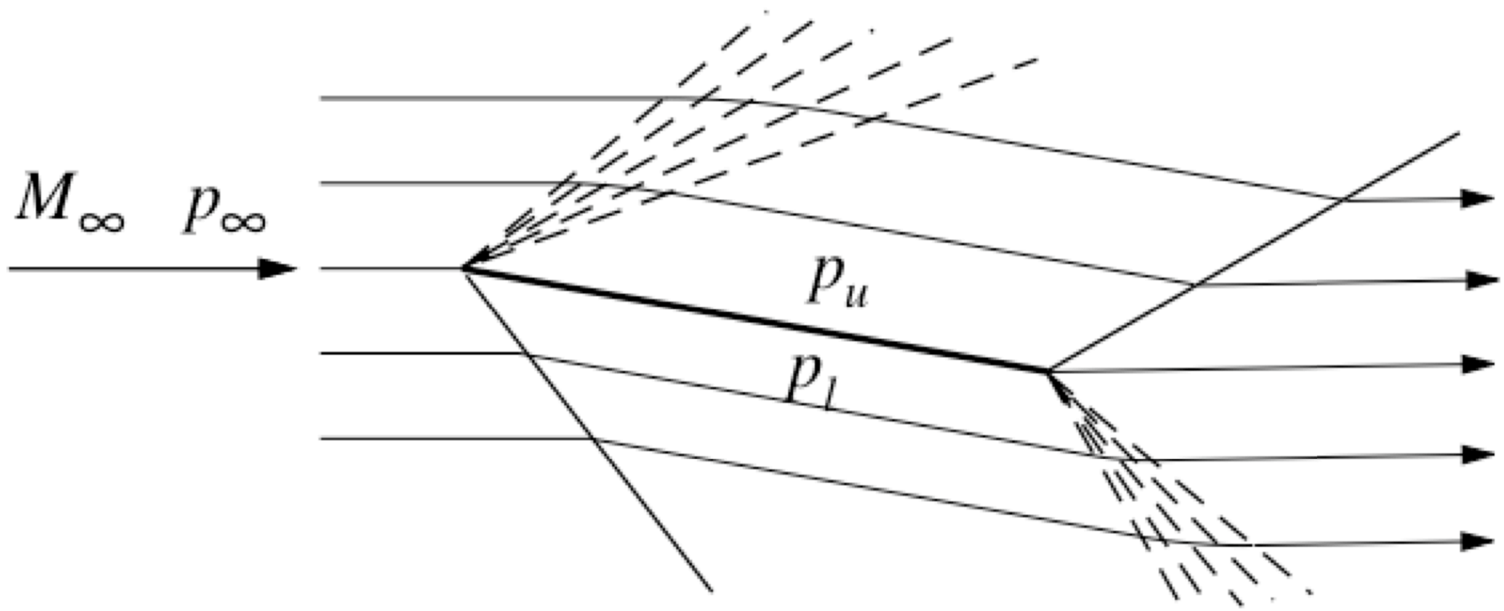
Section 7 Lecture 1:Supersonic Flow Over Flat Plates at Angle of Attack



- Anderson,
  Chapter 4 pp.127-187

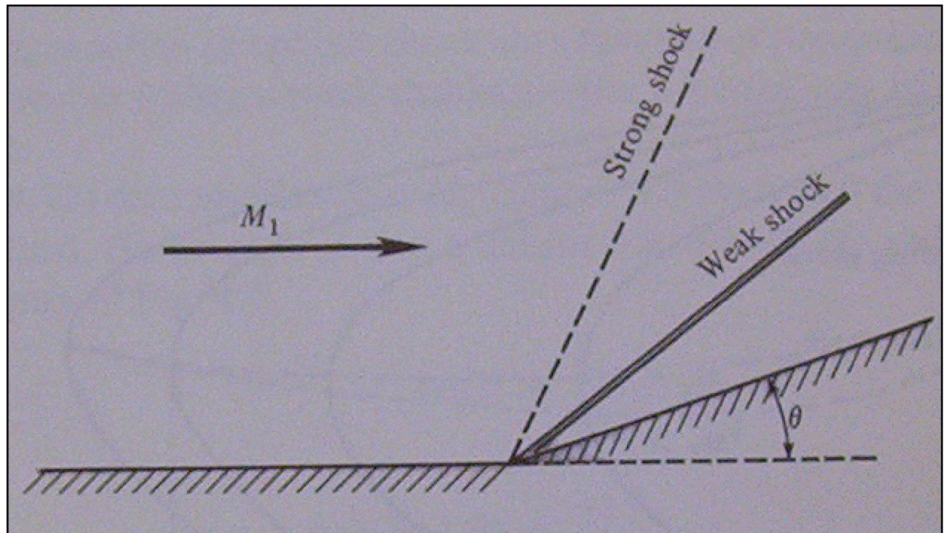
Review: Oblique Shock Wave Angle

- Collect terms

$$\tan(\theta) = \frac{2\tan(\beta)\left\{\left[M_1 \sin(\beta)\right]^2 - 1\right\}}{\tan^2(\beta)\left[2 + M_1^{\,2}\left[\gamma + \cos(2\beta)\right]\right]} = \frac{2\left\{M_1^{\,2}\sin^2(\beta) - 1\right\}}{\tan(\beta)\left[2 + M_1^{\,2}\left[\gamma + \cos(2\beta)\right]\right]}$$

- "Wedge Angle" Given explicitly as function of shock angle and freestream Mach number

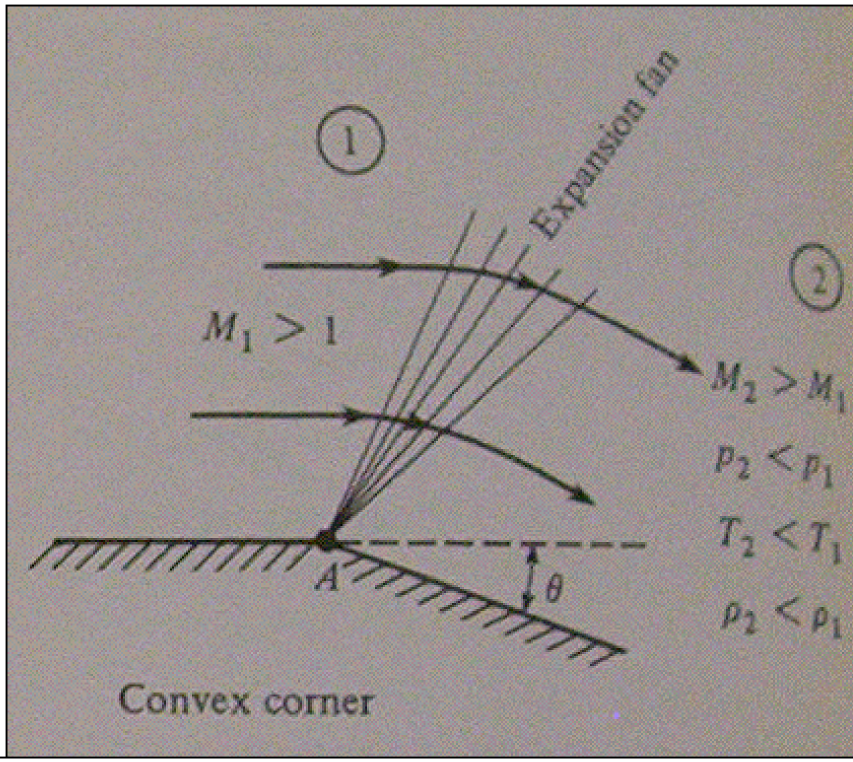
- Two Solutions "weak" and "strong" shock wave … in reality weak shock typically occurs; strong only occurs under very Specialized circumstances .e.g near stagnation point for a detached Shock (Anderson, pp. 138-139, 165,166)



Weak and strong shocks.

Review: Prandtl-Meyer Expansion Waves

$\theta<0$ .. We get an *expansion wave (Prandtl-Meyer)*

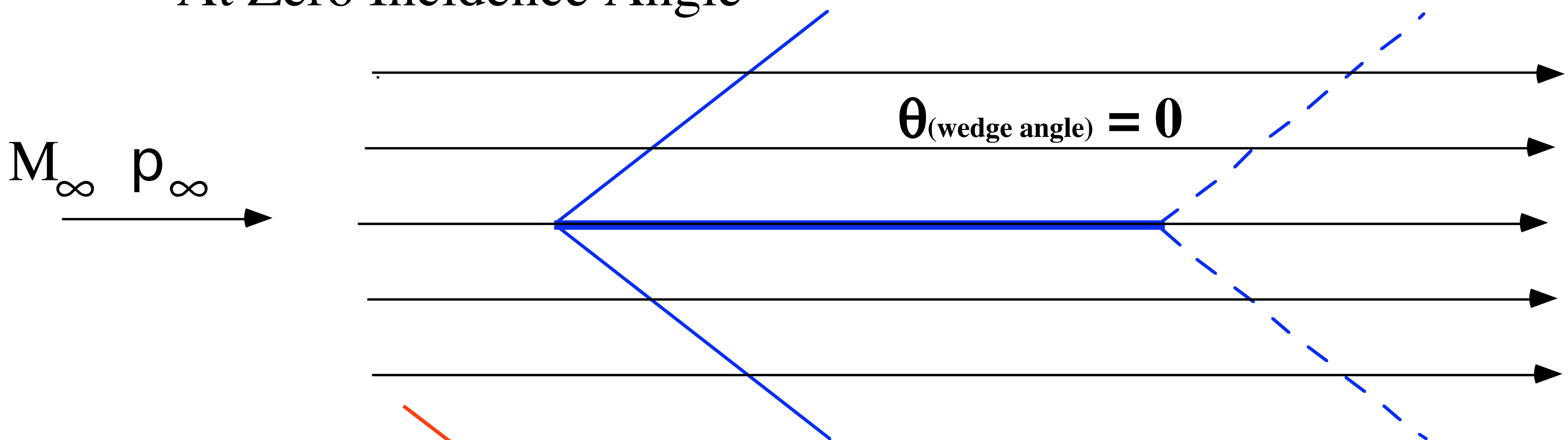


- Flow accelerates around corner
- Continuous flow region …
  sometimes called
  "expansion fan"
- Each mach wave is infinitesimally weak
  isentropic flow region
- Flow stream lines are curved and smooth through fan

$$\theta = \nu(M_2) - \nu(M_1) \rightarrow \nu(M) = \sqrt{\frac{\gamma+1}{\gamma-1}}\tan^{-1}\left\{\sqrt{\frac{\gamma-1}{\gamma+1}\left(M^2-1\right)}\right\} - \tan^{-1}\sqrt{M^2-1}$$

Supersonic Flow Over Flat Plate

- At Zero Incidence Angle



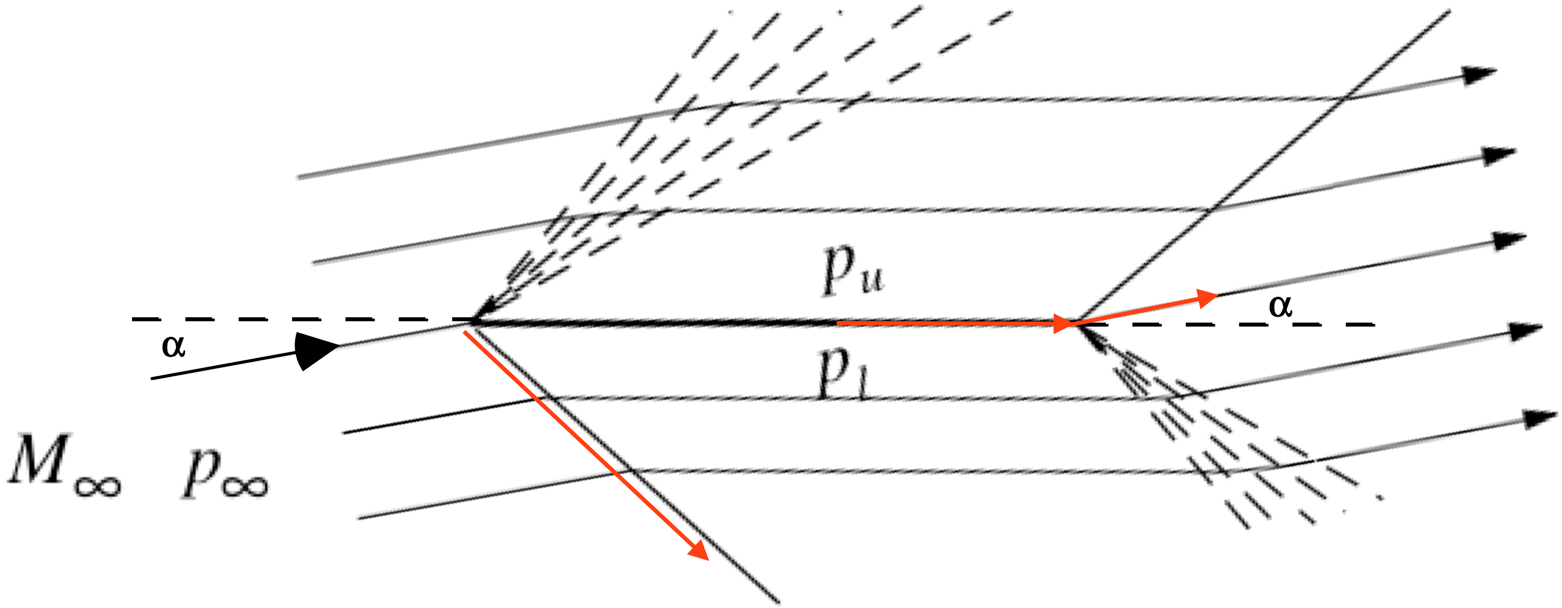
$$\tan(\theta) = 0 = \frac{2\left\{M_1^{\,2}\sin^2(\beta) - 1\right\}}{\tan(\beta)\left[2 + M_1^{\,2}\left[\gamma + \cos(2\beta)\right]\right]} \Rightarrow$$

$$M_1^{\,2}\sin^2(\beta) - 1 = 0 \rightarrow \beta = \sin^{-1}\left[\frac{1}{M}\right] \rightarrow \text{"}Mach..Line\text{"}$$

Supersonic Flow Over Flat Plate

(cont'd)

- But at non-zero incidence angle, α (angle-of-attack)



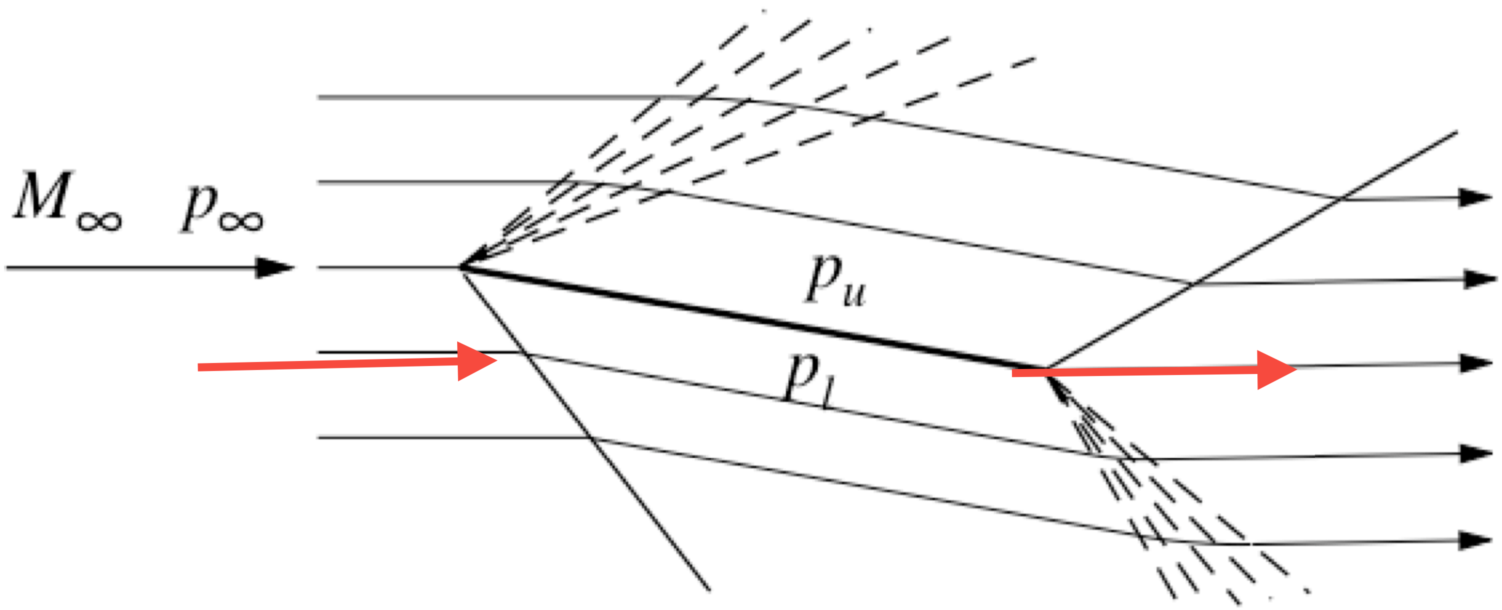
- Free Stream Flow sees Both "wedge" and "expansion" corners, … shocks have finite strength … no longer Mach waves

- Need components of both Oblique shock and expansion wave Theory to analyze flow properties

Supersonic Flow Over Flat Plate

(cont'd)

- Look at Lower Surface first

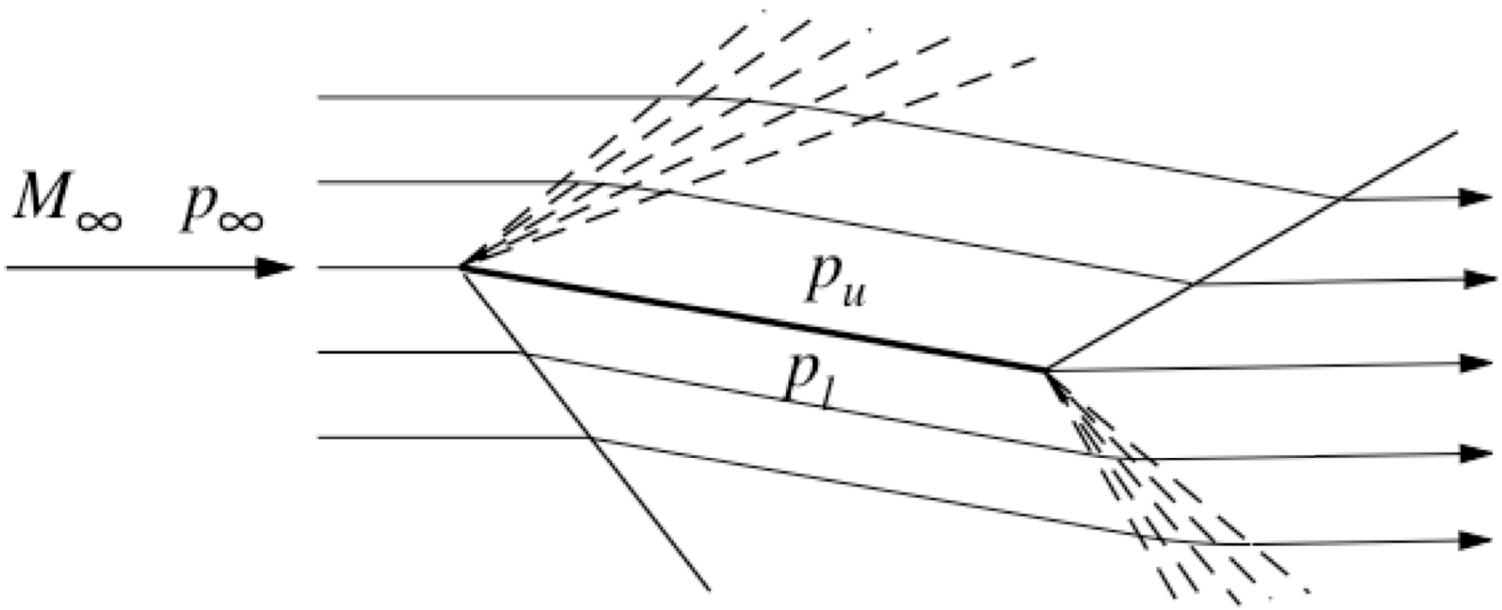


i) Flow sees compression corner first … wedge angle α
ii) Flow sees expansion corner next … expansion angle α
iii) Pressure on Lower surface > p_∞
iv) Mach on Lower Surface < M_∞

Supersonic Flow Over Flat Plate

(cont'd)

- Look at Upper Surface Next



i) Flow sees expansion corner first … expansion angle α

ii) Flow sees compression corner next … wedge angle α

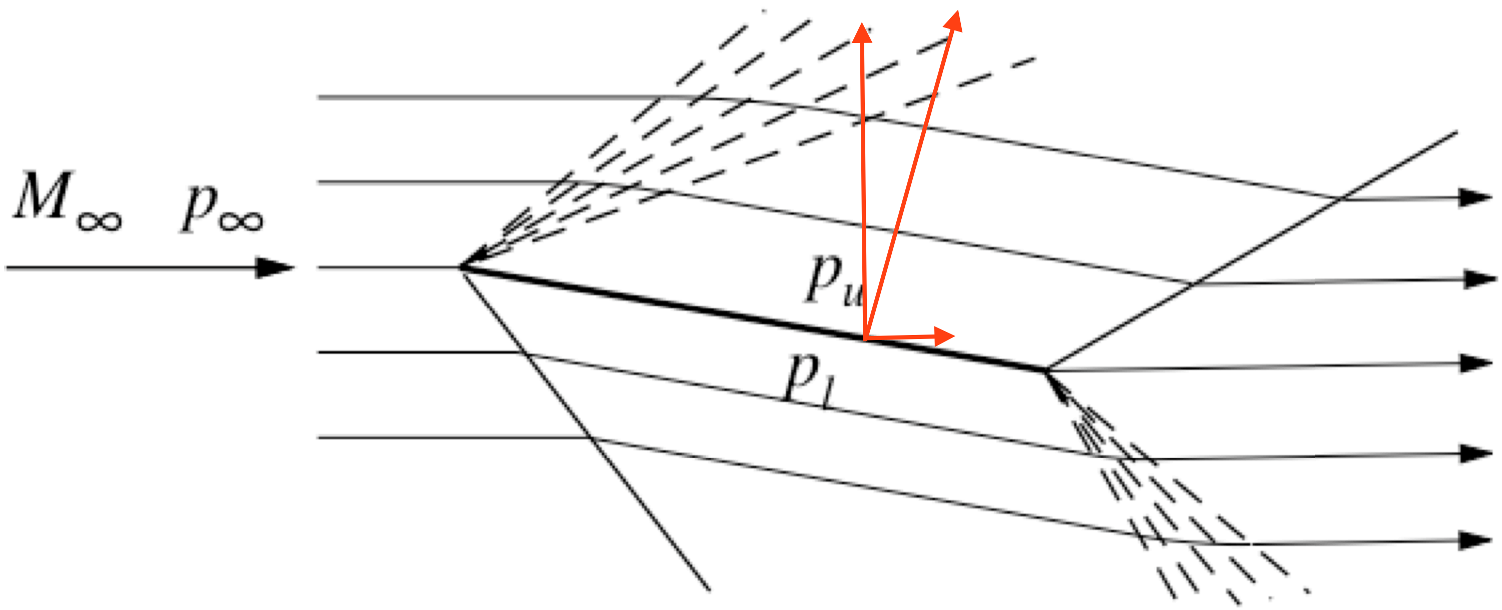
iii) Pressure on Upper surface < p_∞

iv) Mach on Lower Surface > M_∞

Supersonic Flow Over Flat Plate

(cont'd)

- Look at Upper Surface Next

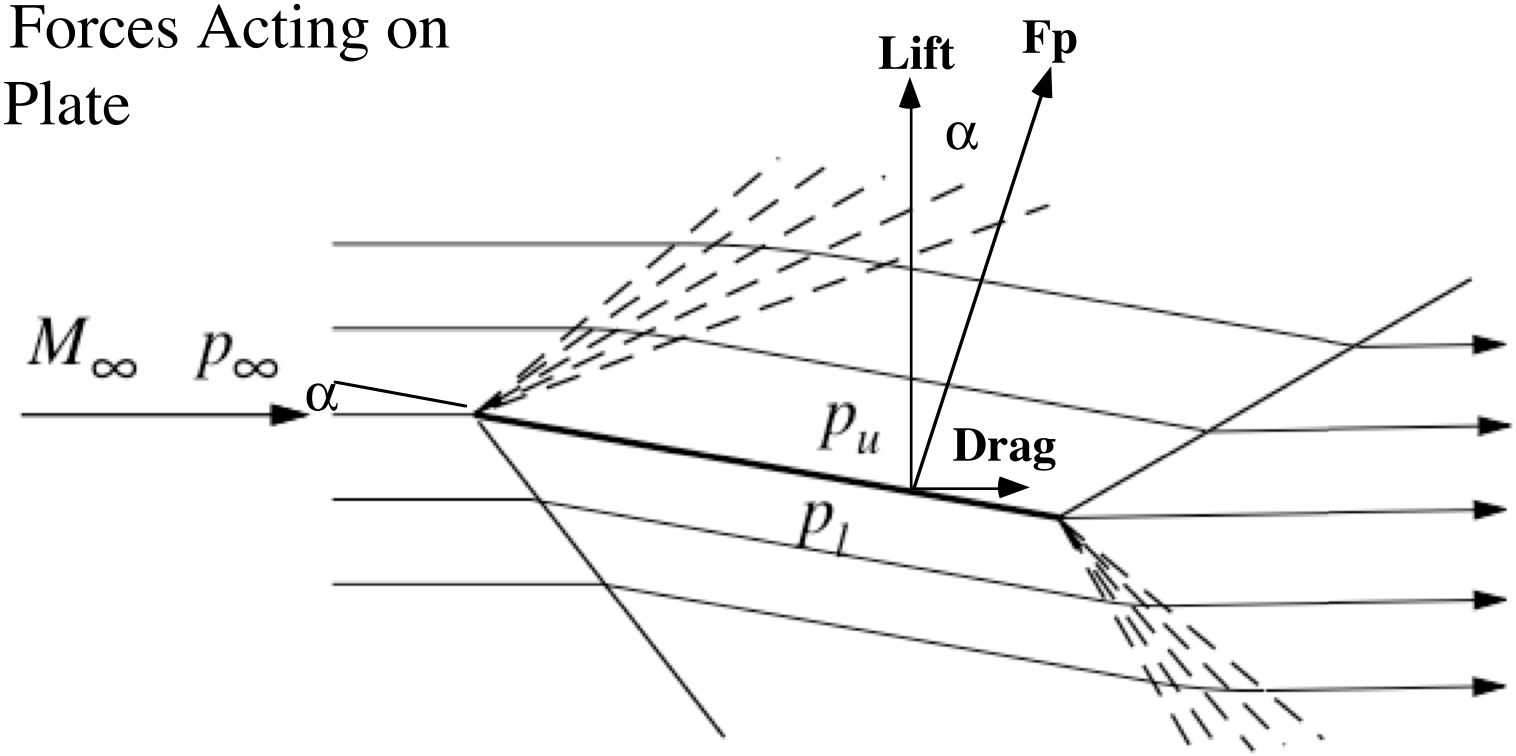


$P_u < P_l$ ... net pressure force on plate … produces *Lift and Drag*

Supersonic Flow Over Flat Plate

(cont'd)

- Forces Acting on Plate



- For Inviscid flow, pressure force can be resolved into Lift and Drag Forces

Supersonic Flow Over Flat Plate

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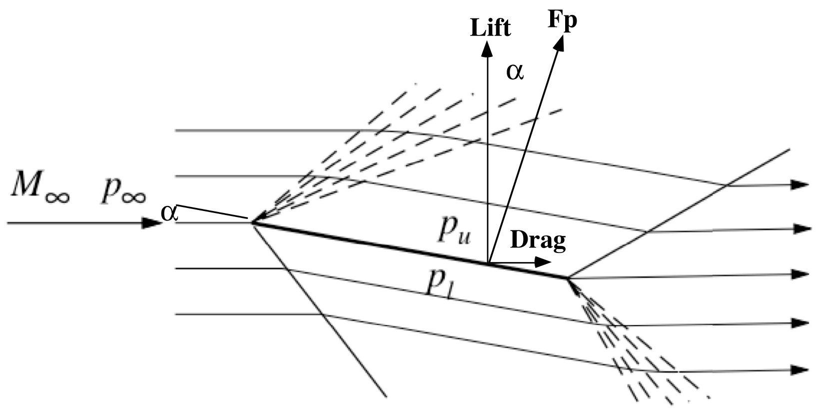


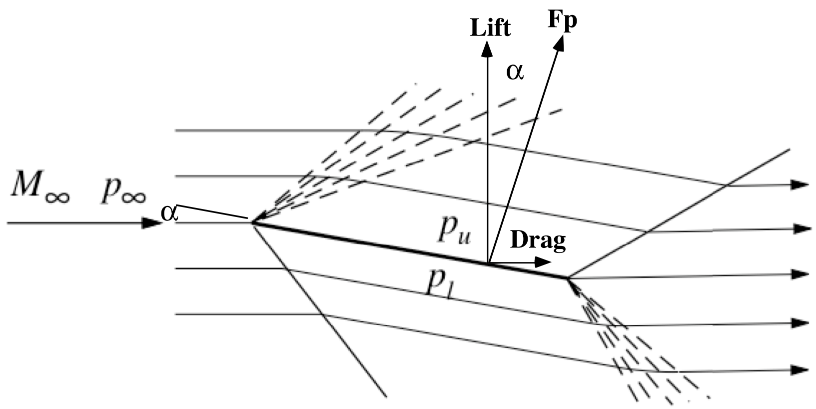
$$F_p = [p_l - p_u] A_{plate} \equiv [p_l - p_u] \bar{c} b \equiv [p_l - p_u] S$$

$$Lift = F_p \cos\alpha = [p_l - p_u] S \cos\alpha \rightarrow C_L = \frac{Lift}{\left(\frac{1}{2}\rho_\infty V_\infty^{\,2}\right) S} = \frac{[p_l - p_u]}{\left(\frac{\gamma}{2} p_\infty M_\infty^{\,2}\right)} \cos\alpha$$

$$Drag = F_p \sin\alpha = [p_l - p_u] S \sin\alpha \rightarrow C_D = \frac{Drag}{\left(\frac{1}{2}\rho_\infty V_\infty^{\,2}\right) S} = \frac{[p_l - p_u]}{\left(\frac{\gamma}{2} p_\infty M_\infty^{\,2}\right)} \sin\alpha$$

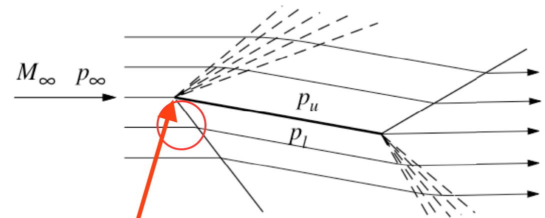
Supersonic Flow Over Flat Plate

(cont'd)

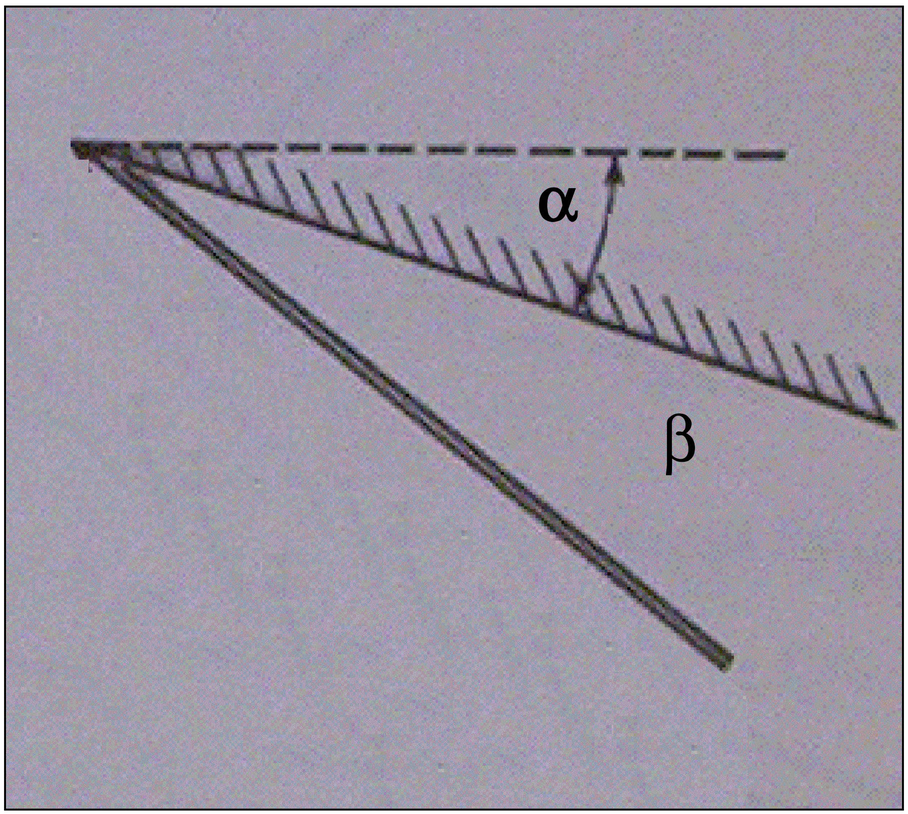


- Calculate Pressures on Upper and Lower Surfaces

Lower Surface



- $\alpha > 0$
- Corner 1: Corner Compression



- Solve for Shock Angle Using Solver

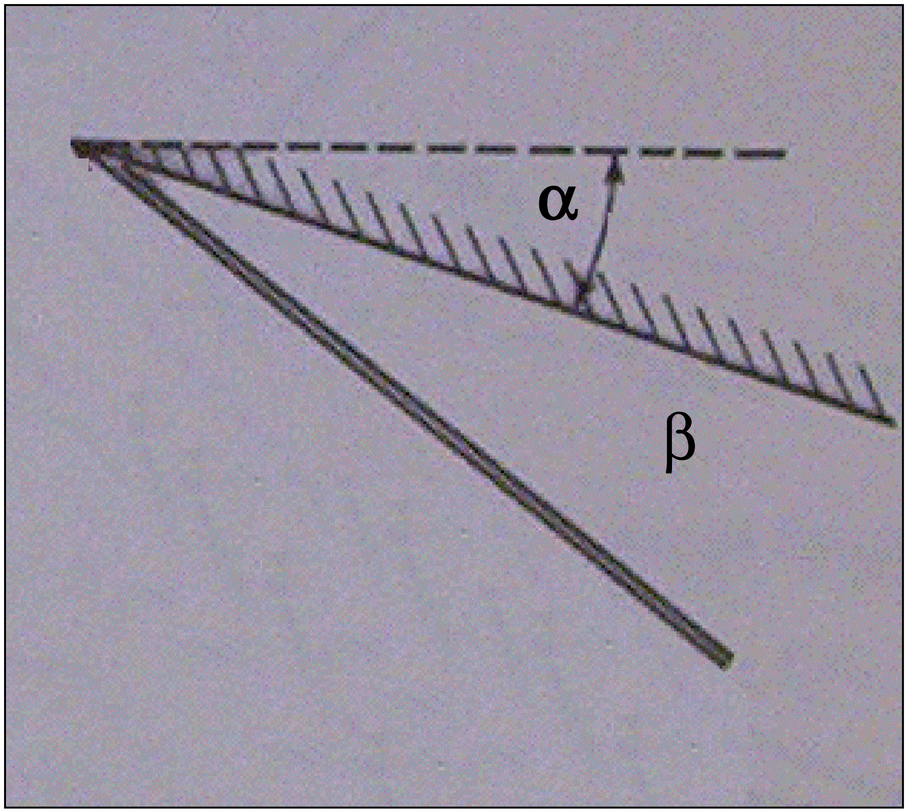
$$\tan(\alpha) = \frac{2\left\{M_1^{\,2}\sin^2(\beta) - 1\right\}}{\tan(\beta)\left[2 + M_1^{\,2}\left[\gamma + \cos(2\beta)\right]\right]}$$

Example:

$$\alpha = 10^o, M_\infty = 2 \Rightarrow \beta = 39.31^o$$

Lower Surface (continued)

- Case 1: $\alpha > 0$
- Corner 1: Corner Compression



- Compute Pressure Ratio Behind Shock

$$\alpha = 10^o, M_\infty = 2 \Rightarrow \beta = 39.31^o$$

$$\frac{p_L}{p_\infty} = 1 + \frac{2\gamma}{(\gamma+1)}\left(\left(M_\infty \sin\beta\right)^2 - 1\right) =$$

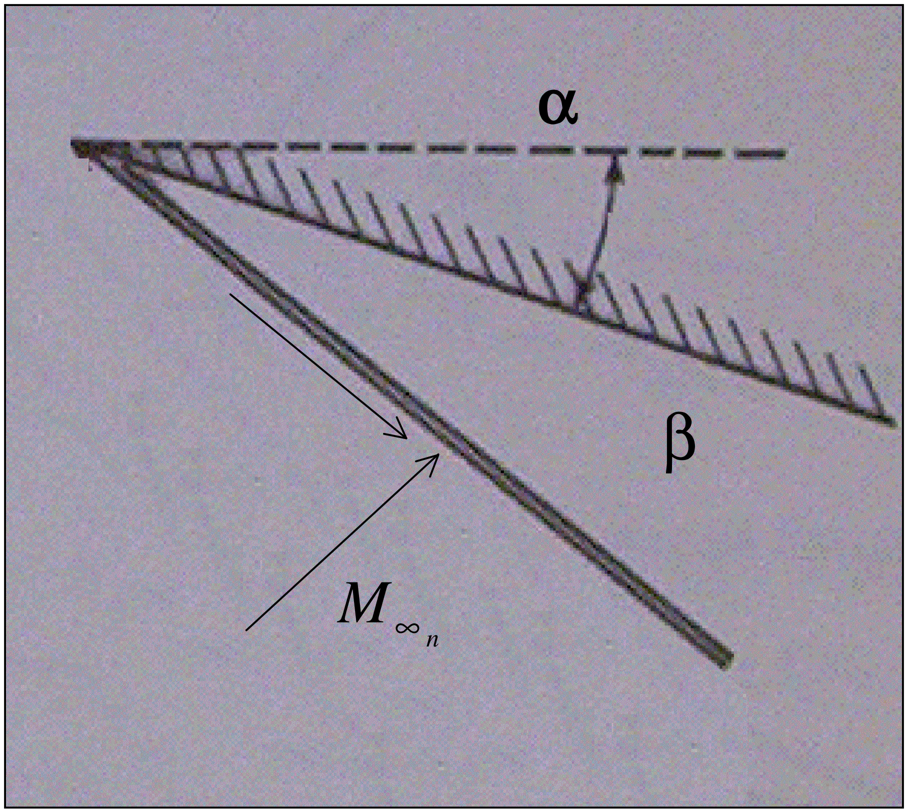
$$1 + \frac{2 \cdot 1.4}{1.4+1}\left(\left(2\sin\left(\frac{\pi}{180}39.3139\right)\right)^2 - 1\right)$$

$$=1.7066$$

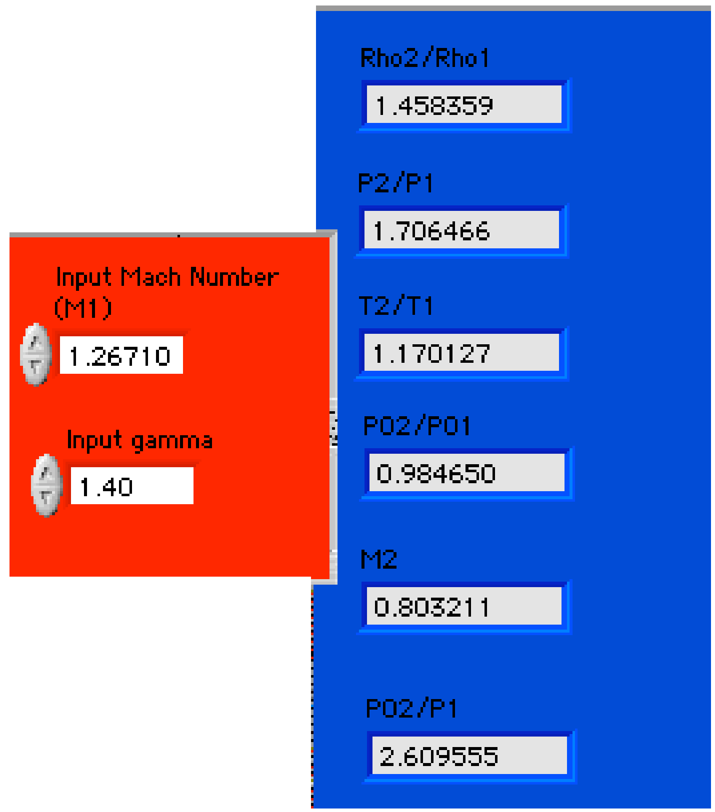
Lower Surface (continued)

- $\alpha > 0$
- Corner 1: Corner Compression

$$M_{\infty_n} = M_\infty \sin\beta = 1.2671$$



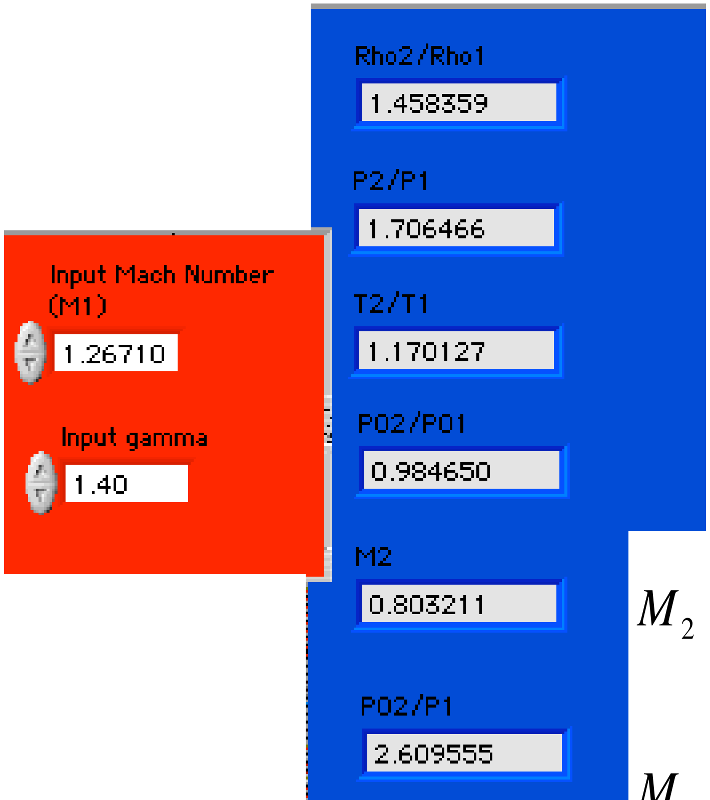
- Or Use Normal Shock Solver with $M_\infty n$ as input



Lower Surface (continued)

- $\alpha > 0$

- Compute Mach number behind shock



$M_2 n = 0.803211$

$$M_{\infty_t} = M_\infty \cos\beta = 1.5474$$

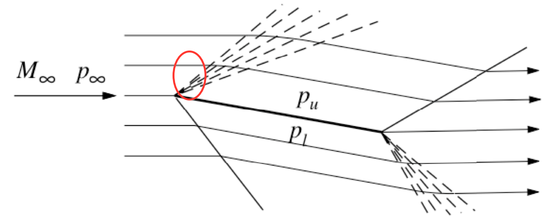
$$V_{\infty t} = V_{2t}$$

$$M_2 = \sqrt{M_{2n}^{\ 2} + M_{2t}^{\ 2}} \rightarrow M_{2t}^{\ 2} = \frac{V_{2t}^{\ 2}}{\gamma R_g T_2} = \frac{V_{\infty t}^{\ 2}}{\gamma R_g T_\infty}\frac{\gamma R_g T_\infty}{\gamma R_g T_2} = \frac{M_{\infty t}^{\ 2}}{T_2/T_1}$$

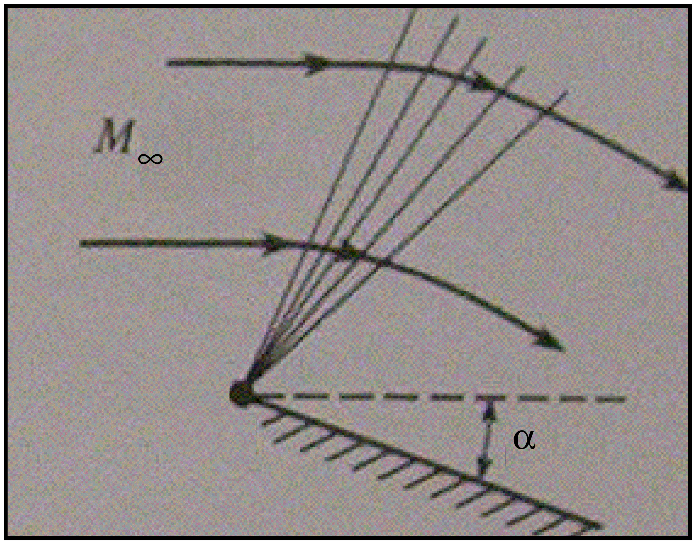
$$M_2 = \sqrt{M_{2n}^{\ 2} + \frac{M_{\infty t}^{\ 2}}{T_2/T_1}} = \left(0.803211^2 + \frac{1.5474^2}{1.1701}\right)^{0.5}$$

$= 1.64052$

Upper Surface



- $\alpha > 0$
- Corner 1: Expansion



- Use Prandtl-Meyer Solver

$$\nu(M_u) = \alpha + \nu(M_\infty) =$$

$$10\frac{\pi}{180} + 0.46041 \quad = 0.63494$$

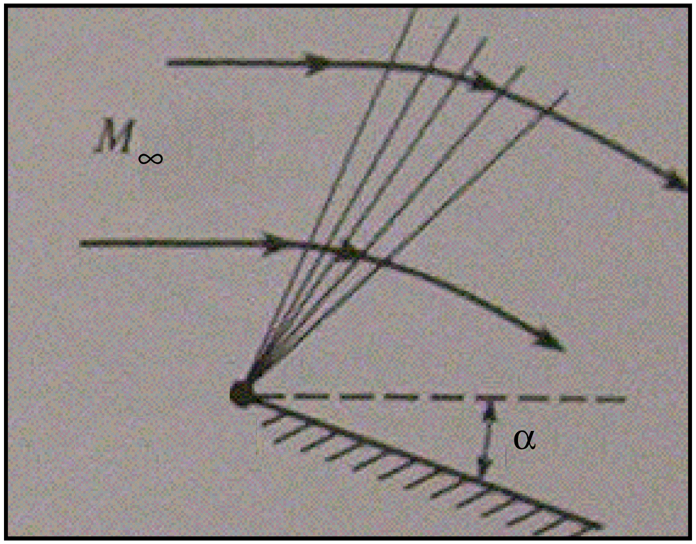
$$\nu(M_u) = \alpha + \nu(M_\infty) \rightarrow$$

$$\nu(M_\infty) = \sqrt{\frac{\gamma+1}{\gamma-1}}\tan^{-1}\left\{\sqrt{\frac{\gamma-1}{\gamma+1}\left(M_\infty^{\ 2}-1\right)}\right\} - \tan^{-1}\sqrt{M_\infty^{\ 2}-1} =$$

$$\left(\frac{1.4+1}{1.4-1}\right)^{0.5} \text{atan}\left(\left(\frac{1.4-1}{1.4+1}\,(2^2-1)\right)^{0.5}\right) - \text{atan}\left(\,(2^2-1)^{0.5}\right) \quad = 0.46041$$

Upper Surface (cont'd)

- $\alpha > 0$



- Solve for mach number on upper surface

$$\nu(M_u) = \alpha + \nu(M_\infty) = 10\frac{\pi}{180} + 0.46041 = 0.63494$$

- Use iterative solver

  - M_u=2.3849

- Flow is isentropic

$$\frac{P_{0u}}{P_{0\infty}} = \frac{p_u\left[1+\frac{\gamma-1}{2}M_u^{\,2}\right]^{\frac{\gamma}{\gamma-1}}}{p_\infty\left[1+\frac{\gamma-1}{2}M_\infty^{\,2}\right]^{\frac{\gamma}{\gamma-1}}} = 1 \Rightarrow \frac{p_u}{p_\infty} = \left[\frac{1+\frac{\gamma-1}{2}M_\infty^{\,2}}{1+\frac{\gamma-1}{2}M_2^{\,2}}\right]^{\frac{\gamma}{\gamma-1}} = \left(\frac{1+\frac{1.4-1}{2}(2)^2}{1+\frac{1.4-1}{2}(2.3849)^2}\right)^{\left(\frac{1.4}{1.4-1}\right)}$$

$$=0.54796$$

Compute Lift and Drag Coefficients

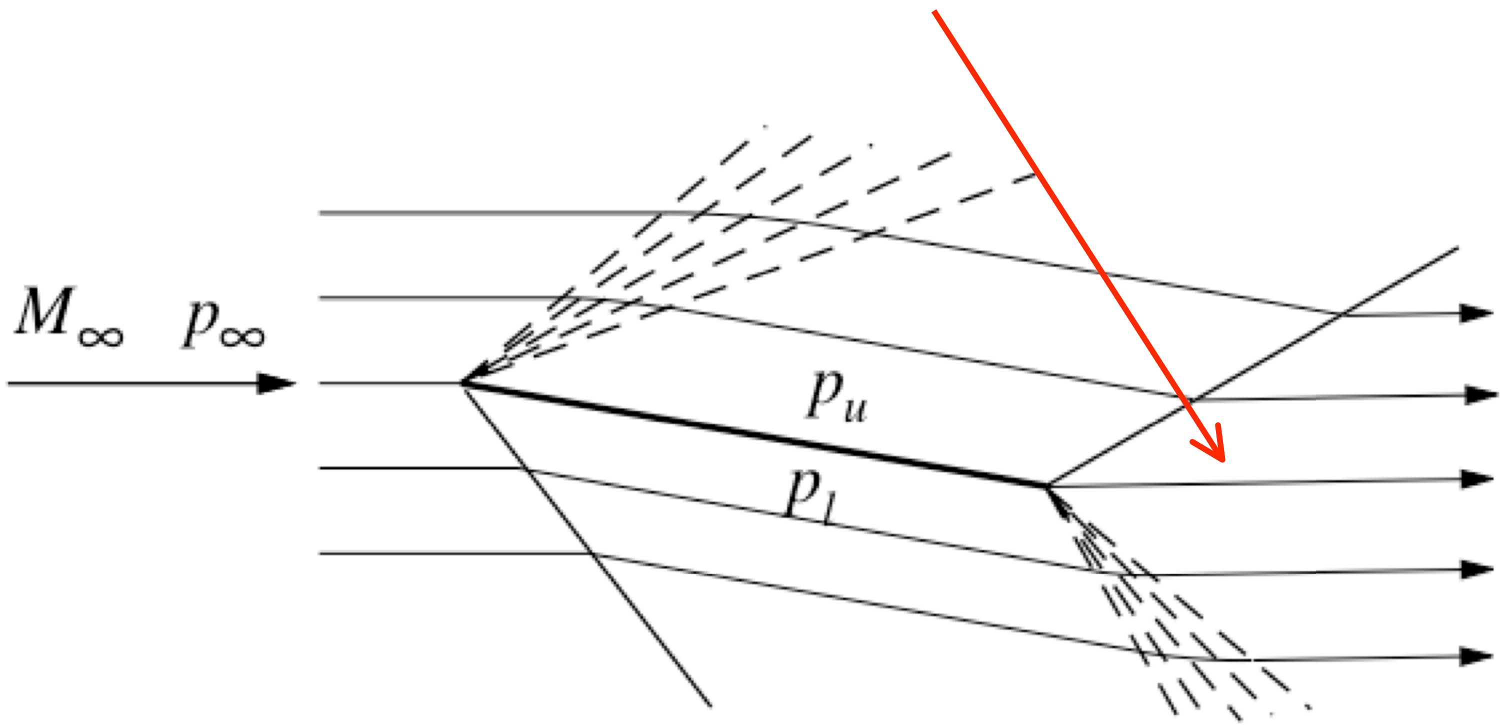
$$C_L = \frac{[p_l - p_u]}{\left(\frac{\gamma}{2} p_\infty M_\infty^{\ 2}\right)} \cos\alpha = \frac{\left[\frac{p_l}{p_\infty} - \frac{p_u}{p_\infty}\right]}{\left(\frac{\gamma}{2} M_\infty^{\ 2}\right)} \cos\alpha = \frac{1.7066 - 0.54796}{\frac{1.4}{2}(2^2)} \cos\left(\frac{\pi}{180} 10\right) = 0.4075$$

$$C_D = \frac{[p_l - p_u]}{\left(\frac{\gamma}{2} p_\infty M_\infty^{\ 2}\right)} \sin\alpha = \frac{\left[\frac{p_l}{p_\infty} - \frac{p_u}{p_\infty}\right]}{\left(\frac{\gamma}{2} M_\infty^{\ 2}\right)} \sin\alpha = \frac{1.7066 - 0.54796}{\frac{1.4}{2}(2^2)} \sin\left(\frac{\pi}{180} 10\right) = 0.07186$$

L/D=5.671 …. Computed L/D of plate without knowing The dimensions of the plate!

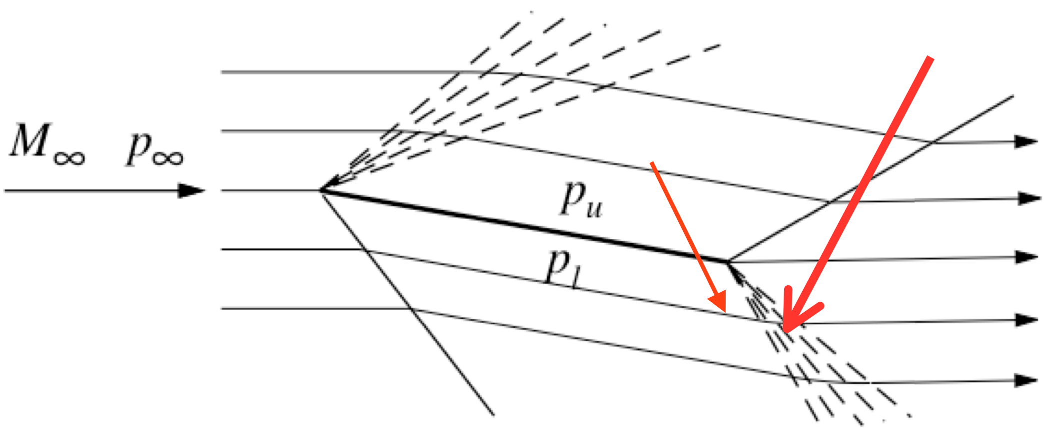
Look at exit Conditions

- Look at exit conditions …



Lower Surface (corner 2)

- $\alpha > 0$
  - Corner 2: Expansion $\quad \alpha = 10^o, M_\infty = 2 \Rightarrow M_2 = 1.64052$



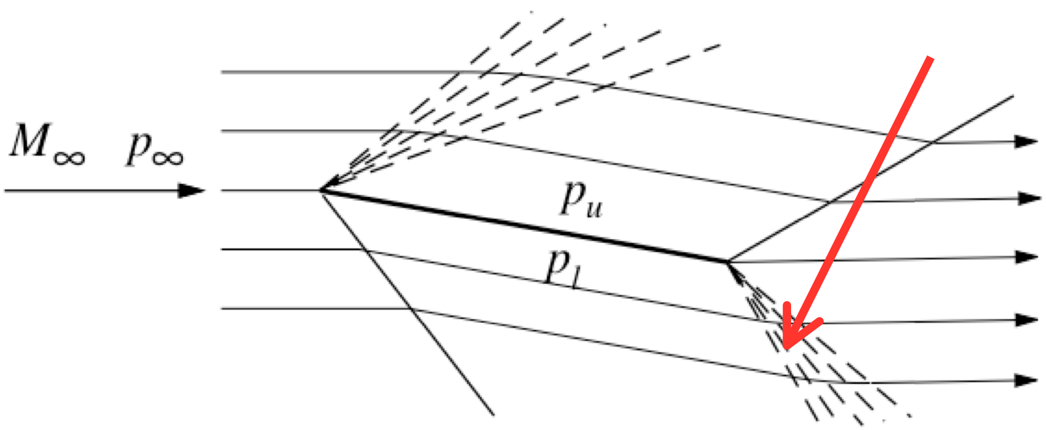
$$\nu(M_2) = \sqrt{\frac{\gamma+1}{\gamma-1}} \tan^{-1}\left\{\sqrt{\frac{\gamma-1}{\gamma+1}\left(M_2{}^2 - 1\right)}\right\} - \tan^{-1}\sqrt{M_\infty{}^2 - 1} =$$

$$\left(\frac{1.4+1}{1.4-1}\right)^{0.5} \operatorname{atan}\left(\left(\left(\frac{1.4-1}{1.4+1}\right)(1.64052^2 - 1)\right)^{0.5}\right) - \operatorname{atan}\left(\left(1.64052^2 - 1\right)\right)^{0.5}$$

$$= 0.280266$$

Lower Surface (corner2) (cont'd)

- $\alpha > 0$
  - Corner 2: Expansion $\alpha = 10^o, M_\infty = 2 \Rightarrow M_2 = 1.64052$

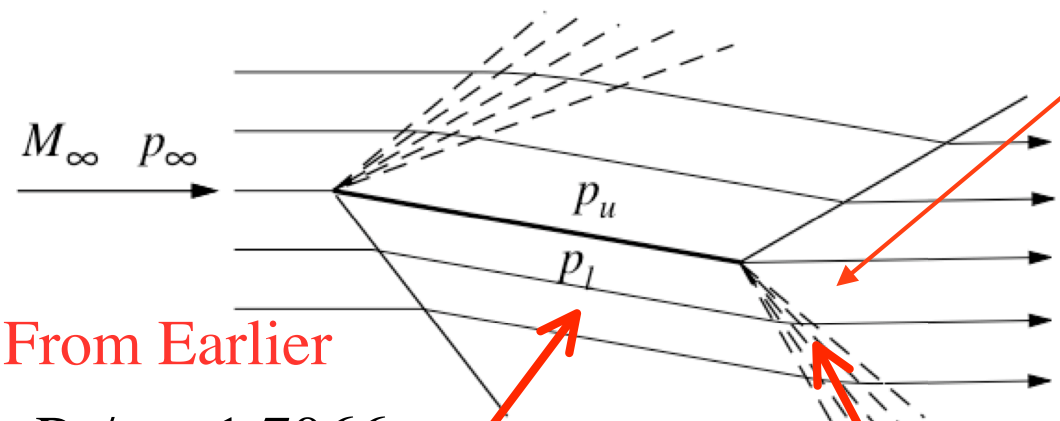


$$\nu(M_{exit}) = \alpha + \nu(M_2) = \left(\frac{\pi}{180}10\right) + 0.2872 = 0.454799$$

- Use iterative solver
  - M_{uexit}=1.98835

Lower Surface (corner2) (cont'd)

- Compute Pressure Ratio
- From earlier, behind oblique shock on Lower Surface



From Earlier

$P_L/p_\infty = 1.7066$

$$\frac{p_{L_{exit}}}{p_\infty} = \left(\frac{p_L}{p_\infty}\right)\left(\frac{p_{L_{exit}}}{p_L}\right) =$$

$$0.5875 \cdot 1.7066 \quad = 1.0026$$

Pressure is Higher!

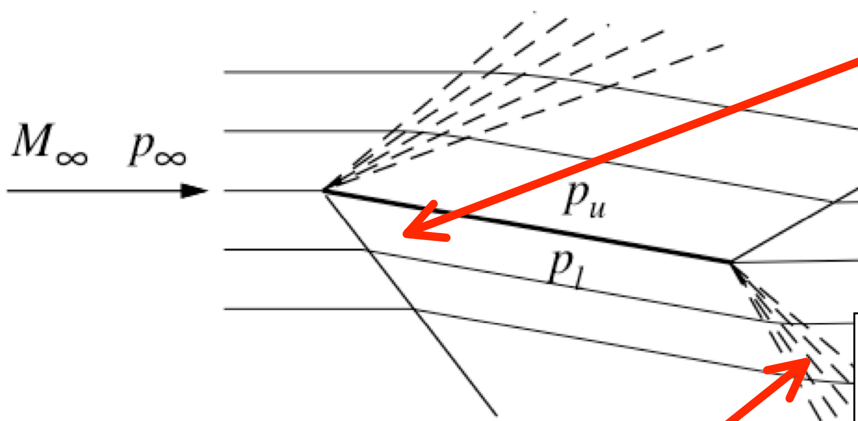
- And since expansion wave is isentropic

$$\frac{p_{L_{exit}}}{p_L} = \left[\frac{1+\frac{\gamma-1}{2}M_L^{\ 2}}{1+\frac{\gamma-1}{2}M_{L_{exit}}^{\ \ 2}}\right]^{\frac{\gamma}{\gamma-1}} = \left(\frac{1+\frac{1.4-1}{2}1.64052^2}{1+\frac{1.4-1}{2}1.9883^2}\right)^{\frac{1.4}{1.4-1}} = 0.5875$$

Lower Surface (corner2) (cont'd)

- Compute Temperature Ratio
- Look at Temperature behind oblique shock

From Earlier $T_L/T_\infty=1.170151$



$$\frac{T_{L_{exit}}}{T_\infty} = \left(\frac{T_{L_{exit}}}{T_L}\right)\left(\frac{T_L}{T_\infty}\right) = (1.17015)(0.85925)$$

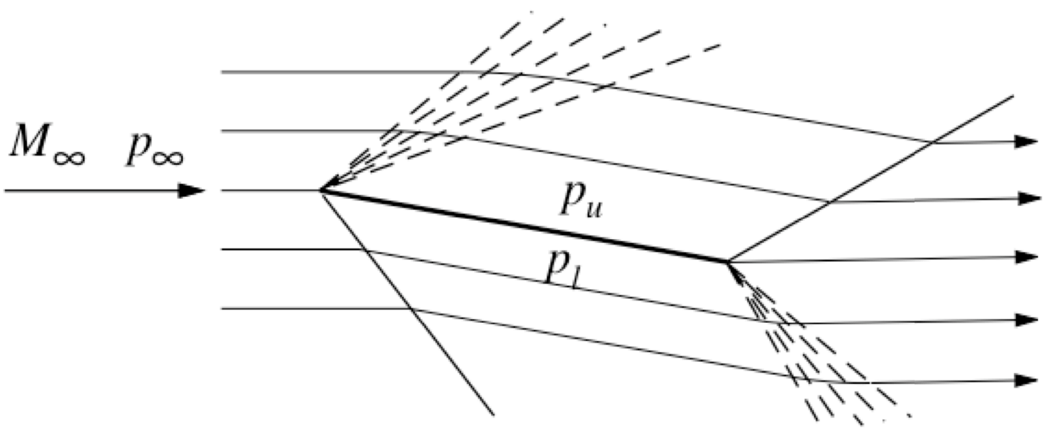
$=1.0052$

- And since expansion wave is isentropic

$$\frac{T_{L_{exit}}}{T_L} = \left[\frac{1+\frac{\gamma-1}{2}M_L{}^2}{1+\frac{\gamma-1}{2}M_{L_{exit}}{}^2}\right] = \left(\frac{1+\frac{1.4-1}{2}1.64052^2}{1+\frac{1.4-1}{2}1.9883^2}\right) = 0.859$$

Lower Surface (corner2) (cont'd)

- Compute Velocity ratio



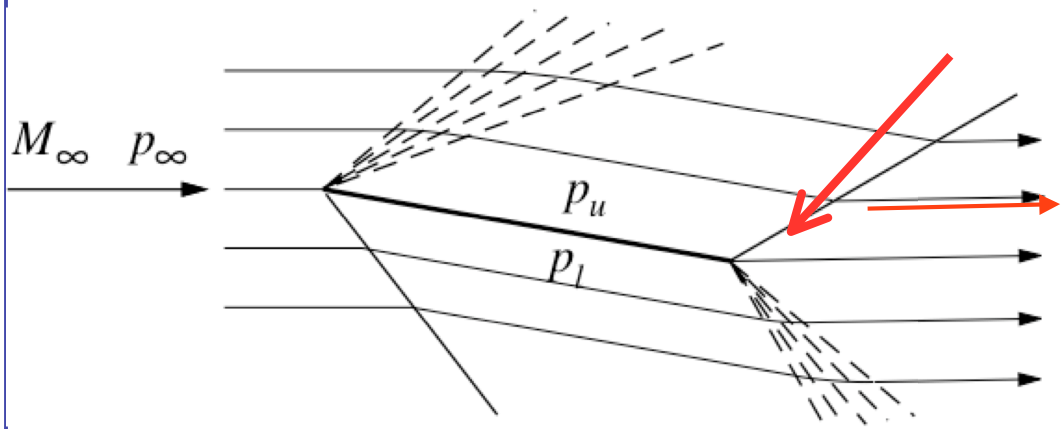
$$\frac{V_{L_{exit}}}{V_\infty} = \frac{M_{L_{exit}}}{M_\infty}\sqrt{\frac{T_{L_{exit}}}{T_\infty}} = \frac{1.9883}{2}\sqrt{1.0052} = 0.9967$$

Velocity is Lower!

Upper Surface (corner 2)

- $\alpha > 0$
- Corner 2: Compression

$$\alpha = 10^o, M_\infty = 2 \Rightarrow M_2 = 2.3849$$



$\beta = 33.211°$

$M_2n = 1.306267$

$M_{uexit}n = 0.7828$

$M_{uexit} = 1.9861796$

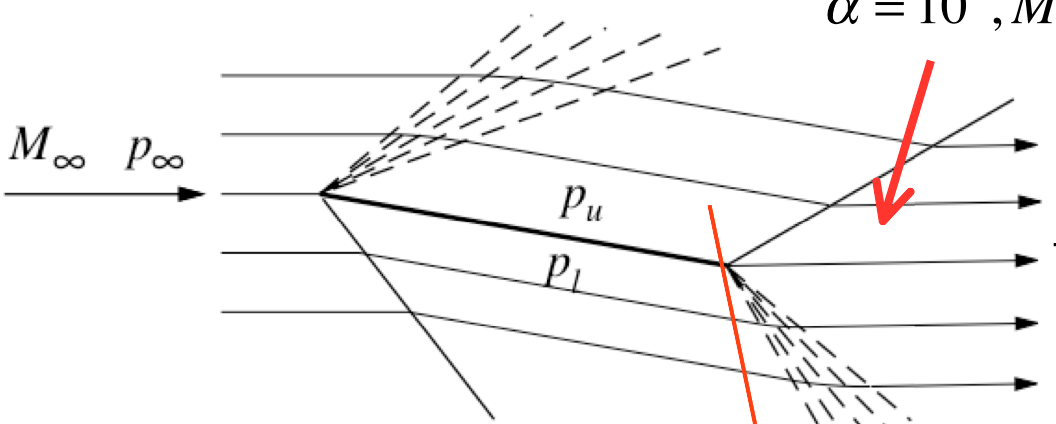
- Solve for Shock Angle Using Solver

$$\tan(\alpha) = \frac{2\left\{M_2^{\,2}\sin^2(\beta_2) - 1\right\}}{\tan(\beta_2)\left[2 + M_2^{\,2}\left[\gamma + \cos(2\beta_2)\right]\right]} \longrightarrow$$

Upper Surface (corner2) (cont'd)

- Now look at pressure ratio across oblique shock

$$\alpha = 10^o, M_\infty = 2 \Rightarrow M_2 = 2.3849$$



$$\frac{p_u}{p_\infty} = \frac{7.824449}{14.27929} = 0.547962$$

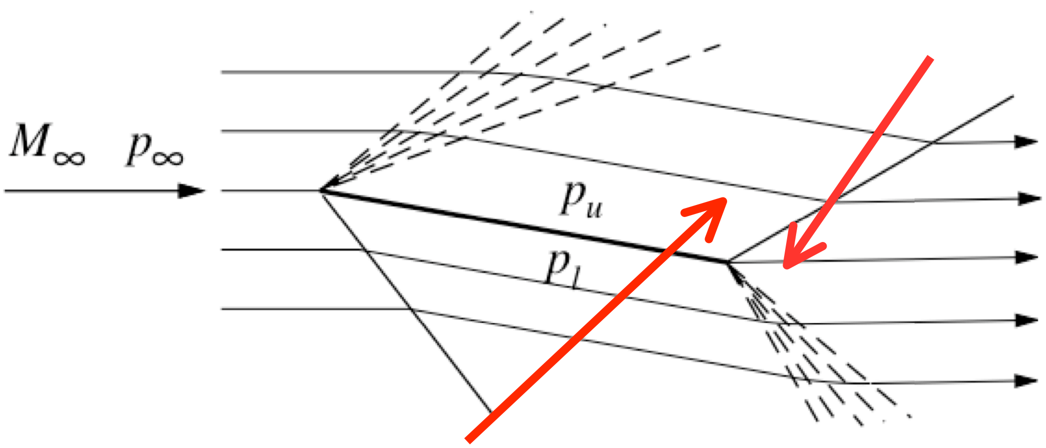
- Across first expansion wave $M_{2u} = 2.3849 \Rightarrow \frac{P_{0u}}{p_u} = 14.27929$

- And $M_{2u} = 2.3849 \Rightarrow \frac{P_{0u}}{p_u} = 14.27929$

$$\frac{p_u}{p_\infty} = \frac{7.824449}{14.27929} = 0.547962$$

Upper Surface (corner2) (cont'd)

- Compute Pressure



- M_2n = 1.306267 … normal shock code

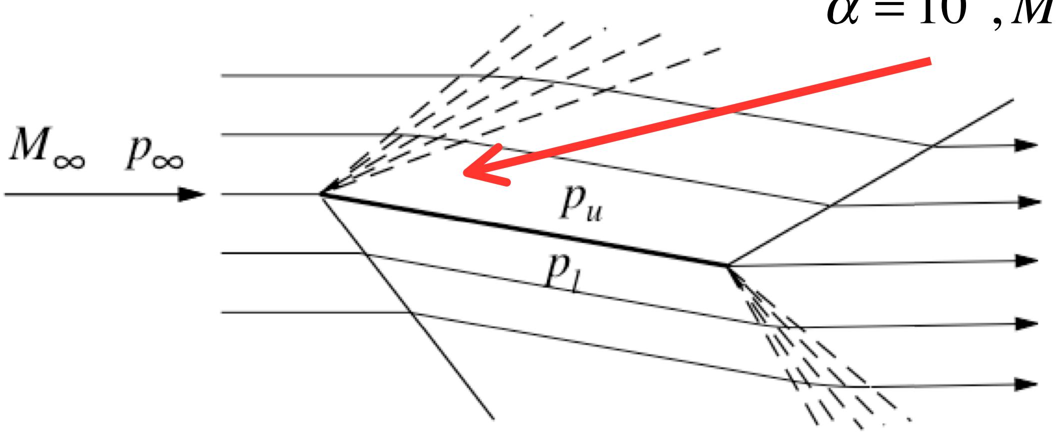
---> $$\frac{p_u exit}{p_u} = 1.824052 \Rightarrow \frac{p_u exit}{p_\infty} = \frac{p_u}{p_\infty}\frac{p_u exit}{p_u} = 0.54796 \cdot 1.824052$$

$\boxed{\textbf{=0.99951 … Here Pressure is Lower}}$

Upper Surface (corner2) (cont'd)

- look at temperature ratio across expansion wave

$\alpha = 10^o, M_\infty = 2 \Rightarrow M_2 = 2.3849$



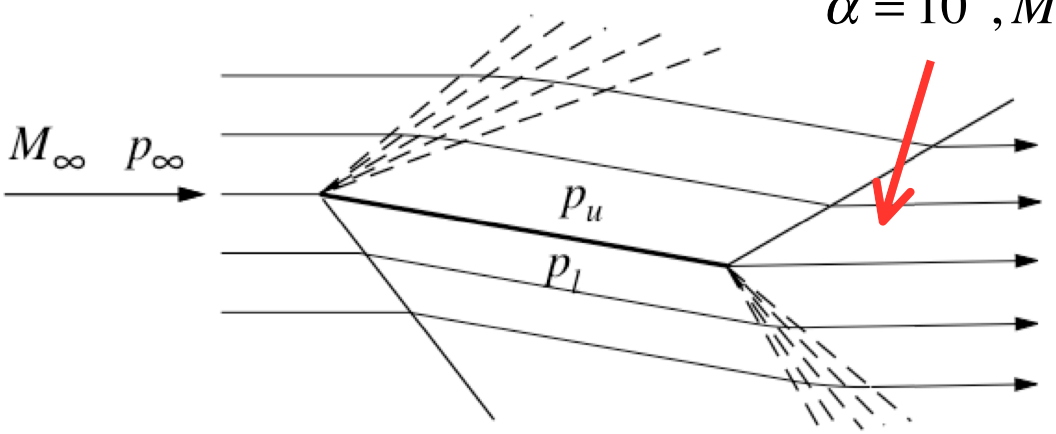
- for isentropic flow

$$\frac{T_{u_{2t}}}{T_\infty} = \left[\frac{1+\frac{\gamma-1}{2}M_\infty{}^2}{1+\frac{\gamma-1}{2}M_{u_2}{}^2}\right] = \frac{1+\frac{1.4-1}{2}2^2}{1+\frac{1.4-1}{2}2.3849^2} = 0.84209$$

Upper Surface (corner2) (cont'd)

- Now look at temperature ratio across oblique shock

$$\alpha = 10^o, M_\infty = 2 \Rightarrow M_2 = 2.3849$$

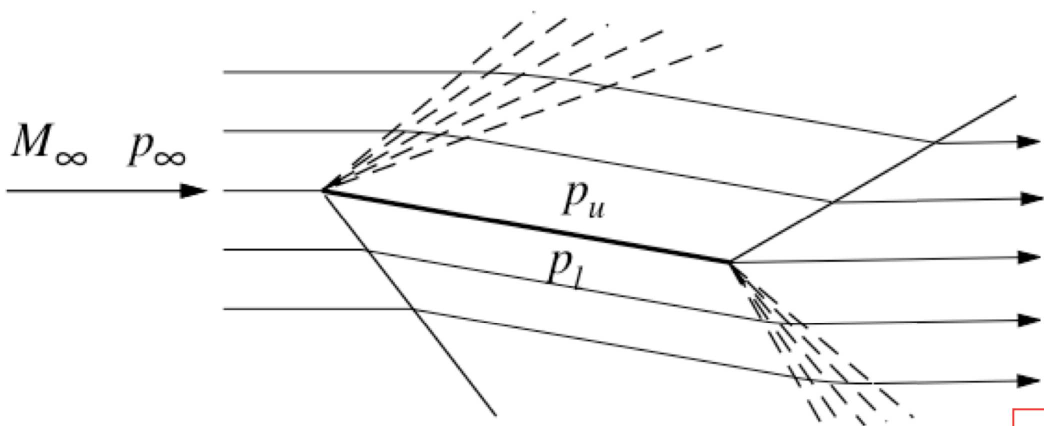


- Use Oblique Shock Solver $\dfrac{T_{u_{exit}}}{T_{u_2}} = 1.194835$

$$\frac{T_{u_{exit}}}{T_\infty} = \left(\frac{T_{u_{exit}}}{T_{u_2}}\right)\left(\frac{T_{u_2}}{T_\infty}\right) = (1.194935)(0.84209) \quad = 1.0062$$

Upper Surface (corner2) (cont'd)

- Compute Velocity ratio



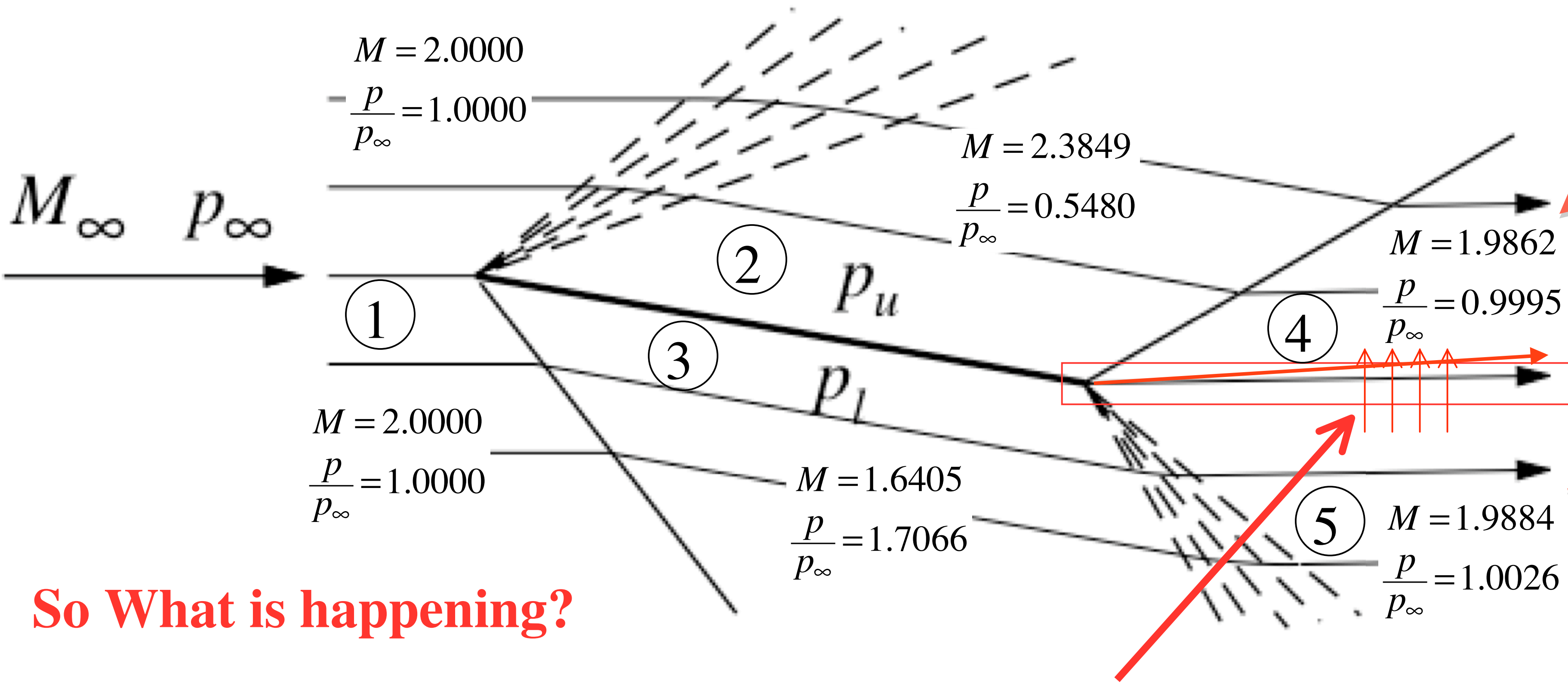
Velocity is Lower

$$\frac{V_{u_{exit}}}{V_\infty} = \frac{M_{u_{exit}}}{M_\infty}\sqrt{\frac{T_{u_{exit}}}{T_\infty}} = \frac{1.9862}{2}\sqrt{1.0062} = 0.9962$$

So What is happening?

Shock-Expansion analysis of flat plate flow

- Summary (M_∞ =2, α=10°)



So What is happening?

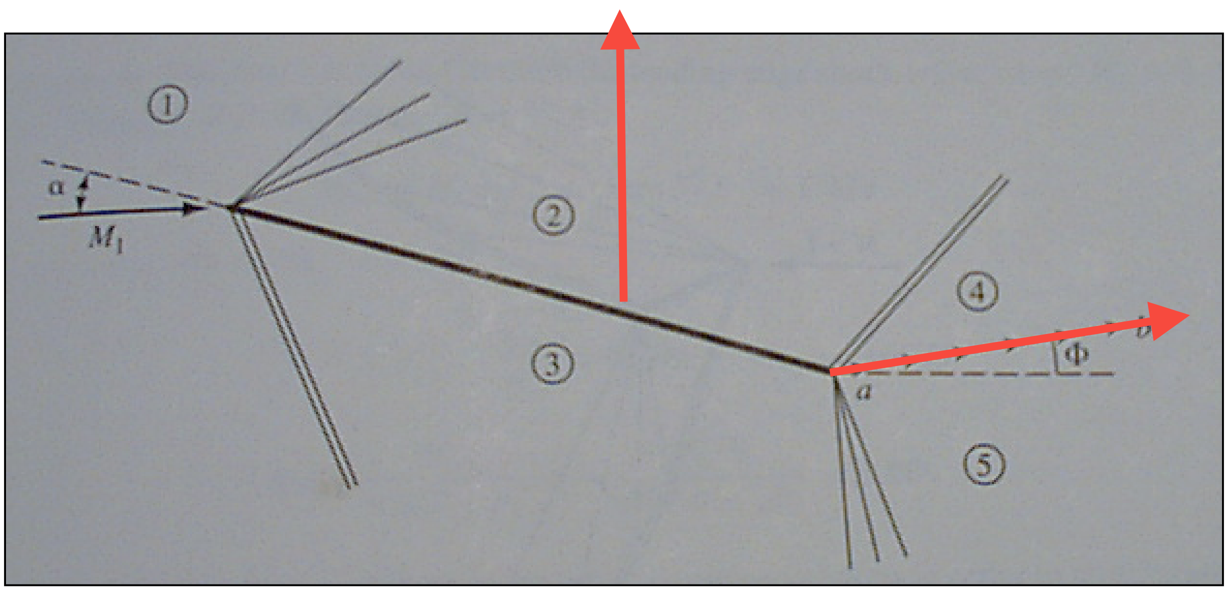
- Vertical Pressure Gradient at exit
- Conservation of Momentum not satisfied

Shock-Expansion analysis of flat plate flow

(cont'd)

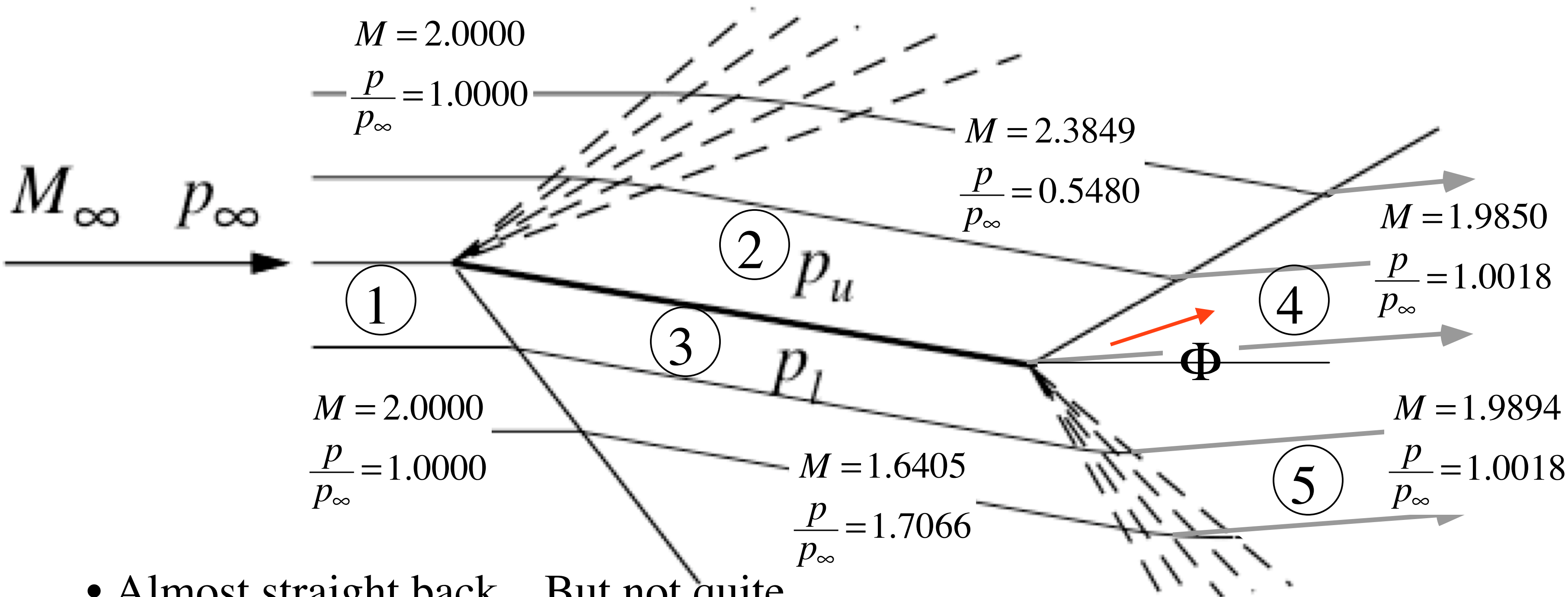
- Flow cannot exit "straight back" from plate

- Pressure Gradient "deflects" flow upward until gradient is relieved … exit angle Φ



Shock-Expansion analysis of flat plate flow

- Pressure gradient =0 when Φ=0.0277°



- Almost straight back .. But not quite

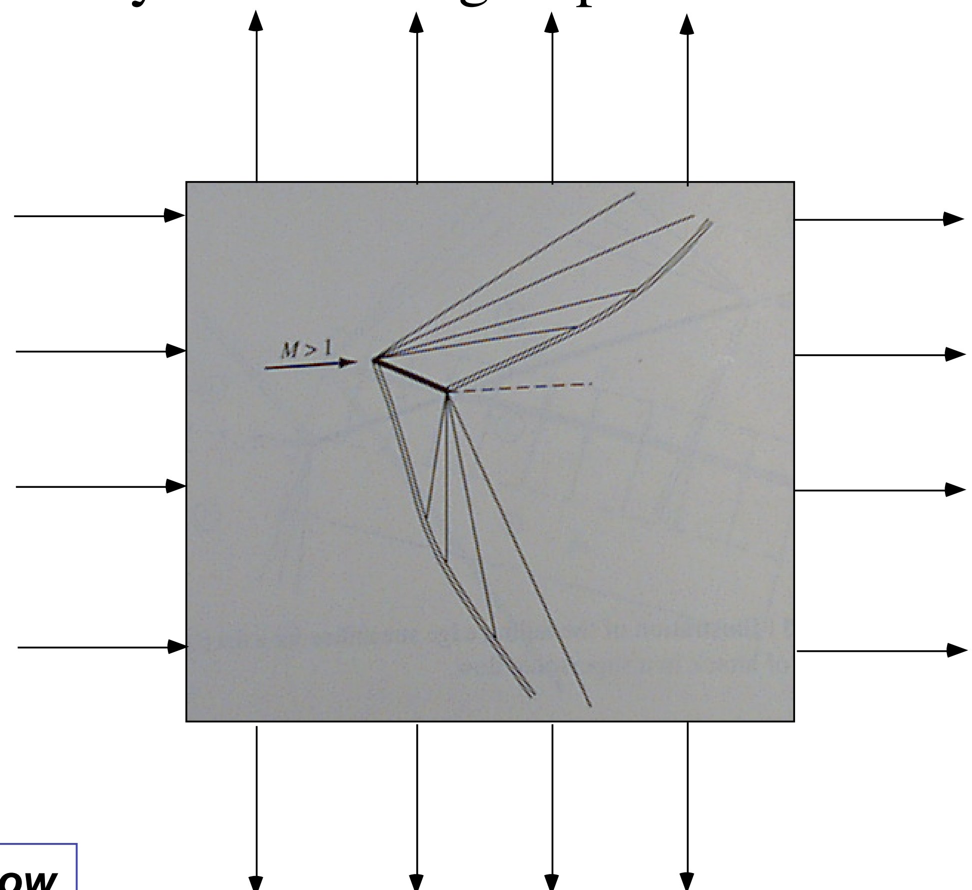
- Unequal velocity above and below exit center stream line called "slip line" .. Region of unequal entropy … *another flow discontinuity*

Far Field Flow Conditions

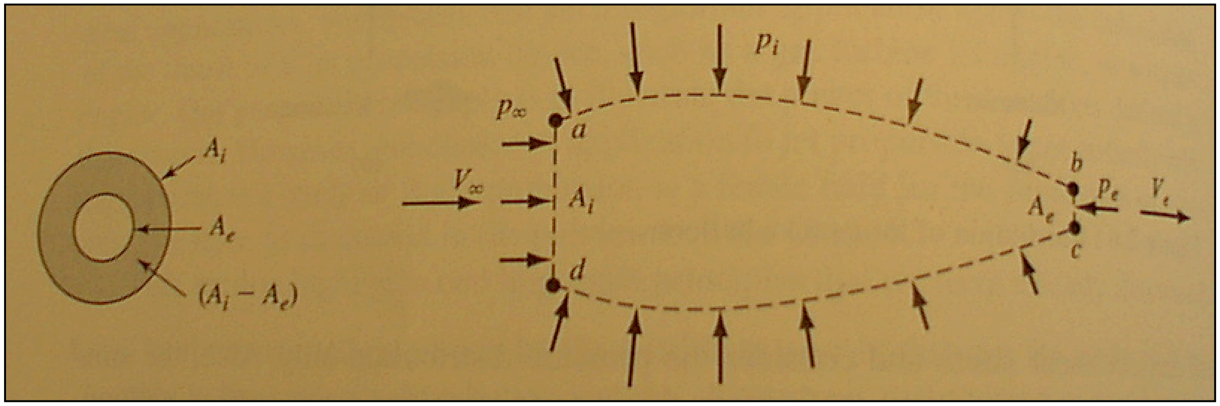
- What happens in "far field"

  Shock and expansion waves interact to produce net down wash (we'll get to this later)

- Momentum defects caused by lift and drag of plate



Far Field
Return to Integral Momentum Equation



For 1-D flow

$$D = -F_x = \left[\dot{m}_\infty V_\infty + p_\infty A_\infty\right] - \left[\dot{m}_e V_e + p_e A_e\right] =$$

$$A_e\left[\frac{A_\infty}{A_e} p_\infty\left(1+\gamma M_\infty^{\ 2}\right) - p_{exit}\left(1+\gamma M_{exit}^{\ 2}\right)\right]$$

Far Field Flow Conditions

- What happens in "far field"

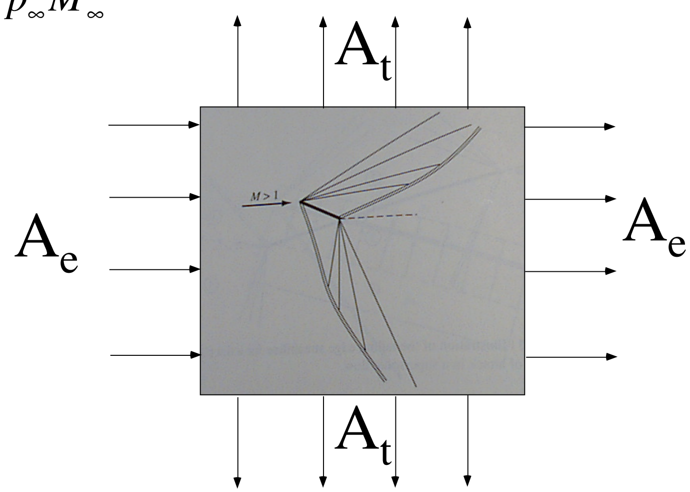
  Shock and expansion waves interact to produce net down wash (we'll get to this later)

- Momentum defects caused by lift and drag of plate

$$C_D = \frac{Drag}{A_e \frac{\gamma}{2} p_\infty M_\infty^{\;2}} = \frac{\left[p_\infty\left(1+\gamma M_\infty^{\;2}\right)\right] - \left[b\int\left\{\left[p_{exit}(y)\left(1+\gamma M_{exit}^{\;2}(y)\right)\right]\right\}dy\right] \Big/ A_e}{\frac{\gamma}{2} p_\infty M_\infty^{\;2}}$$

$$C_L = \frac{Drag}{A_e \frac{\gamma}{2} p_\infty M_\infty^{\;2}} = \frac{\left[b\int\left\{\left[p_{bot}(y)\left(1+\gamma M_{bot}^{\;2}(y)\right)\right]\right\}dx\right] - \left[b\int\left\{\left[p_{top}(y)\left(1+\gamma M_{top}^{\;2}(y)\right)\right]\right\}dx\right]}{A_{exit} \frac{\gamma}{2} p_\infty M_\infty^{\;2}}$$

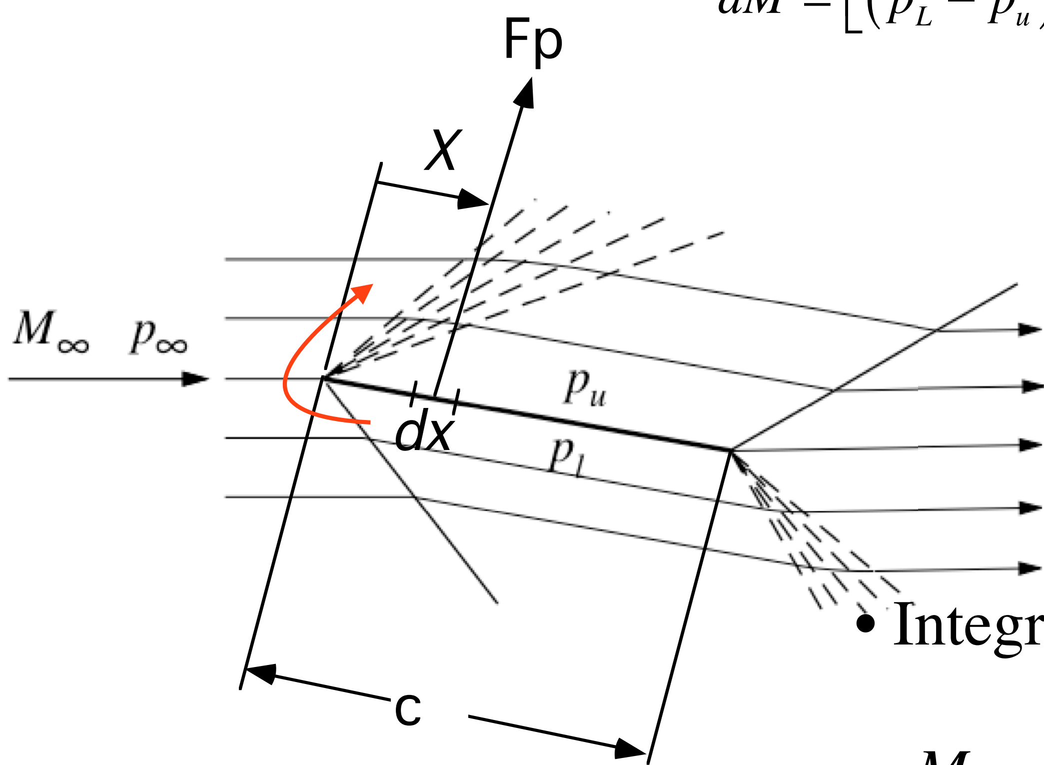
- Not quite ready to do this integral yet



Compute Pitching Moment

- At some distance x, from the leading edge, the incremental aero moment is

$$dM = \left[\left(p_L - p_u \right) b dx \right] x \Rightarrow dF_p = \left(p_L - p_u \right) b dx$$



- Integrating the length of the chord

$$M = b \int_0^C \left(p_L - p_u \right) x dx$$

Pitching Moment (cont'd)

- Defining Pitching Moment Coefficient as

$$C_M = \frac{M}{\frac{1}{2}\rho_\infty V_\infty^{\;2}(bc)c} \Rightarrow C_M = b\int_0^C \frac{(p_L - p_u)x}{\frac{1}{2}\rho_\infty V_\infty^{\;2}(bc)c}dx = \frac{1}{\frac{1}{2}\rho_\infty V_\infty^{\;2}}\int_0^1 (p_L - p_u)\frac{x}{c}\frac{dx}{c}$$

Letting u=x/c ---> du = dx/c ---> substituting gives

$$C_M = \frac{1}{\frac{1}{2}\rho_\infty V_\infty^{\;2}}\int_0^1 (p_L - p_u)udu$$

- For a flat plate $(p_L - p_u)$ is constant along the plate …

$$C_M = \frac{(p_L - p_u)}{\frac{1}{2}\rho_\infty V_\infty^{\;2}}\int_0^1 udu = \frac{(p_L - p_u)}{\frac{1}{2}\rho_\infty V_\infty^{\;2}}\frac{u^2}{2}\Big|_0^1 = \frac{1}{2}\left[\frac{(p_L - p_u)}{\frac{1}{2}\rho_\infty V_\infty^{\;2}}\right]$$

Pitching Moment (cont'd)

- For a Flat Plate, the C_M about the leading edge is

$$C_L = \frac{(p_L - p_u)}{\frac{1}{2}\rho_\infty V_\infty^{\ 2}} \cos\alpha \Rightarrow C_D = \frac{(p_L - p_u)}{\frac{1}{2}\rho_\infty V_\infty^{\ 2}} \sin\alpha$$

$$\sqrt{C_L^{\ 2} + C_D^{\ 2}} = \frac{(p_L - p_u)}{\frac{1}{2}\rho_\infty V_\infty^{\ 2}} \sqrt{\cos^2\alpha + \sin^2\alpha} = \frac{(p_L - p_u)}{\frac{1}{2}\rho_\infty V_\infty^{\ 2}}$$

$$C_{M_{LE}} = \frac{1}{2}\sqrt{C_L^{\ 2} + C_D^{\ 2}}$$

Aerodynamic Center

• The Aerodynamic Center is the point on the wing about which all changes in lift effectively act. In other words when the angle of attack is increased or decreased the total lift and center of pressure move. But, the net effect is as though the change in lift happens at the ac.

• For a cambered airfoil aerodynamic center (a/c) is the point on the wing about which the coefficient of pitching moment is Independent of angle-of-attack

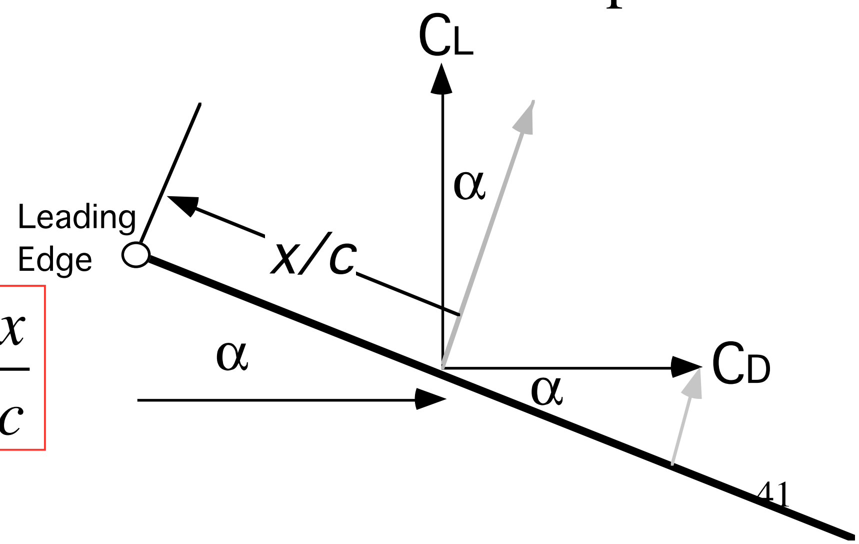
• For a symmetrical airfoil, the pitching moment is zero at the aerodynamic center

Aerodynamic Center (cont'd)

- C_M is independent of angle of attack
(zero for symmetric airfoil)

$$C_{M_{LE}} = \frac{1}{2}\sqrt{C_L^{\,2} + C_D^{\,2}} = \frac{1}{2}\left[\frac{\left(p_L - p_u\right)}{\frac{1}{2}\rho_\infty V_\infty^{\,2}}\right] \Rightarrow \left(p_L - p_u\right) = f(\alpha)$$

- Using the moment transfer theorem .. At some other point on the wing



$$\boxed{C_M = C_{M_{LE}} - \left[C_L \cos\alpha + C_D \sin\alpha\right]\frac{x}{c}}$$

Aerodynamic Center (cont'd)

- But Lift and Drag Coefficient are

$$C_L = \frac{(p_L - p_u)}{\frac{1}{2}\rho_\infty V_\infty^{\ 2}} \cos\alpha \Rightarrow C_D = \frac{(p_L - p_u)}{\frac{1}{2}\rho_\infty V_\infty^{\ 2}} \sin\alpha \Downarrow$$

$$C_M = C_{M_{LE}} - [C_L \cos\alpha + C_D \sin\alpha]\frac{x}{c} = C_{M_{LE}} - \left[\frac{(p_L - p_u)}{\frac{1}{2}\rho_\infty V_\infty^{\ 2}} \cos^2\alpha + \frac{(p_L - p_u)}{\frac{1}{2}\rho_\infty V_\infty^{\ 2}} \sin^2\alpha\right]\frac{x}{c}$$

- Simplifying

$$C_M = C_{M_{LE}} - \frac{(p_L - p_u)}{\frac{1}{2}\rho_\infty V_\infty^{\ 2}} \frac{x}{c} = \frac{1}{2}\sqrt{C_L^{\ 2} + C_D^{\ 2}} - \sqrt{C_L^{\ 2} + C_D^{\ 2}}\,\frac{x}{c} = \sqrt{C_L^{\ 2} + C_D^{\ 2}}\left[\frac{1}{2} - \frac{x}{c}\right]$$

Aerodynamic Center (cont'd)

- But Lift and Drag Coefficient are

$$C_L = \frac{(p_L - p_u)}{\frac{1}{2}\rho_\infty V_\infty^{\ 2}}\cos\alpha \Rightarrow C_D = \frac{(p_L - p_u)}{\frac{1}{2}\rho_\infty V_\infty^{\ 2}}\sin\alpha \Downarrow$$

$$C_M = C_{M_{LE}} - \left[C_L\cos\alpha + C_D\sin\alpha\right]\frac{x}{c} = C_{M_{LE}} - \left[\frac{(p_L - p_u)}{\frac{1}{2}\rho_\infty V_\infty^{\ 2}}\cos^2\alpha + \frac{(p_L - p_u)}{\frac{1}{2}\rho_\infty V_\infty^{\ 2}}\sin^2\alpha\right]\frac{x}{c}$$

- Simplifying

$$C_M = C_{M_{LE}} - \frac{(p_L - p_u)}{\frac{1}{2}\rho_\infty V_\infty^{\ 2}}\frac{x}{c} = \frac{1}{2}\sqrt{C_L^{\ 2} + C_D^{\ 2}} - \sqrt{C_L^{\ 2} + C_D^{\ 2}}\,\frac{x}{c} = \sqrt{C_L^{\ 2} + C_D^{\ 2}}\left[\frac{1}{2} - \frac{x}{c}\right]$$

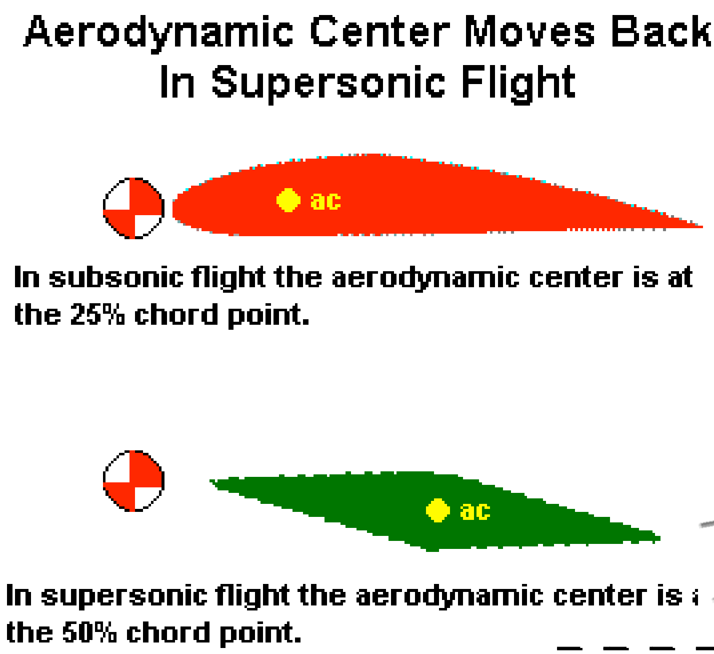
Aerodynamic Center (cont'd)

- At the aerodynamic center C_M is independent of angle of attack *(zero for symmetric airfoil)*

$$0 = \sqrt{C_L{}^2 + C_D{}^2}\left[\frac{1}{2} - \frac{x}{c}\right] \rightarrow \boxed{x = \frac{c}{2}}$$

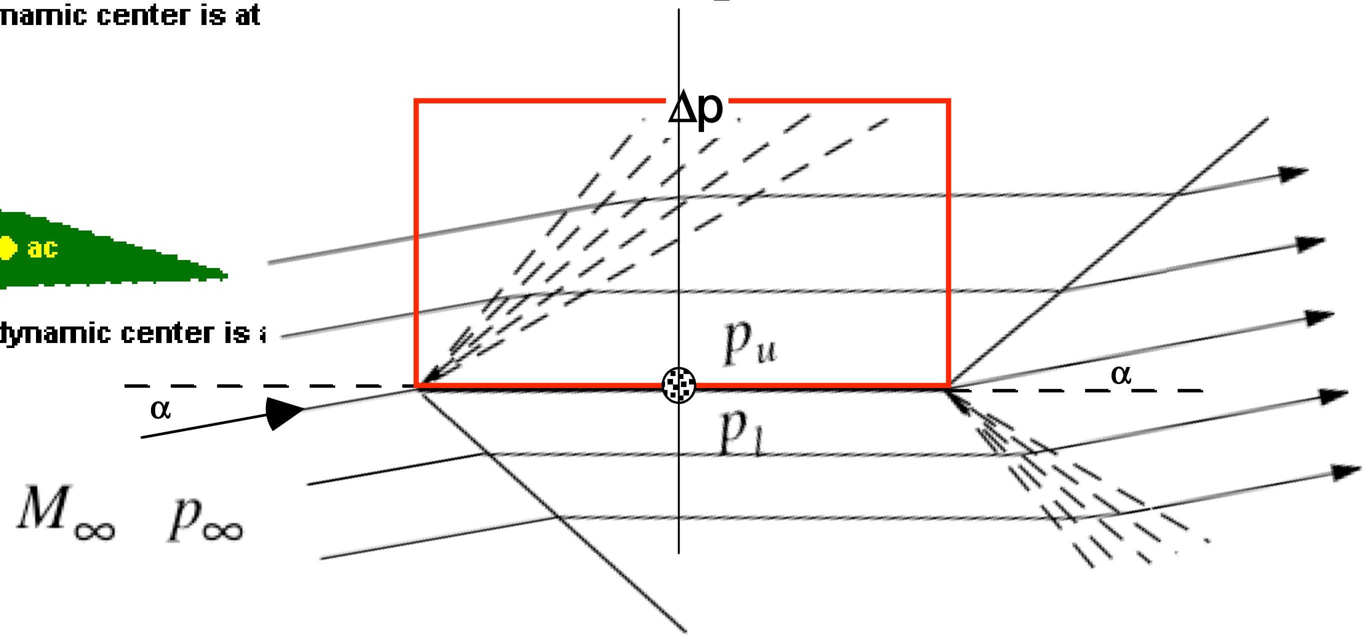
- Supersonically, the mean aerodynamic center is at the half-chord point

Aerodynamic Center (cont'd)



- This Result makes Complete sense

- Half-chord is the balance
Point for the pressure distribution



Aerodynamic Center (cont'd)

• Verify Result Numerically for Our Example

• Moment about leading edge

$$C_M = \frac{1}{\frac{1}{2}\rho_\infty V_\infty^{\ 2}} \int_0^1 \left(p_L - p_u\right) \frac{x}{c}\frac{dx}{c} = \int_0^1 \left(c_{p_L} - c_{p_u}\right) \frac{x}{c}\frac{dx}{c} = \frac{1.7066 - 0.54796}{\frac{1.4}{2}(2^2)} \int_0^1 \frac{x}{c}\frac{dx}{c} =$$

$$0.4138 \left\{ \frac{1}{2}\left(\frac{x}{c}\right)^2 \Big|_0^1 \right\} = \boxed{0.2069}$$

Check!

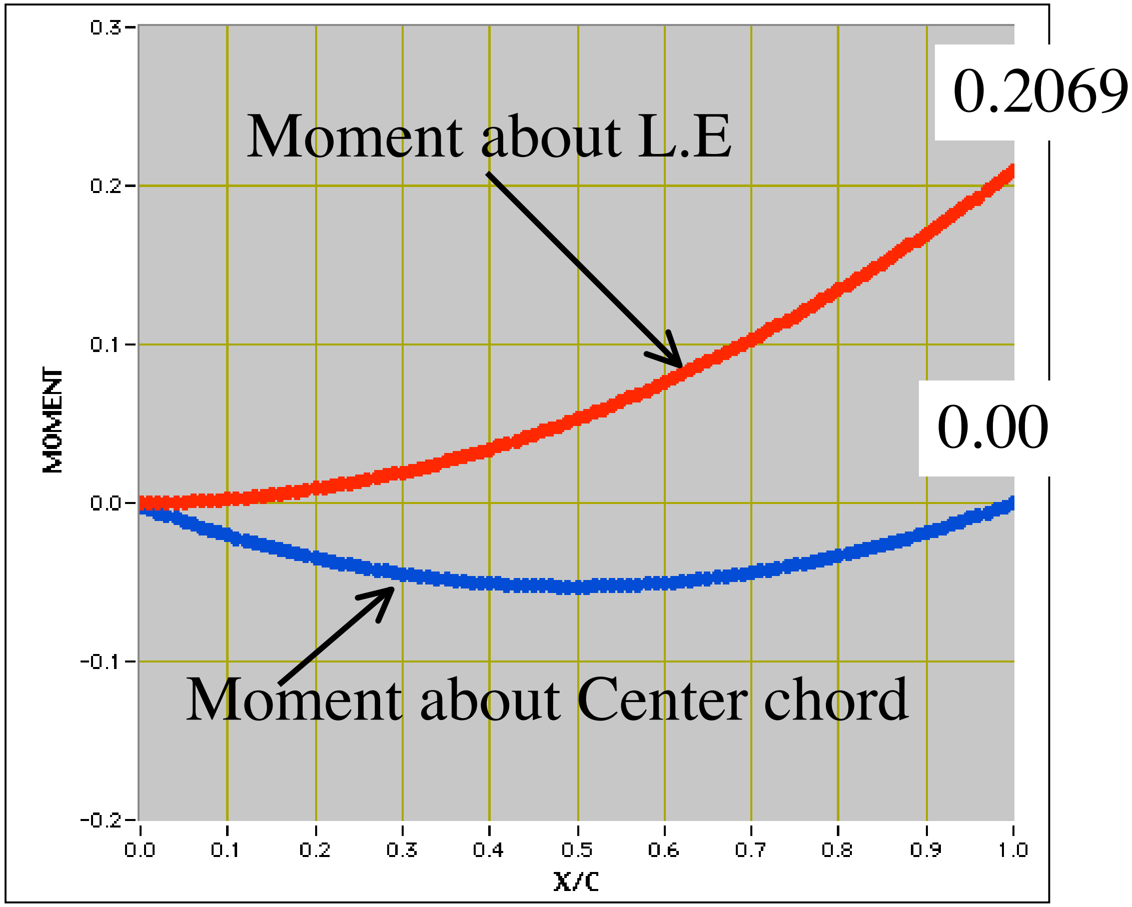
$$\Rightarrow C_{M_{LE}} = \frac{1}{2}\sqrt{C_L^{\ 2} + C_D^{\ 2}} = \frac{1}{2}\sqrt{\left[0.4075^2 + 0.07186^2\right]} = \boxed{0.2069}$$

Aerodynamic Center (cont'd)

• Verify Result Numerically for Our Example

• Plot Moment about leading edge and about center chord *x/c=1/2* … for our example

• moment about c/2 integrates to zero



Aerodynamic Center (cont'd)

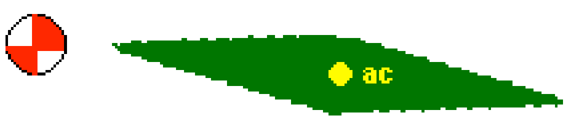
- Contrast this result with a subsonic Wing

- You learned about this in

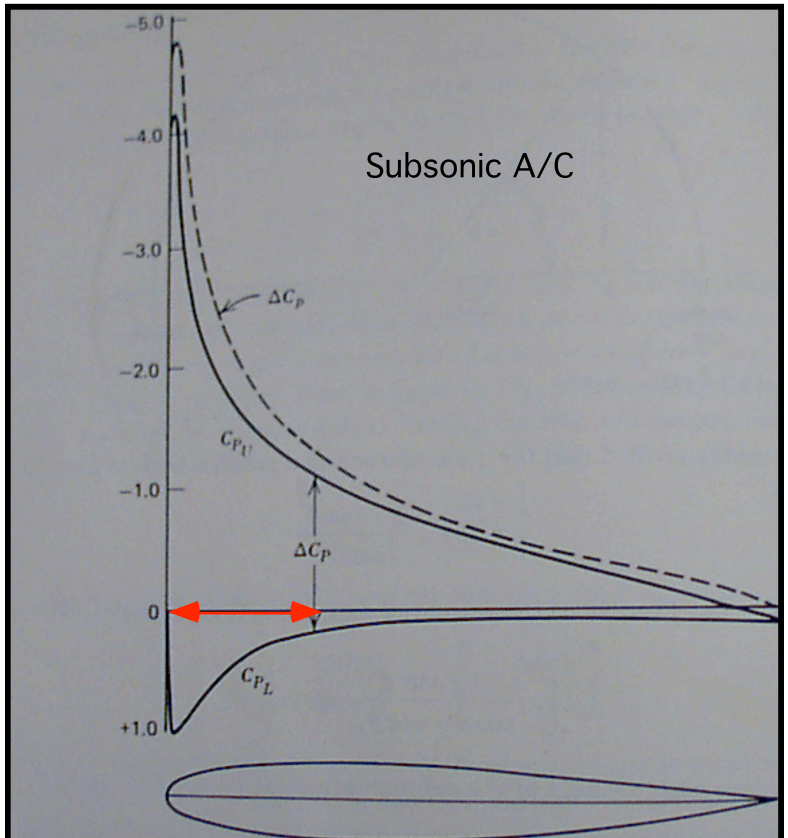
MAE 5500: Aerodynamics

Aerodynamic Center Moves Back In Supersonic Flight

In subsonic flight the aerodynamic center is at the 25% chord point. *(approximate)*



In supersonic flight the aerodynamic center is at the 50% chord point.



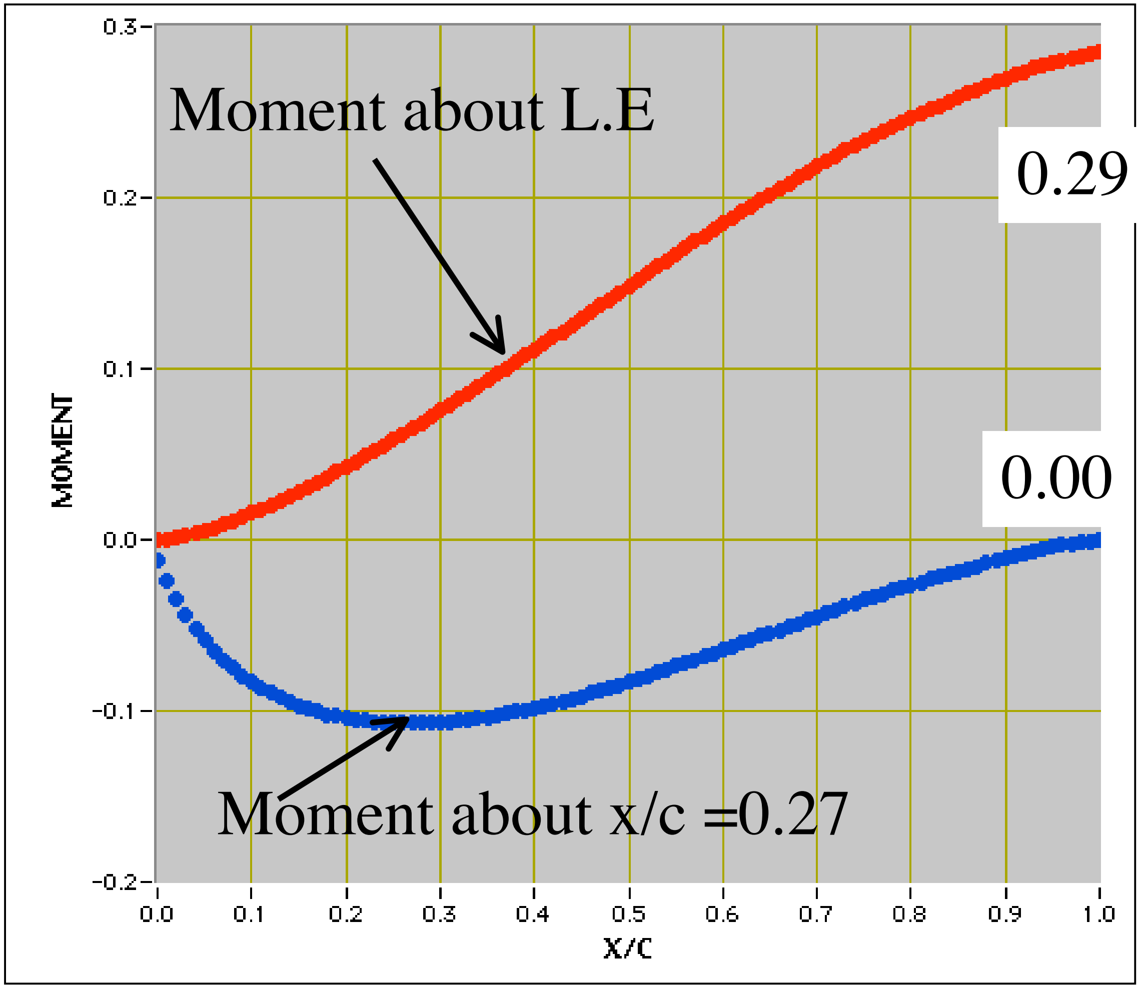
Distribution of pressure coefficient and ΔC_p on NACA 0012 airfoil at $\alpha = 9°$

Aerodynamic Center (cont'd)

• Verify Result Numerically for Subsonic Wing

• Plot Moment about leading edge and about center chord *x/c=0.27* … NACA 12

• moment about 0.27 c integrates to zero



"Mach Tuck" Revisited

• Driven by Combat Needs in WWII, Aircraft airspeeds became increasingly faster.

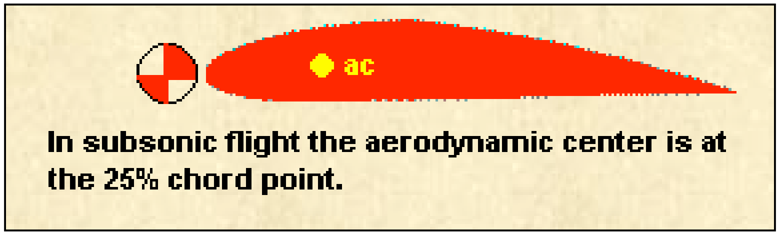
• P-51s, Spitfires and other types were reaching speeds close to that of sound, especially in dives to catch, or escape from, the enemy.

• Pilots began to report control difficulties and unexpected problems, including a strong nose down pitch and a loss of pitch control authority. Often took all of pilot's strength to correct. Some did not make it and dove into the ground, or broke up, as their aircraft exceeded the maximum design speed.

"Mach Tuck" Revisited(cont'd)

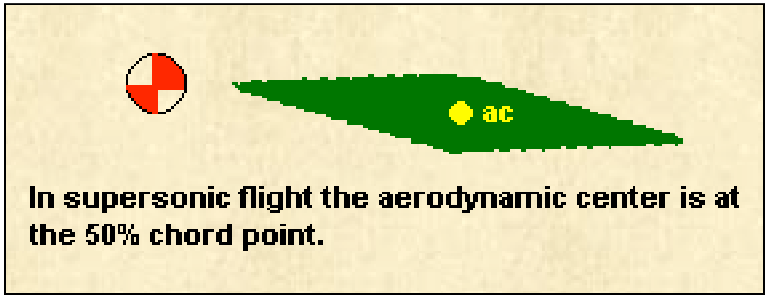
• Nose down pitching moment was a result of Localized Supersonic *Flow and Air Compressibility*

• At low speeds airfoils have an aerodynamic center that is Approximately at the 25% chord point.



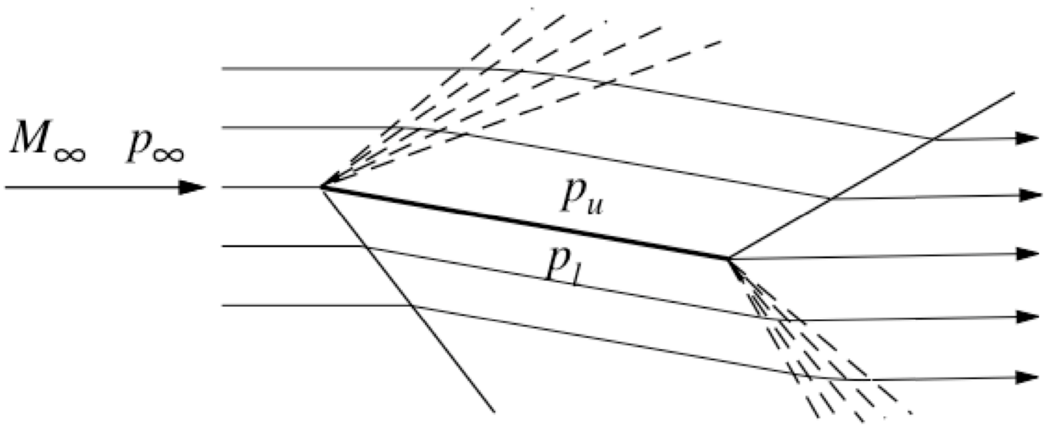
• However, as the aircraft moves into supersonic flight the induced wash ahead of the wing disappears …
… As a result the aerodynamic center moves back to the 50% chord point.

Now we know why!



Credit: Selkirk College Professional Aviation Program

Flat Plate Summary



$$C_L = \frac{\left[\frac{p_l}{p_\infty} - \frac{p_u}{p_\infty}\right]}{\left(\frac{\gamma}{2} M_\infty^{\;2}\right)} \cos\alpha$$

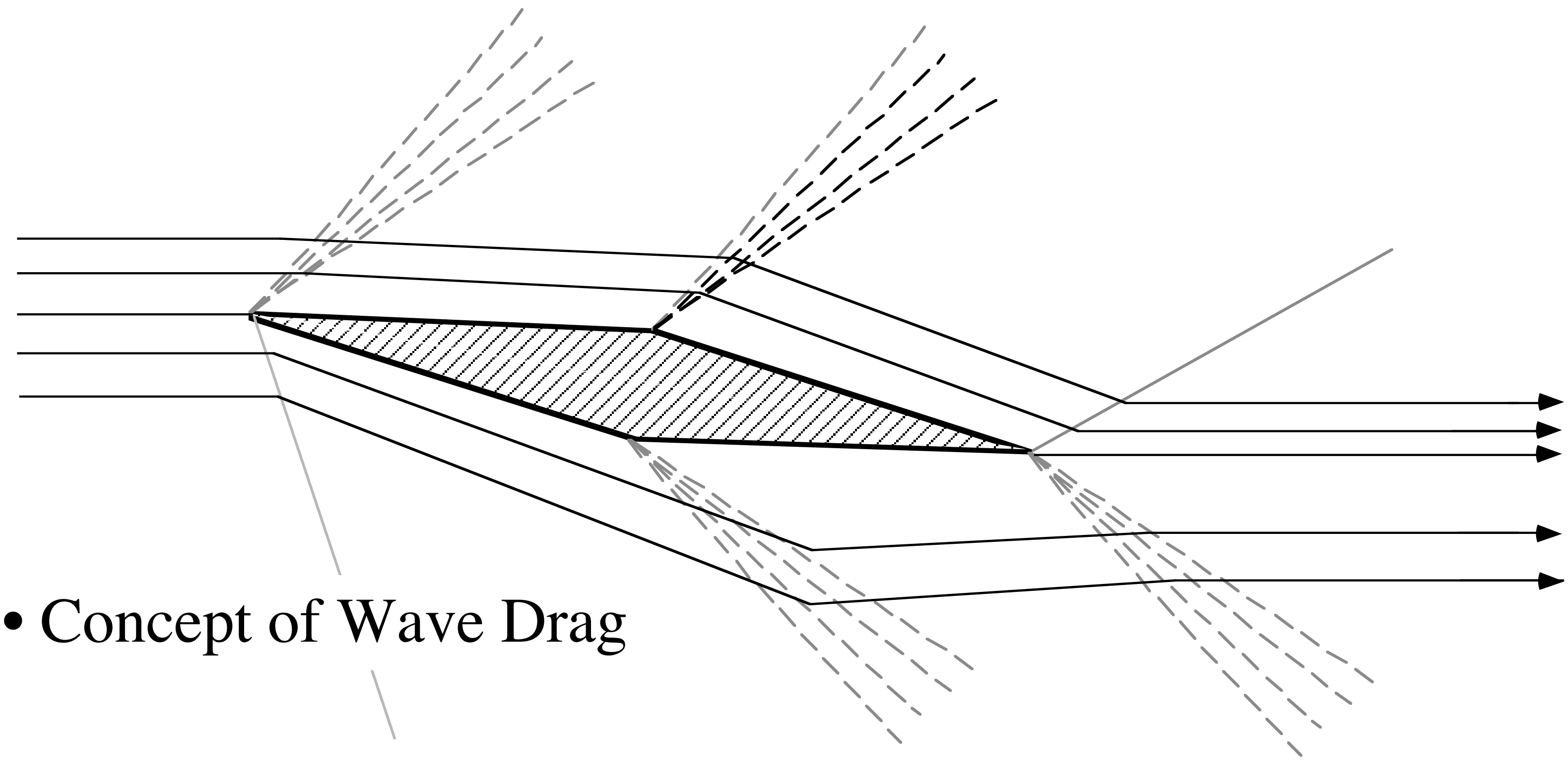
$$C_D = \frac{\left[\frac{p_l}{p_\infty} - \frac{p_u}{p_\infty}\right]}{\left(\frac{\gamma}{2} M_\infty^{\;2}\right)} \sin\alpha$$

- Aerodynamic Center

$$C_M = 0 = \sqrt{C_L^{\;2} + C_D^{\;2}}\left[\frac{1}{2} - \frac{x}{c}\right] \rightarrow \boxed{x = \frac{c}{2}}$$

$$C_{M_{LE}} = \frac{1}{2}\sqrt{C_L^{\;2} + C_D^{\;2}}$$

Next: Supersonic Flow on Finite Thickness Wings



- Concept of Wave Drag