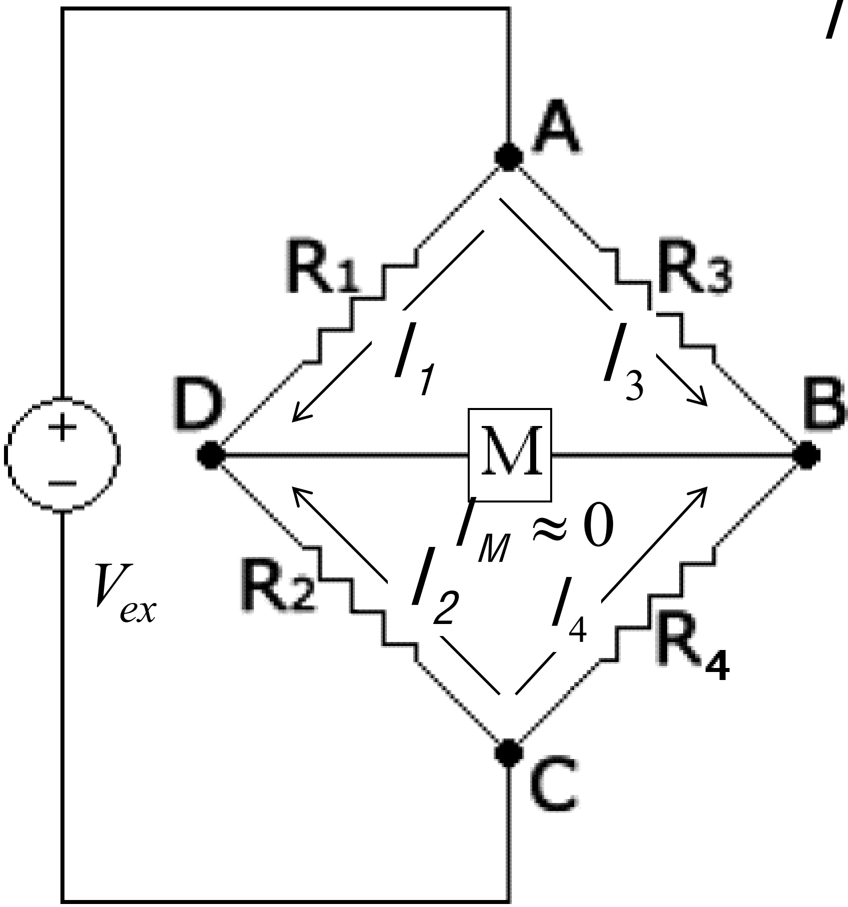
$$V_{ex}\left(\frac{R_x}{R_3 + R_x} - \frac{R_2}{R_1 + R_2}\right) = V_{ex}\left(\frac{R_x(R_1 + R_2) - R_2(R_3 + R_x)}{(R_1 + R_2)(R_3 + R_x)}\right) =$$

$$V_{ex}\frac{R_1 R_x - R_2 R_3}{(R_1 + R_2)(R_3 + R_x)}$$

"Balanced Bridge" (1)

If the Voltage across the meter {D,B}=0

… bridge is said to be "balanced."



- Currents at nodes *{D,B}*

$$I_M \approx 0 \rightarrow \boxed{\begin{array}{l} I_3 = I_4 \\ I_1 = I_2 \end{array}}$$

- Voltage drops in Loops *{ABD}, {BCD}*

$$I_3 R_3 + I_M R_M - I_1 R_1 = 0$$

$$I_4 R_4 - I_2 R_2 - I_M R_M = 0$$

$$\frac{I_3 R_3}{I_4 R_4} = \frac{I_1 R_1}{I_2 R_2}$$

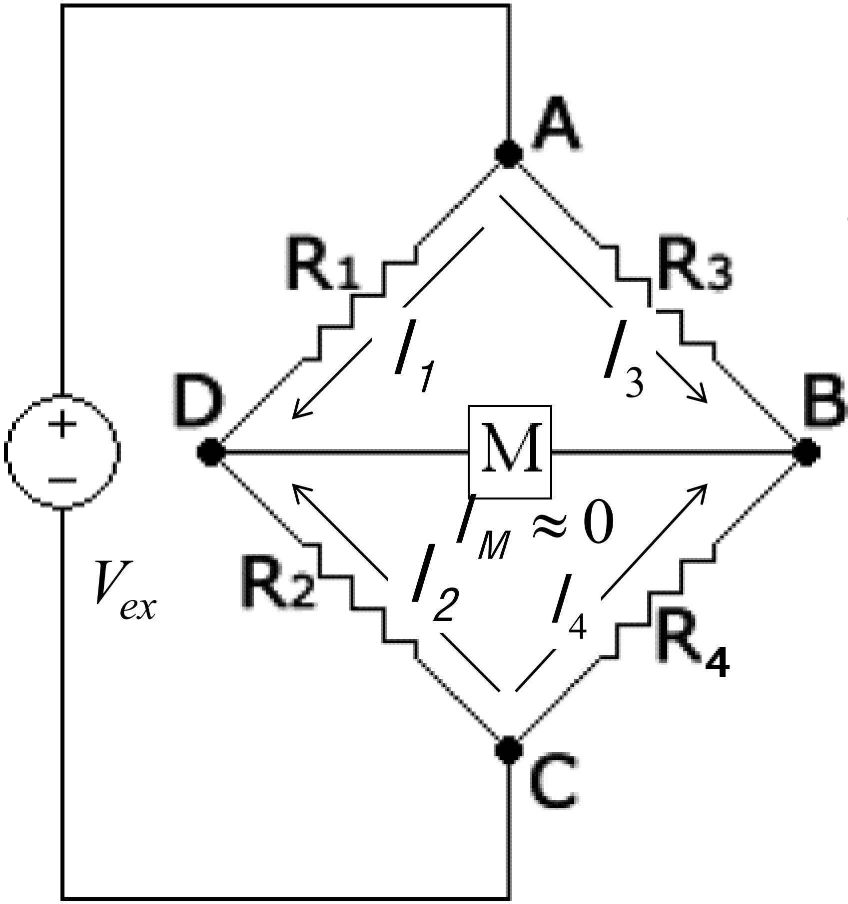
$$\begin{array}{l} I_3 = I_4 \\ I_1 = I_2 \end{array} \rightarrow \boxed{\frac{R_3}{R_4} = \frac{R_1}{R_2}}$$

Balanced Bridge (2)

from previous…. →

$$I_3 R_3 = I_1 R_1 \rightarrow \frac{R_1}{R_3} = \frac{I_3}{I_1}$$

$$I_4 R_4 = I_2 R_2 \rightarrow \frac{R_2}{R_4} = \frac{I_4}{I_2}$$



- Sum currents at Node A …

$$I_1 = -I_3$$

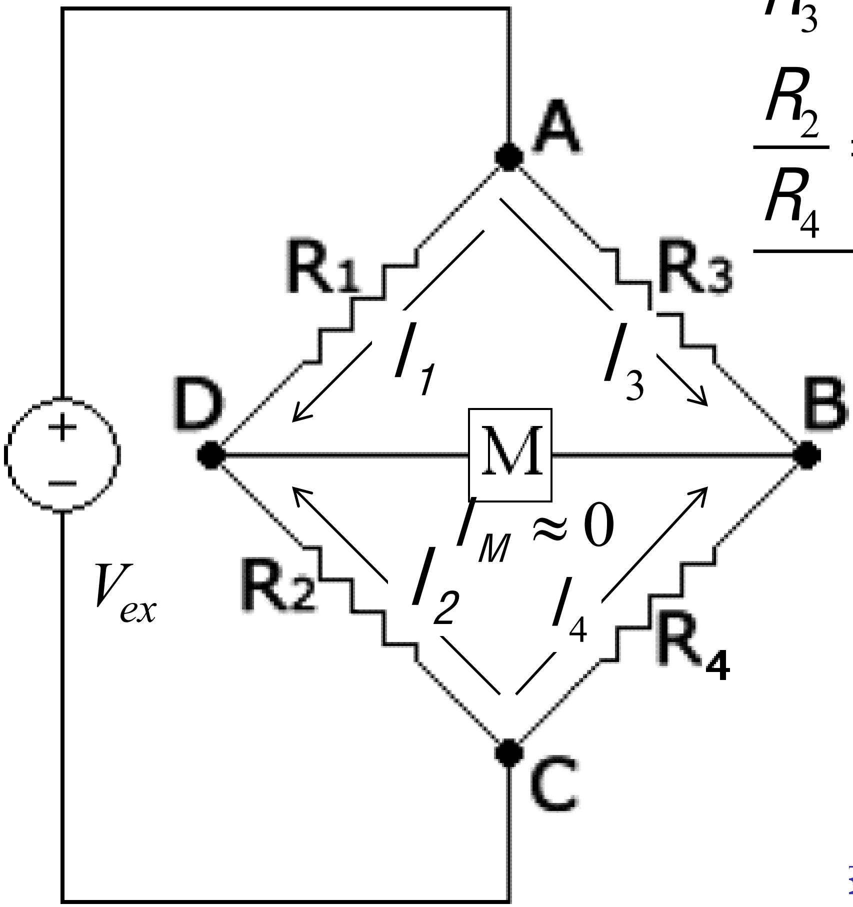
- Sum currents at Node c …

$$I_2 = -I_4$$

Balanced Bridge (3)

from previous….

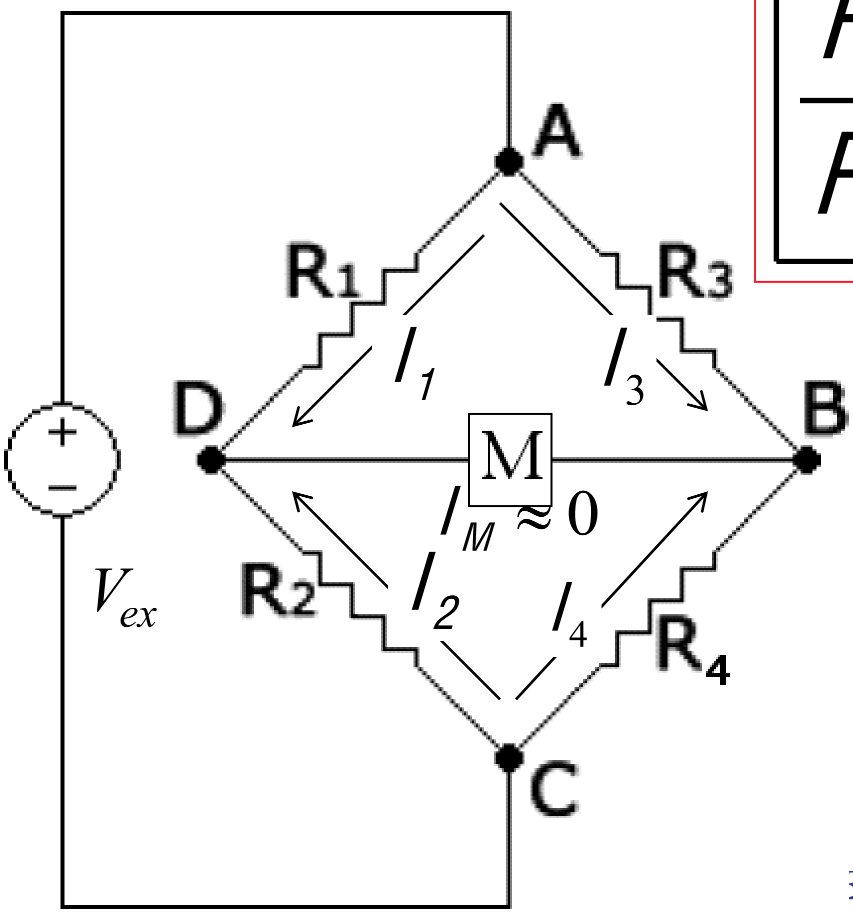
$$\frac{R_1}{R_3} = \frac{I_3}{I_1} \qquad \frac{R_2}{R_4} = \frac{I_4}{I_2} \rightarrow \begin{array}{l} I_1 = -I_3 \\ I_2 = -I_4 \end{array} \rightarrow \frac{R_1}{R_3} = -1 = \frac{R_2}{R_4}$$



$$\frac{R_1}{R_3} = \frac{R_2}{R_4}$$

Balanced Bridge (4)

Conditions for Balanced Bridge ….

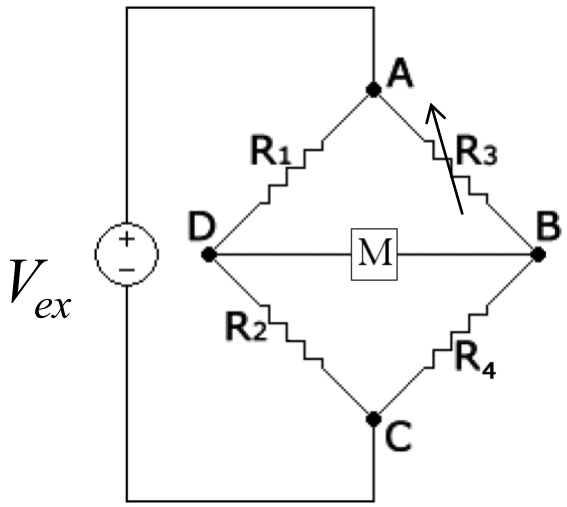


$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

$$\frac{R_1}{R_3} = \frac{R_2}{R_4}$$

- *Ratio of resistances of any two Adjacent arms must equal ratio Of resistance of remaining two Arms …*

Balancing Techniques (1)



- Good way to sense
  An *unknown resistance*

- Solve for unknown resistance

- Adjust R_3 until $I_M = 0$

$$\begin{array}{l} I_3 R_3 = I_1 R_1 \\ I_x R_x = I_2 R_2 \end{array} \rightarrow R_x = \frac{(R_2 I_2)(R_3 I_3)}{(R_1 I_1) I_x} = \left(\frac{R_2 R_3}{R_1}\right)\left(\frac{I_2 I_3}{I_1 I_x}\right) \downarrow$$

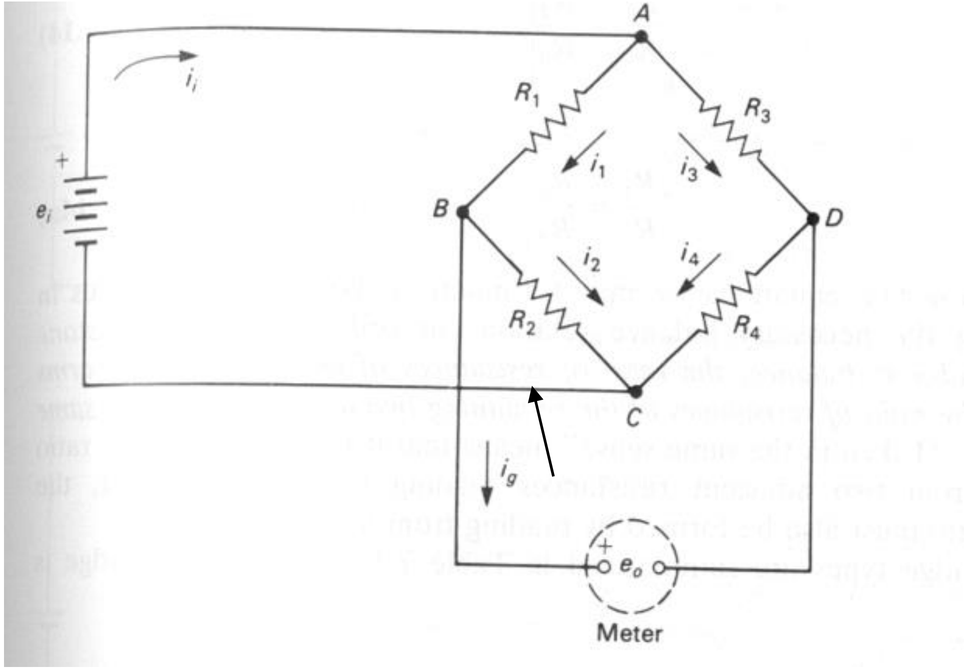
$$\begin{array}{l} I_3 = I_x \\ I_1 = I_2 \end{array} \rightarrow R_x = \left(\frac{R_2 R_3}{R_1}\right)$$

R_3 variable resistance
"rheostat"

Balancing Techniques (2)

If we make R_3 a calibrated variable resistor, then we simply dial it until Meter current = 0.

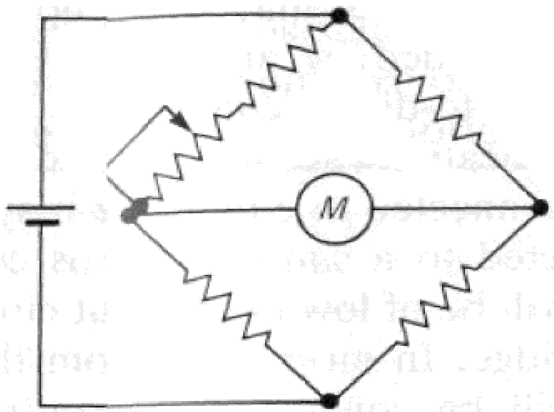
If a change in the measured condition causes a change in R_4, we determine the new value by re-nulling the bridge.



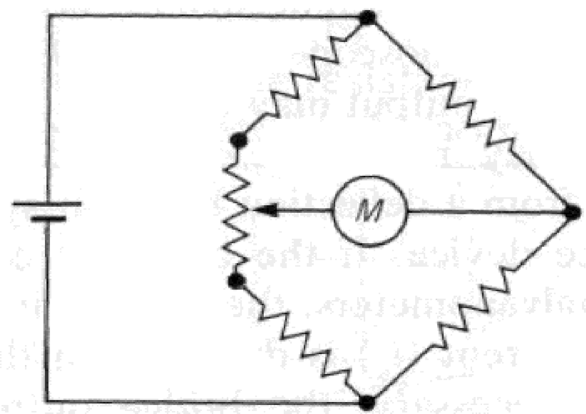
- "Null Method"

Balancing Techniques (2)

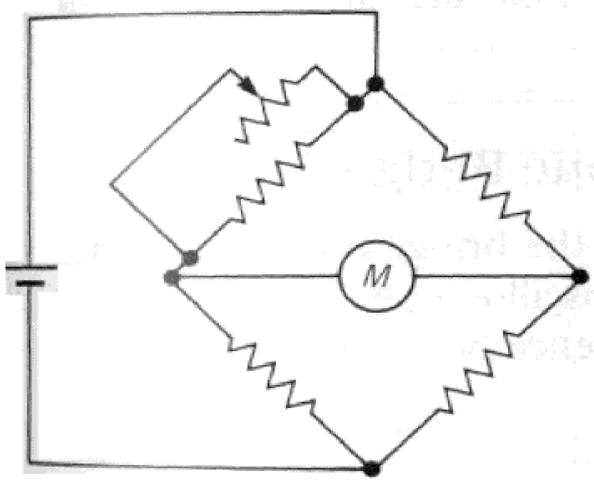
Arrangements used to balance dc resistance bridges



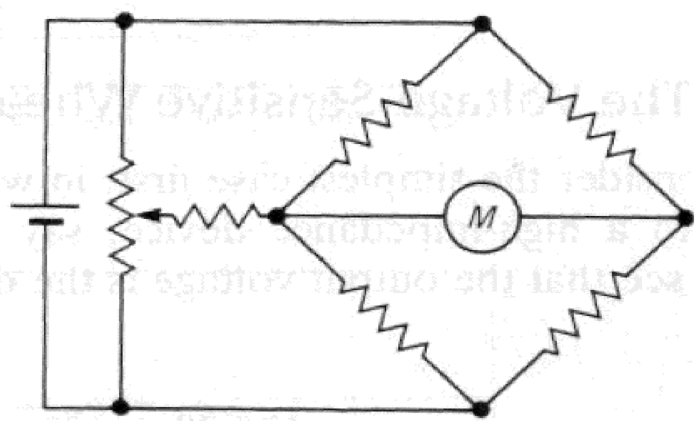
(a) Series balance



(b) Differential series balance



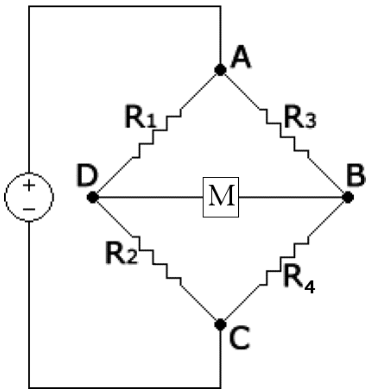
(c) Shunt balance



(d) Differential shunt balance

Resistance Bridge Summary

- *.. Infinite Meter Impedance*



$$\begin{bmatrix} I_M = 0 \\ I_1 = \dfrac{V_{ex}}{(R_1 + R_2)} \\ I_2 = \dfrac{V_{ex}}{(R_1 + R_2)} \\ I_3 = \dfrac{V_{ex}}{(R_3 + R_4)} \\ I_4 = \dfrac{V_{ex}}{(R_3 + R_4)} \end{bmatrix} \rightarrow \begin{bmatrix} V_M = V_{ex}\dfrac{R_1 R_4 - R_2 R_3}{(R_1 + R_2)(R_3 + R_x)} \\ V_1 = V_{ex}\dfrac{R_1}{(R_1 + R_2)} \\ V_2 = V_{ex}\dfrac{R_2}{(R_1 + R_2)} \\ V_3 = V_{ex}\dfrac{R_3}{(R_3 + R_4)} \\ V_4 = V_{ex}\dfrac{R_4}{(R_3 + R_4)} \end{bmatrix} \rightarrow \begin{bmatrix} P_M = 0 \\ P_1 = R_1\left[\dfrac{V_{ex}}{(R_1 + R_2)}\right]^2 \\ P_2 = R_2\left[\dfrac{V_{ex}}{(R_1 + R_2)}\right]^2 \\ P_3 = R_3\left[\dfrac{V_{ex}}{(R_3 + R_4)}\right]^2 \\ P_4 = R_4\left[\dfrac{V_{ex}}{(R_3 + R_4)}\right]^2 \end{bmatrix}$$

Current thru resistor	Voltage drop cross resistor	Power dissipated by resistor