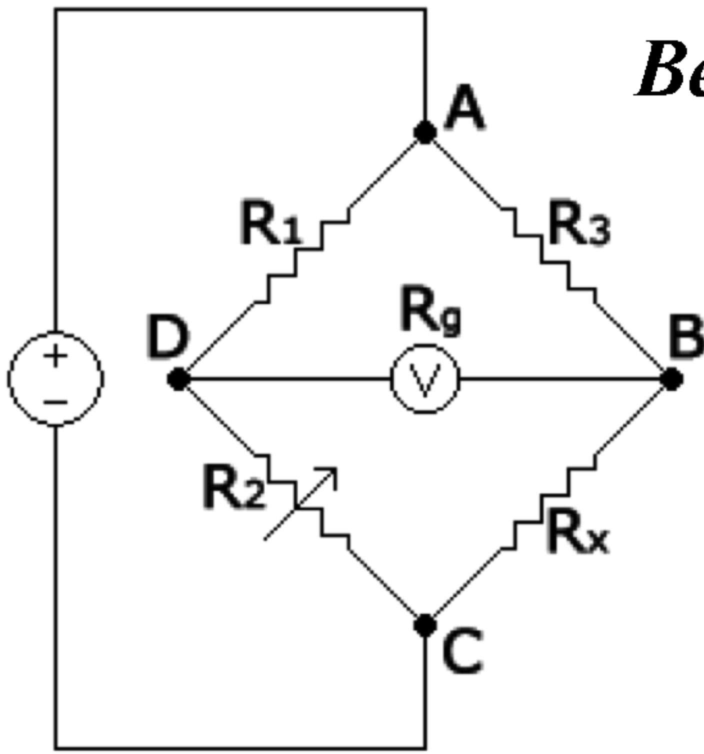
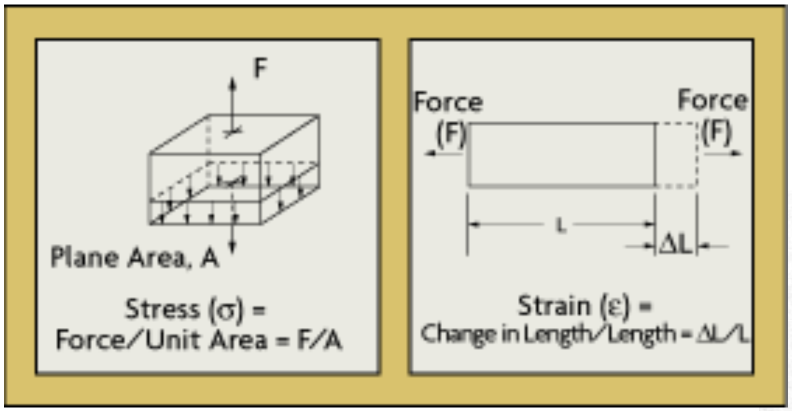
Section 4.2 : Introduction to Using the Wheatstone Bridge for Strain-Based Sensor Measurements



Beckwith Chapter 12

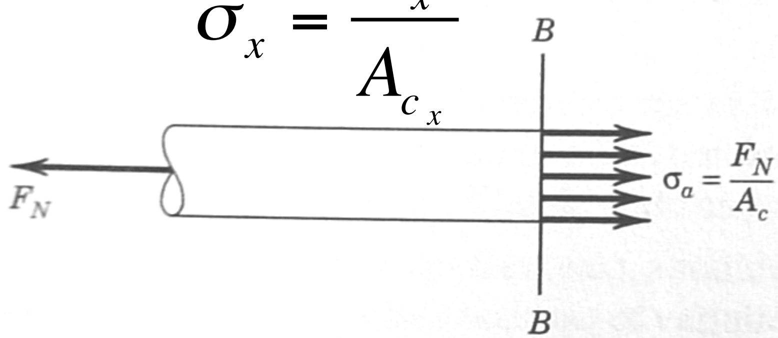
Stress/Strain Basic Definitions (1)

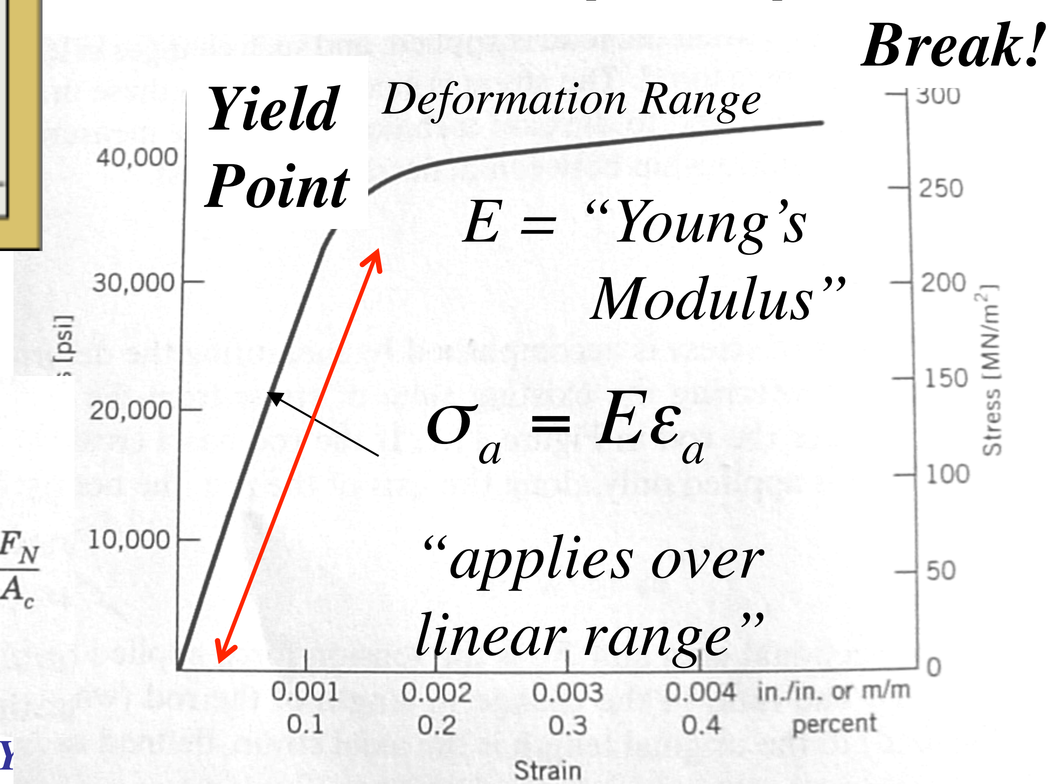
Engineering calculations are often based on stress. If we want to do experiments to confirm our theory, we need to measure the result of stress rather than stress directly. Stress results in the deformation of material, which is called strain. For most engineering materials, there is a rather simple relationship between stress and strain.



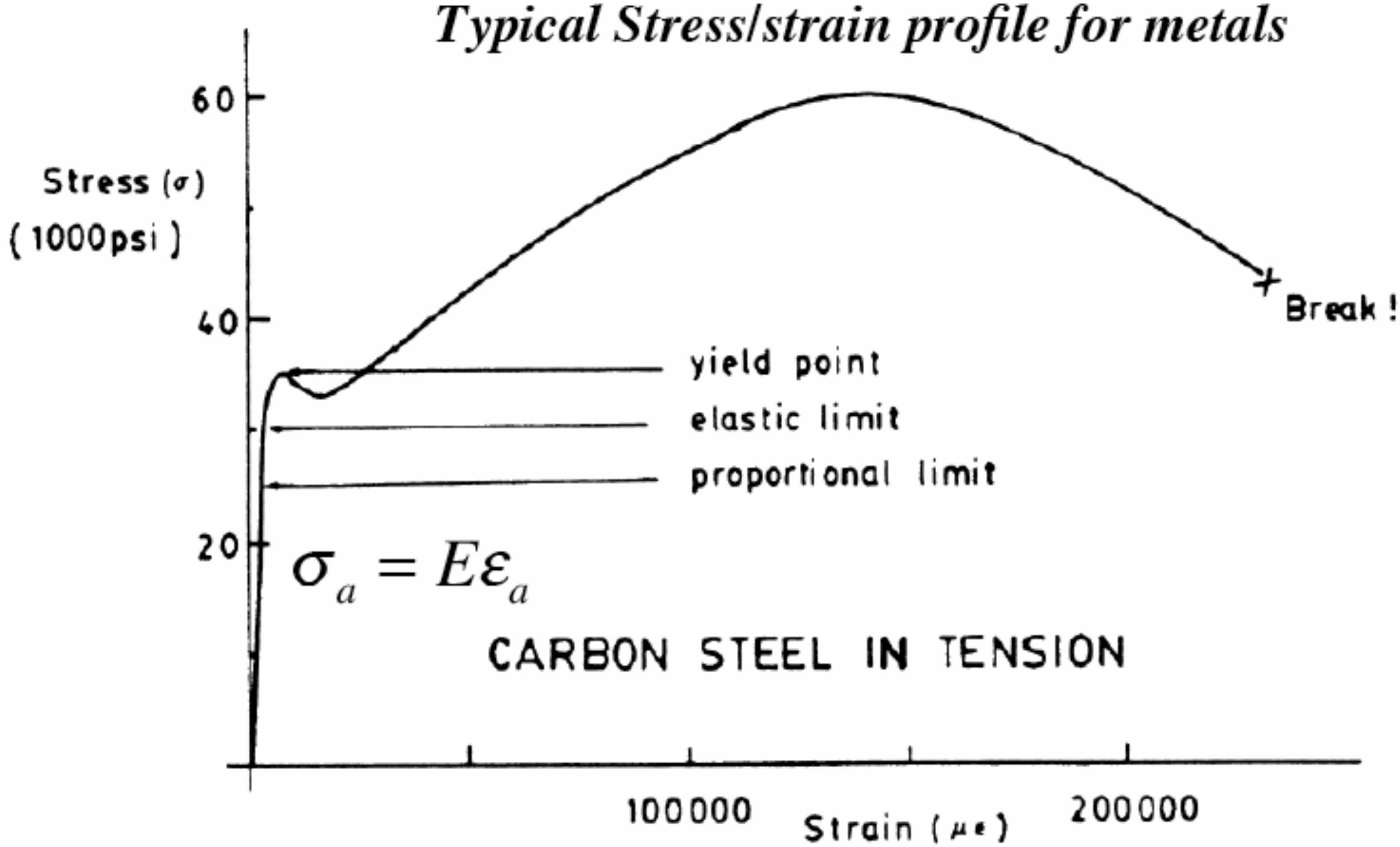
$$\varepsilon_a = \frac{dL}{L} \approx \frac{L_2 - L_1}{L_1} = \frac{\Delta L}{L_1}$$

$$\sigma_x = \frac{F_x}{A_{c_x}}$$

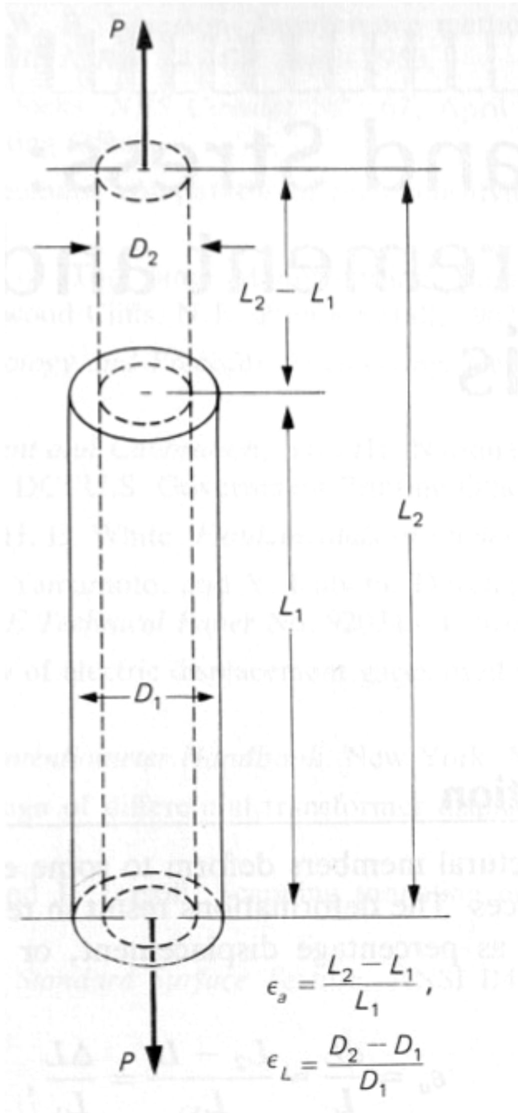




Stress/Strain Basic Definitions (2)

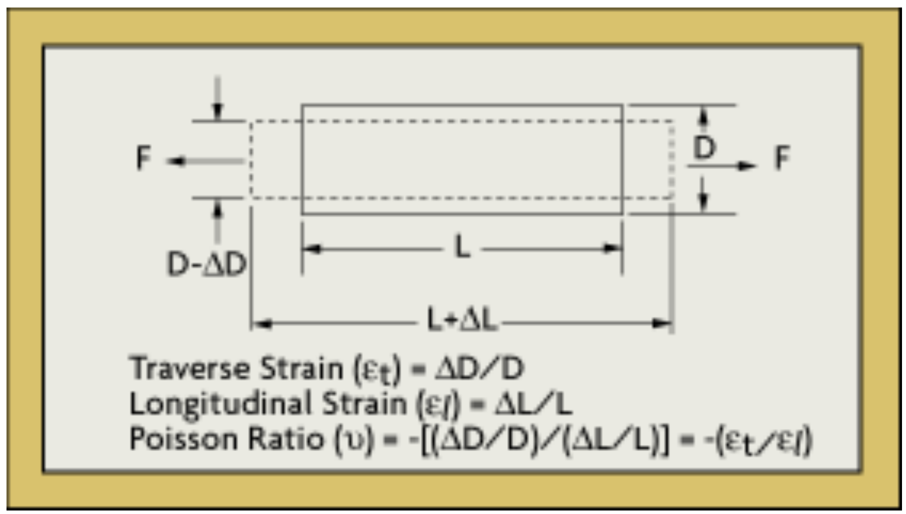


Lateral Strain, Poisson's Ratio (1)



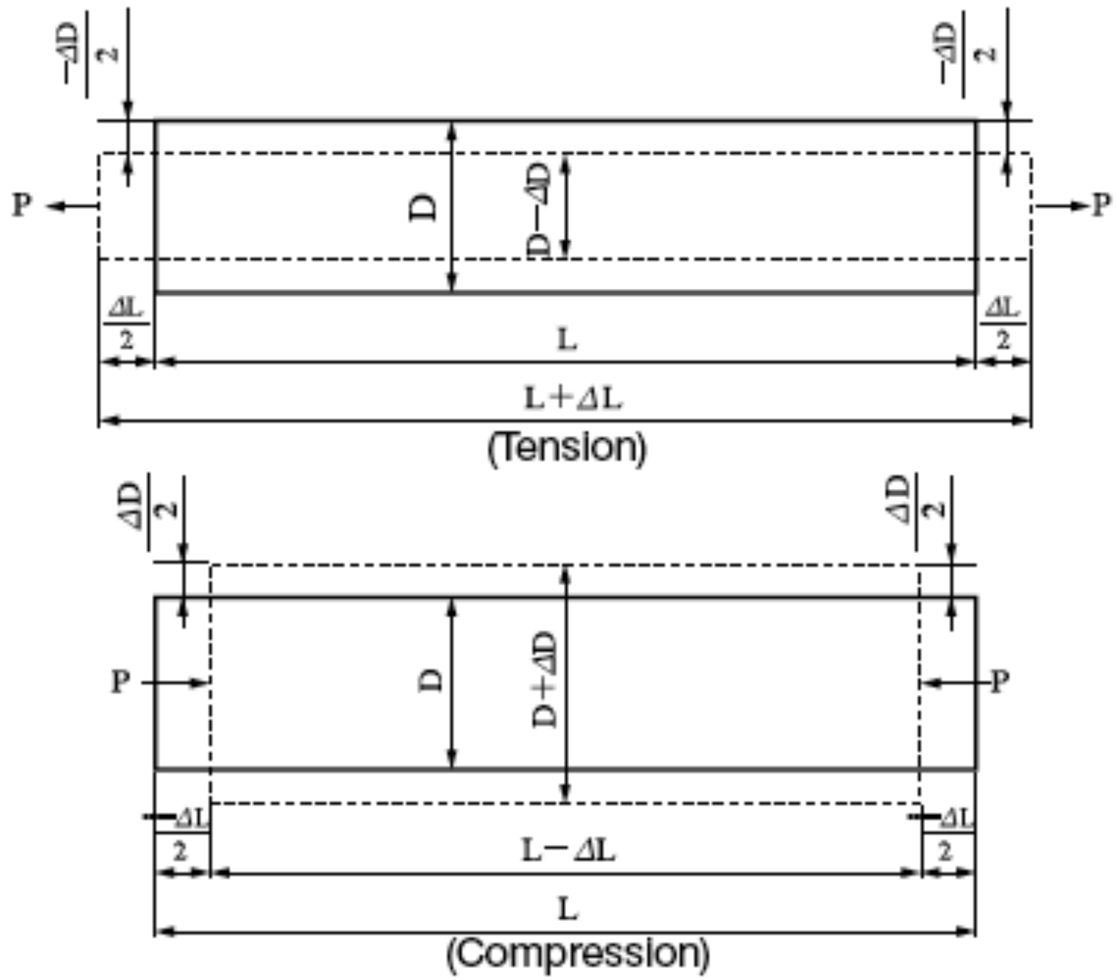
If we stress a rod by pulling on it, and is stretches axially as a result, it will also get thinner. This behavior is quantified by Poisson's ratio:

$$\nu = \frac{\text{lateral strain}}{\text{axial strain}} = -\frac{\varepsilon_L}{\varepsilon_a}$$



Lateral Strain, Poisson's Ratio (2)

Works the opposite direction for compression



Lateral Strain, Poisson's Ratio (3)

$$\varepsilon = \frac{\Delta L}{L}$$

ε: Strain
L: Original length
ΔL: Elongation

$$\nu = \left|\frac{\varepsilon_2}{\varepsilon_1}\right|$$

ν: Poisson's ratio
ε_1: Longitudinal strain $\frac{\Delta L}{L}$ or $-\frac{\Delta L}{L}$
ε_2: Transverse strain $-\frac{\Delta D}{D}$ or $\frac{\Delta D}{D}$

Compression

Tension

E, ν, G are properties of material

Mechanical Properties of Industrial Materials

$$G = \frac{E}{2(1+\nu)}$$

Material	Young's Modulus E (GPa)	Shearing Modulus G (GPa)	Tensile Strength** (MPa)	Poisson's Ratio ν
Carbon steel (C0.1 - 0.25%)	205	78	363 - 441	0.28 - 0.3
Carbon steel (C > 0.25%)	206	79	417 - 569	0.28 - 0.3
Spring steel (quenched)	206 - 211	79 - 81	588 - 1667	0.28 - 0.3
Nickel steel	205	78	549 - 657	0.28 - 0.3
Cast iron	98	40	118 - 235	0.2 - 0.29
Brass (casting)	78	29	147	0.34
Phosphor bronze	118	43	431	0.38
Aluminum	73	27	186 - 500	0.34
Concrete	20 - 29	9 -13	—	0.1

G = Shearing Modulus *(more on this later)*

General Stress States, 2-D

Relate the 2-D stress field to the 2-D strain field.

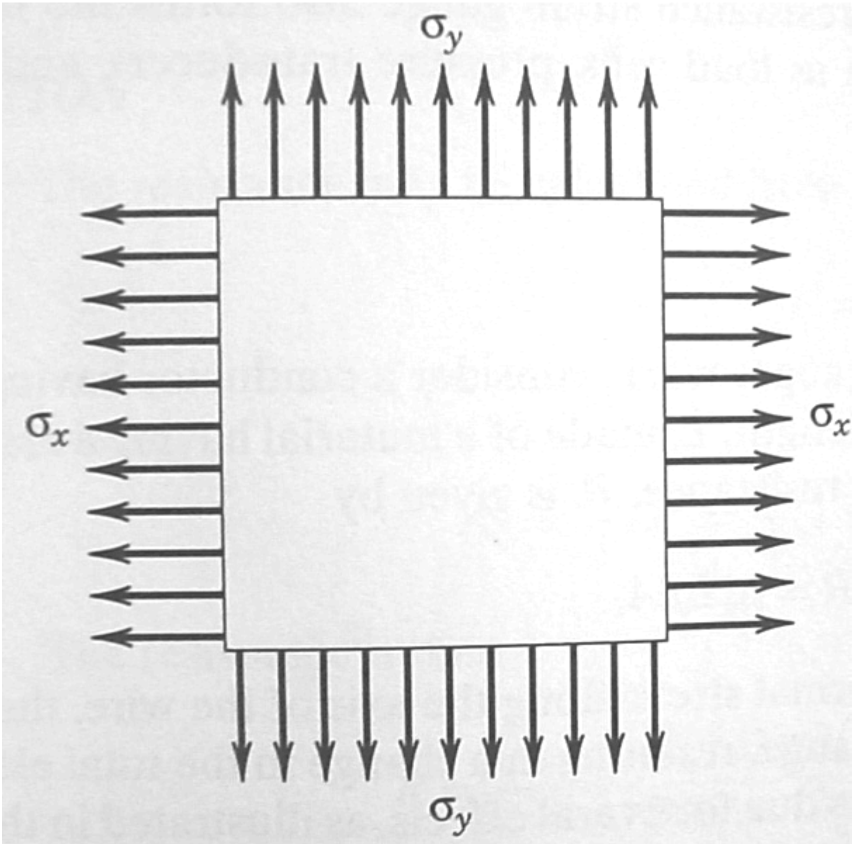
$$\varepsilon_y = \frac{\sigma_y}{E} - \nu \frac{\sigma_x}{E}$$

$$\varepsilon_x = \frac{\sigma_x}{E} - \nu \frac{\sigma_y}{E}$$

$$\sigma_x = \frac{E\left(\varepsilon_x + \nu\varepsilon_y\right)}{1-\nu^2}$$

$$\sigma_y = \frac{E\left(\varepsilon_y + \nu\,\varepsilon_x\right)}{1-\nu^2}$$

- ***two equations, two unknowns***



General Stress States, 3-D

$$\boxed{\begin{aligned} \varepsilon_x &= \frac{1}{E}\left[\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right] \\ \varepsilon_y &= \frac{1}{E}\left[\sigma_y - \nu\left(\sigma_x + \sigma_z\right)\right] \\ \varepsilon_z &= \frac{1}{E}\left[\sigma_z - \nu\left(\sigma_x + \sigma_y\right)\right] \end{aligned}} \longrightarrow$$

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \end{bmatrix} = \frac{1}{E}\begin{bmatrix} 1 & -\nu & -\nu \\ 1 & -\nu & -\nu \\ 1 & -\nu & -\nu \end{bmatrix}\begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \end{bmatrix} \rightarrow$$

$$\boxed{\begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \end{bmatrix} = E\begin{bmatrix} 1 & -\nu & -\nu \\ 1 & -\nu & -\nu \\ 1 & -\nu & -\nu \end{bmatrix}^{-1}\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \end{bmatrix}}$$

- We measure strain in one or more directions and infer the stress state from that. In general, in order to know the 3-D stress state, we would need 3 components of strain. In some cases (like pure axial stress) we may be able to reduce the number of required components.

- 3-equations, 3 unknowns … ***typically a numerical solution***

Strain Gauge (1)

- A **strain gauge** is a sensor used to measure deformation of an object.

- Most common type of strain gauge consists of an insulating flexible backing which supports a metallic foil pattern.

- Gauge is attached to the object by a suitable adhesive.

- As object being tested is deformed, the foil is deformed, causing its electrical resistance to change …. Which should be sensible As a change in current thru sensor …

Strain Gauge (2)

- ***Electrical resistivity*** is a measure of how strongly a material opposes the flow of electric current.

- A low resistivity indicates a material that readily allows the movement of electrical charge.

- The SI unit of electrical resistivity is the ohm meter.

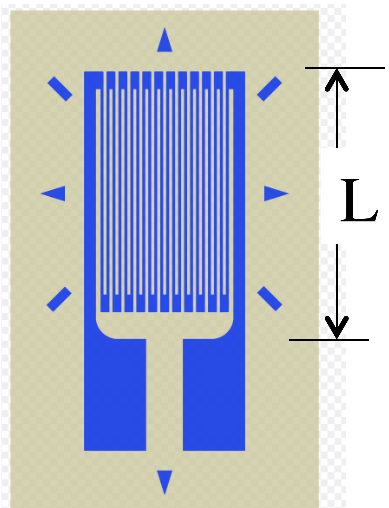
- The ***Resistance*** of Specimen is calculated by

$$R = \rho \cdot \frac{L}{A_c}$$

ρ = resistivity of material (Ω-m)
L = length of specimen (m)
A = cross sectional area of specimen (m^2)

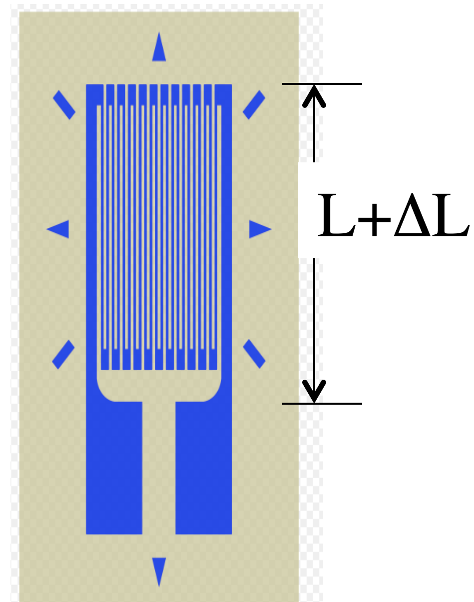
Strain Gauge (3)

• ***Now consider a Strain Gauge of a material with a known Resistivity …. And the design is far more sensitive to strain in the vertical direction than in the horizontal direction.***



• Now stretch the device ….

• Cross section does not change Much … but length changes significantly



$$R = \rho \cdot \frac{L}{A}$$

$$R + \Delta R = \rho \cdot \left(\frac{L + \Delta L}{A - \Delta A} \right)$$

Strain Gauge (4)

- Normalize by R and collect terms

$$\frac{R+\Delta R}{R} = \left(\frac{L+\Delta L}{A-\Delta A}\right)\frac{A}{L} = \left(\frac{L+\Delta L}{L}\right)\frac{A}{A-\Delta A} \rightarrow$$

$$1+\frac{\Delta R}{R} = \left(1+\frac{\Delta L}{L}\right)\left(\frac{A}{A-\Delta A}\right) \rightarrow$$

$$\frac{\Delta R}{R} = 1-\left(\frac{A}{A-\Delta A}\right)+\left(\frac{\Delta L}{L}\right)\left(\frac{A}{A-\Delta A}\right)$$

$$\boxed{1-\left(\frac{A}{A-\Delta A}\right) = \left(\frac{\Delta A}{A-\Delta A}\right) \quad \text{For small deflections} \sim 0}$$

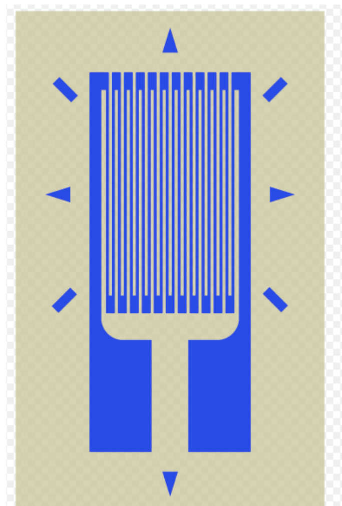
Strain Gauge(5)

- Solving for area ratio

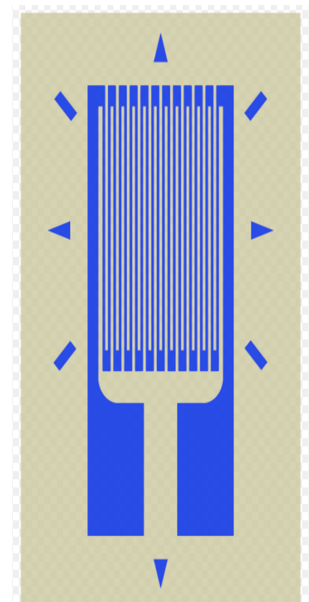
$$\frac{\Delta R}{R} = \left(\frac{\Delta L}{L}\right)\left(\frac{A}{A-\Delta A}\right) \rightarrow \left(\frac{A}{A-\Delta A}\right) \equiv G_F = \frac{\Delta R}{R} \Big/ \varepsilon$$

G_F ---> Gauge factor

ε ---> linear strain



$$\Delta R = G_F \varepsilon R_g$$



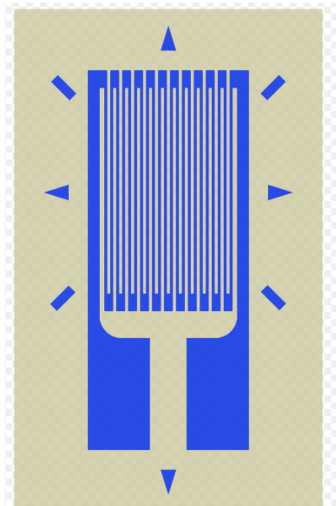
- ***Strain Gauge … change in resistance is proportional To the strain on the sensor***

Strain Gauge(6)

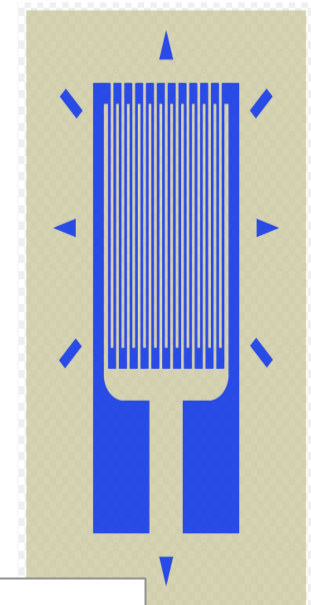
- Solving for area ratio

G_F ---> Gauge factor

ε ---> linear strain



$$\Delta R = G_F \varepsilon R_g$$



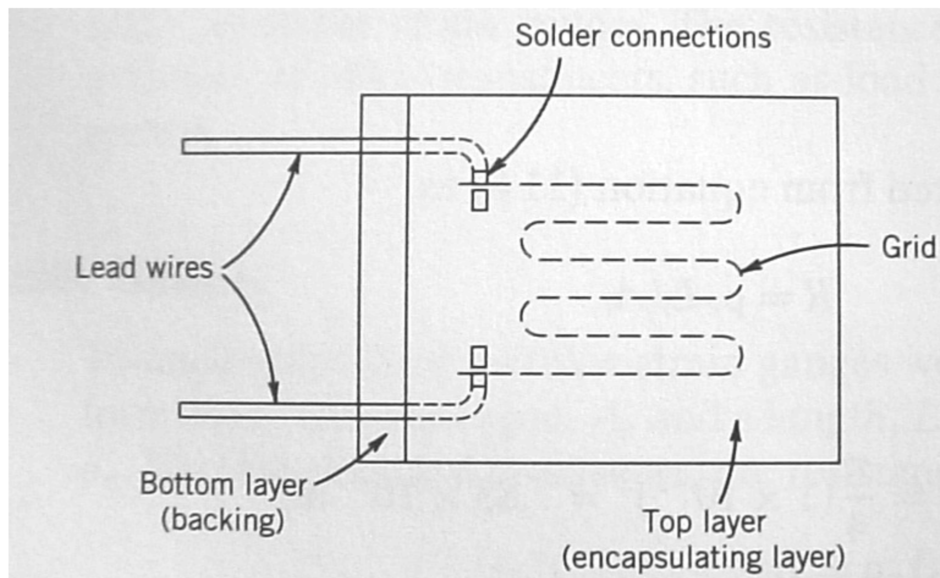
G_F, R Typically supplied by manufacturer via calibration

Most commercial strain gages are 120 Ω, have a gage factor near 2, and can measure 1 microstrain (1 part in a million).

Example Calculation, Nominal Resistance

A typical strain gauge uses constantan (55% copper, 45% nickel) which has a resistivity of 49 X 10^{-8} Ωm. If the diameter is 0.025 mm, how long does it need to be a 120Ω nominal resistance?

$$R = \frac{\rho L}{A_c} \longrightarrow L = 12 \text{ cm}$$



Example Calculation, Change in Resistance

Most commercial strain gages are 120 Ω, have a gage factor near 2, and can measure 1 microstrain (1 part in a million).

$$\varepsilon = \frac{1}{F}\frac{\Delta R}{R}$$

$$\Delta R = 120 \times 2 \times 1E-6 = 0.00024\Omega$$

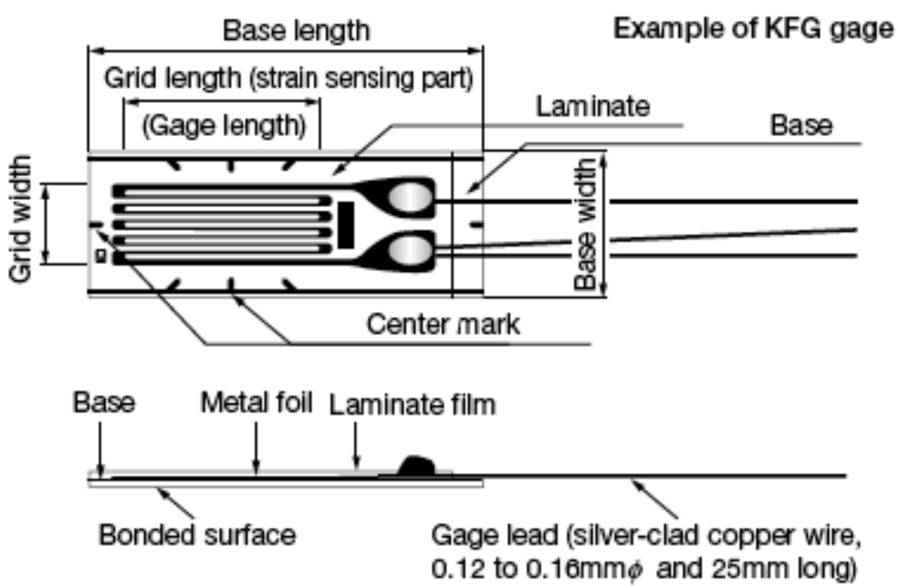
Clearly, our work is cut out for us in terms of the measurement.

Strain Gauge Designs (1)

- Types of strain gages include foil strain gage, wire strain gage and semiconductor strain gage.

- ***Structure of Foil Strain Gage***

The foil strain gage has metal foil photo-etched in a grid pattern on the electric insulator of the thin resin and gage leads attached, as shown in the figure below.

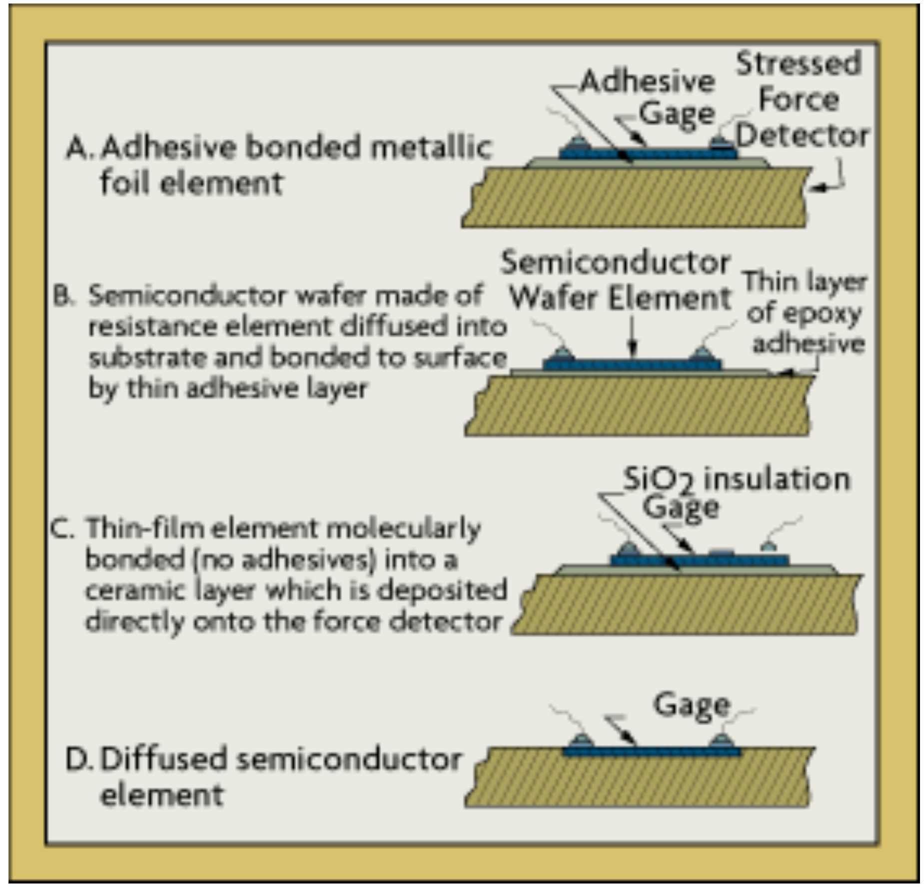


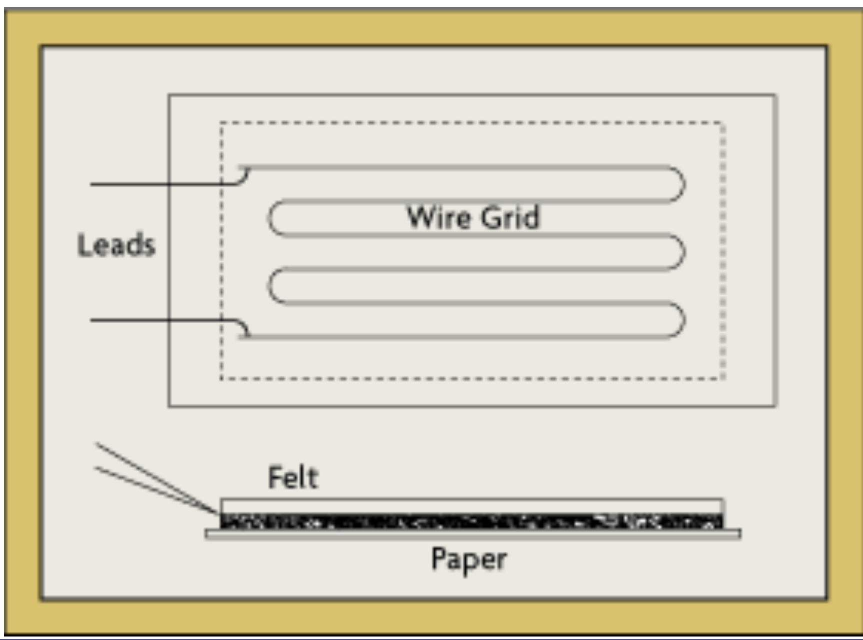
- Strain gage is bonded to measuring specimen with dedicated adhesive. Strain occurring at measuring site is transferred to sensing element via gage base. For accurate measurement, strain gage and adhesive should match Test specimen material and operating Conditions including temperature.

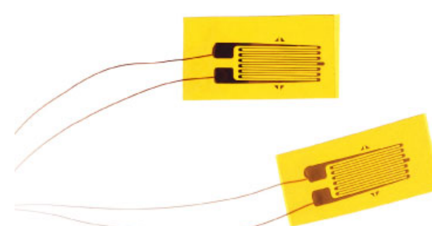
Strain Gauge Designs (2)

Prime Strain Gage Selection Considerations

- Gage Length
- Number of Gages in Gage Pattern
- Arrangement of Gages in Gage Pattern
- Grid Resistance
- Strain Sensitive Alloy
- Carrier Material
- Gage Width
- Solder Tab Type
- Configuration of Solder Tab
- Availability







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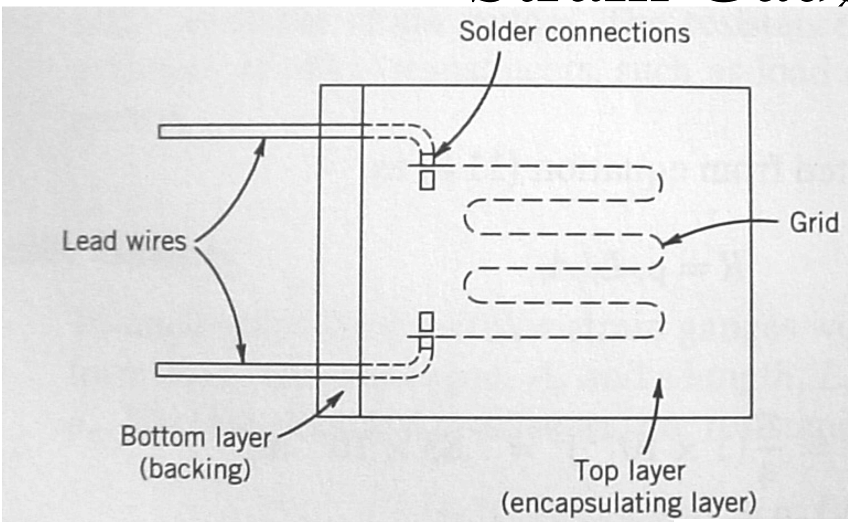
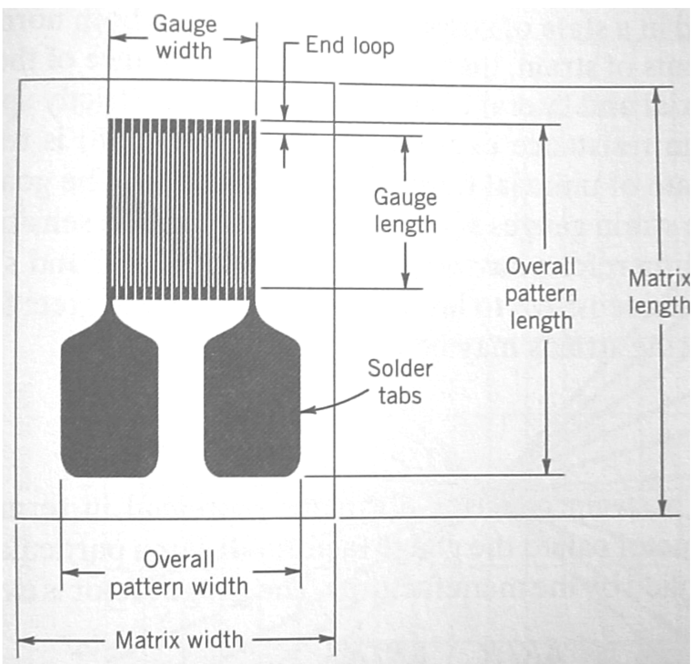
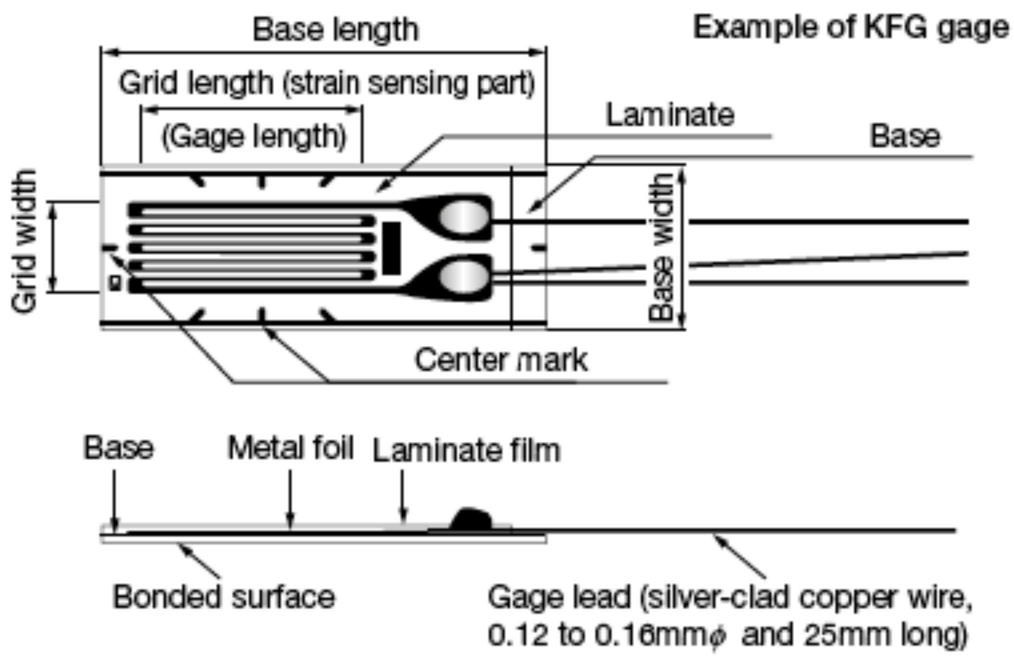
Strain Gauge Designs (3)



- Typical gage

  Layouts





Strain Gauge Designs (4)

Gauge length limits the spatial resolution of the sensor.

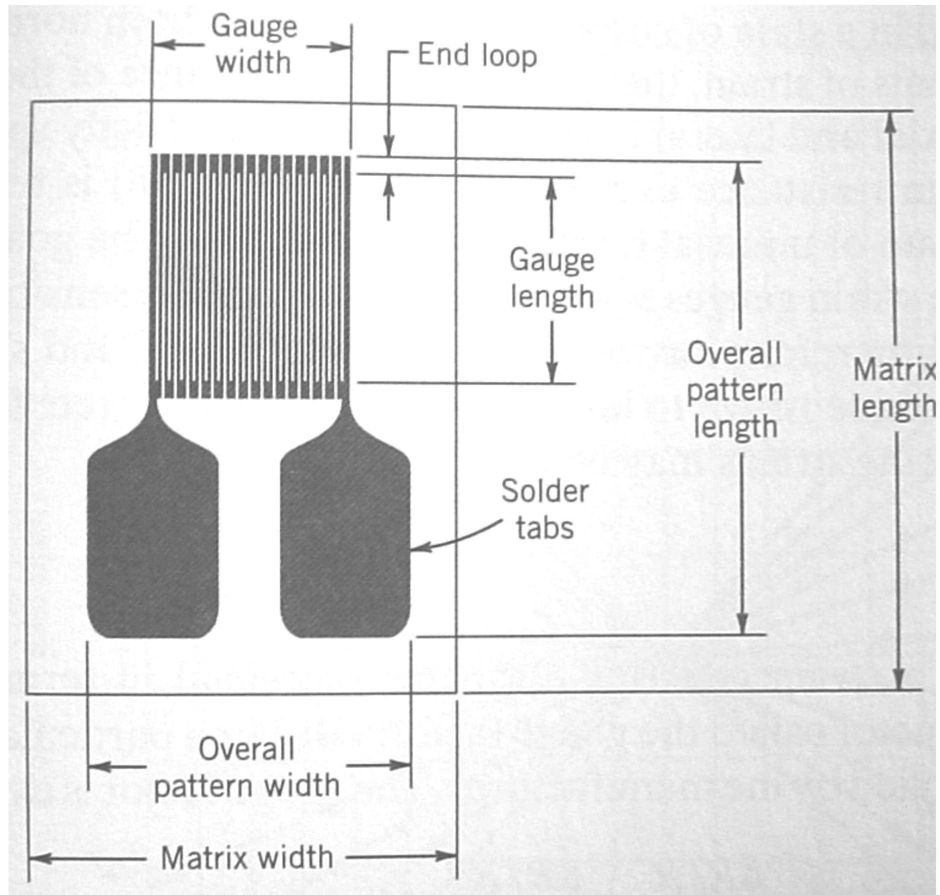
Connection to the bridge is made at the solder tabs.

Required Backing Material Properties:

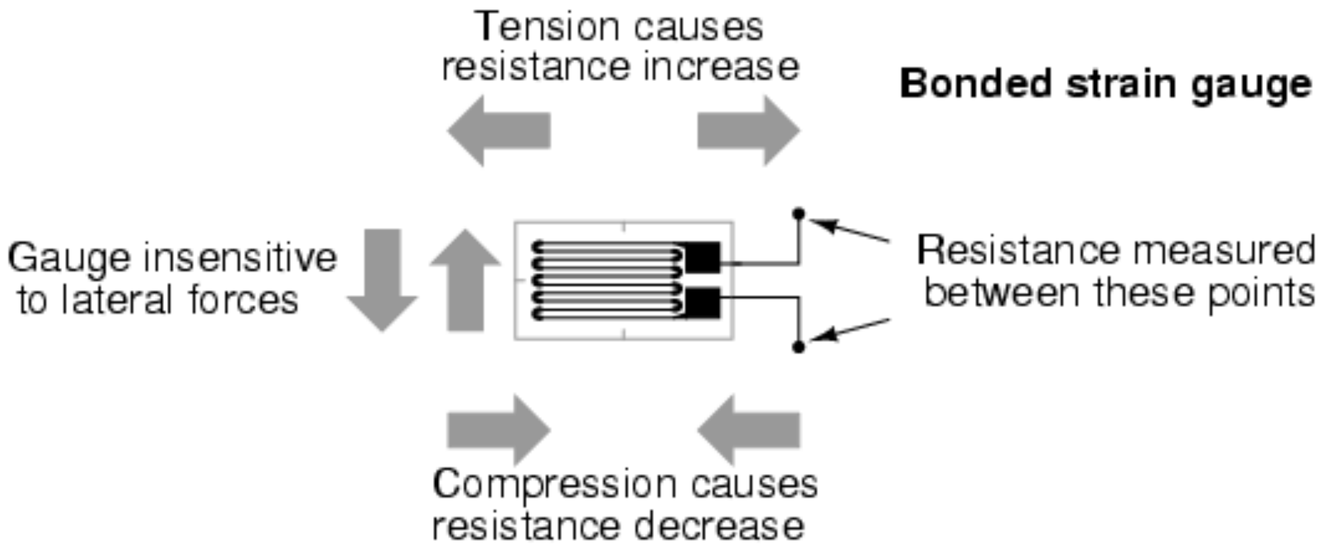
•*Withstand the temperatures encountered*

•*Transmit strain but electrically insulate*

•*Accept the bonding adhesive*



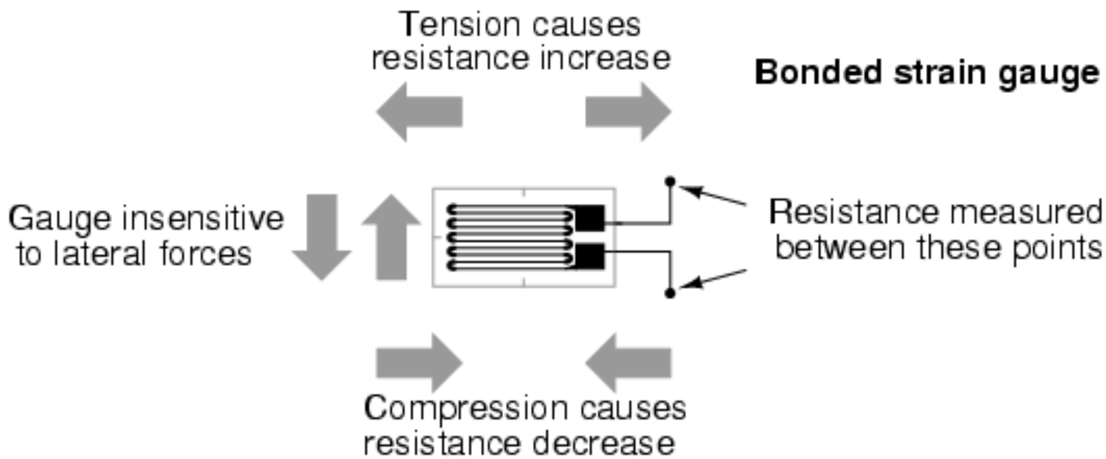
Strain Gauge, Elastic Limits (1)



- After being strained, gauge is designed to "bounce-back" to original shape

- Given limits imposed by the elastic limits of the gauge material and test specimen, Resistance changes only a small fraction of a percent (~0.1%)for full force range of the gauge.

Strain Gauge, Elastic Limits (2)



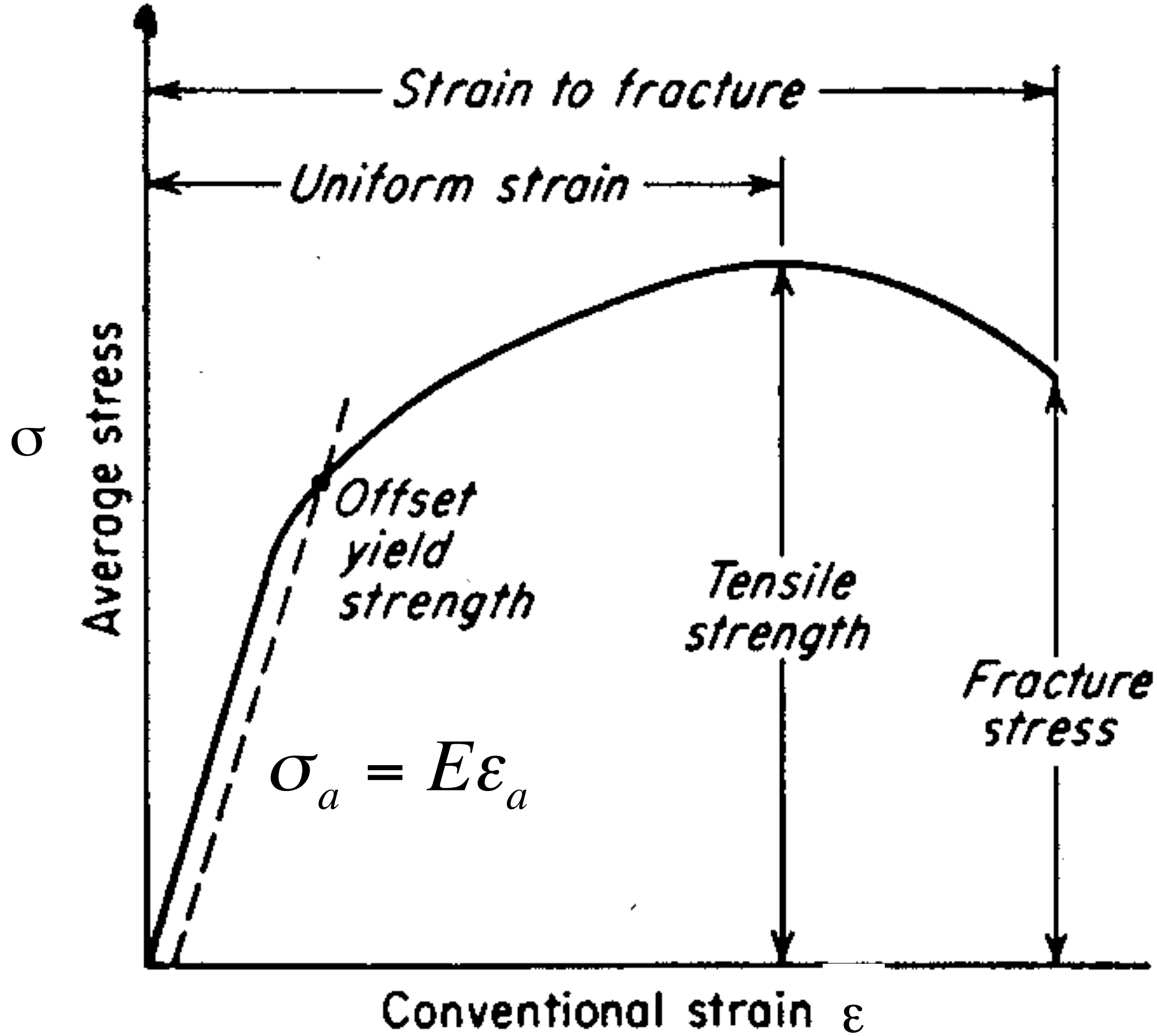
- Forces great enough to induce greater resistance changes permanently deform the test specimen and/or gauge.

- In order to use the strain gauge as a practical instrument, must measure very small changes in resistance with high accuracy.

- Typical strain gauge resistances range from 30 Ω to 100 Ω (unstressed).

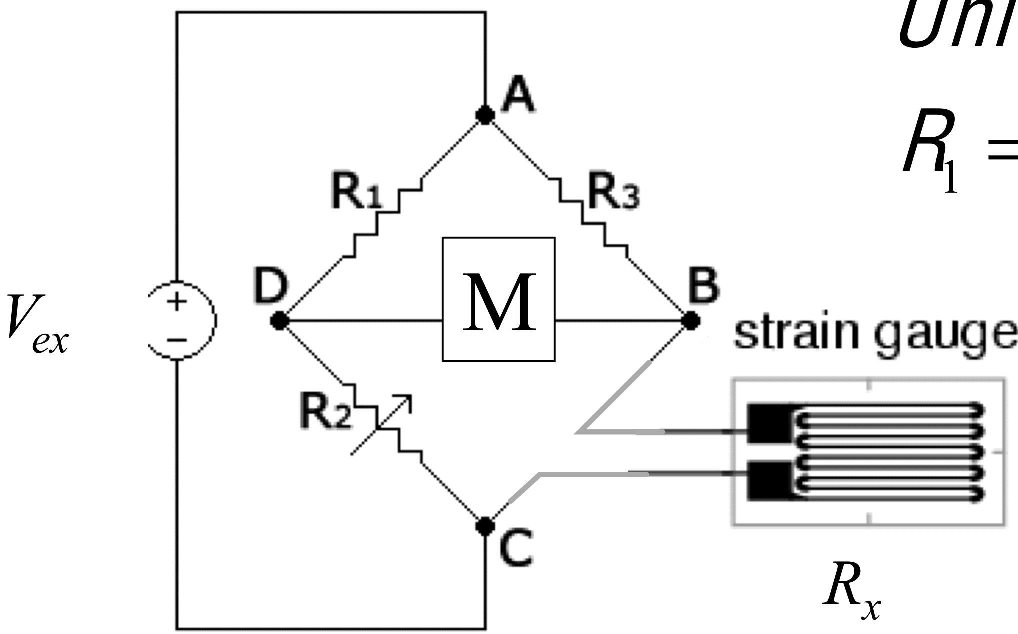
- ***ΔR ~ 0.03 to 0.1 Ω*** … a very difficult task for a multimeter

Strain Gauge, Elastic Limits (3)



Strain Gauge Bridge (1)

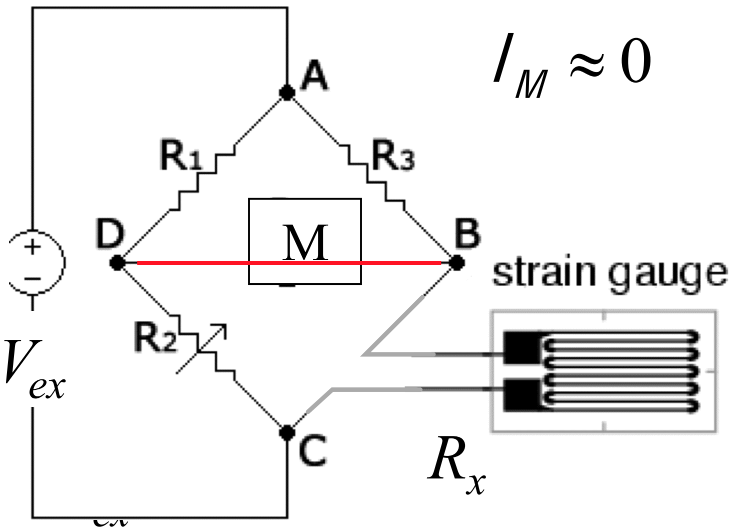
- Need a device that amplifies the changes in resistance
- Consider the circuit now with Rx being provided by a strain Gauge … the gauge is mounted to a strainable substrate And its resistance changes with the strain on the substrate



$$\textit{Unloaded conditions} \rightarrow$$

$$R_1 = R_2 = R_3 = R_x = R_g$$

Strain Gauge Bridge (6)



$$I_M \approx 0$$

- Now let the substrate strain

-> $$R_x = R_g + \Delta R$$

"un Balanced Bridge"

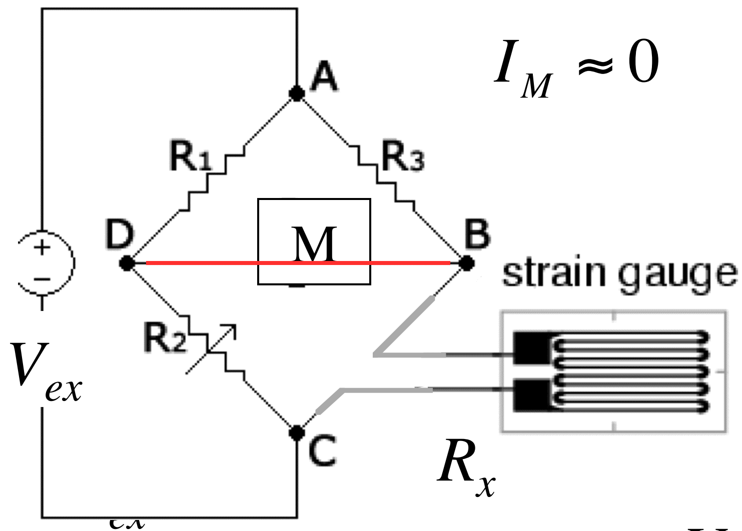
Under strain …

$$\begin{array}{l} R_1 = R_g \\ R_2 = R_g \\ R_3 = R_g \\ R_x = R_g + \Delta R \end{array} \rightarrow V_{BD} = V_{ex}\left(\frac{R_1 R_x - R_2 R_3}{(R_1 + R_2)(R_3 + R_x)}\right) \rightarrow$$

$$V_{BD} = V_{ex}\left(\frac{R_g(R_g + \Delta R) - R_g R_g}{(R_g + R_g)(R_g + R_g + \Delta R)}\right) =$$

$$V_{ex}\left(\frac{R_g^2 + R_g \Delta R - R_g^2}{2R_g(2R_g + \Delta R)}\right) = V_{ex}\left(\frac{\Delta R}{2(2R_g + \Delta R)}\right)$$

Wheatstone Bridge Revisited (3)



$$I_M \approx 0$$

- In terms of Strain properties

$$V_{BD} = V_{ex}\left(\frac{\Delta R / R_g}{4 + 2\Delta R / R_g}\right) \rightarrow \Delta R / R_g = G_F \varepsilon$$

As strain sensor …

"quarter bridge"

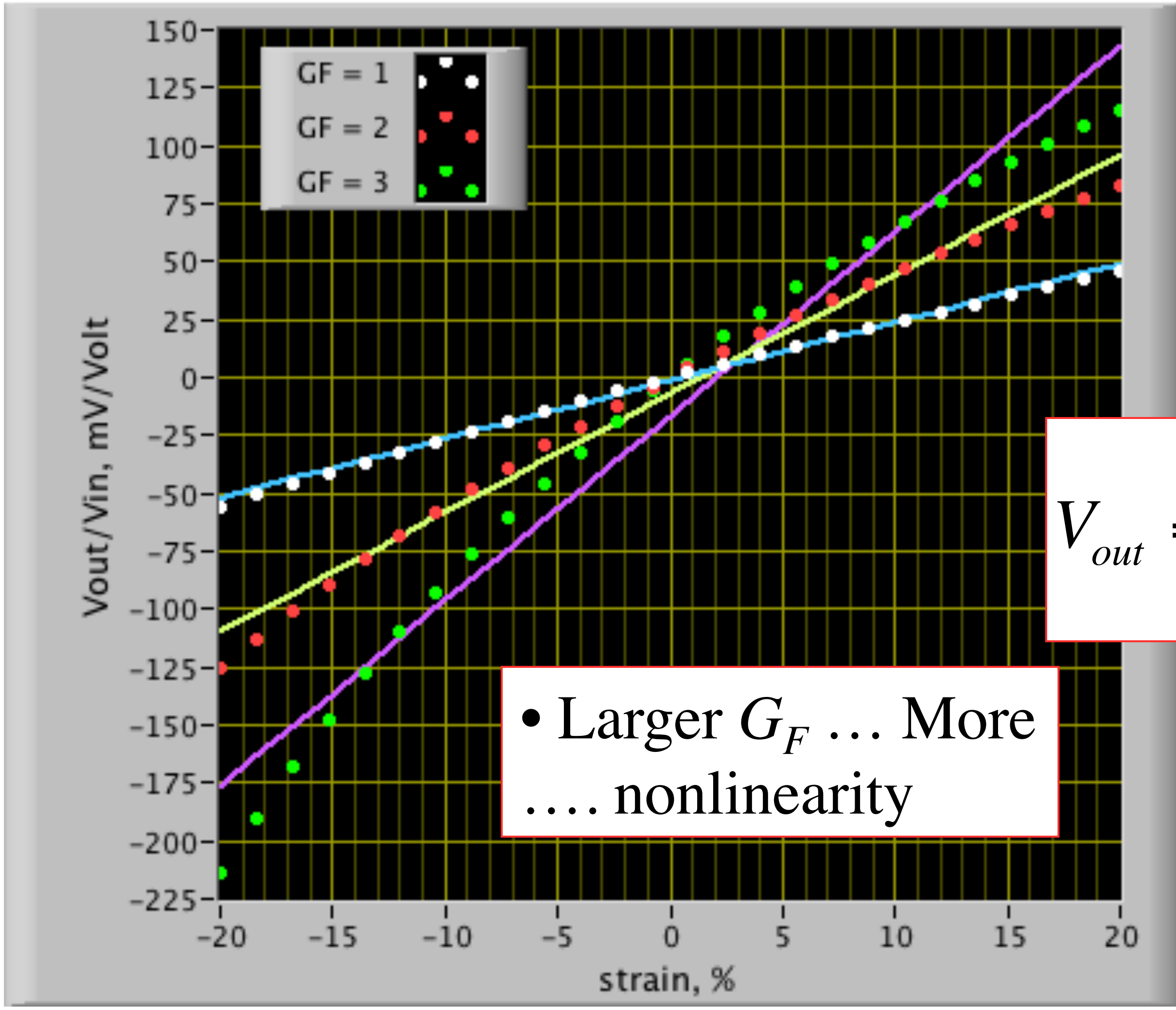
"One arm" bridge

$$\boxed{V_{out} = V_{ex}\left(\frac{G_F \varepsilon}{4 + 2G_F \varepsilon}\right)}$$

- Not Quite linear to Strain

Non-Linearity of Quarter (Single-arm) Bridge

Quarter-Bridge Strain Gauge Response Plot



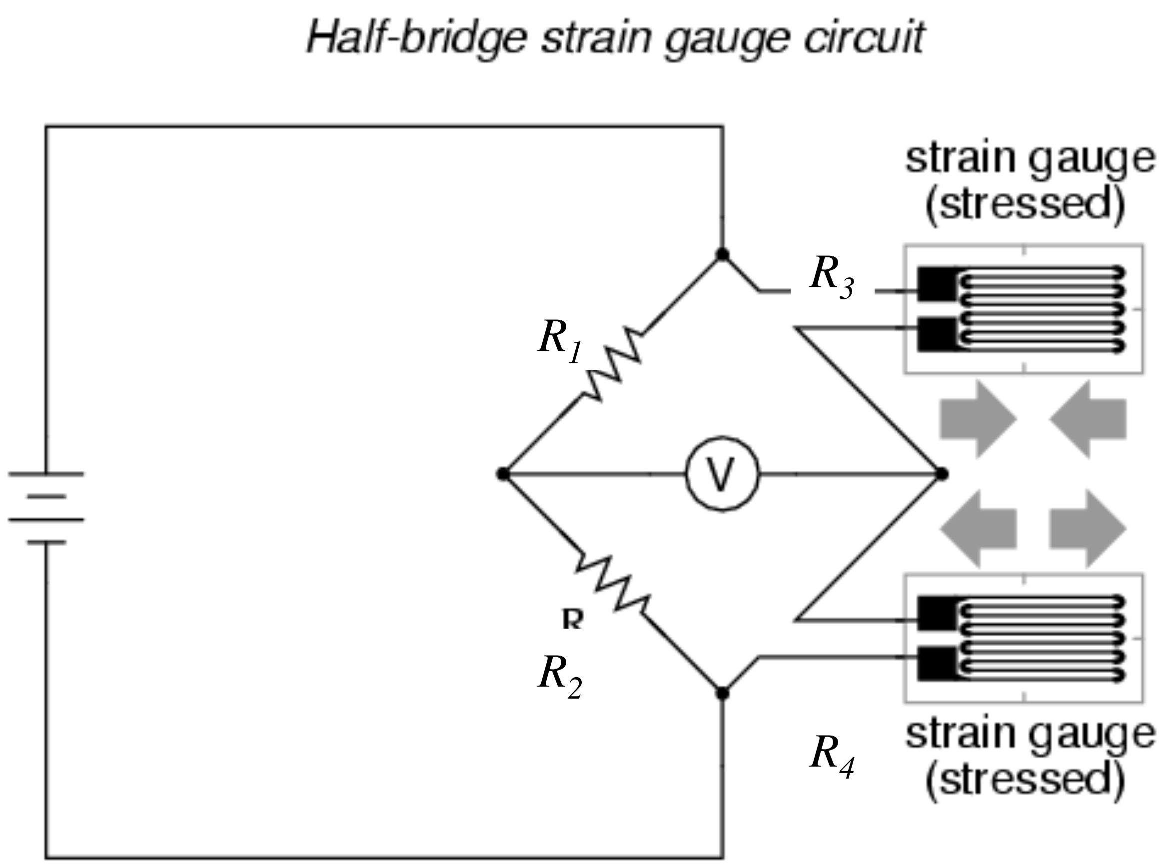
- Compared With best-linear Fits for different Gauge factors

$$V_{out} = V_{ex}\left(\frac{G_F\varepsilon}{4 + 2G_F\varepsilon}\right)$$

- Larger G_F … More …. nonlinearity

Two-Arm (Half) Bridge Configuration (1)

- Look at Configuration below

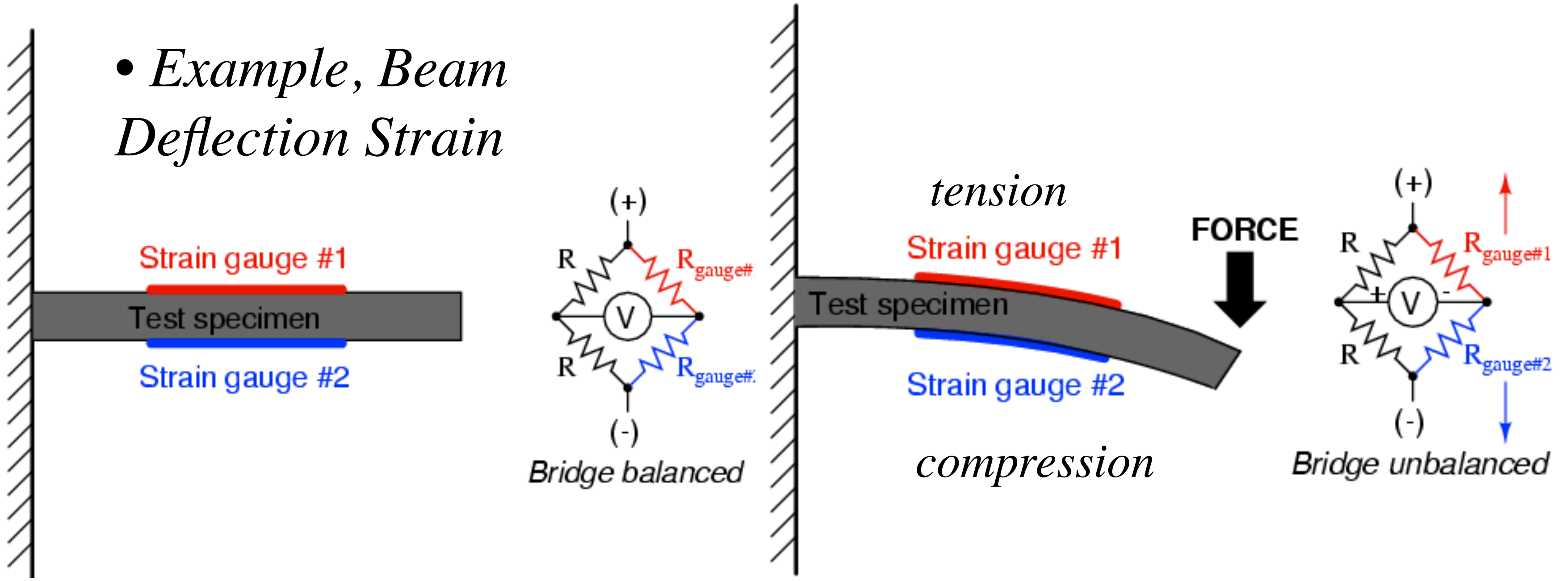


- When would We want to do This arrangement?

- Double sensing element …
"two arm active"

Two-Arm (Half) Bridge Configuration (2)

- *Example, Beam Deflection Strain*



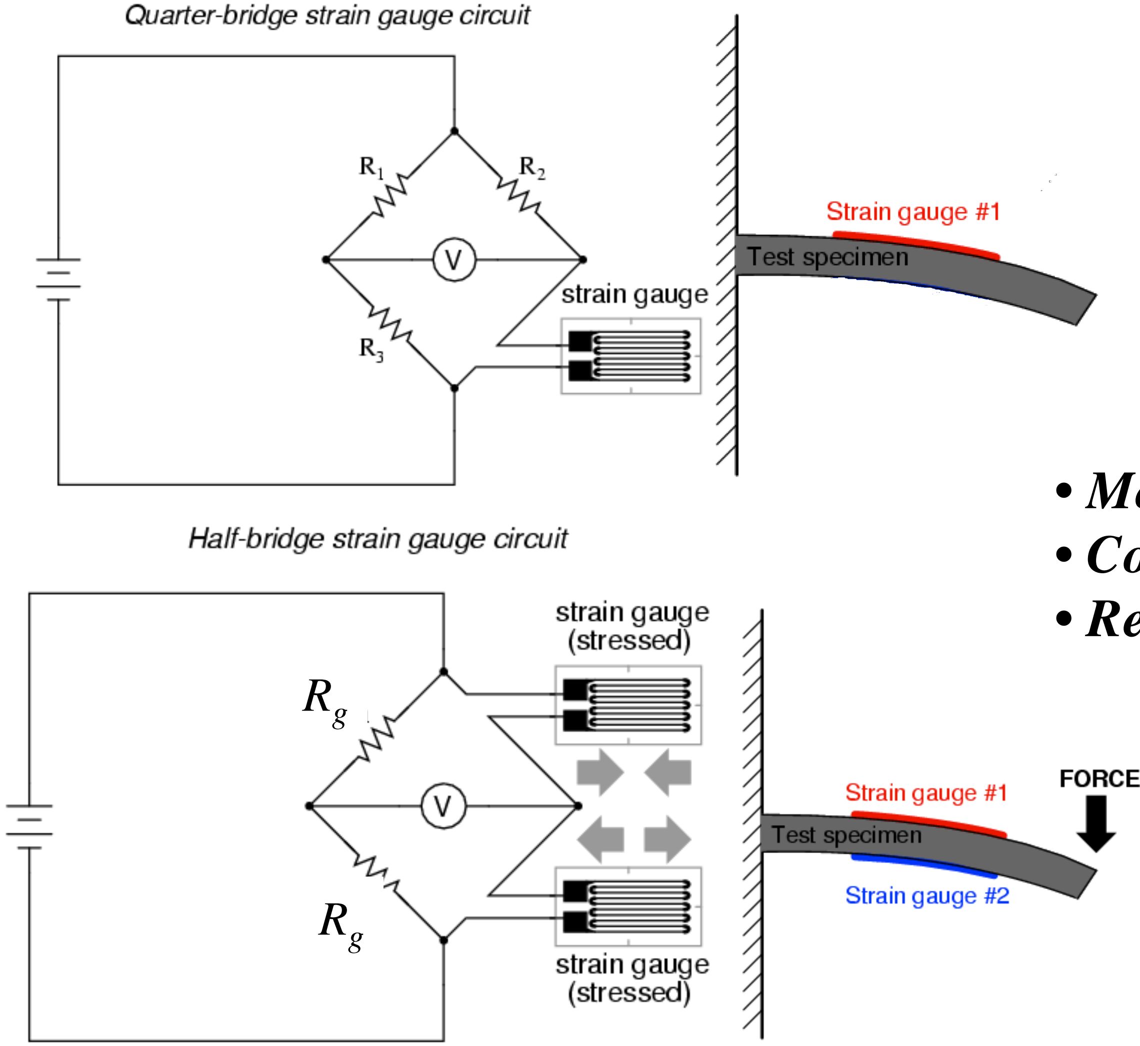
With no force applied to the test specimen, both strain gauges have equal resistance and the bridge circuit is balanced. However, when a downward force is applied to the free end of the specimen, it will bend downward, stretching gauge #1 and compressing gauge #2 at the same time:

$$\boxed{\begin{aligned} R_1 &= R_g \\ R_2 &= R_g \\ R_3 &= R_g + \Delta R \\ R_4 &= R_g - \Delta R \end{aligned}} \rightarrow V_{out} = V_{ex}\left(\frac{R_1 R_4 - R_2 R_3}{(R_1 + R_2)(R_3 + R_4)}\right) =$$

$$= V_{ex}\left(\frac{R_g(R_g - \Delta R) - R_g(R_g + \Delta R)}{(R_g + R_g)(R_g + \Delta R + R_g - \Delta R)}\right) = -V_{ex}\left(\frac{\Delta R}{2R_g}\right)$$

Two-Arm (Half) Bridge Configuration (3)

- Compare quarter and half bridge responses

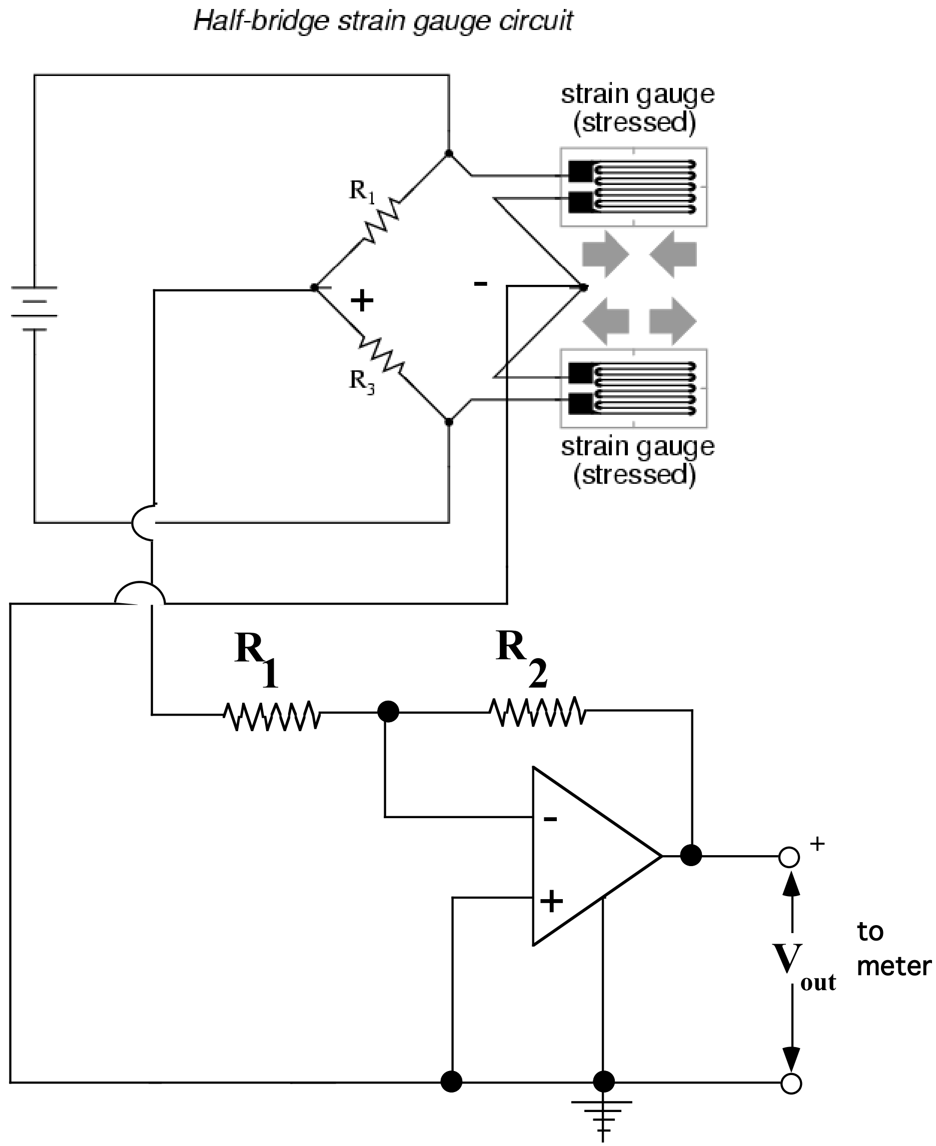


$$V_{out} = V_{ex}\left(\frac{G_F\varepsilon}{4 + 2G_F\varepsilon}\right)$$

- ***More Sensitive Response***
- ***Completely Linear***
- ***Reversed Polarity***

$$V_{out} = -V_{ex}\left(\frac{1}{2}G_F\varepsilon\right)$$

Two-Arm (Half) Bridge Configuration (4)



- **Inverting amplifier comes in handy here**

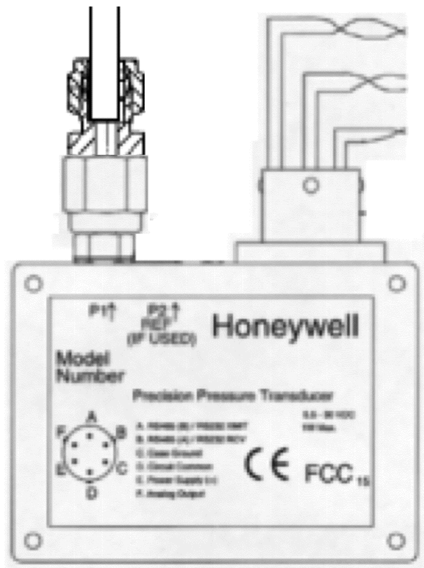
$$V_{out} = -V_{ex}\left(\frac{1}{2}G_F\varepsilon\right)$$

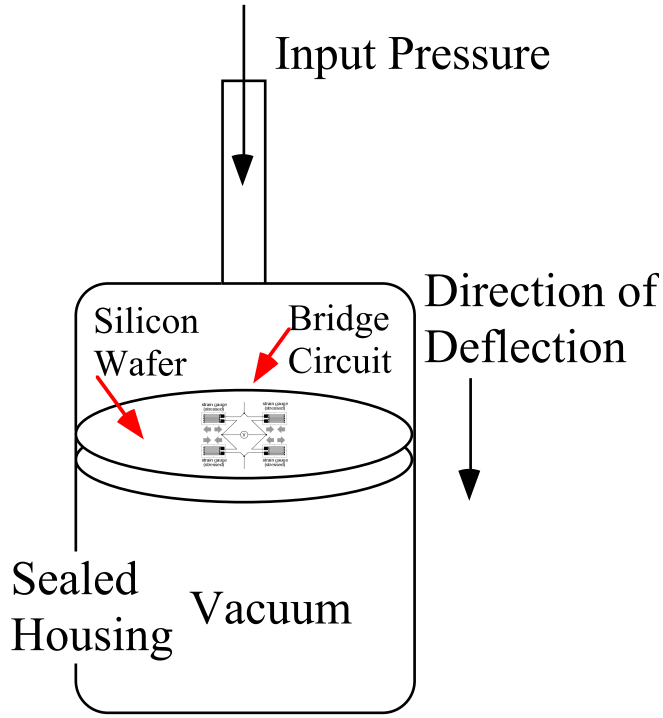
- **Reversed Polarity for Positive strain … couple With inverting Amplifier .. For gain and corrected Polarity**

$$V_{out} = V_{ex}\frac{R_2}{R_1}\left(\frac{1}{2}G_F\varepsilon\right)$$

Four-Arm Active (Full) Bridge (1)

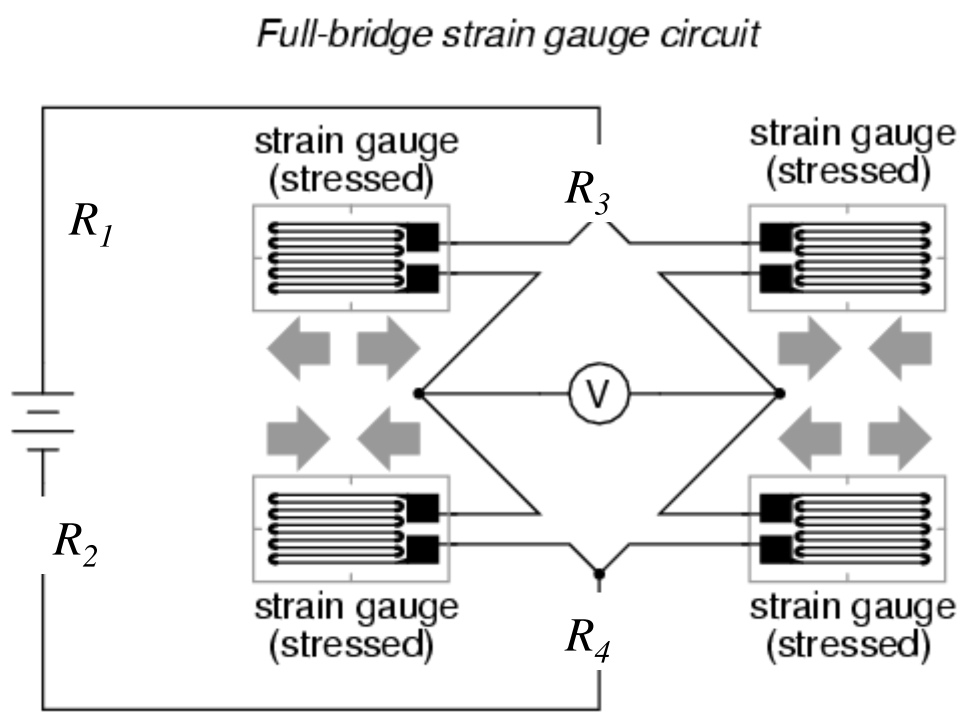
- How about a pressure transducer?





Four-Arm Active (Full) Bridge (2)

- 4-Arm (Full) bridge?



Four-Arm Active (Full) Bridge (3)

- 4-Arm (Full) bridge?

$$\boxed{\begin{aligned} R_1 &= R_g + \Delta R \\ R_2 &= R_g - \Delta R \\ R_3 &= R_g - \Delta R \\ R_4 &= R_g + \Delta R \end{aligned}} \rightarrow V_{out} = V_{ex}\left(\frac{R_1R_4 - R_2R_3}{(R_1+R_2)(R_3+R_4)}\right) =$$

$$= V_{ex}\left(\frac{(R_g+\Delta R)(R_g+\Delta R)-(R_g-\Delta R)(R_g-\Delta R)}{(R_g+\Delta R+(R_g-\Delta R))(R_g-\Delta R+(R_g+\Delta R))}\right) =$$

$$V_{ex}\left(\frac{(R_g{}^2+2\Delta R\cdot R_g+\Delta R^2)-(R_g{}^2-2\Delta R\cdot R_g+\Delta R^2)}{4R_g}\right) =$$

$$V_{ex}\left(\frac{4\Delta R\cdot R_g}{4R_g}\right) = V_{ex}\left(\frac{\Delta R}{R_g}\right) \rightarrow \boxed{V_{out} = V_{ex}\cdot G_F\cdot\varepsilon}$$

- ***Double The sensitivity Of 1/2 bridge***
- ***Linear response***
- ***Strain calibrated As proportional To pressure***

Bridge Constant

- Single Arm Bridge …. $V_{out} = V_{ex}\left(\frac{G_F \varepsilon}{4 + 2G_F \varepsilon}\right)$

  Quarter Bridge

- Dual Arm Bridge …. $\left|V_{out}\right| = V_{ex}\left(\frac{1}{2} G_F \varepsilon\right)$ "roughly doubled sensitivity"

  Half Bridge

- Four Arm Bridge …. $V_{out} = V_{ex} \cdot G_F \cdot \varepsilon$ "roughly quadrupled sensitivity"

  Full Bridge

k = the bridge constant

A = the actual bridge output

B = the output you would get with a single gage.

$$k = \frac{A}{B}$$

Bridge Sensitivity

- Single Arm Bridge …. $V_{out} = V_{ex}\left(\dfrac{G_F\varepsilon}{4+2G_F\varepsilon}\right)$

- Dual Arm Bridge …. $\left|V_{out}\right| = V_{ex}\left(\dfrac{1}{2}G_F\varepsilon\right)$ "roughly doubled sensitivity"

  Roughly doubled sensitivity

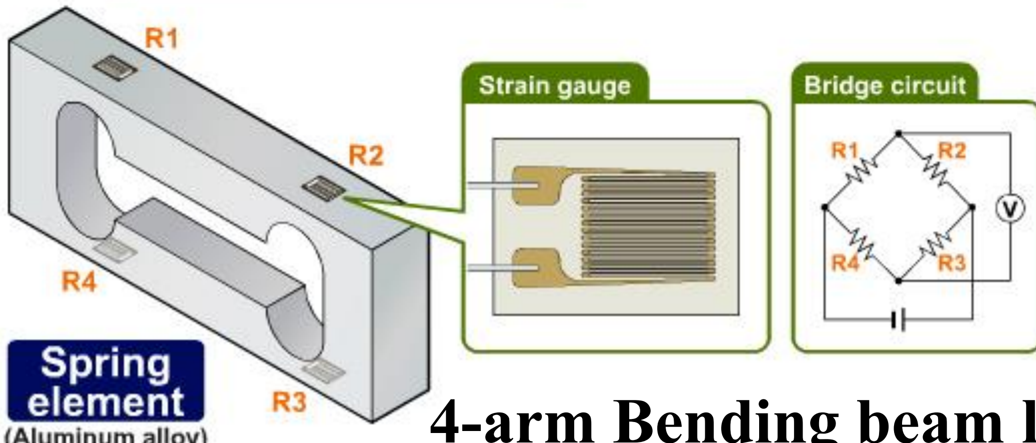
- Four Arm Bridge …. $V_{out} = V_{ex}\cdot G_F\cdot\varepsilon$ "roughly quadrupled sensitivity"

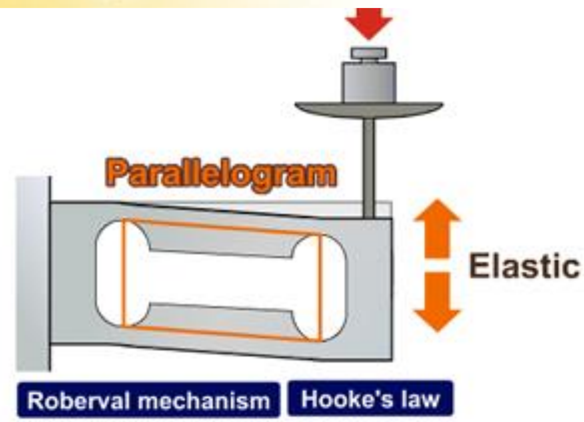
  Roughly quadrupled sensitivity

$$\varepsilon_{1/4} = \frac{4}{G_F}\cdot\left(\frac{V_{out}}{\left[V_{ex}-2\cdot V_{out}\right]}\right) \qquad \varepsilon_{1/2} = \frac{2}{G_F}\cdot\left(\frac{V_{out}}{V_{ex}}\right) \qquad \varepsilon_{full} = \frac{1}{G_F}\cdot\left(\frac{V_{out}}{V_{ex}}\right)$$

Examples of Modern Load Bridge Circuits

4
~~2~~-arm S-beam load cell

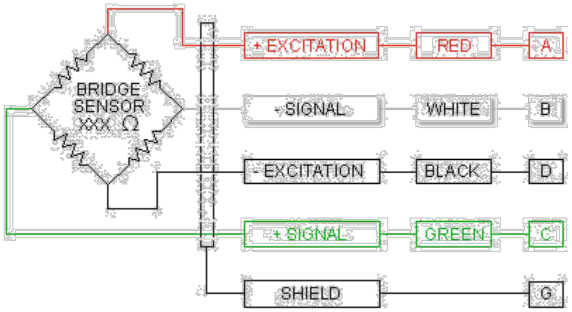


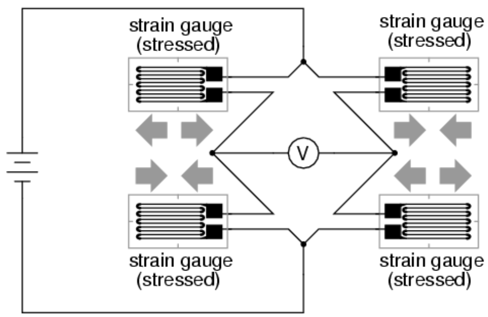


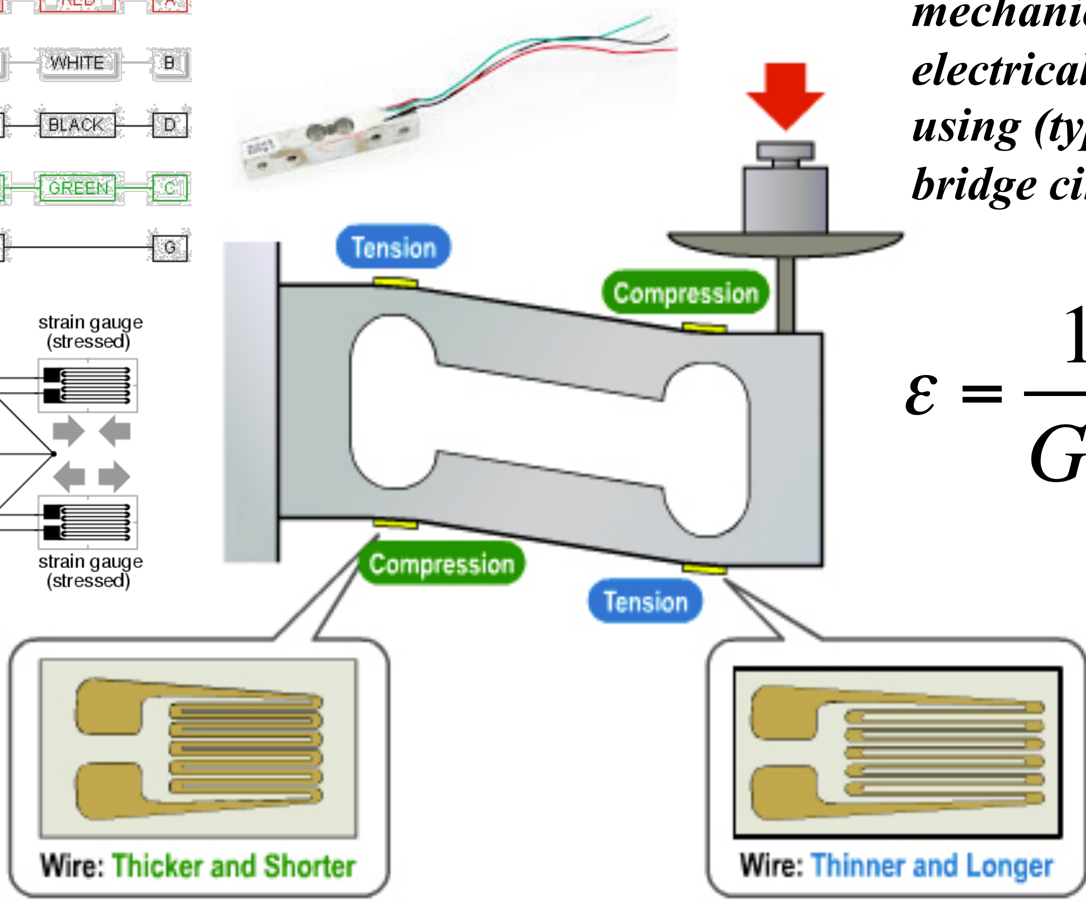
4-arm Bending beam load cell

Load Cells

4-arm active bridge ("Full Bridge")







Load Cell is a type of transducer that converts mechanical force into an electrical output signal using (typically) a full-bridge circuit

$$\varepsilon = \frac{1}{G_F} \cdot \left(\frac{V_{out}}{V_{ex}} \right)$$

Load Cells

Nearly All Use a Full Bridge Resistive Circuit

Types

S Type

Button

Canister

Shear

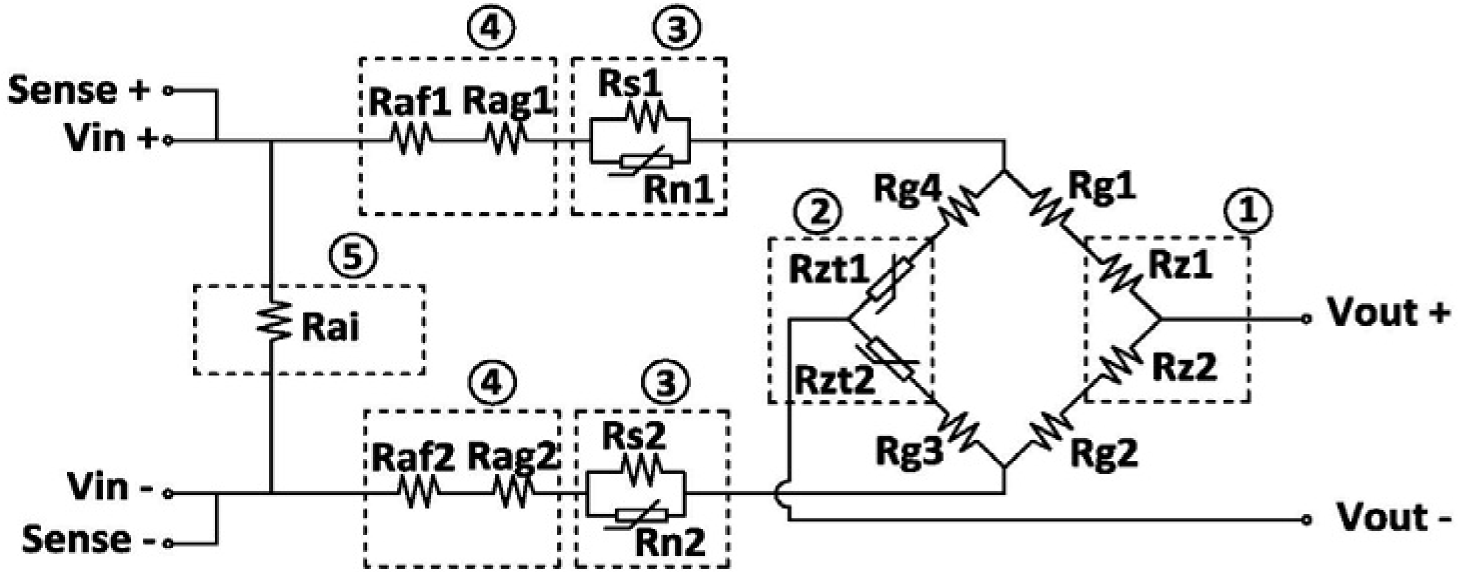
Beam

Examples of Modern Load Bridge Circuits (3)

Complete circuit for a high precision load cell

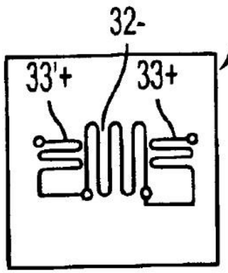
To manufacture a real high precision load cell, it is necessary an additional circuitry to the strain gauges, dedicated to the fine adjustment of the output signal at different loads and also to make the necessary individual thermal compensations during the manufacturing process.

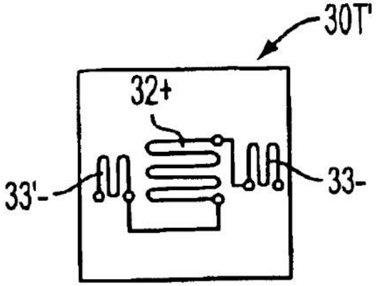
The following wiring diagram allows us to identify different stages, described below.

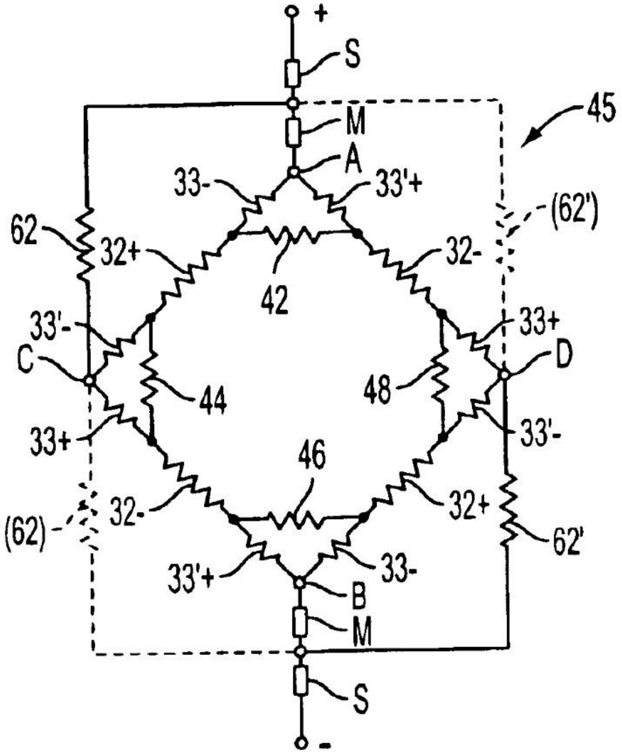


Examples of Modern Bridge Sensing Circuits (2)

Compression column load cell

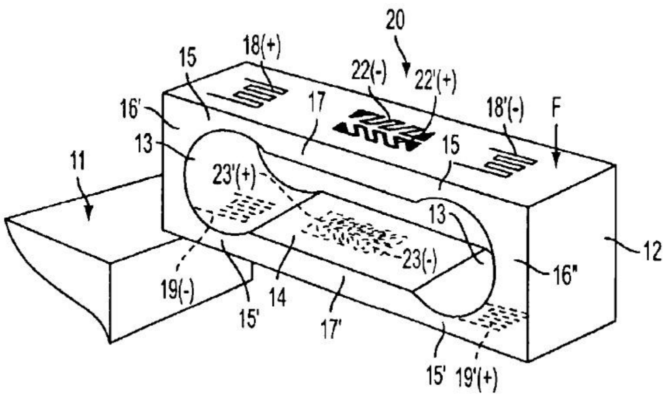


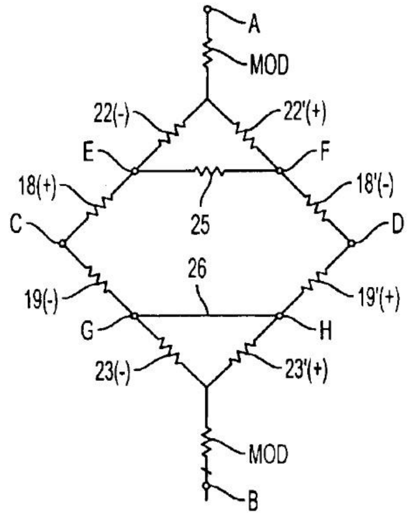




Examples of Modern Bridge Circuits (3)

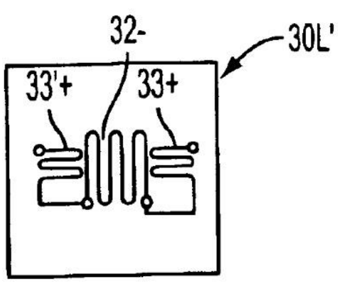
Bending beam load cell with torque sensitivity compensation

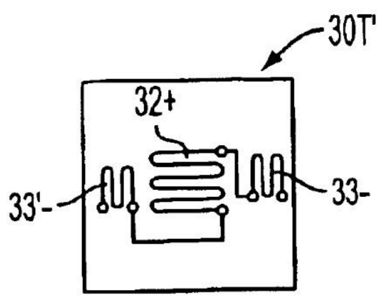


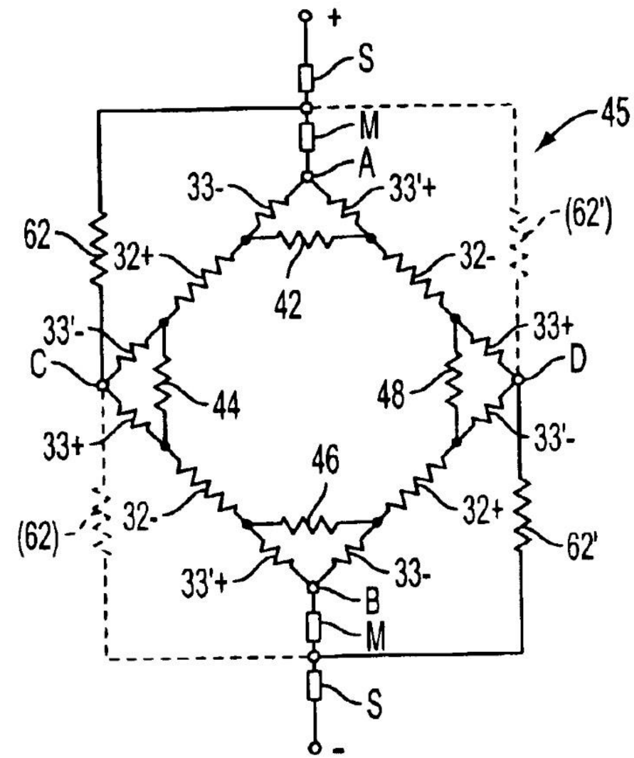


Examples of Modern Bridge Sensing Circuits (2)

Compression column load cell







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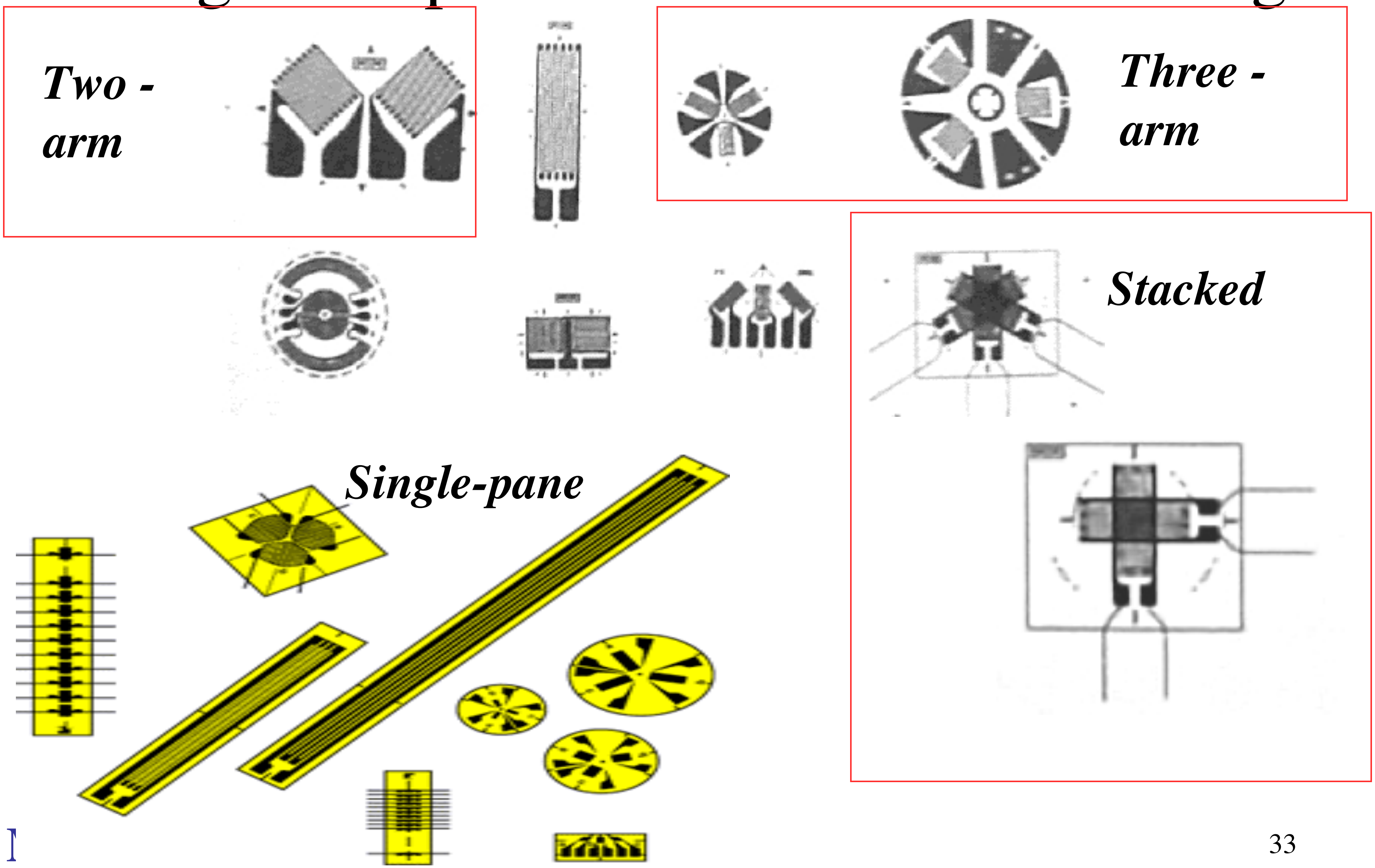
Bridge Components often Built into Gauge



Semiconductor Strain Gauges

The gauges we have been talking about are made of metal. We can also make them out of semiconductors, which is how the strain gauges in our pressure sensors are made. These are dominated by the piezoresistive component of the change in resistance and have several advantages and disadvantages:

Pros:

- Very high gauge factors (up to 200)
- Higher resistance
- Longer fatigue life
- Lower Hysteresis
- Smaller
- High frequency response

Cons:

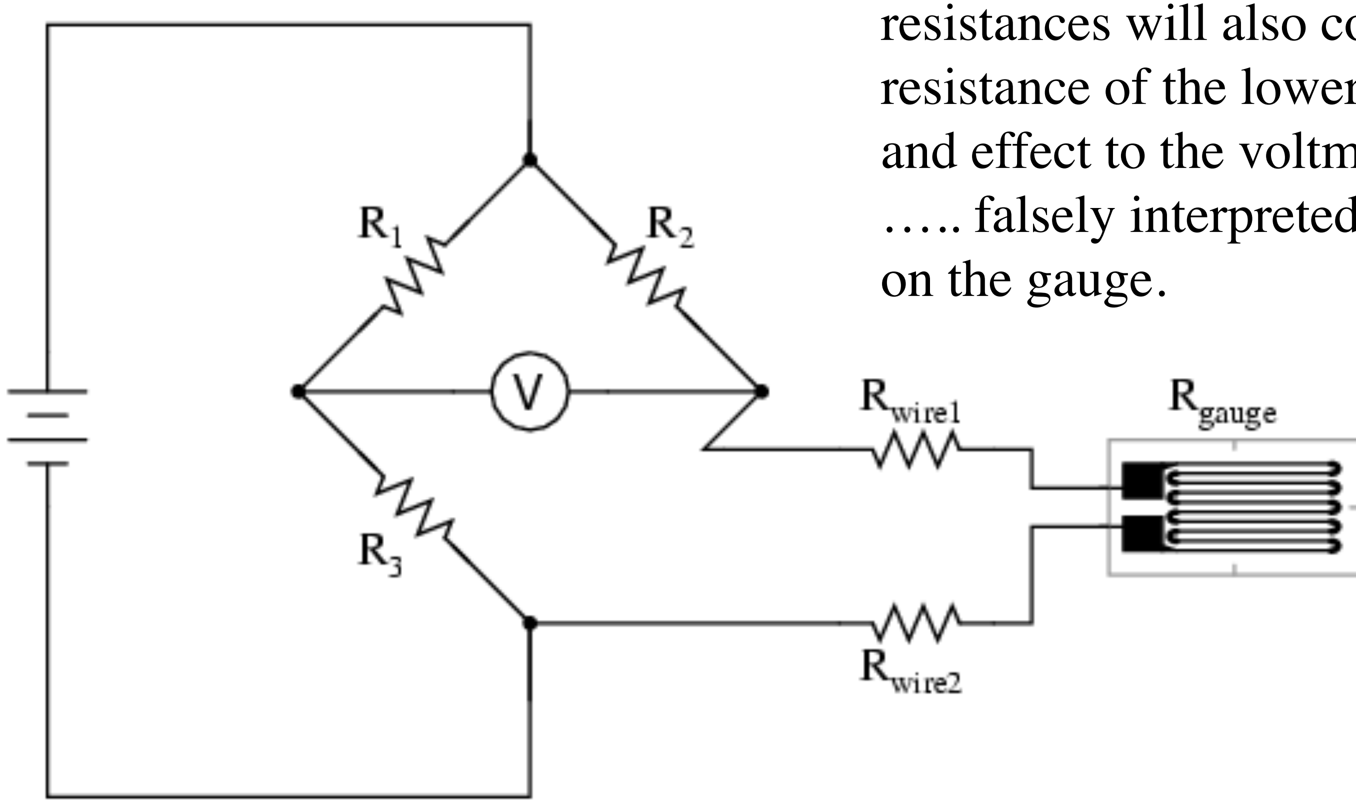
- Temperature sensitivity
- Nonlinear output
- More limited on maximum strain

Mostly used for construction of pressure transducers

Lead Wire Error (1)

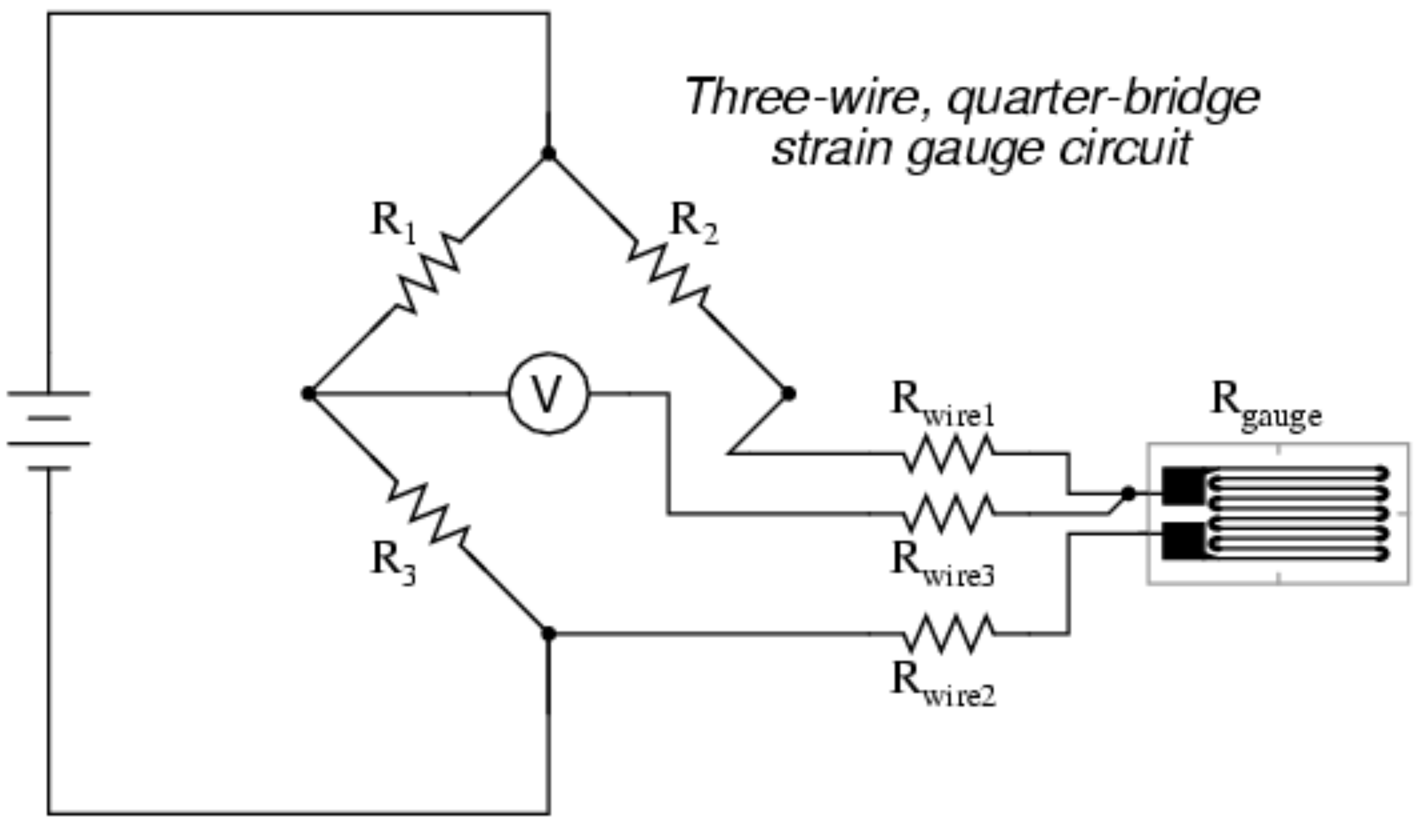
• Since Strain gauges are often Located far from Data acquisition system, resistance of the lead wires to the gauge can potentially contaminate the reading

• …. Lead wire resistances are in series with the strain gauge and their resistances will also contribute to the resistance of the lower half of the bridge, and effect to the voltmeter's indication. ….. falsely interpreted as physical strain on the gauge.



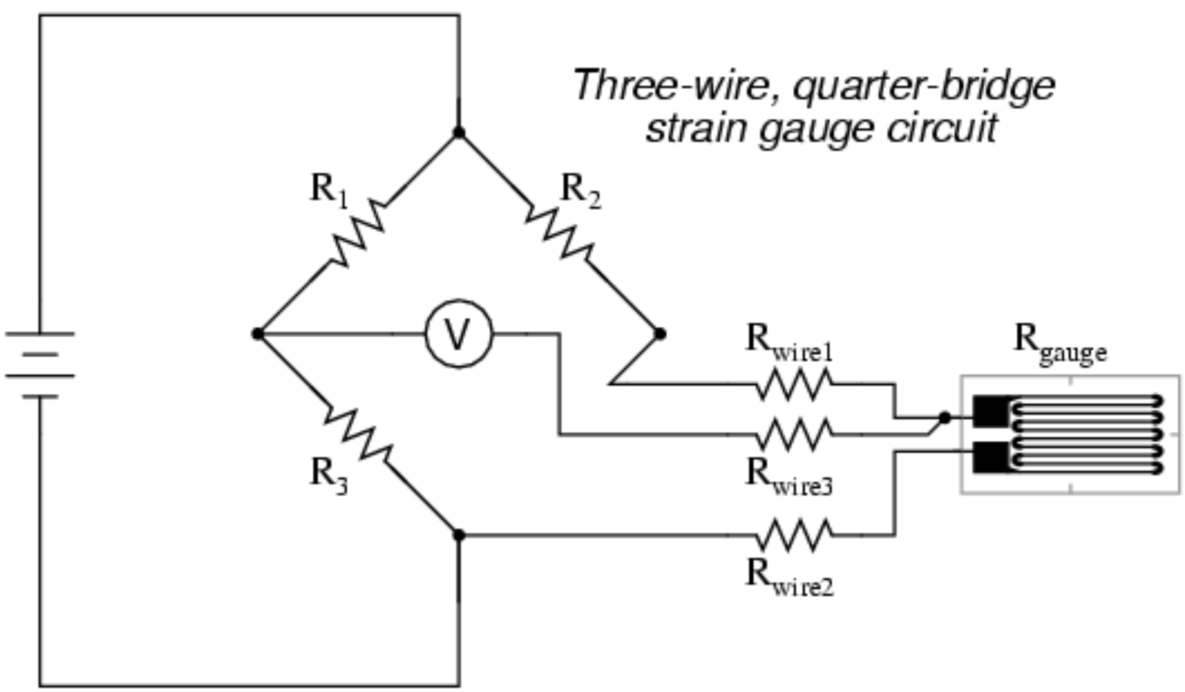
Lead Wire Error (2)

- Lead wire effect can be minimized by a third wire connecting the right side of the meter directly to the upper wire of the strain gauge:



Lead Wire Error (3)

- Wire 3 carries no current (Infinite impedance meter), R_{wire3} will not drop any substantial amount of voltage. Resistance of the top wire *(R_{wire1})* has been "bypassed" with voltmeter connected to top terminal of the strain gauge, leaving only lower wire's resistance (R_{wire2}) to contribute stray resistance in series with gauge.

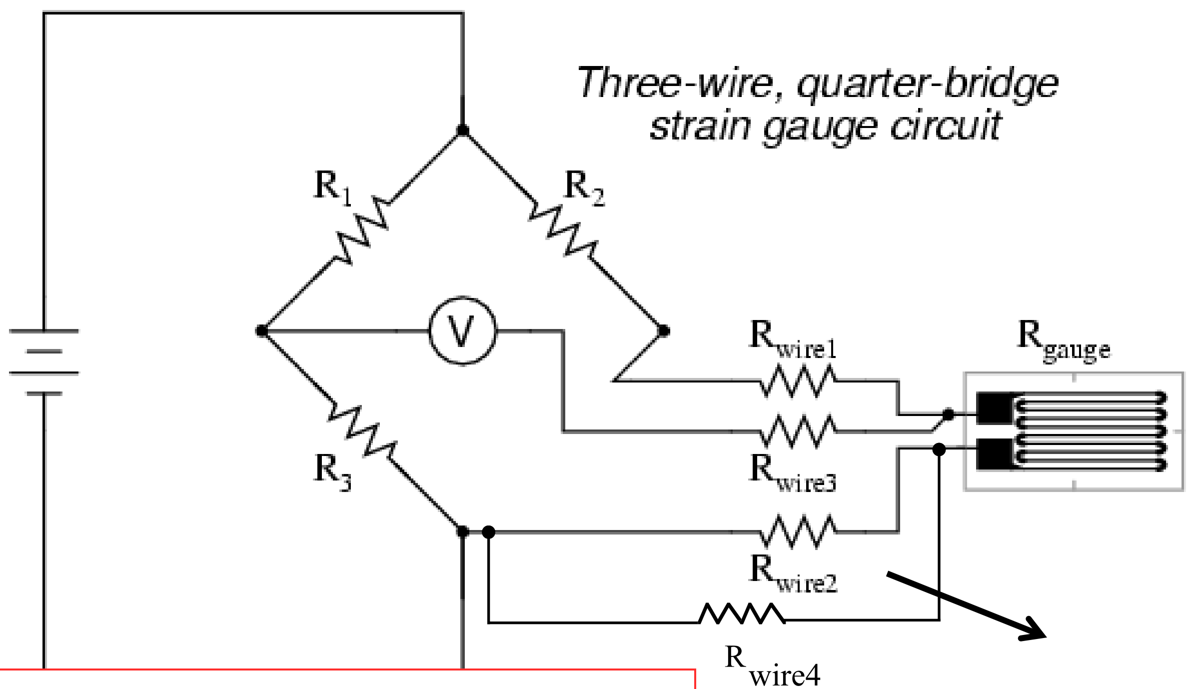


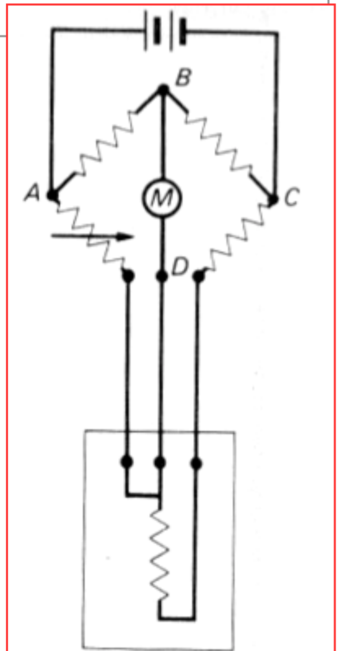
- Not a perfect

Correction … but better than nothing

Lead Wire Error (4)

• Add 4th wire in parallel with wire 2 … and you can reduce the effect of this resistance





(a) Siemens lead arrangement

$$\frac{1}{\frac{1}{0.010}+\frac{1}{0.010}} = 0.005$$

example

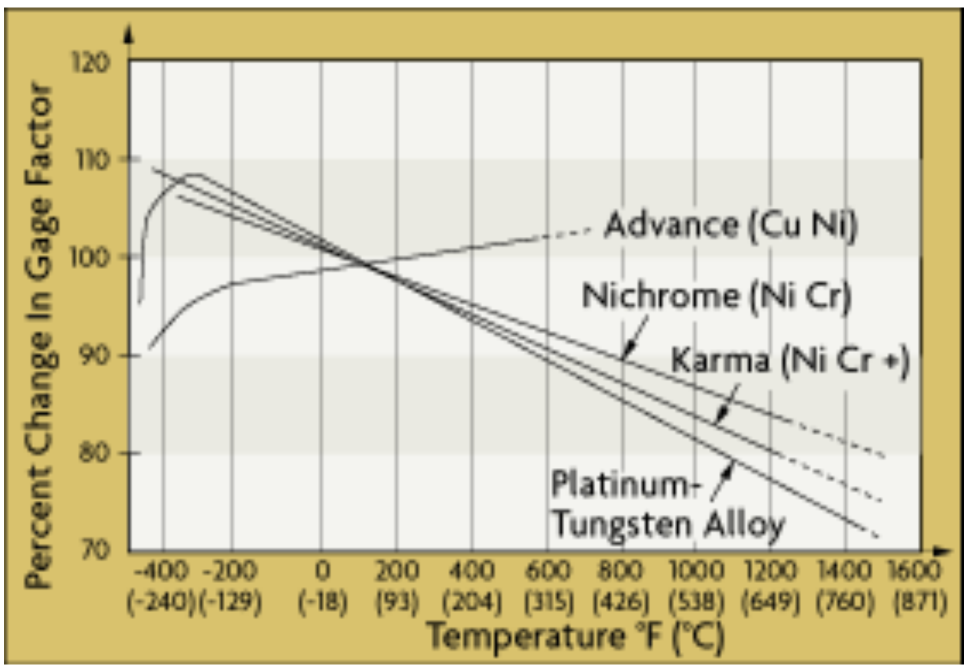
SYSTEMS

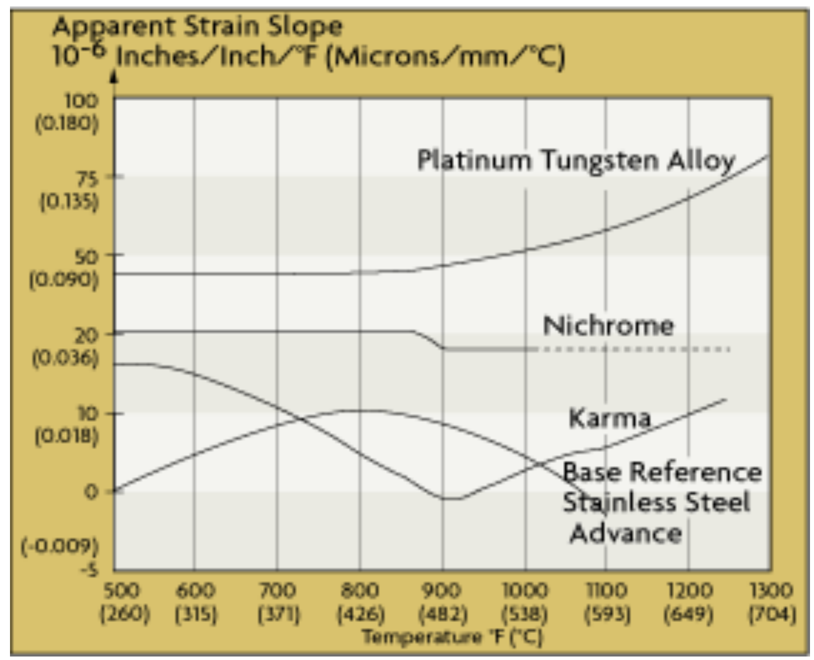
$$R_{wire24_{eq}} = \frac{1}{\frac{1}{R_{wire_2}}+\frac{1}{R_{wire_4}}}$$

Temperature Compensation (1)

• In general, electrical resistivity of metals increases with temperature, while the resistivity of semiconductors decreases with increasing temperature.

• For a strain gauge, this change in resistivity due to temperature Shows up as an "apparent strain" on the sensor output ….





Temperature Compensation (2)

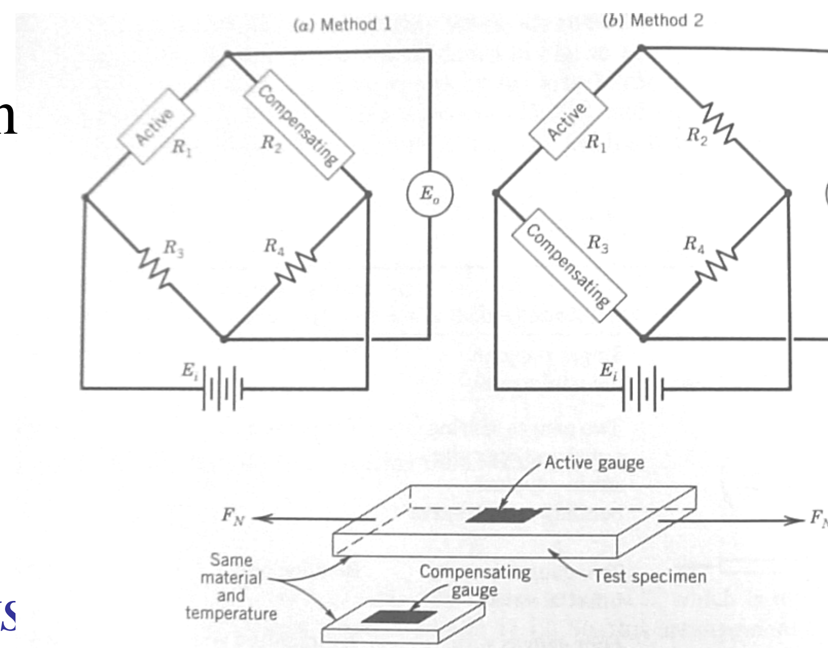
• Apparent strain is any change in gage resistance not caused by strain on the force element. Apparent strain is result of interaction of thermal coefficient of strain gage and difference in expansion between the gage and the test specimen.

• Compensation for apparent strain is necessary if temperature varies while strain is being measured.

• Most common method for compensation is use of multiple gauges

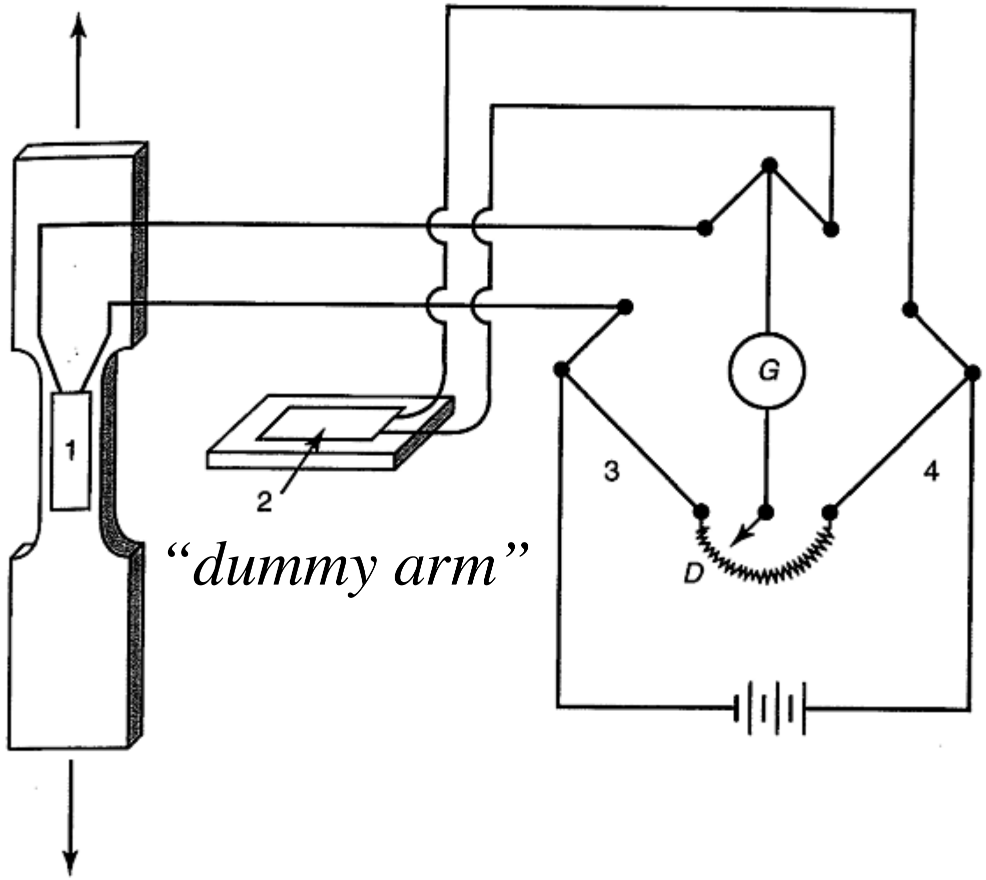
If the temperature of the specimen changes, then both gages will change their resistance similarly

$$\frac{R_1}{R_2} = \frac{R_3}{R_4} \qquad \frac{R_1 + \Delta R_t}{R_2 + \Delta R_t} = \frac{R_3}{R_4}$$



Temperature Compensation (3)

- Another Example of a Temperature Compensated Circuit



"dummy arm"

"active arm"

- Compensation Method works well for most applications

Temperature Compensation (4)

- *Self-compensating gauges*

Suppose test specimen and resistive element of strain gage have linear expansion coefficients β_s and β_g, respectively. Then, strain gage bonded on the surface of object provides a thermally-induced apparent strain ε_T per 1°C expressed as:

α: Resistive temperature coefficient of resistive element

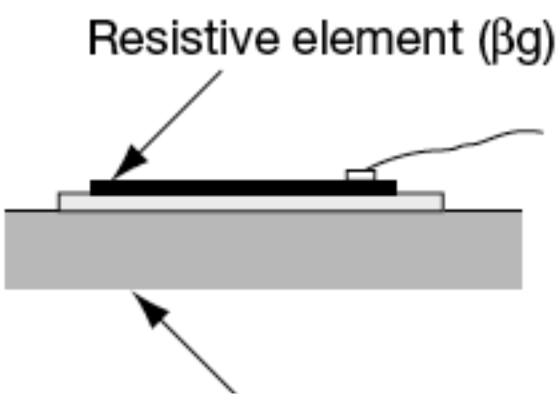
$$\varepsilon_T = \frac{\alpha}{G_F} + \left(\beta_s - \beta_g\right)$$

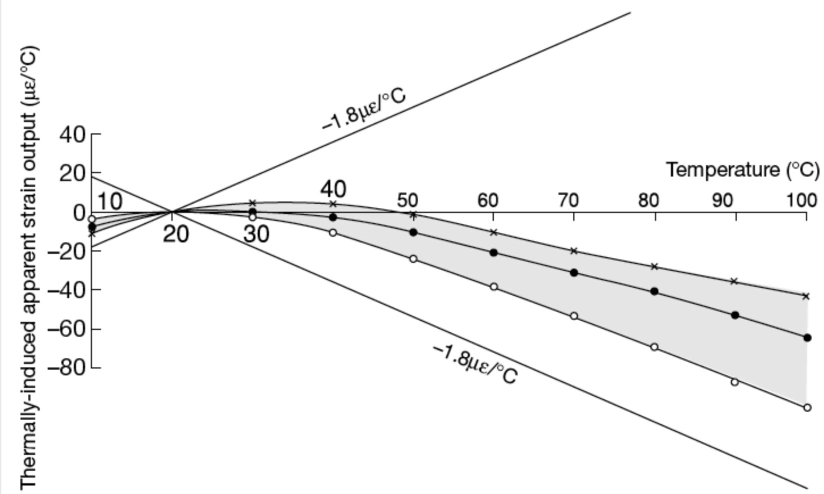
G_F: Gage factor of strain gage

Temperature Compensation (5)

- *Self-compensating gauges*

Self-temperature-compensation gage is designed so that ε_T is driven to approximately zero by designing the the resistive temperature coefficient of the gage's resistive element so that it expands according to the linear expansion coefficient of the test specimen.





Typical Temperature Characteristics of Self-temperature-compensation Foil Gage

Temperature Compensation (6)

- *Self-compensating gauges*

● Linear Expansion Coefficients of Materials (x10^{-6}/°C)

Material	Linear Exp. Coef.	Material	Linear Exp. Coef.
Quartz glass	0.4	Beryllium	11.5
Amber	1.1	Common steel	11.7
Brick	3.0 to 5.0	Inconel X	12.1
Tungsten	4.5	Nickel	13.3
Lumber (grain dir.)	5.0	Gold	14.0
Molybdenum	5.2	SUS 304	16.2
Zirconium	5.4	Beryllium copper	16.7
Cobar	5.9	Copper	16.7
Concrete	6.8 to 12.7	Brass	21.0
Titanium alloy	8.5	2024-T4 aluminum	23.2
Platinum	8.9	2014-T4 aluminum	23.4
Soda-lime glass	9.2	Magnesium alloy	27.0
SUS 631	10.3	Lead	29.0
SUS 630	10.6	Acrylic resin	Approx. 65 to 100
Cast iron	10.8	Polycarbonate	66.6
NiCrMo steel	11.3	Rubber	Approx. 77

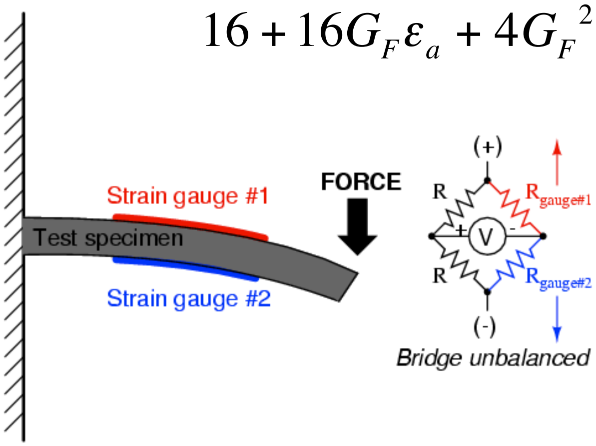
Other Forms of Compensation

Sometimes gauge may be subject to strains other than the one we are interested in. Compensation is removing these effects by using multiple gauges. As an example, say you have a beam under axial stress and a bending moment, and you are interested in the axial stress only:

$$\frac{V_{out}}{V_{ex}} = \left(\frac{G_F\left(\varepsilon_a + \varepsilon_b\right)}{4 + 2G_F\left(\varepsilon_a + \varepsilon_b\right)} \right) + \left(\frac{G_F\left(\varepsilon_a - \varepsilon_b\right)}{4 + 2G_F\left(\varepsilon_a - \varepsilon_b\right)} \right) =$$

$$\frac{G_F\left(\varepsilon_a + \varepsilon_b\right)\left[4 + 2G_F\left(\varepsilon_a - \varepsilon_b\right)\right] + G_F\left(\varepsilon_a - \varepsilon_b\right)\left[4 + 2G_F\left(\varepsilon_a + \varepsilon_b\right)\right]}{\left(4 + 2G_F\left(\varepsilon_a + \varepsilon_b\right)\right)\cdot\left(4 + 2G_F\left(\varepsilon_a - \varepsilon_b\right)\right)}$$

$$= \frac{8G_F\varepsilon_a + 4G_F^{\,2}\left(\varepsilon_a^{\,2} - \varepsilon_b^{\,2}\right)}{16 + 16G_F\varepsilon_a + 4G_F^{\,2}\left(\varepsilon_a^{\,2} - \varepsilon_b^{\,2}\right)} \approx \boxed{\frac{G_F\varepsilon_a}{2 + 2G_F\varepsilon_a}}$$



The two gauges see the same axial strain but opposite bending strains

Gauge Calibration (1)

• Periodic calibration is required to verify the accuracy and linearity of the strain gauges and calibration is also required on a more regular basis to scale the instrument sensitivity, by adjusting the gauge factor or gain, in order to ensure that the output of the strain measuring instrument corresponds to a pre-determined input.

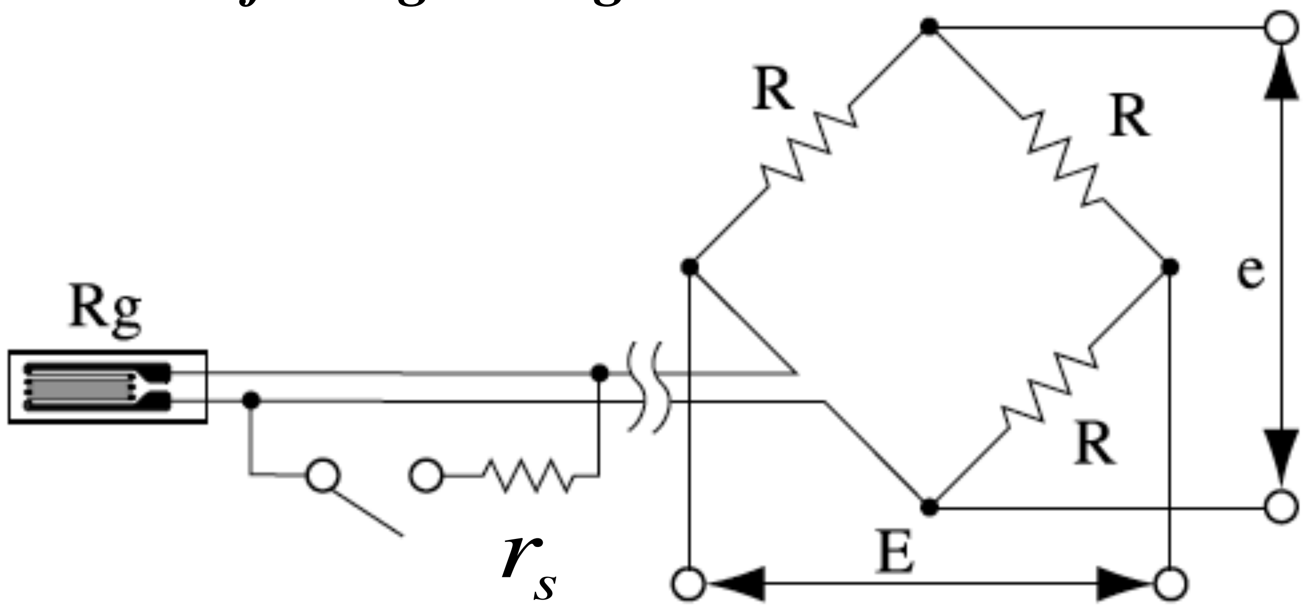
• Shunt calibration is a method of indirect calibration widely used for scaling and verification of a Wheatstone Bridge.

• It is achieved by simulating a strain gauge output by decreasing the resistance of one arm of the bridge. This is achieved by placing a large resistor in parallel with the arm resistance (known as "shunting").

Gauge Calibration (2)

• Since a strain gage is a passive electrical device, there exists a simple, yet effective, Method for checking the calibration of a load cell system in the field or when a means of applying actual forces is unavailable. Inducing an electrical imbalance in the cell's bridge circuit will simulate the bridge imbalance caused by the application of actual strains.

• ***Shunt Calibration Of Gauge/Bridge Circuit***

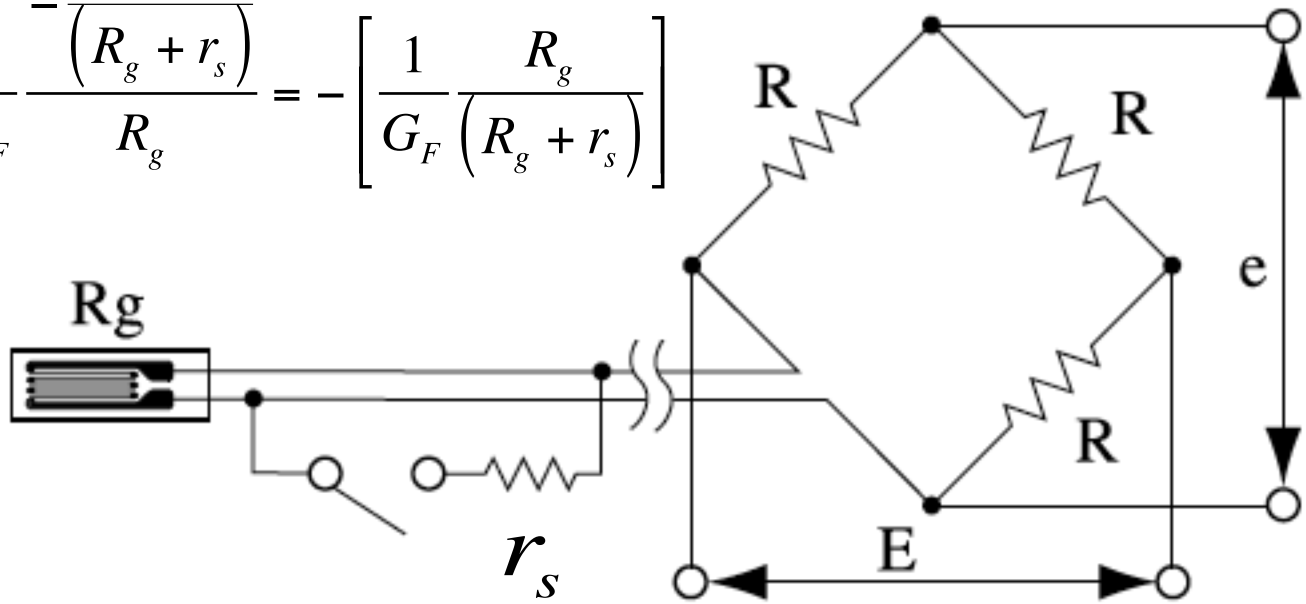


Gauge Calibration (3)

- Shunt Calibration
Of Gauge/Bridge Circuit

$$\Delta R = \frac{R_g r_s}{\left(R_g + r_s\right)} - R_g = -\frac{R_g^2}{\left(R_g + r_s\right)}$$

$$\varepsilon_{eq} = \frac{1}{G_F}\frac{\Delta R_g}{R_g} = \frac{1}{G_F}\frac{-\frac{R_g^2}{\left(R_g + r_s\right)}}{R_g} = -\left[\frac{1}{G_F}\frac{R_g}{\left(R_g + r_s\right)}\right]$$



Gauge Calibration (4)

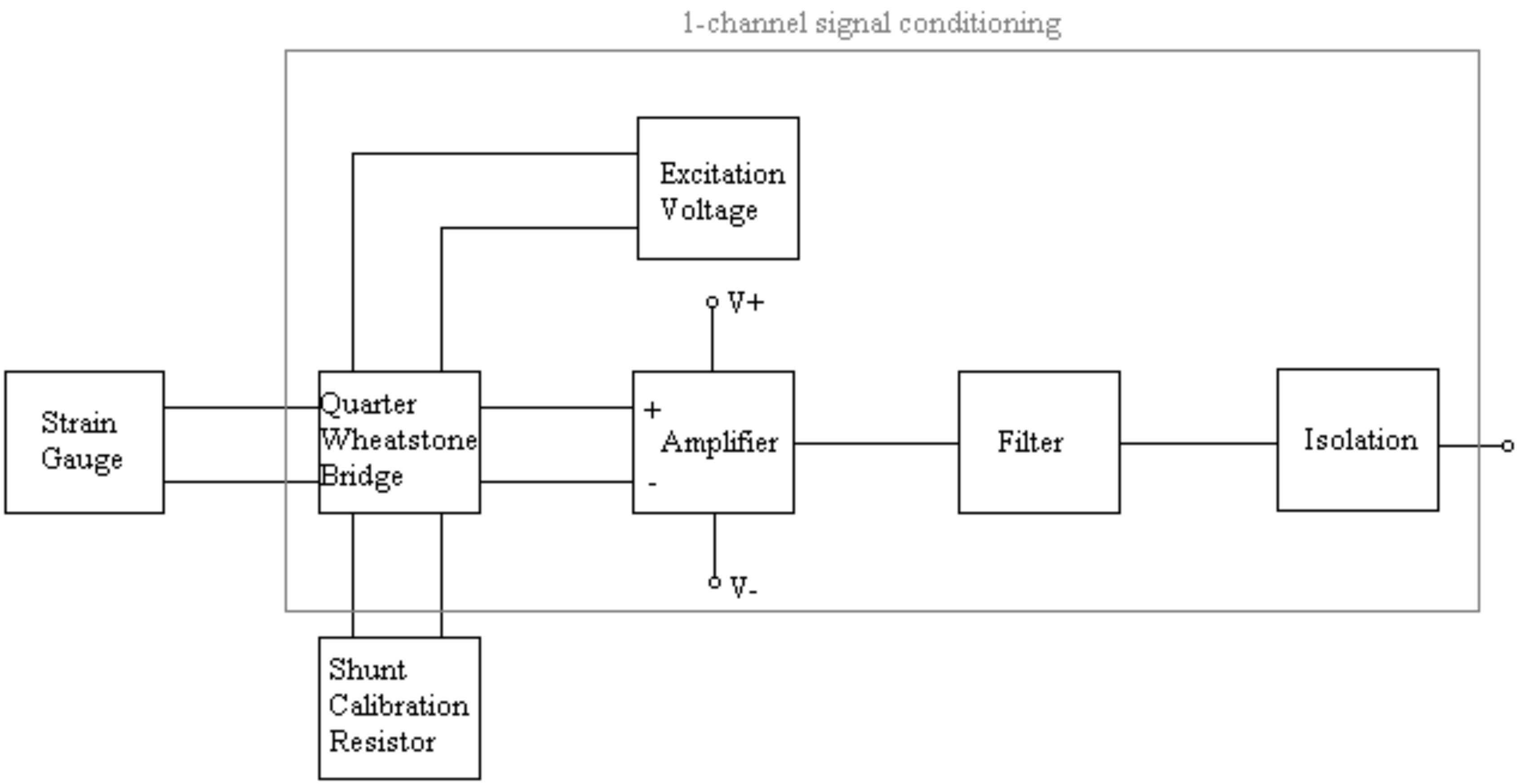
Examples of Calibration Strain Value and Resistance ($Rg = 120\Omega$, $G_F = 2.00$)

Calibration Strain Value	Resistance, r_s (approx.)
100 με	600 kΩ
200 με	300 kΩ
500 με	120 kΩ
1000 με	60 kΩ
2000 με	30 kΩ

$$-\frac{1}{2}\left(\frac{120}{120+600\cdot 10^{3}}\right) = -100 \times 10^{-6}\ \varepsilon$$

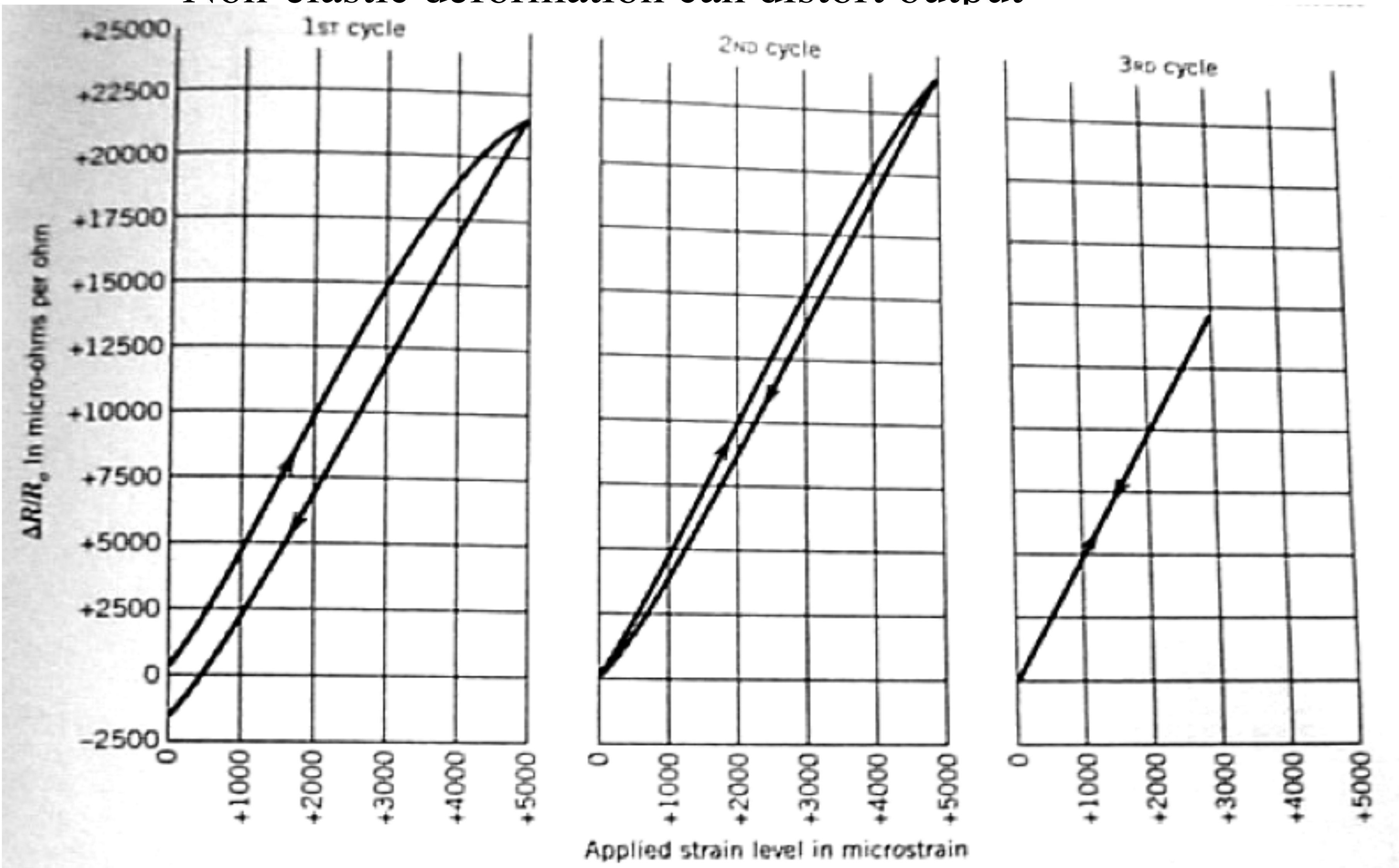
Strain Gauge Signal Conditioning and Calibration

• Output from each strain gauge must be conditioned before it can be passed to a data acquisition device.



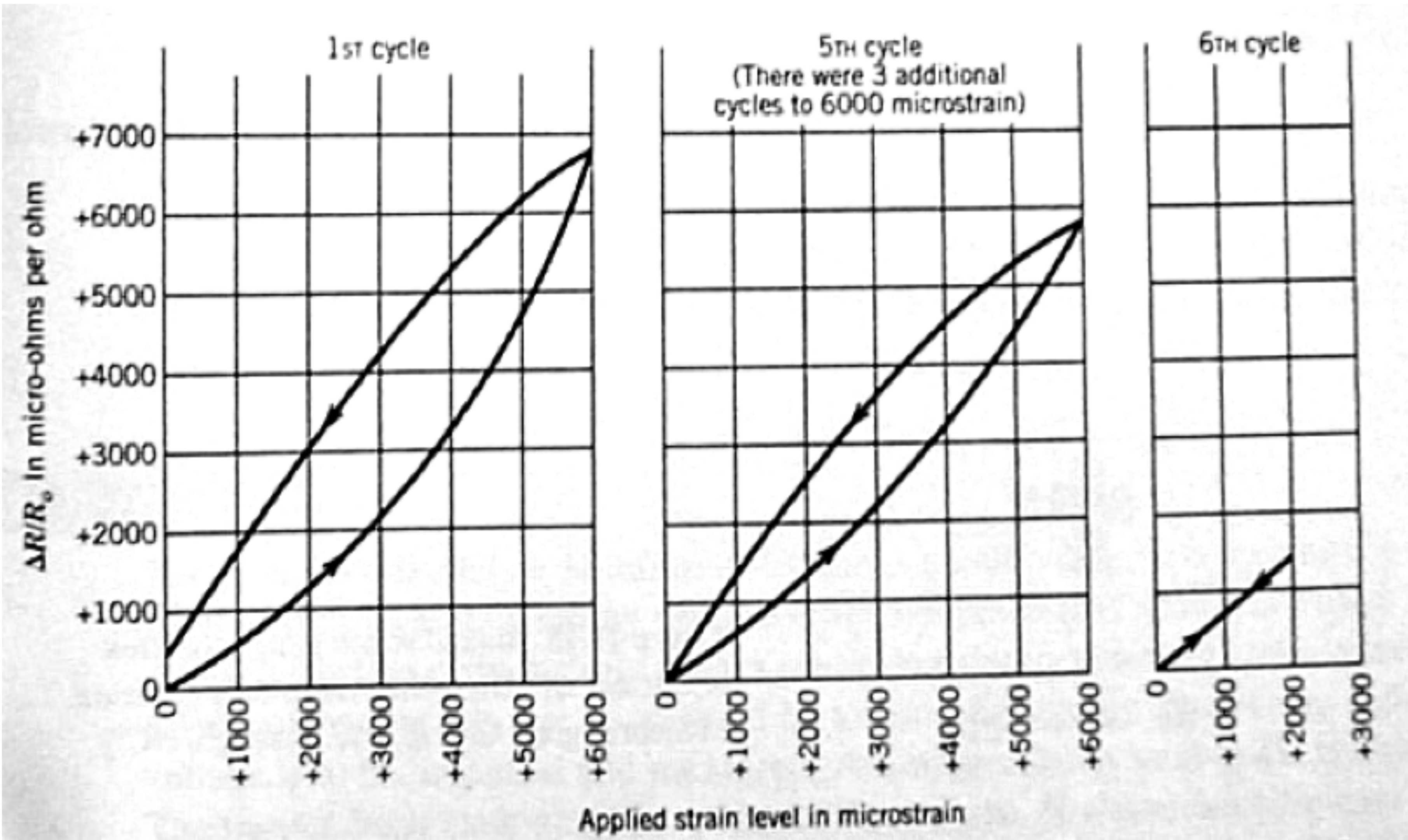
Hysteresis of Strain Gauges (1)

- Non-elastic deformation can distort output



Hysteresis of Strain Gauges (2)

- Non-elastic deformation can distort output

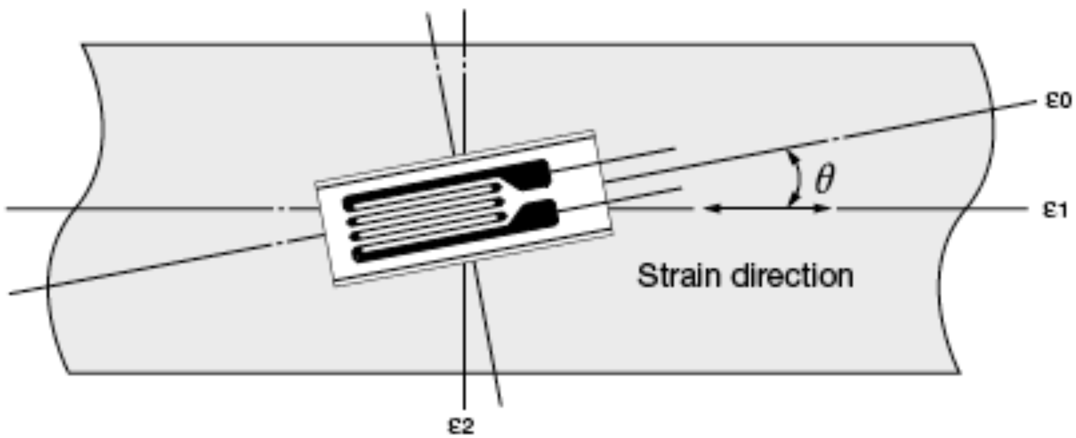


Misalignment Effects

• The strain ε_0 measured by a strain gage that is misaligned by an angle θ from the direction of the principal strain is given by:

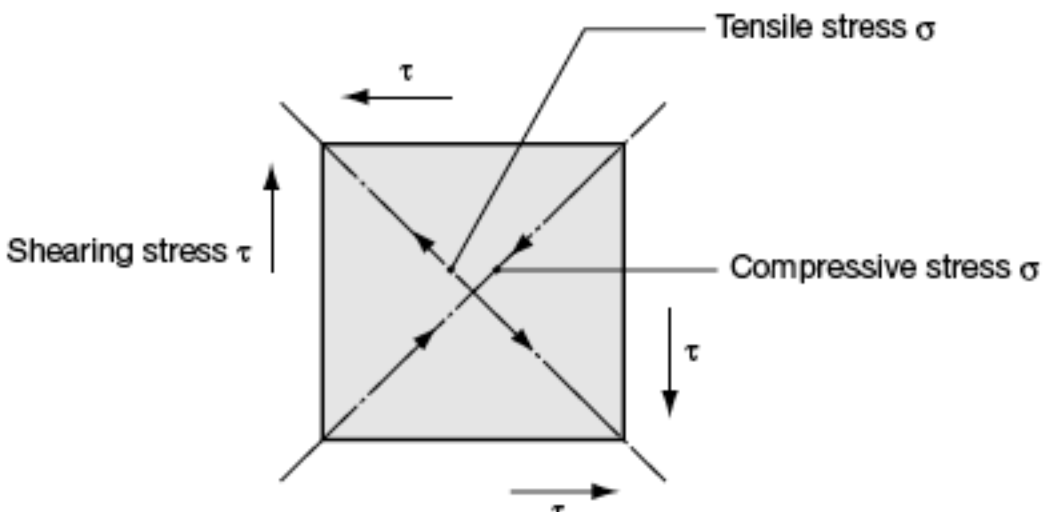
$$\varepsilon_0 = \frac{1}{2}\left\{\left(\varepsilon_1 + \varepsilon_2\right) + \left(\varepsilon_1 - \varepsilon_2\right)\cdot\cos\left(2\theta\right)\right\} \rightarrow \varepsilon_2 = -\nu\cdot\varepsilon_1$$

$$\rightarrow \varepsilon_0 = \frac{1}{2}\varepsilon_1\left\{\left(1-\nu\right) + \left(1+\nu\right)\cdot\cos\left(2\theta\right)\right\}$$



Torsional Strain Measurements (1)

- When a test specimen is twisted, the shearing stress (τ) axis is inclined at 45° from the axial line and has tensile and compressive stress of equal magnitude to the shearing stress.



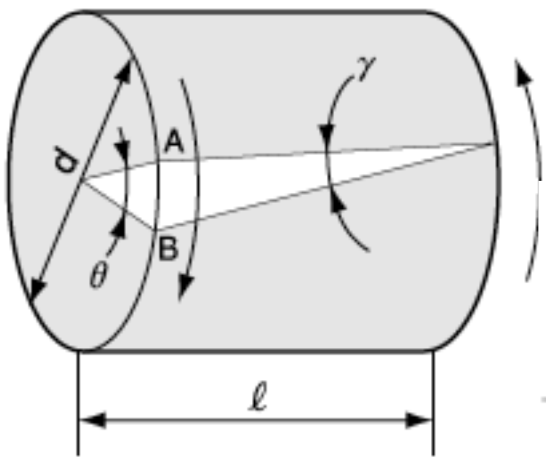
- τ related to shear strain by "Shearing modulus"

$$\gamma = \frac{\tau}{G} \rightarrow G = \frac{E}{2(1+\nu)} \rightarrow \boxed{\begin{array}{c} E \rightarrow Young's\ Modulus \\ \nu \rightarrow Poisson's\ Ratio \end{array}}$$

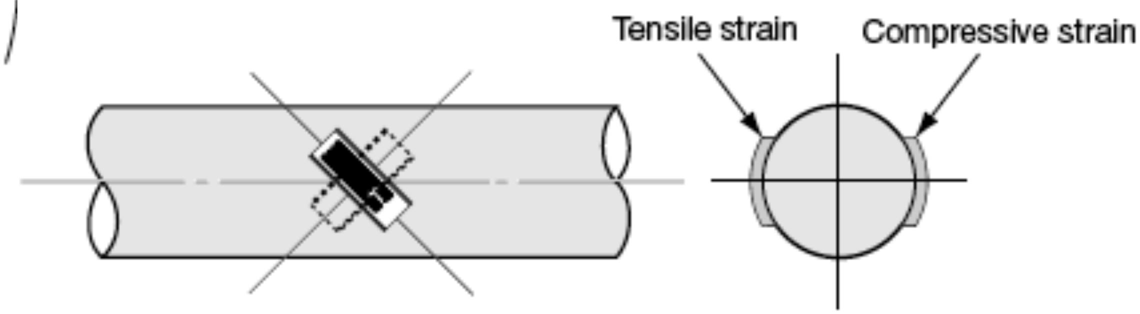
Torsional Strain Measurements (2)

- When the axis is twisted, point A moves to point B, and the torsional angle is θ :

$$\theta = \frac{l\gamma}{d/s} = \frac{l\gamma}{d}$$

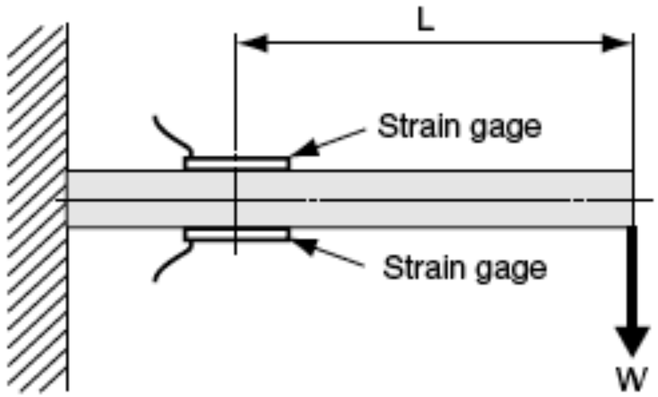


- 2-arm strain gauge bridge mostly used for torsional strain measurments



$$\gamma = \frac{V_{out}}{V_{ex}} \cdot \frac{2}{G_F}$$

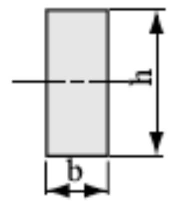
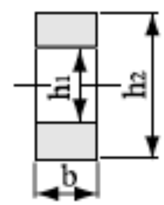
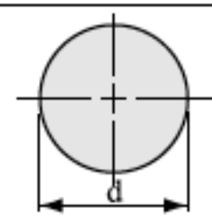
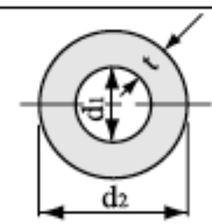
Bending Moment Measurements



$$E \rightarrow Young's\ Modulus$$
$$M \rightarrow Section\ Modulus$$

$$\boxed{\left|\varepsilon\right| = \frac{2}{G_F}\frac{V_{out}}{V_{ex}}}$$

$$M = Z \cdot E \cdot \varepsilon$$

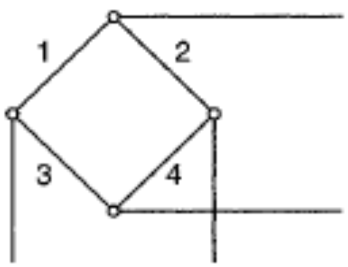
Cross Section	Section Modulus Z
	$\frac{1}{6}bh^2$
	$\frac{1}{6}\cdot\frac{b(h_2^3-h_1^3)}{h^2}$
	$\frac{\pi}{32}d^3$
	$\frac{\pi}{32}\cdot\frac{d_2^4-d_1^4}{d_2}$

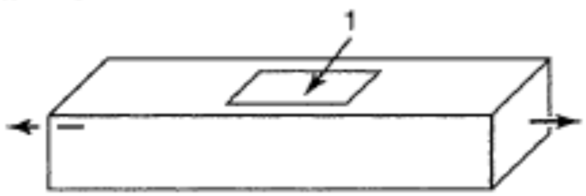
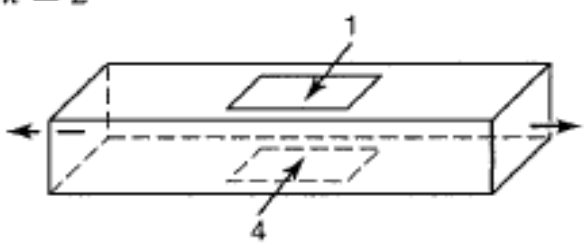
"Laundry List" of Strain Measurement Methods (1)

Standard Bridge Configuration

Requirement for null: $\dfrac{R_1}{R_2} = \dfrac{R_3}{R_4}$

k = Bridge constant = $\dfrac{\text{Output of bridge}}{\text{Output of primary gage}}$



A	$k = 1$ 	Compensates for temperature if "dummy" gage is used in arm 2 or arm 3. Does not compensate for bending.
B	$k = 2$ 	Compensates for bending. Two-arm bridge does not provide temperature compensation. Four-arm bridge ("dummy" gages in arms 2 and 3) provides temperature compensation.

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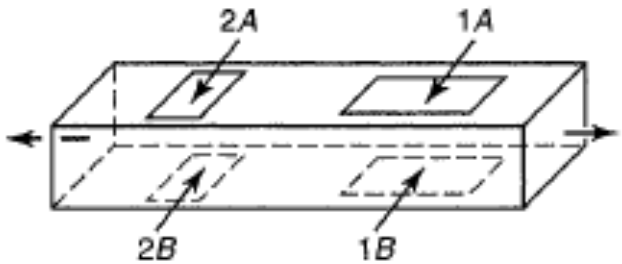
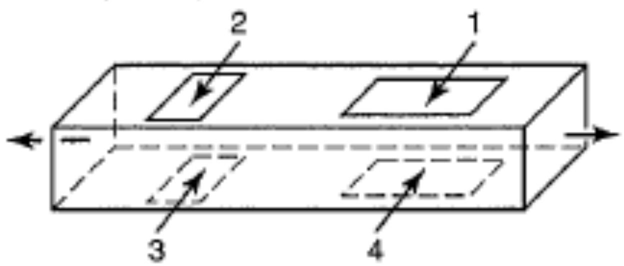
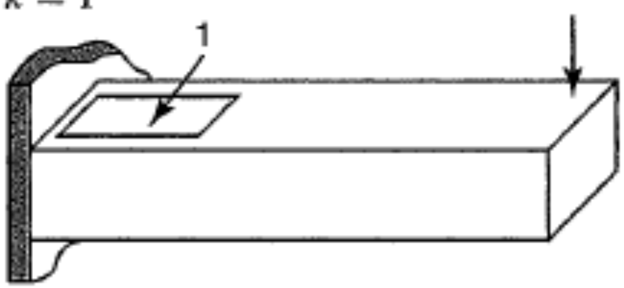
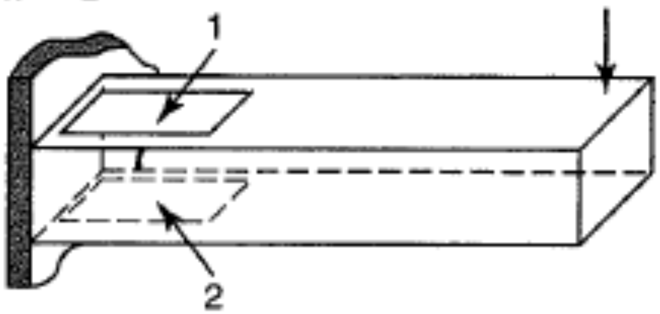
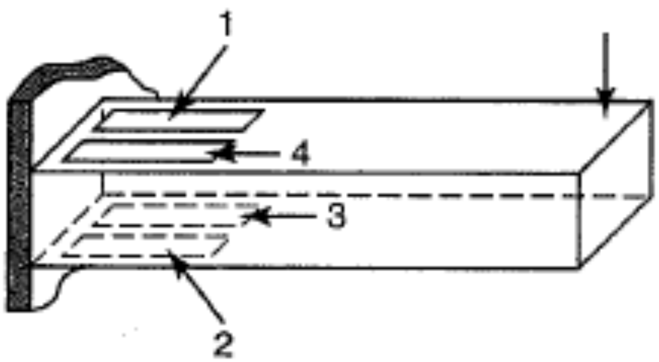
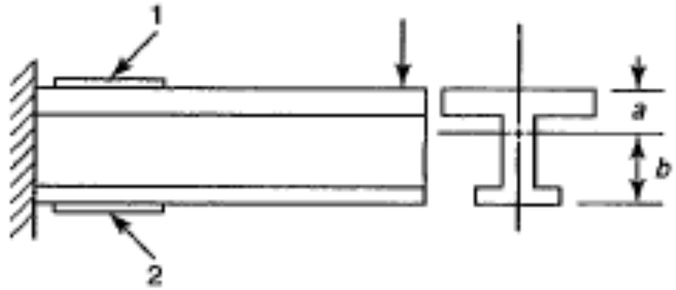
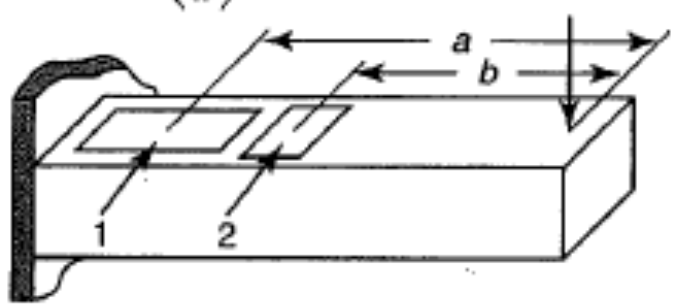
"Laundry List" of Strain Measurement Methods (2)

C	$k = 1 + \nu$ 	Two-arm bridge compensates for temperature and bending.
D	$k = 2(1 + \nu)$ 	Four-arm bridge compensates for temperature and bending.
E	$k = 1$ 	Temperature compensation accomplished when "dummy" gage is used in arm 2 or arm 3. Bridge is also sensitive to axial and torsional components of loading.

“Laundry List” of Strain Measurement Methods (3)

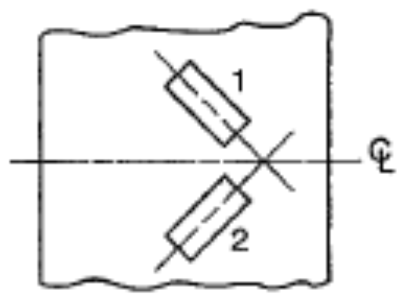
F	$k = 2$ 	Temperature effects and axial and torsional components are compensated.
G	$k = 4$ 	Four-arm bridge. Temperature effects and axial and torsional components are compensated.

"Laundry List" of Strain Measurement Methods (4)

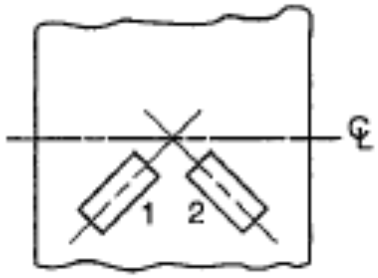
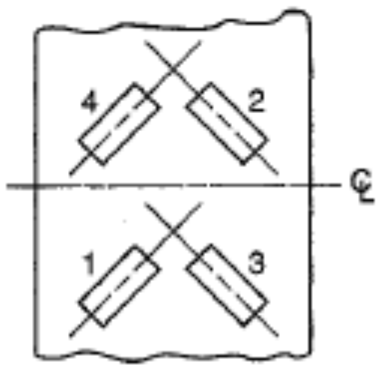
H	$k = \frac{a+b}{a}$ 	Temperature effects and axial and torsional components are compensated.
I	$k = 1 + \left(\frac{b}{a}\right)\nu$ 	Temperature effects are compensated. Axial and torsional load components are not compensated.

"Laundry List" of Strain Measurement Methods (5)

Torsion

J	$k = 2$		Two-arm bridge. Temperature and axial load components are compensated. Bending components are accentuated.

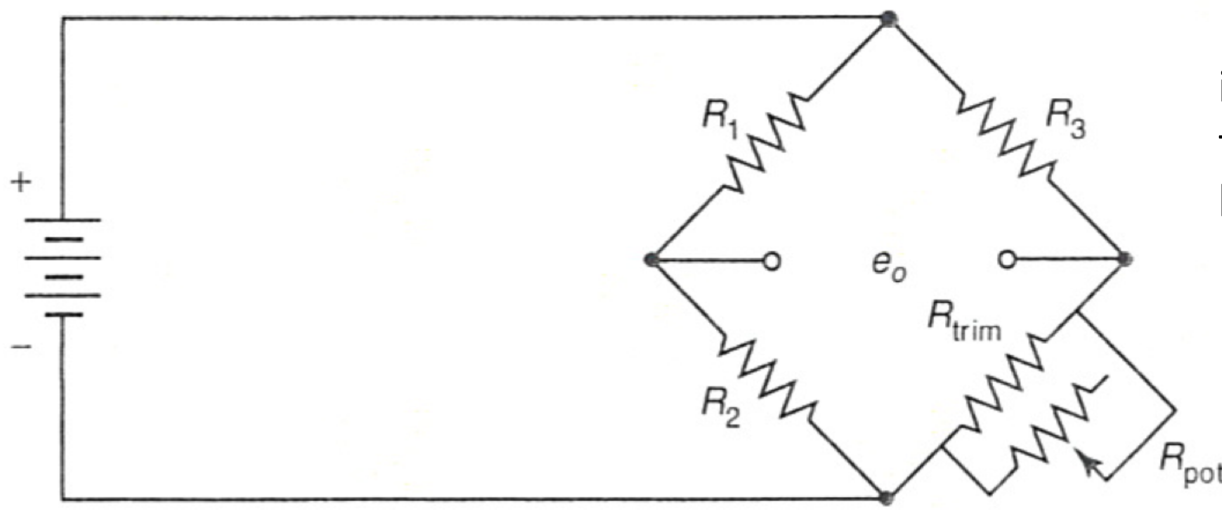
"Laundry List" of Strain Measurement Methods (6)

K	$k = 2$ 	Two-arm bridge. Temperature effects and axial load components are compensated. Relatively insensitive to bending.
L	$k = 4$ 	Four-arm bridge. Sensitive to torsion only. (Gages 1 and 3 are on opposite sides of the shaft from gages 2 and 4.)

Homework 5

Figure below shows a shunt balance arrangement for nulling a Wheatstone bridge. Suppose

1) that $R_1 = R_3 = 120\ \Omega$, $R_{trim} = 127\ \Omega$, and $R_{pot} = 10\ k\Omega$. What is the maximum value of R_2 for which the bridge can be brought into balance by adjusting R_{pot}? What would be the maximum value if $R_1 = 119\ \Omega$ and $R_3 = 121\ \Omega$?



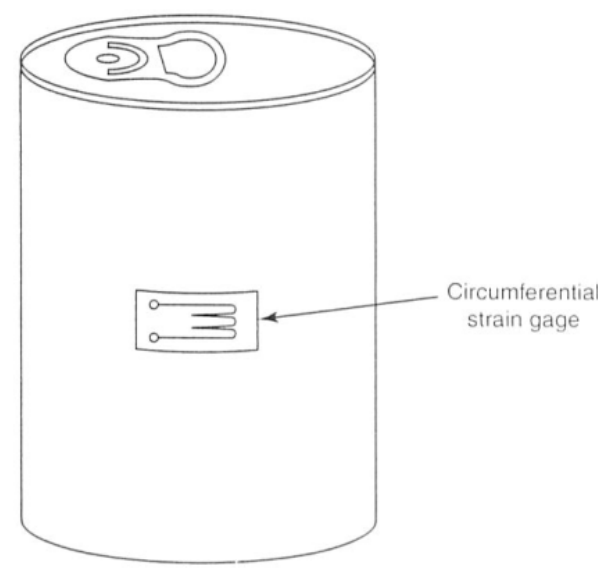
- Assume Pot is fully variable from 0 to 10 kΩ

Circuit for Problem

Homework 5

2) . A mechanical engineering student wishes to determine the internal pressure existing in a diet soda can. She proceeds by carefully mounting a single-element strain gage aligned in circumferential direction on the center of the soda can, as shown in Figure Below After wiring the gage properly to a commercial strain indicator, she "pops" the flip-top lid, which relieves the internal pressure. She notes that the strain indicator reads $-400\ \mu$-strain. If the can body is made of aluminum with a thickness of 0.010 in. and a diameter of 2.25 in., what was the original internal pressure of the sealed can?

Assume $E = 10\ ksi\ (6.89\ GPa)$
assume $v = 0.5$



Homework 5

3) Another student also performed the experiment described in Previous problem Unfortunately, he did not have access to the commercial strain indicator, and instead he had to construct his own Wheatstone bridge circuit. His strain gage had an initial resistance of 120 Ω and a gage factor of 2.05. He used the single gage as one leg of the bridge, which he powered with a 6-V battery. The bridge output was fed to an amplifier (gain = 1000), and the amp's output was read by a voltmeter. The student balanced the bridge circuit before he opened the can. After the can was opened, the voltmeter indicated a voltage of −1.57 V. What was the measured strain for his can?