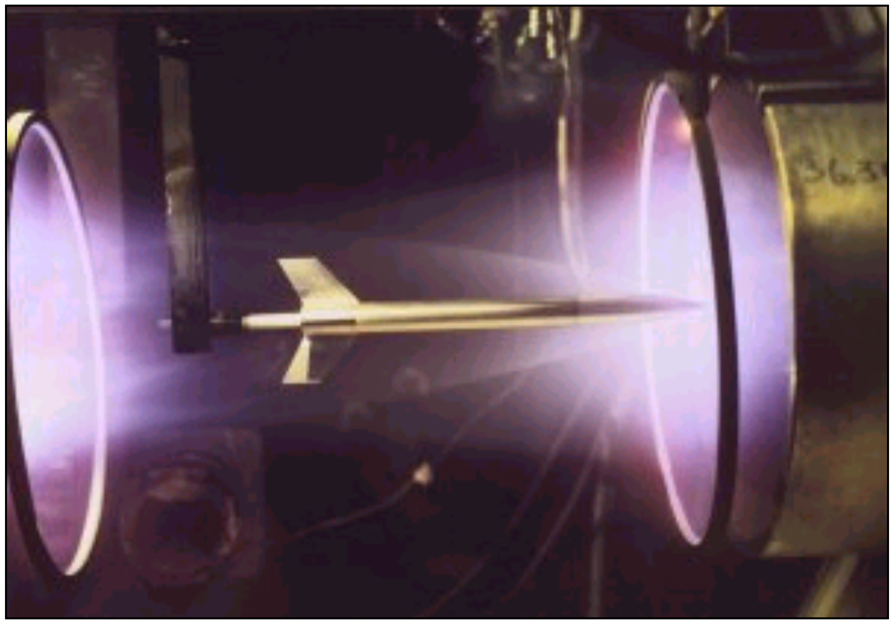
Section1: Compressible Flow: Historical Perspective and Basic Definitions

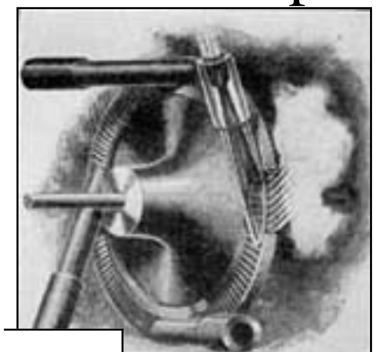


Anderson: Chapter 1 pp. 1-19

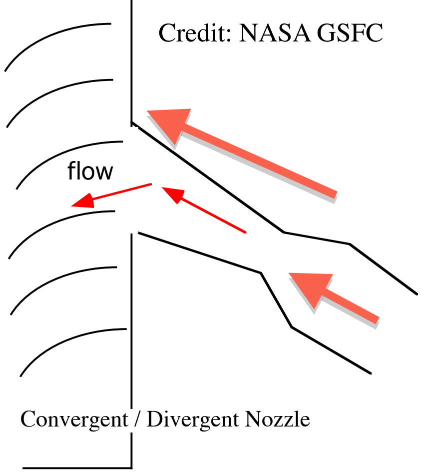
De Laval Nozzle

- High Speed flows often seem "counter-intuitive" when Compared with low speed flows

- Example: Convergent-Divergent Nozzle (De Laval)



Credit: NASA GSFC



Convergent / Divergent Nozzle

In 1897 Swedish Engineer Gustav De Laval designed A turbine wheel powered by 4- steam nozzles

De Laval Discovered that if the steam nozzle first narrowed, and then expanded, the efficiency of the turbine was increased dramatically

Furthermore, the ratio of the minimum area to the inlet and outlet areas was critical for achieving maximum efficiency … **Counter to the "wisdom" of the day**

De Laval Nozzle (cont'd)

• Mechanical Engineers of the 19'th century were Primarily "hydrodynamicists" .. That is they were Familiar with fluids that were incompressible … liquids and Low speed gas flows where fluid density was Essentially constant

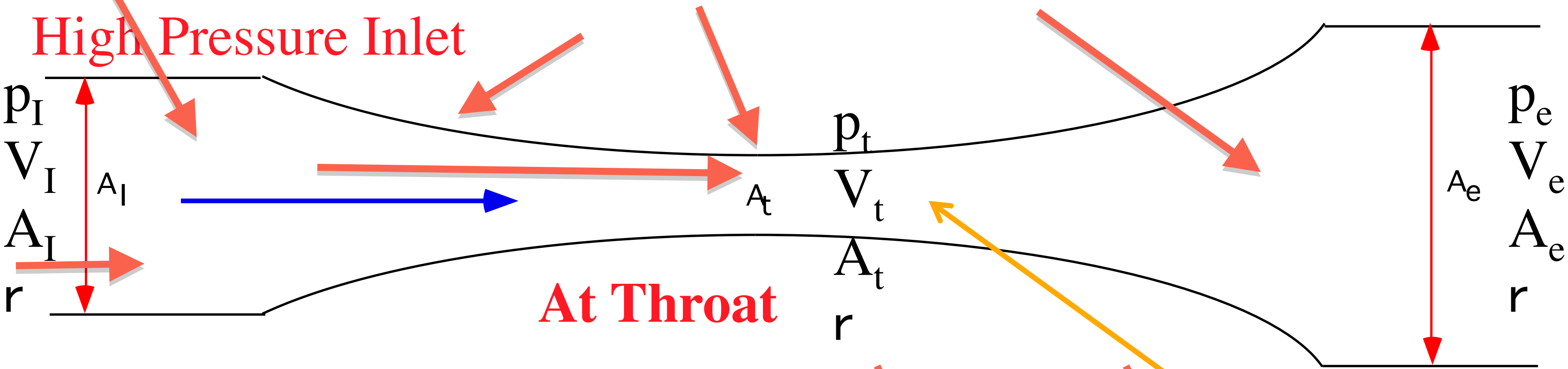
• Primary Principles are Continuity and Bernoulli's Law

Continuity: $\rho \, A \, V = C_1$

Bernoulli: $p + \frac{1}{2} \rho \, V^2 = C_2$

De Laval Nozzle (cont'd)

- When Continuity and Bernoulli are applied to a De Laval Nozzle and density is Assumed constant



$$\cancel{\rho} A_1 V_1 = \cancel{\rho} A_t V_t$$

$$V_t = V_I \frac{A_I}{A_t}$$

Continuity

$$p_t + \frac{1}{2}\rho V_t^2 = p_I + \frac{1}{2}\rho V_I^2$$

$$\frac{p_t - p_I}{\frac{1}{2}\rho V_I^2} = \left[1 - \left[\frac{A_I}{A_t}\right]^2\right]$$

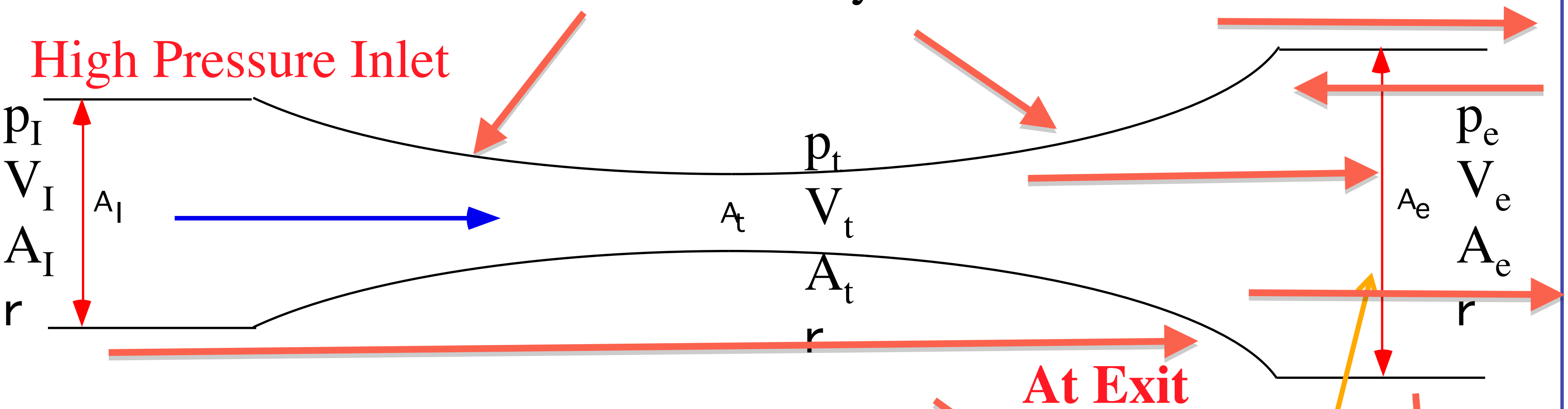
Bernoulli

- Pressure Drop
- Velocity Increases

"classic" Venturi

De Laval Nozzle (cont'd)

• When Continuity and Bernoulli are applied to a De Laval Nozzle and density is Assumed constant



$$\cancel{\rho}\, A_t V_t = \cancel{\rho}\, A_e V_e$$

$$V_e = V_t \frac{A_t}{A_e}$$

Continuity

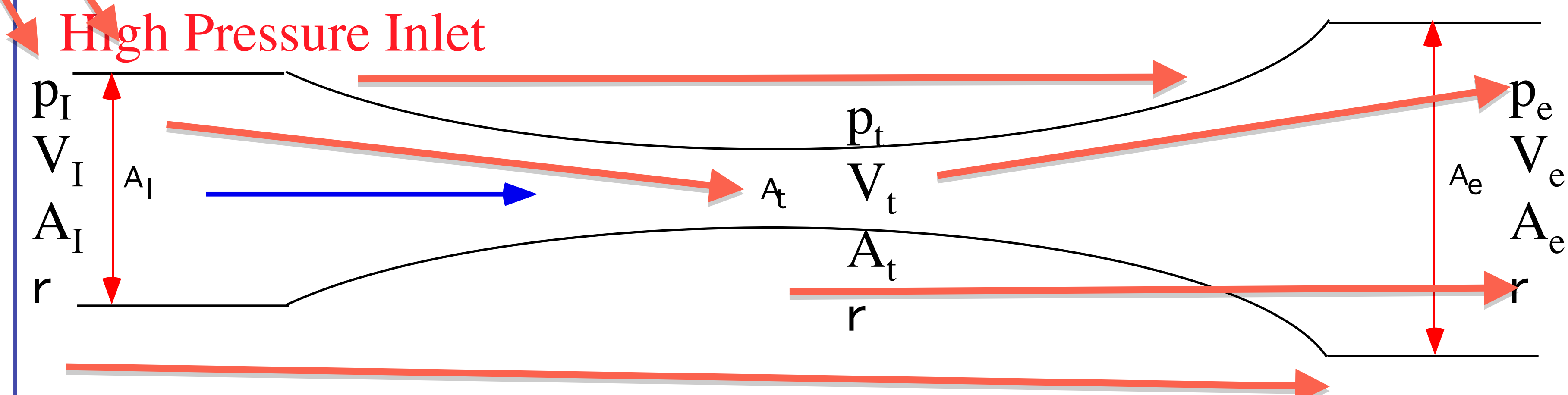
$$p_t + \frac{1}{2}\rho V_t^2 = p_e + \frac{1}{2}\rho V_e^2$$

$$\frac{p_e - p_t}{\frac{1}{2}\rho V_t^2} = \left[1 - \left[\frac{A_t}{A_e}\right]^2\right]$$

Bernoulli

• Pressure Increases
• Velocity Drops

De Laval Nozzle (concluded)



- But De Laval Discovered that when the *Nozzle throat Area was adjusted* downward until the pressure ratio became $p_t / p_I < 0.5484$ -> then the *exit Pressure* dropped *(instead of Rising … compared to the throat pressure)*
And the exit velocity rose (instead of dropping)…
Which is counter to What Bernoulli's law predicts
… he had inadvertently ,,, Generated *supersonic flow!* …

- **fundamental principle that makes rocket motors possible**

Effects of Compressibility Example 2: "Mach Tuck"

• Driven by Combat Needs in WWII, Aircraft airspeeds became increasingly faster.

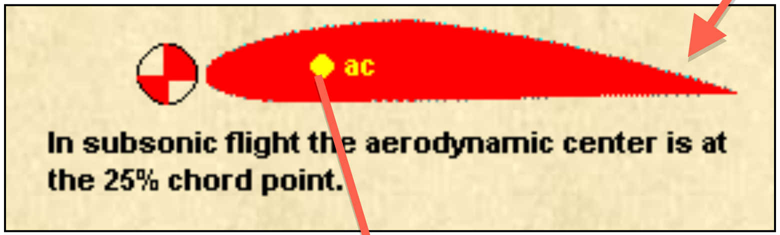
• P-51s, Spitfires and other types were reaching speeds close to that of sound, especially in dives to catch, or escape from, the enemy.

• Pilots began to report control difficulties and unexpected problems, including a strong nose down pitch and a loss of pitch control authority. Often took all of pilot's strength to correct. Some did not make it and dove into the ground, or broke up, as their aircraft exceeded the maximum design speed.

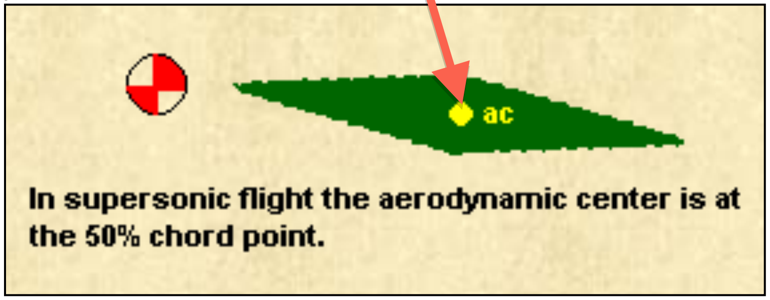
Effects of Compressibility
Example 2: “Mach Tuck” (cont’d)

• Nose down pitching moment was a result of Localized Supersonic *Flow and Air Compressibility*

• At low speeds airfoils have an aerodynamic center that is Approximately at the 25% chord point.



• However, as the aircraft moves into supersonic flight the induced wash ahead of the wing disappears … we’ll learn Why later … As a result the aerodynamic center moves back to the 50% chord point.



Credit: Selkirk College Professional Aviation Program

High Speed Flight: Historical Perspective (cont'd)

• As the aircraft moves into supersonic flight the aerodynamic center suddenly moves back from 25% chord to 50%, the resolved Moment is strongly nose down … a phenomenon known as "Mach Tuck"

• Modern Supersonic aircraft compensate for "Mach Tuck" Using the flight control system, and the effect is insignificant to the pilot.

• However, if the aircraft was never designed to enter supersonic flight (like early subsonic fighters) the nose would pitch down Is severe during the transition through the transonic speed range.

High Speed Flight: Historical Perspective (cont'd)

• Nose down pitching moment was a result of Localized Supersonic *Flow and Air Compressibility*

• *Reduced control* authority was a result of the movement of the Aerodynamic center aft on the aircraft.

• In 1940 NACA commissioned Bell aircraft company to build a special research aircraft for exploring speed range beyond the speed of sound … *the Bell X-1*. *Instrumental in proving these effects*.

• X1 became the first aircraft to fly faster than the speed of sound on October, 14 1947 when Chuck Yeager flew to Mach 1.08

Flow Regimes

• In compressible flow regimes, flow properties vary significantly
From those of lower speed flows

• Understanding these differences is the primary topic of this course

1. Subsonic - All flow everywhere on the aircraft less than local speed of sound.
   M < 0.3 ~ incompressible .... M > 0.8 highly compressible
3. Transonic - Some flow is subsonic and some is supersonic.
   characterized by unsteady chaotic flow fields
5. Supersonic - All flow everywhere on the aircraft is supersonic.
6. Hypersonic - Fluid flows that are Much Higher than sonic velocity

Mechanical Properties of Hypersonic Flow become Independent of Mach Number

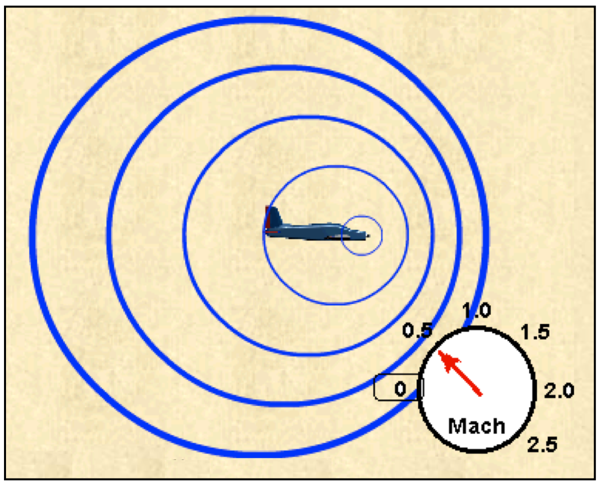
• Key Parameter: *Mach number* --> ratio of airspeed and local speed of sound.
Mach 1~ 573.8 knots at -56 degC (the typical stratosphere temperature.)

Flow Regimes (cont'd)

- As an object moves through a fluid medium it creates pressure waves.

- Pressure waves travel out at the speed of sound which in term depends on gas properties and temperature (more on this later)

- If the object is traveling significantly slower than sonic velocity, then pressure waves travel out uniformly similar to waves on the surface of a pond.



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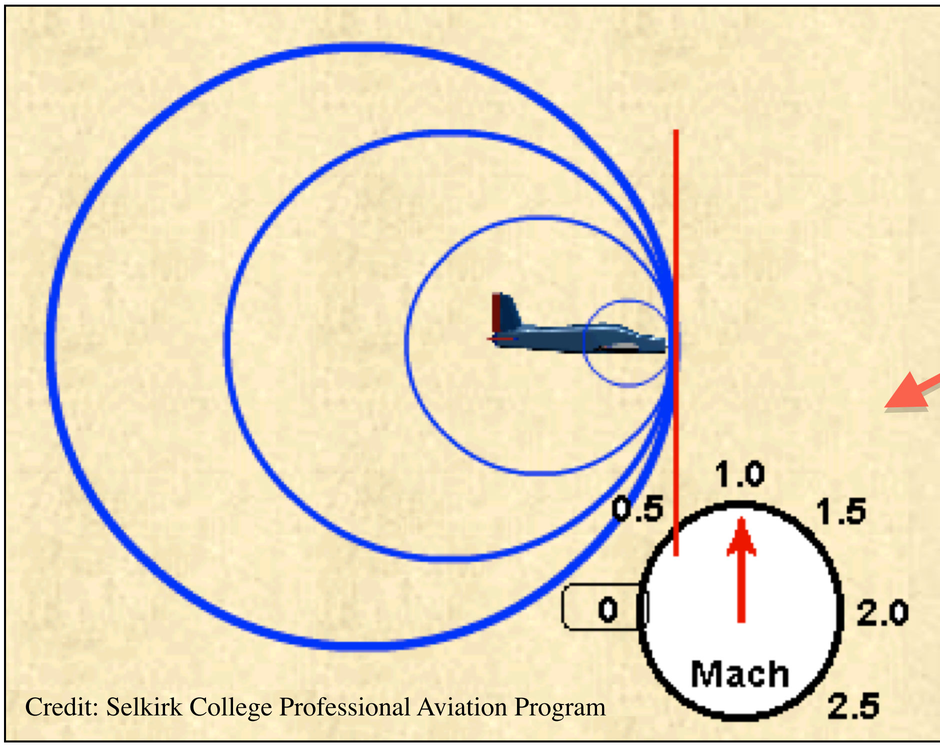
Flow Regimes (cont'd)

• As the object approaches the speed of sound, it begins to catch up with the pressure waves and creates an infinitesimally weak flow discontinuity just ahead of the aircraft



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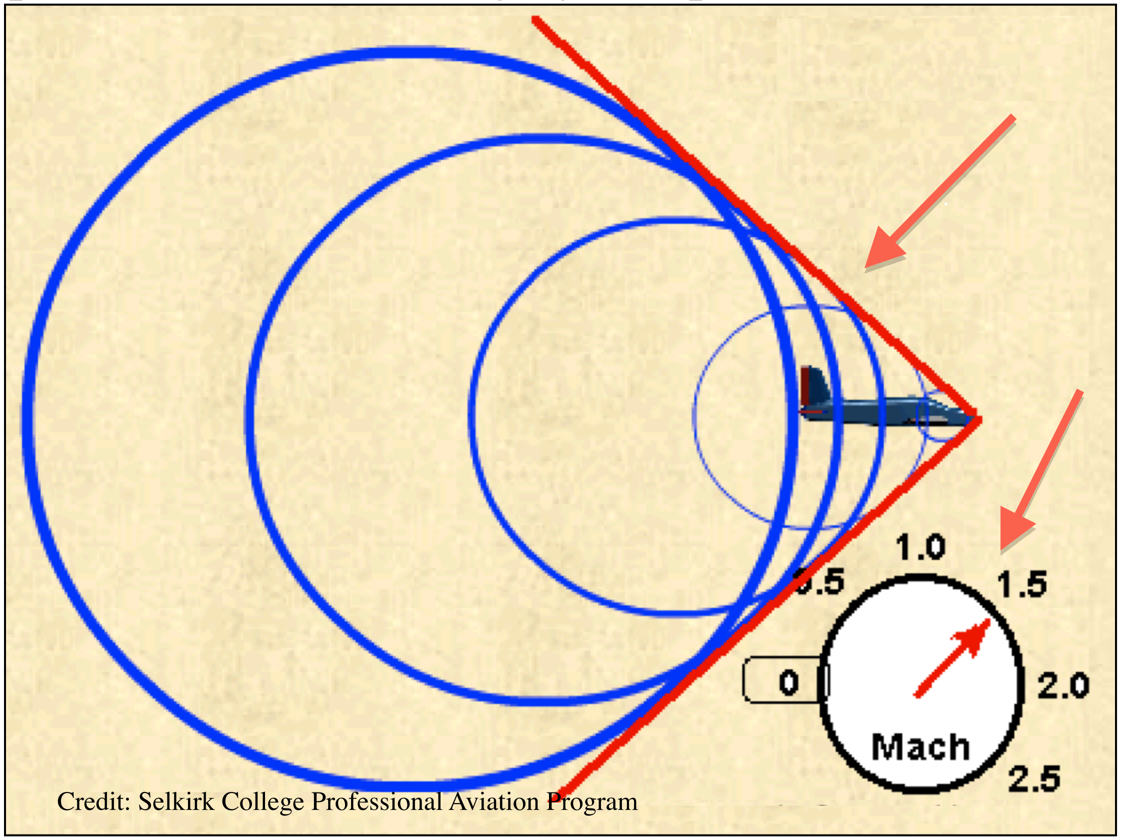
Flow Regimes (cont'd)

• As the vehicle breaks the speed of sound, the infinitesimally weak Shock waves begin to add up along a "Mach Line" and form a strong pressure wave with highly compressed air, called a shockwave.

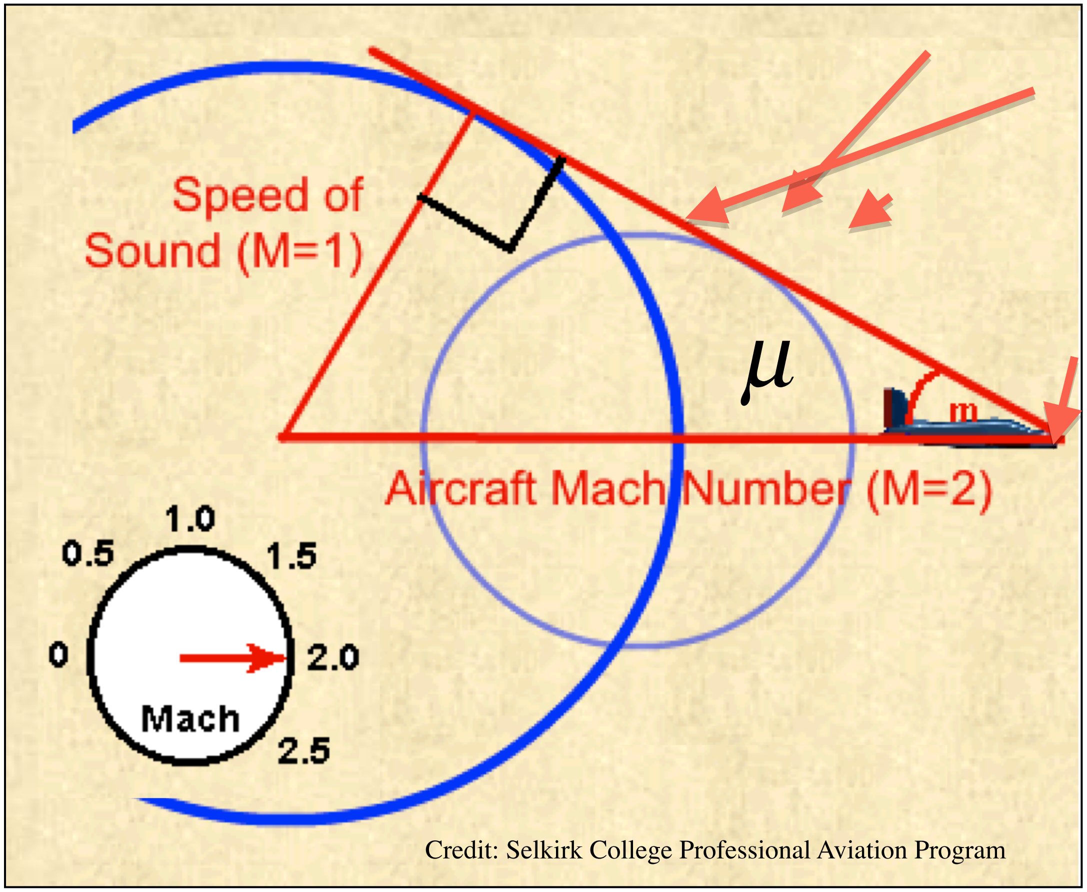


Credit: Selkirk College Professional Aviation Program

• *We'll spend a considerable Portion of the course Understanding The properties of shockwaves*

Flow Regimes (cont'd)

• As Mach number increases, the strength of the shock wave increases and the Angle of the shockwave becomes increasingly severe



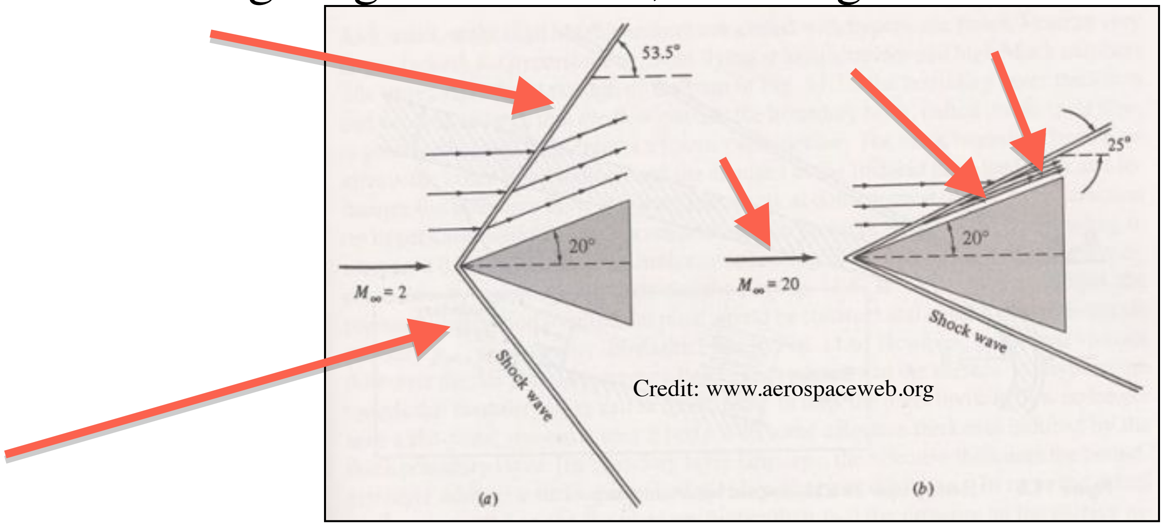
Credit: Selkirk College Professional Aviation Program

• *Mach Angle*

$$\boxed{\mu = \sin^{-1}\left(\frac{1}{M}\right)}$$

Flow Regimes (concluded)

• Finally as *mach number becomes very large* shock wave Is bent so severely that it lies right against vehicle; resulting flow field called *shock layer.*



Credit: www.aerospaceweb.org

• Within shock layer air is heated so much by friction and its own kinetic energy that air molecules ionize.

• This thin layer can produce many complications in vehicle design, and gas dissociation chemistry is essential part of the Flow calculations

• In this Course we will only consider simple approximations for Hypersonic flow

Mathematical Definition of Compressible Flow

- Compressibility is a fluid property
  - Liquids are largely incompressible
  - Gasses are more highly compressible
- Fractional change in volume as pressure is increased

$$\tau = -\frac{1}{v}\frac{dv}{dp}$$

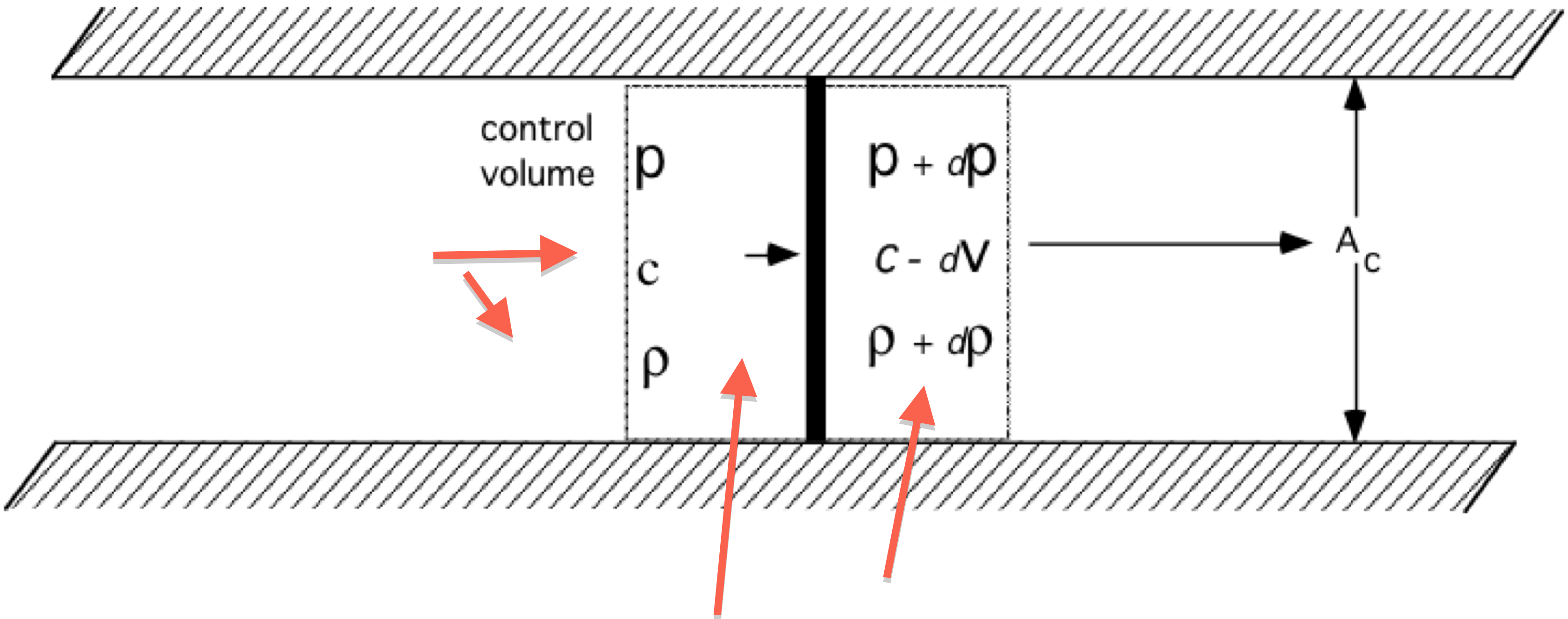
t-> compressibility, *v* gas volume, *p*

Incompressible gas --- infinite t

"Not especially Useful Engineering definition

… but necessary to derive Other more useful expressions

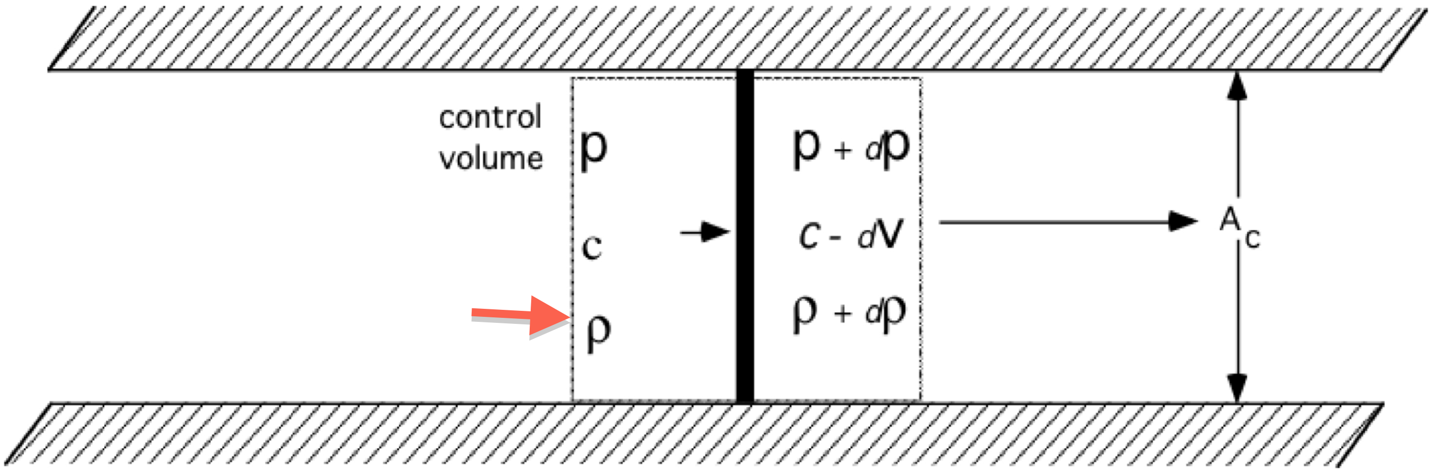
Speed of Sound in a Gas



• Consider flow across an infinitesimal strength compression wave

• Compression results in pressure and density rise and velocity drop

Speed of Sound in a Gas (cont'd)



- Across the control volume we apply Newton's Second law

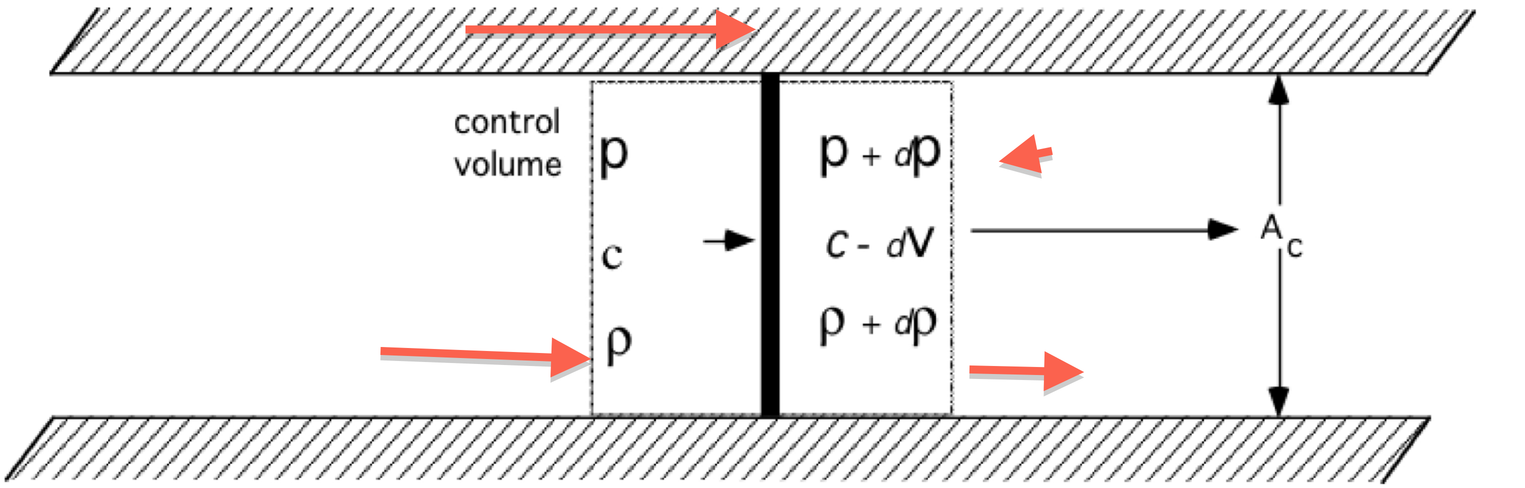
$$F = \frac{d}{dt}\left[mV\right] = \dot{m}\,V + m\,\frac{dV}{dt}$$

- Letting the volume become infinitesimally small

$$\delta F = \dot{m}\,\delta V + \delta m\,\frac{dV}{dt}$$

0

Speed of Sound in a Gas (cont'd)



- But across shock wave ….
the force/momentum balance is …

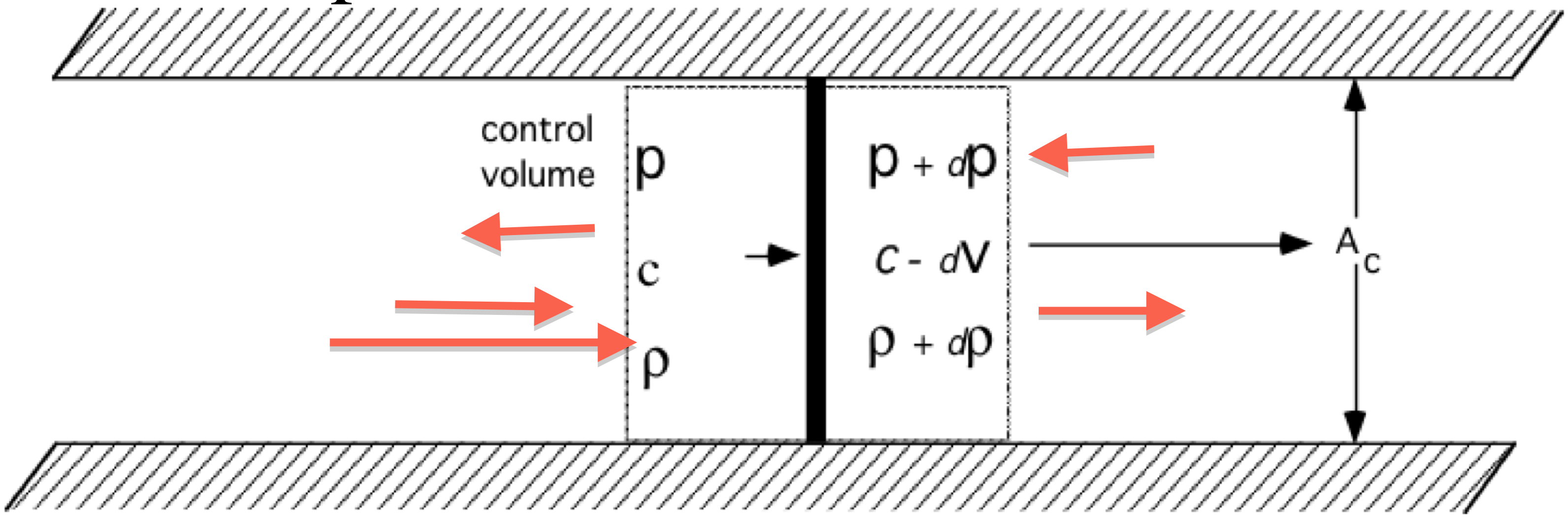
$$\delta F = \dot{m}\,\delta V \rightarrow \boxed{A_c\left[(p+dp)-p\right] = \dot{m}\left[(c-dV)-c\right]}$$

downstream

upstream

- Collecting terms $$A_c \cdot dp = -\dot{m} \cdot dV$$

Speed of Sound in a Gas (cont'd)



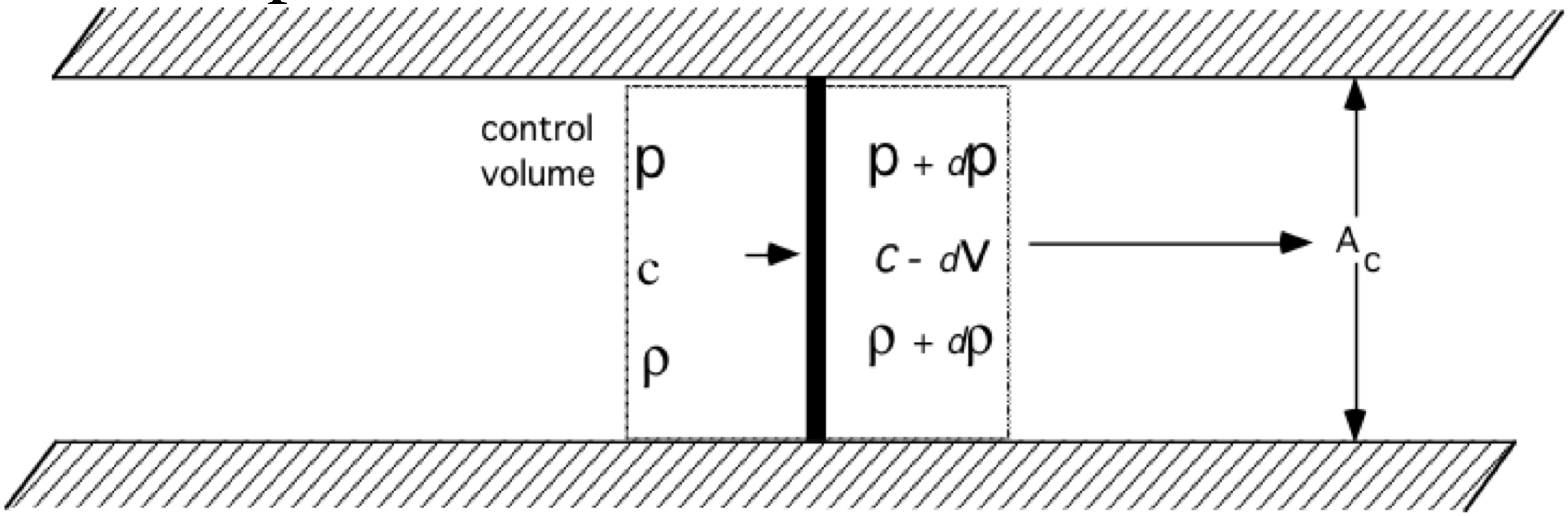
- The constant mass flow into the control volume is

$$\dot{m} = \rho \cdot A_c \cdot c \qquad \Leftrightarrow \textit{check units} \; \frac{kg}{M^3} \cdot M^2 \cdot \frac{M}{\sec} \rightarrow \frac{kg}{\sec} \; \textit{check}!$$

- Combining terms and simplifying ….

$$A_c \cdot dp = -\dot{m} \cdot dV = -\left(\rho \cdot A_c \cdot c\right) \cdot dV \Rightarrow \boxed{dp = -\rho \cdot c \cdot dV}$$

Speed of Sound in a Gas (cont'd)



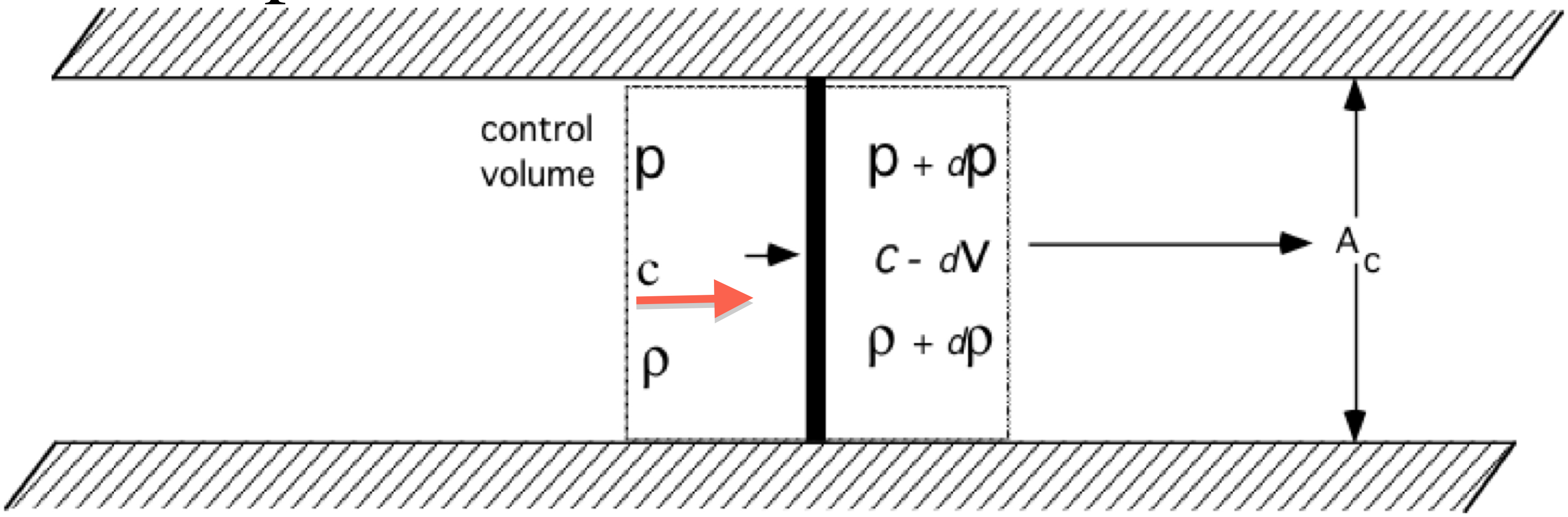
- Applying *Continuity* (conservation of mass) across control volume

$$A_c \cdot \rho \cdot c = A_c \cdot (\rho + d\rho) \cdot (c - dV) \rightarrow \rho \cdot c = \rho \cdot c + d\rho \cdot c - \rho \cdot dV - \underbrace{\cancel{d\rho \cdot dV}}_{0}$$

$$\Downarrow$$

$$\boxed{\frac{d\rho}{\rho} = -\frac{dV}{c}}$$

Speed of Sound in a Gas (cont'd)



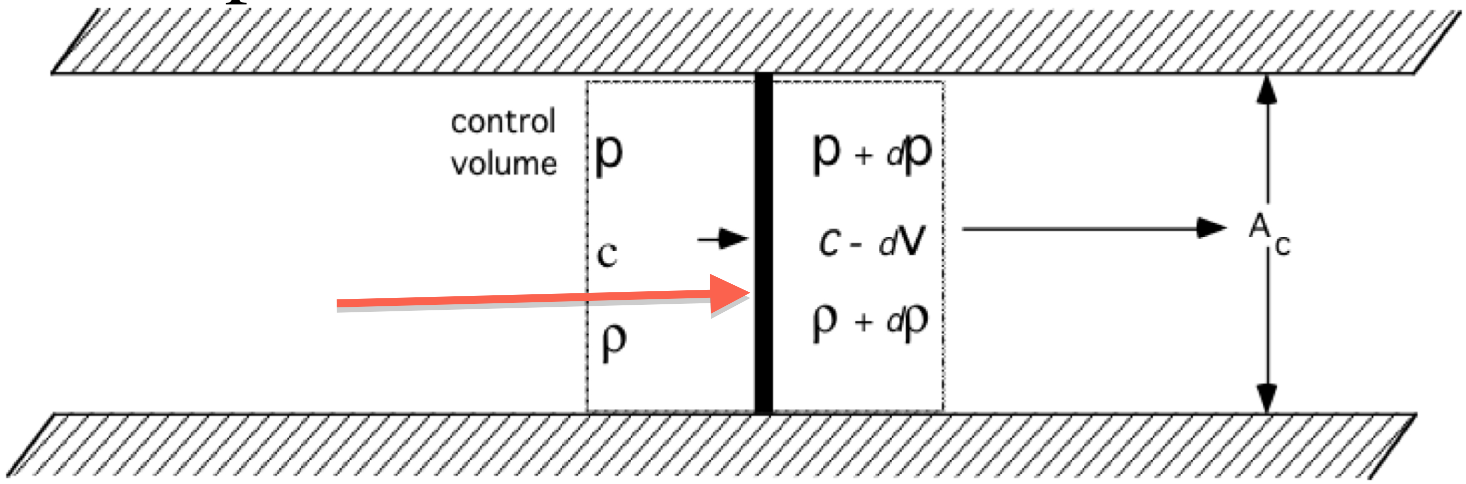
- Collecting terms

$$\boxed{\frac{d\rho}{\rho} = -\frac{dV}{c}} \quad \boxed{dp = -\rho \cdot c \cdot dV}$$

$$\Downarrow$$

$$\frac{\frac{dp}{d\rho}}{\rho} = \frac{-\rho \cdot c \cdot dV}{-\frac{dV}{c}} \Rightarrow \frac{dp}{d\rho} \cdot \rho = \rho \cdot c^2 \Rightarrow c^2 = \frac{dp}{d\rho} \Rightarrow \boxed{c = \sqrt{\frac{dp}{d\rho}}}$$

Speed of Sound in a Gas (cont'd)



- Because shock wave is infinitesimal, process is

1) Adiabatic … no heat loss or addition
2) Reversible … no dissipative phenomena occur
   i.e. entropy is constant

Reversible adiabatic process is referred to as *isentropic*, *and sonic velocity is written as*

$$c = \sqrt{\left(\frac{\partial p}{\partial \rho}\right)_{\Delta s=0}}$$

Speed of Sound in a Gas (cont'd)

- Incompressible Gas … where $\rho = constant \rightarrow \partial\rho = 0$

$$c = \sqrt{\left(\frac{\partial p}{\partial \rho}\right)_{\Delta s=0}}$$

… speed of sound is infinite

- Incompressible gas …*Mach number is zero*

$$\boxed{M = V/c \dashrightarrow V/\infty = 0}$$

- Later we'll get a More workable expression for Sonic velocity for a "perfect gas"

Summary

• Key Concepts:

i) High Speed flows often seem "counter-intuitive" when Compared with low speed flows

ii) Flow regimes

Subsonic - All flow everywhere on the aircraft less than local speed of sound.
Transonic - Some flow is subsonic and some is supersonic.
Supersonic - All flow everywhere on the aircraft is supersonic.
Hypersonic - Fluid flows that are Much Higher than sonic velocity

iii) Mach number - ratio of true airspeed to local speed of sound

iv) Mach Angle … angle of shock wave generated by "point object"

$$\boxed{\mu = \sin^{-1}\left(\frac{1}{M}\right)}$$

v) Sonic Velocity in a gas

$$\boxed{c = \sqrt{\left(\frac{\partial p}{\partial \rho}\right)_{\Delta s=0}}}$$

Next:

The Equation of State and a Review of Thermodynamics