UtahState UNIVERSITY

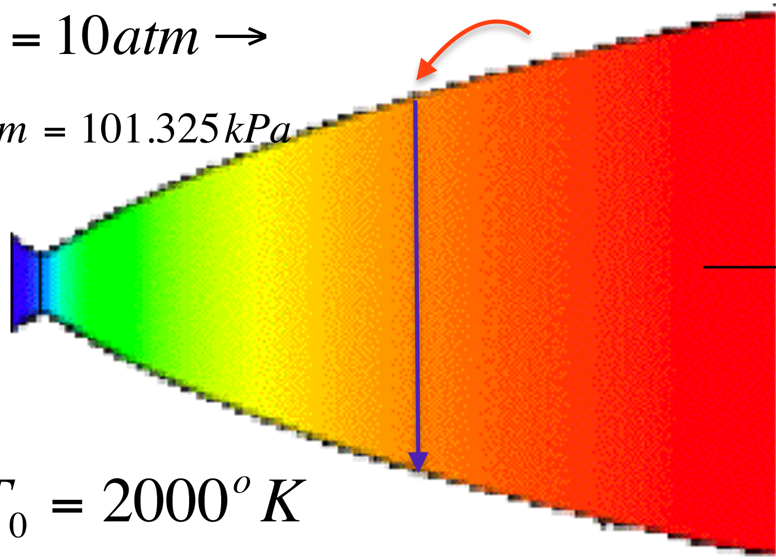
Mechanical & Aerospace Engineering

Project 1 (counts as double homework)

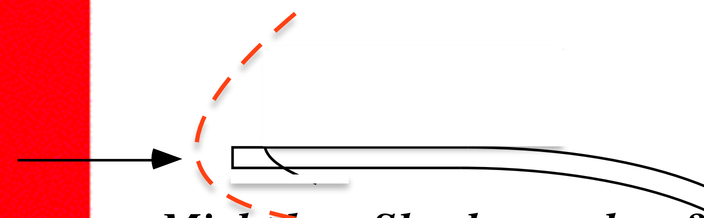
Due Wednesday October 13, 2021

$P_0 = 10 atm \rightarrow$

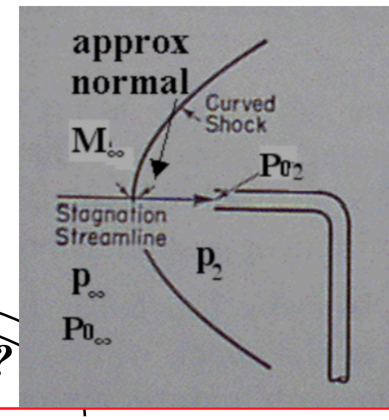
$1 atm = 101.325 kPa$



$P0_2 = 0.5 atm$



Might be a Shock wave here?



$T_0 = 2000^o K$

$A_{throat} = 0.30 m^2$

$A_{exit} = 23.256 m^2$

- Assume γ =1.2, MW = 22

Is nozzle isentropic?

- Calculate

i) *Exit mach,* M_e

ii) *Exit pressure ,*p_e

iii) *Exit Temperature,* T_e

iv) *Mass Flow Through Nozzle, dm/dt*

v) *Thrust of Nozzle @ sea level, & vacuum*

vi) *Compare Non-Ideal Thrust to Thrust of isentropic nozzle @ sea level, & vacuum*

Hmmmmm ! What is happening here?

Hint:

- Compute exit mach number for isentropic nozzle

- Employ normal shock wave equations to determine if there is a shock wave standing in front of Pitot tube

- If nozzle is non isentropic …
You'll have to write a solver for (or use trial and error)

$$\frac{P_{02}}{P_{01}} = \frac{2}{(\gamma+1)\left(\gamma M_1^2 - \frac{(\gamma-1)}{2}\right)^{\frac{1}{\gamma-1}}} \left(\frac{\left[\frac{(\gamma+1)}{2} M_1\right]^2}{\left(1+\frac{\gamma-1}{2} M_1^2\right)} \right)^{\left(\frac{\gamma}{\gamma-1}\right)}$$

… and

$$P_0 \neq constant, A^* \neq constant \rightarrow \left\{P_0 \cdot A^*\right\} = constant$$

Project 1 Summary *(show calculations)*

Operating Condition	Sea Level	Vacuum
Non-Isentropic Nozzle (with Embedded Shock Wave)	Massflow: Shock Position: Mach Before Shock: Exit Mach Number: Exit Pressure: Momentum Thrust: Total Thrust; Isp:	Massflow: Shock Position: Mach Before Shock: Exit Mach Number: Exit Pressure: Momentum Thrust: Total Thrust; Isp:
Isentropic Nozzle (with Idealized Flow, Constant A*)	Massflow: Exit Mach Number: Exit Pressure: Momentum Thrust: Total Thrust; Isp:	Massflow: Exit Mach Number: Exit Pressure: Momentum Thrust: Total Thrust; Isp:

What do These Comparison Tell you?

Solution Procedure Hints

$\rightarrow i)$ *Show that given* P_{0_e} / P_{0_1} *is inconsistent with isentropic nozzle*

$$\rightarrow ii)\ \frac{P_{0_e}}{P_{0_1}} = g\left(M_1\right) \rightarrow \textit{Solve for mach number ahead of shockwave}$$

$$\rightarrow iii)\ \frac{A_1}{A_1^*} = f\left(M_1\right) \rightarrow \textit{Solve for } \frac{A_1}{A_1^*},\ A_1 \textit{ ahead of shockwave}$$

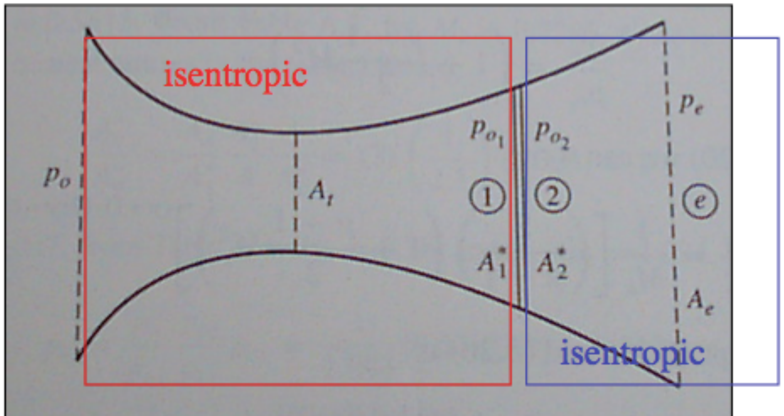
$$\rightarrow iv)\ P_0 A^* = constant \rightarrow \textit{Solve for } A_2^*$$

$$\rightarrow v)\ \frac{A_e}{A_2^*} = f(M_e) \rightarrow \textit{Solve for} \text{ exit plane } \textit{mach number}, p_e, T_e, V_e$$

$\rightarrow$ *calculate performance and compare to isentropic nozzle*

$\rightarrow$ *Hmmmmm, what's happening here?*

Non-Isentropic Nozzle Solve for Mach Number Ahead of Shock Wave

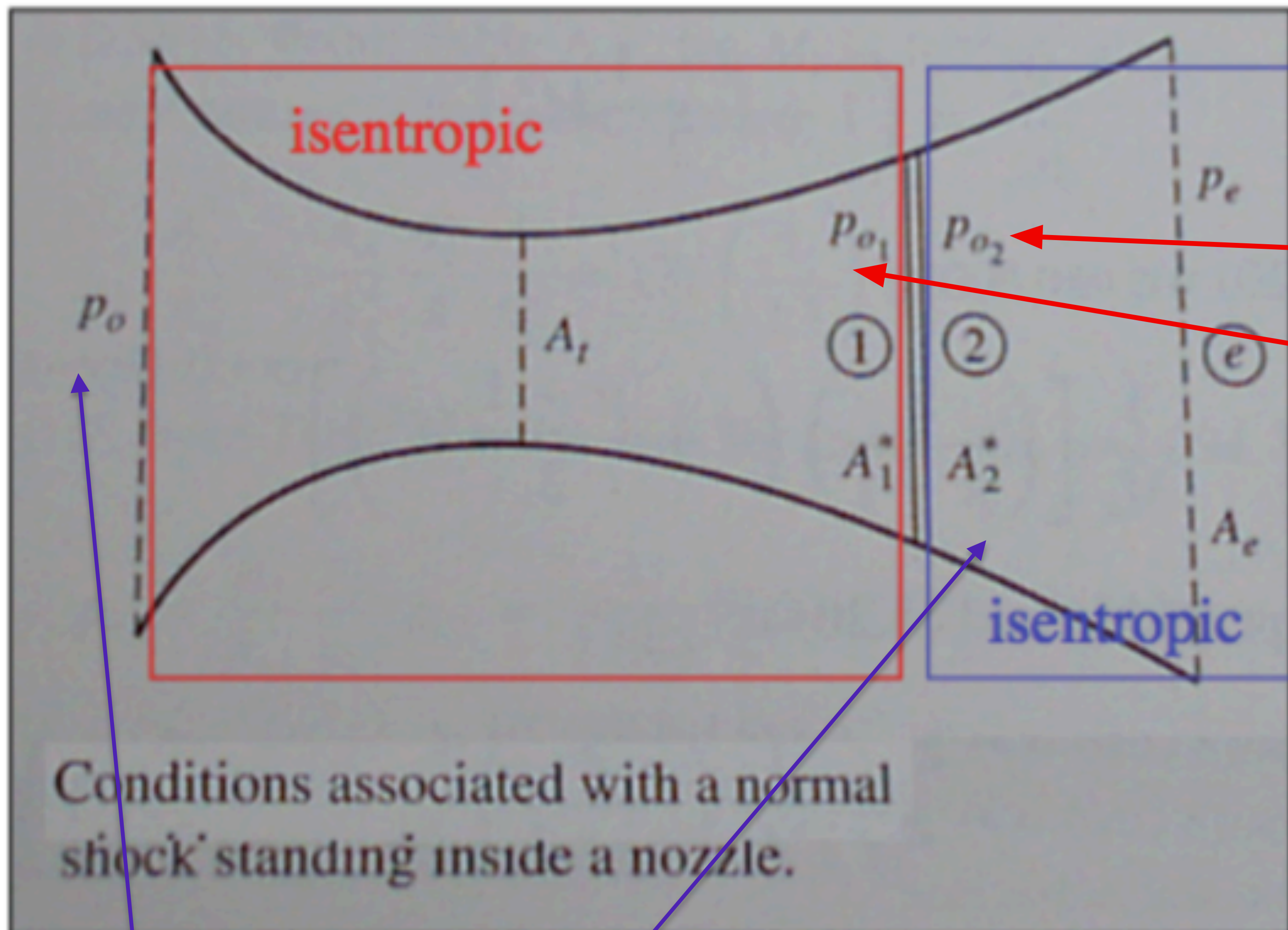


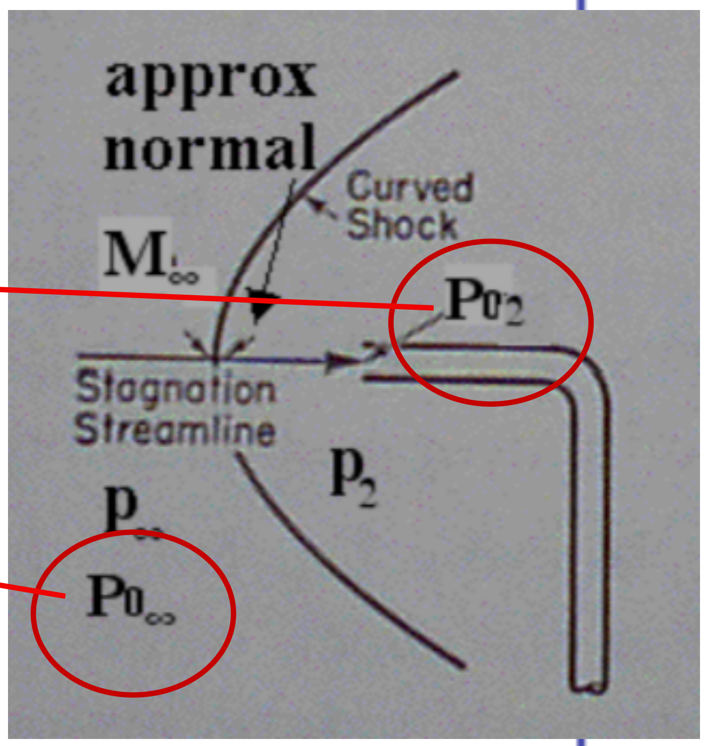
Conditions associated with a normal shock standing inside a nozzle.

- REQUIRES ITERATIVE NUMERICAL SOLVER
- USE Newton's method

$$\frac{P_{02}}{P_{01}} = \frac{2}{(\gamma+1)\left(\gamma M_1^2 - \frac{(\gamma-1)}{2}\right)}^{\frac{1}{\gamma-1}} \left(\frac{\left[\frac{(\gamma+1)}{2} M_1\right]^2}{\left(1+\frac{\gamma-1}{2} M_1^2\right)} \right)^{\left(\frac{\gamma}{\gamma-1}\right)}$$

Non-Isentropic Nozzle Solve for Mach Number Ahead of Shock Wave





- USE Newton's method

For Numerical Solution

$$\frac{P_{02}}{P_{01}} = \frac{2}{(\gamma+1)\left(\gamma M_1^2 - \frac{(\gamma-1)}{2}\right)^{\frac{1}{\gamma-1}}} \left(\frac{\left[\frac{(\gamma+1)}{2} M_1\right]^2}{\left(1+\frac{\gamma-1}{2} M_1^2\right)} \right)^{\left(\frac{\gamma}{\gamma-1}\right)}$$

Non-Isentropic Nozzle Solve for Mach Number Ahead of Shock Wave (concluded)

$$M_{1(j+1)} = M_{1(j)} - \frac{G(M_{1(j)})}{\left(\frac{\partial G}{\partial M_1}\right)_{(j)}} \rightarrow$$

$$G(M_{1(j)}) = \frac{2}{(\gamma+1)\left(\gamma M_{1(j)}^{2} - \frac{(\gamma-1)}{2}\right)^{\frac{1}{\gamma-1}}} \left(\frac{\left[\frac{(\gamma+1)}{2} M_{1(j)}\right]^{2}}{\left(1+\frac{\gamma-1}{2} M_{1(j)}^{2}\right)} \right)^{\left(\frac{\gamma}{\gamma-1}\right)} - \left(\frac{P_{0_2}}{P_{0_1}}\right)$$

$$\left(\frac{\partial G}{\partial M_1}\right)_{(j)} = -\frac{2^{\left(3-\frac{2\gamma}{\gamma-1}\right)} \gamma \left(M_{1(j)}^{2} - 1\right)^{2} \left(\frac{\left[(\gamma+1) M_{1(j)}\right]^{2}}{\left(1+\frac{\gamma-1}{2} M^{2}\right)} \right)^{\left(\frac{\gamma}{\gamma-1}\right)} \left[\frac{1}{2} + \gamma\left(M_{1(j)}^{2} - \frac{1}{2}\right)\right]^{\left(\frac{-1}{\gamma-1}\right)}}{(\gamma+1) M_{1(j)} \left(2 + M_{1(j)}^{2}(\gamma-1)\right)\left[1+\gamma\left(2M_{1(j)}^{2}-1\right)\right]}$$

- Similar to - M(A/A*) - algorithm … given starting mach number Iterate to convergence