Example Calculation

Consider Venturi Flow with a Gaseous Oxygen O_2 with Measured Static Pressures p_1, p_2

$$\begin{bmatrix} p_1 \\ D_1 \\ T_0 \end{bmatrix} = \begin{bmatrix} 3000\ kPa \\ 0.8\ cm \\ 25\ {}^oC \end{bmatrix} \rightarrow \begin{bmatrix} p_2 \\ D_2 \\ C_d \end{bmatrix} = \begin{bmatrix} 2000\ kPa \\ 0.4\ cm \\ 0.95 \end{bmatrix}$$

$$A = \frac{\pi}{4} D^2$$

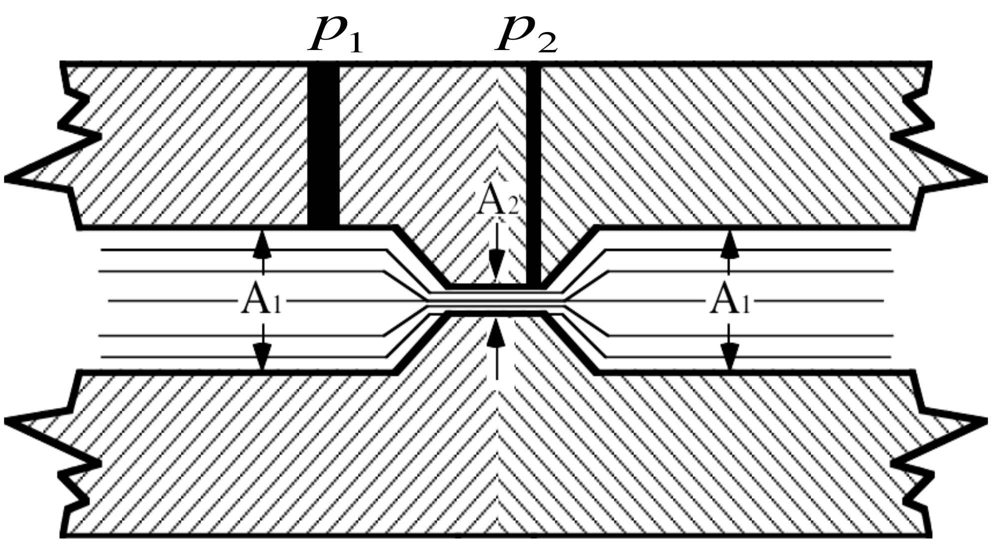
• ***Part 2: Assuming Isentropic Flow .. Calculate the Massflow and Flow Velocity Through Venturi at stations (1) and (2)***

$\gamma = 1.4$, $Mw = 32.0$ *kg/kg-mol*

Example Calculation (2)

Compare results of compressible Venturi calculations to values calculated using incompressible Venturi Equation Massflow, V_1, V_2

- *Assume $T_1 = T_0 = 25\ ^oC$*
- *Base incompressible Density on p_1*
- *Use Cd=0.95 for calculations*



$$V_2 = \sqrt{\frac{2\cdot(p_1 - p_2)}{\rho\cdot\left(1-\left(\frac{A_2}{A_1}\right)^2\right)}} \rightarrow \dot{m} = A_2 \cdot \sqrt{\frac{2\cdot\rho\cdot(p_1 - p_2)}{\left(1-\left(\frac{A_2}{A_1}\right)^2\right)}}$$

Incompressible Venturi Equations

Example Calculation (3)

- **Check to see if Venturi is Choked**

$$\frac{p_1}{p_2} = \frac{3000_{kPa}}{2000_{kPa}} = 1.5$$

$$\left(\frac{\gamma+1}{2}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{1.4+1}{2}\right)^{\frac{1.4}{(1.4-1)}} = 1.89293$$

Subcritical Flow!

Example Calculation (3)

- **First Calculate Stagnation Pressure**

$$P_0 = \left[\frac{\left(\frac{A_1}{A_2}\right)^2 \cdot (p_1)^{\frac{\gamma+1}{\gamma}} - (p_2)^{\frac{\gamma+1}{\gamma}}}{\left(\frac{A_1}{A_2}\right)^2 \cdot (p_1)^{\frac{2}{\gamma}} - (p_2)^{\frac{2}{\gamma}}} \right]^{\frac{\gamma}{\gamma-1}} =$$

$$Subcritical\ Flow: \left(\frac{p}{P_0}\right) > \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\left(\frac{\left(\frac{0.8}{0.4}\right)^4 3000^{\frac{1.4+1}{1.4}} - 2000^{\frac{1.4+1}{1.4}}}{\left(\frac{0.8}{0.4}\right)^4 3000^{2/1.4} - 2000^{2/1.4}} \right)^{\frac{1.4}{(1.4-1)}} = 3041.89 \quad kPa$$

Example Calculation (4)

- **Now Calculate Massflow through Venturi**

$$\dot{m} = A_2 \cdot \sqrt{\left(\frac{2\gamma}{\gamma-1}\right)\cdot\left(\frac{P_0^{\,2}}{R_g \cdot T_0}\right)\cdot\left[\left(\frac{p_2}{P_0}\right)^{\frac{2}{\gamma}} - \left(\frac{p_2}{P_0}\right)^{\frac{\gamma+1}{\gamma}}\right]} =$$

$$0.95\left(\frac{\left(0.4^2\,\frac{\pi}{4}\right)}{100^2}\right)\left[\left(\frac{2\cdot 1.4}{1.4-1}\right)\left(\frac{(3041.89\cdot 10^3)^2}{\frac{8314.4612}{32}\,(273.15+25)}\right)\left[\left(\frac{2000}{3041.89}\right)^{2/1.4} - \left(\frac{2000}{3041.89}\right)^{\frac{1.4+1}{1.4}}\right]\right]^{0.5} 1000$$

= 85.971 g/sec

Example Calculation (5)

- **Calculate Flow Velocities at station (1)**

$$M_1 = \left(\sqrt{ \left(\frac{2}{\gamma - 1} \right) \cdot \left(\left(\frac{P_0}{p_1} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right) } \right) = \left(\left(\frac{2}{1.4-1} \right) \left(\left(\frac{3041.89}{3000} \right)^{\left(\frac{1.4-1}{1.4} \right)} - 1 \right) \right)^{0.5} = 0.140886$$

$$V_1 = M_1 \cdot \sqrt{\gamma \cdot R_g \cdot T_1} = M_1 \cdot \sqrt{ \frac{\gamma \cdot R_g \cdot T_0}{1 + \frac{\gamma - 1}{2} M_1 2} } =$$

$$0.1408 \left(\frac{1.4 \frac{8314.4612}{32} (273.15 + 25)}{1 + \left(\frac{1.4-1}{2} \right) 0.1408^2} \right)^{0.5} = 46.2772 \ \textit{m/sec}$$

Example Calculation (6)

- **Calculate Flow Velocities at station (2)**

$$M_2 = \left(\sqrt{\left(\frac{2}{\gamma-1}\right) \cdot \left(\left(\frac{P_0}{p_2}\right)^{\frac{\gamma-1}{\gamma}} - 1 \right)} \right) = \left(\left(\frac{2}{1.4-1}\right) \left(\left(\frac{3041.89}{2000}\right)^{\left(\frac{1.4-1}{1.4}\right)} - 1 \right) \right)^{0.5} = 0.797752$$

$$V_2 = M_2 \cdot \sqrt{\gamma \cdot R_g \cdot T_2} = M_2 \cdot \sqrt{\frac{\gamma \cdot R_g \cdot T_0}{1 + \frac{\gamma-1}{2} M_2^2}} =$$

$$0.7977 \left(\frac{1.4 \frac{8314.4612}{32} (273.15 + 25)}{1 + \left(\frac{1.4-1}{2}\right) 0.7977^2} \right)^{0.5} = 247.429 \ \textit{m/sec}$$

Example Calculation (7)

- **Calculate Massflow Using Incompressible Venturi Equations**

$$\dot{m} = A_2 \cdot \sqrt{\frac{2 \cdot \rho \cdot (p_1 - p_2)}{\left(1 - \left(\frac{A_2}{A_1}\right)^2\right)}} =$$

$$\left(0.95\left(\frac{0.4}{100}\right)^2\right)\frac{\pi}{4}\left(\frac{2\,\frac{3000 \cdot 10^3}{\frac{8314.4612}{32}(25 + 273.15)}(3000 - 2000)\,10^3}{1 - \left(\frac{0.4}{0.8}\right)^4}\right)^{0.5} 1000 = 108.509$$

g/sec

Example Calculation (8)

- **Calculate Inlet Velocity Using Incompressible Venturi Equations**

$$V_1 = \frac{\dot{m}}{\rho \cdot A_1} = \frac{A_2}{A_1} \cdot \sqrt{\frac{2 \cdot (p_1 - p_2)}{\rho \cdot \left(1 - \left(\frac{A_2}{A_1}\right)^2\right)}} =$$

$$\left(\frac{0.4}{0.8}\right)^2 \left(\frac{2\,(3000 - 2000)\,10^3}{\left(\frac{3000 \cdot 10^3}{\frac{8314.46}{32}(25 + 273.15)}\right)\left(1 - \left(\frac{0.4}{0.8}\right)^4\right)} \right)^{0.5} = 58.677$$

m/sec

Example Calculation (8)

- **Calculate Throat Velocity Using Incompressible Venturi Equations**

$$V_2 = \frac{\dot{m}}{\rho \cdot A_2} = \sqrt{\frac{2\cdot(p_1 - p_2)}{\rho\cdot\left(1-\left(\frac{A_2}{A_1}\right)^2\right)}} =$$

$$\left(\frac{2\,(3000-2000)\,10^3}{\left(\frac{3000\cdot 10^3}{\frac{8314.46}{32}\,(25+273.15)}\right)\left(1-\left(\frac{0.4}{0.8}\right)^4\right)}\right)^{0.5} = 234.708$$

m/sec

Compressible vs. Incompressible Comparisons

Flow Conditions	Stagnation Pressure	Massflow Calculation	Mach Number	Flow Velocity
Compressible Station 1 (inlet)	*3041.89 kPa*	*85.97 g/sec*	*0.1408*	*46.28 m/sec*
Station 2 (throat)	-	-	*0.7878*	*247.43 m/sec*
Incompressible Station 1 (inlet)	*3000 kPa*	*108.51g/sec*	--	*58.68 m/sec*
Station 2 (throat)	-	-	--	*234.71 m/sec*

Quite a big! Difference!, Incompressible Equations Significantly Overestimate massflow ~ 23%