CHAPTER 6

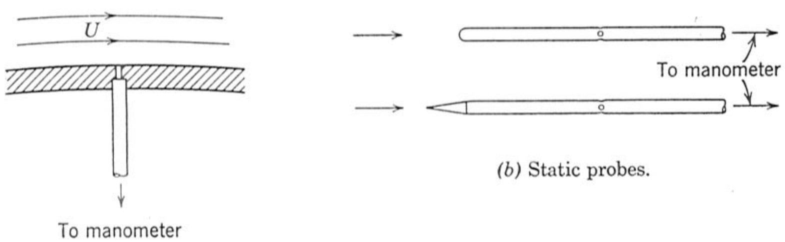
Methods of Measurement

6·1 Introduction

This chapter is an outline of methods for measuring the variables of compressible flow. It is by no means possible to include all possible measurements; only the better-developed ones are discussed, and these only briefly. The discussion is mainly from the point of view of an observer working with a wind tunnel, but many of the methods are, of course, applicable, or adaptable, to the problem of measurement in flight.

6·2 Static Pressure

The pressure on an aerodynamic surface, such as a wind tunnel wall or airfoil, may be measured by means of a small orifice drilled normal (or nearly so) to the surface, and connected to a manometer or other pressure



(*a*) Surface pressure orifice.

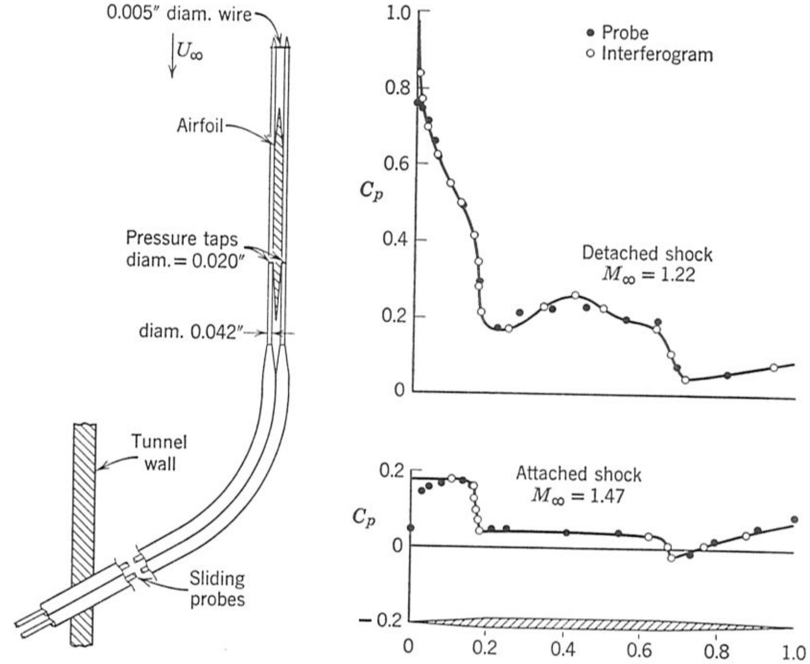
(*b*) Static probes.

FIG. 6·1 Measurement of static pressure.

measuring device (Fig. 6·1*a*). The orifice must be small, and free of imperfections (such as burrs), in order not to disturb the flow locally. A safe rule is that the diameter should be small (say ⅕) compared to the boundary-layer thickness, unless governed by some other factor, such as surface curvature or resolution of pressure gradient. Diameters usually range from 0.01 in., on small models, to 0.1 in. on large installations.

This principle is also used for measuring static pressure at an interior point of the flow, by introducing into the flow a *static probe* (Fig. 6·1*b*) having an orifice at the point of measurement. To disturb the flow as little as possible, the probe must be slender, and aligned parallel to the local

flow direction. The nose region will in any case be disturbed, and so the orifice must be downstream of its "region of influence" (10 to 15 diameters). The point of measurement is at the orifice, not at the nose. The sensitivity to yaw may be decreased by using several holes around the circumference, opening to a common manometer lead, so that an average pressure is



(*a*) Sketch of the probe configuration for a 3% thick airfoil.

(*b*) Pressure distributions as obtained from probes and interferograms.

FIG. 6·2 Measurement of surface pressure, with a static probe. From W. W. Willmarth, *J. Aeronaut. Sci.*, *20* (1953), p. 438.

measured. Even with this precaution it is usually not permissible for the yaw angle to exceed 5°, if 1% accuracy is required.

The round-nosed probe shown in Fig. 6·1*b* may be used for either subsonic or supersonic work, although the one with conical nose is usually preferred for the supersonic case.

Long, slender static probes may also be used for the measurement of pressure on surfaces in which it is not feasible to install orifices, for example, very thin airfoils, as illustrated in Fig. 6·2.

Supersonic static probes are subject to the type of interference from shock waves that is illustrated in Fig. 6·3. This shows a pressure survey

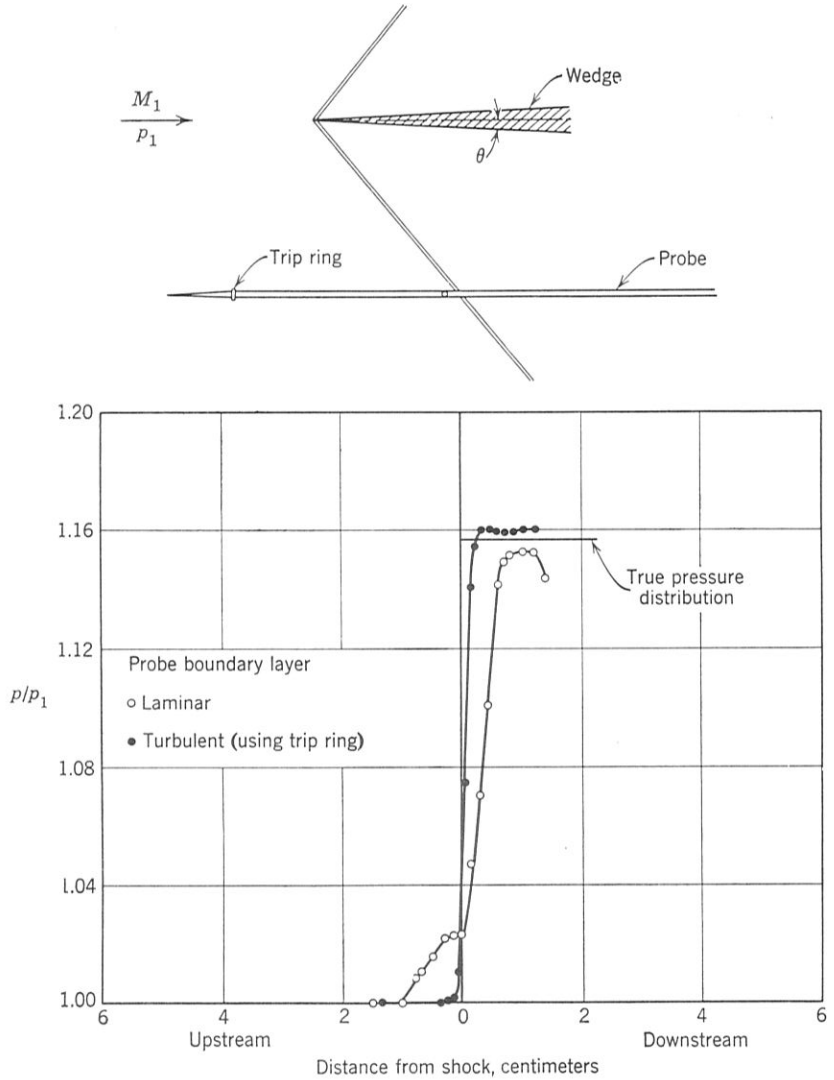


Fig. 6·3 Effect of probe boundary layer on static-pressure measurements through a shock wave. $\theta = 3°$; $M_1 = 1.354$. From Liepmann, Roshko, and Dhawan, "On Reflection of Shock Waves from Boundary Layers," *NACA Rep.* 1100 (1952).

made with a static probe through a weak, oblique shock. The true pressure distribution is a "step," the shock thickness being much too small to show up on this scale, but the probe cannot measure this, since its boundary layer cannot support a large pressure gradient. It interacts with the shock

in such a way as to relieve the pressure gradient which it feels, the result being an upstream and downstream "influence" of the pressure rise. The effect on a laminar boundary layer is much greater than on a turbulent one, as indicated by the improvement obtained with a turbulent boundary layer, tripped by means of a small ring on the nose of the probe.

6·3 Total Pressure

The total or stagnation pressure is defined in Article 2·4. It is a measure of the entropy of the fluid. If the fluid at a given point of the flow has experienced only *isentropic* changes in its past history, its total pressure is equal to the reservoir pressure p_0, and may simply be measured in the settling chamber. If, on the other hand, entropy-changing conditions have

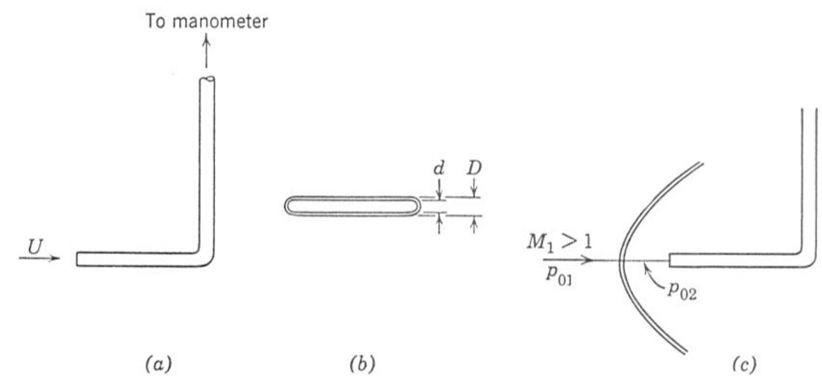


Fig. 6·4 Pitot probes. (*a*) Simple pitot tube; (*b*) front view of tube with flattened opening for boundary-layer work; (*c*) pitot probe in supersonic flow.

been encountered, the *local* pitot pressure p'_0 is different from p_0, and must be obtained from a local measurement. This occurs, for instance, if the fluid has traversed shock waves, or entered boundary layers or wakes, or if external heat has been added.

A pitot tube, for measuring the total pressure, consists of nothing more than an open-ended tube (Fig. 6·4) aligned parallel to the flow, with the open end "facing" the stream. The other end is connected to a manometer. The fluid in the tube is at rest. Except at very low Reynolds numbers (based on tube diameter), the deceleration to rest is isentropic, and the pressure in the tube is the stagnation pressure of the flow at the position defined by the mouth of the tube.

The limits on the size of the opening are similar to those for static pressure orifices.

The pitot probe is less sensitive than the static probe to flow alignment.

A simple open-ended probe, like that shown in Fig. 6·4, gives 1% accuracy up to yaw angles of 20°. Tubes with rounded nose and small orifice (compared to external diameter) are more sensitive.

Pitot tubes are often used for measuring the distribution of stagnation pressure in a boundary layer. The gradient normal to the surface is large, but good accuracy is obtainable with a flattened pitot tube like that shown in Fig. 6·4*b*. The small opening height d gives good resolution, and the large width improves response time. It is quite practicable (with a little patience) to build tubes having dimensions as small as $D = 0.003$ in. and $d = 0.001$ in. The tube is traversed normal to the surface by a suitable micrometer drive, and its position is measured by a micrometer head or a traversing microscope.

In supersonic flow a pitot tube does not indicate the local total pressure, since a detached shock wave stands ahead of the tube (Fig. 6·4*c*). On the stagnation streamline the shock is normal and the ratio of true total pressure to measured pitot pressure is given by Eq. 2·54. Slightly rewritten, this is

$$\frac{p_{01}}{p_{02}} = \left(\frac{2\gamma}{\gamma+1}M_1^2 - \frac{\gamma-1}{\gamma+1}\right)^{1/(\gamma-1)} \left(\frac{1+\frac{\gamma-1}{2}M_1^2}{\frac{\gamma+1}{2}M_1^2}\right)^{\gamma/(\gamma-1)} \tag{6·1}$$

If M_1 is known, then the true stagnation pressure may be easily computed from this relation. Otherwise an additional measurement is needed.

6·4 Mach Number from Pressure Measurements

The Mach number is one of the most important parameters in compressible flow. It may be obtained from any one of the many flow relationships in which it appears, if the other quantities in the relationship may be measured. Among the most useful of these are the ones involving pressures.

If the fluid at the point of measurement has undergone only isentropic changes, its stagnation pressure may be assumed to be the reservoir pressure, p_0. A measurement of the *static pressure* p then determines the Mach number from the isentropic relation,

$$\frac{p_0}{p} = \left(1 + \frac{\gamma-1}{2}M^2\right)^{\gamma/(\gamma-1)} \tag{6·2}$$

This relation, valid for subsonic and supersonic flow, is used for the determination of Mach number distribution along an aerodynamic surface, using surface static holes, and for flow-field Mach number, using a static probe.

In isentropic *supersonic* flow, the local Mach number may be measured by a pitot tube, using Eq. 6·1, with p_{01} equal to the reservoir pressure. This method is quite as sensitive as the preceding one based on static pressure. It must be certain that no condensation has occurred in the flow ahead of the point of measurement, for this affects the stagnation pressure. Alternately, this measurement, together with a supplementary measurement of Mach number, may be used for *detecting* condensation.

In measurements from aircraft, or in nonisentropic flow, the "reservoir" conditions are not known, and it is necessary to obtain *both* the static pressure and the pitot pressure. If the flow is *subsonic*, the pitot pressure is the true total pressure, and the Mach number is obtained from Eq. 6·2. But if the flow is *supersonic*, the indicated pitot pressure is p_{02}, the stagnation pressure behind the normal shock. The Mach number may be obtained by dividing Eq. 6·1 by Eq. 6·2 in order to eliminate p_0 ($\equiv p_{01}$). This gives

$$\frac{p}{p_{02}} = \frac{\left(\frac{2\gamma}{\gamma+1}M_1^2 - \frac{\gamma-1}{\gamma+1}\right)^{1/(\gamma-1)}}{\left(\frac{\gamma+1}{2}M_1^2\right)^{\gamma/(\gamma-1)}} \tag{6·3}$$

which is called the *Rayleigh supersonic pitot formula*.

The *dynamic pressure* is usually obtained from the static pressure and Mach number, using Eq. 2·39,

$$\frac{1}{2}\rho u^2 = \frac{\gamma}{2}pM^2$$

6·5 Wedge and Cone Measurements

In supersonic flow, it is sometimes convenient to use a wedge or cone instead of a static probe. The pressure on the wedge surface may be related to the flow conditions by the simple oblique shock relations (Chart 2), whereas for the cone the corresponding conical shock theory is used (Fig. 4·27). Cones have the advantage that the detachment Mach number is lower.

For instance, in the case of a wedge symmetrically aligned with the flow, the ratio between the pressure p_a, on the wedge surface, and the stream pressure p_1, may be obtained from the oblique shock charts. When multiplied by p_1/p_0, from Eq. 6·2, a relation is obtained in the form

$$\frac{p_a}{p_0} = f(M_1, \theta)$$

where θ is the half-angle of the wedge and p_0 is the local stagnation pressure.

The wedge may also be used to find the flow inclination by measuring the difference of pressure, $p_b - p_a$, on the two sides. Using the shock charts, it is possible to plot the ratios

$$\frac{p_b + p_a}{2p_0} \quad \text{and} \quad \frac{p_b - p_a}{2p_0}$$

as functions of the Mach number for a given wedge of angle 2θ, and for various flow inclinations α. The measurement of p_a and p_b then gives both Mach number and flow inclination.

For wedges of *small* nose angle, at small inclination, the approximate oblique shock relations (Article 4·7) may be used to obtain these functions in simple, closed form.

The Mach number and flow inclination may also be obtained from a measurement of the *shock wave angles*. This is not usually as accurate or as convenient as the pressure measurements.

6·6 Velocity

The Mach number may be converted into other dimensionless speed ratios by means of the relations developed in Chapter 2. Thus Eq. 2·37*a* gives

$$\left(\frac{u}{a^*}\right)^2 = M^{*2} = \frac{\gamma + 1}{(2/M^2) + (\gamma - 1)} \tag{6·4a}$$

Since

$$\left(\frac{a^*}{a_0}\right)^2 = \frac{2}{\gamma + 1} \tag{6·5}$$

it follows that

$$\left(\frac{u}{a_0}\right)^2 = \frac{2}{(2/M^2) + (\gamma - 1)} \tag{6·4b}$$

In Eqs. 6·4*a* and 6·4*b* a_0 and a^* must be the *local* values. If the flow is adiabatic up to the point of measurement, these local values are equal to the reservoir values, and may be measured in the supply section. They may be obtained from a measurement of the reservoir temperature T_0, from the perfect gas relation

$$a_0^2 = \gamma R T_0$$

If there is heat addition ahead of the point of measurement, or if the point is in a nonequilibrium region such as a boundary layer, or if the measurement is being made from an aircraft, it is necessary to make a *local* measurement of T_0. This problem is discussed in the following article.

Using pressures, the determination of velocity requires the measurement of three local quantities, p, p_0, and T_0. In special cases one or more of these measurements may be avoided, as already noted, by using the settling chamber values. For velocity profiles through a boundary layer, it is usually possible to assume that p is constant throughout; it may then be simply measured at a surface orifice.

Methods for *direct* measurement of velocity are based mainly on tracer techniques, using, for example, ions or illuminated particles for tracers.

They are seldom used, owing largely to the technical inconveniences, and also because they do not provide a *local* measurement, since the flight must be timed over a finite distance.

Other methods use velocity-sensitive elements which are *calibrated*. One of these, the hot-wire anemometer, is discussed in Article 6·19.

6·7 Temperature and Heat Transfer Measurements

There is no method for the direct measurement of *static* temperature T in a moving fluid. The temperature indicated by any thermometer immersed in the fluid is higher than T, for there is an increase of temperature through the boundary layer, from the static temperature T at the edge to the *recovery* temperature T_r at the surface. T_r will in general be different on different parts of the surface, depending on geometry, Reynolds number, etc., and the thermometer will indicate a *mean* recovery temperature.†

Indirect determinations of T may be obtained from measurements of pressure and density, using the equation of state; if conditions are isentropic, one of these is sufficient. An *independent*, direct determination may be obtained from a measurement of the acoustic speed, a, using the relation

$$a^2 = \gamma R T \tag{6·6}$$

The value of a is obtained by producing weak pressure pulses (sound waves), of known frequency, in the flow, photographing them by one of the optical techniques described later in this chapter, and measuring the spacing (wavelength) between pulses. In the measurement of wavelength, the velocity of the fluid relative to which the pulses are propagating must be taken into account. This difficulty, together with the fact that an averaging distance is involved, has so far prevented any general adoption of the method.

The measurement of the stagnation or *total* temperature T_0 is in principle simple. The temperature inside a pitot tube, where the flow is brought to rest at equilibrium, should be the stagnation temperature, in either subsonic or supersonic flow, and could be measured by a thermometer placed inside the tube. The difficulty is that equilibrium does not actually exist, since heat is conducted (and radiated) away by the thermometer and the probe walls.‡ This reduces the temperature of the stagnant fluid to a value T_r lower than T_0. It depends on the probe configuration, conductivity of the walls, flow conditions on the outer walls, etc.

Figure 6·5 shows an example of a total temperature probe which uses a thermocouple for a sensing element. The shield and support are designed

†In principle, a direct static temperature measurement could be obtained by allowing the thermometer to move *with* the fluid.

‡This departure from thermal equilibrium affects only the temperature and density, not the pressure.

to keep the rate of heat loss by conduction and radiation to a minimum. To replenish some of the lost energy a small flow through the probe is permitted, by means of a vent hole. With such designs, it has been possible to obtain values of the *recovery factor*,

$$r = \frac{T_r - T_1}{T_0 - T_1}$$

very nearly equal to 1, and, what is more important, to keep r constant over a fairly wide range of conditions, for the probe is a *calibrated* instrument.

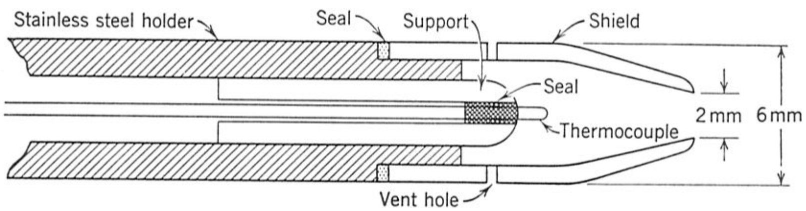


FIG. 6·5 Design of total temperature probe. From Eva M. Winkler, *J. Appl. Phys.*, *25* (1954), p. 231.

For boundary layer profiles a probe with flattened opening, like the pitot probe described in Article 6·3, may be employed.

Inasmuch as the total temperature probe is a *calibrated* instrument, it is possible to use a much simpler calibrated probe, namely, a fine resistance wire aligned normal to the flow. The wire temperature may be determined from a measurement of its resistance (see Article 6·19).

Measurements of *wall temperature*, for use in heat transfer or boundary layer studies, may be made with thermometer elements, such as thermocouples or resistance wires, embedded in the surface. The principal requirement is that the element should not change the conditions of heat transfer that would exist if it were absent. By embedding several thermocouples *in* the wall, at various distances from the surface, data for a temperature gradient may be obtained† and the local heat transfer to the surface may be determined.

Another technique for measurement of heat transfer between the wall and the flow is the *transient* method, in which the rate of temperature change of a portion of the wall provides a measure of the heat transfer through the surface. The heat capacity of the wall material must be known, and it is necessary to minimize heat transfer through sections other than the surface, or to estimate its effect (see Exercise 6·5).

†Lobb, Winkler, and Persh, *J. Aeronaut. Sci.*, *22* (1955), p. 1.

6·8 Density Measurements

If, in addition to the pressure p, there is a measurement of the density ρ, considerable information about the state of the flow becomes available. For instance, a local measurement of p, p_0, and ρ defines all the other flow variables, as follows. The speed of sound is calculated from

$$a = \sqrt{\frac{\gamma p}{\rho}}$$

and the velocity from

$$u = aM$$

where M is obtained from the pressure relation (Eq. 6·2). The local values of T and T_0 are also determined from

$$p = R\rho T$$

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} M^2$$

In this example, the measurement of density supplements the pressure measurements, but sometimes the measurement of density is the *only* one required. For instance, in isentropic flow (p_0 and T_0 known throughout) a measurement of density gives the pressure and Mach number from

$$\frac{p}{p_0} = \left(\frac{\rho}{\rho_0}\right)^{\gamma}$$

$$\frac{\rho_0}{\rho} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{1/(\gamma-1)} \qquad (6·7)$$

The methods for measuring, or visualizing, the density field depend almost invariably on the effects of the fluid density on some form of electromagnetic radiation. The methods may be broadly classified as those which depend on the *refractive index* (optical methods), on *absorption*, and on *emission*, respectively. Of these, the optical methods are by far the best developed and the ones most widely used. The next articles are devoted to them; the others are briefly discussed in Article 6·17.

6·9 Index of Refraction

The three principal optical methods, schlieren, shadowgraph, and interferometer, depend on the fact that the *speed of light* varies with the density of the medium through which it is passing. The speed c, in any medium, is related to the speed c_0, in vacuum, by the *index of refraction*,

$$n = c_0/c \qquad (6·8)$$