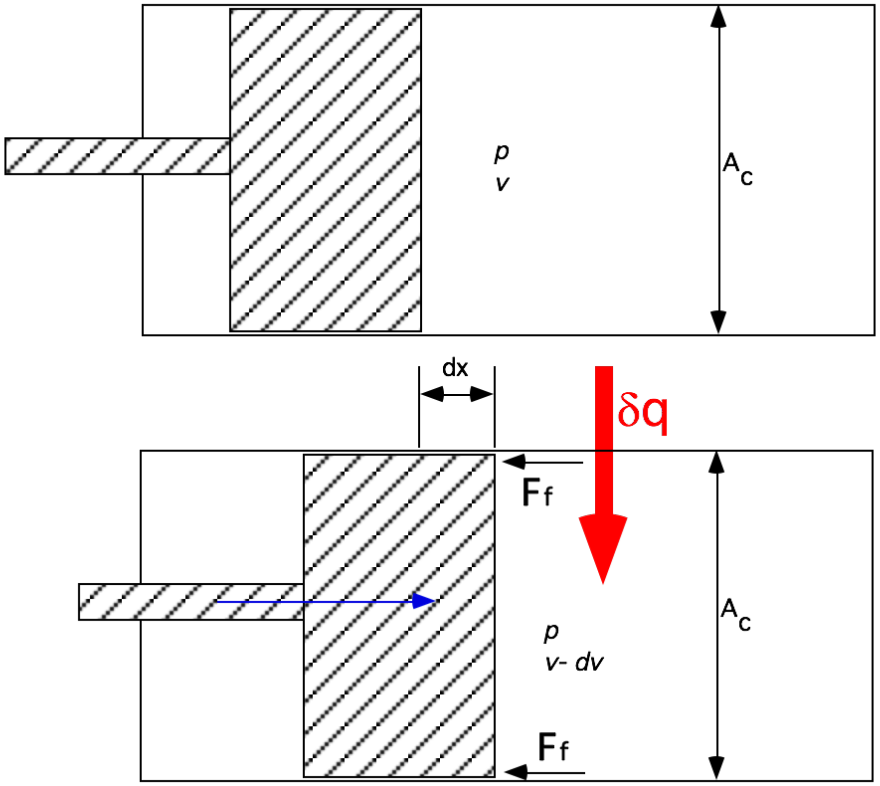
Section1, Lecture 2

Equation of State, Review of Thermodynamics, Isentropic Processes

work performed by piston on system
with heat addition



Anderson: Chapter 1 pp. 19-39

Review

- Key Concepts:

i) High Speed flows often seem "counter-intuitive" when Compared with low speed flows

ii) Flow regimes

Subsonic - All flow everywhere on the aircraft less than local speed of sound.
Transonic - Some flow is subsonic and some is supersonic.
Supersonic - All flow everywhere on the aircraft is supersonic.
Hypersonic - Fluid flows that are Much Higher than sonic velocity

iii) Mach number - ratio of true airspeed to local speed of sound

iv) Mach Angle … angle of shock wave generated by "point object"

$$\boxed{\sin(\mu) = \frac{1}{M}}$$

v) Sonic Velocity in a gas

$$\boxed{c = \sqrt{\left(\frac{\partial p}{\partial \rho}\right)_{\Delta s=0}}}$$

Equation of State for a Perfect Gas

- Relationship Between pressure, temperature, and density derived empirically in Modern form by John Dalton

- Theoretically derived by Ludwig Boltzmann using statistical Thermodynamics

- In perfect gas … intermolecular (van der Waals) forces are neglected


John Dalton

$$p\,V = n\,\mathrm{R_u}\,\mathrm{T}$$

- p - pressure acting on gas
- V - volume of gas in system
- n - Number of moles of gas in system
- $\mathrm{R_u}$ - Universal gas constant
- T - Temperature of gas

p
v
T
n

1-mole --> 6.02×10^{23}
Avagadro's number

Equation of State for a Perfect Gas (cont'd)

- Re organizing the equation of state

$$p = \frac{M}{V}\frac{\mathfrak{n}}{M}R_u T = \rho\frac{R_u}{M/\mathfrak{n}}T = \rho\frac{R_u}{M_w}T = \rho R_g T$$

- p - pressure acting on gas
- V - volume of gas in system
- n - Number of moles of gas in system
- R_u - Universal gas constant
- T - Temperature of gas
- M_w- Molecular weight of gas
- R_g - Gas Specific Constant
- M - Mass of gas contained in volume

$$\boxed{\frac{M}{V} \equiv v = \frac{1}{\rho}}$$

$$\boxed{\frac{M}{n} \equiv M_w} \rightarrow R_g = \frac{R_u}{M_w}$$

- Useful working form for Gas Dynamics

$$\boxed{p = \rho R_g T}$$

Equation of State for a Perfect Gas (cont'd)

- Numerical Values for Universal Gas Constant

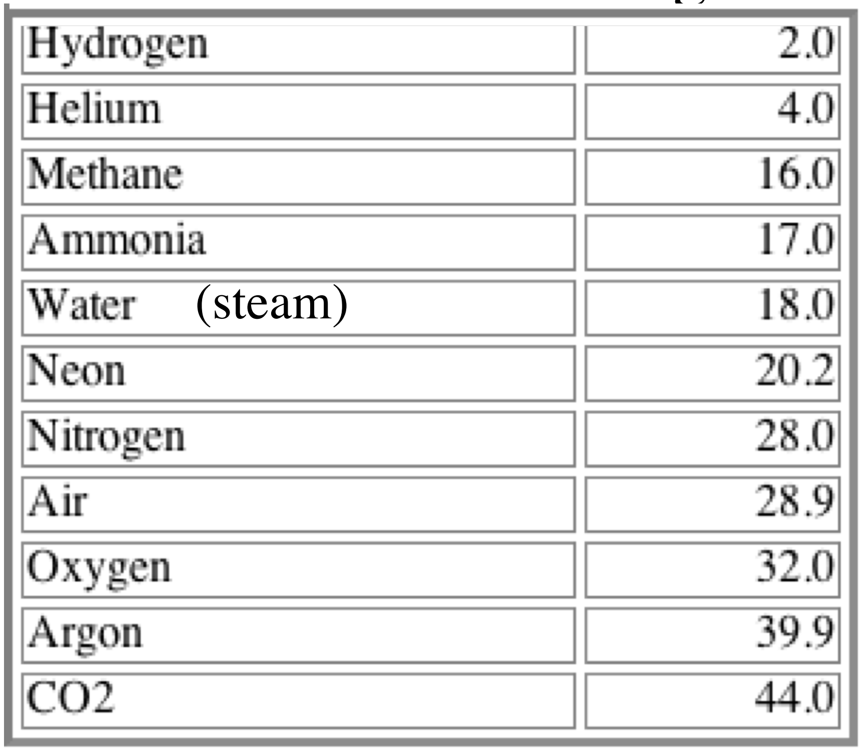
$R_u = 1545.40$ ft-lbf/°R-(lbm-mole)

$R_u = 49722.01$ ft-lbf/°R-(slug-mole)

$R_u = 8314.4612$ J/°K-(kg-mole)

- Gas Specific constant is Universal constant divided by the average molecular weight of the gas

- Molecular of various gases



Hydrogen	2.0
Helium	4.0
Methane	16.0
Ammonia	17.0
Water (steam)	18.0
Neon	20.2
Nitrogen	28.0
Air	28.9
Oxygen	32.0
Argon	39.9
CO2	44.0

Equation of State for a Perfect Gas (concluded)

- Molecular weight of Air

Average molecular weight of the gases in the atmosphere.
Air on earth at sea level is a mixture of approximately 78% nitrogen, 21%oxygen, with the remaining one percent a mix of argon, carbon dioxide, neon, helium and other rare gases,

~ 28.96443 kg/kg-mole

- Numerical Values for Air Specific Gas Constant

$R_g = 53.355$ ft-lbf/°R-(lbm)
$R_g = 1716.658$ ft-lbf/°R-(slug)
$R_g = 287.056$ J/°K-(kg)

Specific Heats, Internal Energy, and Enthalpy (cont'd)

- *e* is the internal energy per unit mass

The ***total kinetic and potential energy*** associated with the motions and relative positions of the molecules a gas, solid, or liquid. An increase in internal energy results in a rise in temperature or a change in phase

- *h* is the enthalpy per unit mass

The measure of the *heat content of a chemical or physical system*.

$$h = e + p \cdot v$$

$$\rightarrow dh = d(e + p \cdot v) = \overset{\text{Change in energy}}{de} + \underbrace{dp \cdot v + p \cdot dv}_{\frac{work}{mass}}$$

Symbols "*e*" --> "*u*" often Used interchangeably In thermodynamics texts

Anderson uses "*e*"

I mostly will use "*e*"

But sometimes I will slip Up and use "u"

h is the specific enthalpy, *(SI - joules/kg)*
e is the specific internal energy, *(SI – joules/kg)*
P is the pressure of the system, (SI - pascals)
v is the specific volume, *(SI cubic metres/kg)* $= 1/\rho$

Specific Heats, Internal Energy, and Enthalpy

(cont'd)

- Specific heat definitions:
  - Constant Volume:

$$c_v = \left(\frac{de}{dT}\right)_v$$

$$\boxed{c_p} = \left(\frac{dh}{dt}\right)_p = \frac{d}{dT}\left(e + pv\right) = \frac{de}{dT} + \frac{d}{dT}\left(R_g T\right) = \boxed{c_v + R_g}$$

  - Constant Pressure:

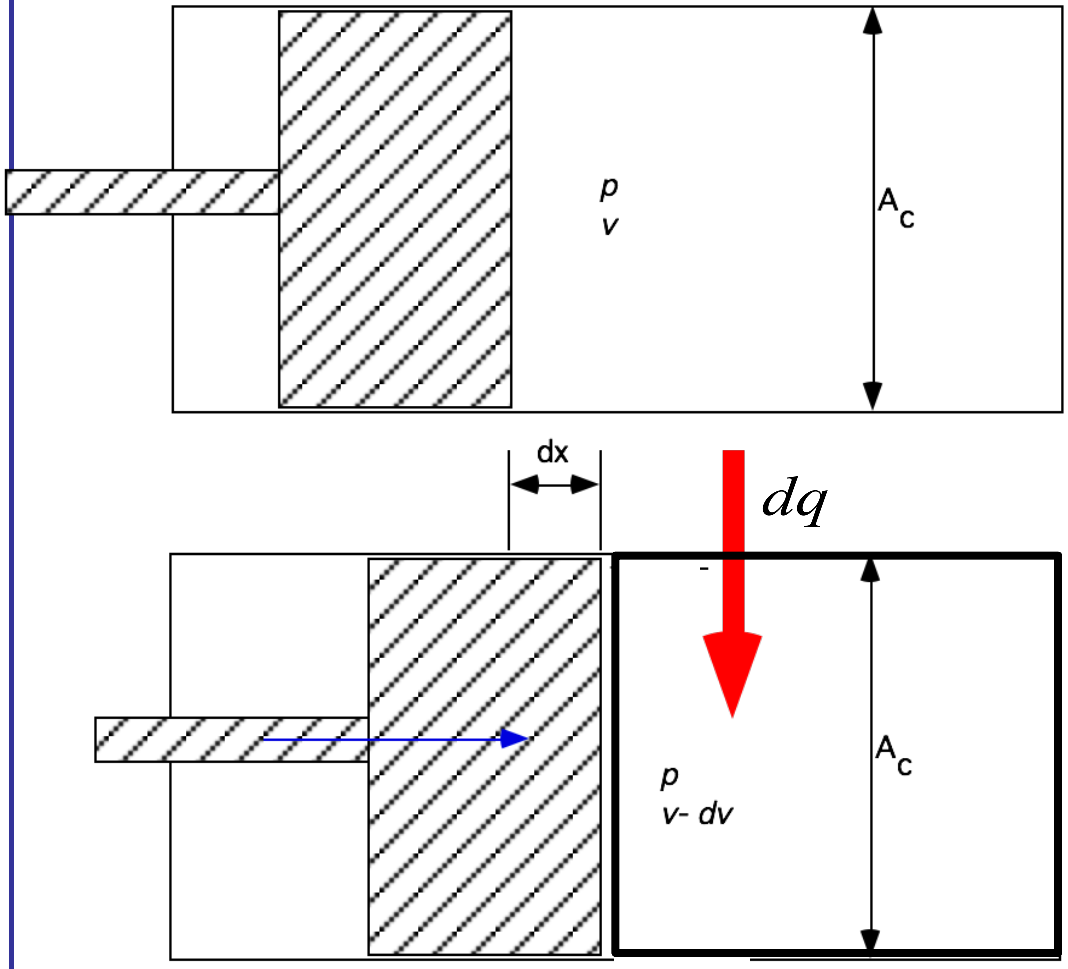
$$c_p = \left(\frac{dh}{dT}\right)_p$$

Universal relationship applies to all perfect gases

$$\boxed{\boxed{c_p = c_v + R_g}}$$

First Law of Thermodynamics

work performed by piston on system
with heat addition



- Change in internal energy = heat added - work performed *(ignore frictional energy loss)*

$$de = dq - \delta\left(w_{ork}\right) \rightarrow \quad dq = \frac{\dot q}{\dot m} \qquad \delta\left(w_{ork}\right) = p \cdot \frac{A_c \cdot dx}{m}$$

$$\rightarrow \delta\left(w_{ork}\right) = p \cdot \frac{A_c \cdot dx}{m} = p \cdot dv \qquad \frac{A_c \cdot dx}{m} = dv$$

$$\boxed{de = dq - p \cdot dv}$$

- In terms of enthalpy

$$dh = de + v \cdot dp + p \cdot dv = \left(dq - p \cdot dv\right) + v \cdot dp + p \cdot dv$$

$$\rightarrow \boxed{dh = dq + v \cdot dp}$$

First Law of Thermodynamics (continued)

- Recall from earlier

1) Adiabatic Process… no heat loss or addition
2) Reversible Process … no dissipative phenomena occur
3) *Reversible adiabatic process* is referred to as *isentropic*

- Reversible, non-adiabatic

$$\boxed{\begin{aligned} de &= dq - p \cdot dv \\ dh &= dq + v \cdot dp \end{aligned}}$$

- Reversible, Adiabatic (isentropic)

$$\boxed{\begin{aligned} de &= -p \cdot dv \\ dh &= \quad v \cdot dp \end{aligned}}$$

$$\boxed{v = \frac{1}{\rho}}$$

$$c_v = \left(\frac{de}{dT}\right)_v$$

$$c_p = \left(\frac{dh}{dT}\right)_p$$

Second Law of Thermodynamics (cont'd)

- Thermodynamic entropy, *s*, is a measure of the amount of energy in a physical system that cannot be used to do work.

- It is a relative measure of the disorder present in a system.
(SI unit of entropy is J/°K)

Do NOT! Confuse enthalpy and entropy

$$Tds = \underset{\substack{\uparrow \\ \text{Reversible heat addition}}}{dq} + \underset{\substack{\uparrow \\ \text{Effect of dissipative processes}}}{Tds_{irrev}}$$

- Calculate change in entropy using first, second laws:

First Law: $$de = dq - pdv - dw_{irrev}$$

Second Law: $$Tds = dq + Tds_{irrev}$$

Second Law of Thermodynamics (cont'd)

For a reversible process

$$de = dq - pdv$$

$$Tds = dq \longrightarrow Tds = de + pdv$$

Writing *e* in terms of enthalpy (***h = e + p v***)

$$dh = de + pdv + vdp$$

and

Second Law: $Tds = de + pdv$

First Law: $dh = de + pdv + vdp$

$$dh = Tds - pdv + pdv + vdp =$$

$$\boxed{dh = Tds + vdp} \longleftarrow$$

Second Law of Thermodynamics (cont'd)

Solving for *ds* and integrating between state 1 and state 2

$$ds = \frac{dh}{T} - \frac{vdp}{T} \longrightarrow s_2 - s_1 = \int_1^2 \left[\frac{dh}{T} - \frac{vdp}{T} \right]$$

But from earlier definition $c_p = \left(\frac{dh}{dT} \right)_p \longrightarrow s_2 - s_1 = \int_1^2 \left[\frac{c_p dT}{T} - \frac{vdp}{T} \right]$

From the equation of state

$$p = \rho R_g T = \frac{1}{v} R_g T$$

$$v = \frac{R_g T}{p}$$

Second Law of Thermodynamics (concluded)

- Substituting equation of state into entropy integral

$$\boxed{v = \frac{R_g T}{p}}$$

$$s_2 - s_1 = \int_1^2 \left[\frac{c_p dT}{T} - \frac{v dp}{T}\right] = \int_1^2 \left[\frac{c_p dT}{T} - \frac{\frac{R_g T}{p} dp}{T}\right] =$$

$$s_2 - s_1 = \int_1^2 \left[\frac{c_p dT}{T} - R_g \frac{dp}{p}\right]$$

for thermally perfect gas
c_p , R_g ~ constant

$$\boxed{\int_1^2 c_p \frac{dT}{T} = c_p \left[\ln(T_2) - \ln(T_1)\right] = c_p \ln\left(\frac{T_2}{T_1}\right)}$$

$$\boxed{\int_1^2 R_g \frac{dp}{p} = R_g \left[\ln(p_2) - \ln(p_1)\right] = R_g \ln\left(\frac{p_2}{p_1}\right)}$$

Second Law of Thermodynamics (concluded)

- Substituting into entropy integral

Valid for adiabatic Process ($\delta q=0$)

$$\boxed{\int_1^2 c_p \frac{dT}{T} = c_p\left[\ln(T_2)-\ln(T_1)\right] = c_p \ln\left(\frac{T_2}{T_1}\right)}$$

$$\boxed{\int_1^2 R_g \frac{dp}{p} = R_g\left[\ln(p_2)-\ln(p_1)\right] = R_g \ln\left(\frac{p_2}{p_1}\right)}$$

$$s_2 - s_1 = \int_1^2 \left[\frac{c_p dT}{T} - R_g \frac{dp}{p}\right]$$

$$\boxed{s_2 - s_1 = c_p \ln\left(\frac{T_2}{T_1}\right) - R_g \ln\left(\frac{p_2}{p_1}\right)}$$

Isentropic Process Relationships

- For an isentropic process, (adiabatic, reversible) dq , $ds_{irrev} = 0$

$$ds = \cancel{\frac{dq}{T}} + \cancel{ds_{irrev}} \quad \mathbf{= 0 \text{ ---> }} \quad s_2 - s_1 = 0 = c_p \ln\left(\frac{T_2}{T_1}\right) - R_g \ln\left(\frac{p_2}{p_1}\right)$$

and
$$c_p \ln\left[\frac{T_2}{T_1}\right] - R_g \ln\left[\frac{p_2}{p_1}\right] \longrightarrow \left[\frac{T_2}{T_1}\right]^{c_p} = \left[\frac{p_2}{p_1}\right]^{R_g}$$

Solving for p_2 in terms of T_2

$$\frac{p_2}{p_1} = \left[\frac{T_2}{T_1}\right]^{\frac{c_p}{R_g}} = \left[\frac{T_2}{T_1}\right]^{\frac{c_p}{c_p - c_v}}$$

$$\boxed{c_p = c_v + R_g}$$

Isentropic Process Relationships (cont'd)

- Defining a *ratio of specific heats*, $\gamma = \frac{c_p}{c_v}$

More "later"

$$\boxed{\frac{p_2}{p_1} = \left[\frac{T_2}{T_1}\right]^{\frac{\gamma}{\gamma-1}}}$$

- Applying the equation of state

$$\frac{p_2}{p_1} = \frac{\rho_2 R_g T_2}{\rho_1 R_g T_1} \rightarrow \frac{T_2}{T_1} = \frac{p_2}{p_1}\frac{\rho_1}{\rho_2}$$

$\delta s = 0$

!

- and

$$\left[\frac{p_2}{p_1}\right]^{1-\frac{\gamma}{\gamma-1}} = \left[\frac{p_2}{p_1}\right]^{\frac{-1}{\gamma-1}} = \left[\frac{\rho_2}{\rho_1}\right]^{\frac{-\gamma}{\gamma-1}} - \triangleright \boxed{\left[\frac{p_2}{p_1}\right] = \left[\frac{\rho_2}{\rho_1}\right]^{\gamma}}$$

Isentropic Process Relationships (continued)

- Returning for the expression for sonic velocity

$$c = \sqrt{\left[\frac{\partial p}{\partial \rho}\right]_{\Delta s = 0}} \;-\!\triangleright \left[\frac{p_2}{p_1}\right] = \left[\frac{\rho_2}{\rho_1}\right]^{\gamma}$$

For isentropic process (*ds = 0*)

$$\frac{p_2}{p_1} = \left[\frac{\rho_2}{\rho_1}\right]^{\gamma} \rightarrow p_2 = \boxed{\frac{p_1}{\rho_1^{\gamma}}} \cdot \rho_2^{\gamma} \rightarrow \boxed{p = k \cdot \rho^{\gamma}}$$

$$\left(\frac{\delta p}{\delta \rho}\right)_s = \gamma k \rho^{\gamma - 1} = \gamma k \rho^{\gamma} \frac{1}{\rho} = \gamma p \frac{1}{\rho} = \gamma R_g T \;-\!\triangleright \boxed{c = \sqrt{\gamma R_g T}}$$

Isentropic Process Relationships (concluded)

- Returning for the expression for sonic velocity

$$c = \sqrt{\left[\frac{\partial p}{\partial \rho}\right]_{\Delta s = 0}} \quad - \triangleright \quad \boxed{c = \sqrt{\gamma R_g T}}$$

For isentropic process (*ds = 0*)

Definition for Mach number

$$M = V/C = \boxed{\frac{V}{\sqrt{\gamma R_g T}}}$$

Working expressions for
Sonic velocity and Mach number!

Thermodynamic properties of a mixed gas

- Molecular weight:

$$\bar{M}_w = \frac{\sum_j n_j (M_w)_j}{\sum_j n_j} \rightarrow$$

j --- constituent gases

ideal gas law → $pV = nR_u T \rightarrow n = \frac{p}{R_u T} V$

@constant temperature and pressure → $n_j \approx V_j$

"Dalton's law"

$$\bar{M}_w = \frac{\sum_j V_j (M_w)_j}{\sum_j V_j} = \sum_j \left(\frac{V_j}{\sum_k V_k} (M_w)_j \right) = \sum_j \left(V_{frac_j} M_{w_j} \right)$$

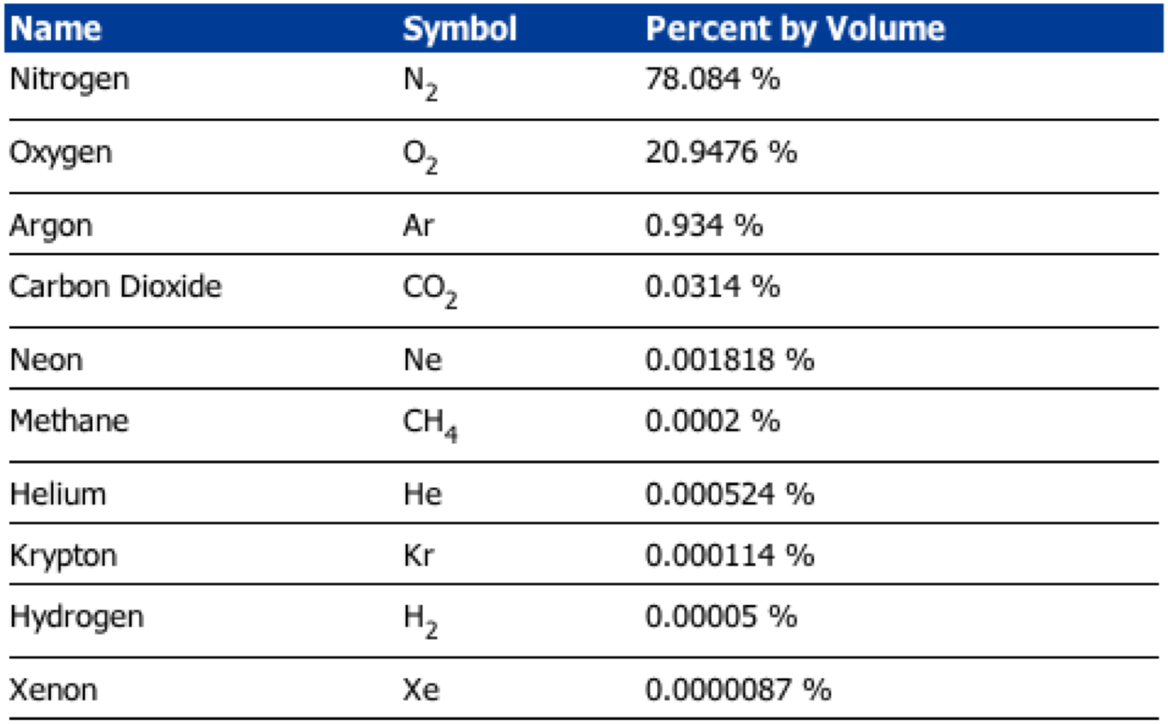
$$\boxed{V_{frac_j} = \frac{V_j}{\sum_k V_k}}$$

Thermodynamic properties of a mixed gas

(cond't)

- Example: air

$$\bar{M}_w = \sum_j \left(V_{frac_j} M_{w_j} \right)$$



Name	Symbol	Percent by Volume
Nitrogen	N_2	78.084 %
Oxygen	O_2	20.9476 %
Argon	Ar	0.934 %
Carbon Dioxide	CO_2	0.0314 %
Neon	Ne	0.001818 %
Methane	CH_4	0.0002 %
Helium	He	0.000524 %
Krypton	Kr	0.000114 %
Hydrogen	H_2	0.00005 %
Xenon	Xe	0.0000087 %

$M_{w_{air}}$ =

$$2 \cdot 14 \cdot 0.78084 + 2 \cdot 16 \cdot 0.209476 + 40 \cdot 0.00934 + (12 + 2 \cdot 16)\ 0.000314$$

+ …. = 28.96443 kg/kg-mole

$$\dashrightarrow R_g = \frac{8314.4612}{28.9644} \boxed{= 287.056 \text{ J/}^{o}\text{K-(kg)}}$$

Thermodynamic properties of a mixed gas

(cond't)

- Specific heat

$$\bar{c}_p{}' = \frac{\sum_j n_j \left(c_p{}'\right)_j}{\sum_j n_j} = \sum_j \left(V_{frac_j} \left(c_p{}'\right)_j\right) \rightarrow c_p{}' = \text{Molar specific heat} \left(\frac{\text{J}}{\text{kg-mol}^\circ K}\right)$$

$$\bar{c}_p = \frac{\bar{c}_p{}'}{\bar{M}_w} = \frac{1}{\bar{M}_w} \sum_j \left(V_{frac_j} \left(c_p{}'\right)_j\right) \rightarrow mass\ specific\ heat \left(\frac{\text{J}}{\text{kg}^\circ K}\right)$$

$$c_{p_j} = \frac{c_p{}'_j}{M_w} \longrightarrow \boxed{\bar{c}_p = \frac{1}{\bar{M}_w} \sum_j \left(V_{frac_j} M_{w_j} c_{p_j}\right)}$$

Same holds for c_v

Thermodynamic properties of a mixed gas

(cond't)

- Specific heat example air

Gas	**Molar Property** c_v(J/kg-mol^{0}K)
Ar	12500
He	12500
CO	20700
H2	20400
HCl	21400
N2	20610
NO	20900
O2	21700
Cl2	24800
CO2	28200
N2O	28500

Data at 15° C and 1 atmosphere.

$$c_{v_{air}} = \frac{20610\cdot 0.78084 + 21700\cdot 0.209476 + 12500\cdot 0.00934 + 28200\cdot 0.000314}{28.9541} + \ldots = 717.64 \text{ J/kg-}^{o}\text{K}$$

$$c_{p_{air}} = \left(R_g + c_v\right)_{air} =$$

$$\left(287.056 + 717.64\right)_{\text{J/kg-}^\circ\text{K}} = 1004.696_{\text{J/kg-}^\circ\text{K}}$$

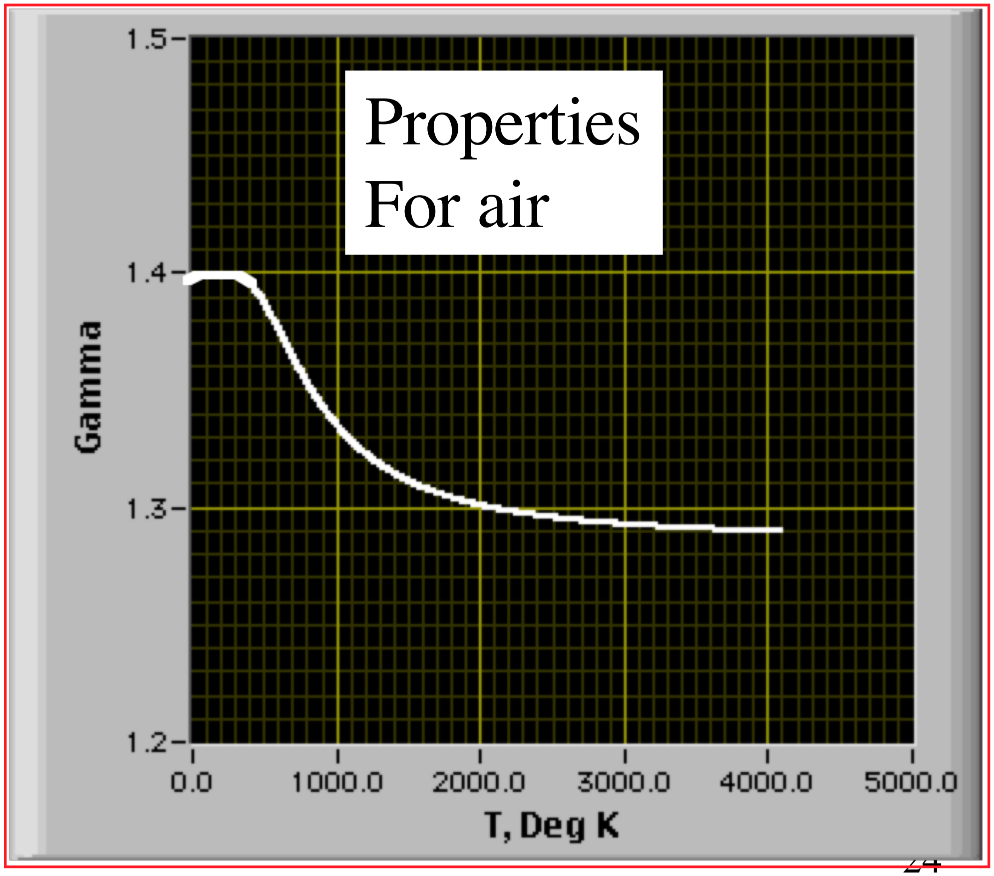
$$\gamma = c_p/c_v = 1007.696/717.64 = 1.400$$

Ratio of Specific Heats

- γ varies as a function of temperature and drops off significantly at higher temperatures *(molecular spin mode)*
- γ *effects become significant for hypersonic flow*

$$\gamma = \frac{c_p}{c_v}$$

- For *a lot* of this class…we will use $\gamma = 1.40$

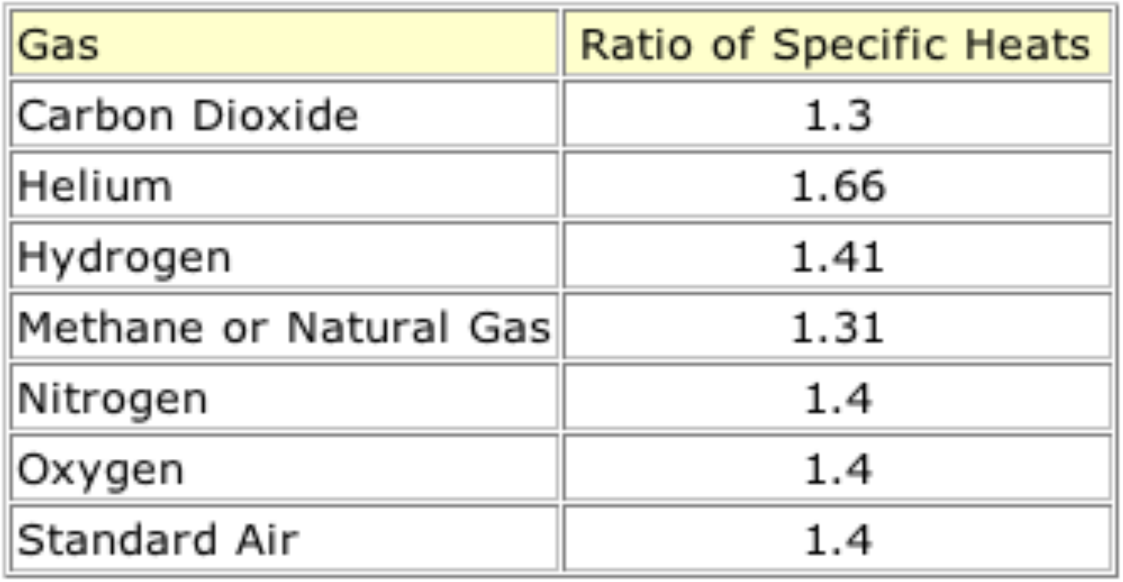


- Careful …… for propulsion (where combustion occurs) invariably… $\gamma < 1.40$

Ratio of Specific Heats (concluded)

$$\gamma = \frac{c_p}{c_v}$$ key parameter for compressible flow analysis

Approximate Specific Heat Ratio for Various Gases, at moderate temperatures



Gas	Ratio of Specific Heats
Carbon Dioxide	1.3
Helium	1.66
Hydrogen	1.41
Methane or Natural Gas	1.31
Nitrogen	1.4
Oxygen	1.4
Standard Air	1.4

Water (steam) 1.2855

Thermodynamics Summary

- Equation of State: $\boxed{p = \rho R_g T - \triangleright\ R_g = \frac{R_u}{M_w}}$

- R_u = 8314.4126 J/^{0}K-(kg-mole)
- $R_{g\ (air)}$ = 287.056 J/^{0}K-(kg-mole)

- Relationship of R_g to specific heats, γ

$$\boxed{c_p = c_v + R_g} \qquad \boxed{\gamma = \frac{c_p}{c_v}}$$

- Internal Energy and Enthalpy

$$\boxed{h = e + Pv} \qquad \boxed{c_v = \left(\frac{de}{dT}\right)_v} \qquad \boxed{c_p = \left(\frac{dh}{dT}\right)_p}$$

Thermodynamics Summary (cont'd)

- First Law of Thermodynamics, *reversible process*

$$\boxed{de = dq - pdv} \qquad \boxed{dh = dq + vdp}$$

- First Law of Thermodynamics, *isentropic process*

(adiabatic, reversible)

$$\boxed{de = -pdv} \qquad \boxed{dh = vdp}$$

Thermodynamics Summary (cont'd)

- Second Law of Thermodynamics, *reversible process*

$$\boxed{s_2 - s_1 = c_p \ln_2\left[\frac{T_2}{T_1}\right] - R_g \ln\left[\frac{p_2}{p_1}\right]}$$

- Second Law of Thermodynamics, *isentropic process*

(adiabatic, reversible) ------> $s_2 - s_1 = 0$

$$\boxed{\frac{p_2}{p_1} = \left[\frac{T_2}{T_1}\right]^{\frac{\gamma}{\gamma-1}}} \qquad \boxed{\left[\frac{p_2}{p_1}\right] = \left[\frac{\rho_2}{\rho_1}\right]^{\gamma}}$$

Thermodynamics Summary (cont'd)

- Speed of Sound for calorically Perfect gas

$$c = \sqrt{\left[\frac{\partial p}{\partial \rho}\right]_{\Delta s = 0}} \boxed{= \sqrt{\gamma R_g T}}$$

- Mathematic definition of Mach Number

$$\boxed{M = \frac{V}{\sqrt{\gamma R_g T}}}$$

Thermodynamics Summary (concluded)

- For a mixed gas

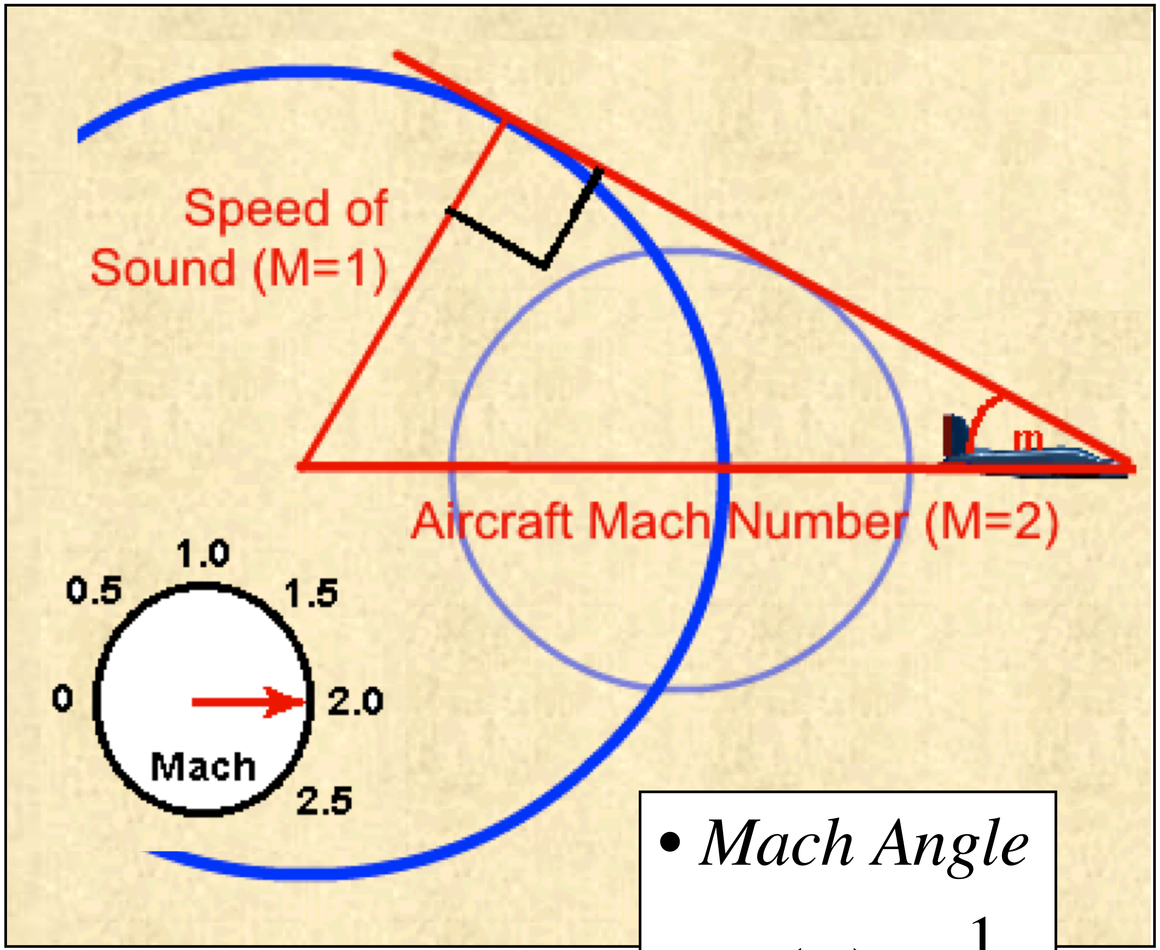
$$\bar{M}_w = \frac{\sum_j V_j \left(M_w\right)_j}{\sum_j V_j} = \sum_j \left(\frac{V_j}{\sum_k V_k} \left(M_w\right)_j \right) = \sum_j \left(V_{frac_j} \left(M_w\right)_j \right)$$

$$\bar{c}_p' = \frac{\sum_j n_j \left(c_p'\right)_j}{\sum_j n_j} = \sum_j \left(V_{frac_j} \left(c_p'\right)_j \right) \rightarrow c_p' = \text{Molar specific heat} \; \left(\frac{\text{J}}{\text{kg-mol}^\circ K}\right)$$

$$\bar{c}_p = \frac{\bar{c}_p'}{\bar{M}_w} = \frac{1}{\bar{M}_w} \sum_j \left(V_{frac_j} M_{w_j} c_{p_j} \right) \rightarrow \textit{mass specific heat} \; \left(\frac{\text{J}}{\text{kg}^\circ K}\right)$$

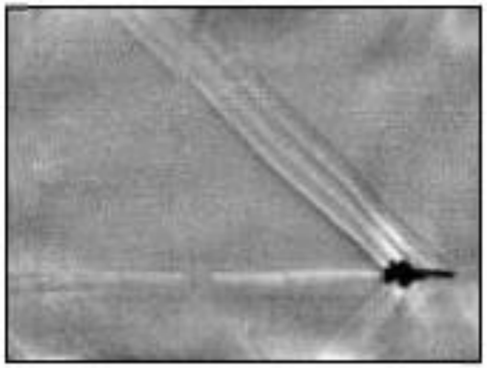
Same holds for c_v

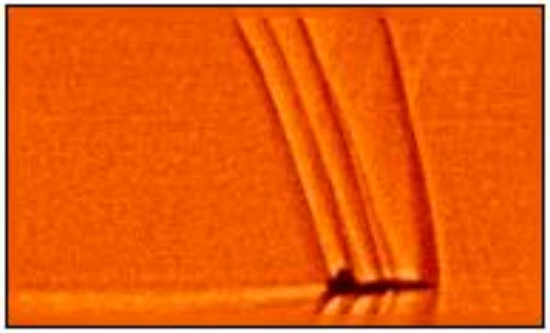
Supersonic Flow Field Examples



- *Mach Angle*

$$\sin(\mu) = \frac{1}{M}$$





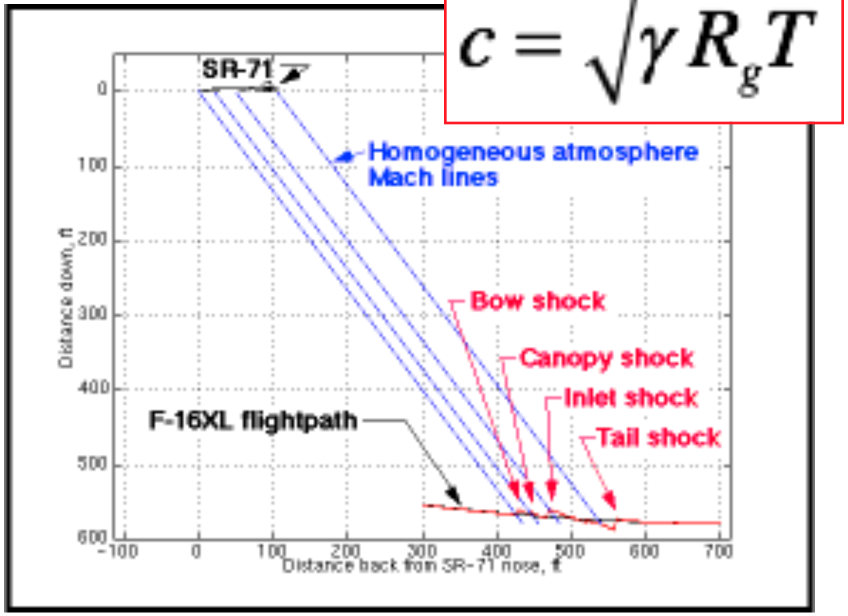
- Which aircraft
Is moving faster?

SR-71 Near Field Shock Wave Patterns

PRELIMINARY AIRBORNE MEASUREMENTS FOR THE SR-71 SONIC BOOM PROPAGATION EXPERIMENT

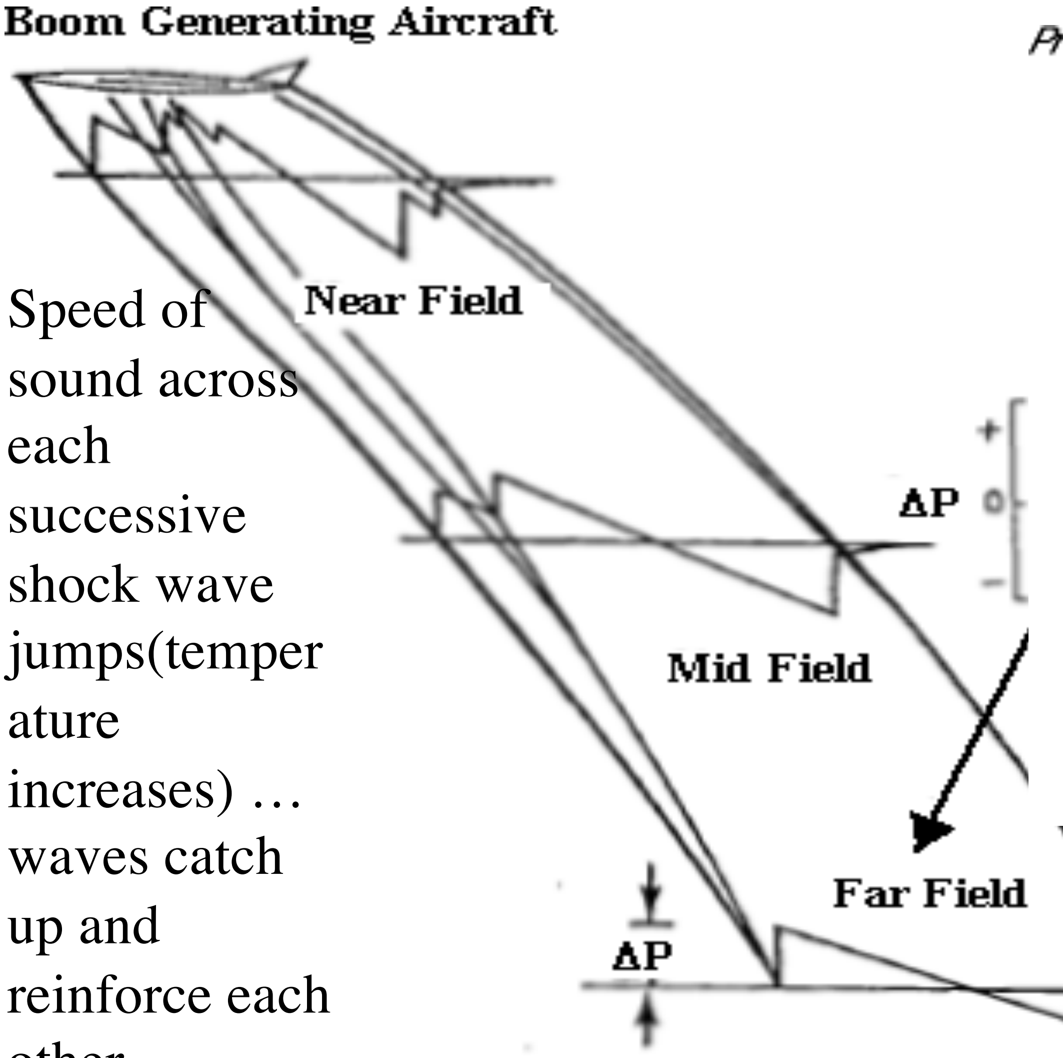
Stephen A. Whitmore
Edward A. Haering Jr.
L. J. Ehernberger

$$c = \sqrt{\gamma R_g T}$$



Speed of sound across each successive shock wave is Higher (temperature increases) … wave catch up and Reinforce each other

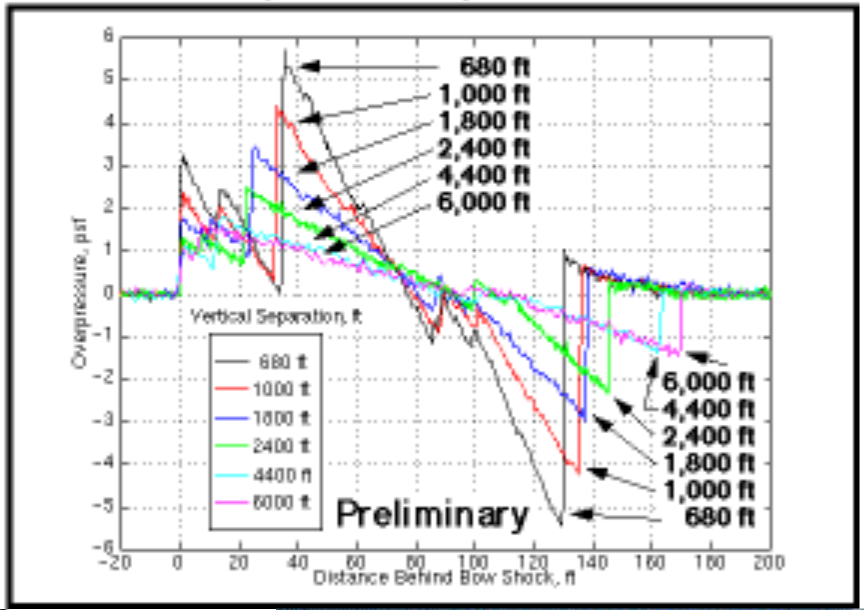
Effect of Local Speed of Sound on Shock Wave Propagation



Speed of sound across each successive shock wave jumps(temperature increases) … waves catch up and reinforce each other

Probing Data at Mach 1.25 and Alt=44,000 ft

- As vertical separation increases,
  - Tail shock moves aft
  - Overpressures decrease
  - Inlet and canopy shocks move forward
- Plume pressures collapse to one curve



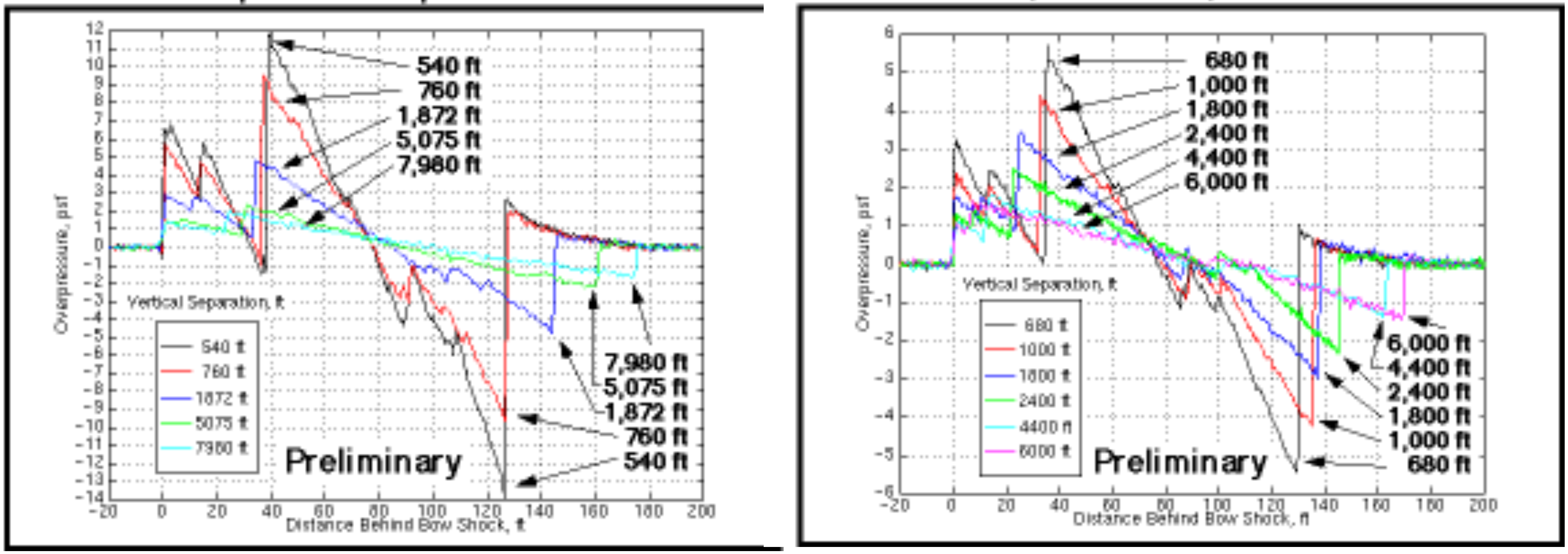
Effect of Local Speed of Sound on Shock Wave Propagation (2)

Probing Data at Mach 1.25 and Alt=31,000 ft

- As vertical separation increases,
  - Tail shock moves aft
  - Overpressures decrease
  - Inlet and canopy shocks move forward
- Plume pressures collapse to one curve

Probing Data at Mach 1.25 and Alt=44,000 ft

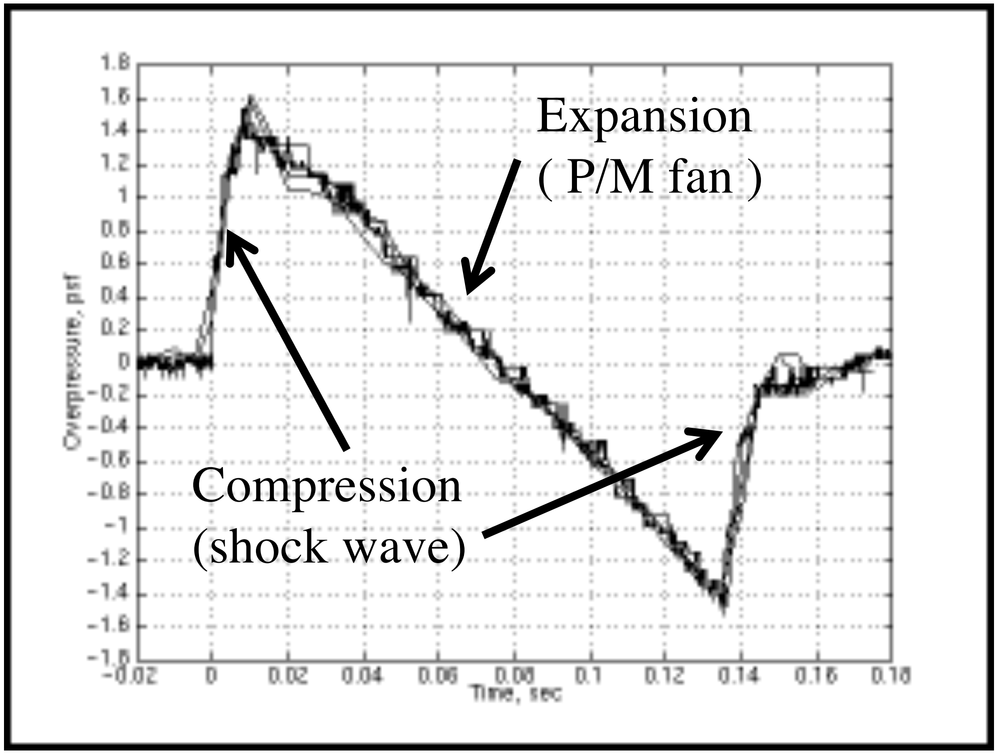
- As vertical separation increases,
  - Tail shock moves aft
  - Overpressures decrease
  - Inlet and canopy shocks move forward
- Plume pressures collapse to one curve



SR-71 near and mid-field shock wave overpressure signatures

Sonic Boom N-wave

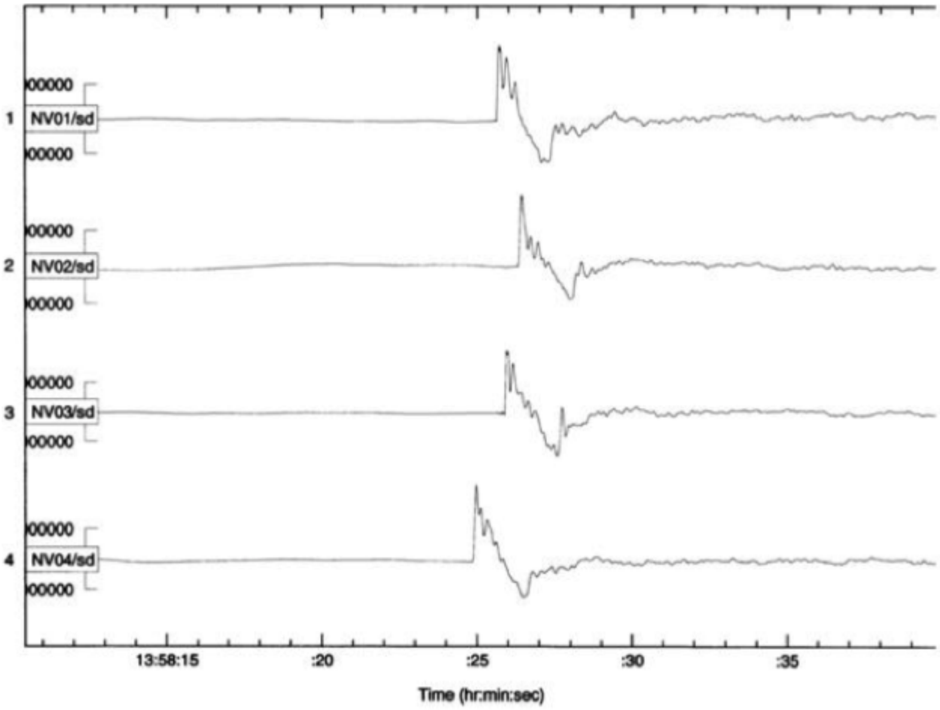
By time boom reaches ground, shocks
Have coalesced into familiar sounding
N-waves "boom-boom" … *entirely a result*
of local sonic velocity changing with temperature



Expansion
(P/M fan)

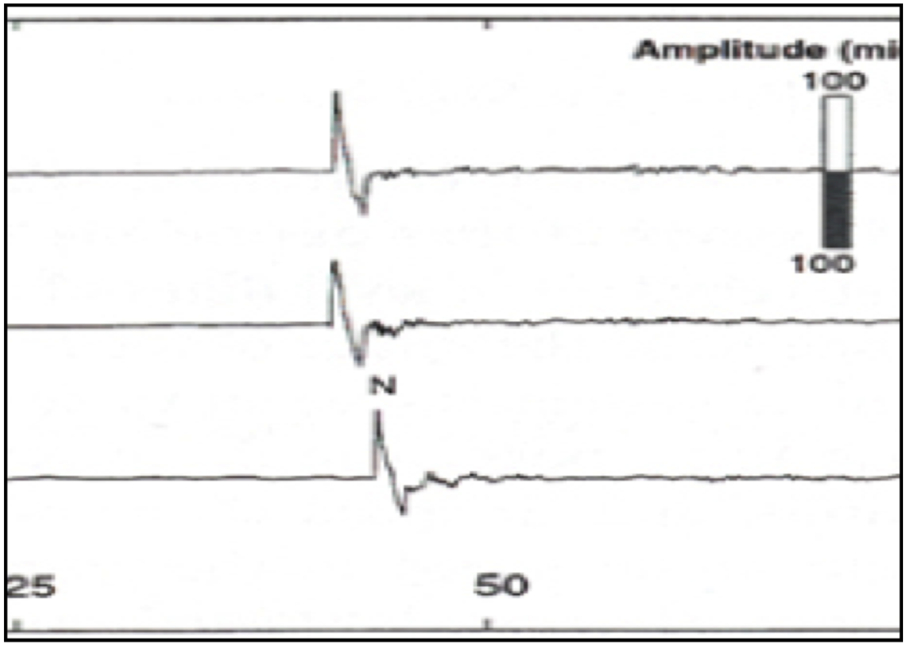
Compression
(shock wave)

Speed of Sound Can Tell You a Lot



Shuttle Columbia, STS-107 Mission, Sonic Boom during Re-entry, Nevada Station

- Observe the difference?

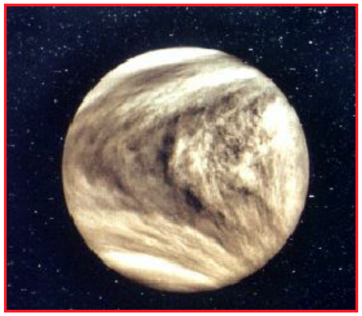


Time (seconds) From 08:37:06 UTC on February 21, 1997

Shuttle Discovery, STS-81 Mission, Sonic Boom during Re-entry

Homework, Section 1

- A sample return Probe is being sent on a 1-year mission from Earth to Mars Via Venus Using "aero-gravity" assist *(both both gravity And aerodynamics at Venus used to turn the corner to Mars)*



- The aero-assist maneuver at Venus is performed at An altitude of *110 km* above the Surface at a peak Atmospheric velocity of *13.09 km/sec*

- ***At 110 km altitude, the ambient temperature is*** **147.63°K**

- Assume that the Venusian atmospheric Composition by volume is {97% CO_2, 3% N_2}

- Calculate the Probe Mach number at the Venus aero-assist interface

c_p At -125 ° C

CO_2 ~ 0.845kJ/kg ° K

N_2 ~ 0.995kJ/kg ° K

Note Units!

Homework, Section 1 (cont'd)

- Show that for a reversible process

$$s_2 - s_1 = c_v \ln\left[\frac{T_2}{T_1}\right] + R_g \ln\left[\frac{\rho_1}{\rho_2}\right]$$

- and that for a reversible, adiabatic process

$$\left[\frac{T_2}{T_1}\right] = \left[\frac{\rho_2}{\rho_1}\right]^{\gamma - 1}$$

Homework, Section 1 (cont'd)

- Show that for an ideal gas the following Useful relationships *hold*

$$\boxed{\begin{aligned} c_p &= \frac{\gamma}{\gamma - 1} R_g \\ c_v &= \frac{1}{\gamma - 1} R_g \end{aligned}}$$