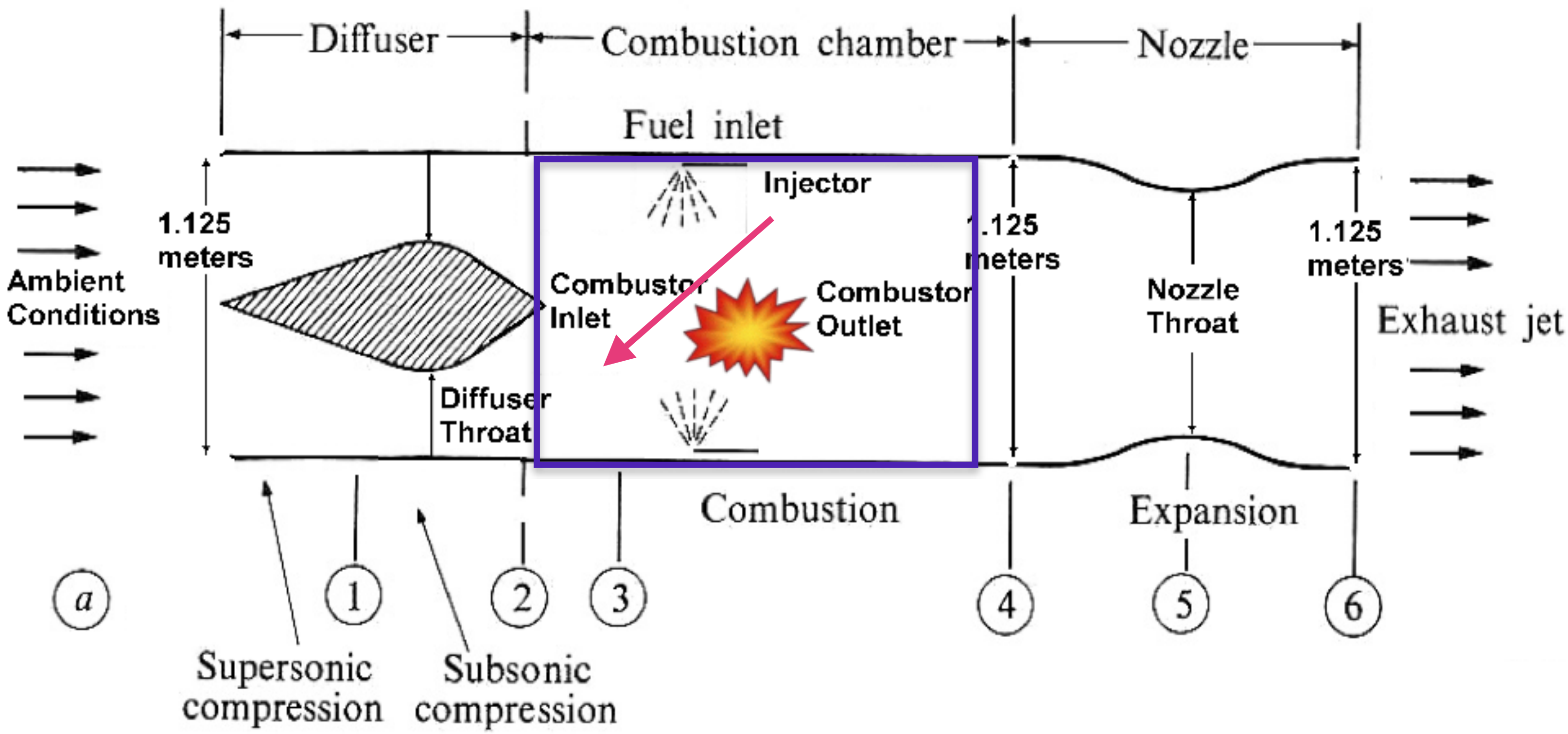
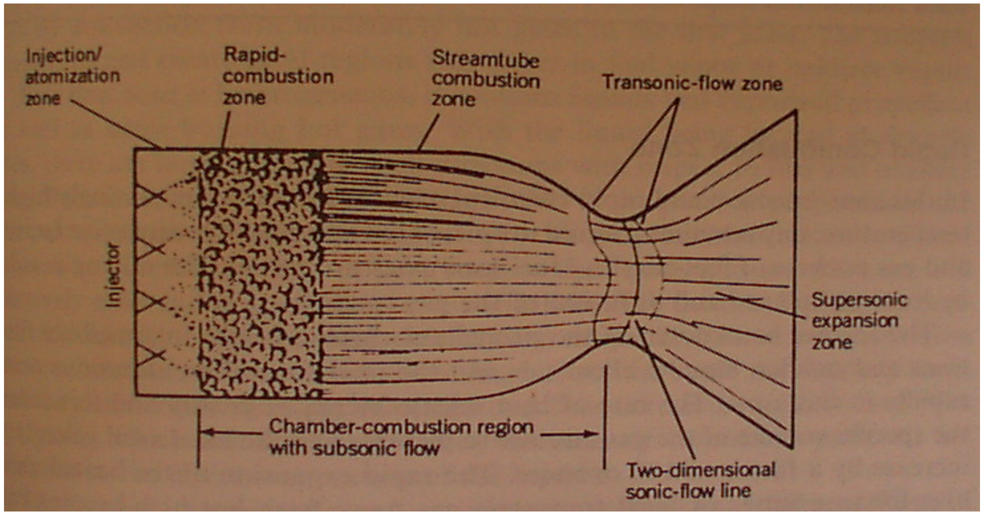
Non-Adiabatic, Quasi-1-D Flow



Anderson, Pages 102-111

Chamber pressure

- Why can we assume that the Rocket Chamber pressure is Approximately stagnation pressure in an isentropic Nozzle?



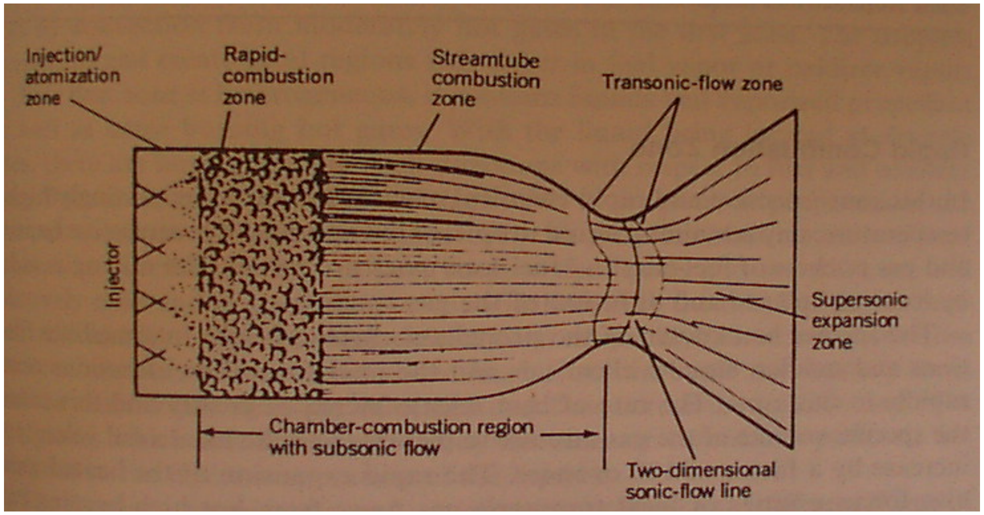
- Less than 0.6% error In Assumption

- Combustion Velocity is initially in all directions .. .Little net axial velocity ~ $M_{chamber}$ ~0.1 …

$$\frac{P_{chamber}}{P_{0_{chamber}}} \approx \frac{1}{\left[1+\frac{1.2-1}{2}0.1^2\right]^{\frac{1.2}{1.2-1}}} = 0.994 \rightarrow$$

Throat pressure

- As long as nozzle is isentropic then stagnation pressure Remains constant …



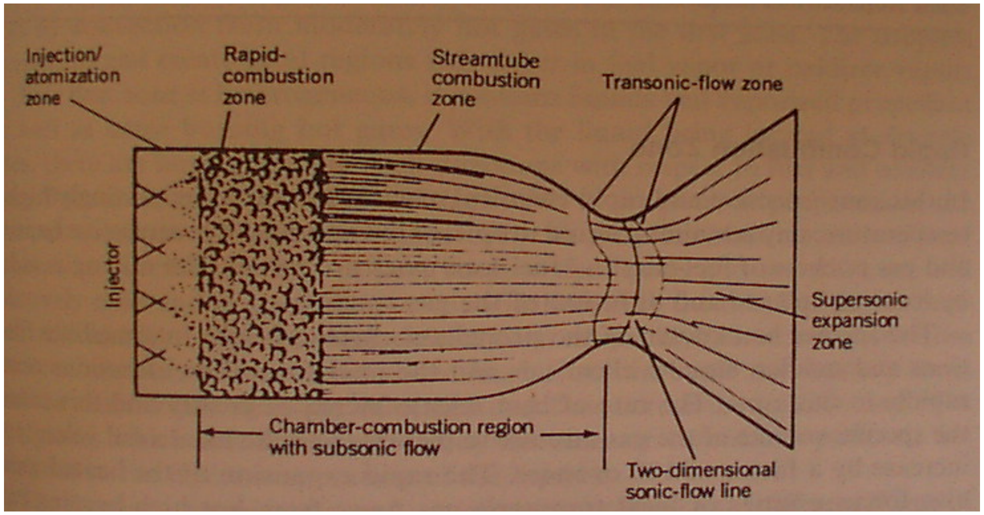
- *Stagnation pressure Loss is a measure of Process irreversibility*

- Compare ratio of static and stagnation pressure at Throat

$$\frac{P_{throat}}{P_{0_{chamber}}} \approx \frac{1}{\left[1+\frac{1.2-1}{2}1.0^2\right]^{\frac{1.2}{1.2-1}}} = 0.5645 \rightarrow$$

Chamber temperature

• Combustion Flame temperature Temperature of Endothermic reaction of propellants



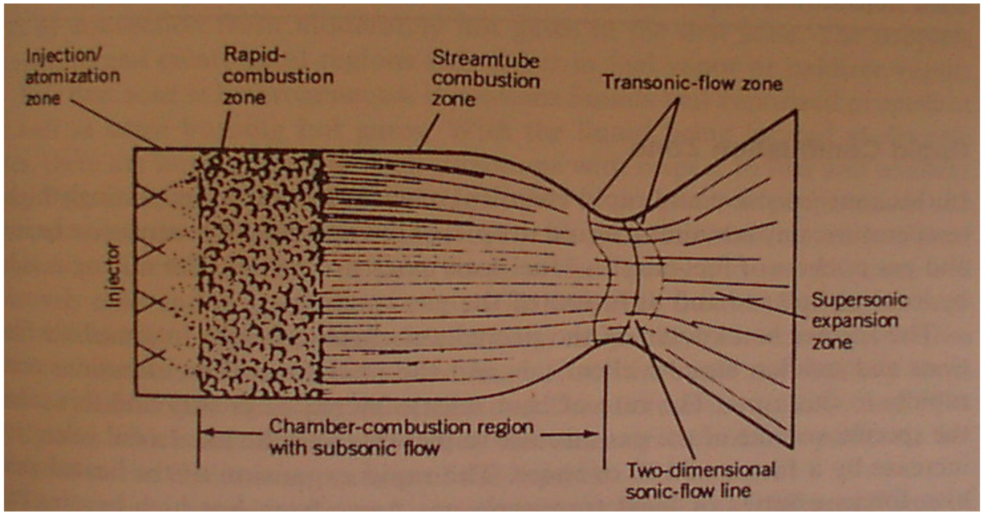
• Less than 0.01% error In Assumption

• Combustion Velocity is initially in all directions .. .Little net axial velocity ~ $M_{chamber}$ ~0.1 …

$$\frac{T_{flame}}{T_{0_{chamber}}} \approx \frac{1}{\left[1 + \frac{1.2-1}{2} 0.1^2\right]} = 0.9990 \rightarrow$$

Throat temperature

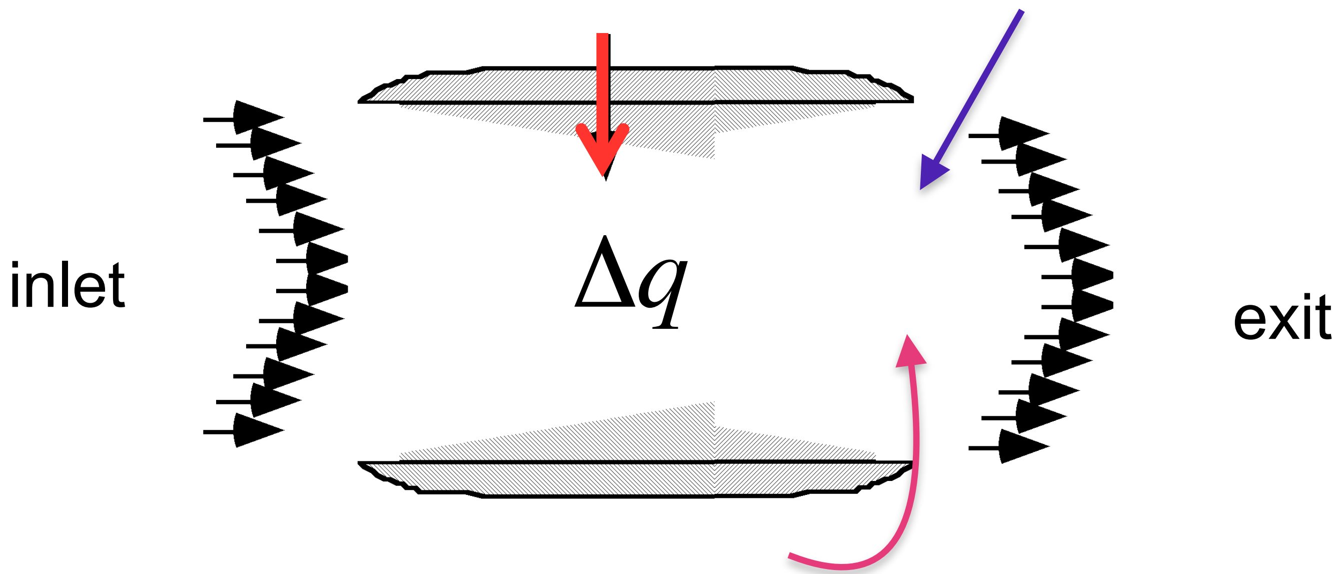
- ***As long as nozzle is adiabatic then stagnation pressure Remains constant …***



- Compare ratio of static and stagnation temperature at Throat

$$\frac{T_{throat}}{T_{0_{chamber}}} \approx \frac{1}{\left[1 + \frac{1.2 - 1}{2} 1.0^2\right]} = 0.9091 \rightarrow$$

What happens in Non-adiabatic flow? (1)



- From Conservation of Energy

$$\Delta q + h_i + \frac{V_i^2}{2} = h_e + \frac{V_e^2}{2} \rightarrow$$

$$\frac{\dot{Q}}{\dot{m}} \equiv \Delta q$$

$$\Delta q \sim \frac{Watts}{kg / \sec} = \frac{J}{kg}$$

What happens in Non-adiabatic flow? (2)

- For Calorically perfect gas $h = c_p T$

$$\Delta q + h_i + \frac{V_i^2}{2} = h_e + \frac{V_e^2}{2} \rightarrow \Delta q + C_p T_i + \frac{V_i^2}{2} = C_p T_e + \frac{V_e^2}{2} \rightarrow$$

$$\Delta q + C_p T_i \left[1 + \frac{V_i^2}{2 C_p T_i}\right] = C_p T_e \left[1 + \frac{V_e^2}{2 C_p T_e}\right] \rightarrow$$

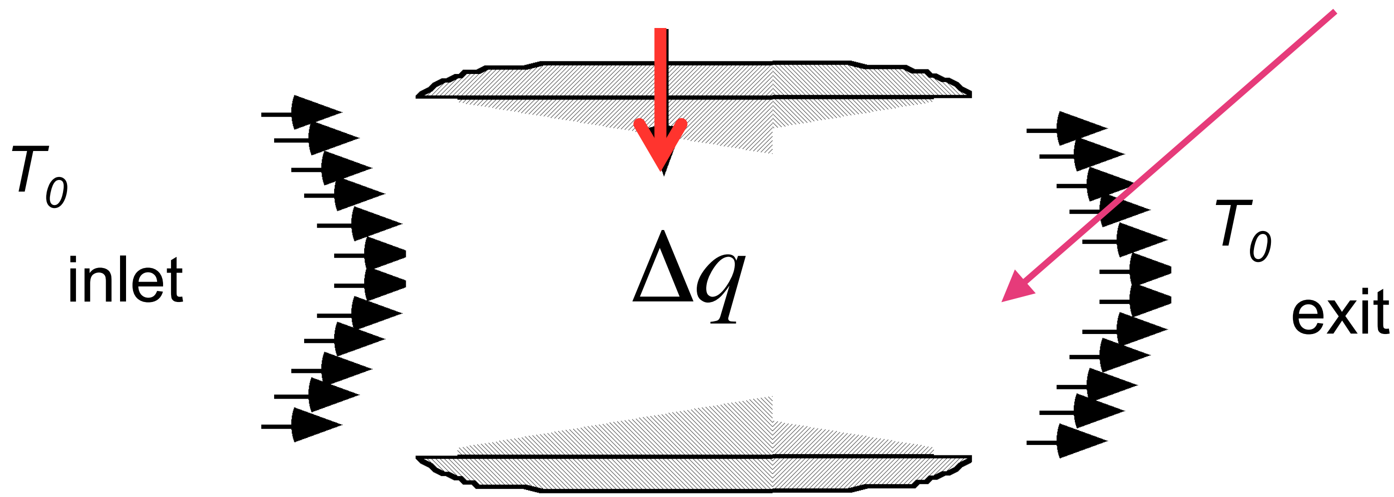
Cp = g*Rg/(g-1)

$$\Delta q + C_p T_i \left[1 + \frac{\gamma - 1}{2} \frac{V_i^2}{\gamma R_g T}\right] = C_p T_e \left[1 + \frac{\gamma - 1}{2} \frac{V_e^2}{\gamma R_g T}\right]$$

$$\Delta q + C_p T_i \left[1 + \frac{\gamma - 1}{2} M_i^2\right] = C_p T_e \left[1 + \frac{\gamma - 1}{2} M_e^2\right] \rightarrow$$

$$\boxed{\Delta q = C_p \left[T_{0_{exit}} - T_{0inlet}\right]}$$

What happens in Non-adiabatic flow? (2)



- heat addition boosts stagnation temperature!

$$T_{0_{exit}} = T_{0_{inlet}} + \frac{1}{c_p}\left(\Delta q\right)$$

What about the rest of the story? (1)

- Quasi 1-D continuity equation

$$\rho_{inlet} V_{inlet} = \rho_{exit} V_{exit} \rightarrow \frac{\rho_{inlet}}{\rho_{exit}} = \frac{V_{exit}}{V_{inlet}} \rightarrow \boxed{\frac{p_{inlet}}{p_{exit}} \frac{T_{exit}}{T_{inlet}} = \frac{V_{exit}}{V_{inlet}}}$$

- Quasi 1-D momentum equation

$$\boxed{\frac{p_{exit}}{p_{inlet}} = \frac{\left[1 + \gamma M_{inlet}^{\;2}\right]}{\left[1 + \gamma M_{exit}^{\;2}\right]}} \downarrow$$

- Plug Continuity into Momentum eqn.

$$\frac{p_{exit}}{p_{inlet}} = \frac{\left[1 + \gamma M_{inlet}^{\;2}\right]}{\left[1 + \gamma M_{exit}^{\;2}\right]} = \frac{T_{exit}}{T_{inlet}} \frac{V_{inlet}}{V_{exit}}$$

What about the rest of the story? (2)

- Rearranging …

$$\frac{T_{exit}}{T_{inlet}} = \frac{V_{exit}}{V_{inlet}} \frac{\left[1+\gamma M_{inlet}^{2}\right]}{\left[1+\gamma M_{exit}^{2}\right]} = \sqrt{\frac{T_{exit}}{T_{inlet}}} \left[\frac{V_{exit} / \sqrt{\gamma R_g T_{exit}}}{V_{inlet} / \sqrt{\gamma R_g T_{inlet}}}\right] \frac{\left[1+\gamma M_{inlet}^{2}\right]}{\left[1+\gamma M_{exit}^{2}\right]} \rightarrow$$

$$\sqrt{\frac{T_{exit}}{T_{inlet}}} = \left[\frac{M_{exit}}{M_{inlet}}\right] \frac{\left[1+\gamma M_{inlet}^{2}\right]}{\left[1+\gamma M_{exit}^{2}\right]} \rightarrow \boxed{\frac{T_{exit}}{T_{inlet}} = \left(\left[\frac{M_{exit}}{M_{inlet}}\right] \frac{\left[1+\gamma M_{inlet}^{2}\right]}{\left[1+\gamma M_{exit}^{2}\right]}\right)^2}$$

What about the rest of the story? (3)

- Since … $\frac{\rho_{exit}}{\rho_{inlet}} = \frac{p_{exit}}{p_{inlet}} \frac{T_{inlet}}{T_{exit}}$

$$\frac{\rho_{exit}}{\rho_{inlet}} = \frac{p_{exit}}{p_{inlet}} \frac{T_{inlet}}{T_{exit}} = \frac{\left[1+\gamma M_{inlet}^{\ 2}\right]}{\left[1+\gamma M_{exit}^{\ 2}\right]} \left(\left[\frac{M_{inlet}}{M_{exit}}\right] \frac{\left[1+\gamma M_{exit}^{\ 2}\right]}{\left[1+\gamma M_{inlet}^{\ 2}\right]} \right)^2 \rightarrow$$

$$\boxed{\frac{\rho_{exit}}{\rho_{inlet}} = \left[\frac{M_{inlet}}{M_{exit}}\right]^2 \frac{\left[1+\gamma M_{exit}^{\ 2}\right]}{\left[1+\gamma M_{inlet}^{\ 2}\right]}}$$

What about the rest of the story? (3)

- also Since …

$$\frac{p_{exit}}{p_{inlet}} = \frac{\left[1+\gamma M_{inlet}^{2}\right]}{\left[1+\gamma M_{exit}^{2}\right]} \downarrow$$

- assuming flow is isentropic in either side of the heat addition …

$$\frac{P_{0_{exit}}}{P_{0_{inlet}}} = \frac{p_{exit}}{p_{inlet}} \frac{\left[1+\frac{\gamma-1}{2}M_{exit}^{2}\right]^{\frac{\gamma}{\gamma-1}}}{\left[1+\frac{\gamma-1}{2}M_{inlet}^{2}\right]^{\frac{\gamma}{\gamma-1}}} \Downarrow$$

$$\boxed{\frac{P_{0_{exit}}}{P_{0_{inlet}}} = \frac{\left[1+\gamma M_{inlet}^{2}\right]}{\left[1+\gamma M_{exit}^{2}\right]} \left[\frac{1+\frac{\gamma-1}{2}M_{exit}^{2}}{1+\frac{\gamma-1}{2}M_{inlet}^{2}}\right]^{\frac{\gamma}{\gamma-1}}}$$

What about the rest of the story? (4)

- Similarly for stagnation temperature

$$\frac{T_{exit}}{T_{inlet}} = \frac{T_{0_{exit}}\left[1+\frac{\gamma-1}{2}M_{inlet}^{\;2}\right]}{T_{0_{inlet}}\left[1+\frac{\gamma-1}{2}M_{exit}^{\;2}\right]}$$

$$\left(\left[\frac{M_{exit}}{M_{inlet}}\right]\frac{\left[1+\gamma M_{inlet}^{\;2}\right]}{\left[1+\gamma M_{exit}^{\;2}\right]}\right)^2 = \frac{T_{0_{exit}}\left[1+\frac{\gamma-1}{2}M_{inlet}^{\;2}\right]}{T_{0_{inlet}}\left[1+\frac{\gamma-1}{2}M_{exit}^{\;2}\right]}$$

- and from earlier

$$\frac{T_{exit}}{T_{inlet}} = \left(\left[\frac{M_{exit}}{M_{inlet}}\right]\frac{\left[1+\gamma M_{inlet}^{\;2}\right]}{\left[1+\gamma M_{exit}^{\;2}\right]}\right)^2$$

$$\frac{M_{exit}^{\;2}\left[1+\frac{\gamma-1}{2}M_{exit}^{\;2}\right]}{\left[1+\gamma M_{exit}^{\;2}\right]^2} = \left[\frac{T_{0_{exit}}}{T_{0_{inlet}}}\right]\frac{M_{inlet}^{\;2}\left[1+\frac{\gamma-1}{2}M_{inlet}^{\;2}\right]}{\left[1+\gamma M_{inlet}^{\;2}\right]^2}$$

Solve for M_{exit} (1)

$$\frac{M_{exit}^{2}\left[1+\frac{\gamma-1}{2}M_{exit}^{2}\right]}{\left[1+\gamma M_{exit}^{2}\right]^{2}} = \left[\frac{T_{0_{exit}}}{T_{0_{inlet}}}\right]\frac{M_{inlet}^{2}\left[1+\frac{\gamma-1}{2}M_{inlet}^{2}\right]}{\left[1+\gamma M_{inlet}^{2}\right]^{2}}$$

- Let

$$F(M_{inlet}) = \left[\frac{T_{0_{exit}}}{T_{0_{inlet}}}\right]\frac{M_{inlet}^{2}\left[1+\frac{\gamma-1}{2}M_{inlet}^{2}\right]}{\left[1+\gamma M_{inlet}^{2}\right]^{2}}$$

Solve for M_{exit} (2)

- Regroup in terms of M_{exit}^2

$$\frac{M_{exit}^2\left[1+\frac{\gamma-1}{2}M_{exit}^2\right]}{\left[1+\gamma M_{exit}^2\right]^2}=\left[\frac{T_{0_{exit}}}{T_{0_{inlet}}}\right]\frac{M_1^2\left[1+\frac{\gamma-1}{2}M_{inlet}^2\right]}{\left[1+\gamma M_{inlet}^2\right]^2}\downarrow$$

$$M_{exit}^2\left[1+\frac{\gamma-1}{2}M_{exit}^2\right]-F(M_{inlet})\left[1+\gamma M_{exit}^2\right]^2=0$$

Solve for M_{exit} (3)

- Collect terms in powers of M_{exit}

$$\frac{\gamma-1}{2}M_{exit}^{4}+M_{exit}^{2}-F(M_{inlet})\left[1+\gamma M_{exit}^{2}\right]^{2}=0$$

$$\frac{\gamma-1}{2}M_{exit}^{4}+M_{exit}^{2}-F(M_{inlet})\left[1+2\gamma M_{exit}^{2}+\gamma^{2}M_{exit}^{4}\right]=0$$

$$\left[\frac{\gamma-1}{2}-\gamma^{2}F(M_{inlet})\right]M_{exit}^{4}+\left[1-F(M_{inlet})2\gamma\right]M_{exit}^{2}-F(M_{inlet})=0$$

- **Quartic Equation, … but quadratic in M_2^2**

Solve for M_{exit} (4)

$$\left[\frac{\gamma-1}{2}-\gamma^2 F(M_{inlet})\right]M_{exit}^{\ 4}+\left[1-F(M_{inlet})2\gamma\right]M_{exit}^{\ 2}-F(M_{inlet})=0$$

$\rightarrow$ *use quadratic formula*

$$M_{exit}=\sqrt{\frac{-\left[1-F(M_{inlet})2\gamma\right]\pm\sqrt{\left[1-F(M_{inlet})2\gamma\right]^2+4\left[\frac{\gamma-1}{2}-\gamma^2 F(M_{inlet})\right]F(M_{inlet})}}{2\left[\frac{\gamma-1}{2}-\gamma^2 F(M_{inlet})\right]}}$$

two solutions...pick subsonic solution if $(M_{inlet}<1,\ \Delta q>0)$

...pick supersonic solution if $(M_{inlet}>1,\ \Delta q>0)$

Change in Entropy

- Since … we have already shown

$$\frac{T_{0_{exit}}}{T_{0_{inlet}}} = \frac{2\cdot(\gamma+1)\cdot M_{exit}^{2}\cdot\left[1+\frac{\gamma-1}{2}M_{exit}^{2}\right]}{\left[1+\gamma M_{exit}^{2}\right]^{2}} \times \frac{\left[1+\gamma M_{inlet}^{2}\right]^{2}}{2\cdot(\gamma+1)\cdot M_{inlet}^{2}\cdot\left[1+\frac{\gamma-1}{2}M_{inlet}^{2}\right]}$$

$$\frac{P_{0_{exit}}}{P_{0_{inlet}}} = \frac{\left[1+\frac{\gamma-1}{2}M_{exit}^{2}\right]^{\frac{\gamma}{\gamma-1}}}{\left[1+\gamma M_{exit}^{2}\right]} \times \frac{\left[1+\gamma M_{inlet}^{2}\right]}{\left[1+\frac{\gamma-1}{2}M_{inlet}^{2}\right]^{\frac{\gamma}{\gamma-1}}}$$

- And Since … we have also already shown

$$s_{exit} - s_{inlet} = c_p \cdot \ln\left[\frac{T_{0_{exit}}}{T_{0_{inlet}}}\right] - R_g \cdot \ln\left[\frac{P_{0_{exit}}}{P_{0_{inlet}}}\right]$$

Remember for a normal shockwave?

$$s_2 - s_1 = -R_g \ln\left[\frac{P_{02}}{P_{01}}\right] \Rightarrow \frac{P_{02}}{P_{01}} = e^{-\left(\frac{s_2 - s_1}{R_g}\right)}$$

Change in Entropy (2)

- Collect Terms in Entropy Equation

$$\frac{s_{exit} - s_{inlet}}{c_p} = \left(\ln\left[\frac{T_{0_{exit}}}{T_{0_{inlet}}}\right] - \frac{R_g}{c_p} \cdot \ln\left[\frac{P_{0_{exit}}}{P_{0_{inlet}}}\right] \right) = \left(\ln\left[\frac{T_{0_{exit}}}{T_{0_{inlet}}}\right] - \frac{\gamma - 1}{\gamma} \cdot \ln\left[\frac{P_{0_{exit}}}{P_{0_{inlet}}}\right] \right) = \ln\left[\frac{\frac{T_{0_{exit}}}{T_{0_{inlet}}}}{\left(\frac{P_{0_{exit}}}{P_{0_{inlet}}}\right)^{\frac{\gamma-1}{\gamma}}}\right]$$

- Substitute Temperature and Pressure Ratios

$$\frac{s_{exit} - s_{inlet}}{c_p} = \ln\left[\frac{\frac{M_{exit}^2 \cdot \left[1 + \frac{\gamma - 1}{2} M_{exit}^2\right]}{\left[1 + \gamma M_{exit}^2\right]^2} \times \frac{\left[1 + \gamma M_{inlet}^2\right]^2}{M_{inlet}^2 \cdot \left[1 + \frac{\gamma - 1}{2} M_{inlet}^2\right]}}{\left(\frac{\left[1 + \frac{\gamma - 1}{2} M_{exit}^2\right]^{\frac{\gamma}{\gamma-1}}}{\left[1 + \gamma M_{exit}^2\right]} \times \frac{\left[1 + \gamma M_{inlet}^2\right]}{\left[1 + \frac{\gamma - 1}{2} M_{inlet}^2\right]^{\frac{\gamma}{\gamma-1}}}\right)^{\frac{\gamma-1}{\gamma}}}\right]$$

Change in Entropy (3)

- Collecting Terms Again

$$\frac{s_{exit}-s_{inlet}}{c_p}=\ln\left[\frac{\dfrac{M_{exit}^2\cdot\left[1+\frac{\gamma-1}{2}M_{exit}^2\right]}{\left[1+\gamma M_{exit}^2\right]^2}\times\dfrac{\left[1+\gamma M_{inlet}^2\right]^2}{M_{inlet}^2\cdot\left[1+\frac{\gamma-1}{2}M_{inlet}^2\right]}}{\left(\dfrac{\left[1+\frac{\gamma-1}{2}M_{exit}^2\right]^{\frac{\gamma}{\gamma-1}}}{\left[1+\gamma M_{exit}^2\right]}\times\dfrac{\left[1+\gamma M_{inlet}^2\right]}{\left[1+\frac{\gamma-1}{2}M_{inlet}^2\right]^{\frac{\gamma}{\gamma-1}}}\right)^{\frac{\gamma-1}{\gamma}}}\right]$$

$$=\ln\left(\frac{M_{exit}^2}{\left[1+\gamma M_{exit}^2\right]^{2-\frac{\gamma-1}{\gamma}}}\times\frac{\left[1+\gamma M_{inlet}^2\right]^{2-\frac{\gamma-1}{\gamma}}}{M_{inlet}^2}\right)=\ln\left(\left(\frac{M_{exit}}{M_{inlet}}\right)^2\left[\frac{1+\gamma M_{inlet}^2}{1+\gamma M_{exit}^2}\right]^{\frac{\gamma+1}{\gamma}}\right)$$

Equation Says a lot about what conditions result at combustor exit

Calculation Procedure ….. (1)

- Given T_{0inlet}, P_{0inlet}, γ, c_p, R_g, A_{exit}/A_*, $\left(\Delta q = \dfrac{\dot{q}}{\dot{m}}\right)$

- Compute $T_{0_{exit}} / T_{0_{inlet}}$ $$T_{0_{exit}} = T_{0_{inlet}} + \frac{1}{c_p}(\Delta q)$$

- Compute M_{exit} from quartic (be sure to chose correct solution !)

$$\left[\frac{\gamma - 1}{2} - \gamma^2 F(M_{inlet})\right] M_{exit}^4 + \left[1 - F(M_{inlet}) 2\gamma\right] M_{exit}^2 - F(M_{inlet}) = 0$$

$$F(M_{inlet}) = \left[\frac{T_{0exit}}{T_{0_{inlet}}}\right] \frac{M_{inlet}^2 \left[1 + \frac{\gamma - 1}{2} M_{inlet}^2\right]}{\left[1 + \gamma M_{inlet}^2\right]^2}$$

Calculation Procedure ….. (2)

- ***Calculation procedure:*** ***cont'd***

- 5) Compute T_{exit}

$$\frac{T_{exit}}{T_{inlet}} = \left(\left[\frac{M_{exit}}{M_{inlet}} \right] \frac{\left[1 + \gamma M_{inlet}^{\ 2}\right]}{\left[1 + \gamma M_{exit}^{\ 2}\right]} \right)^2$$

- 6) Compute p_{exit}

$$\frac{p_{exit}}{p_{inlet}} = \frac{\left[1 + \gamma M_{inlet}^{\ 2}\right]}{\left[1 + \gamma M_{exit}^{\ 2}\right]} \downarrow$$

- 7) Compute $P_{0\ exit}$

$$\frac{P_{0_{exit}}}{P_{0_{inlet}}} = \frac{\left[1 + \gamma M_{inlet}^{\ 2}\right]}{\left[1 + \gamma M_{exit}^{\ 2}\right]} \left[\frac{1 + \frac{\gamma - 1}{2} M_{exit}^{\ 2}}{1 + \frac{\gamma - 1}{2} M_{inlet}^{\ 2}} \right]^{\frac{\gamma}{\gamma - 1}}$$

Calculation Procedure ….. (2)

- ***Calculation procedure:*** *cont'd*

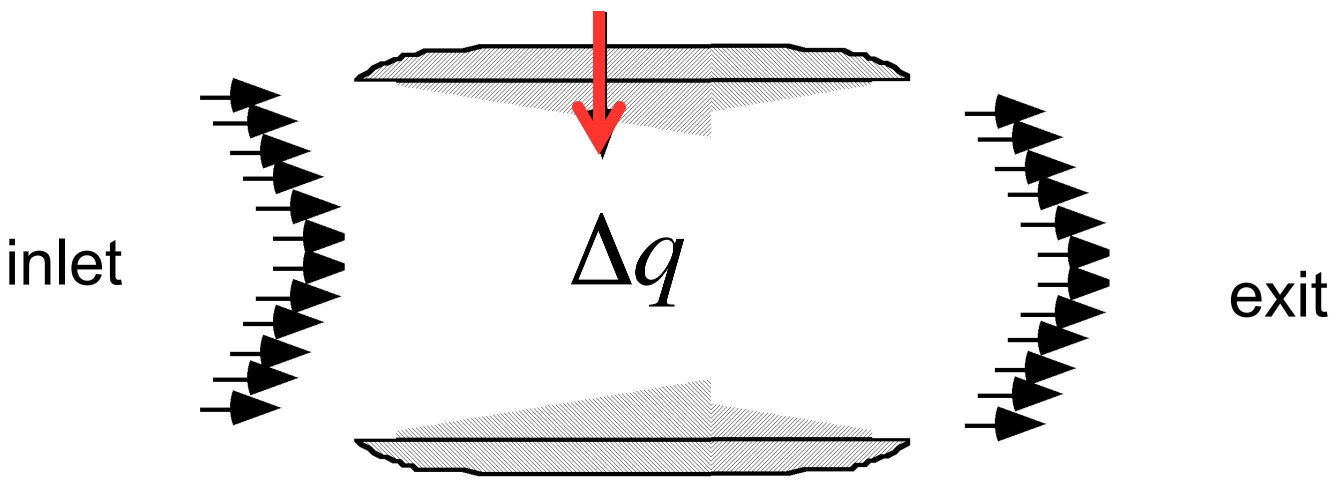
- 8) Compute V_{exit}

$$V_{exit} = M_{exit} \times \sqrt{\gamma R_g T_{exit}}$$

- 9) Compute non-adiabatic exit properties, … Thrust, Isp, etc

Example calculation (1)

- Ramjet Adds Fuel and Combustion adds Heat to Input Air …



$$Continuity \Rightarrow \rho_1 V_1 + \dot{m}_f / A = \rho_2 V_2$$

.. Assume fuel mass flow is negligible compared to air intake

… Constant 1 meter cross section diameter

Example calculation (1)

Incoming Air to Ramjet

• Molecular weight	=	28.96443 kg/kg-mole
• γ	=	1.40
• R_g	=	287.056 J/°K-(kg)
• T_∞	=	216.65 °K
• p_∞	=	19.330 kPa
• V_∞	=	118.03 m/sec
• R_u	=	8314.4126 J/°K-(kg-mol
• $\Delta q = \dot{Q}/\dot{m} =$	=	190 kWatt/(kg/sec)

- Engine flow path is constant 1 meter diameter cross section at *both* inlet and exit, assume constant flow chemistry

Example calculation (3)

i) Calculate Stagnation temperature ratio across engine

ii) Calculate Exit Mach Number (hint use above to solve for M_2)

iii) Calculate Exit Stagnation Pressure, static pressure and static temperature

iv) Calculate mass flow required to choke exit compare to actual exit mass flow

v) What heat load is required to choke exit

Inlet conditions

- Specific heat (c_p) $$c_p - c_v = R_g \Rightarrow c_p = \frac{\gamma}{\gamma - 1} R_g$$

$$c_p = \frac{1.4}{1.4 - 1} 287.056 = 1004.696 \text{ J/}^o\text{K-(kg)}$$

- Mach

$$M_{inlet} = \frac{V_{inlet}}{\sqrt{\gamma R_g T_{inlet}}} = \frac{118.03}{(1.4 \cdot 287.056 \cdot 216.65)^{0.5}} = 0.400$$

- Stagnation temperature

$$T_{0_{inlet}} = \left(T_{inlet} + \frac{V_{inlet}^{\,2}}{2 c_{p_{inlet}}} \right) = \left(216.65 + \frac{118.03^2}{2 \cdot 1004.696} \right) = 223.583\text{K}$$

Exit Stagnation Temperature, Temperature Ratio

- Solve for Stagnation Temperature at Engine Exit

$$T_{0_{exit}} = \frac{q + c_p\left(T_{0_{inlet}}\right)}{c_p} = \frac{(190 \cdot 10^3 + 1004.696\,(223.583))}{1004.696} = 412.695\ ^oK$$

$$\boxed{\frac{T_{0_{exit}}}{T_{0_{inlet}}} = 1.8458}$$

M_{exit} Solution

- Solve for Coefficients of equation first

$$\left[\frac{\gamma-1}{2}-\gamma^2 F(M_{inlet})\right]M_{exit}^{\ 4}+\left[1-F(M_{inlet})2\gamma\right]M_{exit}^{\ 2}-F(M_{inlet})=0$$

$$M_{inlet}=\frac{V_{inlet}}{\sqrt{\gamma R_g T_{inlet}}}=\frac{118.03}{(1.4\cdot 287.056\cdot 216.65)^{0.5}}=0.40$$

$$\frac{T_{0_{exit}}}{T_{0_{inlet}}}=1.8458$$

- Solve for Coefficients of equation first

Calculate for $F(M_{inlet})$

$$F(M_{inlet})=\left[\frac{T_{0exit}}{T_{0_{inlet}}}\right]\frac{M_{inlet}^{\ 2}\left[1+\frac{\gamma-1}{2}M_{inlet}^{\ 2}\right]}{\left[1+\gamma M_{inlet}^{\ 2}\right]^2}=1.8458\left(\frac{(0.4^2)\left(1+\frac{1.4-1}{2}0.4^2\right)}{(1+1.4\cdot 0.4^2)^2}\right)=0.20344$$

M_{exit} Solution (2)

- Solve for Coefficients of equation first

$$\left[\frac{\gamma - 1}{2} - \gamma^2 F(M_{inlet})\right] M_{exit}^4 + \left[1 - F(M_{inlet}) 2\gamma\right] M_{exit}^2 - F(M_{inlet}) = 0$$

$$F(M_{inlet}) = \left[\frac{T_{0exit}}{T_{0_{inlet}}}\right] \frac{M_{inlet}^2 \left[1 + \frac{\gamma - 1}{2} M_{inlet}^2\right]}{\left[1 + \gamma M_{inlet}^2\right]^2} = \ = 0.20344$$

$$\left[\frac{\gamma - 1}{2} - \gamma^2 F(M_{inlet})\right] = \quad \frac{1.4 - 1}{2} - 1.4^2 0.20344 \quad = -0.19874$$

$$\left[1 - F(M_{inlet}) 2\gamma\right] = \quad 1 - 0.20344 \cdot 2 \cdot 1.4 \quad = 0.430369$$

M_{exit} Solution (3)

- $$\left[\frac{\gamma-1}{2}-\gamma^2 F(M_{inlet})\right]M_{exit}^4+\left[1-F(M_{inlet})2\gamma\right]M_{exit}^2-F(M_{inlet})=0$$

$$-0.19874\,M_2^4+0.430369\,M_2^2-0.20344=0$$

$$M_{exit}=\left(\frac{-0.430369 \pm \left(0.430369^2-4\,(-0.20344)\,(-0.19874)\right)^{0.5}}{-2\cdot 0.19874}\right)^{0.5}$$

- Root 1 $M_{exit}=0.83495$
- Root 2 $M_{exit}=1.21175$

Which Root?

Stagnation pressure

- Calculate the Corresponding Stagnation Pressure ratios!!

$$\frac{P_{0_{exit}}}{P_{0_{inlet}}} = \frac{\left[1+\gamma M_{inlet}^2\right]}{\left[1+\gamma M_{exit}^2\right]}\left[\frac{1+\frac{\gamma-1}{2}M_{exit}^2}{1+\frac{\gamma-1}{2}M_{inlet}^2}\right]^{\frac{\gamma}{\gamma-1}}$$

- Root 1

$M_{exit}=0.83495$ $\quad P_{0_{exit}}/P_{0_{inlet}} = \frac{(1+1.4\cdot 0.4^2)}{(1+1.4(0.83495)^2)}\left(\frac{1+\frac{(1.4-1)}{2}(0.83495)^2}{1+\frac{1.4-1}{2}0.4^2}\right)^{\frac{1.4}{1.4-1}} = 0.8761$

- Root 2

$M_{exit}=1.21175$ $\quad P_{0_{exit}}/P_{0_{inlet}} = \frac{(1+1.4(0.4)^2)}{(1+1.4(1.21175)^2)}\left(\frac{1+\frac{(1.4-1)}{2}(1.21175)^2}{1+\frac{1.4-1}{2}(0.4)^2}\right)^{\frac{1.4}{1.4-1}} = 0.8834$

- Nature prefers the solution that gives the largest increase in entropy (stagnation pressure loss) ! …. **Root 1**

Check Change in entropy

$$s_{exit} - s_{inlet} = c_p \ln\left(\frac{T_{0_{exit}}}{T_{0_{inlet}}}\right) - R_g \ln\left(\frac{P_{0\,exit}}{P_{0\,inlet}}\right) =$$

- Root 1

M_{exit}=0.83495

$$1004.696 \ln(1.3954) - 287.056 \ln(0.8761)$$

= 372.76 J/kg-K

- Root 2

M_{exit}=1.21175

$$1004.696 \ln(1.3954) - 287.056 \ln(0.8834)$$

= 370.334 J/kg-K

From Anderson: Page 101

- Nature prefers the solution that gives the largest increase in entropy (*stagnation pressure loss*) ! …. **Root 1**

Choking Heat Load …

- what heat load chokes exit? (M_{exit} = 1)

$$\left[\frac{T0_{exit}}{T0_{inlet}}\right] = \frac{\dfrac{M_{exit}^{2}\left[1+\dfrac{\gamma-1}{2}M_{exit}^{2}\right]}{\left[1+\gamma M_{exit}^{2}\right]^{2}}}{\dfrac{M_{inlet}^{2}\left[1+\dfrac{\gamma-1}{2}M_{inlet}^{2}\right]}{\left[1+\gamma M_{inlet}^{2}\right]^{2}}} = \frac{\left(\dfrac{(1^2)\left(1+\dfrac{1.4-1}{2}1^2\right)}{(1+1.4\cdot 1^2)^2}\right)}{\left(\dfrac{(0.400^2)\left(1+\dfrac{1.4-1}{2}0.400^2\right)}{(1+1.4\cdot 0.400^2)^2}\right)} = 1.8903$$

Stagnation temperature at Choked exit

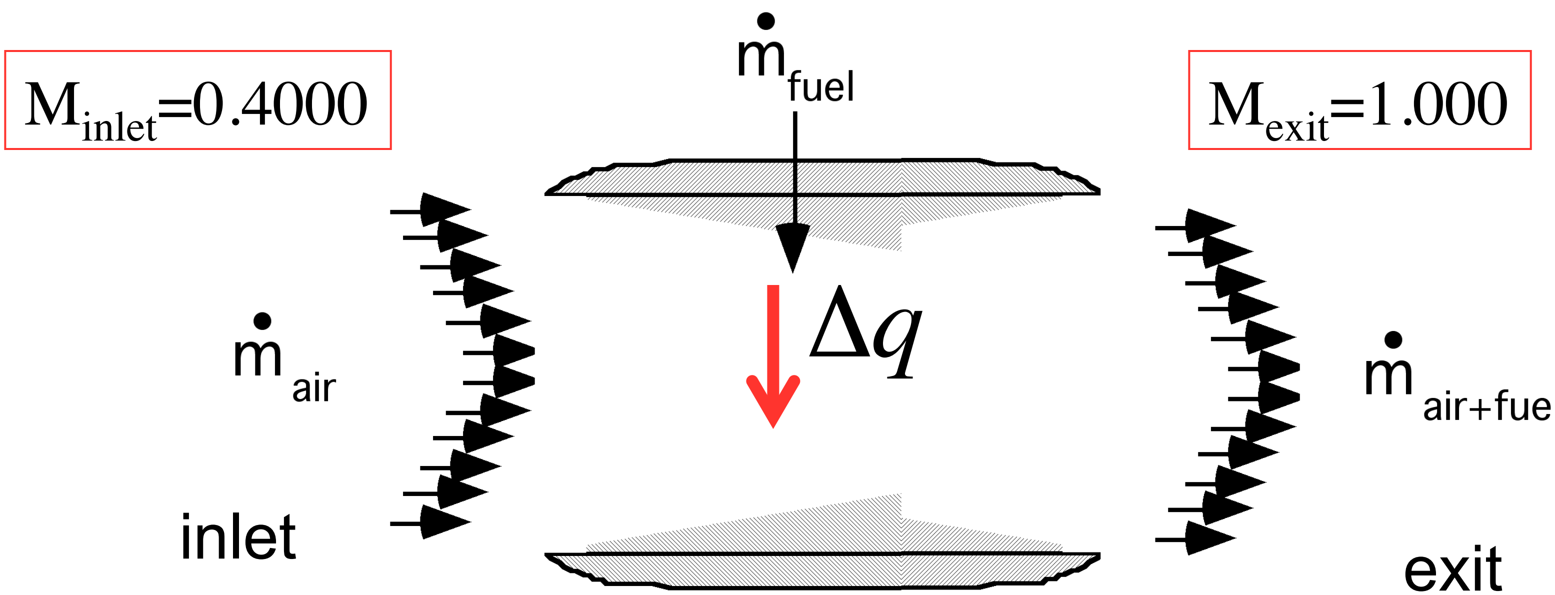
$$\left(T_{0_{exit}}\right)_{choked} = 1.8903\cdot 223.583 = 422.64^{o}K$$

Choking Heat Load … (2)

$$\Delta q = C_p\left[T_{0_{exit}} - T_{0\,inlet}\right] = \frac{1004.696}{1000}\left(1.89022\cdot 223.6 - 223.6\right)$$

$$= 199.975 \text{ kJ/kg}$$

Just a little more heat and we were choke!



What happens for Supersonic Inlet Flow?

- **suppose --> V inlet = 725 m/sec**

$$M_{inlet} = \frac{V_{inlet}}{\sqrt{\gamma R_g T_{inlet}}} = \frac{725}{(1.4 \cdot 287.056 \cdot 216.65)^{0.5}} = 2.457$$

$$T_{0_{inlet}} = \left(T_{inlet} + \frac{V_{inlet}^{\ 2}}{2c_{p\,inlet}} \right) = \left(216.65 + \frac{725^2}{2 \cdot 1004.696} \right) = 478.234\ \text{K}$$

Exit Stagnation Temperature, Temperature Ratio

- Solve for Stagnation Temperature at Engine Exit

$$T_{0_{exit}} = \frac{q + c_p\left(T_{0_{inlet}}\right)}{c_p} = \frac{(190 \cdot 10^3 + 1004.696\,(478.234))}{1004.696} = 667.35\ {}^oK$$

$$\boxed{\frac{T_{0_{exit}}}{T_{0_{inlet}}} = 1.3954}$$

→Combustor temperature ratio drops
→When compared to subsonic case

M_{exit} Solution

- Solve for Coefficients of equation first

$$\left[\frac{\gamma-1}{2}-\gamma^2 F(M_{inlet})\right]M_{exit}^{\;4}+\left[1-F(M_{inlet})2\gamma\right]M_{exit}^{\;2}-F(M_{inlet})=0$$

$$M_{inlet}=\frac{V_{inlet}}{\sqrt{\gamma R_g T_{inlet}}}=\frac{725}{(1.4\cdot 287.056\cdot 216.65)^{0.5}}=2.457$$

$$\frac{T_{0_{exit}}}{T_{0_{inlet}}}=1.3954$$

- Solve for Coefficients of equation first

Calculate for $F(M_{inlet})$

$$F(M_{inlet})=\left[\frac{T_{0exit}}{T_{0_{inlet}}}\right]\frac{M_{inlet}^{\;2}\left[1+\frac{\gamma-1}{2}M_{inlet}^{\;2}\right]}{\left[1+\gamma M_{inlet}^{\;2}\right]^2}=1.3954\left(\frac{(2.457^2)\left(1+\frac{1.4-1}{2}2.457^2\right)}{(1+1.4\cdot 2.457^2)^2}\right)=0.20815$$

M_{exit} Solution (2)

- Solve for Coefficients of equation first

$$\left[\frac{\gamma-1}{2}-\gamma^2 F(M_{inlet})\right]M_{exit}^4+\left[1-F(M_{inlet})2\gamma\right]M_{exit}^2-F(M_{inlet})=0$$

$$F(M_{inlet})=\left[\frac{T_{0exit}}{T_{0_{inlet}}}\right]\frac{M_{inlet}^2\left[1+\frac{\gamma-1}{2}M_{inlet}^2\right]}{\left[1+\gamma M_{inlet}^2\right]^2} = \quad = 0.20815$$

$$\left[\frac{\gamma-1}{2}-\gamma^2 F(M_{inlet})\right]= \frac{1.4-1}{2}-1.4^2 0.20815 \quad = -0.20798$$

$$\left[1-F(M_{inlet})2\gamma\right]= 1-0.20815\cdot 2\cdot 1.4 \quad = 0.41717$$

M_{exit} Solution (3)

- $$\left[\frac{\gamma-1}{2}-\gamma^2 F(M_{inlet})\right]M_{exit}^4+\left[1-F(M_{inlet})2\gamma\right]M_{exit}^2-F(M_{inlet})=0$$

$$-0.20798\,M_2^4+0.4171M_2^2-0.20815=0$$

$$M_{exit}=\left(\frac{-0.417171\pm(0.417171^2-4\,(-0.20815)\,(-0.20798))^{0.5}}{-2\cdot 0.20798}\right)^{0.5}$$

- Root 1 $M_{exit}=0.96553$

Which Root?

- Root 2 $M_{exit}=1.03613$

Either way we have decelerated
The flow from inlet Mach = 2.457

Stagnation pressure

- Calculate the Corresponding Stagnation Pressure ratios!!

$$\frac{P_{0_{exit}}}{P_{0_{inlet}}} = \frac{\left[1+\gamma M_{inlet}^{2}\right]}{\left[1+\gamma M_{exit}^{2}\right]} \left[\frac{1+\frac{\gamma-1}{2}M_{exit}^{2}}{1+\frac{\gamma-1}{2}M_{inlet}^{2}}\right]^{\frac{\gamma}{\gamma-1}}$$

- Root 1

$M_{exit}=0.96553$

$$P_{0exit}/P_{0inlet} = \frac{(1+1.4\cdot 2.457^2)}{(1+1.4(0.96553)^2)}\left(\frac{1+\frac{(1.4-1)}{2}(0.96553)^2}{1+\frac{1.4-1}{2}2.457^2}\right)^{\frac{1.4}{1.4-1}} = 0.4667$$

- Root 2

$M_{exit}=1.03613$

$$P_{0exit}/P_{0inlet} = \frac{(1+1.4\cdot 2.457^2)}{(1+1.4(1.03613)^2)}\left(\frac{1+\frac{(1.4-1)}{2}(1.03613)^2}{1+\frac{1.4-1}{2}2.457^2}\right)^{\frac{1.4}{1.4-1}} = 0.4668$$

- Nature prefers the solution that gives the largest increase in entropy (stagnation pressure loss) ! …. **Root 2**

Choking Heat Load …

- what heat load chokes exit? (M_{exit} = 1)

$$\left[\frac{T0_{exit}}{T0_{inlet}}\right] = \frac{\dfrac{M_{exit}{}^2\left[1+\dfrac{\gamma-1}{2}M_{exit}{}^2\right]}{\left[1+\gamma M_{exit}{}^2\right]^2}}{\dfrac{M_{inlet}{}^2\left[1+\dfrac{\gamma-1}{2}M_{inlet}{}^2\right]}{\left[1+\gamma M_{inlet}{}^2\right]^2}} = \frac{\left(\dfrac{(1^2)\left(1+\dfrac{1.4-1}{2}1^2\right)}{(1+1.4\cdot 1^2)^2}\right)}{\left(\dfrac{(2.457^2)\left(1+\dfrac{1.4-1}{2}2.457^2\right)}{(1+1.4\cdot 2.457^2)^2}\right)} = 1.3966$$

Stagnation temperature at Choked exit

$$\left(T_{0_{exit}}\right)_{choked} = 1.3966\cdot 478.234 = 667.90^o K$$

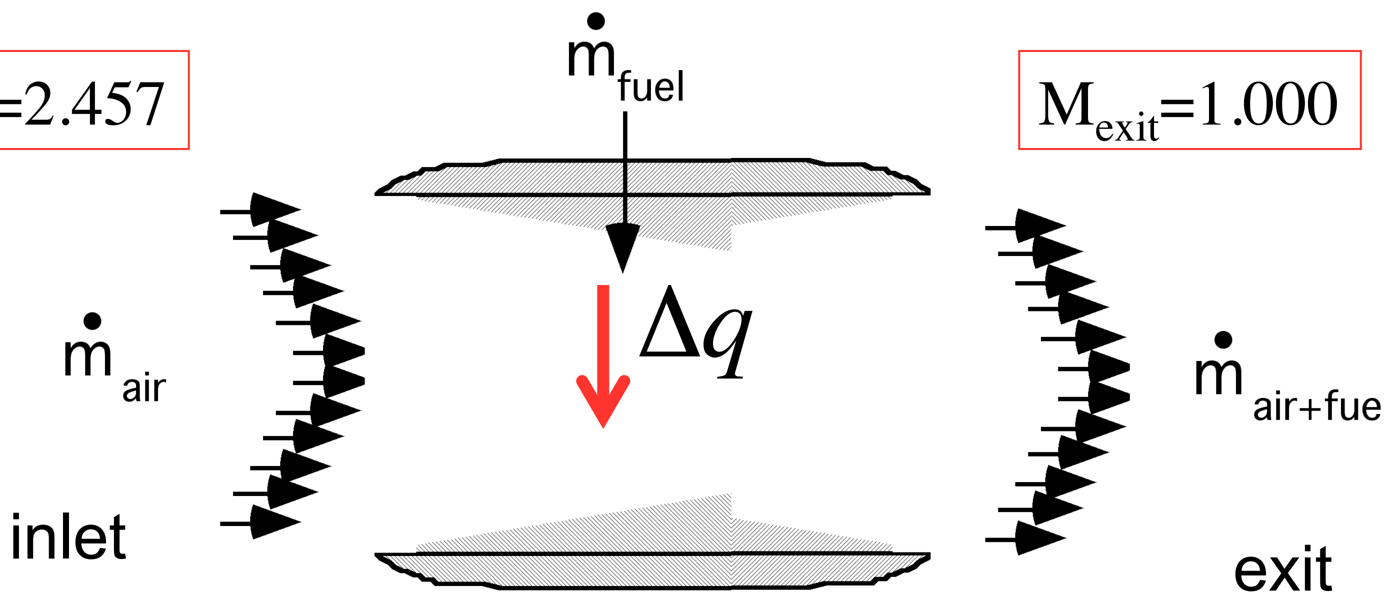
Choking Heat Load … (2)

$$\Delta q = C_p \left[T_{0_{exit}} - T_{0\,inlet} \right] = \frac{1004.696\,(667.9 - 478.234)}{1000}$$

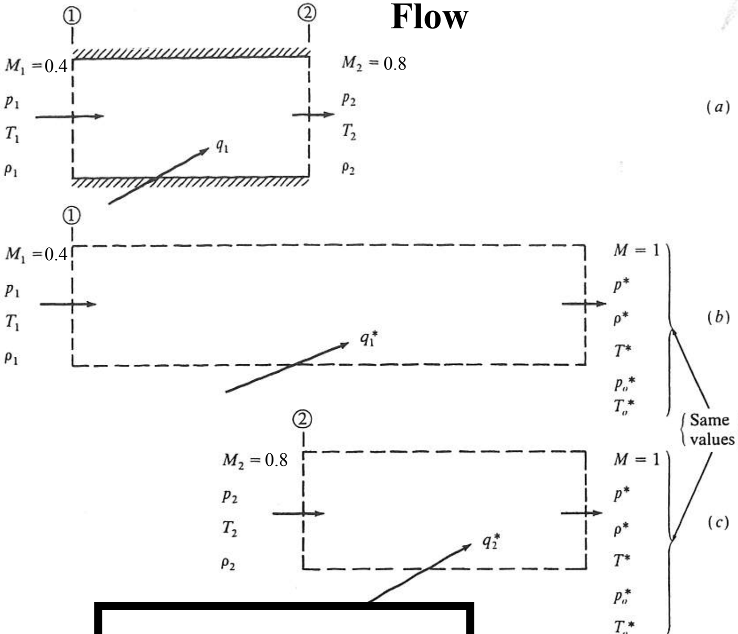
$$= 190.575 \text{ kJ/kg}$$

M_{inlet}=2.457

M_{exit}=1.000

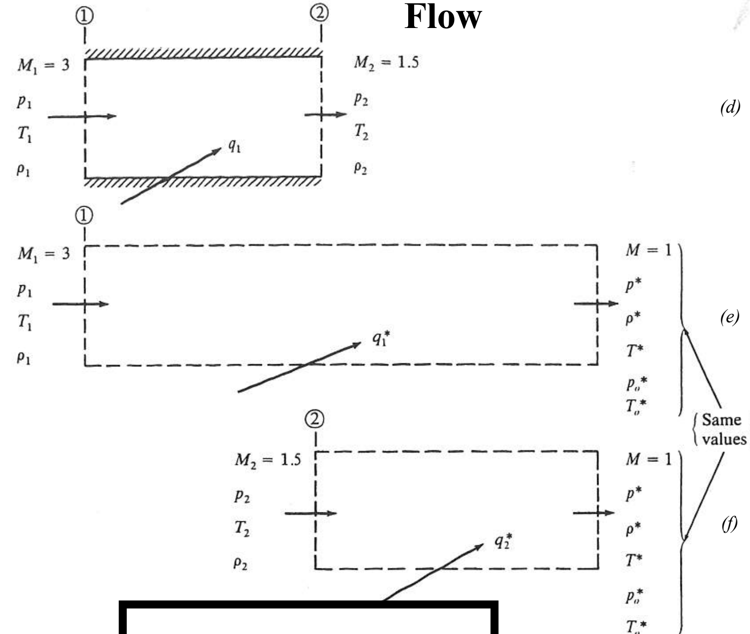


Heat Addition to Subersonic Flow



$$q_1 + q_2^* = q_1^*$$

Heat Addition to Supersonic Flow

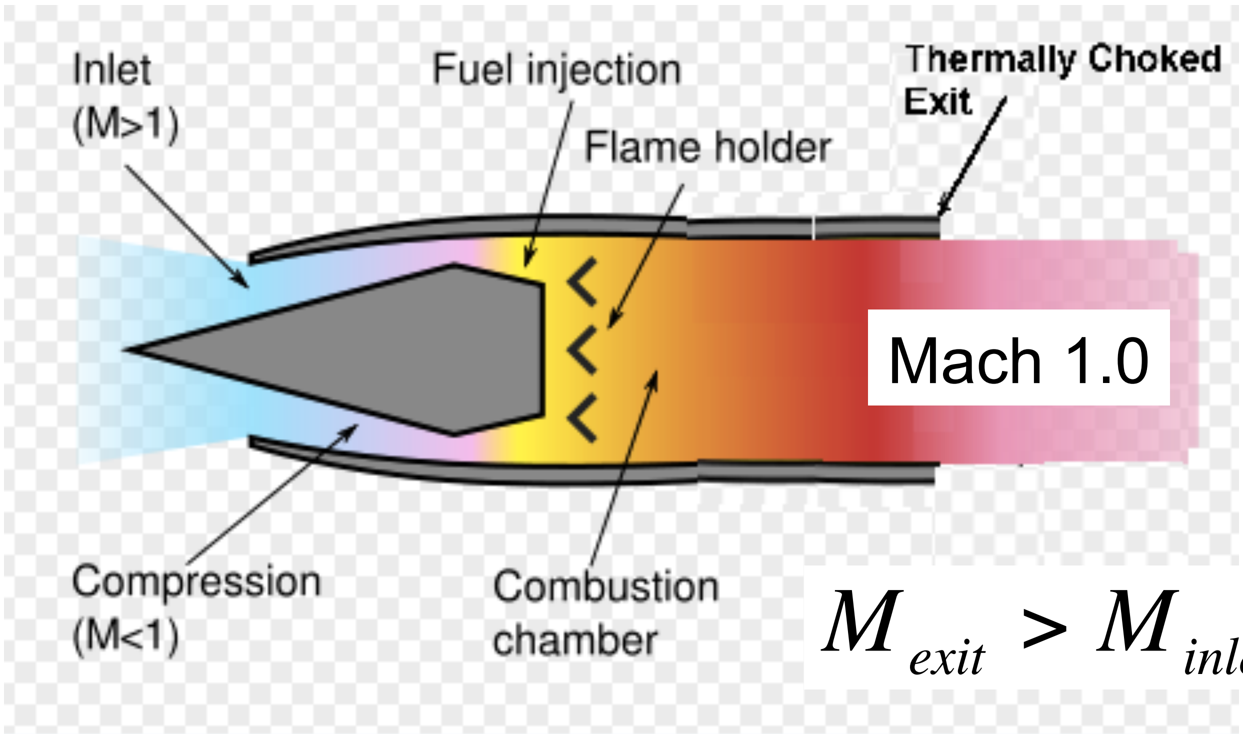


$$q_1 + q_2^* = q_1^*$$

$$M_{inlet} < 1 \Leftrightarrow +\Delta q \ increases \ Mach \rightarrow M_{exit} > M_{inlet}$$

$$M_{inlet} > 1 \Leftrightarrow +\Delta q \ decreases \ Mach \rightarrow M_{exit} < M_{inlet}$$

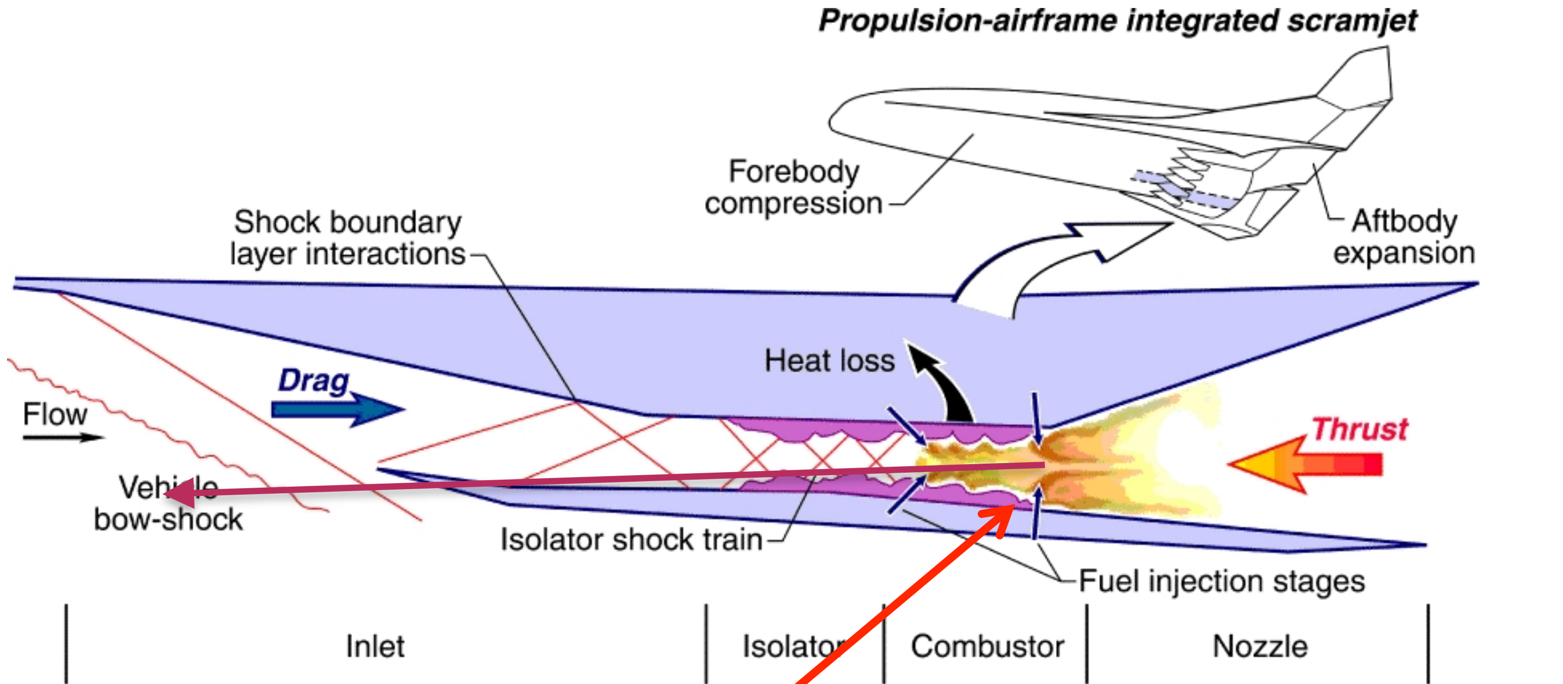
Thermal Choking in Ramjet



$$M_{exit} > M_{inlet}$$

Non-adiabatic flow is accelerated to mach 1 without divergent nozzle by adding heating

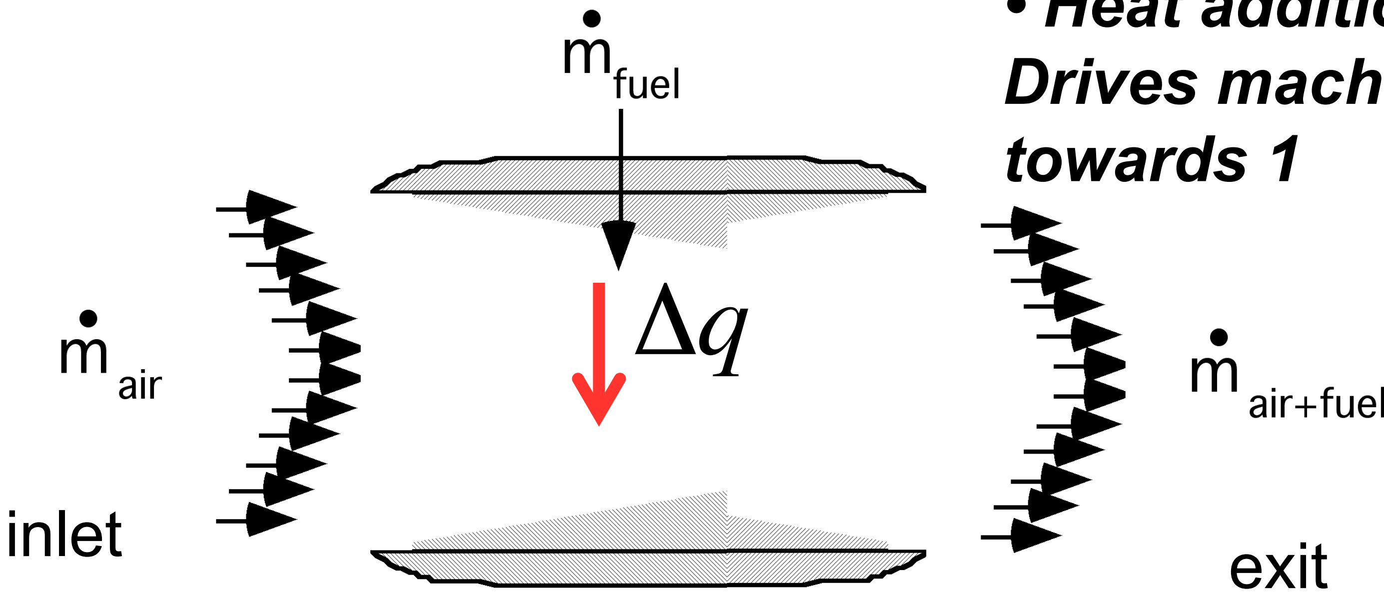
Thermal Choking in SCRAMjet



$$M_{exit} < M_{inlet}$$

Approaches Mach 1.0

Thermal Choking Summary



- ***Heat addition always Drives mach number towards 1***

$$M_{inlet} < 1 \Leftrightarrow +\Delta q \ increases \ Mach \rightarrow M_{exit} > M_{inlet}$$

$$M_{inlet} > 1 \Leftrightarrow +\Delta q \ decreases \ Mach \rightarrow M_{exit} < M_{inlet}$$

$$M_{inlet} < 1 \Leftrightarrow -\Delta q \ decreases \ Mach \rightarrow M_{exit} < M_{inlet}$$

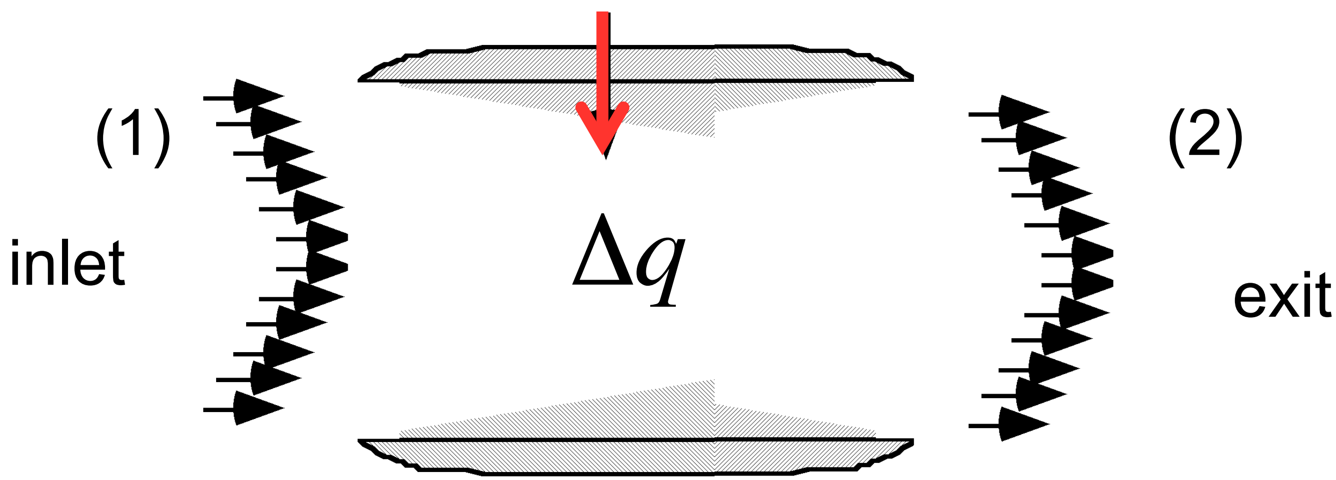
$$M_{inlet} > 1 \Leftrightarrow -\Delta q \ increases \ Mach \rightarrow M_{exit} > M_{inlet}$$

Revisit Thermal Choking

- Previously we showed that

$$\frac{T_{0_{exit}}}{T_{0_{inlet}}} = \frac{2\cdot(\gamma+1)\cdot M_{exit}{}^2\cdot\left[1+\frac{\gamma-1}{2}M_{exit}{}^2\right]}{\left[1+\gamma M_{exit}{}^2\right]^2} \times \frac{\left[1+\gamma M_{inlet}{}^2\right]^2}{2\cdot(\gamma+1)\cdot M_{inlet}{}^2\cdot\left[1+\frac{\gamma-1}{2}M_{inlet}{}^2\right]}$$

- Ramjet Adds Fuel and Combustion adds Heat to Input Air …



Revisit Thermal Choking (2)

- Rewrite previous in terms of new subscripts (1) and (2)

$$\left.\begin{matrix} exit -> (2) \\ inlet -> (1) \end{matrix}\right| \Rightarrow \left[\frac{T0_{exit}}{T0_{inlet}}\right] \equiv \left[\frac{T0_{(2)}}{T0_{(1)}}\right] = \frac{\dfrac{M_{(2)}{}^2\left[1+\dfrac{\gamma-1}{2}M_{(2)}{}^2\right]}{\left[1+\gamma M_{(2)}{}^2\right]^2}}{\dfrac{M_{(1)}{}^2\left[1+\dfrac{\gamma-1}{2}M_{(1)}{}^2\right]}{\left[1+\gamma M_{(1)}{}^2\right]^2}}$$

$$\left[\frac{T0_{(2)}}{T0_{(1)}}\right] = 1 + \frac{\Delta q_{1\rightarrow 2}}{c_p \cdot T0_{(1)}}$$

Heat Addition corresponds to increase in stagnation temperature *(change from state(1) to state (2))*

Revisit Thermal Choking (3)

- Now given M_{exit} and $T0_{exit}$, consider the heat required to thermally choke the nozzle combustor exit

$$\left.\begin{array}{l} unchoked\ exit\ state -> (2) \\ choked\ exit\ state -> (*) \end{array}\right| \Rightarrow \left[\frac{T0_{exit\ unchoked}}{T0_{exit\ choked}}\right] = \left[\frac{T0_{(2)}}{T0_{(*)}}\right] = \frac{\dfrac{M_{(2)}^2\left[1+\frac{\gamma-1}{2}M_{(2)}^2\right]}{\left[1+\gamma M_{(2)}^2\right]^2}}{\dfrac{(1)^2\left[1+\frac{\gamma-1}{2}(1)^2\right]}{\left[1+\gamma(1)^2\right]^2}} =$$

$$= \frac{M_{(2)}^2\left[1+\frac{\gamma-1}{2}M_{(2)}^2\right]}{\left[1+\gamma M_{(2)}^2\right]^2} \times \frac{\left[1+\gamma\right]^2}{\left[\frac{\gamma+1}{2}\right]} = 2(1+\gamma)\cdot\frac{M_{(2)}^2\left[1+\frac{\gamma-1}{2}M_{(2)}^2\right]}{\left[1+\gamma M_{(2)}^2\right]^2}$$

Revisit Thermal Choking (3)

- Thus, for a given exit stagnation temperature, the maximum (choke point) stagnation temperature and heat loads are

$$\left[\frac{T0_{\substack{exit\\unchoked}}}{T0_{\substack{exit\\choked}}}\right]=\left[\frac{T0_{\substack{exit\\unchoked}}}{T0^{(*)}}\right]=\left[\frac{T0_{\substack{exit\\unchoked}}}{T0_{(1)}}\right]\Bigg/\left[\frac{T0^{(*)}}{T0_{(1)}}\right]=\frac{1+\dfrac{\Delta q_{1\to 2(unchoked)}}{c_p\cdot T0_{(1)}}}{1+\dfrac{\Delta q^{(*)}}{c_p\cdot T0_{(1)}}}\to$$

$$\left[\frac{T0_{\substack{exit\\unchoked}}}{T0^{(*)}}\right]=\frac{1+\dfrac{\Delta q_{1\to 2(unchoked)}}{c_p\cdot T0_{(1)}}}{1+\dfrac{\Delta q^{(*)}}{c_p\cdot T0_{(1)}}}=2(1+\gamma)\cdot\frac{M_{\substack{exit\\unchoked}}^{2}\left[1+\dfrac{\gamma-1}{2}M_{\substack{exit\\unchoked}}^{2}\right]}{\left[1+\gamma M_{\substack{exit\\unchoked}}^{2}\right]^2}$$

Revisit Thermal Choking (4)

- In an Identical Manner .. can show that for thermally choked flow

$$\frac{T_{\substack{exit\\unchoked}}}{T_{exit}^{(*)}} = \frac{T_{0\,\substack{exit\\unchoked}}}{\left[1+\frac{\gamma-1}{2}M_{\substack{exit\\unchoked}}^{2}\right]} \Bigg/ \frac{T_{0\,exit}^{(*)}}{\left[1+\frac{\gamma-1}{2}(1)^2\right]} = \frac{T_{0\,\substack{exit\\unchoked}}}{T_{0\,exit}^{(*)}} \times \frac{1}{2}\frac{\gamma+1}{\left[1+\frac{\gamma-1}{2}M_{\substack{exit\\unchoked}}^{2}\right]}$$

Substituting ..

$$\left[\frac{T_{0\,\substack{exit\\unchoked}}}{T_{0\,exit}^{(*)}}\right] = 2(1+\gamma)\cdot\frac{M_{\substack{exit\\unchoked}}^{2}\left[1+\frac{\gamma-1}{2}M_{\substack{exit\\unchoked}}^{2}\right]}{\left[1+\gamma M_{\substack{exit\\unchoked}}^{2}\right]^2}$$

- and Simplifying …

$$\left[\frac{T_{\substack{exit\\unchoked}}}{T_{exit}^{(*)}}\right] = 2(1+\gamma)\cdot\frac{M_{\substack{exit\\unchoked}}^{2}\left[1+\frac{\gamma-1}{2}M_{\substack{exit\\unchoked}}^{2}\right]}{\left[1+\gamma M_{\substack{exit\\unchoked}}^{2}\right]^2}\times\frac{1}{2}\frac{\gamma+1}{\left[1+\frac{\gamma-1}{2}M_{\substack{exit\\unchoked}}^{2}\right]} = \frac{\left((1+\gamma)\cdot M_{\substack{exit\\unchoked}}\right)^2}{\left[1+\gamma M_{\substack{exit\\unchoked}}^{2}\right]^2}$$

Revisit Thermal Choking (5)

• More Generally … for a Mach number .. *M ….The relationships of the* unchoked to the thermally choked stagnation and static temperature ratios are …

$$\Rightarrow H_0^{(*)} \equiv \left[\frac{T0_{unchoked}}{T0^{(*)}}\right] = 2(1+\gamma)\cdot\frac{M_{unchoked}^{\;2}\left[1+\frac{\gamma-1}{2}M_{unchoked}^{\;2}\right]}{\left[1+\gamma M_{unchoked}^{\;2}\right]^2}$$

$$\Rightarrow H^{(*)} \equiv \left[\frac{T_{\substack{exit\\unchoked}}}{T_{exit}^{(*)}}\right] = \frac{\left((1+\gamma)\cdot M_{\substack{exit\\unchoked}}\right)^2}{\left[1+\gamma\cdot M_{\substack{exit\\unchoked}}^{\;2}\right]^2}$$

Ratios entirely a function of Unchoked Mach Number

Check Against Earlier results …

- Revisit ***Subsonic Problem*** on Slides 22-35

M_{exit}=0.83495 (unchoked)
$T_{0\ inlet} = 223.583\ ^oK$
$T_{0\ exit} = 412.695\ ^oK$ (unchoked)

$$\left[\frac{T0_{unchoked}}{T0^{(*)}}\right] = 2(1+\gamma)\cdot\frac{M_{unchoked}^{\ 2}\left[1+\frac{\gamma-1}{2}M_{unchoked}^{\ 2}\right]}{\left[1+\gamma M_{unchoked}^{\ 2}\right]^2}$$

- Calculate $T_0{}^*$=

$$\frac{412.695}{\dfrac{2\,(1.4+1)\;(0.83495^2)\left(1+\dfrac{1.4-1}{2}0.83495^2\right)}{(1+1.4\,(0.83495^2))^{\,2}}} = 422.623\ \text{K}$$

Check!

- Calculate required heat load

$$\Delta q^* = \frac{\dot{Q}^*}{\dot{m}} = c_p\left(T_0^* - T_{0_{inlet}}\right) = \frac{1004.696\,(422.623-223.583)}{1000} = 199.975\ \text{kJ/kg}$$

Check Against Earlier results … (2)

- Revisit ***Supersonic Problem*** on Slides 41-43

M_{exit}=1.03613 (unchoked)
$T_{0\ inlet}$ = 478.243 °K
$T_{0\ exit}$ = 667.35 °K (unchoked)

$$\left[\frac{T_{0_{unchoked}}}{T_0^{(*)}}\right] = 2(1+\gamma)\cdot\frac{M_{unchoked}^2\left[1+\frac{\gamma-1}{2}M_{unchoked}^2\right]}{\left[1+\gamma M_{unchoked}^2\right]^2}$$

- Calculate T_0^{*}=

$$\frac{667.35}{\frac{2\,(1.4+1)\,1.03613^2\left(1+\frac{1.4-1}{2}\,(1.03613^2)\right)}{(1+1.4\,(1.03613^2))^2}} = 667.927\ \text{K}$$

Check!

- Calculate required heat load

$$\Delta q^* = \frac{\dot{Q}^*}{\dot{m}} = c_p\left(T_0^* - T_{0_{inlet}}\right) = \frac{1004.696\,(667.927-478.243)}{1000} = 190.575$$

kg

What about Entropy?

- Previously (***slide 20***) we showed that

$$s_{exit} - s_{inlet} = c_p \cdot \ln\left[\frac{T_{0_{exit}}}{T_{0_{inlet}}}\right] - R_g \cdot \ln\left[\frac{P_{0_{exit}}}{P_{0_{inlet}}}\right]$$

$$s_2 - s_1 = -R_g \ln\left[\frac{P_{02}}{P_{01}}\right] \Rightarrow \boxed{\frac{P_{02}}{P_{01}} = e^{-\left(\frac{s_2 - s_1}{R_g}\right)}}$$

And also ...

Remember for a normal shockwave?

$$\frac{s_{exit} - s_{inlet}}{c_p} = \ln\left(\left(\frac{M_{exit}}{M_{inlet}}\right)^2 \left[\frac{1+\gamma M_{inlet}^2}{1+\gamma M_{exit}^2}\right]^{\frac{\gamma+1}{\gamma}}\right)$$

- In an Identical Manner to $T_0^{(*)}$ .. can show for thermally choked flow

$$\frac{s_{\substack{exit \\ unchoked}} - s^{(*)}_{exit}}{c_p} = \ln\left[\frac{M^2_{\substack{exit \\ unchoked}}}{(1)^2} \cdot \left(\frac{1+\gamma\cdot(1)^2}{\left[1+\gamma M^2_{\substack{exit \\ unchoked}}\right]}\right)^{\frac{\gamma+1}{\gamma}}\right] = \ln\left[M^2_{\substack{exit \\ unchoked}} \cdot \left(\frac{1+\gamma}{\left[1+\gamma M^2_{\substack{exit \\ unchoked}}\right]}\right)^{\frac{\gamma+1}{\gamma}}\right]$$

"Rayleigh" Equations

- Collected together the General set of parametric equations are

$$H_0^{(*)} = \frac{c_p \cdot T_0}{c_p \cdot T_0^{(*)}} = 2(1+\gamma)\cdot\frac{M^2\left[1+\frac{\gamma-1}{2}M^2\right]}{\left[1+\gamma M^2\right]^2}$$

$$H^{(*)} \equiv \left[\frac{c_p \cdot T}{c_p \cdot T^{(*)}}\right] = \left[\frac{(1+\gamma)\cdot M}{1+\gamma \cdot M^2}\right]^2$$

$$\frac{s-s^{(*)}}{c_p} = \ln\left[M^2 \cdot \left(\frac{1+\gamma}{\left[1+\gamma \cdot M^2\right]}\right)^{\frac{\gamma+1}{\gamma}}\right]$$

$$\frac{\Delta q^{(*)}}{c_p \cdot T_o} = \frac{T_0^*}{T_o} - 1 = \frac{1-H_0^*}{H_0^*}$$

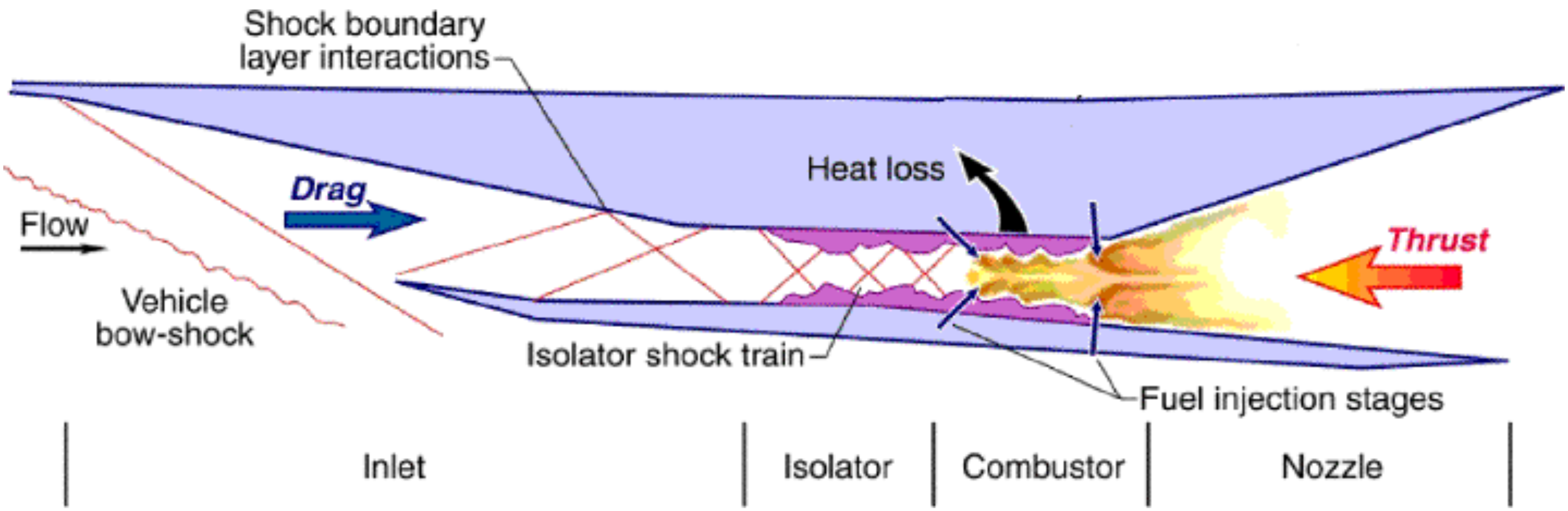
Assumes constant c_p

- Set of Parametric curves with property ratios as a function of Mach number
- Relates current conditions to those that occur for thermal choke point
- Entirely at function of Current Mach number
- Prescribed heat addition Necessary to thermally choke flow

Thermal Choking Illustrated

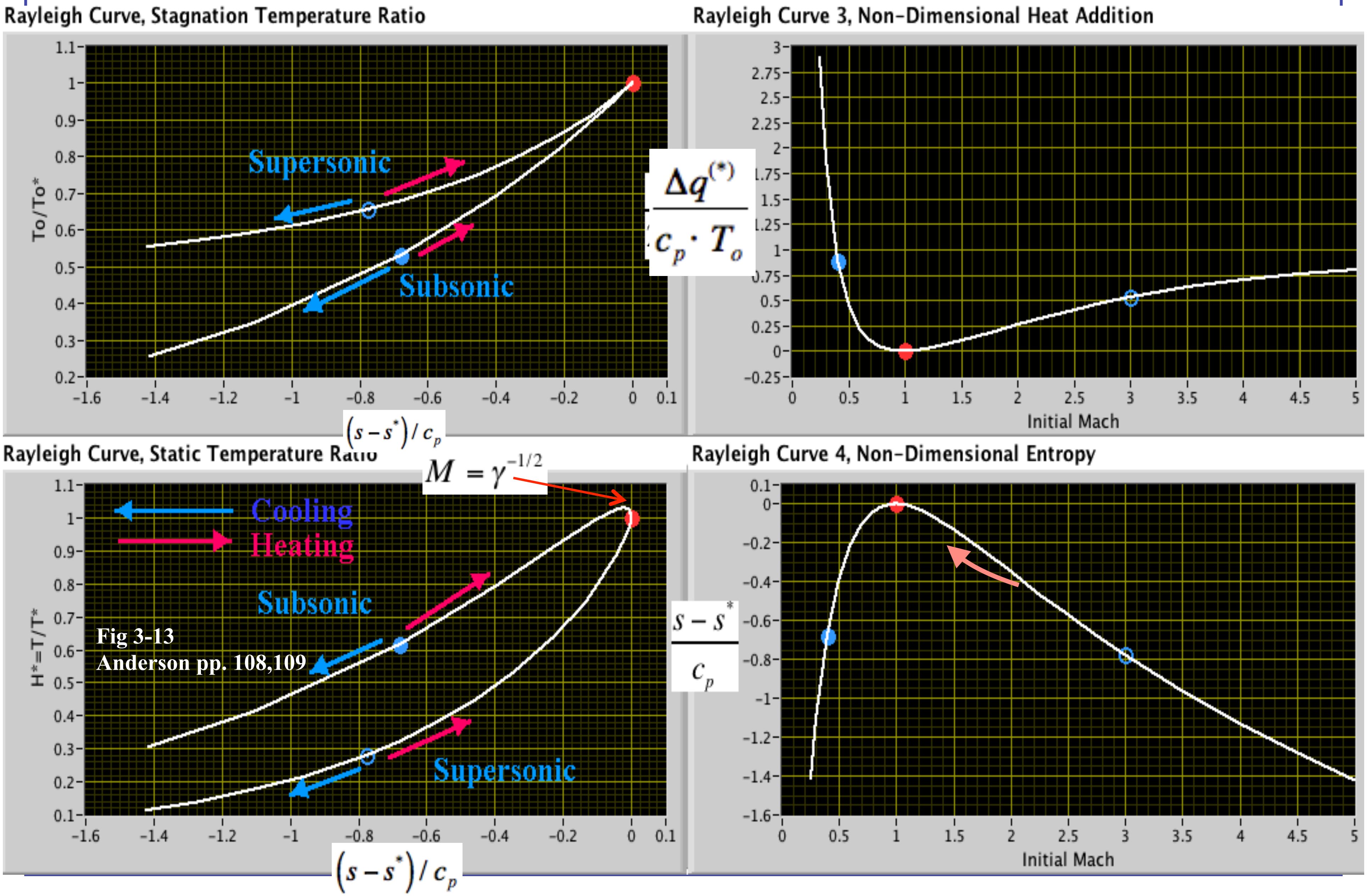
- What if we keep engine flow path supersonic to minimize stagnation pressure loss?

- How do we keep the Inflow supersonic?



- Series of very weak (highly oblique) shockwaves and expansion shocks keep the flow supersonic throughout the engine

Rayleigh Flow Curves (4)



Rayleigh Enthalpy/Entropy Curve (Anderson pp. 108,109)

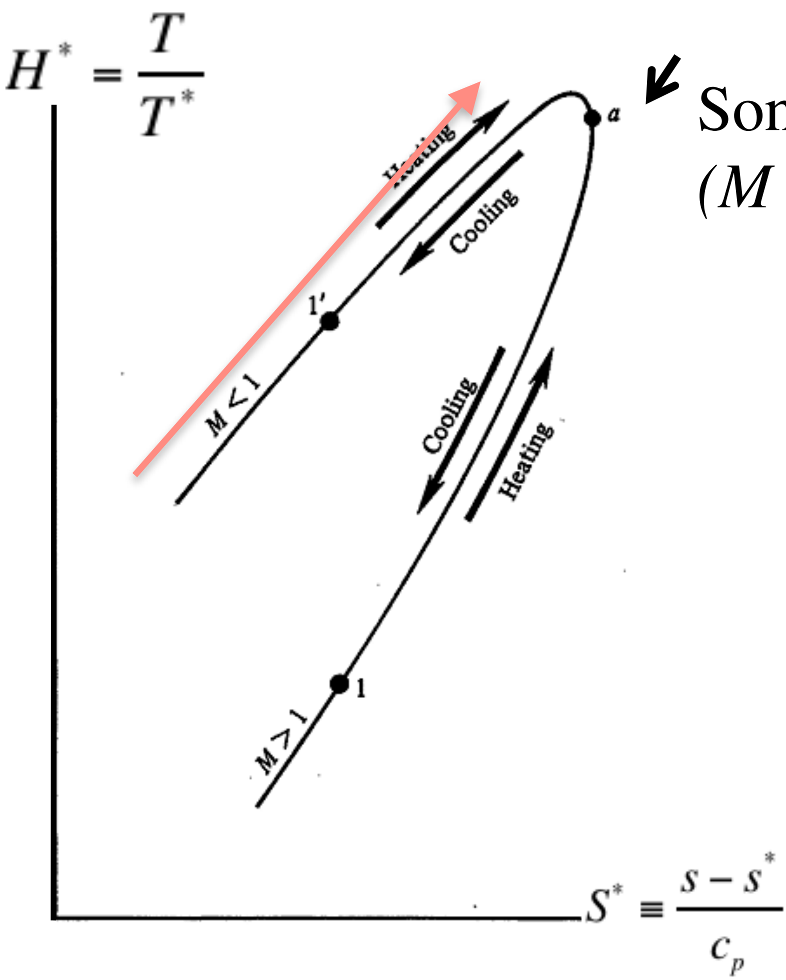


Figure 3.13 | The Rayleigh curve.

Sonic Point
(M = 1)

Figure 3-13 From Anderson

- ***Maximum entropy change occurs at sonic point***

- ***Heating to supersonic Mach numbers starting from subsonic flow violates second law of thermodynamics***

- ***Remember that?***

Rayleigh Curve (Anderson pp. 108,109)

1. For *supersonic flow* in region 1, i.e., $M_1 > 1$, when heat is added

a. Mach number decreases, $M_2 < M_1$

b. Pressure increases, $p_2 > p_1$

c. Temperature increases, $T_2 > T_1$

d. Total temperature increases, $T_{o_2} > T_{o_1}$

e. Total pressure decreases, $p_{o_2} < p_{o_1}$

f. Velocity decreases, $u_2 < u_1$

2. For *subsonic* flow in region 1, i.e., $M_1 < 1$, when heat is added

a. Mach number increases, $M_2 > M_1$

b. Pressure decreases, $p_2 < p_1$

c. Temperature increases for $M_1 < \gamma^{-1/2}$ and decreases for $M_1 > \gamma^{-1/2}$

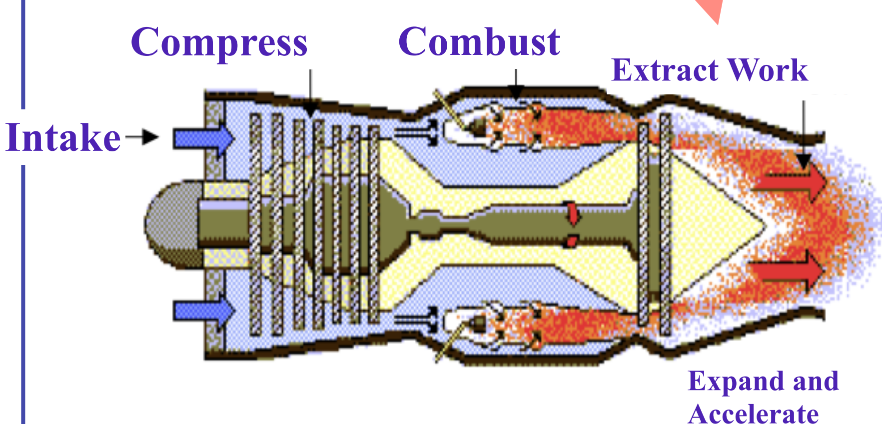
d. Total temperature increases, $T_{o_2} > T_{o_1}$

e. Total pressure decreases, $p_{o_2} < p_{o_1}$

f. Velocity increases, $u_2 > u_1$

Brayton Cycle for Airbreathing Combustion

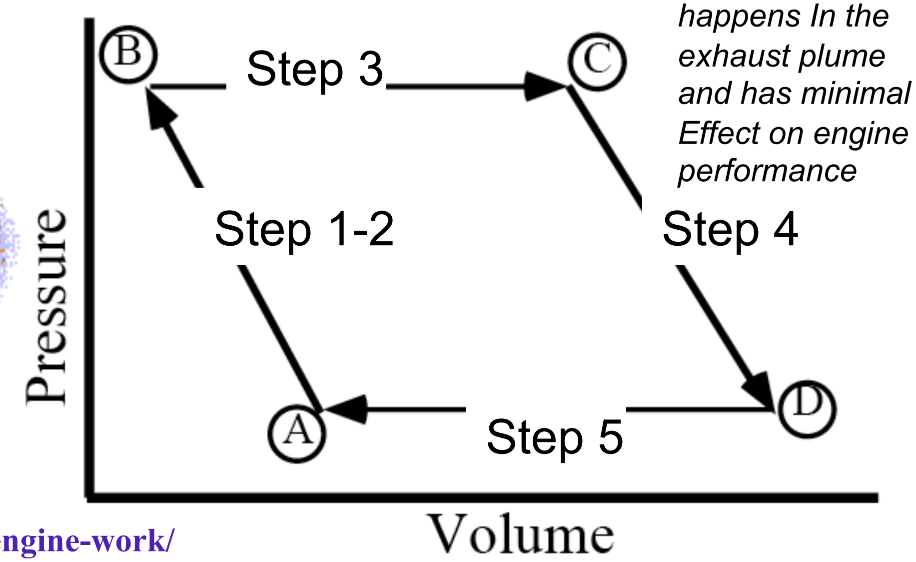
Step	Process
1) Intake	Isentropic Compression
2) Compress the Air	Adiabatic Compression
3) Add heat	Constant Pressure Combustion
4) Extract work	Isentropic Expansion in Nozzle
5) Exhaust	Heat extraction by surroundings



USAF Test Pilot School Slide, Not Mine!

https://cosmosmagazine.com/technology/how-does-a-jet-engine-work/

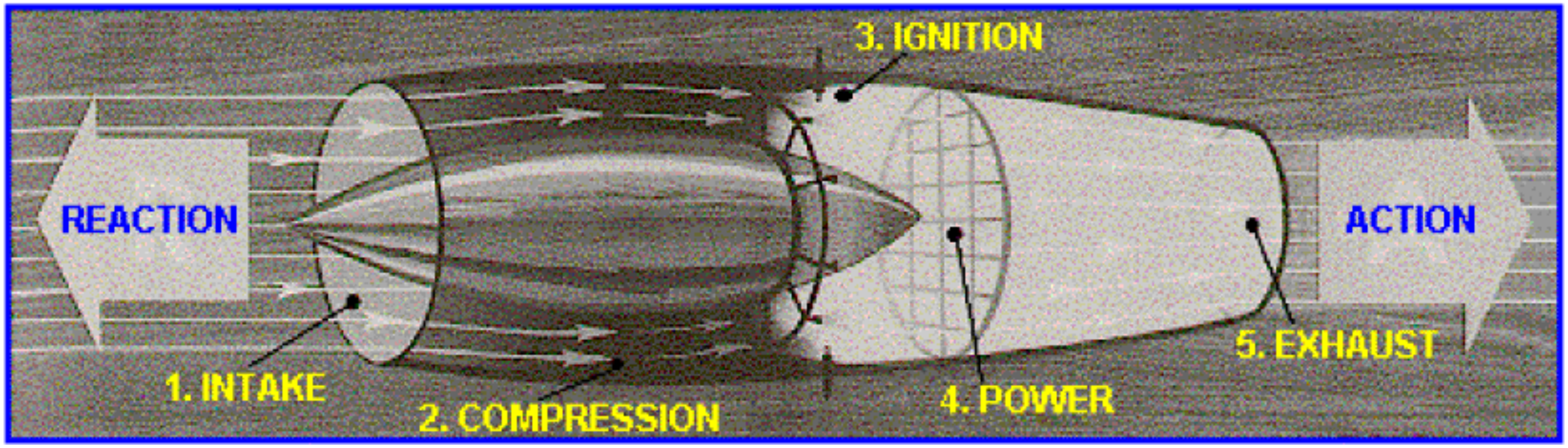
… step 5 above happens In the exhaust plume and has minimal Effect on engine performance

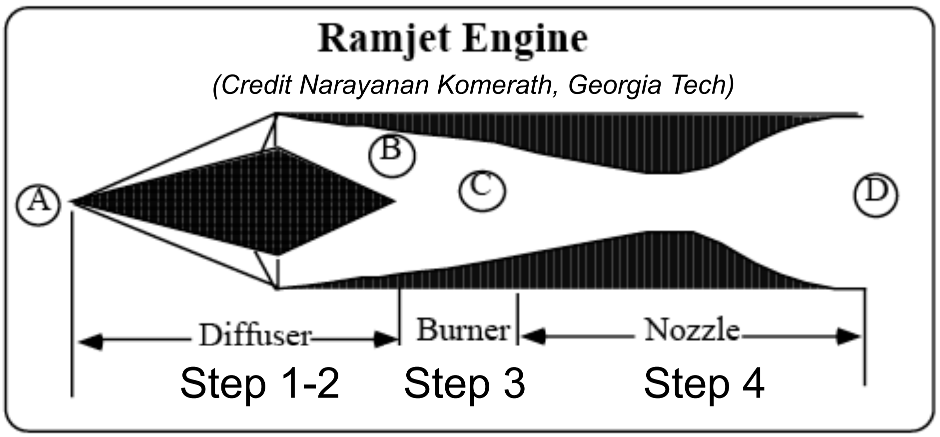


(Credit Narayanan Komerath, Georgia Tech)

Brayton Cycle for Airbreathing Combustion (cont'd)

- Ramjet Engine Combustion Cycle Steps





- Compression and Power extraction steps are performed passively in Ramjet

"Starting" a Constant Geometry Ramjet Inlet

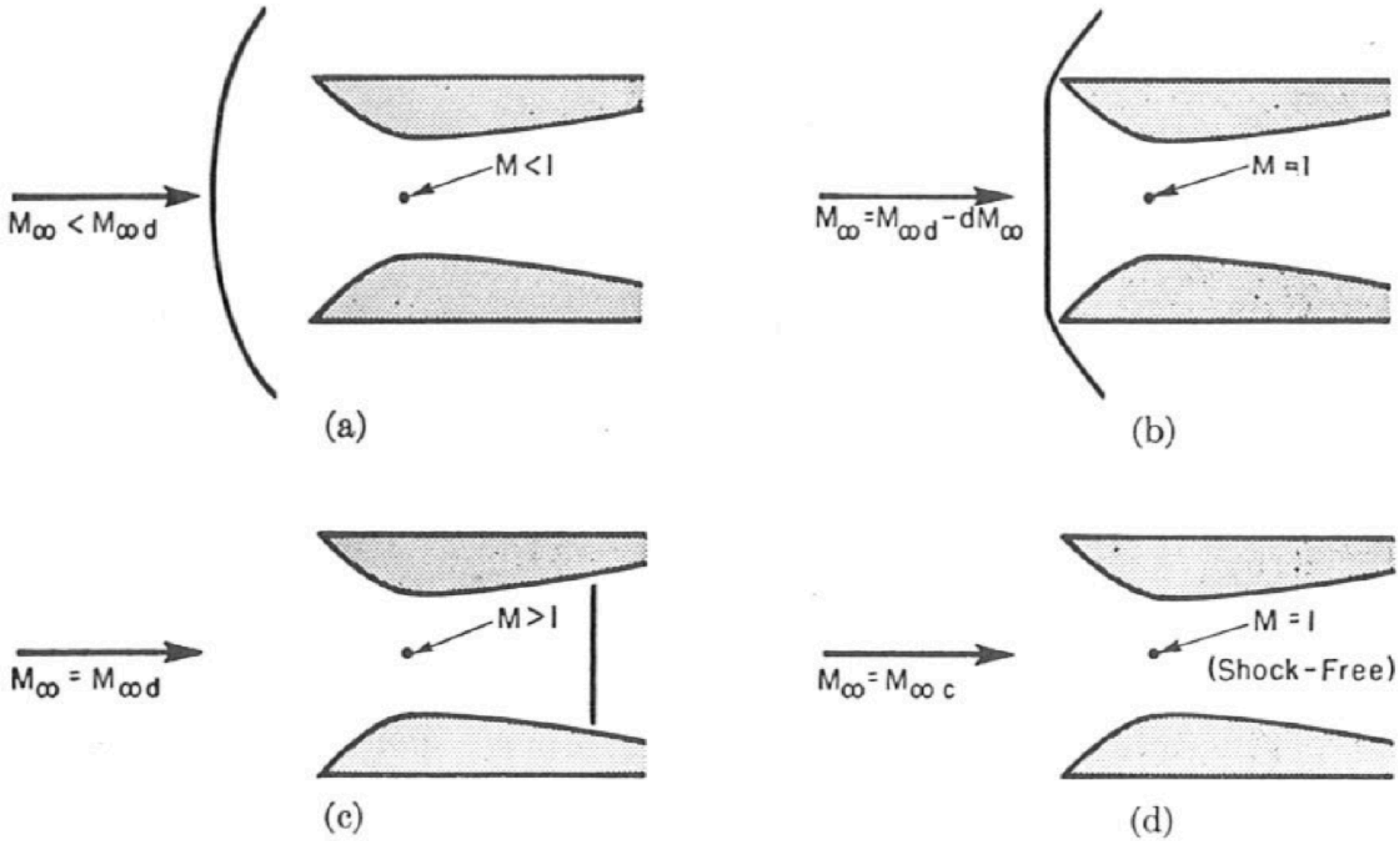
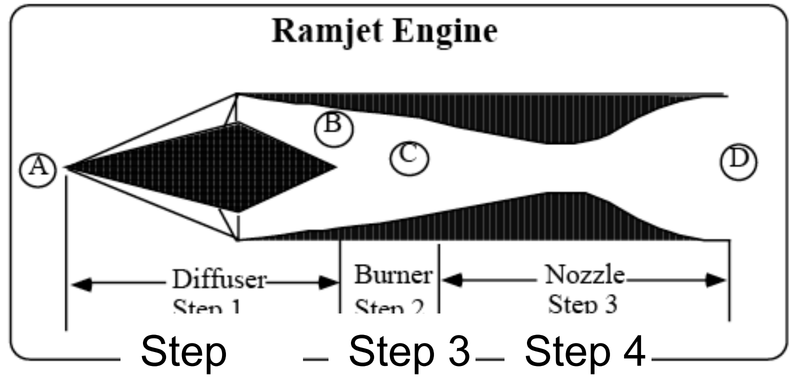


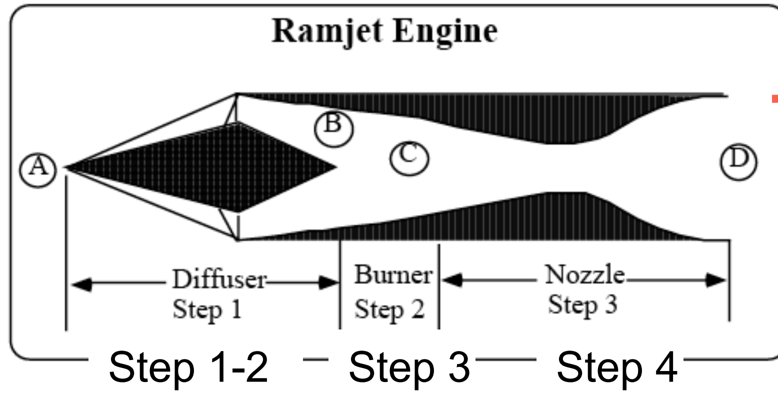
FIG. 5.33. Overspeed starting of fixed-geometry supersonic inlet, designed for free-stream Mach Number $M_{\infty a}$, and having contraction ratio $(A_2/A_1)_c$.

Ideal Ramjet Cycle Analysis

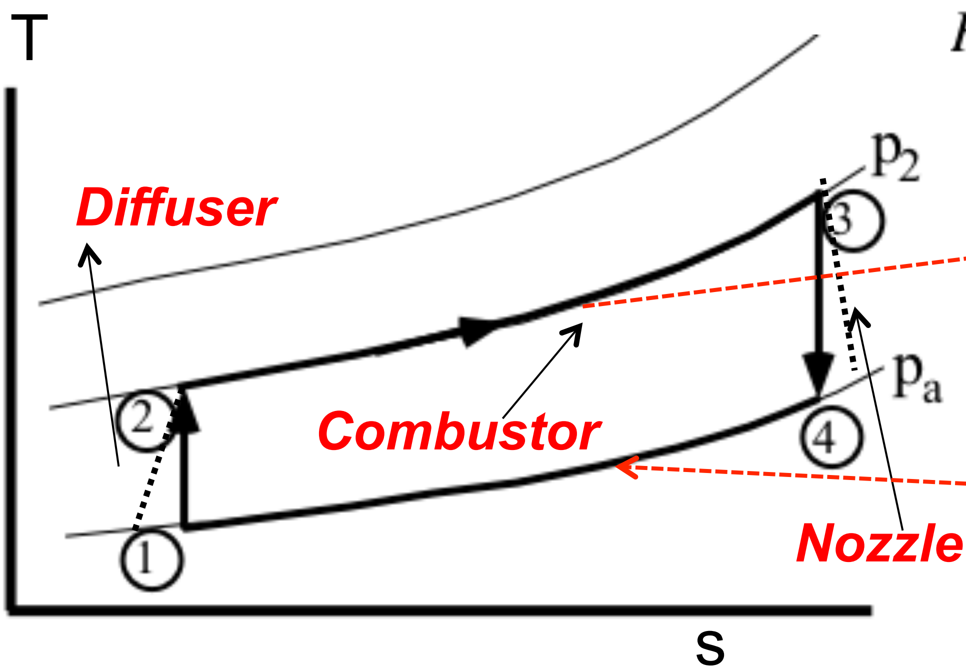


Region	Process	Ideal Behavior	Real Behavior
A to 1(inlet)	Isentropic flow	P_0, T_0 constant $\Delta s = 0$	P_0 drop $\Delta s > 0$
1-2 (diffuser)	Adiabatic Compression	P, T increase P_0 drop	P_0 drop $\Delta s > 0$
2-3 (burner)	Heat Addition	P_0 constant, T_0 Increase $\Delta s = \left(\frac{\Delta q}{T}\right)_{rev} > 0$	P_0 drop $\Delta s > \left(\frac{\Delta q}{T}\right)_{rev}$
3-4 (nozzle)	Isentropic expansion	T_0, P_0 constant $\Delta s = 0$	$\Delta s > 0$ P_0 drop

Ideal Ramjet Cycle Analysis



T-s Diagram



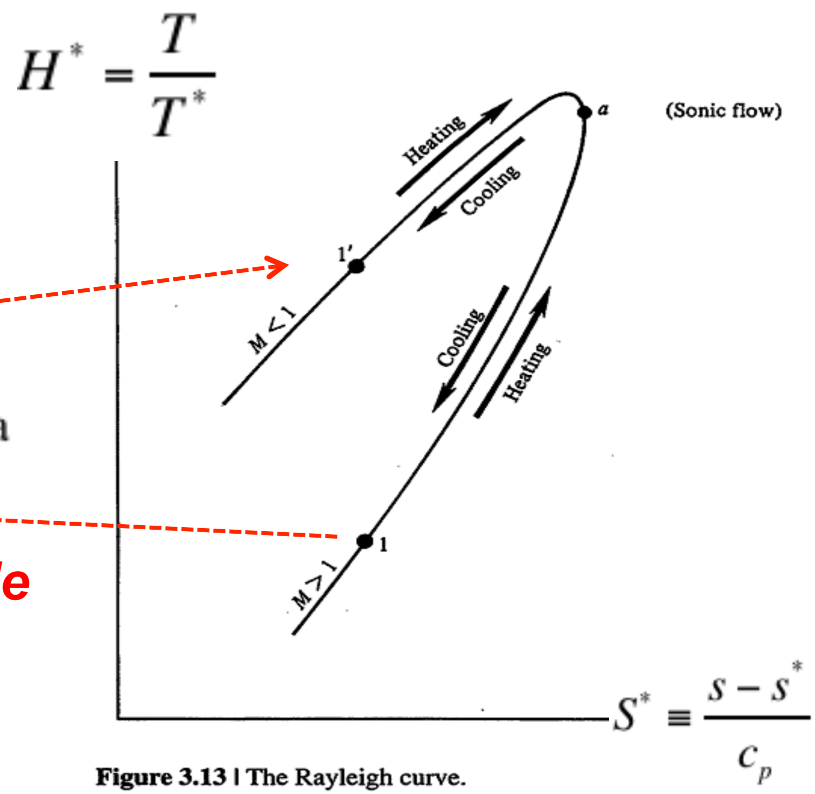


Figure 3.13 | The Rayleigh curve.

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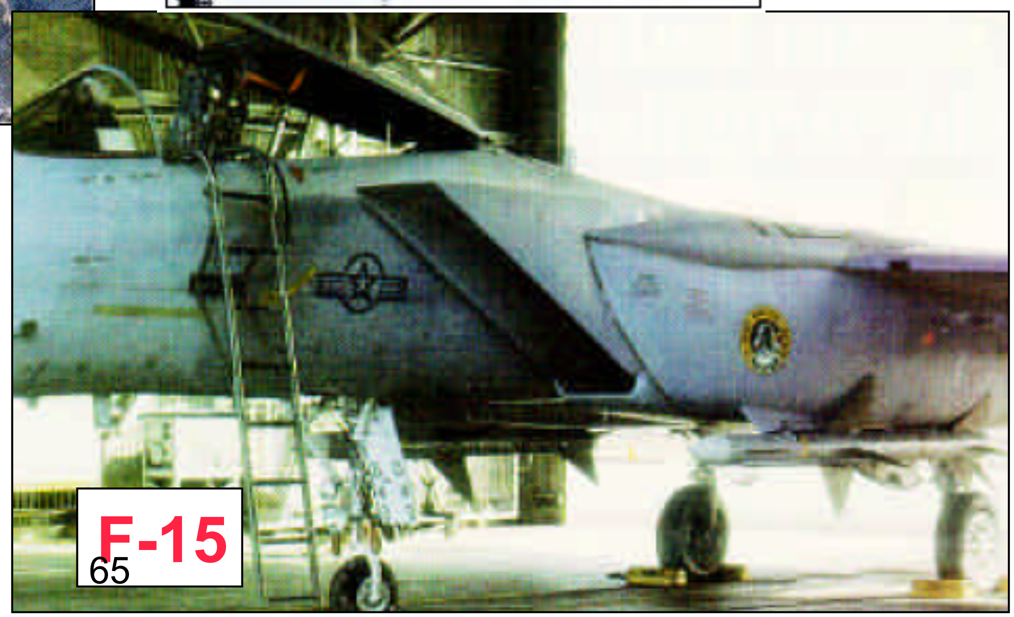
Mechanical & Aerospace Engineering

Appendix I, Inlets and Diffusers



"Starting" a Constant Geometry Inlet

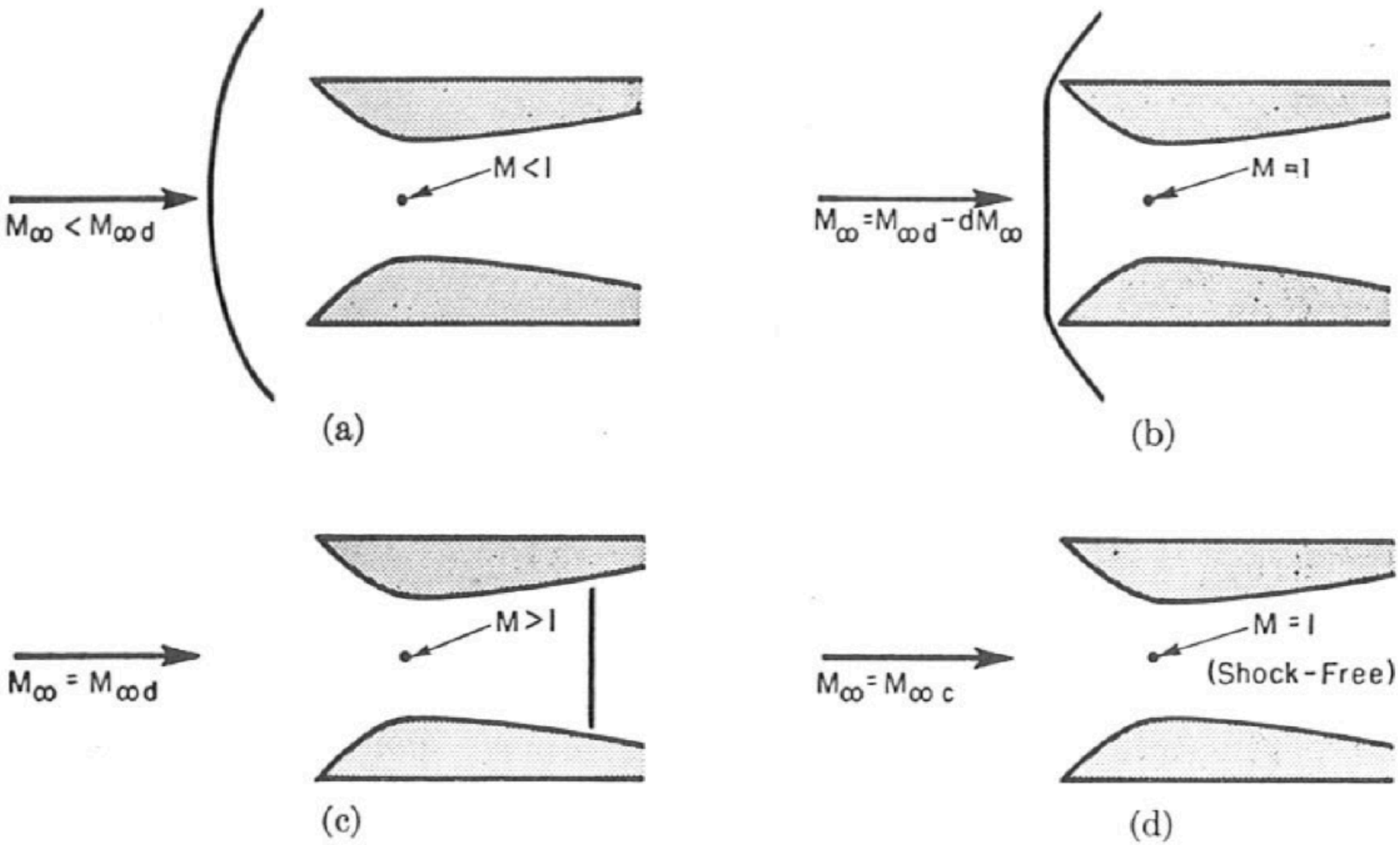


FIG. 5.33. Overspeed starting of fixed-geometry supersonic inlet, designed for free-stream Mach Number $M_{\infty d}$, and having contraction ratio $(A_2/A_1)_c$.

"Starting" a Variable Geometry Inlet

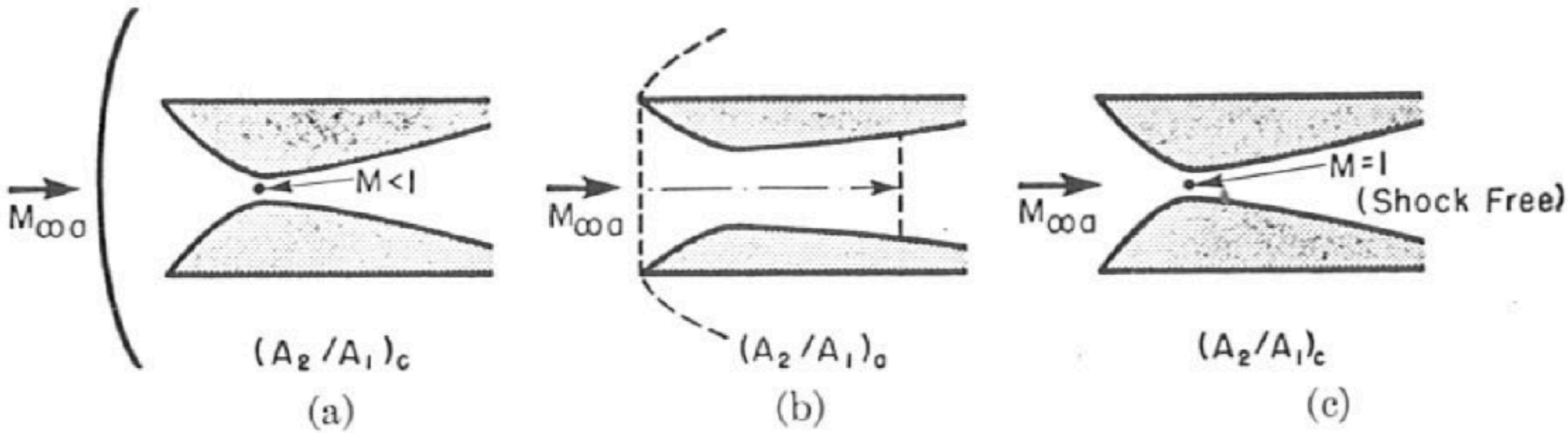
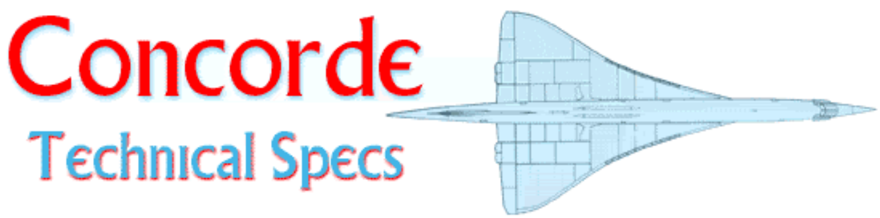
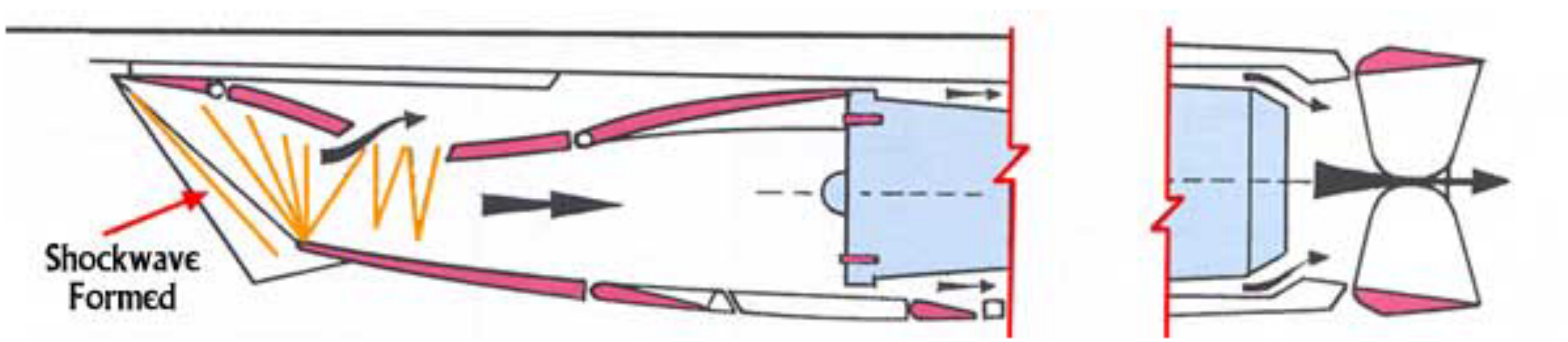


FIG. 5.34. Starting of variable-geometry supersonic inlet, designed for free-stream Mach Number $M_{\infty a}$.

Condorde Inlet Design

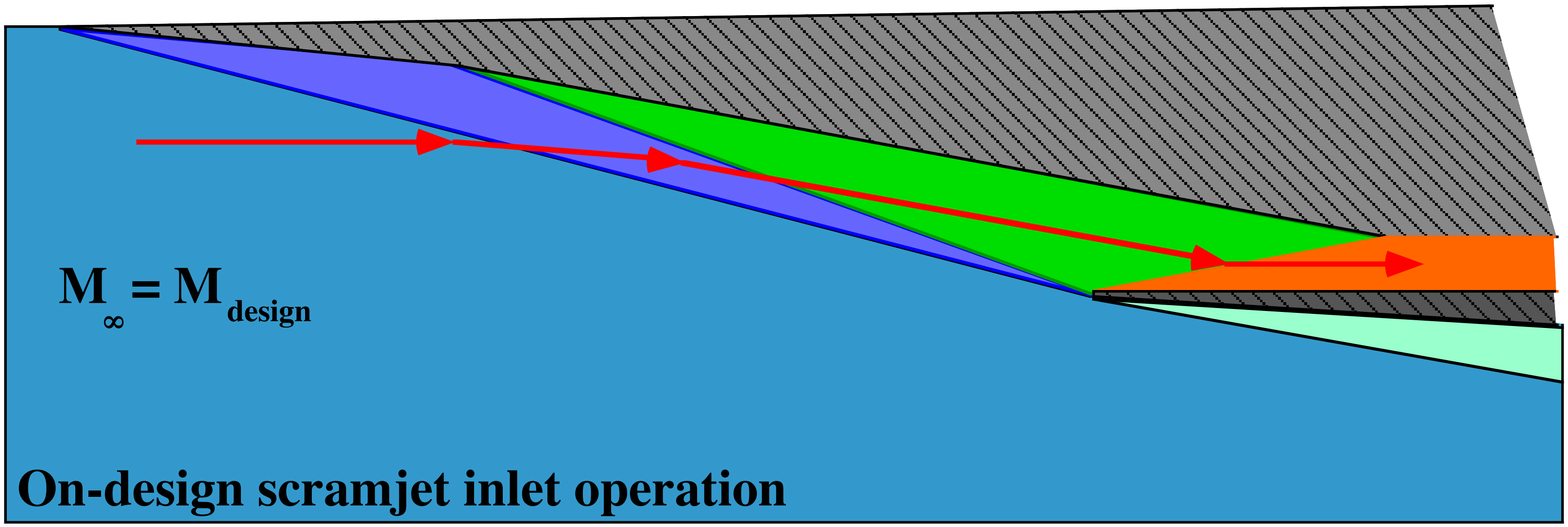


- Mach 2 Cruise



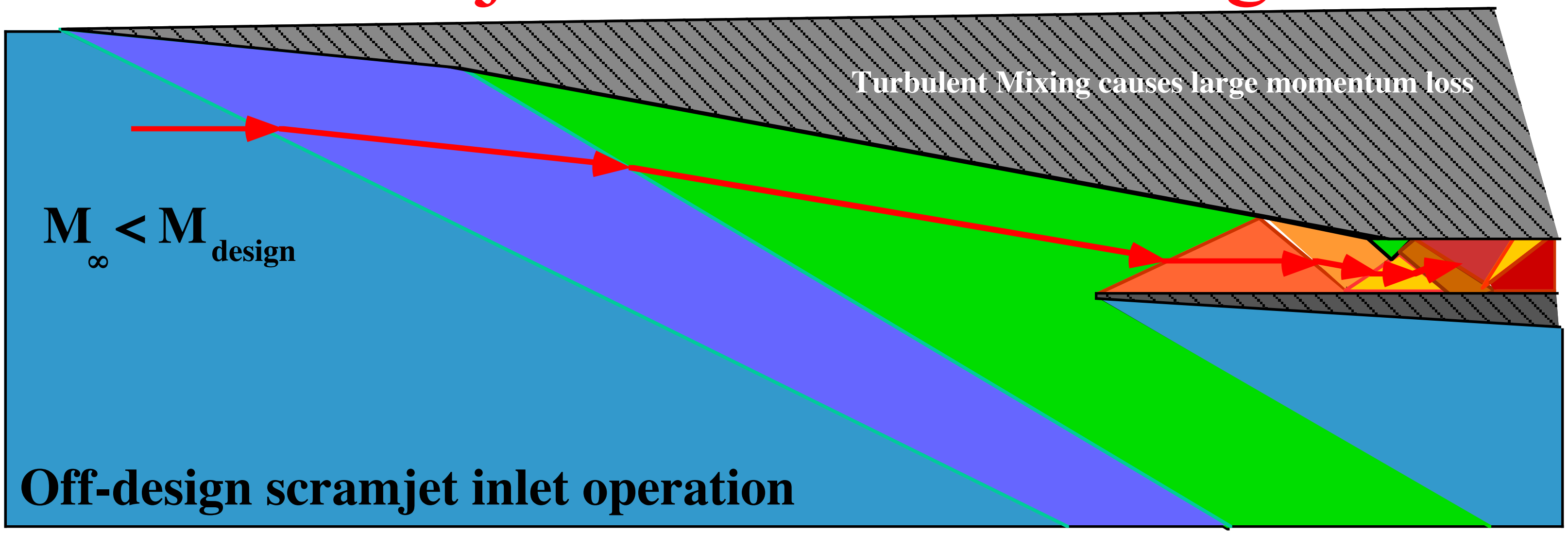
Credit: http://www.concordesst.com/powerplant.html

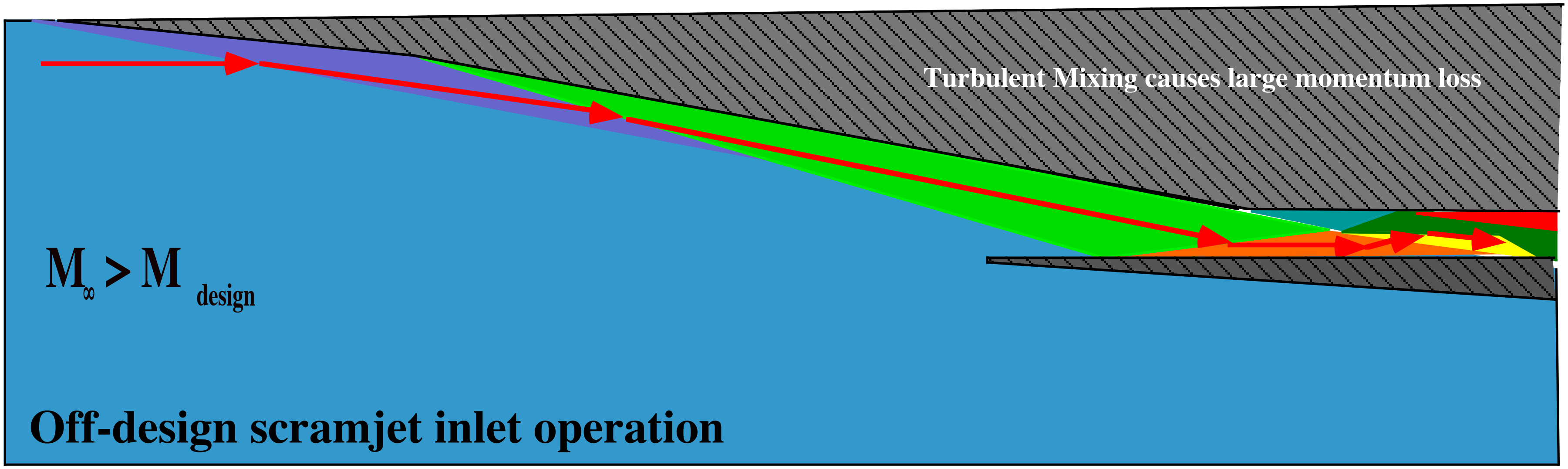
Scramjet Inlet "Point Design"



- SCRAMjets are *VERY …*
  Sensitive to inlet Mach number

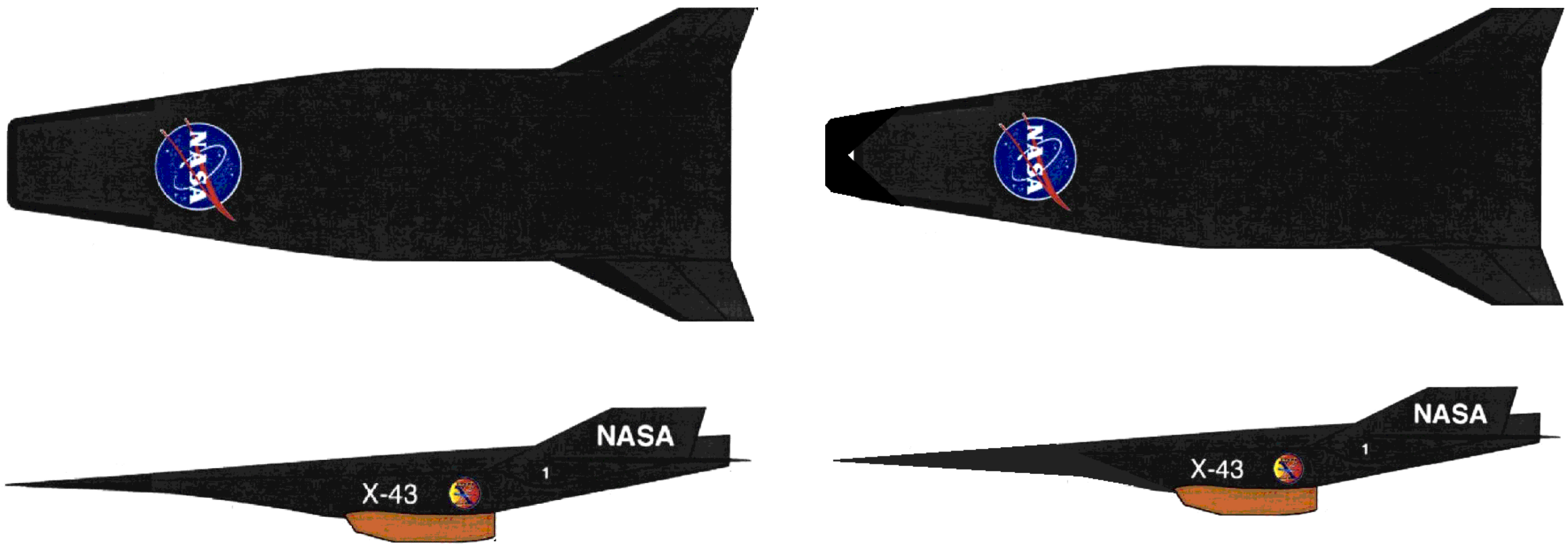
Scramjet Inlet “Point Design”(cont’d)





X-43A Side by Side Comparison

- Subtle but important shape differences Mach 10 Inlet likely would not start at Mach 7



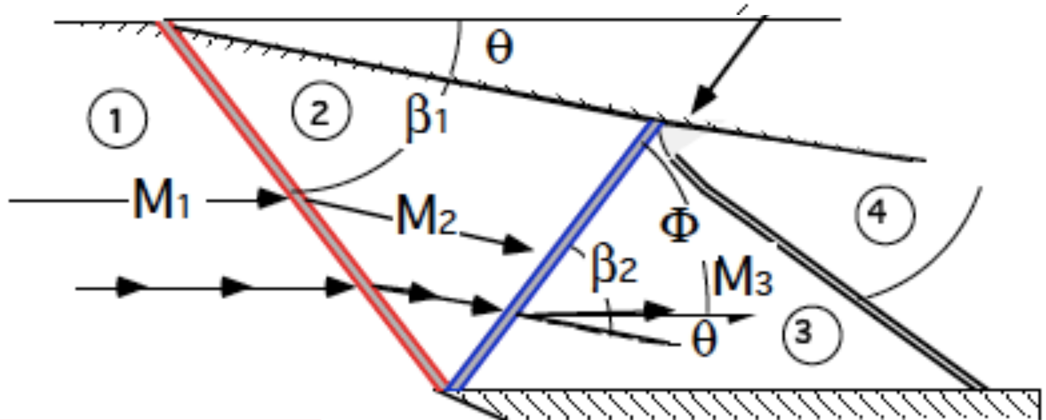
Mach 7 Vehicle

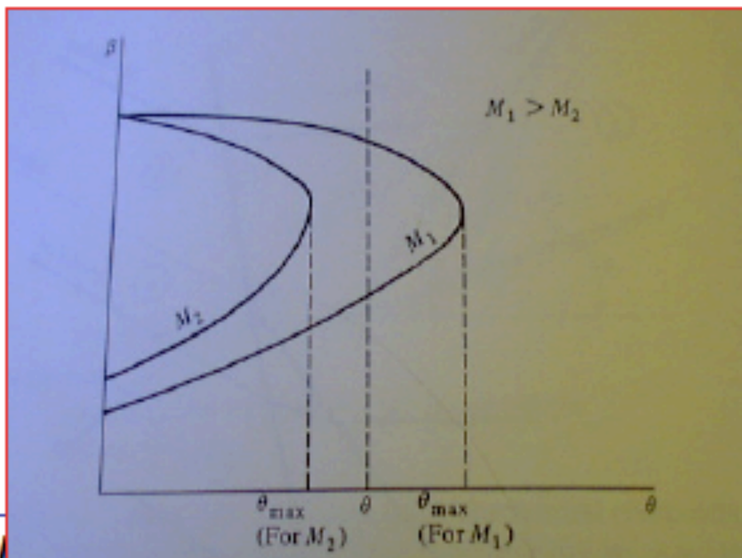
Mach 10 Vehicle

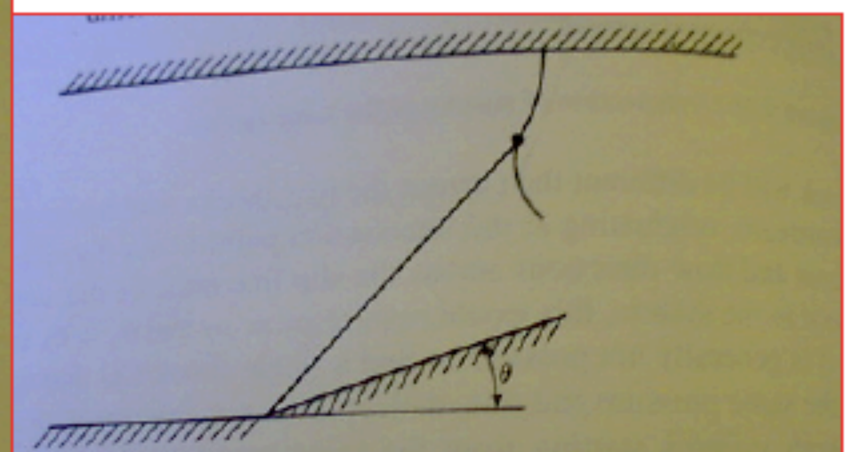
Off Design Operation

(cont'd)

Mach reflection ... localized strong shockwave ... starts bad train of events leading to flow separation and possible unstart

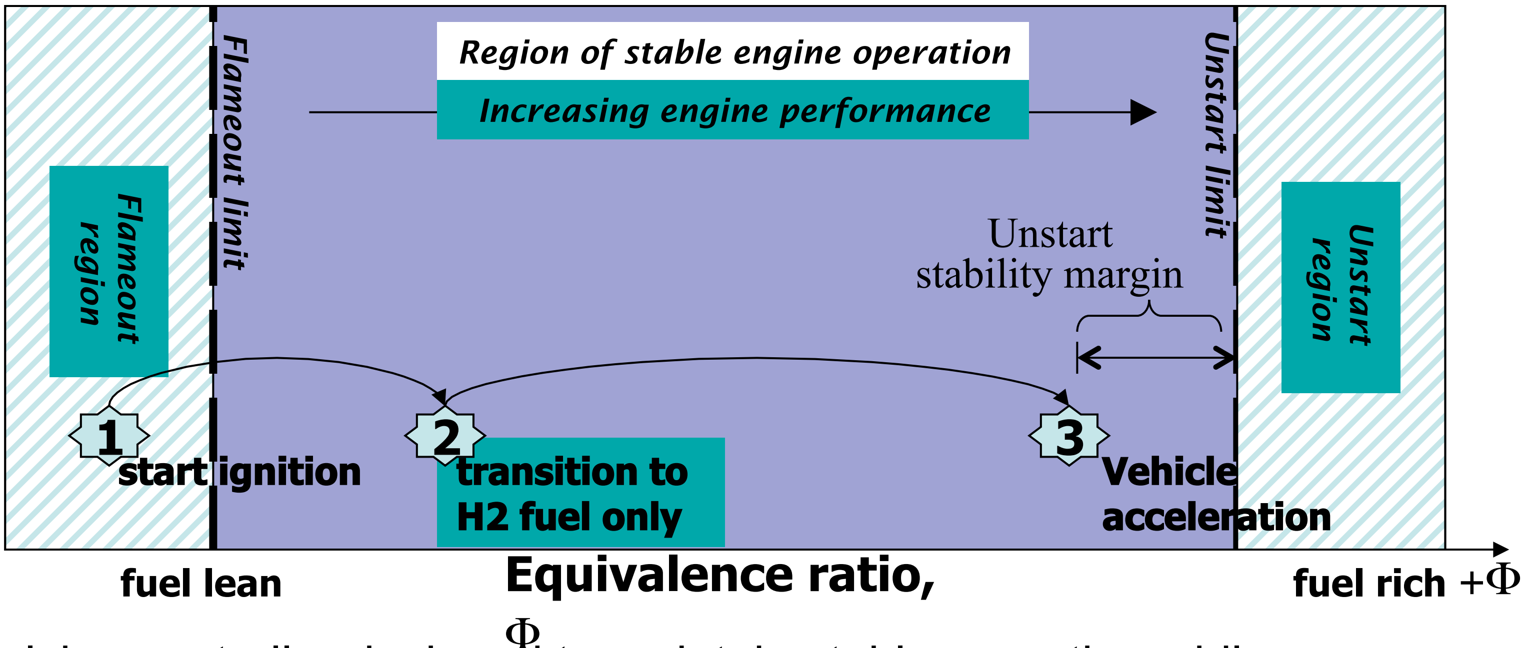






ScrRamjet design issues

• Supersonic flow places stringent requirements on the pressure and temperature of the flow, and requires that the fuel injection and mixing be extremely efficient.



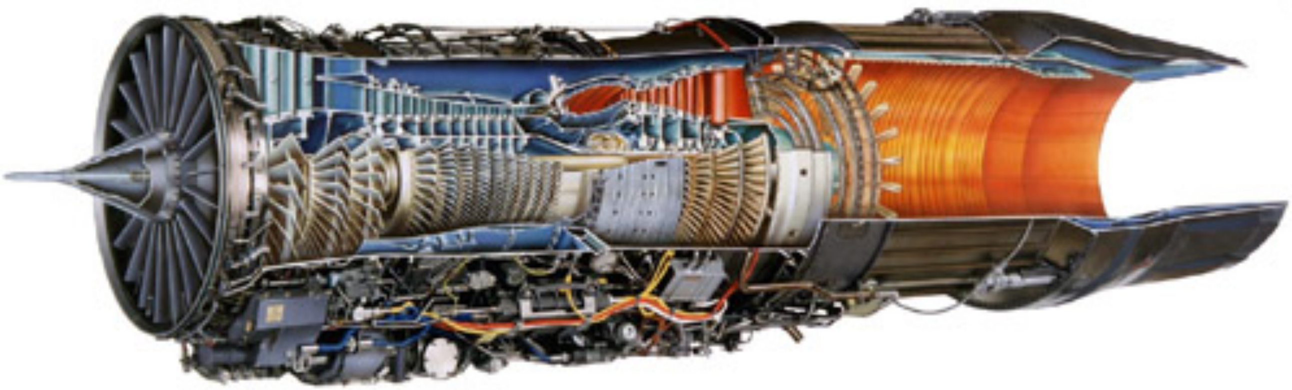
•Propulsion controller designed to maintain stable operation while achieving necessary performance

-- Flameout - low fuel flow condition where hydrogen-only combustion is not sustained

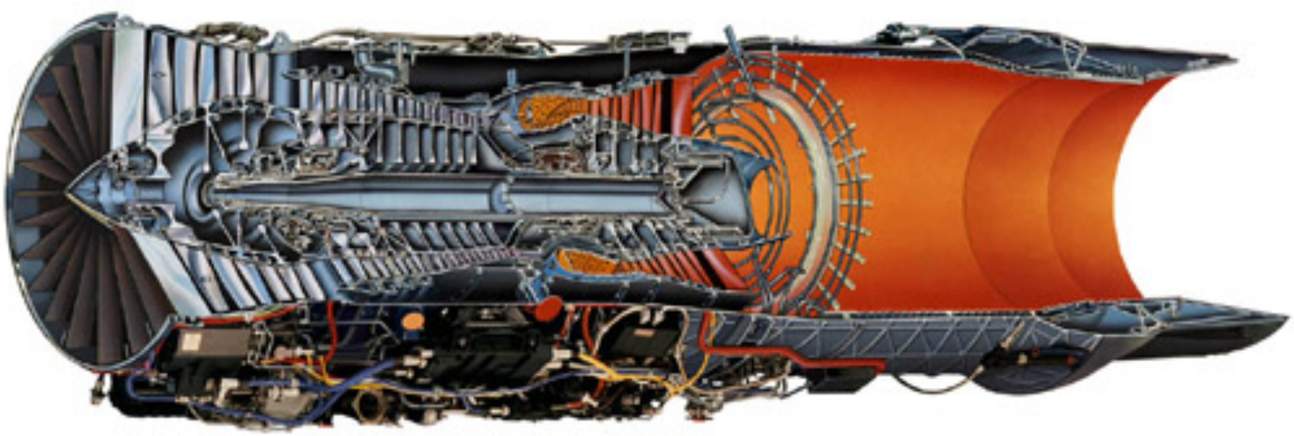
-- Unstart - high fuel flow condition where shock train moves through isolator causing causing normal shock to Occur .. Choked Nozzle result … with flow spill out … likely that shockwave forced back up to inlet .. *Very bad*

Typical Turbine Engine Design (F-15)

F100-PW-220



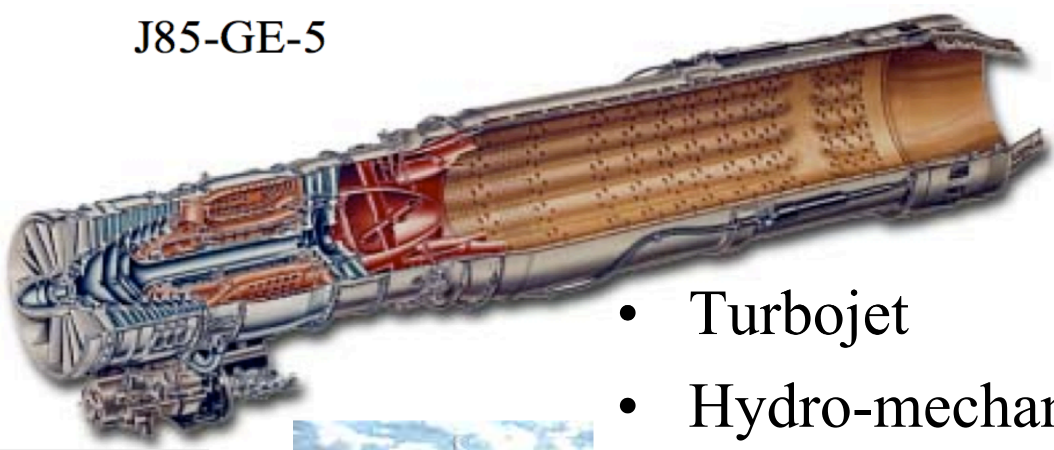
F100-PW-229

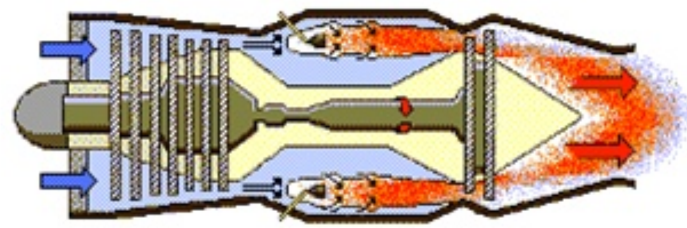


Incredibly Complex Thermo-Mechanical design

Typical Turbine Engine Design (T-38, F-5)

J85-GE-5





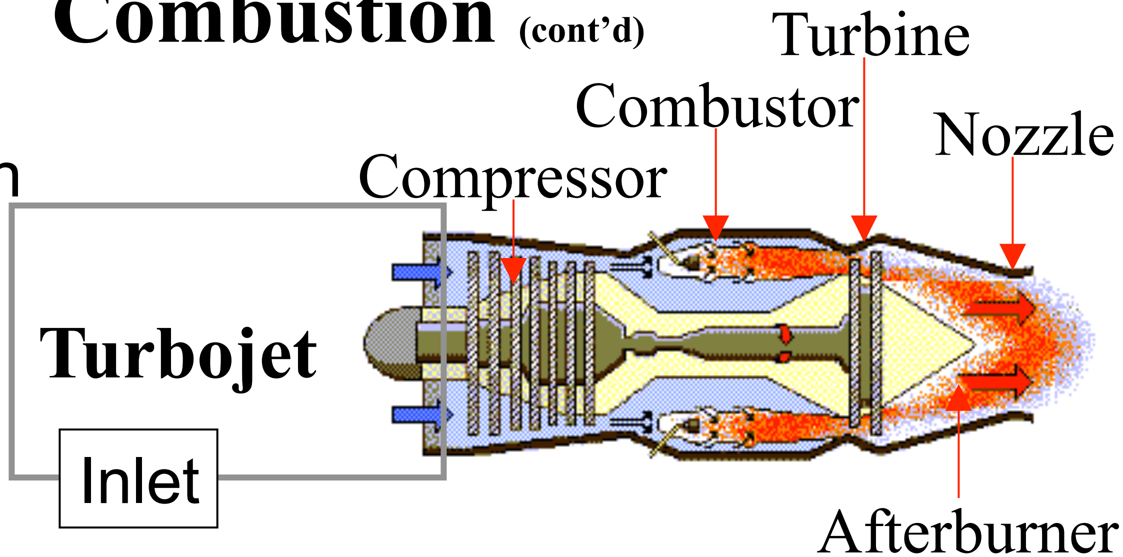
- Turbojet
- Hydro-mechanical controller
- Single Spool
- Max thrust 3,850 lbs/Mil thrust 2680 lbs
- Weight 584 lbs
- Overall pressure ratio 6.8 to 1
- First entered operational service 1961

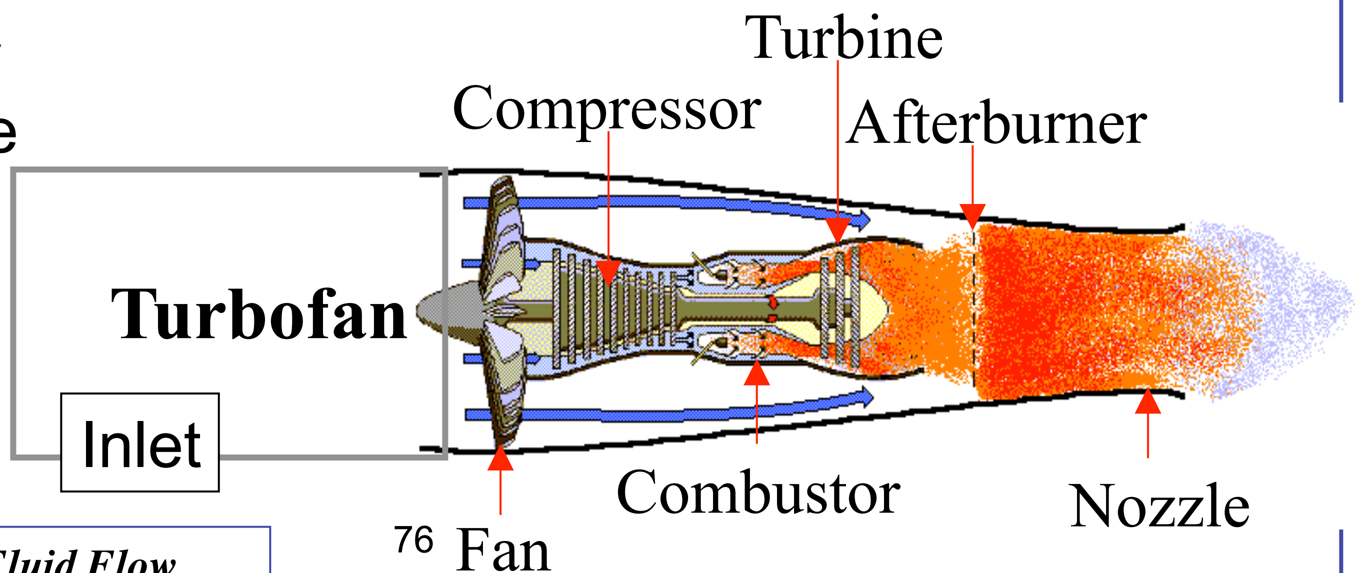
http://en.wikipedia.org/wiki/General_Electric_J85

Brayton Cycle for Airbreathing Combustion (cont'd)

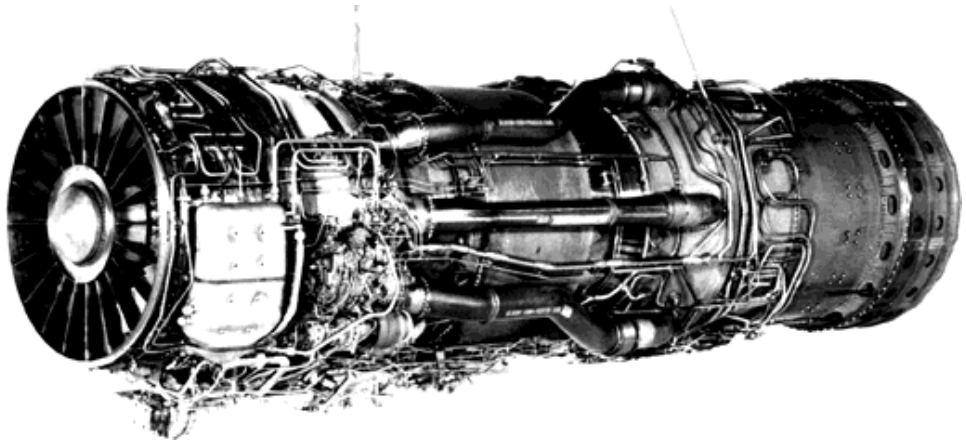
- Turbojet/Turbo fan engines Combustion Cycle steps

Compression and Power extraction steps use Turbo-machinery To augment cycle





Pratt and Whitney J58 Turbojet/Ramjet Combined Cycle Engine



Part Turbo-jet

Part Ram-jet

Dual Burner … Same flow path

Pratt and Whitney J58 Turbojet/Ramjet Combined Cycle Engine (cont'd)

• Above mach 3 a portion of the flow bypasses the turbine and burns Directly in afterburner providing about 80% or thrust …

• At lower speeds the engine operates as a normal supersonic Turbojet … same nozzle used by both operational modes