# Final Project

• Derive a Compressibility correction for laminar skin friction coefficient

Based on parabolic velocity profile

$$\left(C_{D_{fric}}\right)_{compressible} = \frac{1.328}{\left(\frac{\rho(T_{avg})\cdot c\cdot V_\infty}{\mu(T_{avg})}\right)^{\frac{1}{2}}}$$

$$\frac{u_{(y/\delta)}}{V_e} = \left[\frac{2y}{\delta} - \left(\frac{y}{\delta}\right)^2\right]$$

$$\rho(T_{avg}) = \rho_\infty \frac{T_\infty}{T_{avg}}$$

$$\mu(T_{avg}) = \mu(T_\infty)\left(\frac{T_{avg}}{T_\infty}\right)^{3/2}\left(\frac{T_\infty + 120^\circ\text{K}}{T_{avg} + 120^\circ\text{K}}\right)$$

• Approximate the Average Temperature Of boundary layer By Integrating Across Boundary layer And dividing by boundary layer height *(δ)*

$$T\left(\frac{y}{\delta}\right) = T_\infty + R_f \frac{V_\infty^2}{2C_p}\left(1 - \frac{1}{R_f}\left[\frac{u(y/\delta)}{V_\infty}\right]^2\right)$$

$$T_{avg} \approx \frac{1}{\delta}\int_0^\delta T(y)dy = \frac{1}{\delta}\int_0^\delta T(y)dy = \int_0^1 T\left(\frac{y}{\delta}\right) d\left(\frac{y}{\delta}\right)$$

$$\frac{\rho_\infty \cdot c\cdot V_\infty}{\mu_\infty} < 500{,}000$$

***Question: is $R_f$=1 a good assumption for laminar flow?***

# *Laminar Model Form*

$$\left(C_D\right)_{\substack{Laminar\\Incompessibe}} = \frac{1.328}{R_{e_\infty}^{1/2}} \rightarrow \boxed{\left(C_D\right)_{\substack{Laminar\\Compressible}} = \frac{\left(C_D\right)_{\substack{Laminar\\Incompessibe}}}{f\left(T_{avg}, T_\infty\right)}}$$

$$T_{avg} = T_\infty + \frac{V_\infty^2}{2 \cdot c_p} \cdot R_f \cdot \int_0^1 \left\{1 - \frac{1}{R_f} \cdot \left(\frac{u}{V_\infty}\left(\tau\right)\right)^2\right\} d\left(\tau\right)$$

$$\frac{u}{V_\infty}\left(\tau\right) = 2 \cdot \tau - \tau^2$$

$$\rightarrow \tau = \frac{y}{\delta}$$

# Final Project (cont'd)

- Transitional Model

$$\left(C_{D_{transition}}\right)_{compressible} = \frac{\left(C_{D_{transition}}\right)_{incompressible}}{\left(\left(\frac{T_\infty}{T_{avg}}\right)^{5/2}\left(\frac{T_{avg}+120^{\circ}\mathrm{K}}{T_\infty+120^{\circ}\mathrm{K}}\right)\right)^{\frac{1}{7}}}$$

- For Compressible Transitional Model ….. Use turbulent velocity profile, turbulent flow compressibility correction

$$\frac{u_{(y/\delta)}}{V_e} = \left(\frac{y}{\delta}\right)^{\frac{1}{7}}$$

$$500{,}000 \le \frac{\rho_\infty \cdot c \cdot V_\infty}{\mu_\infty} < 10^7$$

# Final Project (cont'd)

• Code up "DOUBLE-WEDGE" Aerofoil Model Supersonic flow … (use methods of section 7.2. …)

• Build skin friction model for accounting for Laminar / Transitional and Turbulent conditions, Adiabatic Compressibility

***• Assume Unity Prandtl number .. We'll find out that this assumption is not very good for laminar flow***

• Use appropriate Compressibility ***correction for laminar or or turbulent flow (for transitional flow Use Turbulent Correction)***

• Compute $L/D_{max}$ for 2° half angle wing , 2 meter chord ***(plot vs alpha pick max)***

*i) Inviscid flow*
*ii) Viscous flow, turbulent only, 1/7th power law*
*iii) Transitional flow (laminar, transitional, and/or turbulent .. Mach/altitude dependent, turbulent **n** that varies with Reynolds number )*

# Final Project (cont'd)

• ***Assume Unity Prandtl number***

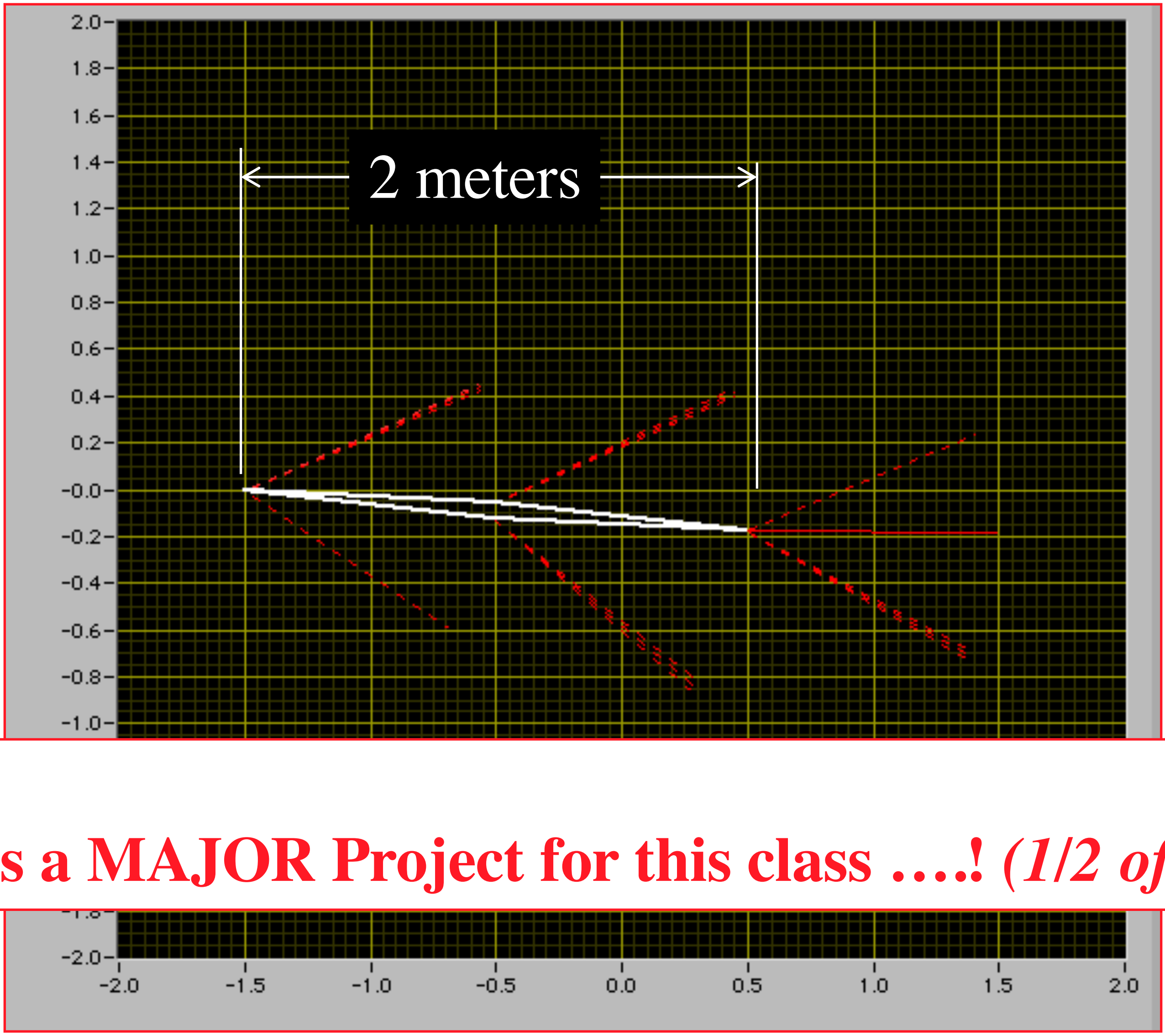
• Use appropriate Compressibility correction for laminar or turbulent flow *(transitional flow .. Use turbulent compressibility model)*

• Plot as function of Mach number at 10 km, 20 km, 30 km, 40km, 50 km altitudes *(ignore exit angle at end of wing … no effect on L/D) …*

*2 meter chord to plate …*
*For* ($1.25 < M_\infty < 10$), …. assume normal air properties for $\gamma$, $R_g$, $c_p$ … etc.

• Identify laminar, transitional and Turbulent regimes on plot

# Final Project (cont'd)




- **This a MAJOR Project for this class ….! *(1/2 of midterm)***

# Final Project (cont'd)

• Things to consider …

-Skin friction coefficient was normalized using the area of one slide of the plate …for our model … ***we have friction on both sides of the wing (approximated as a flat plate)*** **…** be sure to account for this "two-sidedness" accordingly … i.e. convert to drag first add up terms and then normalize by total planform area of wing … *b x c*

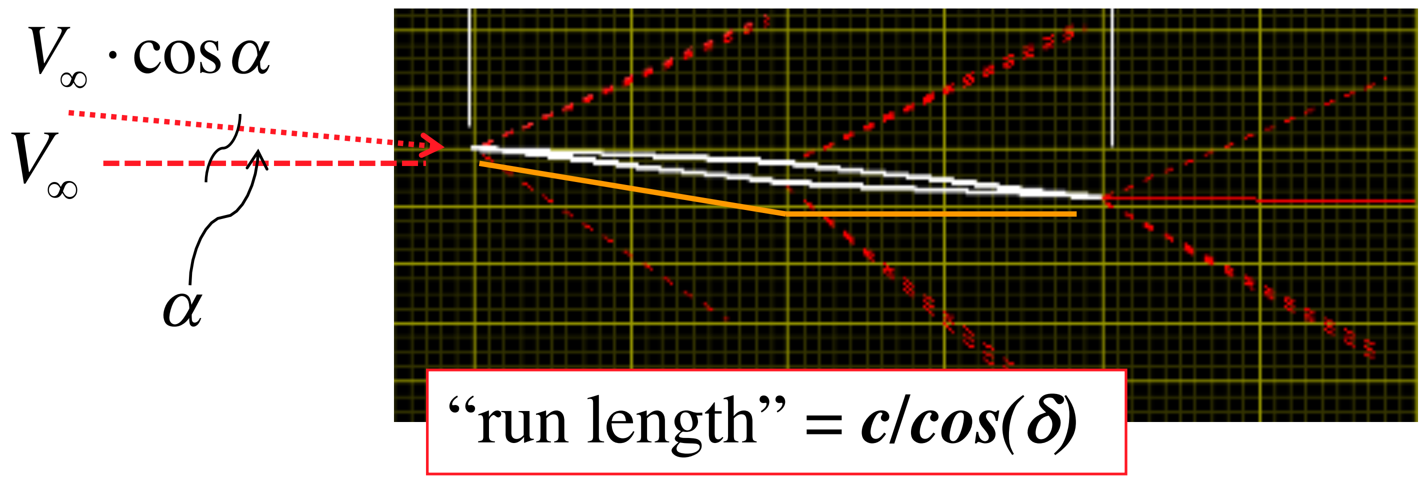
-- Approximate "*c*" in Reynolds number calculations by "run length" along plate in direction of incoming flow and flow parallel to plate axis …..

-- Always use freestream Reynolds number based on flow along wing center axis for inviscid B.L. Calculations

-- Assume no skin drag contribution to lift .

# Final Project (cont'd)

$$R_e(\infty) = \frac{\rho_\infty \cdot V_{\infty\|} \cdot C_{\substack{run \\ length}}}{\mu_\infty} = \frac{\rho_\infty \cdot (V_\infty \cdot \cos\alpha) \cdot \dfrac{c_{plate}}{\cos\delta}}{\mu_\infty} = \frac{\rho_\infty \cdot V_\infty \cdot \left(c_{plate} \cdot \dfrac{\cos\alpha}{\cos\delta}\right)}{\mu_\infty}$$




*Wing has two sides that contribute to skin drag*

# Final Project (cont'd)

Things to consider …

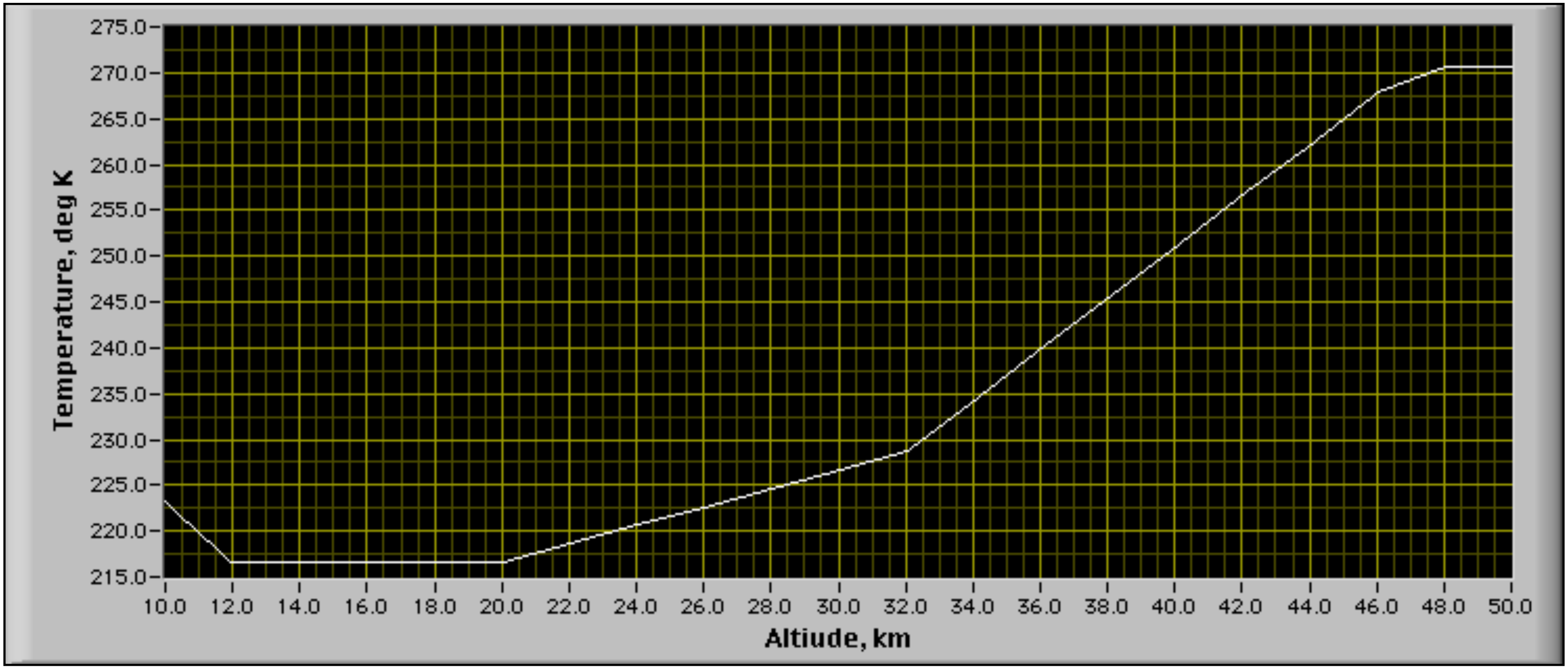
- Include a summary up front on your "design philosophy" … why you chose to model this wing as you did … what were your results, why?

I.e. I want a detailed Report! here

- **This a MAJOR Project for this class ….! *(= 1/2 of midterm)***

# Final Project (cont'd)

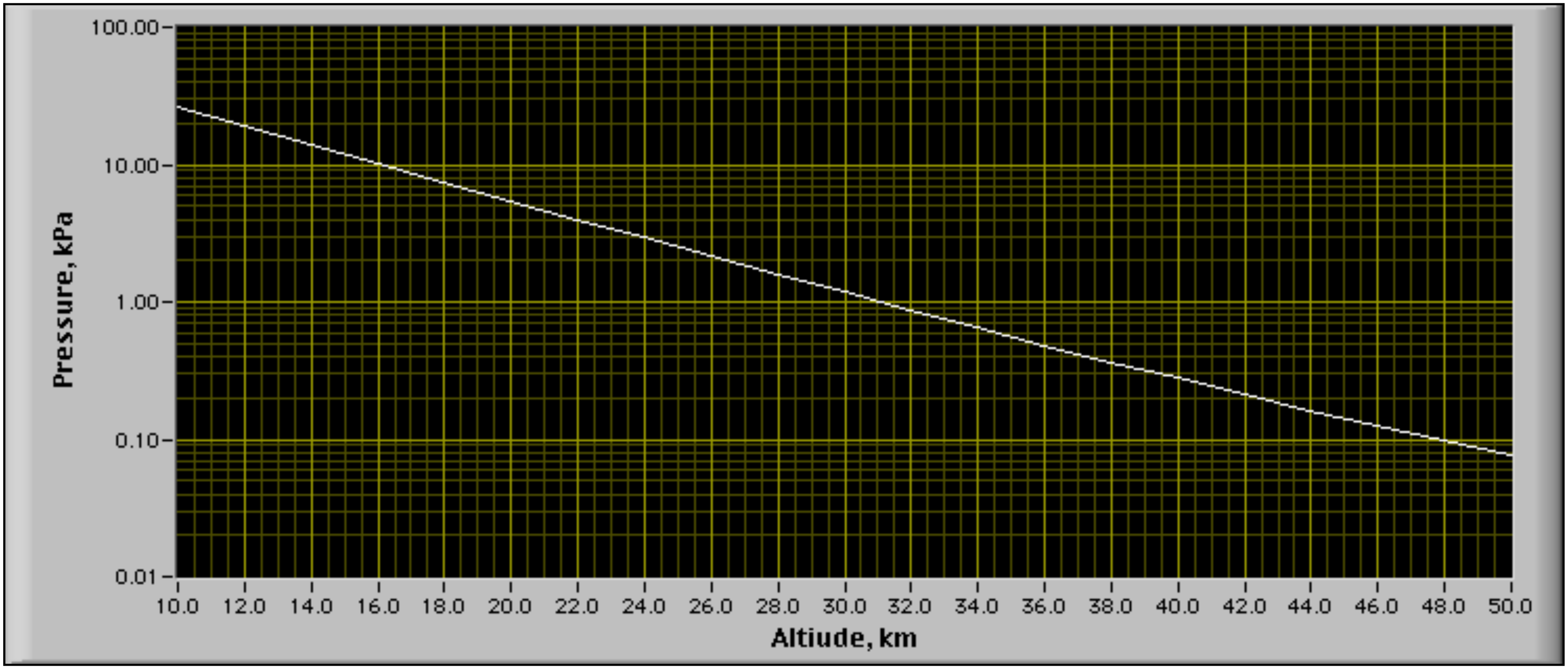
*See: http://www.neng.usu.edu/classes/mae/5420/Compressible_fluids/section8/StandardAtmosphere.txt*




Key data, Ambient Temperature, °K VS ALTITUDE

# Final Project (cont'd)

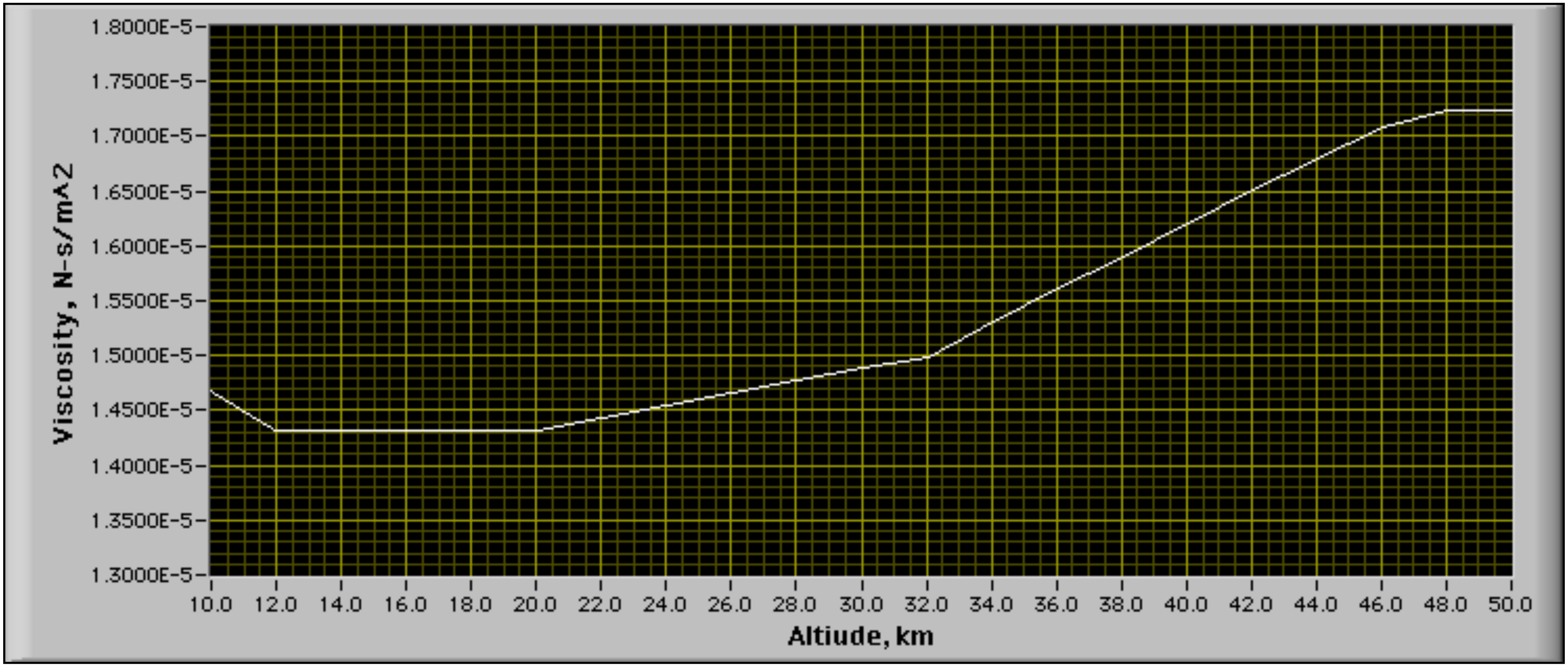
*See: http://www.neng.usu.edu/classes/mae/5420/Compressible_fluids/section8/StandardAtmosphere.txt*




Key data, Ambient Pressure, kPa VS ALTITUDE

# Final Project (cont'd)

*See: http://www.neng.usu.edu/classes/mae/5420/Compressible_fluids/section8/StandardAtmosphere.txt*




Key data, viscosity ($\mu$), Nt-sec/m$^2$ VS ALTITUDE