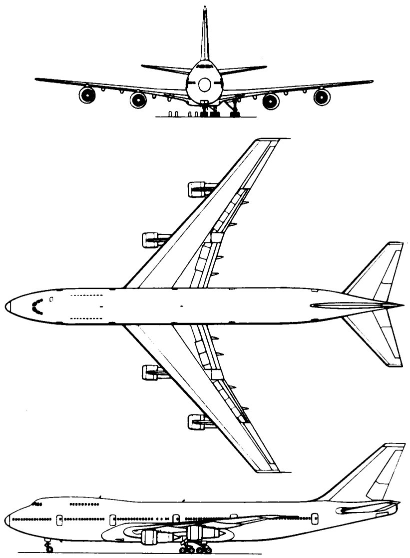
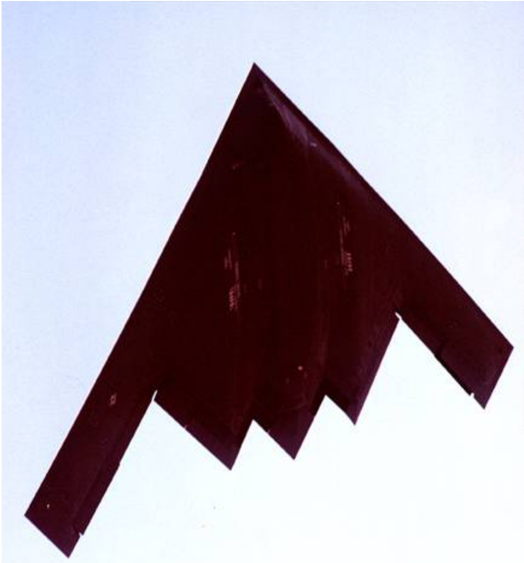
Section 7, Lecture 3:

Effects of Wing Sweep

- All modern high-speed aircraft have swept wings: WHY?

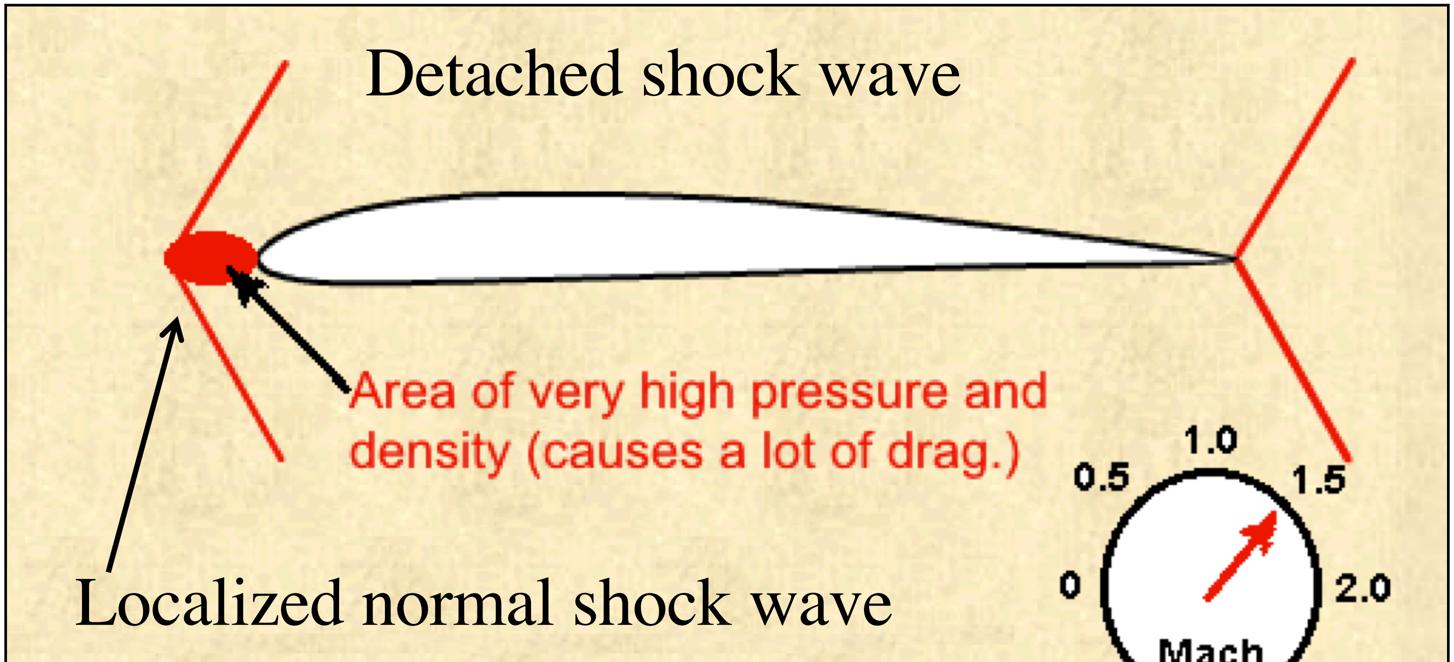




- Not in Anderson

Supersonic Airfoils (revisited)

- Normal Shock wave formed off the front of a blunt leading causes significant drag

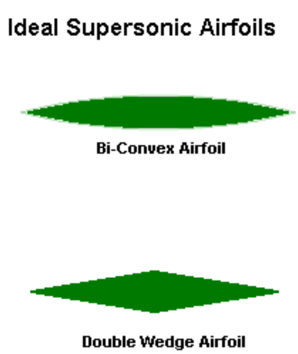


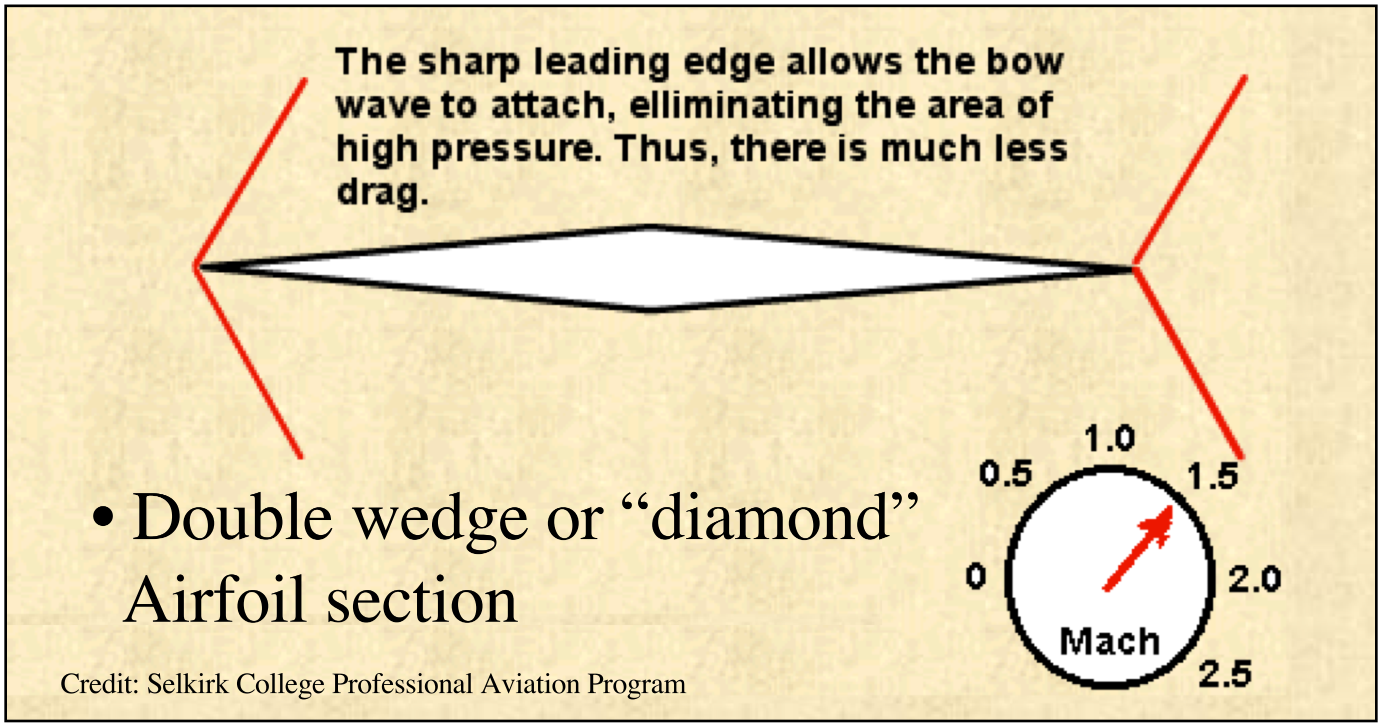
Credit: Selkirk College Professional Aviation Program

Supersonic Airfoils (revisited, 2)

• To eliminate this leading edge drag caused by detached bow wave Supersonic wings are typically quite sharp at the leading edge

• Design feature allows oblique wave to attach to the leading edge eliminating the area of high pressure ahead of the wing.



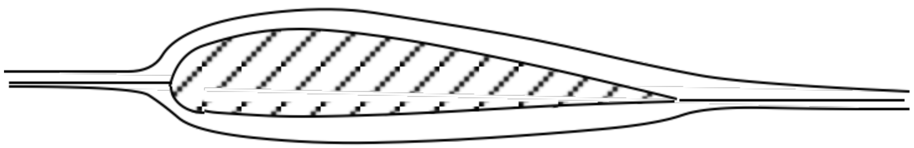


• Double wedge or “diamond” Airfoil section

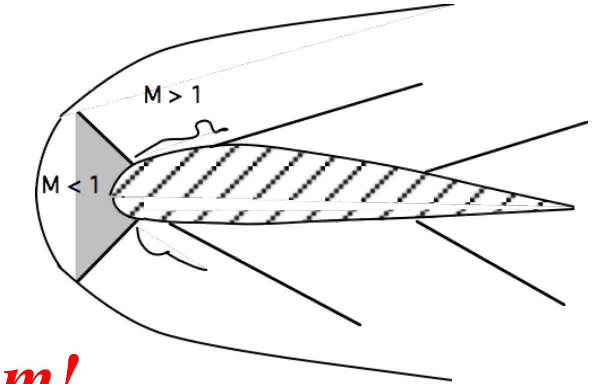
Credit: Selkirk College Professional Aviation Program

Wing Design 101

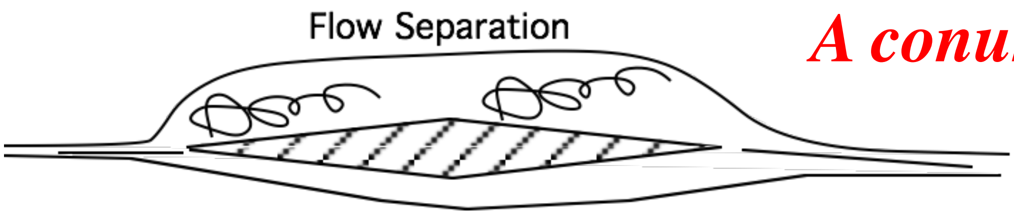
- Subsonic Wing in Subsonic Flow



- Subsonic Wing in Supersonic Flow

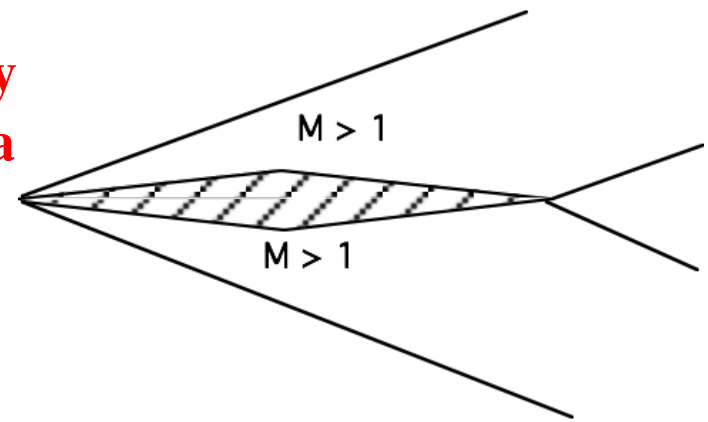


- Supersonic Wing in Subsonic Flow



A conundrum!

- Supersonic Wing in Supersonic Flow



- **Wings that work well sub-sonically generally don’t work well supersonically, and vice-versa**

→ **Leading edge** Wing-sweep can overcome problem with poor performance of sharp leading edge wing in subsonic flight.

Wing Design 101 (2)

• Compromise High-Sweep Delta design generates lift at low speeds by increasing the angle-of-attack, but also has sufficient sweepback and slenderness to perform very efficiently at high speeds.

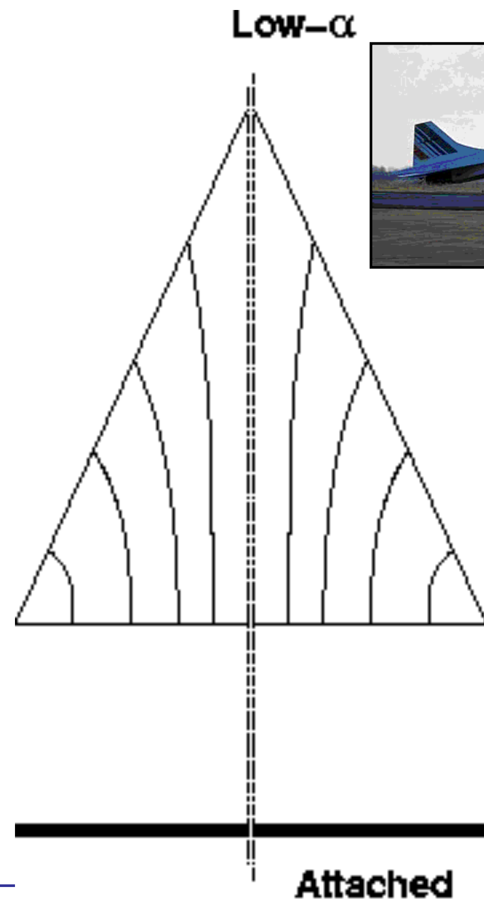
• On a traditional aircraft wing a trailing vortex is formed only at the wing tips.

• On a delta-wing at low speeds, the vortex is formed along the entire wing surface and produces most of the lift.

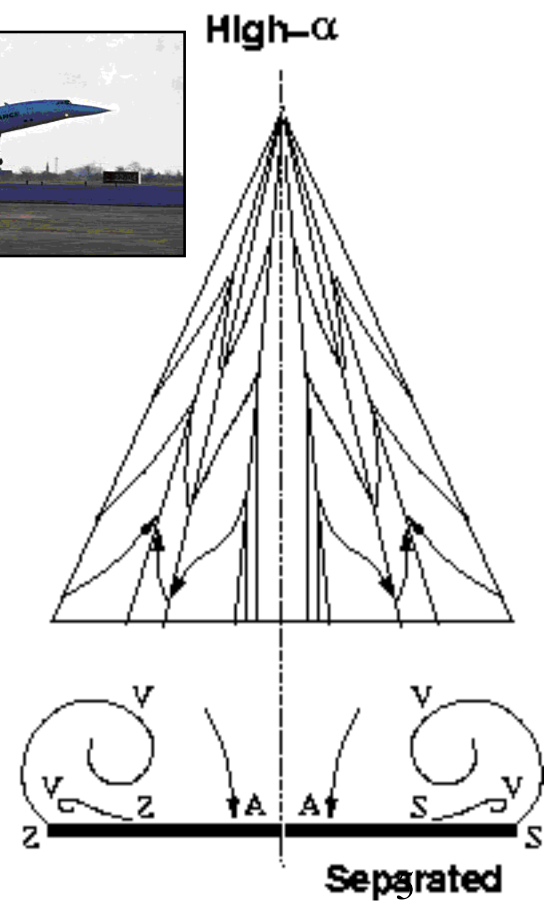
• Vortical-lift generation at high angles-of-attack is fundamental for reentry vehicles like the Space Shuttle to be able to fly at slow speeds.

• Highly-Swept Delta-Wing design … works "pretty well" in both flow regimes

Supersonic



Subsonic



Wing Design 101 (3)

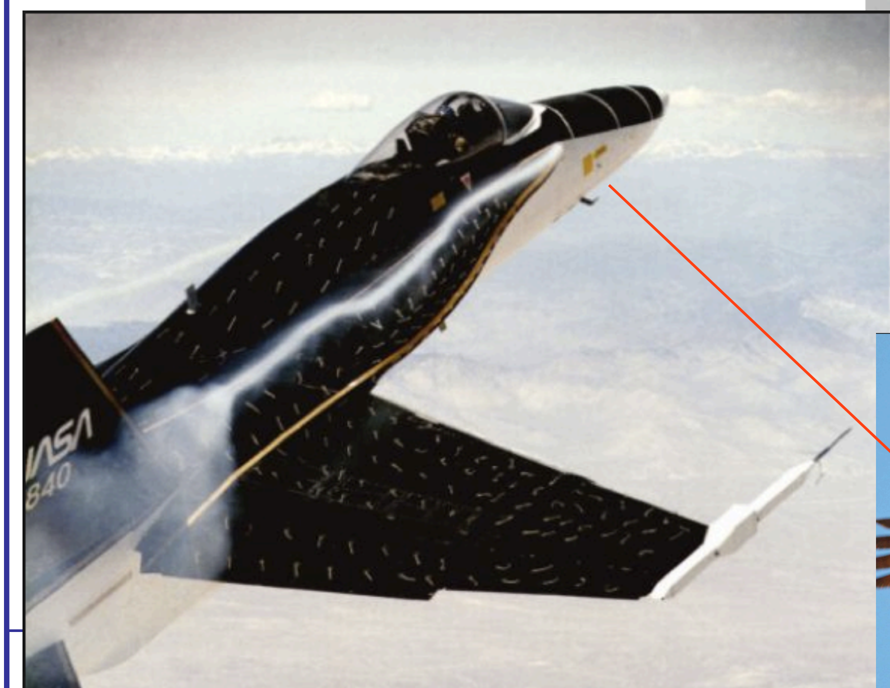
UtahState UNIVERSITY — Mechanical & Aerospace Engineering

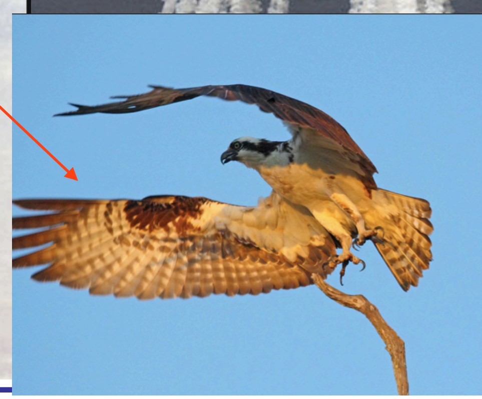
• In subsonic flow at high a the boundary layer can't follow the sharp curve around the leading edge and separates from the surface, as it roles up into a leading edge vortex (LEV) that produces lift on the upper surface.

• On a wing without sweep the LEV can only stay on the wing for a few seconds before it expands to the point where it has to separate from the wing and then the wing stalls, this event is called "dynamic stall".

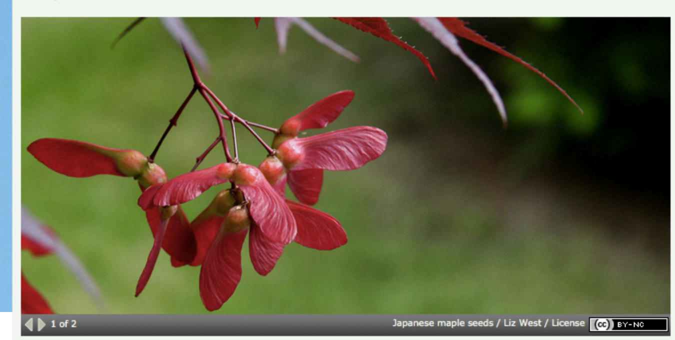
• As wing sweep increases the LEV remains on the wing longer before breaking down until the sweep angle ireaches 55 degrees.

• At 55 degrees sweep the spanwise component of flow is strong enough to push the axial component of the vortex toward the tip thereby creating a stabile, non expanding, cone shaped vortex.



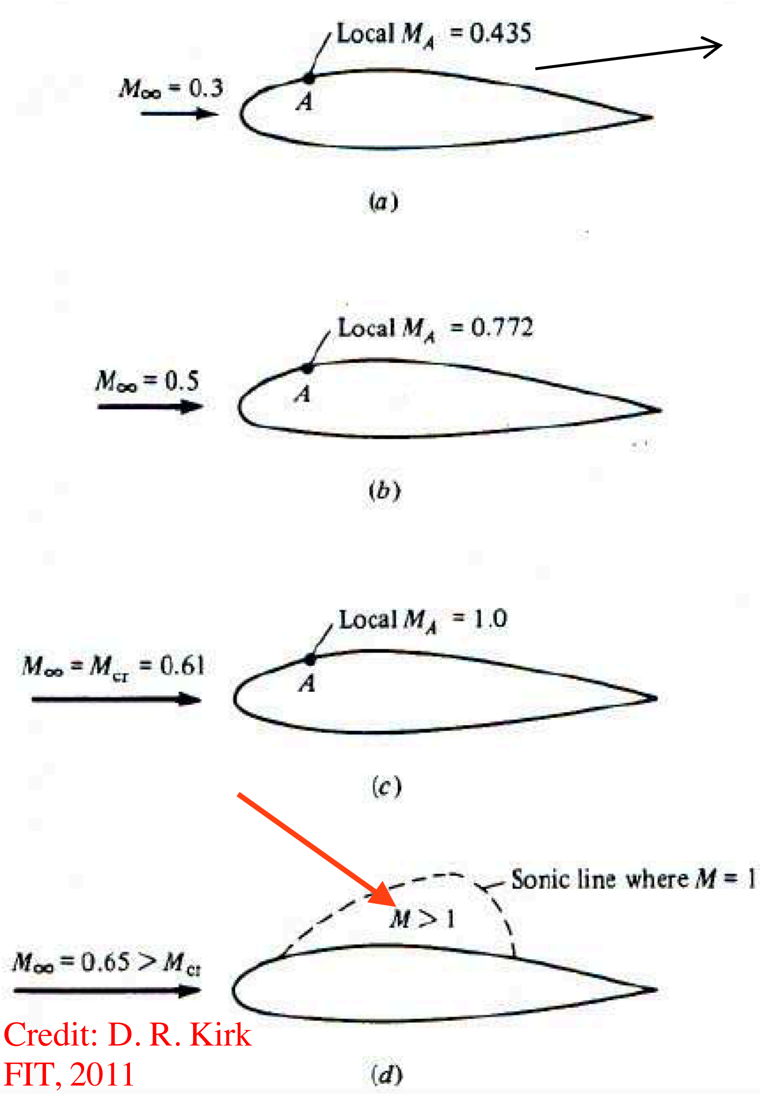


Leading edge of seed creates vortex lift: maple



Critical Mach Number

- As air expands around top surface near leading edge, upper surface velocity and Mach increases



Credit: D. R. Kirk FIT, 2011

» Local $M > M_\infty$

• Flow over airfoil may have sonic regions even though freestream $M_\infty < 1$

…

• Critical Mach number, M_{cr}, is Freestream Mach number at which the local Mach number at some point on the airfoil becomes sonic.

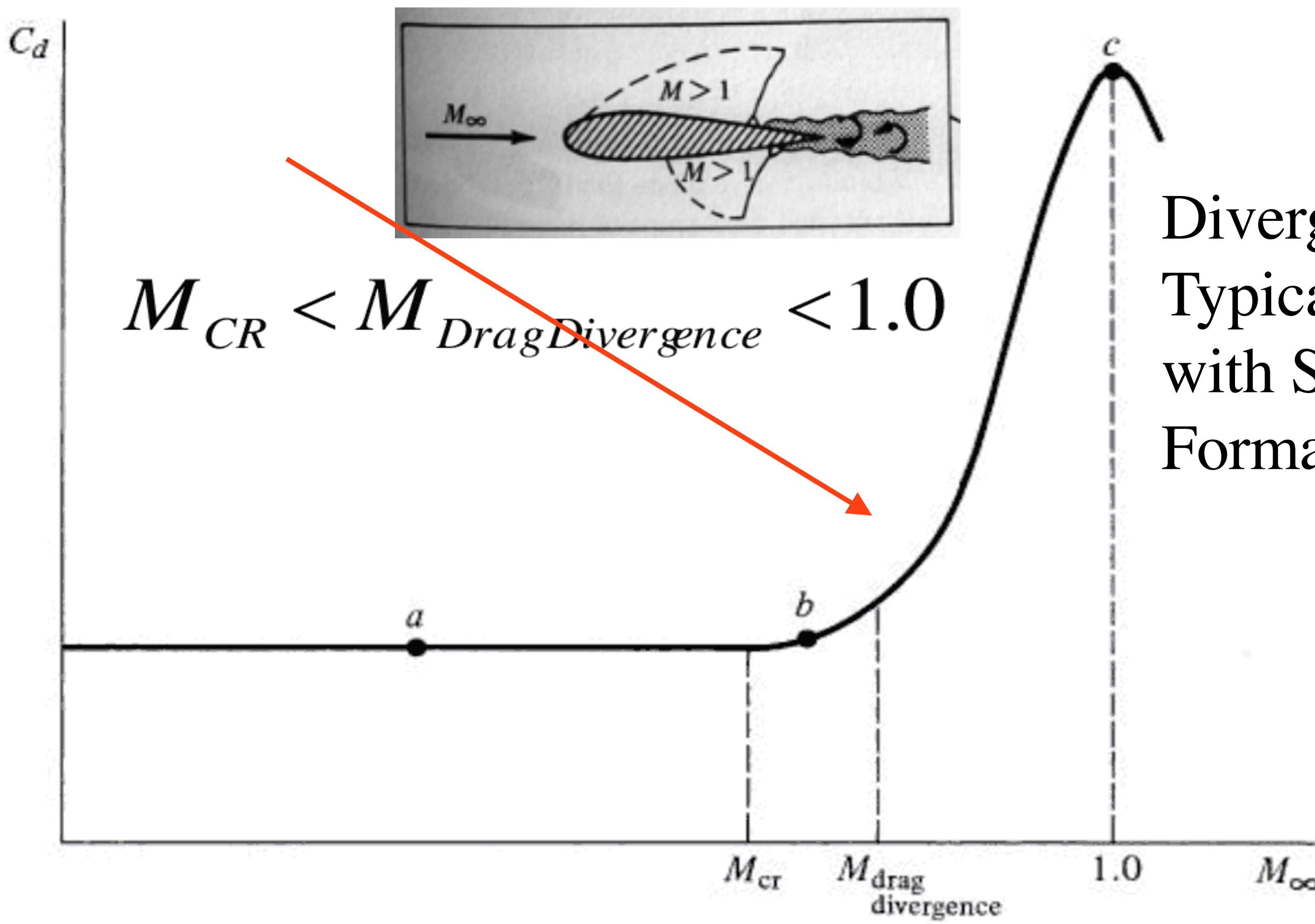
Beyond M_{cr} a substantial swath of upper surface > M =1

• *Drag on Airfoil begins to rise From its incompressible value once M_{cr} is reached*

• Corresponding pressure coefficient is known as critical pressure coefficient, $C_{P\,cr}$

Divergence Mach Number

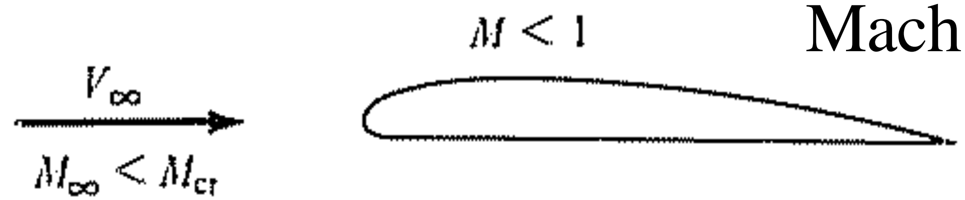
A second point of more rapid drag rise on the airfoil occurs when at Mach number slightly greater than M_{cr}, and the corresponding freestream Mach number is referred as the "Drag Mach Number," $M_{drag-divergence}$



$$M_{CR} < M_{DragDivergence} < 1.0$$

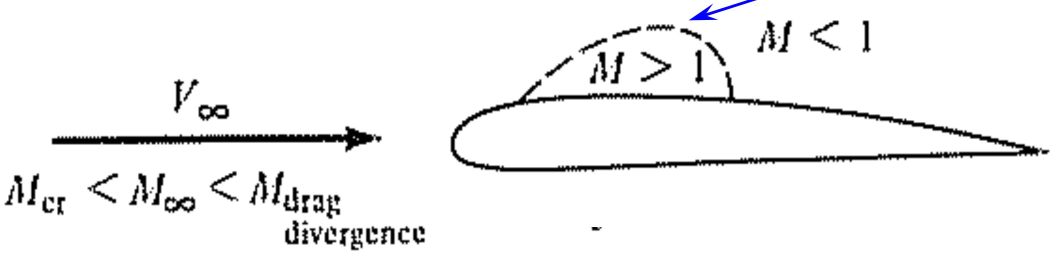
Divergence Drag Typically Associated with Shock Wave Formation

CRITICAL, DIVERGENCE Mach Numbers

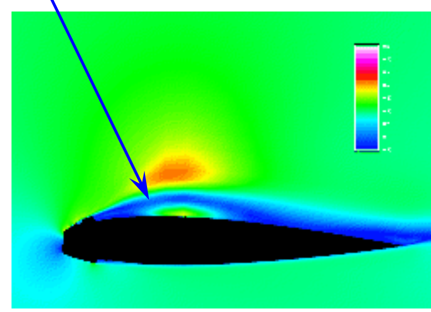


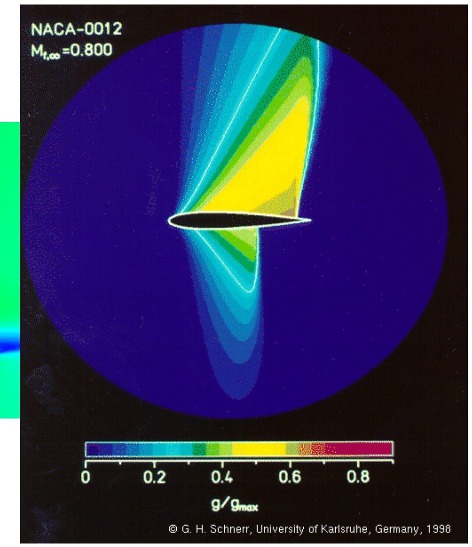
(a)

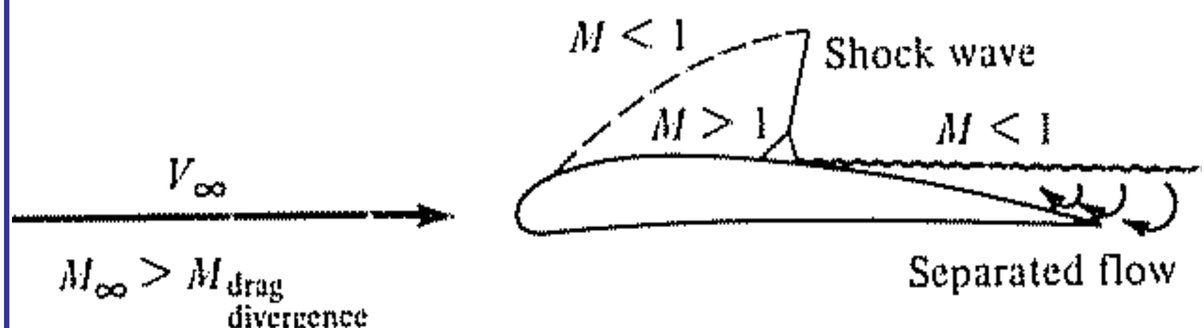
‘bubble’ of supersonic flow

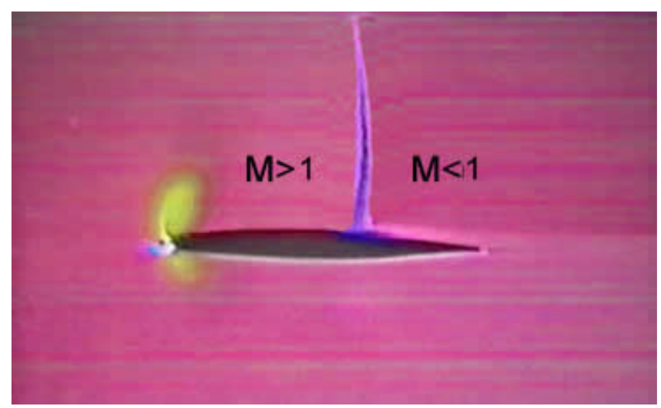


(b)





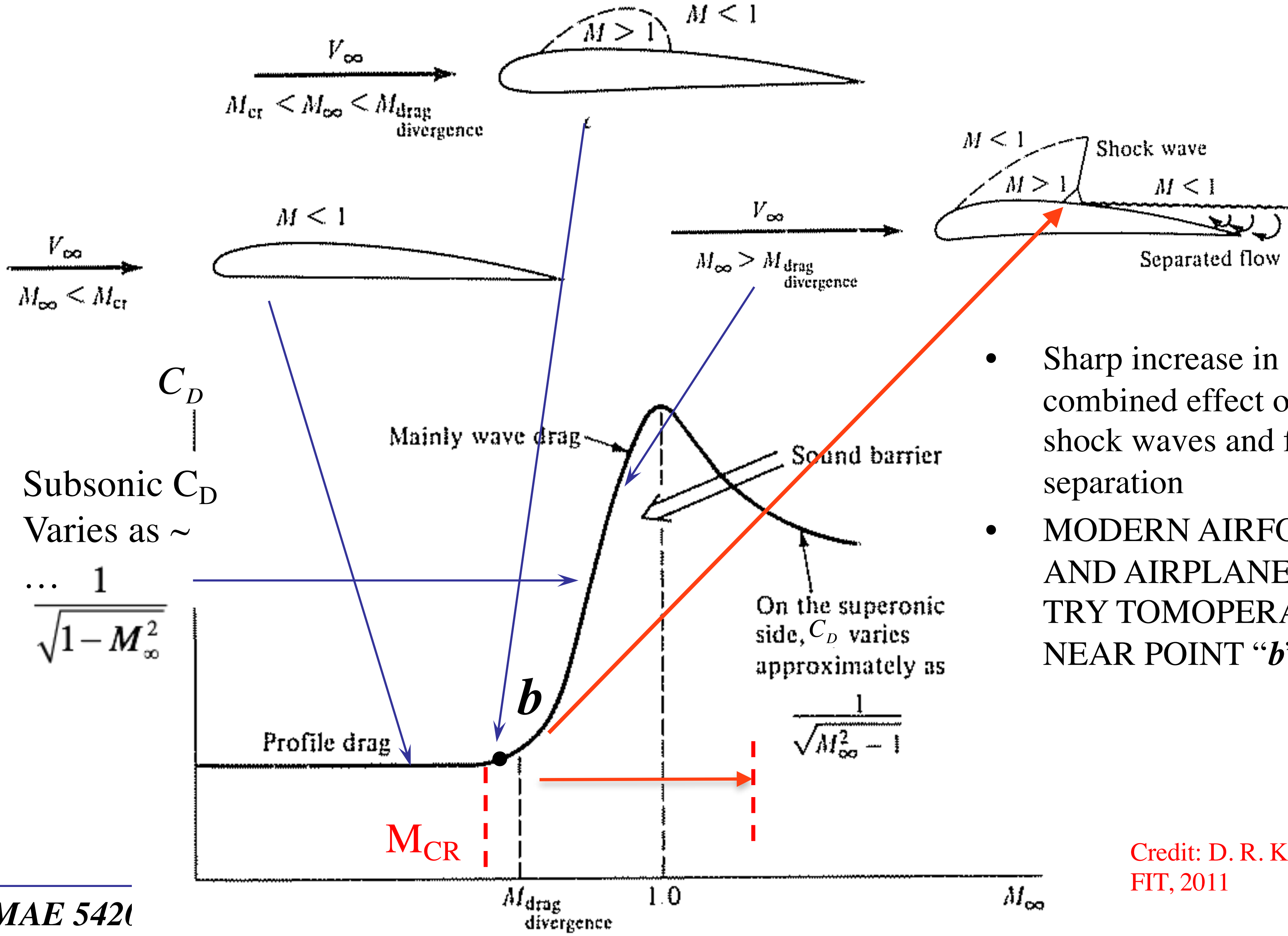




(c)

Credit: D. R. Kirk
FIT, 2011

CRITICAL, DIVERGENCE Mach Numbers (2)



- Sharp increase in C_D is combined effect of shock waves and flow separation
- MODERN AIRFOILS AND AIRPLANES TRY TOMOPERATE NEAR POINT "***b***"

Credit: D. R. Kirk
FIT, 2011

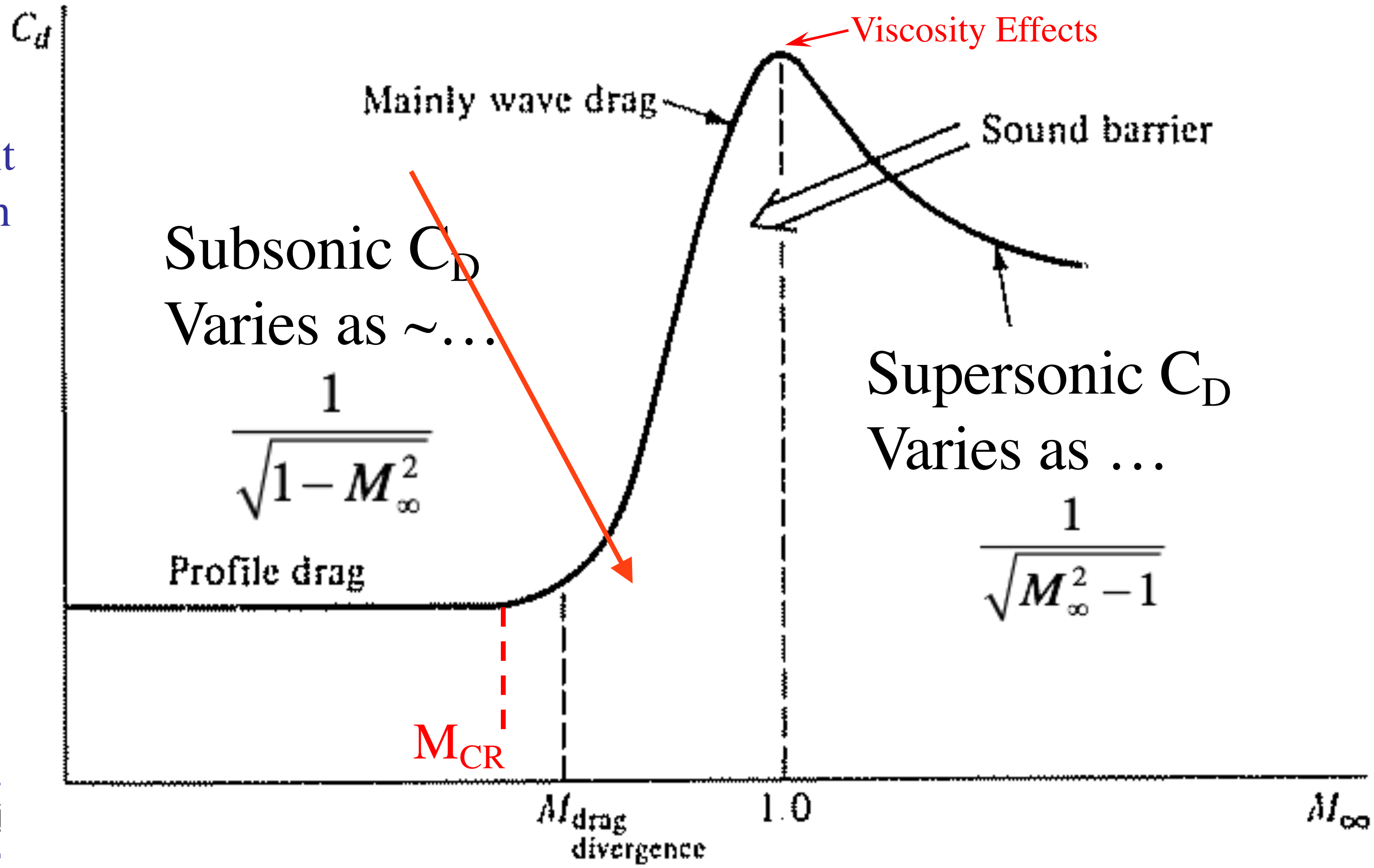
CRITICAL, DIVERGENCE Mach Numbers (3)

$$D = \boxed{D_{friction} + D_{pressure}} + \boxed{D_{wave}}$$

$$C_D = \boxed{C_{D,fric} + C_{D,press}} + \boxed{C_{D,wave}}$$

Only at transonic and supersonic speeds
D_{wave}= 0 for subsonic speeds below $M_{drag\text{-}divergence}$

Profile Drag
Profile Drag coefficient relatively constant with M_∞ at subsonic speeds



Subsonic C_D
Varies as ~…

$$\frac{1}{\sqrt{1-M_\infty^2}}$$

Supersonic C_D
Varies as …

$$\frac{1}{\sqrt{M_\infty^2-1}}$$

Credit: D. R. Kirk
FIT, 2011

Correcting for Compressibility

• Several Simple Transformations exist that allows us to take compressible transonic flow and map back to an “equivalent” incompressible body

• Equivalently, compressibility corrections allow the pressure coefficient of an incompressible airfoil to be transformed into compressible flow on the same body. Since inviscid lift and drag are related directly to the pressure coefficient, similar corrections hold.

• Transformations are written as a function of Freestream Mach number.

$$\left\{C_L, C_D, C_p\right\}_{M_\infty} \equiv \left\{C_L, C_D, C_p\right\}_{M=0} \cdot f\left(M_\infty\right)$$

Correcting for Compressibility (2)

- ***Prandtl Glauert Rule: FIRST-ORDER CORRECTION,*** the pressure coefficient, i.e. profile (pressure) drag at any point on a thin airfoil surface in a subsonic compressible flow is related to the pressure coefficient at the same point on the same airfoil in incompressible flow by

$$\left\{C_L, C_D, C_p\right\}_{M_\infty} \equiv \frac{\left\{C_L, C_D, C_p\right\}_{M=0}}{\sqrt{1-M_\infty^2}}$$

- Correction valid from approximately M_{crit} to about $M=0.9$

- *Correction Not Valid in Supersonic Flow*

- *Applies to Wave and Profile Drag Only*

Correcting for Compressibility (3)

- ***Karman-Tsien Rule: FIRST-ORDER CORRECTION***, the pressure coefficient, i.e. profile (pressure) drag at any point on a thin airfoil surface in a subsonic compressible flow is related to the pressure coefficient at the same point on the same airfoil in incompressible flow by

$$\left\{C_L, C_D, C_p\right\}_{M_\infty} \equiv \frac{\left\{C_L, C_D, C_p\right\}_{M=0}}{\sqrt{1-M_\infty^2} + \dfrac{M_\infty^2}{1+\sqrt{1-M_\infty^2}} \cdot \dfrac{\left\{C_L, C_D, C_p\right\}_{M=0}}{2}}$$

- Correction valid from approximately M_{crit} to about $M=0.98$

- *Correction Not Valid in Supersonic Flow*

- *Applies to Wave and Profile Drag Only*

Correcting for Compressibility (4)

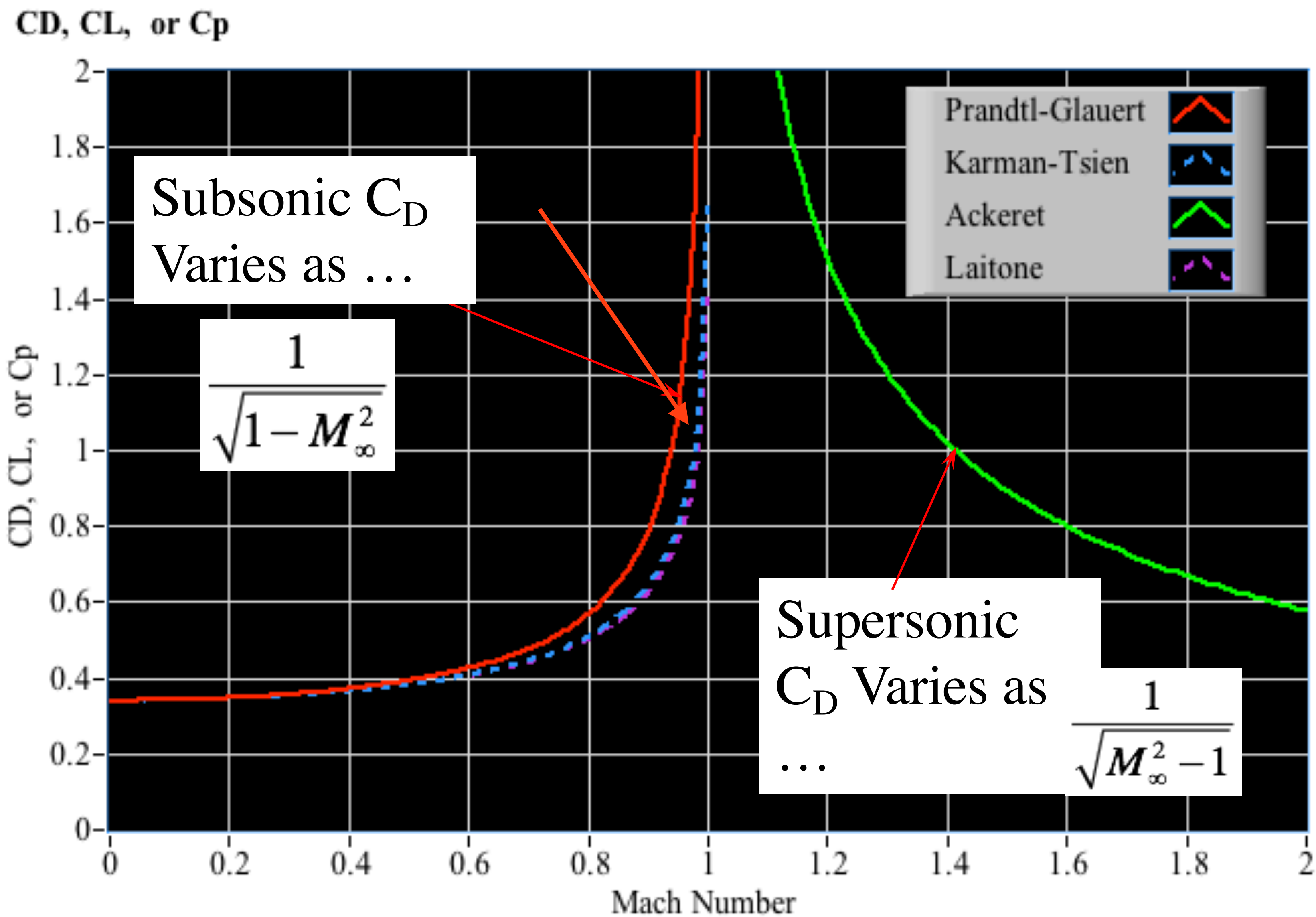
- ***Laitone's Rule:*** Better Accounting for Isentropic Compressibility, and Heating of Local airflow

$$\left\{C_L, C_D, C_p\right\}_{M_\infty} \equiv \frac{\left\{C_L, C_D, C_p\right\}_{M=0}}{\sqrt{1-M_\infty^2} + \frac{M_\infty^2\left(1+\frac{\gamma-1}{2}M_\infty^2\right)}{1+\sqrt{1-M_\infty^2}} \cdot \frac{\left\{C_L, C_D, C_p\right\}_{M=0}}{2}}$$

- ***Ackeret Rule:*** the pressure coefficient, i.e. profile +wave drag at any point on a thin Airfoil surface in a supersonic flow at M_2 is related to the pressure coefficient at M_1 at the same point on the Airfoil by (*Applies to Wave/Profile Drag Only*)

$$\left\{C_L, C_D, C_P\right\}_{M_2} = \left\{C_L, C_D, C_P\right\}_{M_1} \cdot \frac{\sqrt{M_1^2 - 1}}{\sqrt{M_2^2 - 1}}$$

Compressibility Transformations (5)



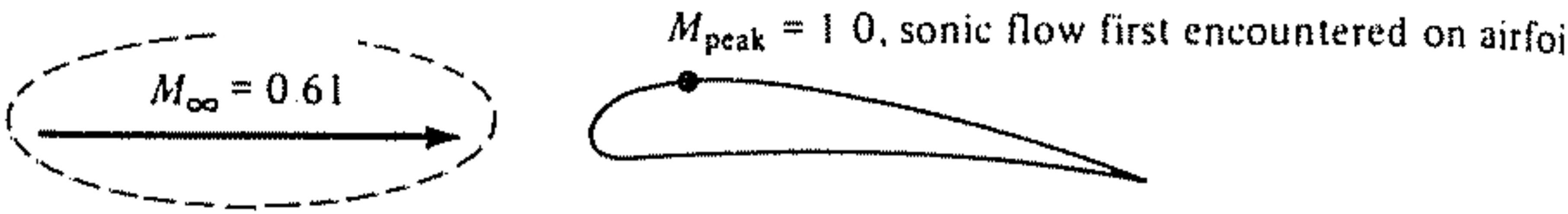
Subsonic C_D Varies as …

$$\frac{1}{\sqrt{1-M_\infty^2}}$$

Supersonic C_D Varies as …

$$\frac{1}{\sqrt{M_\infty^2-1}}$$

Calculating Critical Mach Number



Critical Mach number for the airfoil

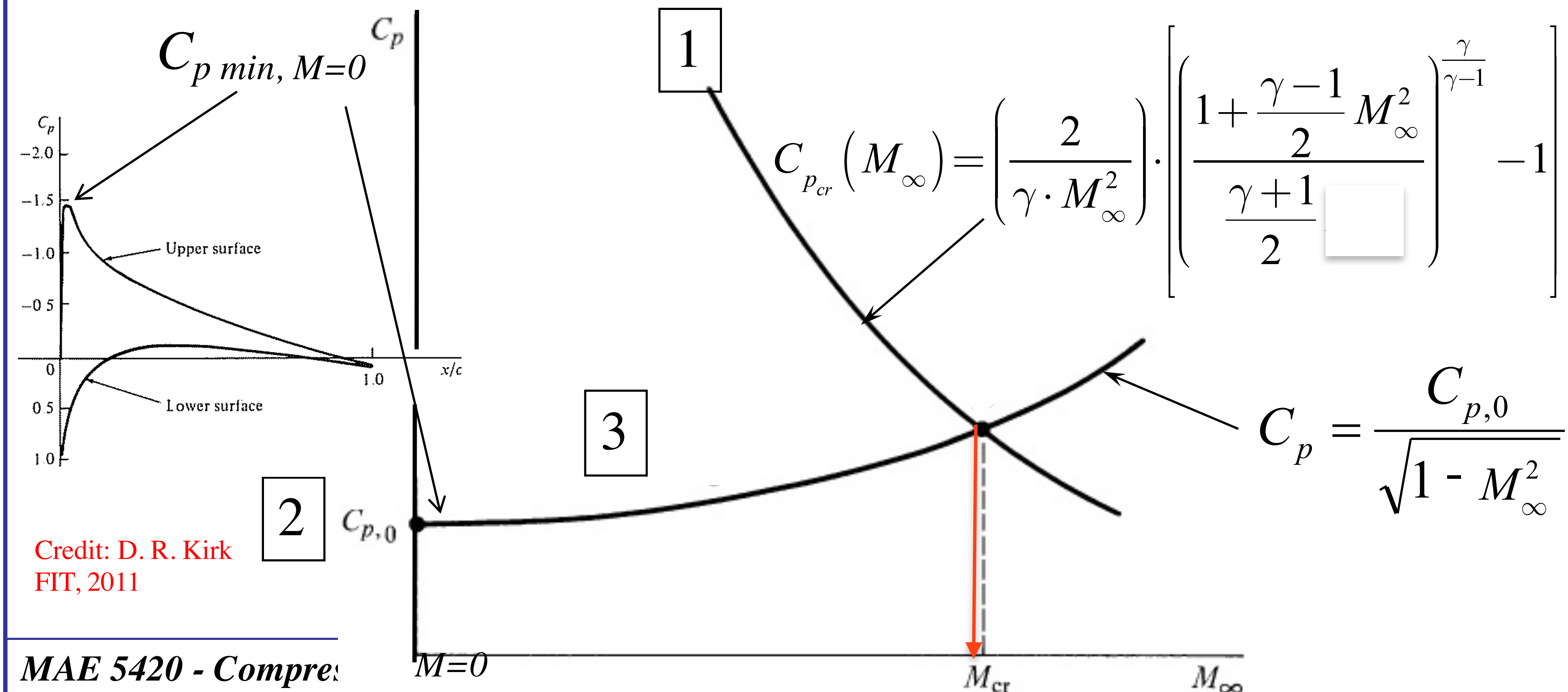
Calculate M_{CR} ... Freestream Mach number that leads to M=1 on upper surface

$$C_{p_{M=1}} = \frac{p_{M=1} - p_\infty}{\bar{q}} = \frac{p_{M=1} - p_\infty}{\frac{\gamma}{2} \cdot p_\infty \cdot M_\infty^2} = \left(\frac{1}{\frac{\gamma}{2} \cdot M_\infty^2} \right) \cdot \left(\frac{p_{M=1}}{p_\infty} - 1 \right)$$

$$\rightarrow \frac{p_{M=1}}{p_\infty} = \frac{p_{M=1}}{P_{0_\infty}} \cdot \frac{P_{0_\infty}}{p_\infty} = \left(\frac{1 + \frac{\gamma - 1}{2} M_{cr}^2}{1 + \frac{\gamma - 1}{2} (1)^2} \right)^{\frac{\gamma}{\gamma - 1}} \rightarrow C_{p_{cr}} = \left(\frac{2}{\gamma \cdot M_{cr}^2} \right) \left[\left(\frac{1 + \frac{\gamma - 1}{2} M_{cr}^2}{\frac{\gamma + 1}{2}} \right)^{\frac{\gamma}{\gamma - 1}} - 1 \right]$$

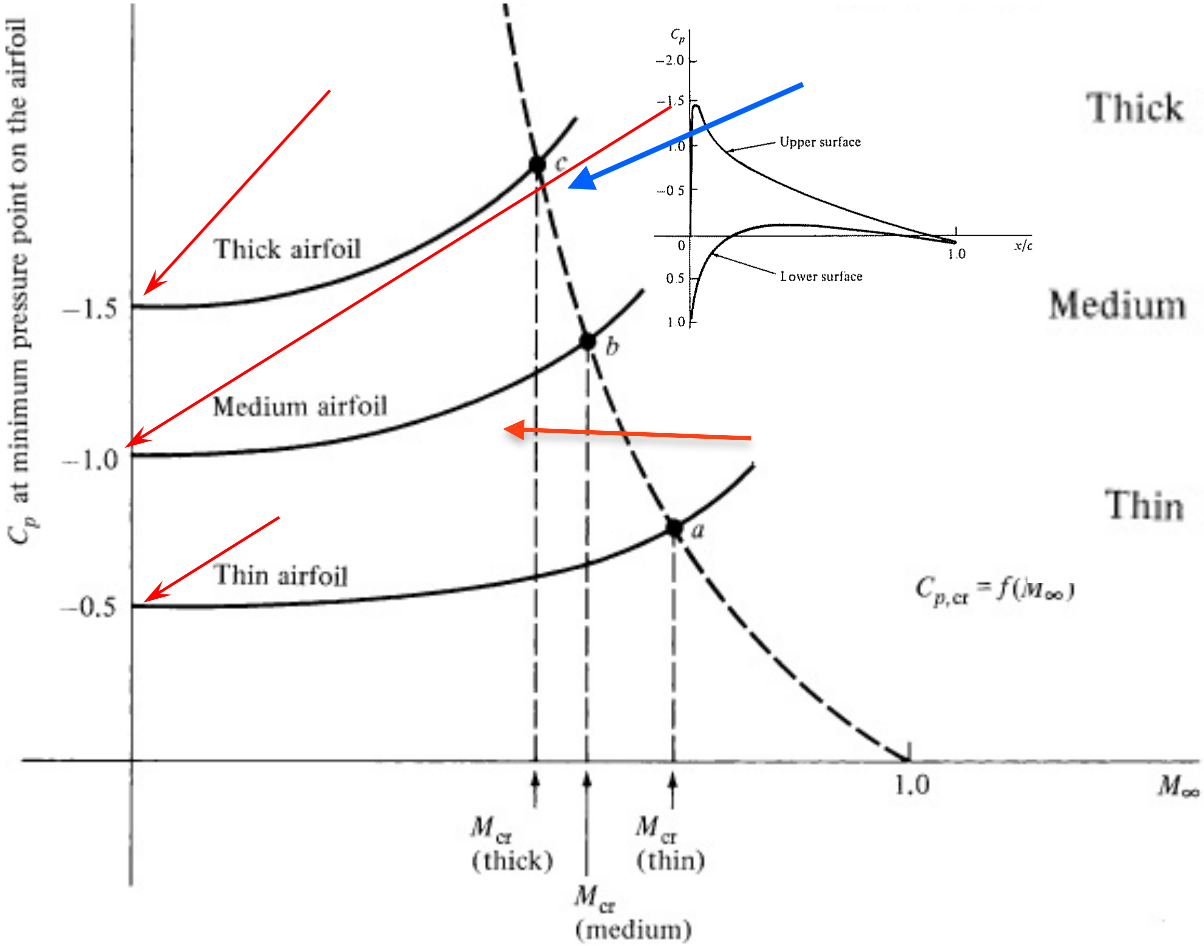
Calculate M_{CR}

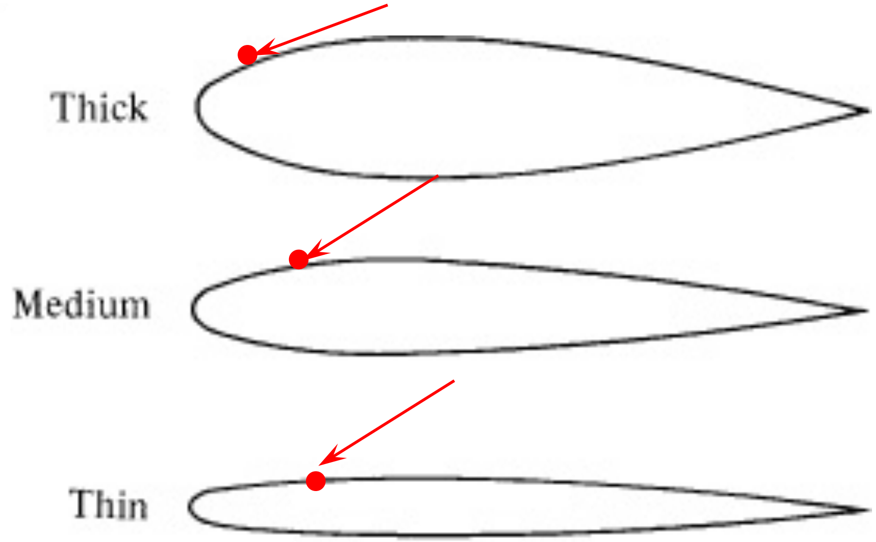
1. Plot curve of $C_{p,cr}$ vs. M_∞
2. Obtain incompressible value of $C_{p\,min}$ at minimum pressure point on given airfoil
3. Use any compressibility correction and plot C_P vs. M_∞
   - Intersection of these two curves represents point corresponding to sonic flow at minimum pressure location on airfoil
   - Value of M_∞ at this intersection is M_{CR}



Credit: D. R. Kirk FIT, 2011

Effect of Wing Thickness on M_{CR} (1)



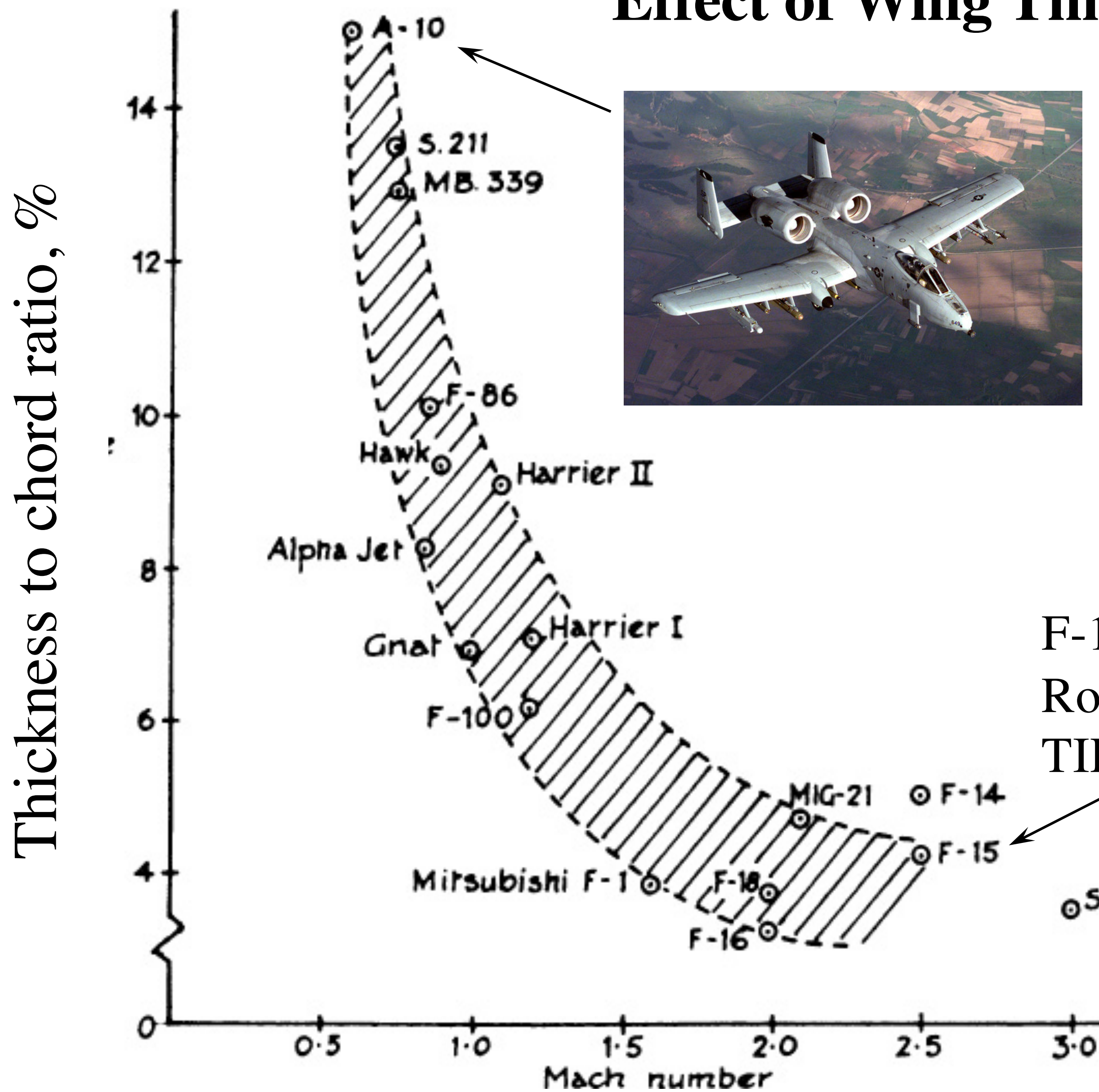


Note: thickness is relative to chord in all cases
Ex. NACA 0012 → 12 %

- Thick airfoils have a lower critical Mach number than thin airfoils
- Desirable to have MCR as high as possible
- Implication for design → high speed wings usually design with thin airfoils

Credit: D. R. Kirk
FIT, 2011

Effect of Wing Thickness on $M_{CR\ (2)}$



A-10
Root: NACA 6716
TIP: NACA 6713

F-15
Root: NACA 64A(.055)5.9
TIP: NACA 64A203

- **Faster top speed, Finer wing cross section**

Credit: D. R. Kirk
FIT, 2011

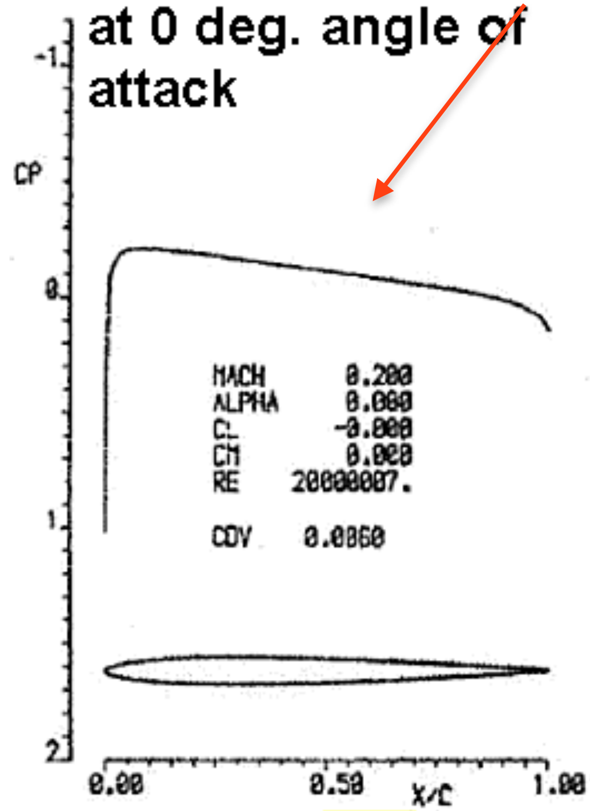
Example Calculation for Critical Mach Number

- Consider the pressure distributions on NACA 0006 and 0018 Airfoil sections at zero angle of attack and with no wing sweep

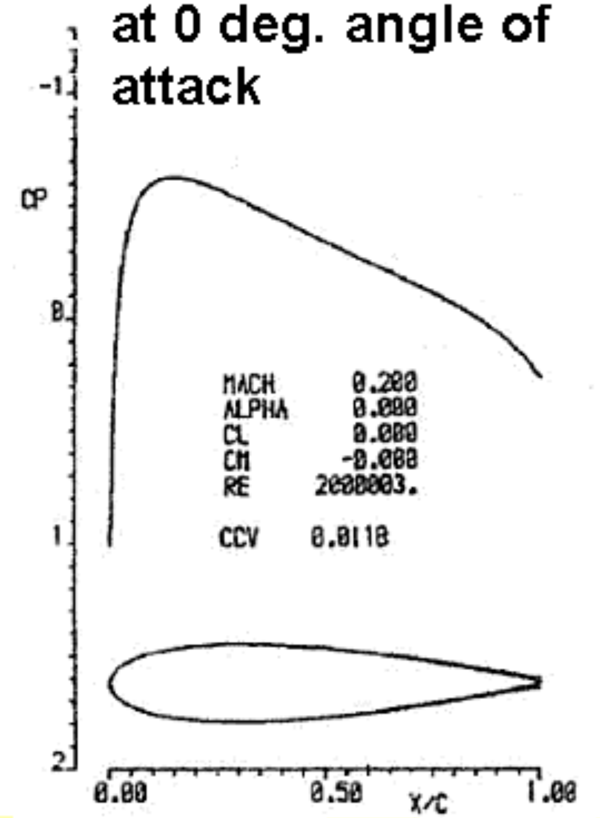
- Calculate the critical drag rise (subsonic) mach number on each section

- Compare results … what can you infer?

NACA 0006 Airfoil at 0 deg. angle of attack



NACA 0018 Airfoil at 0 deg. angle of attack

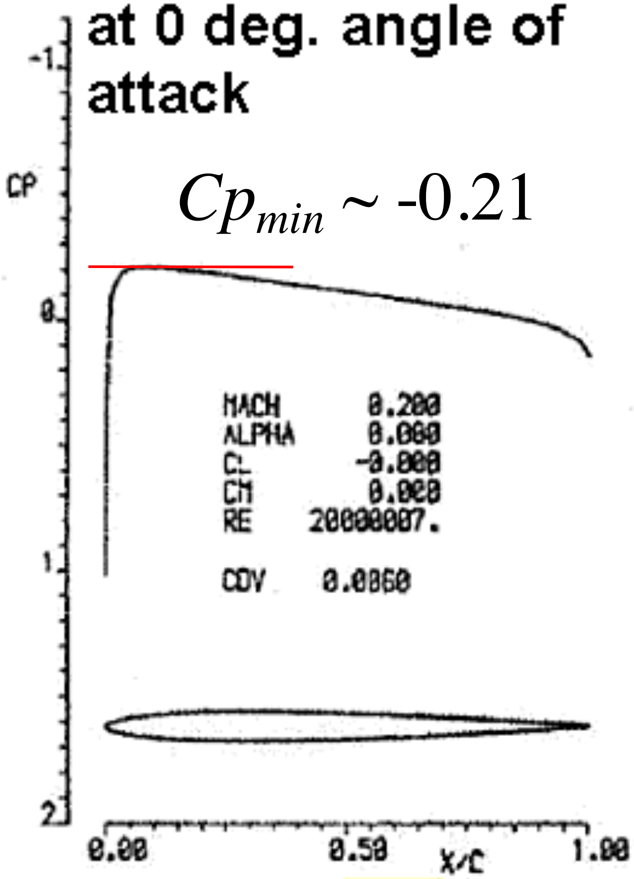


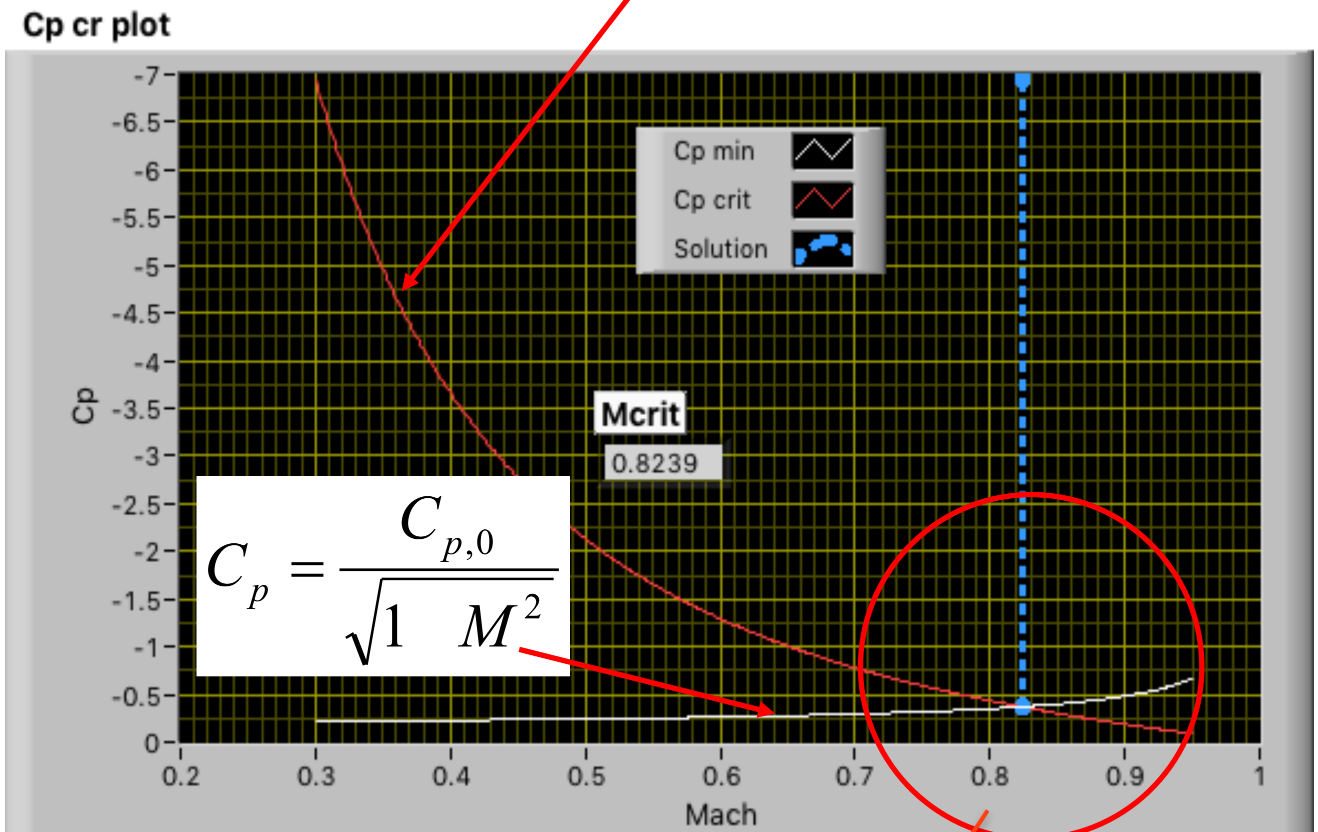
NACA 0006 at $0^o \alpha$

$$C_{p_{cr}}\left(M_\infty\right)=\left(\frac{2}{\gamma\cdot M_\infty^2}\right)\cdot\left[\left(\frac{1+\frac{\gamma-1}{2}M_\infty^2}{\frac{\gamma+1}{2}M_\infty^2}\right)^{\frac{\gamma}{\gamma-1}}-1\right]$$

NACA 0006 Airfoil at 0 deg. angle of attack

$Cp_{min} \sim -0.21$





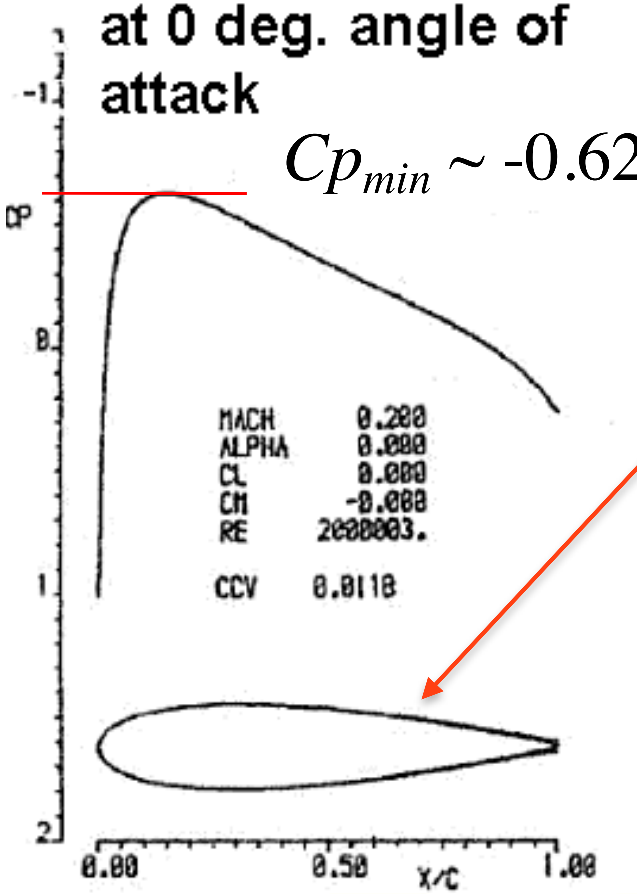
$$C_p=\frac{C_{p,0}}{\sqrt{1-M^2}}$$

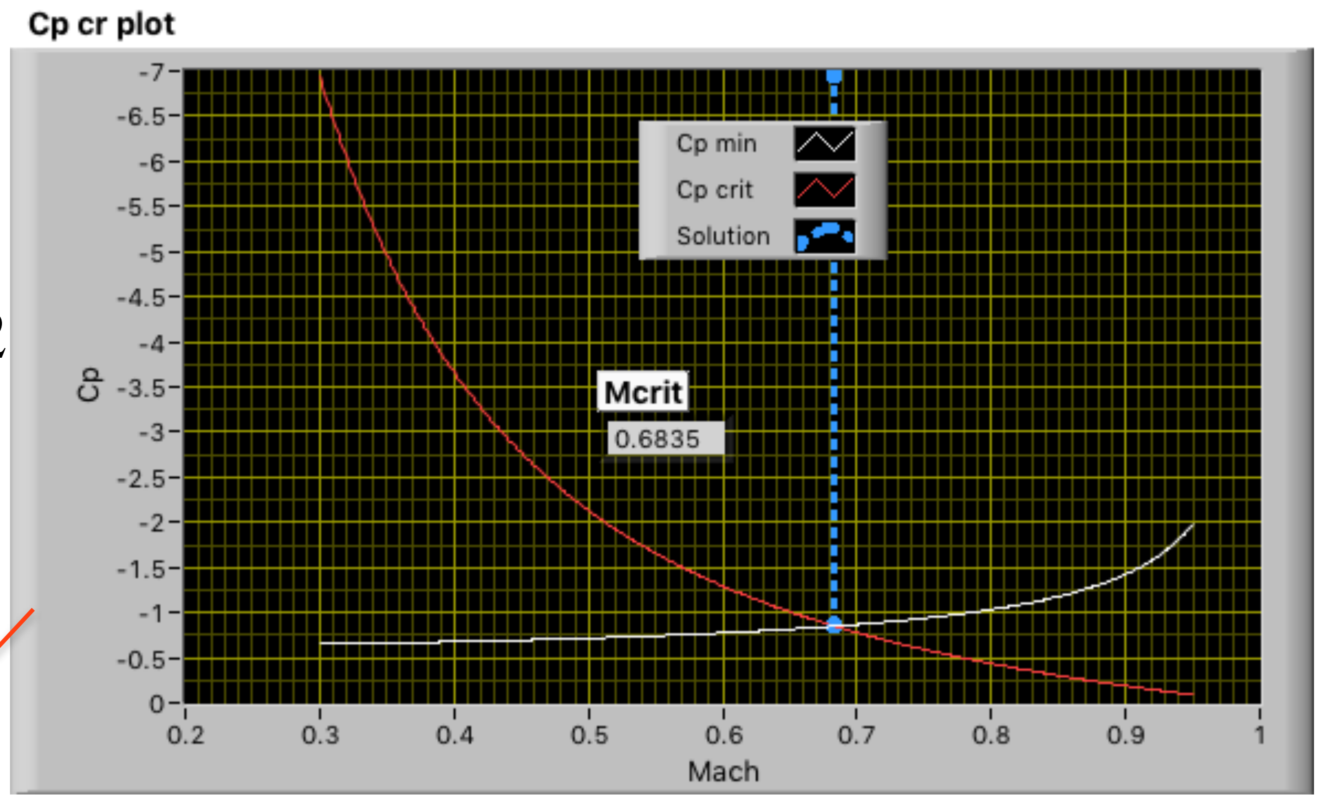
$M_{crit} = 0.8239$

NACA 0018 at $0^o \alpha$

NACA 0018 Airfoil at 0 deg. angle of attack

$Cp_{min} \sim -0.62$





→Clearly the 3 x Thickness lowers
→Critical Mach Significantly

$$M_{crit} = 0.6835$$

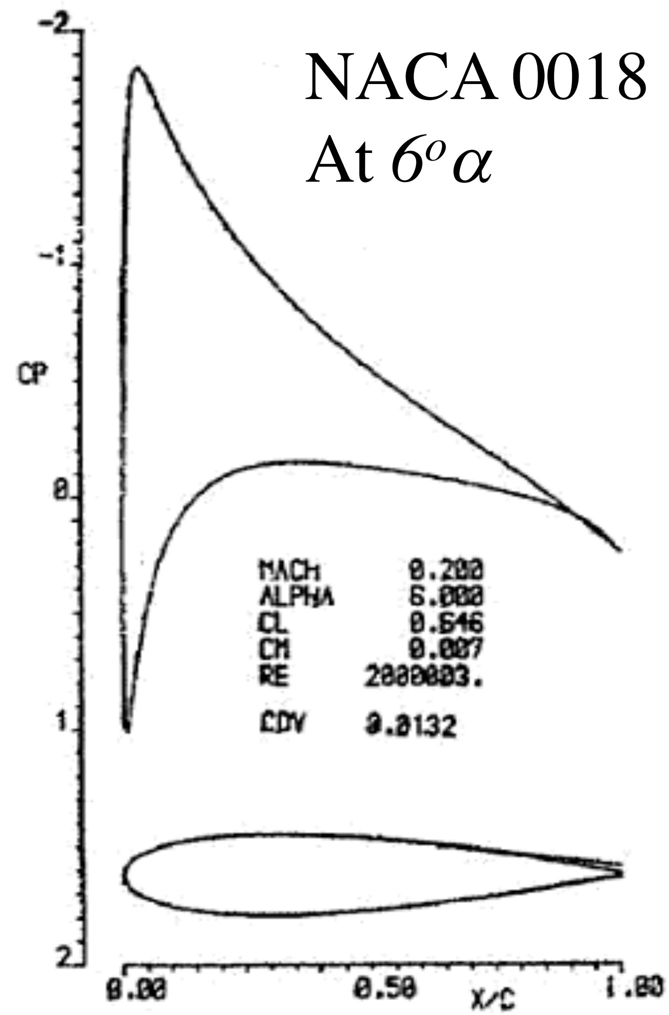
Example Calculation for Critical Mach Number @ Angle of Attack

- Now consider the NACA 0018 airfoil section at 6 degrees angle of attack

- Calculate the critical drag rise (subsonic) Mach number

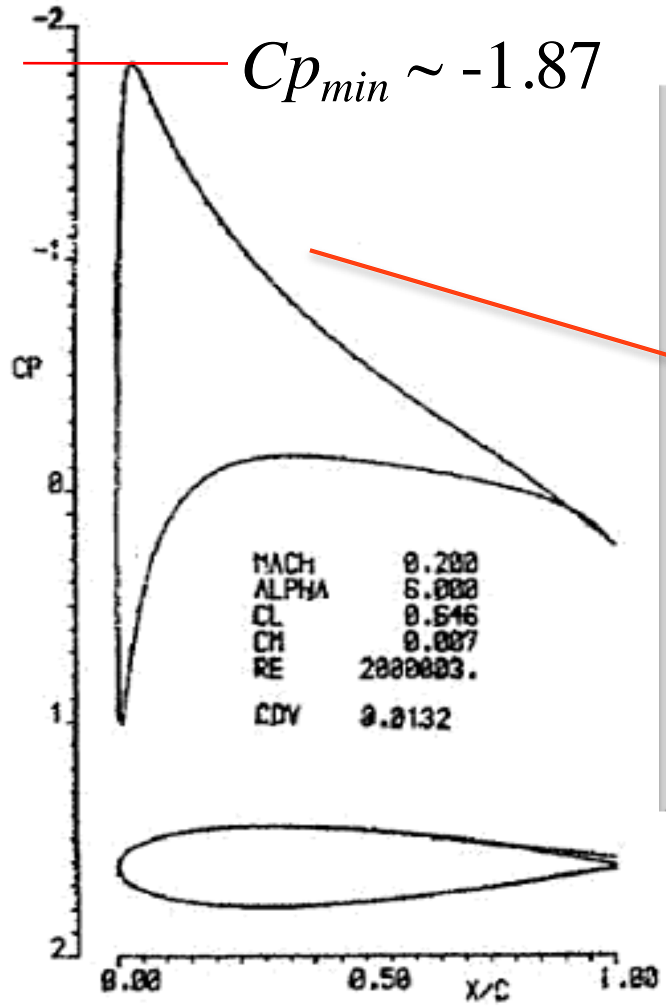
- Compare with 0°α results … what can you infer?

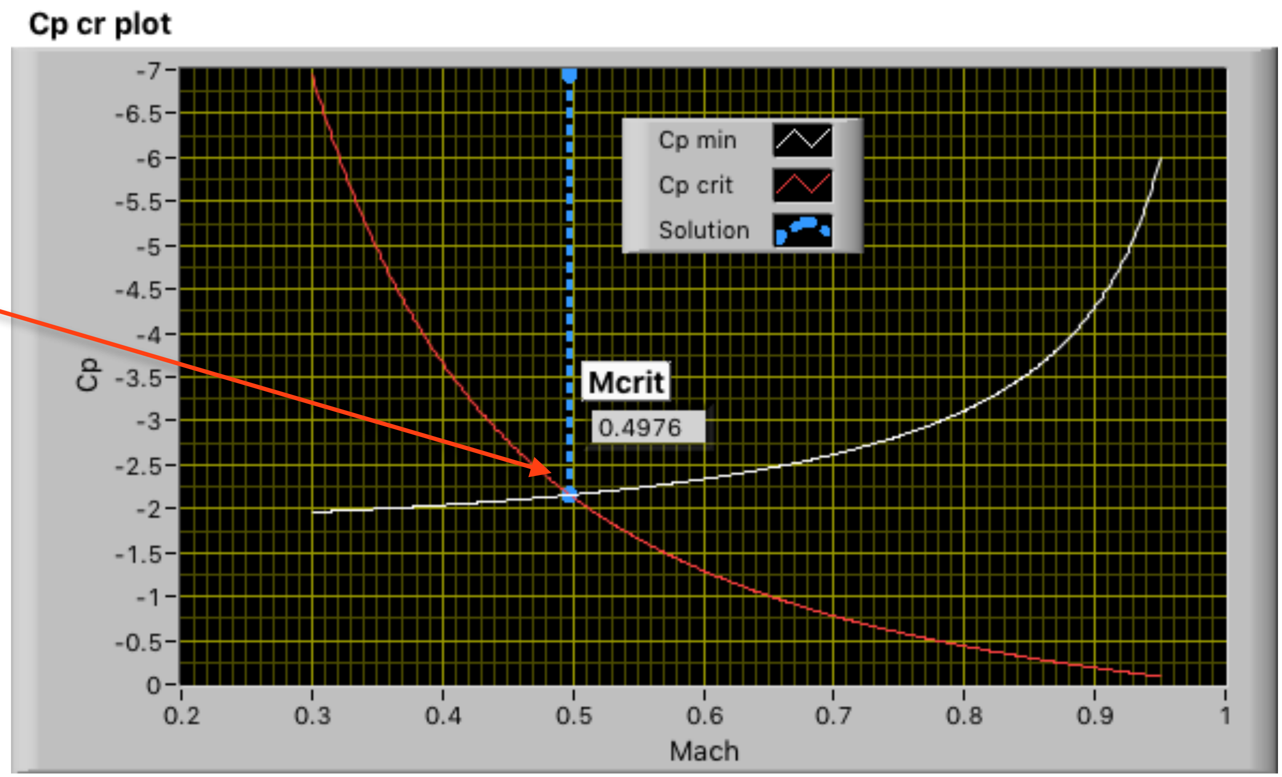
NACA 0018 airfoil
At $6^o \alpha$



NACA 0018 at $6^o \alpha$

$Cp_{min} \sim -1.87$





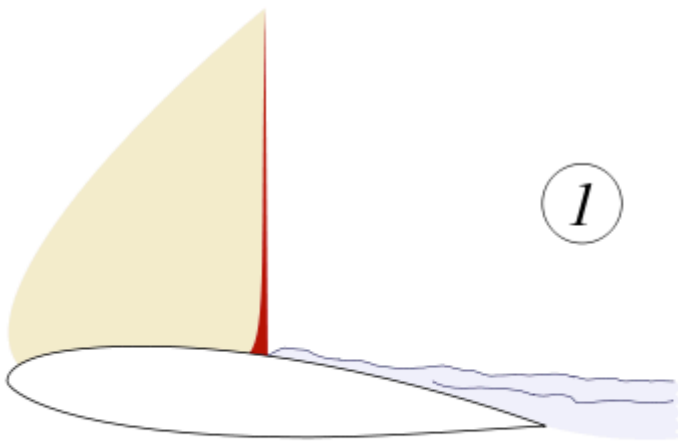
→Increased angle of attack, lowers upper surface pressure and as a result Lowers critical mach number considerably

$$M_{crit} = 0.4976$$

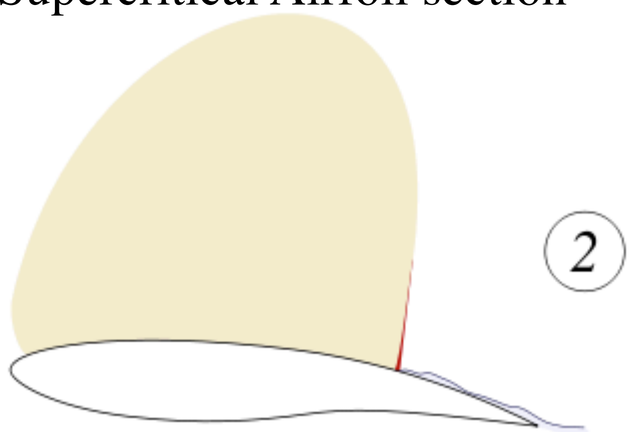
Effect of Wing Thickness on $M_{CR\ (3)}$

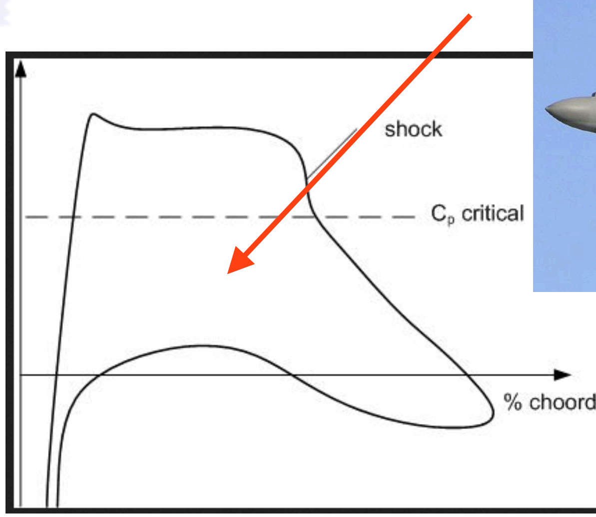
• Whitcomb Supercritical Airfoil

Conventional Airfoil section



• Supercritical Airfoil section

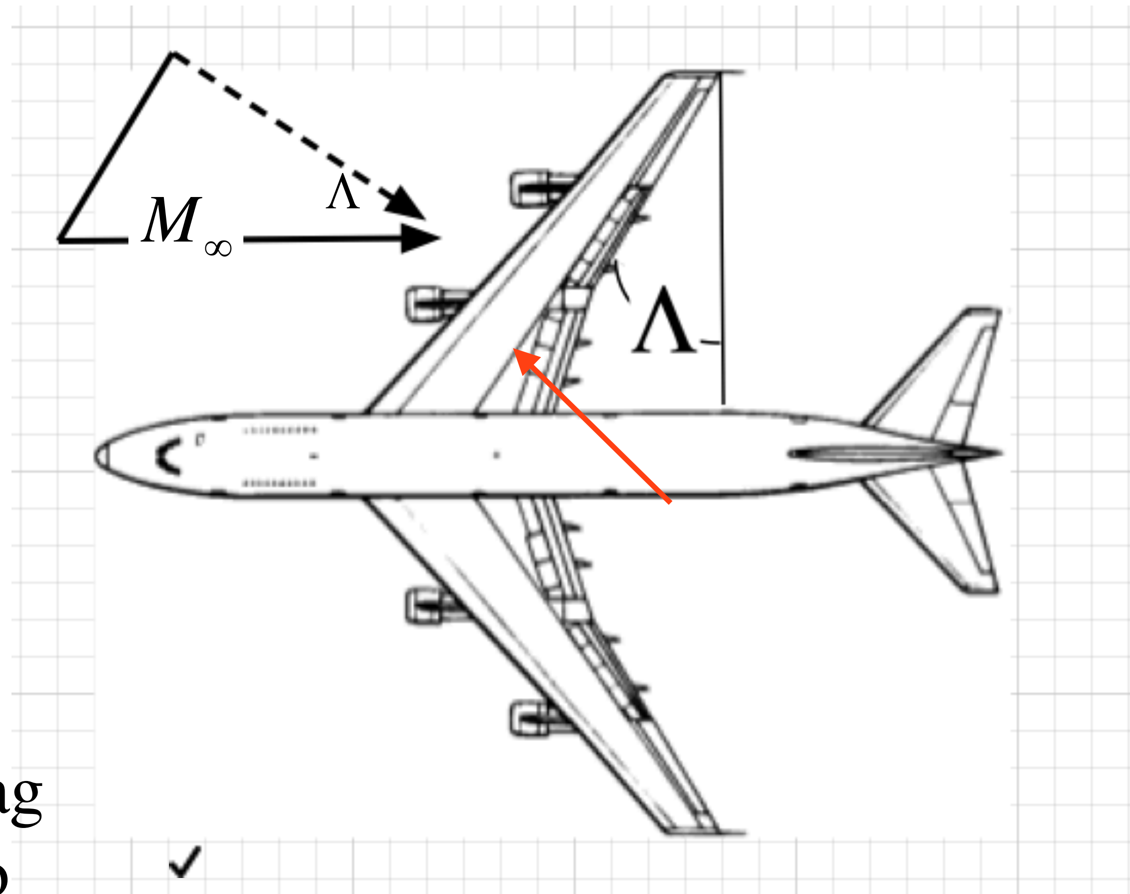




Shock wave onset delayed by shape of wing upper surface

Effect of Wing Sweep on M_{CR}

$$M_{\infty n} = M_{\infty} \cdot \cos\Lambda$$

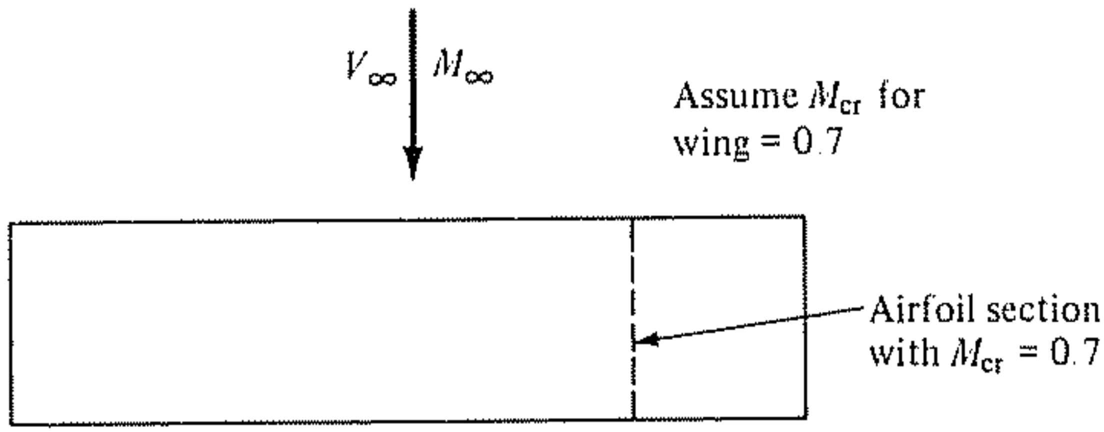


- Wing "sees" only Component of Mach number normal to leading edge

- By sweeping wings of **subsonic** aircraft, drag divergence is delayed to higher Mach numbers

Effect of Wing Sweep on $M_{CR\ (2)}$

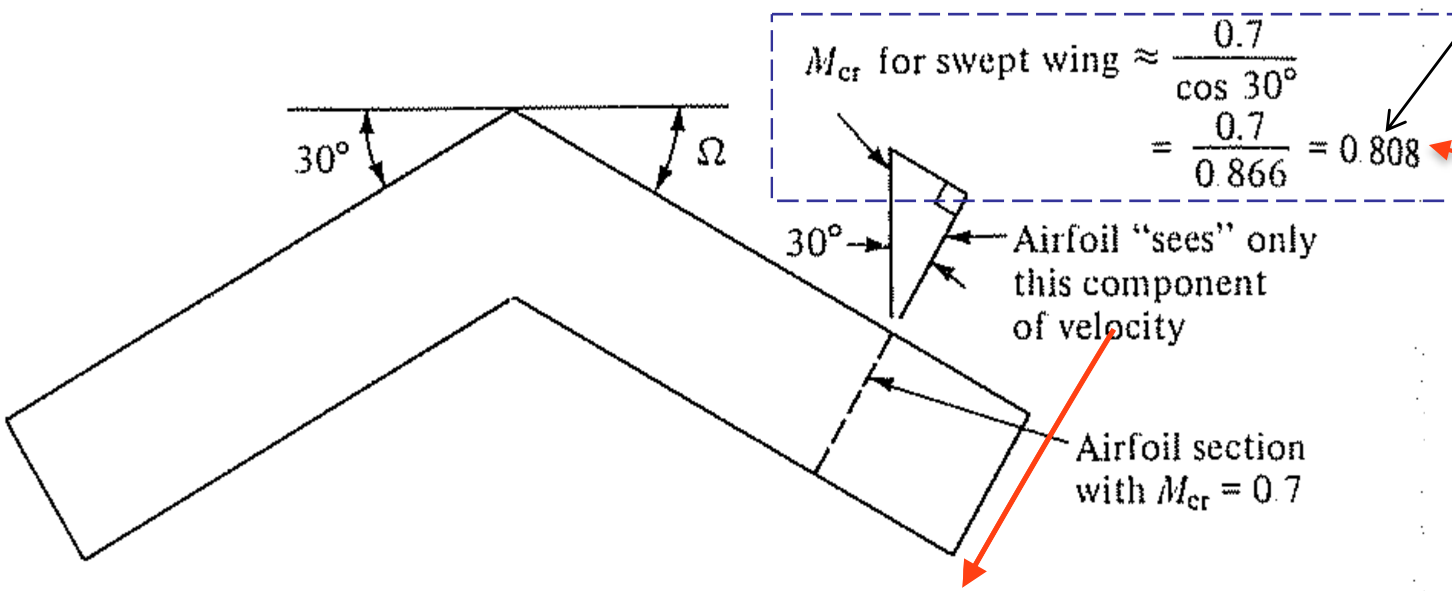
$$M_{\infty n} = M_{\infty} \cdot \cos \Lambda$$



Now sweep the same wing by 30°

(*a*)

Allows Higher M_∞ before transonic drag divergence

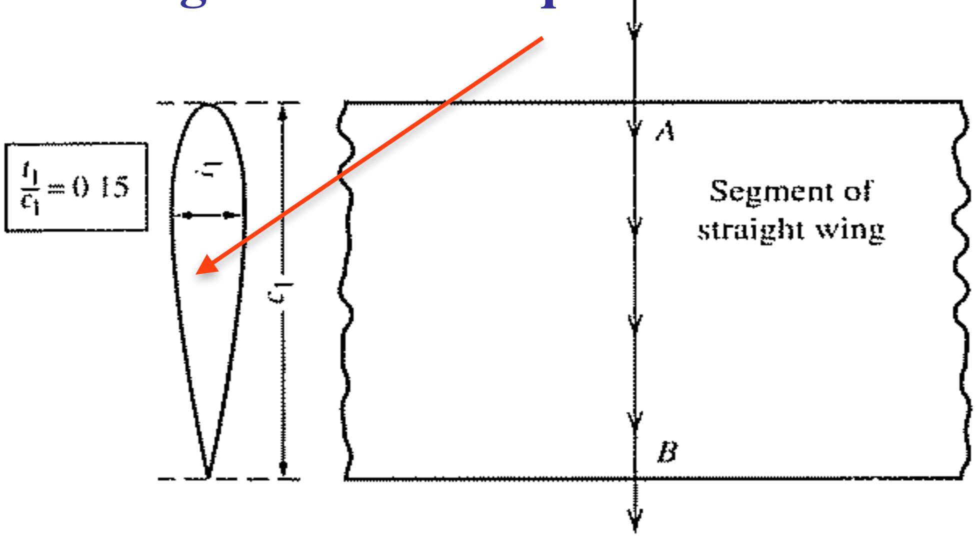


(*b*)

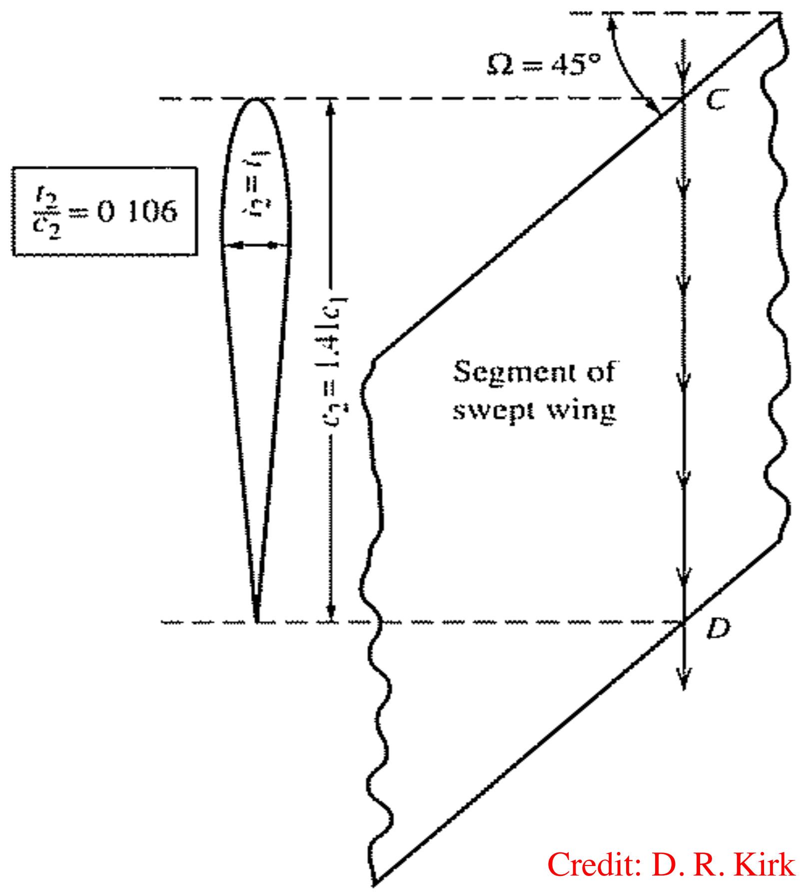
Credit: D. R. Kirk
FIT, 2011

Effect of Wing Sweep on M_{CR} (3)

- Swept Airfoil:
  - Airfoil has same thickness but longer effective chord
  - **Effective airfoil section is thinner**
  - Making airfoil thinner increases critical Mach number
- **Sweeping wing usually reduces lift for subsonic flight .. Thus required larger wing surface for equivalent lift**



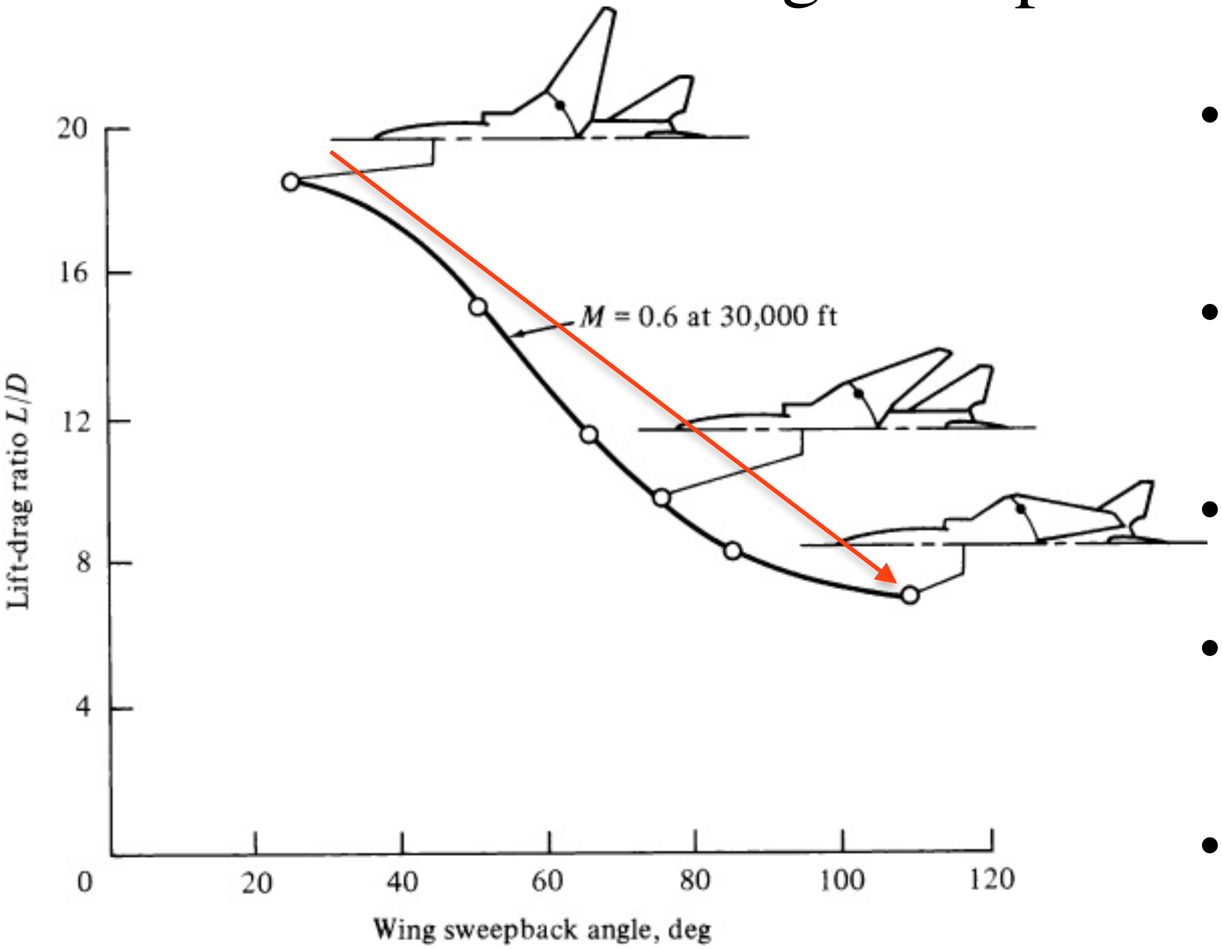
(a)



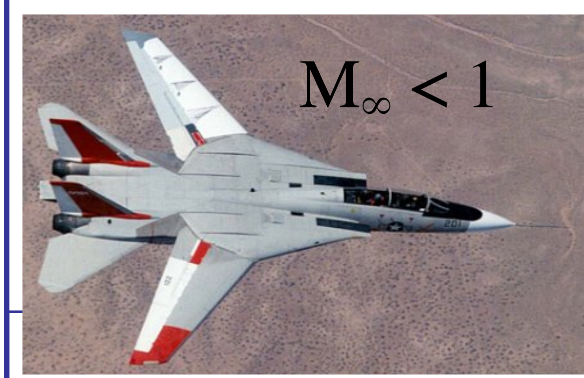
(b)

Credit: D. R. Kirk
FIT, 2011

Wing Sweep in Subsonic Flow



- Wing sweep beneficial in that it increases drag-divergence Mach number
- Increasing wing sweep also reduces the lift coefficient
- Significantly *reduces* L/D for *Subsonic Conditions*
- Significant Decrease in wave Drag for Supersonic Conditions
- *Improved Supersonic L/D*



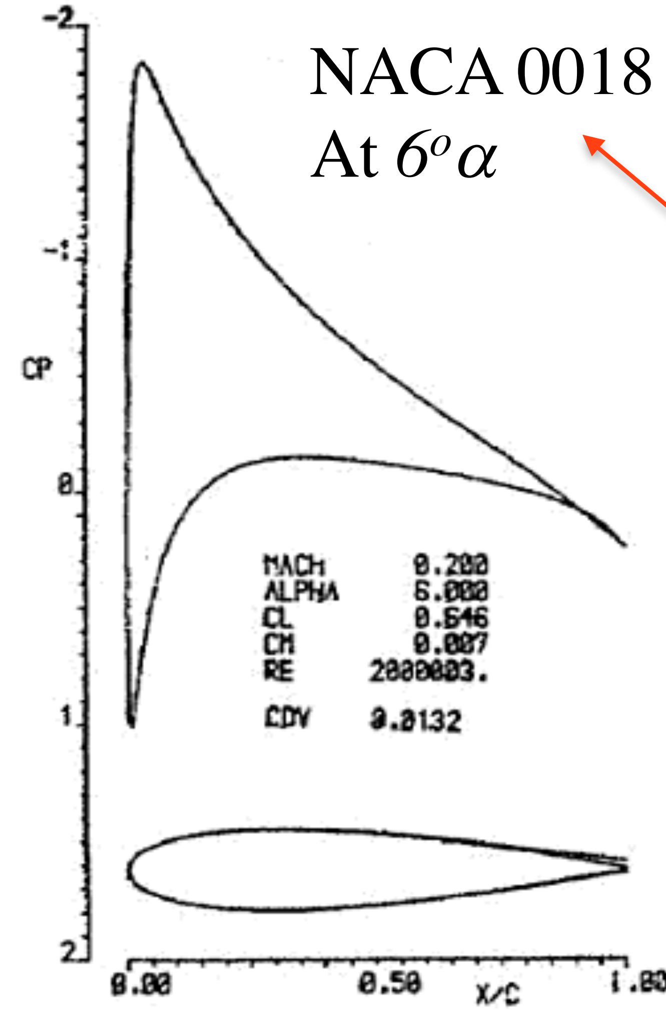
Credit: D. R. Kirk
FIT, 2011

Example Calculation for WING SWEEP EFFECT ON Critical Mach Number

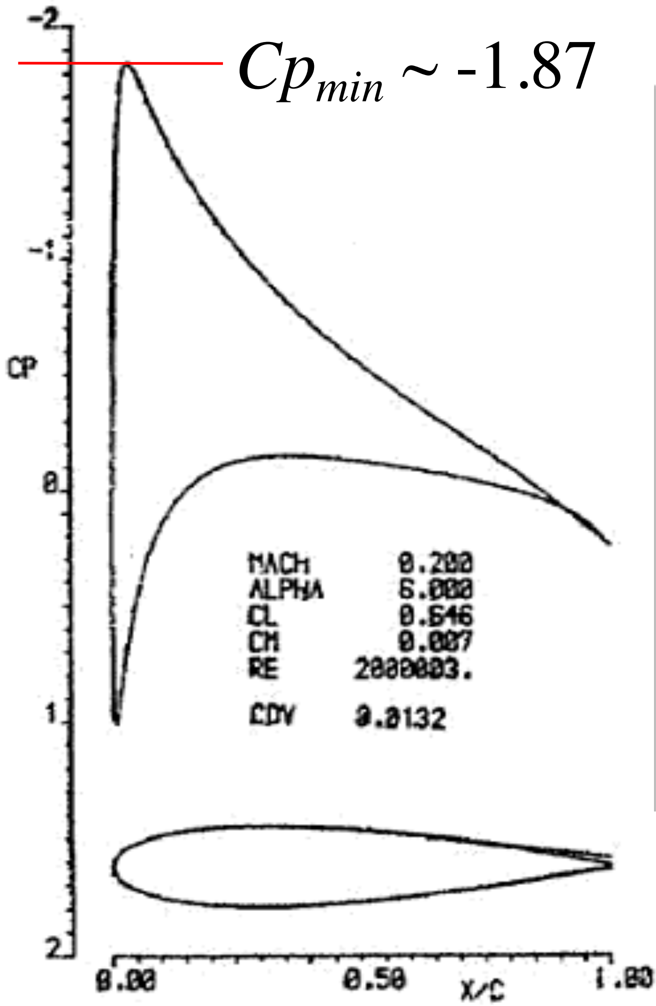
- Now consider the NACA 0018 airfoil section at 6 degrees angle of attack

- Calculate the critical drag rise mach number assuming a 30^o Λ leading edge wing sweep

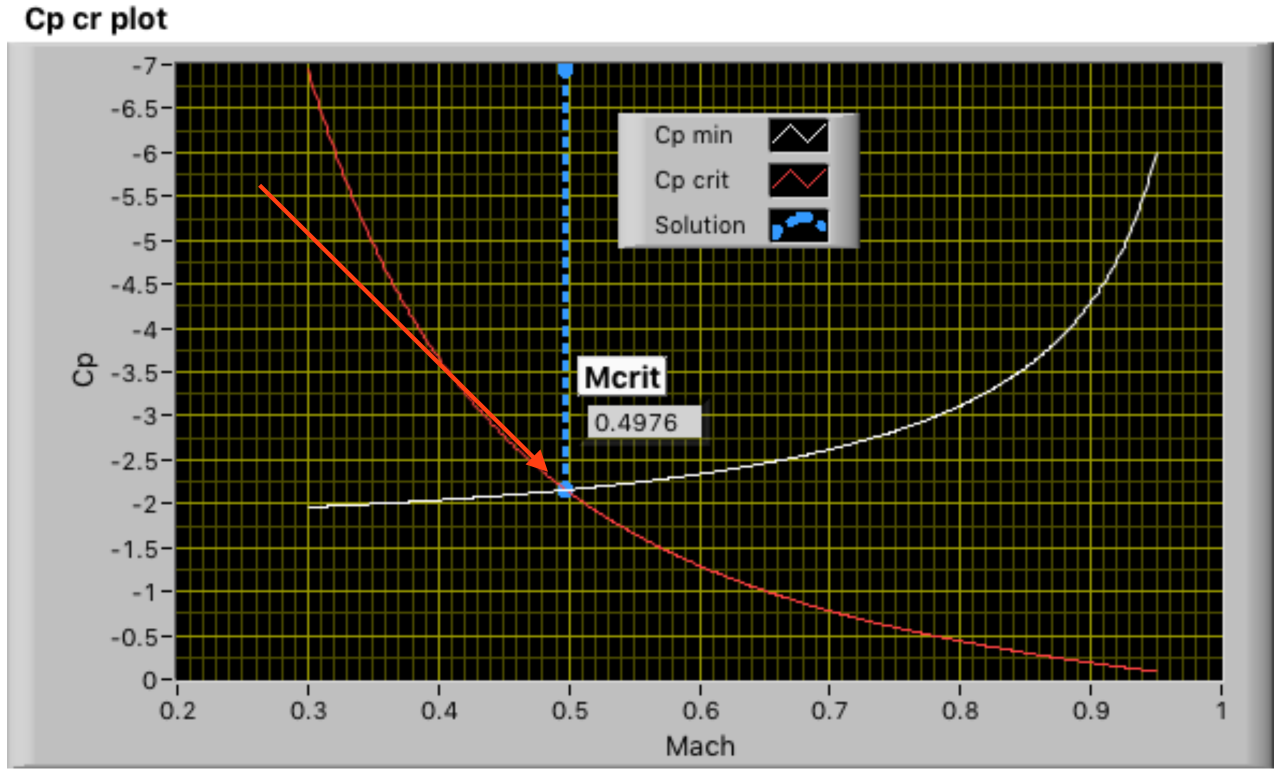
NACA 0018 airfoil
At $6^o \alpha$



From Previous, NACA 0018 at $6^o \alpha$

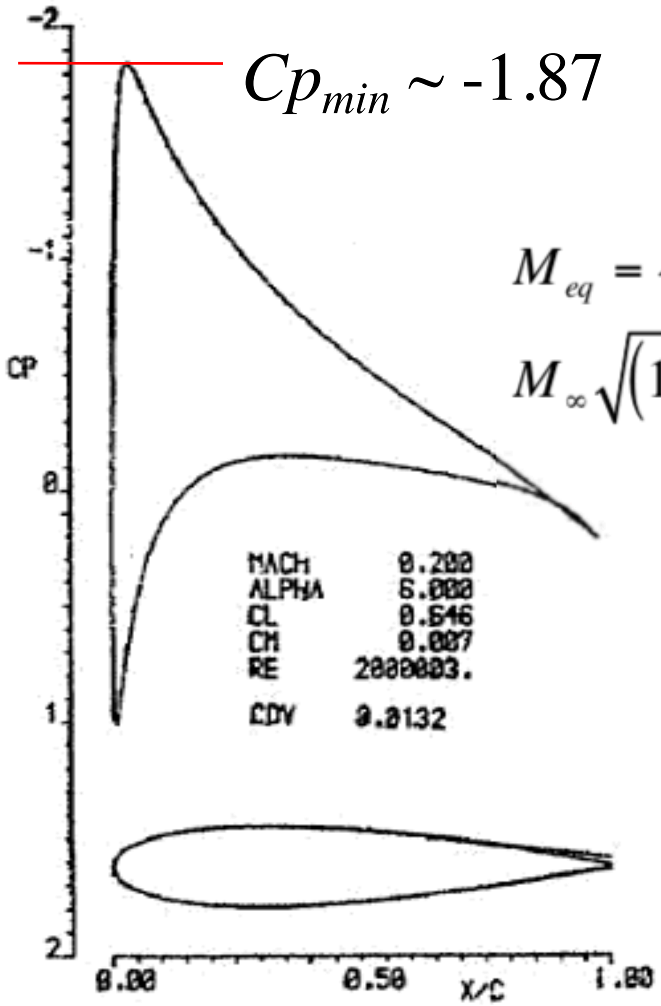


$Cp_{min} \sim -1.87$



$$M_{crit} = 0.4976$$

NACA 0018 at $6^o \alpha$ *with 30 deg. Sweep*



$Cp_{min} \sim -1.87$

→ $30^o\ \Lambda$ leading edge wing sweep

$$M_{eq} = \sqrt{M_{\perp}^{\ 2} + M_{vert}^{\ 2}} = \sqrt{\left(M_\infty \sin\alpha\right)^2 + \left(M_\infty \cos\alpha \cos\Lambda\right)^2} =$$

$$M_\infty\sqrt{\left(1-\cos^2\alpha\right) + \cos^2\alpha\left(1-\sin^2\Lambda\right)} = M_\infty\sqrt{1-\sin^2\Lambda\cos^2\alpha}$$

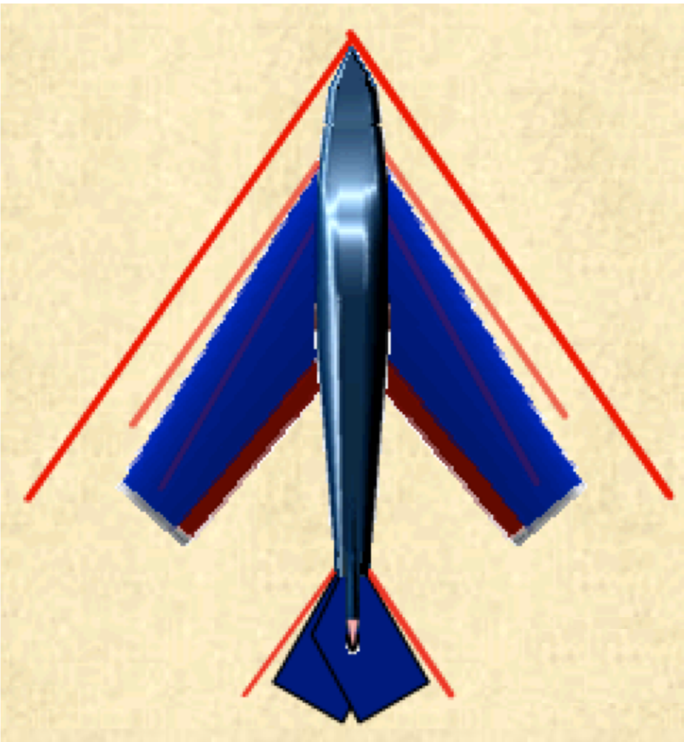
$$\left(M_{crit}\right)_\Lambda = \frac{\left(M_{crit}\right)_{0^o}}{\sqrt{1-\sin^2\Lambda\cdot\cos^2\alpha}} =$$

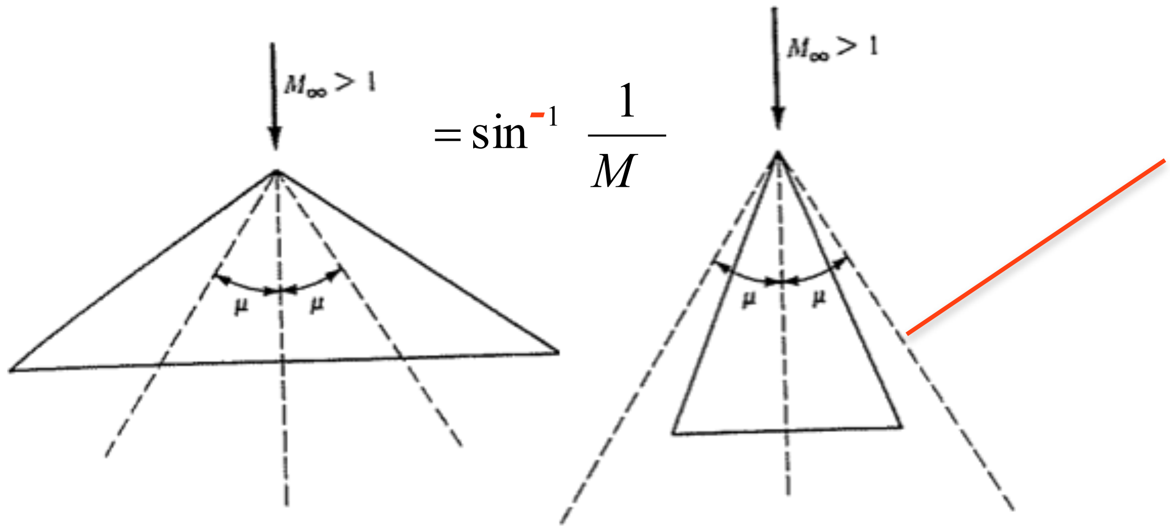
$$\frac{0.498}{\left(1-\left(\sin\left(\frac{\pi}{180}30\right)\cdot\cos\left(\frac{\pi}{180}6\right)\right)^2\right)^{0.5}} = 0.5740$$

15% Increase

@ zero sweep M_{crit} = 0.4976

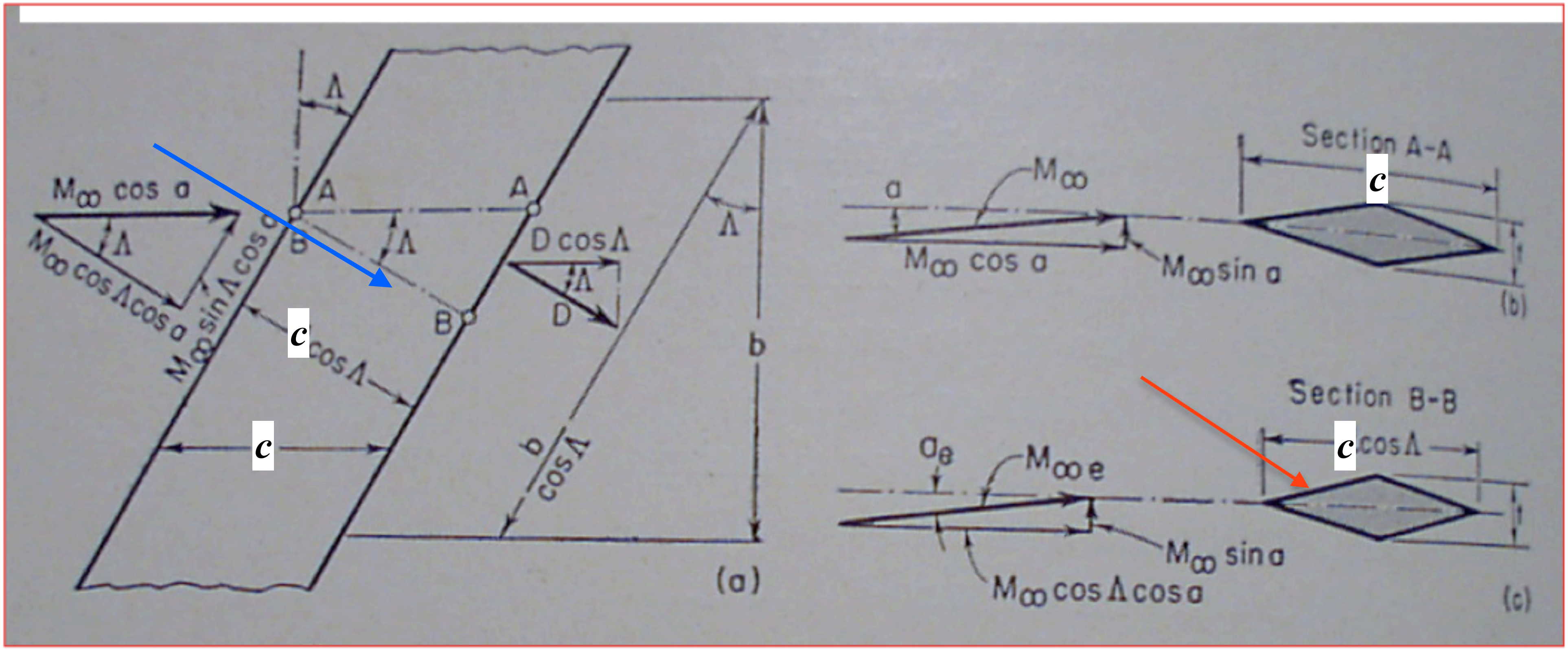
Effects of Wing Sweep in Supersonic Flow





- If leading edge of swept wing is outside Mach cone, component of Mach number normal to leading edge is supersonic → Large Wave Drag
- If leading edge of swept wing is inside Mach cone, component of Mach number normal to leading edge is subsonic → Reduced Wave Drag
- For supersonic flight, swept wings reduce wave drag
- Increase L/D

Equivalent Flow on Swept Wing at Angle of Attack (1)



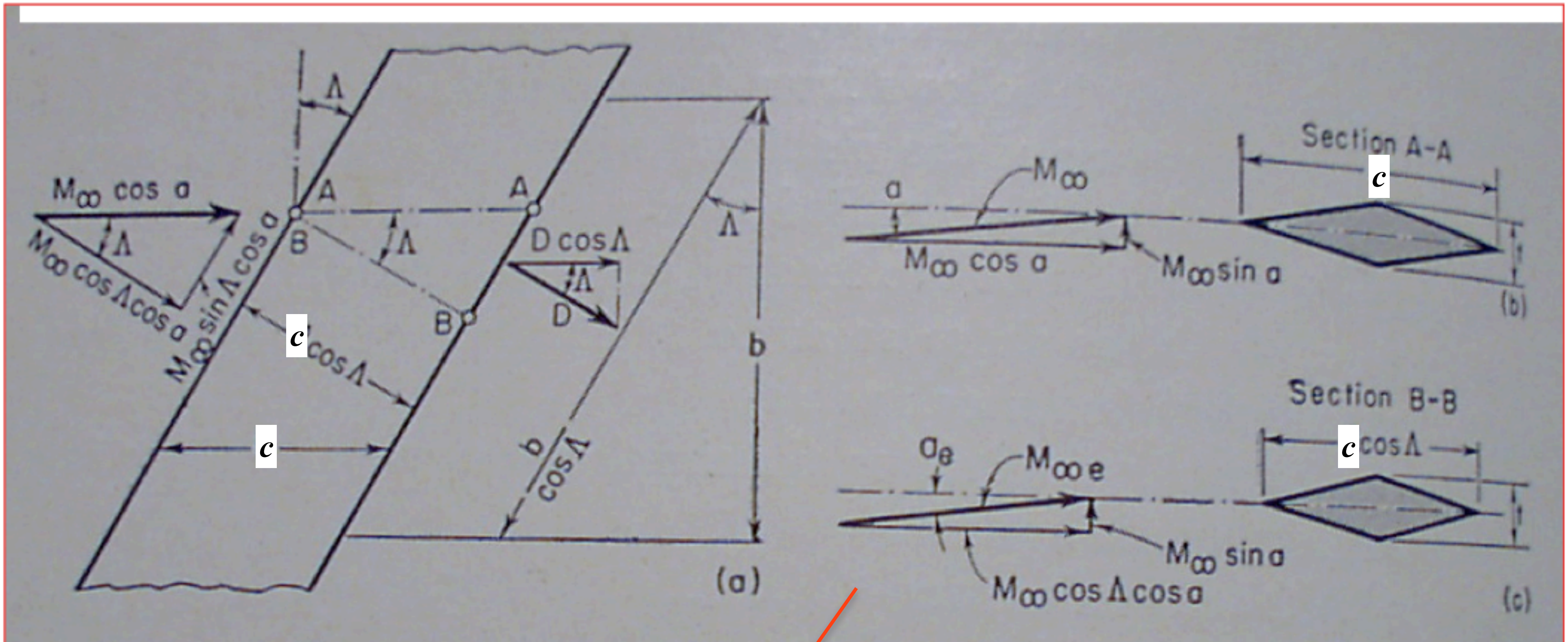
- Freestream Mach number resolved into 3 components

  i) vertical to wing …

  ii) in plane of wing, but tangent to leading edge

  iii) in plane of wing, but normal to leading edge

Equivalent Flow on Swept Wing at Angle of Attack (2)

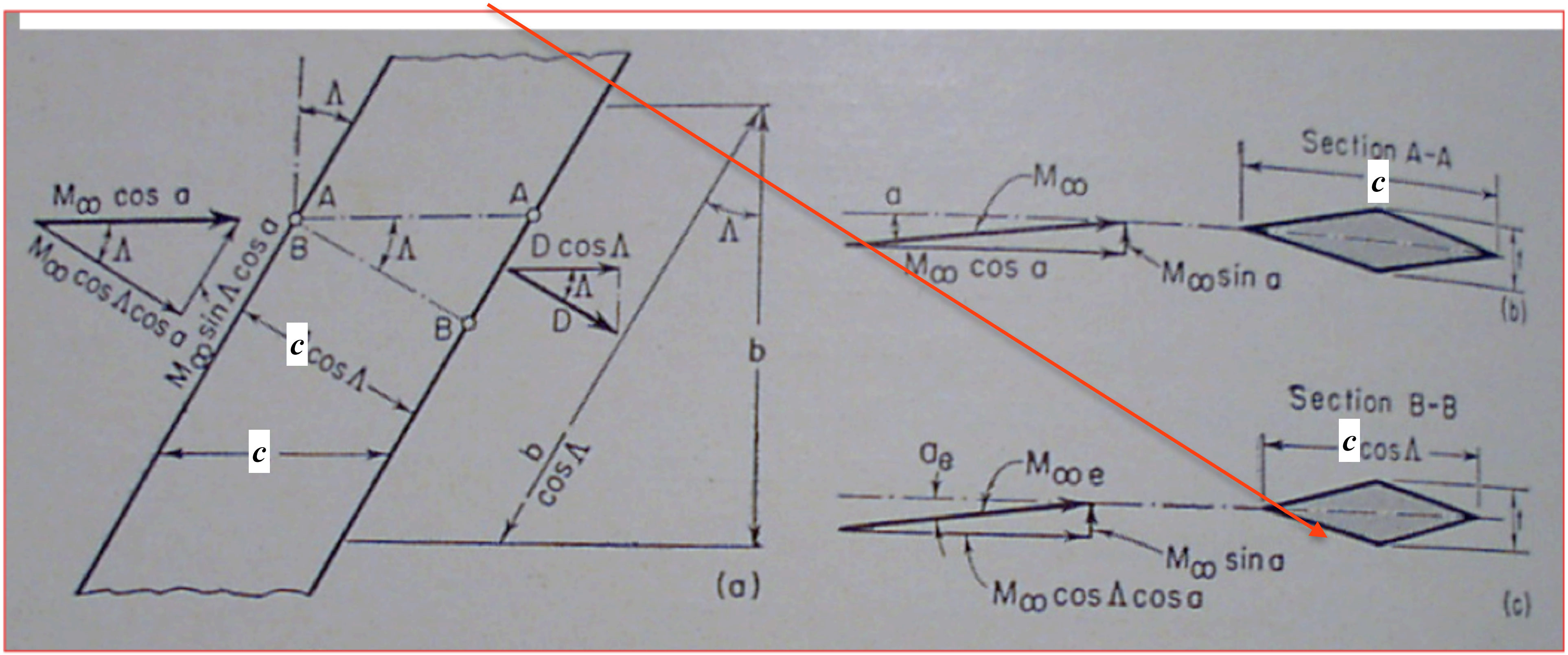


$$i)M_{vert} = M_\infty \sin\alpha$$

$$ii)M_{\parallel} = M_\infty \cos\alpha \sin\Lambda$$

$$ii)M_{\perp} = M_\infty \cos\alpha \cos\Lambda$$

Equivalent Flow on Swept Wing at Angle of Attack (3)

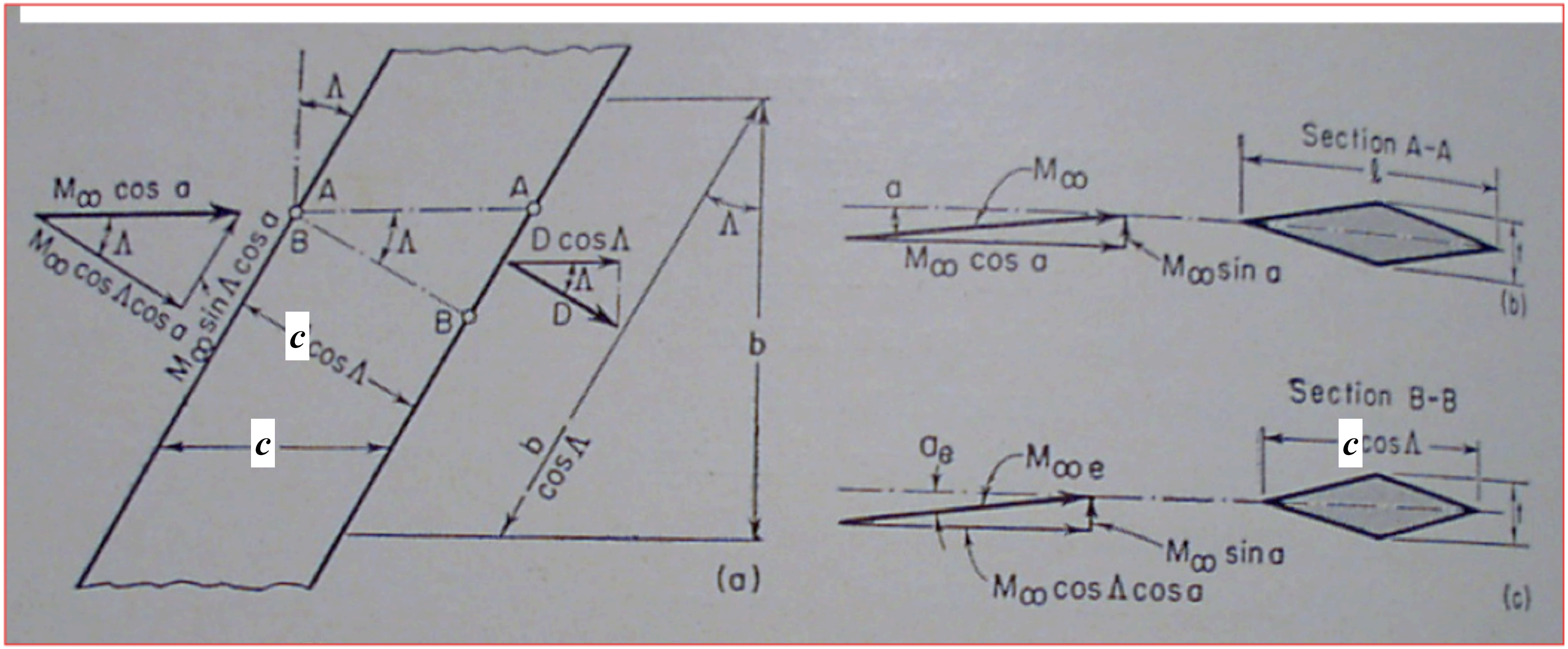


- Equivalent Mach Number normal to leading edge

$$M_{eq} = \sqrt{M_\perp{}^2 + M_{vert}{}^2} = \sqrt{\left(M_\infty \sin\alpha\right)^2 + \left(M_\infty \cos\alpha \cos\Lambda\right)^2} =$$

$$M_\infty \sqrt{\left(1-\cos^2\alpha\right) + \cos^2\alpha\left(1-\sin^2\Lambda\right)} = M_\infty \sqrt{1-\sin^2\Lambda\cos^2\alpha}$$

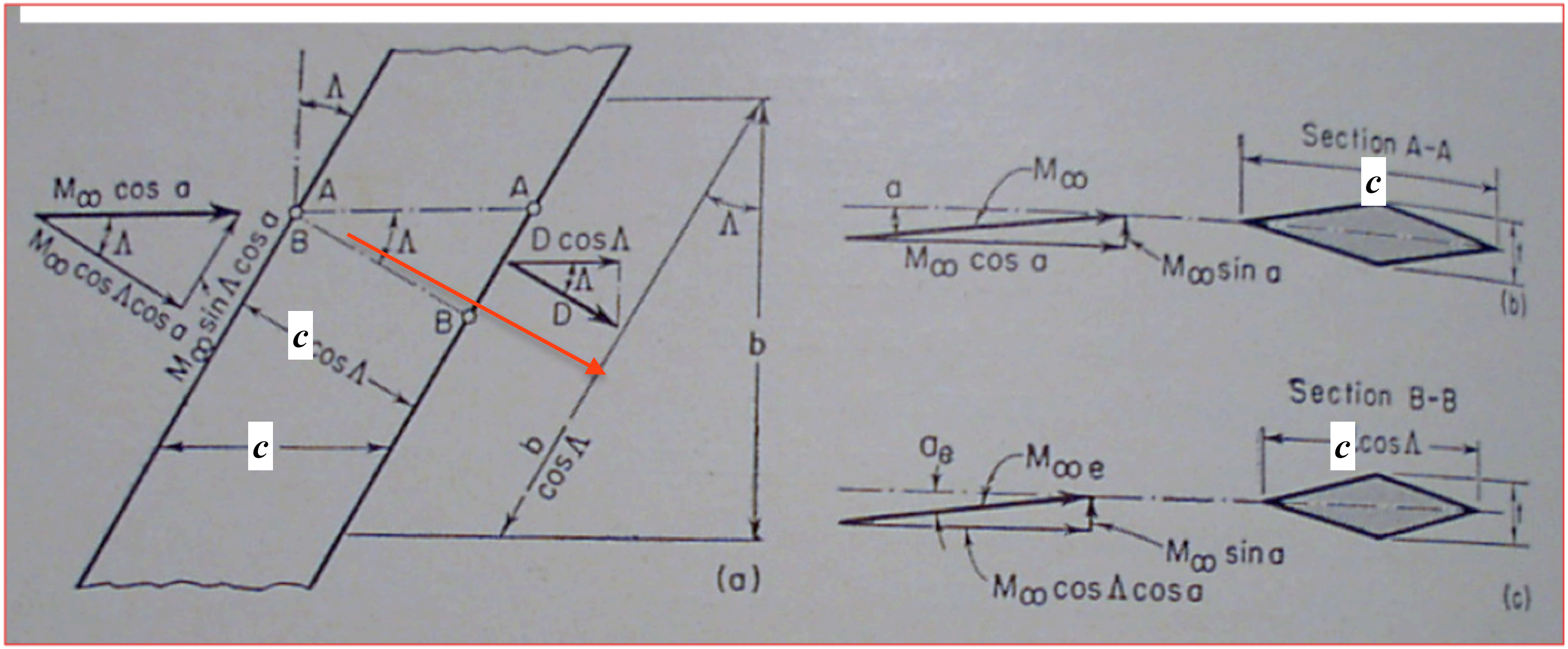
Equivalent Flow on Swept Wing at Angle of Attack (4)



- Equivalent angle of attack normal to leading edge

$$\tan\left(\alpha_{eq}\right) = \frac{M_{vert}}{M_\perp} = \frac{M_\infty \sin\alpha}{M_\infty \cos\alpha \cos\Lambda} = \frac{\tan\left(\alpha\right)}{\cos\Lambda}$$

Equivalent Flow on Swept Wing at Angle of Attack (5)



- Equivalent chord and span

$$c_{eq} = c\left[\cos\Lambda\right]$$

$$b_{eq} = \frac{b}{\cos\Lambda}$$

- Chord is shortened
- Span is lengthened

Equivalent Flow on Swept Wing at Angle of Attack (6)

- **Equivalent Lift Coefficient *(normal to leading edge)***

$$C_{L_{eq}} = \frac{L}{\frac{\gamma}{2} p_\infty M_{eq}{}^2 c[\cos\Lambda]\left[\frac{b}{\cos\Lambda}\right]} = \frac{L}{\frac{\gamma}{2} p_\infty M_{eq}{}^2 cb} =$$

$$\frac{L}{\frac{\gamma}{2} p_\infty M_\infty{}^2 cb\left(1-\sin^2\Lambda\cos^2\alpha\right)} = \frac{C_L}{\left(1-\sin^2\Lambda\cos^2\alpha\right)}$$

- **Equivalent Drag Coefficient *(normal to leading edge)***

$$C_{D_{eq}} = \frac{D/\cos\Lambda}{\frac{\gamma}{2} p_\infty M_{eq}{}^2 c[\cos\Lambda]\left[\frac{b}{\cos\Lambda}\right]} = \frac{D/\cos\Lambda}{\frac{\gamma}{2} p_\infty M_{eq}{}^2 cb} =$$

$$\frac{D/\cos\Lambda}{\frac{\gamma}{2} p_\infty M_\infty{}^2 cb\left(1-\sin^2\Lambda\cos^2\alpha\right)} = \frac{C_D/\cos\Lambda}{\left(1-\sin^2\Lambda\cos^2\alpha\right)}$$

Equivalent Flow on Swept Wing at Angle of Attack (6)

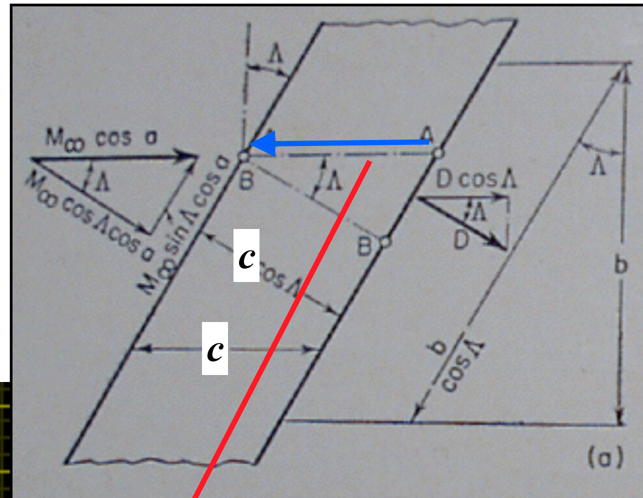
- Solve for true (whole wing) C_L, C_D, L/D

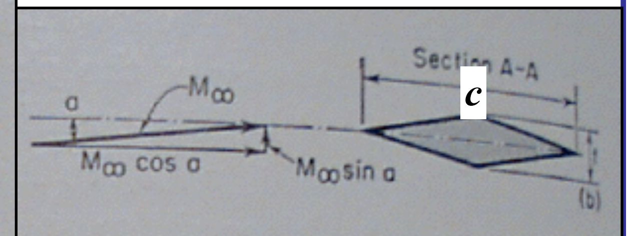
$$C_L = C_{L_{eq}}\left(1 - \sin^2\Lambda\cos^2\alpha\right)$$
$$C_D = C_{D_{eq}}\cos\Lambda\left(1 - \sin^2\Lambda\cos^2\alpha\right) \rightarrow \boxed{\frac{L}{D} = \frac{\left(\frac{L}{D}\right)_{eq}}{\cos\Lambda}}$$

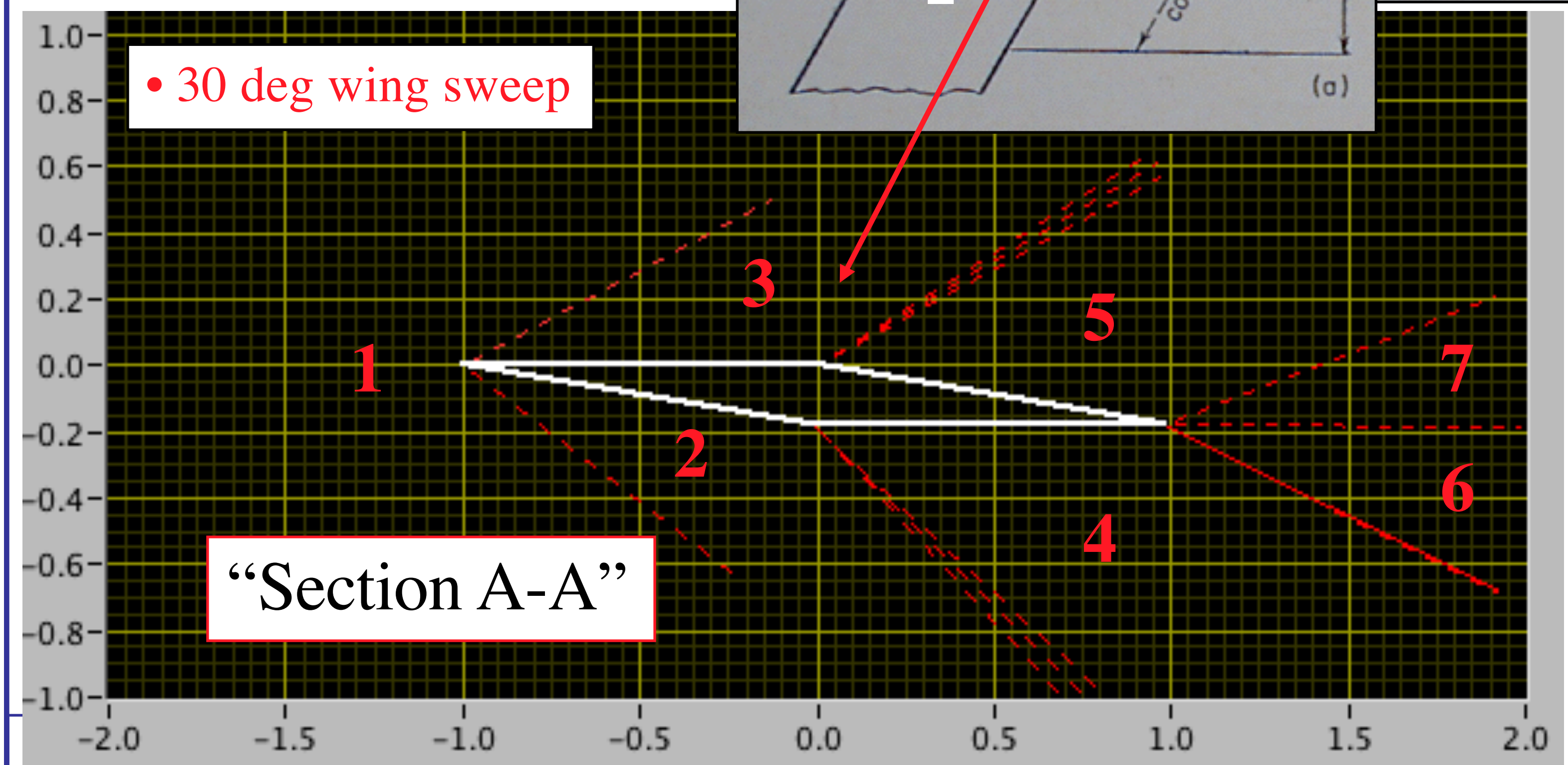
- Whole Wing *Lift Coefficient* is reduced by sweep
- Lift-to-Drag ratio *L/D increased* by *1/ cos Λ*

Example: Symmetric Double-wedge Airfoil

- 5° ramps, $M_\infty = 2.00$
- Total chord: 2 meters
- Fineness ratio (t/c): 0.08749
- $\alpha = 5°$



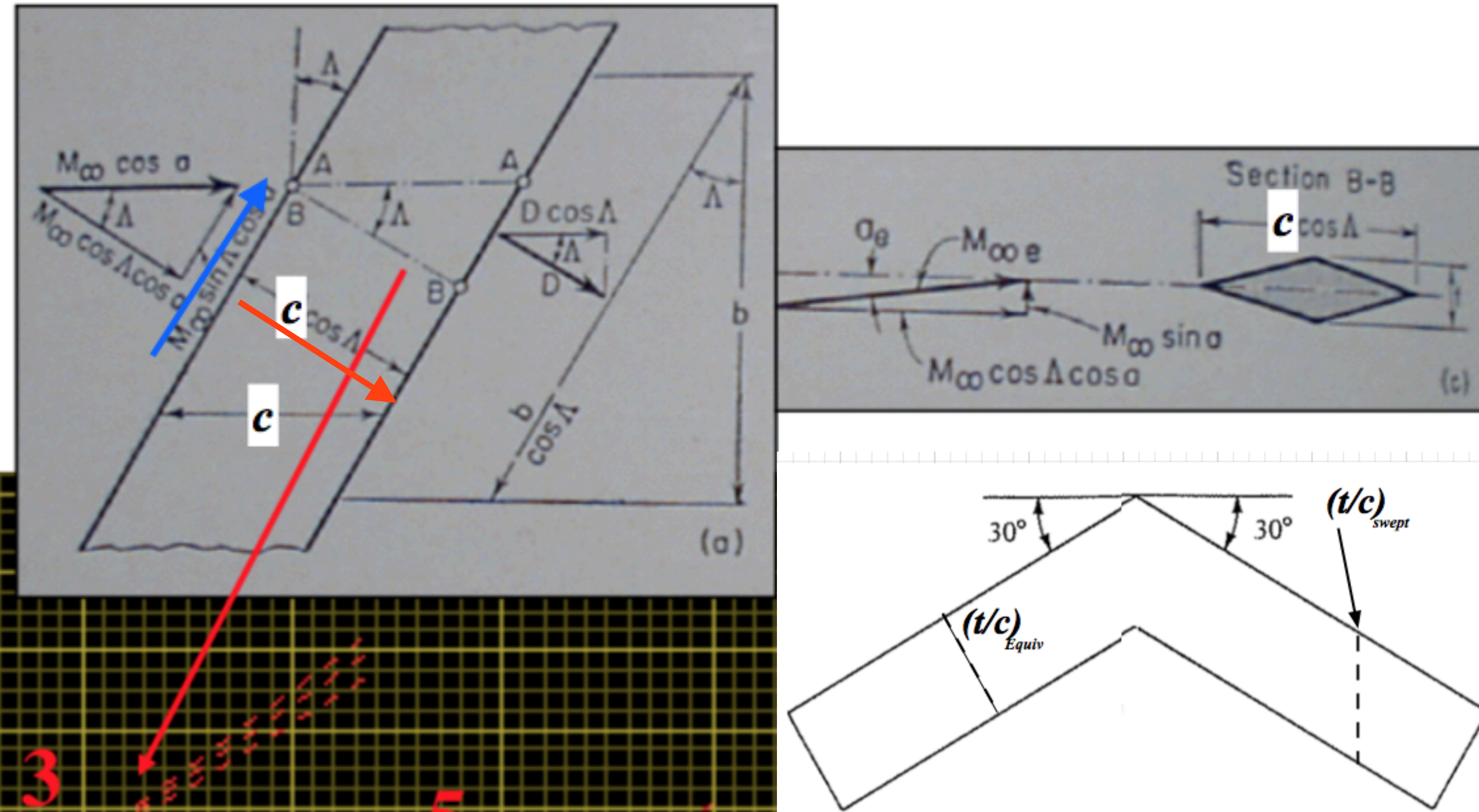


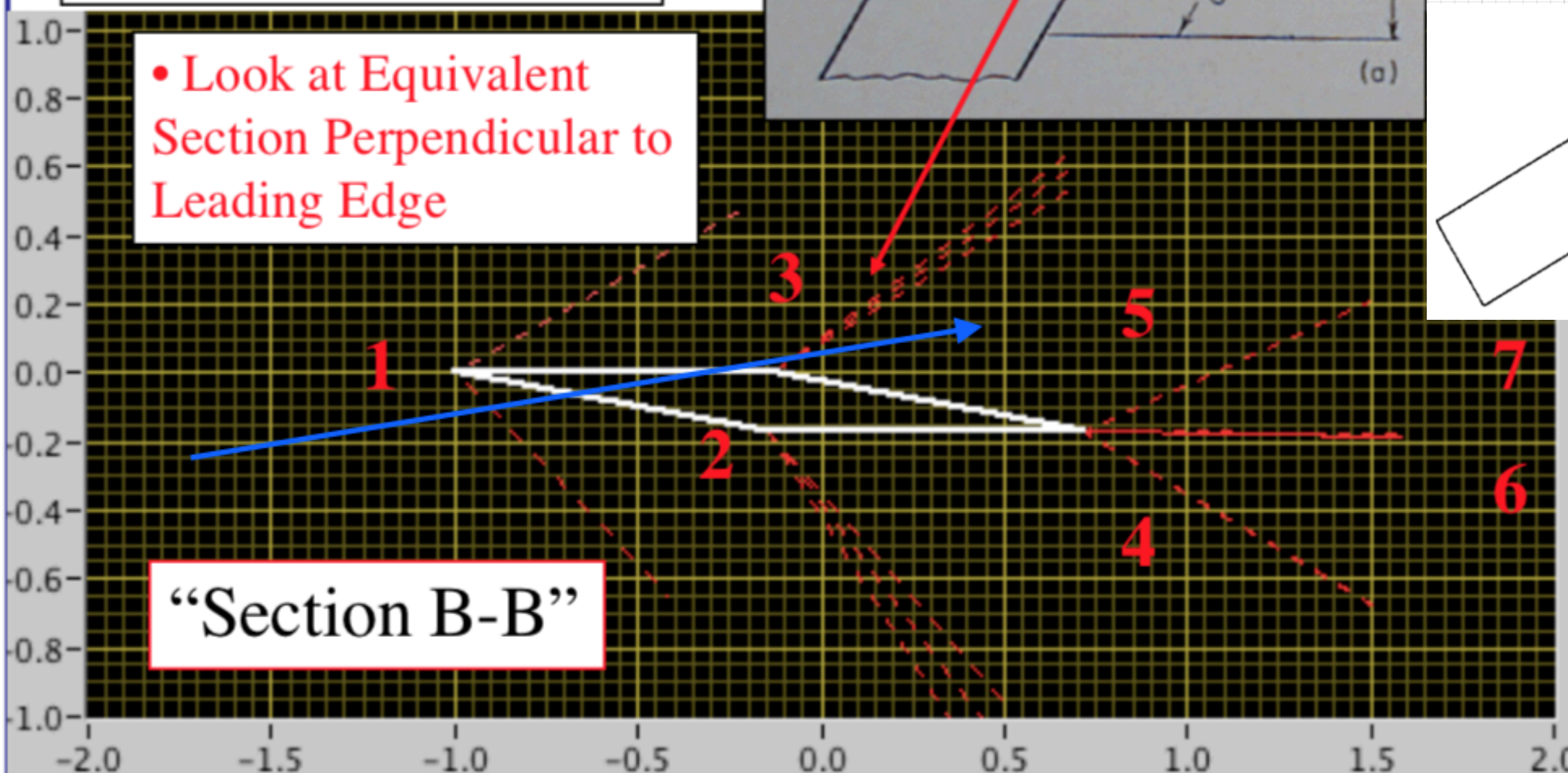


Example: Symmetric Double-wedge Airfoil (2)

Equivalent Airfoil

- 5.769° ramps, $M_{eq} = 1.7342$
- Equivalent Chord: 1.732 meters
- Equiviv. Fineness ratio (t/c): ***0.101026***
- $\alpha_{eq} = 5.769°$

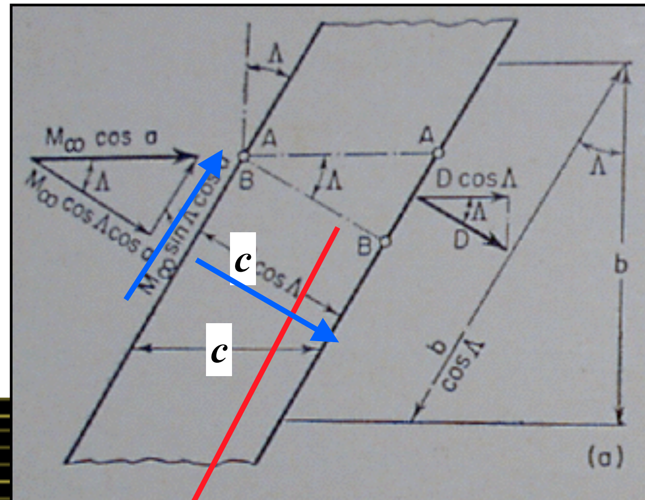


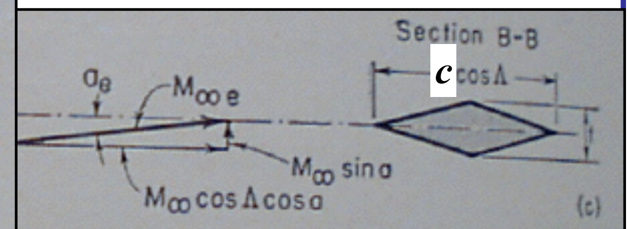


Example: Symmetric Double-wedge Airfoil (2)

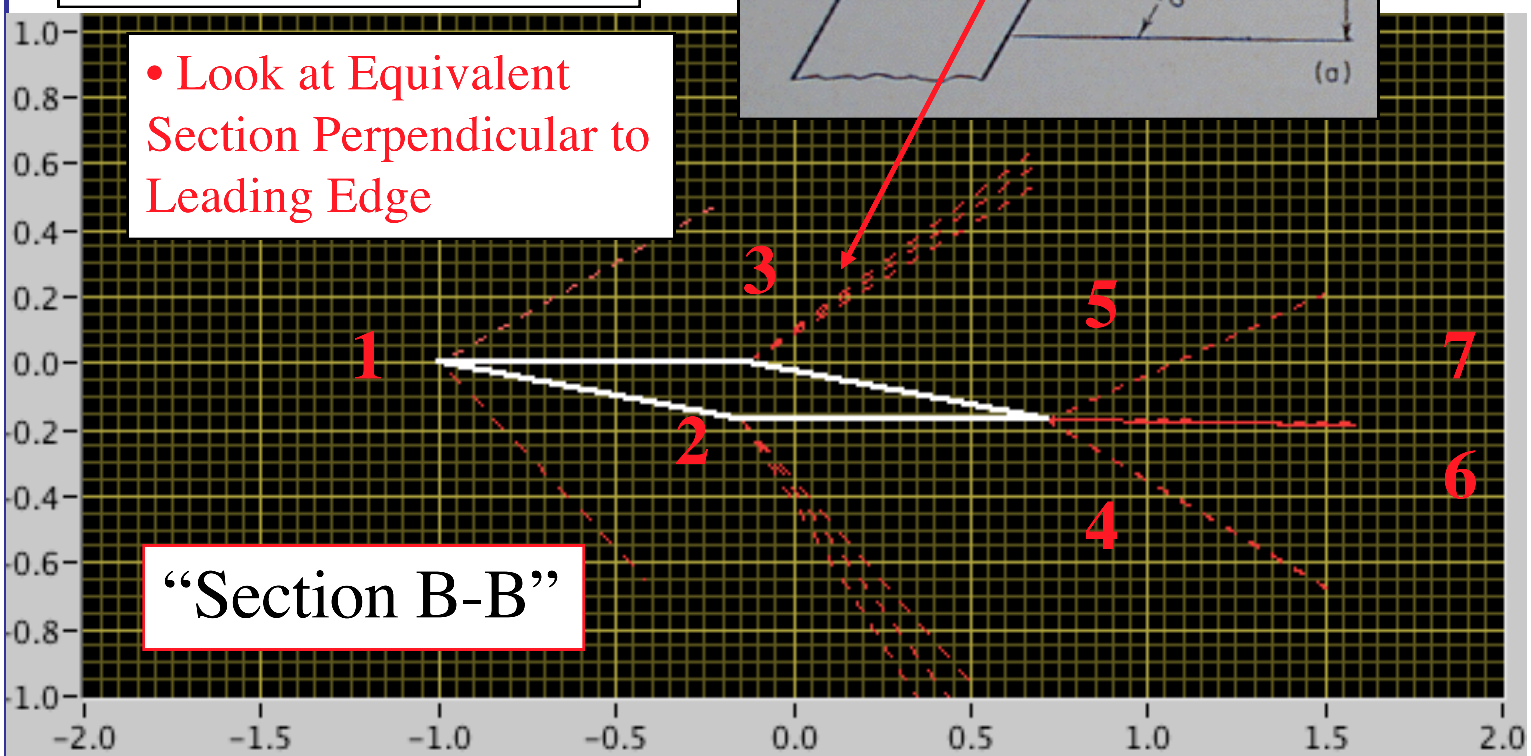
Equivalent Airfoil

- 5.769° ramps, M_{eq} = 1.7342
- Equivalent Chord: 1.732 meters
- Equiviv. Fineness ratio (t/c): ***0.101026***
- α_{eq} = 5.769°





- Look at Equivalent Section Perpendicular to Leading Edge



"Section B-B"

Example: Swept Symmetric Double-wedge Airfoil (3)

Equivalent Airfoil

- 5.769° ramps, $M_{eq} = 1.7342$
- Equivalent Chord: 1.732 meters
- Equiviv. Fineness ratio (t/c): 1 ***0.101026***
- $\alpha_{eq} = 5.769°$

$$M_{eq} = M_{\infty}\sqrt{1-\sin^2\Lambda\cos^2\alpha} =$$

$$2\left(1-\left(\left(\sin\left(\frac{\pi}{180}30\right)\right)\cos\left(\frac{\pi}{180}5\right)\right)^2\right)^{0.5} = 1.7342$$

- Equivalent Section Perpendicular to Leading Edge

$$\alpha_{eq} = \tan^{-1}\left[\frac{\tan(\alpha)}{\cos\Lambda}\right] = \frac{180}{\pi}\,\mathrm{atan}\left(\frac{\tan\left(\frac{\pi}{180}5\right)}{\cos\left(\frac{\pi}{180}30\right)}\right)$$

$$= 5.769°$$

$$c_{eq} = c\left[\cos\Lambda\right] = 2\cos\left(\frac{\pi}{180}30\right)$$

$$= 1.732$$

Example: Swept Symmetric Double-wedge Airfoil (4)

Equivalent Airfoil

- 5.769° ramps, $M_{eq} = 1.7342$
- Equivalent Chord: 1.732 meters
- Equiviv. Fineness ratio (t/c): 1.01026
- $\alpha_{eq} = 5.769°$

Cumulative Lower Surface Properties

P2/P1	T2/T1	P02/P01
1.7150	1.1719	0.984224
P3/P1	**T3/T1**	**P03/P01**
1.0441	1.0170	0.984224
P4/P1	**T4/T1**	**P04/P01**
0.9737	0.9969	0.984224

Cumulative Upper Surface Properties

P2/P1	T2/T1	P02/P01
0.9608	0.9886	1.000000
P3/P1	**T3/T1**	**P03/P01**
0.5525	0.8441	1.000000
P4/P1	**T4/T1**	**P04/P01**
1.0311	1.0159	0.975744

Total CL	Total CD	Total L/D	Lref
0.29212	0.05205	5.61239	1.7320

$$b_{equiv} = \frac{b}{\cos\Lambda}$$

- Equivalent Section Perpendicular to Leading Edge

$$\begin{bmatrix} C_D \\ C_L \end{bmatrix}_{eq} = \left(\frac{1}{2\cos\delta}\cdot\frac{1}{\frac{\gamma}{2}M_{eq}^2}\right)\begin{bmatrix} \left(\frac{P_2}{P_\infty}-\frac{P_5}{P_\infty}\right)\sin\left(\delta+\alpha_{eq}\right)+\left(\frac{P_3}{P_\infty}-\frac{P_4}{P_\infty}\right)\sin\left(\delta-\alpha_{eq}\right) \\ \left(\frac{P_2}{P_\infty}-\frac{P_5}{P_\infty}\right)\cdot\cos\left(\delta+\alpha_{eq}\right)+\left(\frac{P_4}{P_\infty}-\frac{P_3}{P_\infty}\right)\cos\left(\delta-\alpha_{eq}\right) \end{bmatrix}$$

Example: Swept Symmetric Double-wedge Airfoil (5)

Equivalent Airfoil

Total CL	0.29212
Total CD	0.05205
Total L/D	5.61239
Lref	1.7320

- Compute Lift, Drag Coefficient, L/D of Swept Wing

$$C_L = C_{L_{eq}}\left(1-\sin^2\Lambda\cos^2\alpha\right) =$$

$$0.29212\left(1-\left(\sin\left(\frac{\pi}{180}30\right)\cdot\cos\left(\frac{\pi}{180}5.769\right)\right)^2\right) = 0.219828$$

Some Loss of lift

$$C_D = C_{D_{eq}}\cos\Lambda\left(1-\sin^2\Lambda\cos^2\alpha\right) =$$

$$0.05205\left(\cos\left(\frac{\pi}{180}30\right)\right)\left(1-\left(\sin\left(\frac{\pi}{180}30\right)\cdot\cos\left(\frac{\pi}{180}5.769\right)\right)^2\right) = 0.0339213$$

Significantly higher drop in wave drag!

- 30 deg wing sweep

$\rightarrow$ $L/D_{\text{swept wing}} = 0.219828/0.0339213 = 6.4805$

Example: Swept Symmetric Double-wedge Airfoil (6)

• ***Compare to Unswept airfoil with same Swept geometric cross Section,***

- 5deg ramps, M = 2.00
- Total chord: 2 meters
- Fineness ratio (t/c): 0.08749
- α = 5 deg

$$
\begin{bmatrix} C_D \\ C_L \end{bmatrix} = \left(\frac{1}{2\cos\delta} \cdot \frac{1}{\frac{\gamma}{2} M_\infty^2} \right) \times
$$

$$
\begin{bmatrix} \left(\frac{P_2}{P_\infty} - \frac{P_5}{P_\infty} \right) \sin(\delta+\alpha) + \left(\frac{P_3}{P_\infty} - \frac{P_4}{P_\infty} \right) \sin(\delta-\alpha) \\ \left(\frac{P_2}{P_\infty} - \frac{P_5}{P_\infty} \right) \cdot \cos(\delta+\alpha) + \left(\frac{P_4}{P_\infty} - \frac{P_3}{P_\infty} \right) \cos(\delta-\alpha) \end{bmatrix}
$$

• 0 deg wing sweep

Cumulative Upper Surface Properties

P2/P1	T2/T1	P02/P01
1.0000	1.0000	1.000000
P3/P1	**T3/T1**	**P03/P01**
0.5480	0.8421	1.000000
P4/P1	**T4/T1**	**P04/P01**
1.0326[illegible]	1.0167	0.974609

Cumulative Lower Surface Properties

P2/P1	T2/T1	P02/P01
1.7066	1.1702	0.984644
P3/P1	**T3/T1**	**P03/P01**
1.0026	1.0052	0.984644
P4/P1	**T4/T1**	**P04/P01**
0.9702[illegible]	0.9958	0.984644

Total CL	0.20500
Total CD	0.03606
Total L/D	5.68441
Lref	1.9924

Example: Swept Symmetric Double-wedge Airfoil (cont'd)

- Unswept Wing

C_L:	0.2367
C_D:	0.4164
L/D:	5.6844

- 30 ^{deg} Swept Wing

C_L:	0.2198
C_D:	0.0339
L/D:	6.4805

→ Coefficients scaled to have same wing planform area as swept wing

$$S_{wing} = \frac{\bar{c} \cdot b}{\cos \Lambda}$$

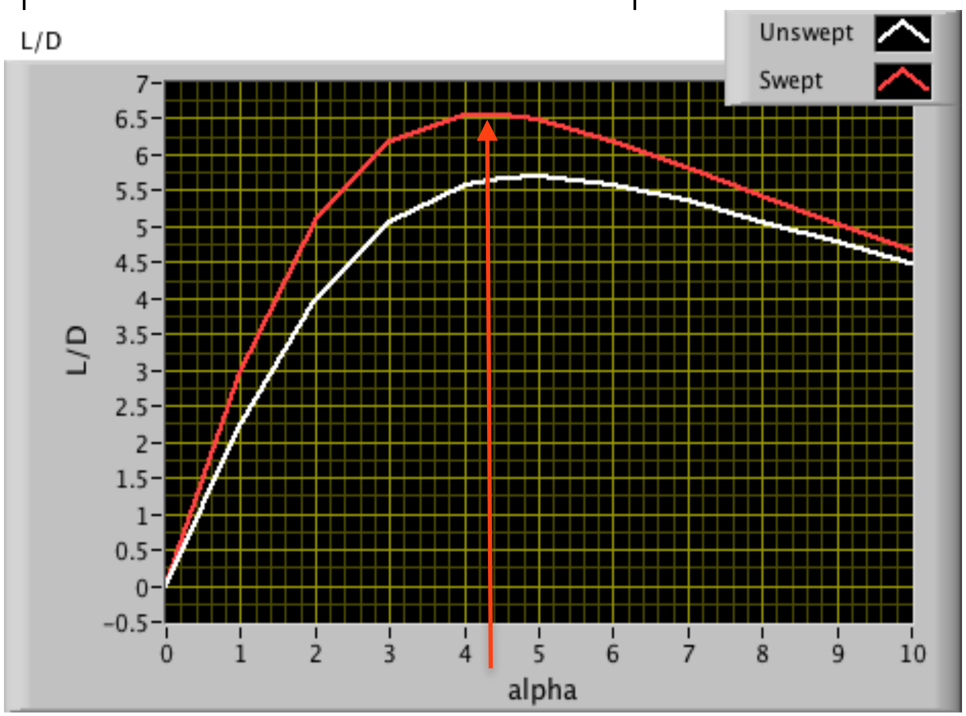
- WOW! … > 10% IMPROVEMENT in L/D by Wing Sweep

UtahState UNIVERSITY

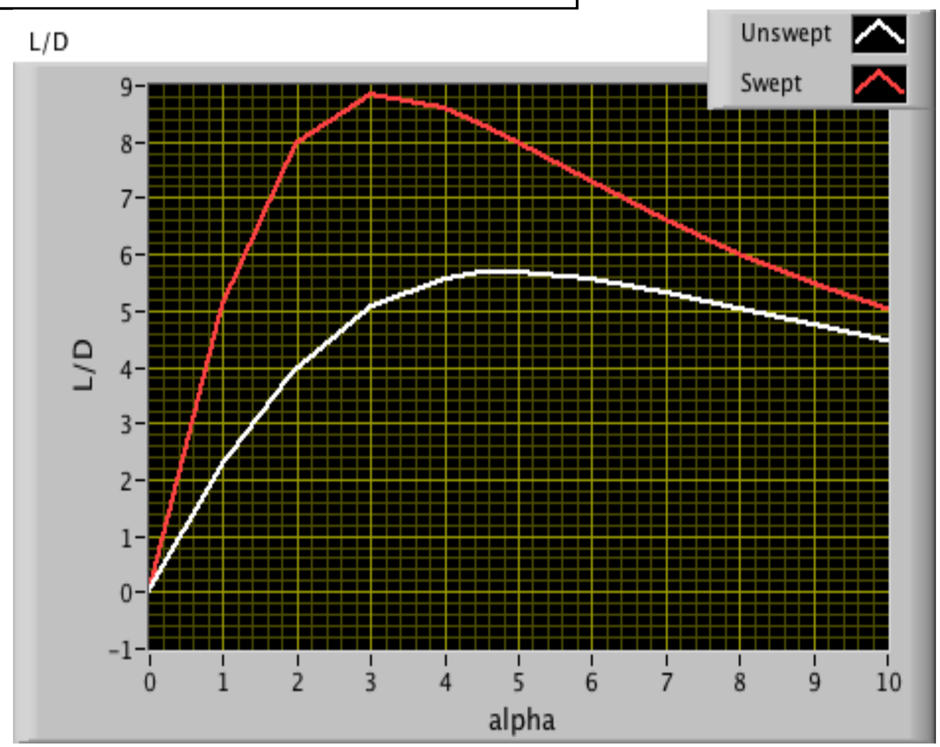
Mechanical & Aerospace Engineering

Example: Swept Symmetric Double-wedge Airfoil (cont'd)

- Mach = 2.0
- 30 deg wing sweep

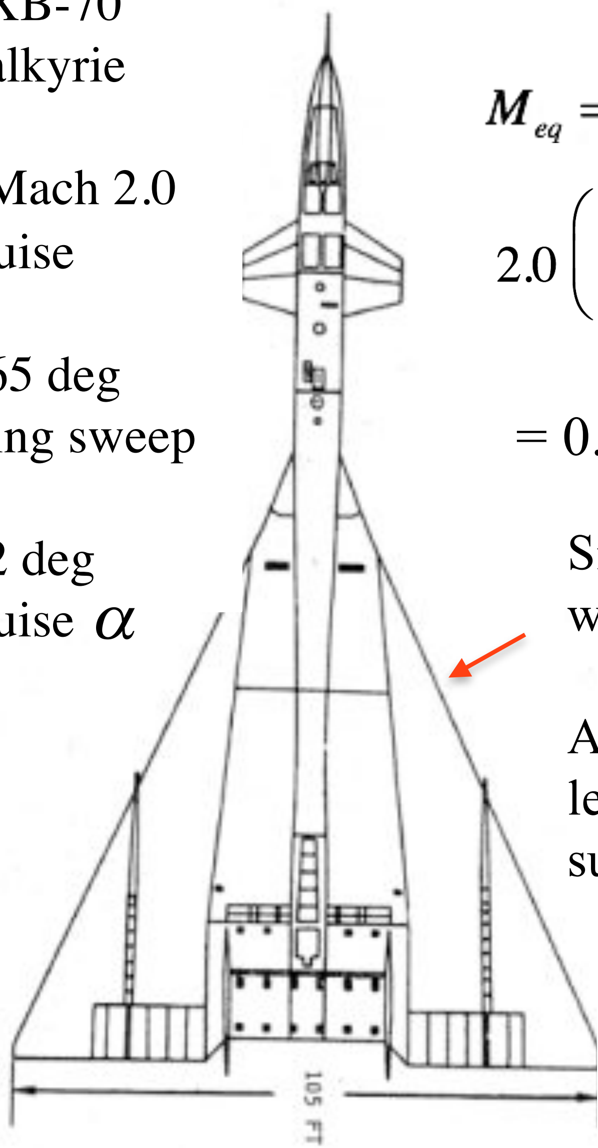


- Mach = 3.0
- 60 deg wing sweep



Other Supersonic Advantages of Wing Sweep

- XB-70 Valkyrie
- Mach 2.0 cruise
- 65 deg wing sweep
- 2 deg cruise α



$$M_{eq} = \sqrt{M_{\perp}^{2} + M_{vert}^{2}} = \sqrt{\left(M_{\infty}\sin\alpha\right)^{2} + \left(M_{\infty}\cos\alpha\cos\Lambda\right)^{2}} =$$

$$2.0\left(\left(\cos\left(\frac{\pi}{180}(65)\right)\cdot\cos\left(\frac{\pi}{180}2\right)\right)^{2} + \sin^{2}\left(\frac{\pi}{180}2\right)\right)^{0.5}$$

$$= 0.847 \quad \text{.. Subsonic leading edge}$$

Significantly weaker wing shock

Allows more rounded leading edge for improved subsonic performance