Homework 6, part 1

Consider Venturi Flow with a Gaseous Oxygen O_2 with Measured Static Pressures p_1, p_2

$$\begin{bmatrix} p_1 \\ D_1 \\ T_0 \end{bmatrix} = \begin{bmatrix} 80\ kPa \\ 1\ cm \\ 300\ {}^oK \end{bmatrix} \rightarrow \begin{bmatrix} p_2 \\ D_2 \end{bmatrix} = \begin{bmatrix} 60 kPa \\ 0.5\ cm \end{bmatrix}$$

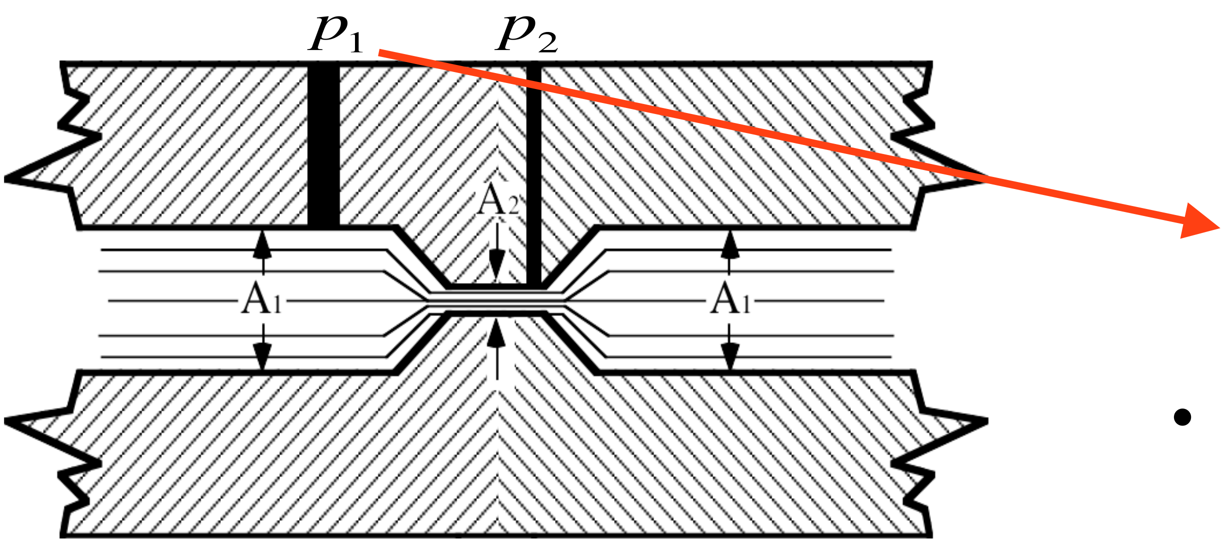
$$A = \frac{\pi}{4} D^2$$

- ***Part 2: Assuming Isentropic Flow .. Calculate the Massflow and Flow Velocity Through Venturi at stations (1) and (2)***

$\gamma = 1.4$, ***Mw = 32.0 kg/kg-mol***

Homework 6, part 1

Compare results of compressible Venturi calculations to values calculated using incompressible Venturi Equation Massflow, V_1, V_2



- *Assume $T_1 = T_0 =$ 300 K Base incompressible Density on p_1 from previous calculation*
- *Use Cd=1 for calculations*

$$V_2 = \sqrt{\frac{2\cdot(p_1 - p_2)}{\rho\cdot\left(1-\left(\frac{A_2}{A_1}\right)^2\right)}} \rightarrow \dot{m} = A_2 \cdot \sqrt{\frac{2\cdot\rho\cdot(p_1 - p_2)}{\left(1-\left(\frac{A_2}{A_1}\right)^2\right)}}$$

Incompressible Venturi Equations

Homework 6, part 2

- Show That for supersonic free stream conditions

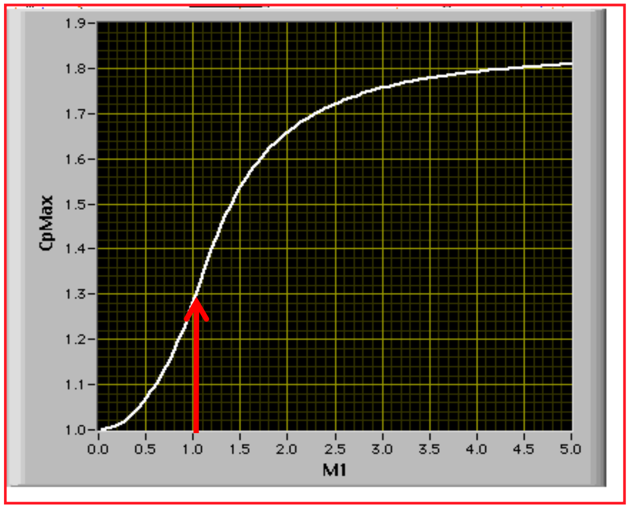
$$C_{P_{\max}} = \frac{P_{0_2} - p_\infty}{\left(\frac{1}{2}\cdot\rho_\infty\cdot V_\infty^{\ 2}\right)} = \frac{1}{\frac{\gamma}{2}M_\infty^{\ 2}}\left\{\frac{\left(\frac{\gamma+1}{2}M_\infty^{\ 2}\right)^{\frac{\gamma}{\gamma-1}}}{\left(\frac{2\gamma}{\gamma+1}M_\infty^{\ 2}-\frac{\gamma-1}{\gamma+1}\right)^{\frac{1}{\gamma-1}}}-1\right\}$$

- Prove that when …. *M→0* …. *Cp_{Max} → 1.0*

  Careful here … subsonic flow when M=0!

Homework 6, part 2

- Verify that Cp_{Max} is a continuous curve at *Mach 1*.



i.e. .. Show that When … $M_\infty \rightarrow 1$

$$\lim_{M_\infty \to 1} \left(C_{P_{\max}}\right)_{supersonic} = \frac{1}{\frac{\gamma}{2} M_\infty^2} \left\{ \frac{\left(\frac{\gamma+1}{2} M_\infty^2\right)^{\frac{\gamma}{\gamma-1}}}{\left(\frac{2\gamma}{\gamma+1} M_\infty^2 - \frac{\gamma-1}{\gamma+1}\right)^{\frac{1}{\gamma-1}}} - 1 \right\} =$$

$$\lim_{M_\infty \to 1} \left(C_{P_{\max}}\right)_{subsonic} = \left[\frac{\left(1 + \frac{\gamma-1}{2} M_\infty^2\right)^{\frac{\gamma}{\gamma-1}} - 1}{\frac{\gamma}{2} M_\infty^2} \right] = \frac{2}{\gamma}\left[\left(\frac{\gamma+1}{2}\right)^{\frac{\gamma}{\gamma-1}} - 1\right]$$

Homework 6: Part 3

- Write iterative Solver for "Rayleigh Pitot Equation" .. Given pressure ratio …. Calculate Mach number

$$\frac{P_{0_2}}{p_\infty} = \left\{ \frac{\left(\frac{\gamma+1}{2} M_\infty{}^2\right)^{\left(\frac{\gamma}{\gamma-1}\right)}}{\left(\frac{2\gamma}{(\gamma+1)} M_\infty{}^2 - \frac{(\gamma-1)}{(\gamma+1)}\right)^{\left(\frac{1}{\gamma-1}\right)}} \right\}$$

Supersonic Flow

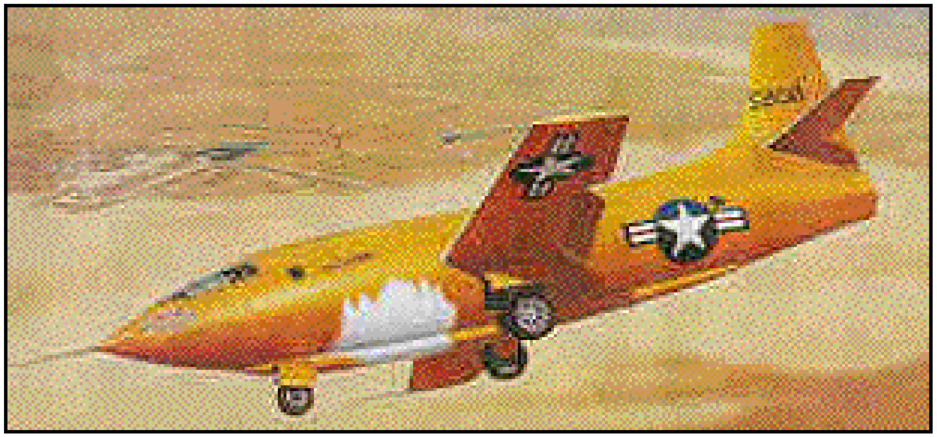
$$\frac{P_0}{p_\infty} = \left(1 + \frac{\gamma-1}{2} M_\infty{}^2\right)^{\frac{\gamma}{\gamma-1}}$$

Supersonic Flow

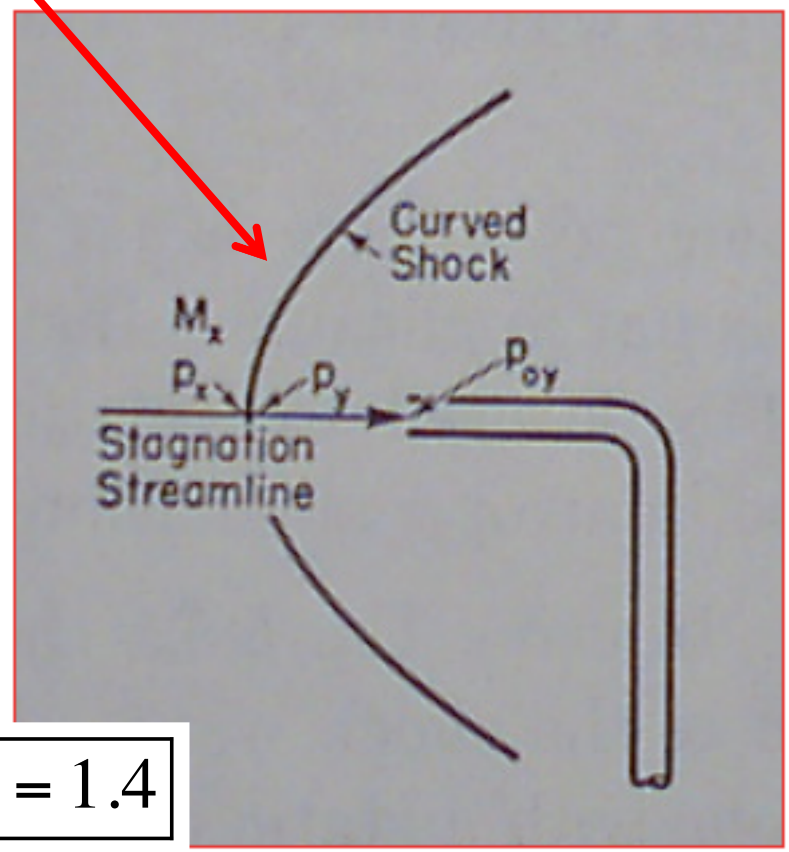
Carefully comment your code .. Hand in code with assignment …. Make program "smart enough" to solve for both supersonic and subsonic free stream conditions

Homework 6: Part 3

- Consider a Pitot / Static Probe on X-1 experimental rocket plane



- Probe measures *free stream static pressure* and *local impact pressure* .. *May or MAY NOT! Be shockwave in front of probe*



- Use Rayleigh-Pitot Code to compute free stream Mach number for following conditions

$a)P_{0_y} = 1.22 \times 10^5\ Nt/m^2 \rightarrow p_x = 1.01 \times 10^5\ Nt/m^2$

$b)P_{0_y} = 7222\ lbf/ft^2 \rightarrow p_x = 2116\ lbf/ft^2$

$c)P_{0_y} = 13107\ lbf/ft^2 \rightarrow p_x = 1020\ lbf/ft^2$

$assume...\gamma = 1.4$

Homework 6: Part 3

- Key Piece of Knowledge

$$\frac{\partial}{\partial M}\left[\frac{\left(\frac{\gamma+1}{2}M^2\right)^{\left(\frac{\gamma}{\gamma-1}\right)}}{\left(\frac{2\gamma}{(\gamma+1)}M^2-\frac{(\gamma-1)}{(\gamma+1)}\right)^{\left(\frac{1}{\gamma-1}\right)}}\right]=\frac{\gamma M\left(2M^2-1\right)\left[M^2\frac{(\gamma+1)}{2}\right]^{\left(\frac{1}{\gamma-1}\right)}}{\left(\frac{2\gamma}{(\gamma+1)}M^2-\frac{\gamma-1}{(\gamma+1)}\right)^{\left(\frac{\gamma}{\gamma-1}\right)}}$$