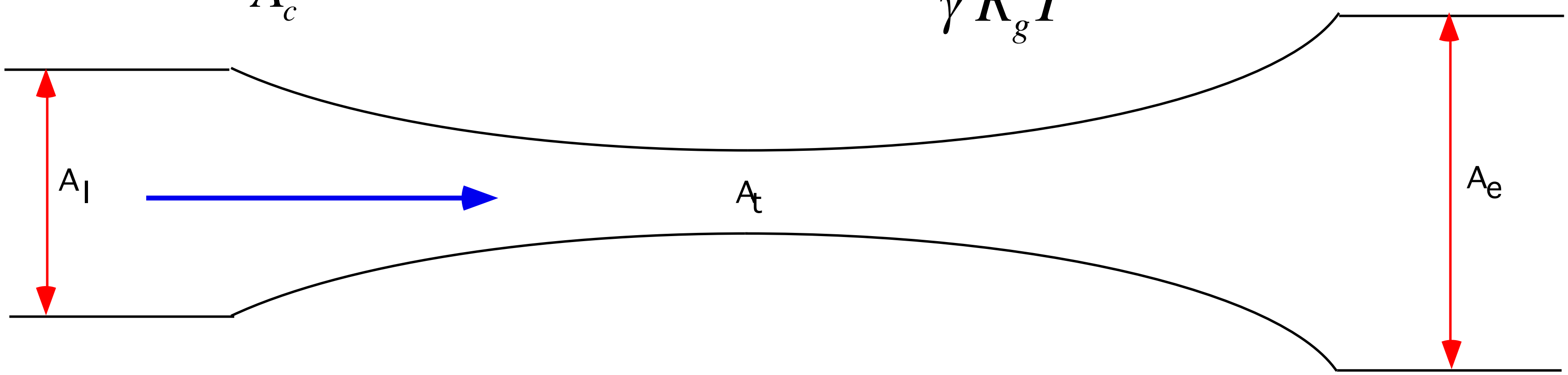
Homework 3

- Solve for M^* in terms of M → ***Plot result with M as independent variable***

- Solve for the Mass Flow per Unit area in a 1-D, steady, isentropic duct flow field as function of T_0, P_0, M, γ, R_g *(hint start with continuity)*

$$\frac{\dot{m}}{A_c} = \rho V \qquad then\ let \ldots.\ \rho = \frac{\gamma P}{\gamma R_g T}$$



Homework 3 (cont'd)

i.e. … Show that …. for Quasi 1-D isentropic flow

$$\frac{\dot{m}}{A} = \sqrt{\frac{\gamma}{R_g}} \frac{P_0}{\sqrt{T_0}} \frac{M}{\left[1 + \frac{(\gamma - 1)}{2} M^2\right]^{\frac{\gamma+1}{2(\gamma-1)}}}$$

- Allowing that for isentropic flow .. Also show that for quasi 1-D flow

$$\frac{T_0}{T} = \left(1 + \frac{\gamma - 1}{2} M^2\right)$$

$$\frac{P_0}{p} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma-1}}$$

$$\rightarrow \frac{\dot{m}}{A} = P_0 \sqrt{\frac{2\gamma}{(\gamma - 1)(R_g \cdot T_0)} \left[\left(\frac{p}{P_0}\right)^{\frac{2}{\gamma}} - \left(\frac{p}{P_0}\right)^{\frac{\gamma+1}{\gamma}}\right]}$$

Homework 3 (cont'd)

Show that …. In general for Quasi 1-D flow

$$\frac{\dot{m}}{A} = \sqrt{\frac{\gamma}{R_g}}\frac{p_0}{\sqrt{T_0}}\frac{M}{\left[1+\frac{(\gamma-1)}{2}M^2\right]^{\frac{\gamma+1}{2(\gamma-1)}}}$$

- show that massflow per unit area has a maximum value when M=1

$$\frac{\dot{m}}{A^*} = \sqrt{\frac{\gamma}{R_g}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}}\frac{p_0}{\sqrt{T_0}}$$

Hint:

$$set.....\frac{\partial}{\partial M}\left(\frac{\dot{m}}{A}\frac{\sqrt{R_g T_0}}{P_0}\right) =$$

$$\frac{\partial}{\partial M}\left(\frac{\sqrt{\gamma}\cdot M}{\left(1+\frac{\gamma-1}{2}M^2\right)^{\frac{\gamma+1}{2(\gamma-1)}}}\right) = 0....\&\ solve\ for\ M$$

Homework (cont'd)

• *Plot* $\frac{\dot{m}}{A_c}\frac{\sqrt{T_0}}{p_0}\sqrt{R_g}$ *as a function of mach number*

.. Does this plot agree with result on previous page?

• *At what mach number does* $\frac{\dot{m}}{A_c}\frac{\sqrt{T_0}}{p_0}\sqrt{R_g}$ *have the greatest value*

• *What does this result imply?*

Assume $\gamma = 1.4$, $R_g = 287.056$ j/kg-°K