$$\int \frac{\sqrt{e^{mx}-1}}{1+b\cdot e^{mx}}dx = \frac{\frac{2\cdot(b+1)}{m}\tan^{-1}\sqrt{\left(\frac{b}{b+1}\right)\cdot\left(e^{mx}-1\right)}}{\sqrt{b+1}\cdot\sqrt{b}} - \left(\frac{2}{m}\right)\tan^{-1}\sqrt{e^{mx}-1} =$$

$$\frac{2}{m}\left[\left(\sqrt{\frac{b+1}{b}}\right)\tan^{-1}\sqrt{\left(\frac{b}{b+1}\right)\cdot\left(e^{mx}-1\right)} - \tan^{-1}\sqrt{e^{mx}-1}\right]$$

$$\rightarrow\left\{m=2,..b=\frac{\gamma-1}{2},...e^{mx}=M^2\rightarrow\right\}$$

$$\int \frac{\sqrt{e^{mx}-1}}{1+b\cdot e^{mx}}dx = \int \frac{\sqrt{M^2-1}}{1+\frac{\gamma-1}{2}\cdot M^2}\frac{dM}{M} =$$

$$\frac{2}{2}\left[\left(\sqrt{\frac{\frac{\gamma-1}{2}+1}{\frac{\gamma-1}{2}}}\right)\tan^{-1}\sqrt{\left(\frac{\frac{\gamma-1}{2}}{\frac{\gamma-1}{2}+1}\right)\cdot\left(M^2-1\right)} - \tan^{-1}\sqrt{M^2-1}\right] =$$

$$\boxed{\left[\left(\sqrt{\frac{\gamma+1}{\gamma-1}}\right)\tan^{-1}\sqrt{\frac{\gamma+1}{\gamma-1}\cdot\left(M^2-1\right)} - \tan^{-1}\sqrt{M^2-1}\right]}$$

$$\int \frac{\sqrt{e^{mx}-1}}{1+b\cdot e^{mx}}dx = \int \frac{\left(e^{mx}-1\right)}{\left(1+b\cdot e^{mx}\right)\cdot\sqrt{e^{mx}-1}}dx = \int \frac{\left(e^{mx}+b\cdot e^{mx}\right)-\left(1+b\cdot e^{mx}\right)}{\left(1+b\cdot e^{mx}\right)\cdot\sqrt{e^{mx}-1}}dx =$$

$$\left[\int \frac{\left(e^{mx}+b\cdot e^{mx}\right)}{\left(1+b\cdot e^{mx}\right)\cdot\sqrt{e^{mx}-1}} - \frac{\left(1+b\cdot e^{mx}\right)}{\left(1+b\cdot e^{mx}\right)\cdot\sqrt{e^{mx}-1}}\right]dx = \left[\int \frac{(1+b)\cdot e^{mx}}{\left(1+b\cdot e^{mx}\right)\cdot\sqrt{e^{mx}-1}} - \frac{1}{\sqrt{e^{mx}-1}}\right]dx$$

$\rightarrow Part1:$

$$\int \frac{(1+b)\cdot e^{mx}}{\left(1+b\cdot e^{mx}\right)\cdot\sqrt{e^{mx}-1}}dx \rightarrow \left|\begin{array}{c}\vartheta = e^{mx}\\ d\vartheta = m\cdot e^{mx}\cdot dx \rightarrow dx = \frac{d\vartheta}{m\cdot e^{mx}} = \frac{1}{m}\cdot\frac{d\vartheta}{\vartheta}\end{array}\right.$$

$$\rightarrow substitute....\frac{(1+b)\cdot e^{mx}}{\left(1+b\cdot e^{mx}\right)\cdot\sqrt{e^{mx}-1}}dx = \frac{(1+b)\cdot\vartheta}{(1+b\cdot\vartheta)\cdot\sqrt{\vartheta-1}}\cdot\frac{1}{m}\cdot\frac{d\vartheta}{\vartheta} = \frac{1}{m}\cdot\frac{(1+b)}{(1+b\cdot\vartheta)\cdot\sqrt{\vartheta-1}}\cdot d\vartheta$$

$$\rightarrow substitute...\left|\begin{array}{c}\nu = \sqrt{\vartheta-1}\rightarrow\vartheta=\nu^2+1\\ d\nu = \frac{1}{2}\cdot\frac{d\vartheta}{\sqrt{\vartheta-1}}\rightarrow d\vartheta = 2\cdot\nu\cdot d\nu\end{array}\right.$$

$$\rightarrow\int \frac{(1+b)\cdot e^{mx}}{\left(1+b\cdot e^{mx}\right)\cdot\sqrt{e^{mx}-1}}dx = \int\frac{1}{m}\cdot\frac{(1+b)}{\left(1+b\cdot\left(\nu^2+1\right)\right)\cdot\nu}\cdot 2\cdot\nu\cdot d\nu = \frac{2}{m}\cdot\int\frac{d\nu}{\left(1+\left(\frac{b}{1+b}\right)\cdot\nu^2\right)}$$

$$\rightarrow substitute...\left|\begin{array}{c}w = \sqrt{\left(\frac{b}{1+b}\right)\cdot\nu^2} = \sqrt{\left(\frac{b}{1+b}\right)}\cdot\nu\\ dw = \sqrt{\left(\frac{b}{1+b}\right)}\cdot d\nu \rightarrow dw\cdot\sqrt{\frac{b+1}{b}}\end{array}\right. \rightarrow\int \frac{(1+b)\cdot e^{mx}}{\left(1+b\cdot e^{mx}\right)\cdot\sqrt{e^{mx}-1}}dx = \frac{2}{m}\cdot\sqrt{\frac{b+1}{b}}\int\frac{dw}{\left(1+w^2\right)}$$

$$\rightarrow\int \frac{(1+b)\cdot e^{mx}}{\left(1+b\cdot e^{mx}\right)\cdot\sqrt{e^{mx}-1}}dx = \frac{2}{m}\cdot\sqrt{\frac{b+1}{b}}\tan^{-1}(w) = \frac{2}{m}\cdot\sqrt{\frac{b+1}{b}}\tan^{-1}\left(\sqrt{\left(\frac{b}{1+b}\right)}\cdot\nu\right) =$$

$$\frac{2}{m}\cdot\sqrt{\frac{b+1}{b}}\tan^{-1}\left(\sqrt{\left(\frac{b}{1+b}\right)}\cdot\sqrt{\vartheta-1}\right) = \boxed{\frac{2}{m}\cdot\sqrt{\frac{b+1}{b}}\tan^{-1}\left(\sqrt{\left(\frac{b}{1+b}\right)}\cdot\sqrt{e^{mx}-1}\right)}$$

$\rightarrow Part2:$

$$\int\frac{1}{\sqrt{e^{mx}-1}}dx \rightarrow \left|\begin{array}{c}\vartheta = e^{mx}\\ d\vartheta = m\cdot e^{mx}\cdot dx \rightarrow dx = \frac{d\vartheta}{m\cdot e^{mx}} = \frac{1}{m}\cdot\frac{d\vartheta}{\vartheta}\end{array}\right.$$

$$\int\frac{1}{\sqrt{e^{mx}-1}}dx = \frac{1}{m}\int\frac{1}{\vartheta\cdot\sqrt{\vartheta-1}}d\vartheta \rightarrow \left|\begin{array}{c}\nu = \sqrt{\vartheta-1}\rightarrow\vartheta=\nu^2+1\\ d\nu = \frac{1}{2}\cdot\frac{d\vartheta}{\sqrt{\vartheta-1}}\rightarrow d\vartheta = 2\cdot\nu\cdot d\nu\end{array}\right.$$

$$\int\frac{1}{\sqrt{e^{mx}-1}}dx = \frac{1}{m}\int\frac{2\cdot\nu}{\left(\nu^2+1\right)\cdot\nu}d\nu = \frac{2}{m}\int\frac{1}{\left(\nu^2+1\right)}d\nu = \frac{2}{m}\tan^{-1}\nu$$

$$= \frac{2}{m}\tan^{-1}\sqrt{\vartheta-1} = \frac{2}{m}\tan^{-1}\sqrt{e^{mx}-1}$$

$\rightarrow collect\ parts\ 1\ and\ 2$

$$\boxed{\int\frac{\sqrt{e^{mx}-1}}{1+b\cdot e^{mx}}dx = \frac{2}{m}\cdot\left[\sqrt{\frac{b+1}{b}}\tan^{-1}\left(\sqrt{\left(\frac{b}{1+b}\right)}\cdot\sqrt{e^{mx}-1}\right) - \tan^{-1}\sqrt{e^{mx}-1}\right]} ...Q.E.D!$$