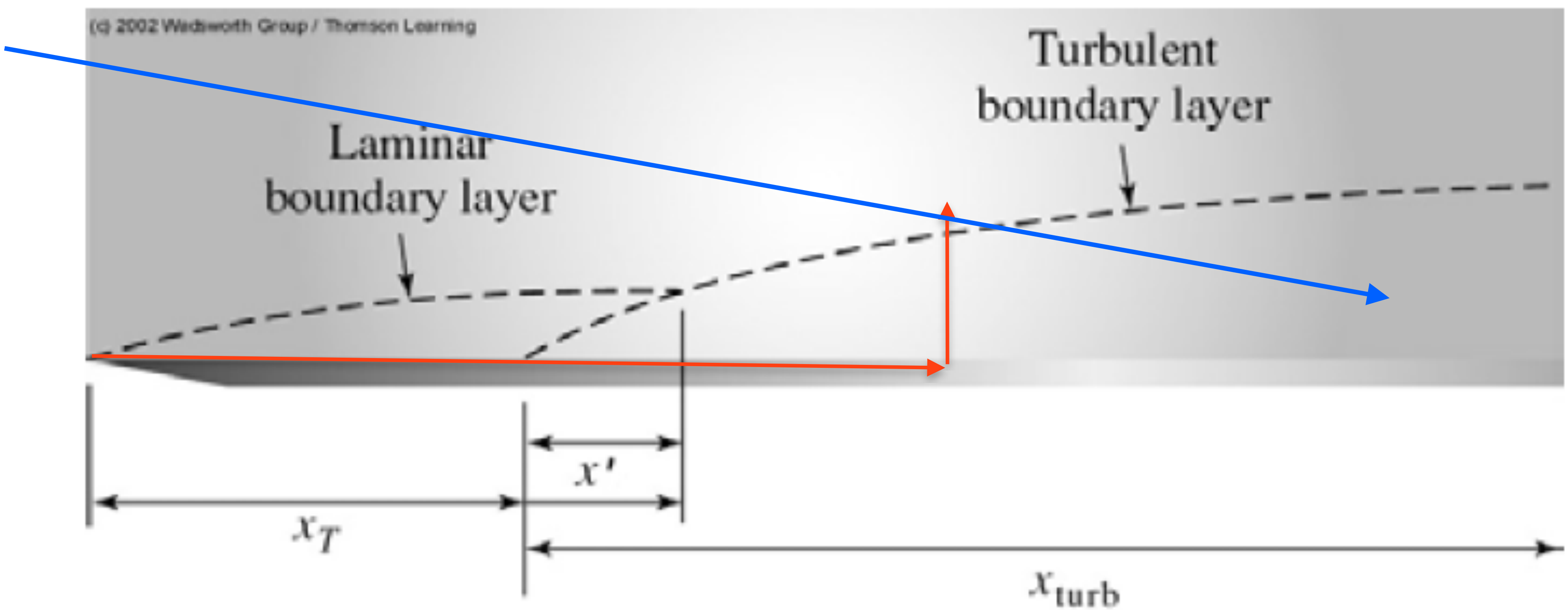
Section 8, Lecture 1

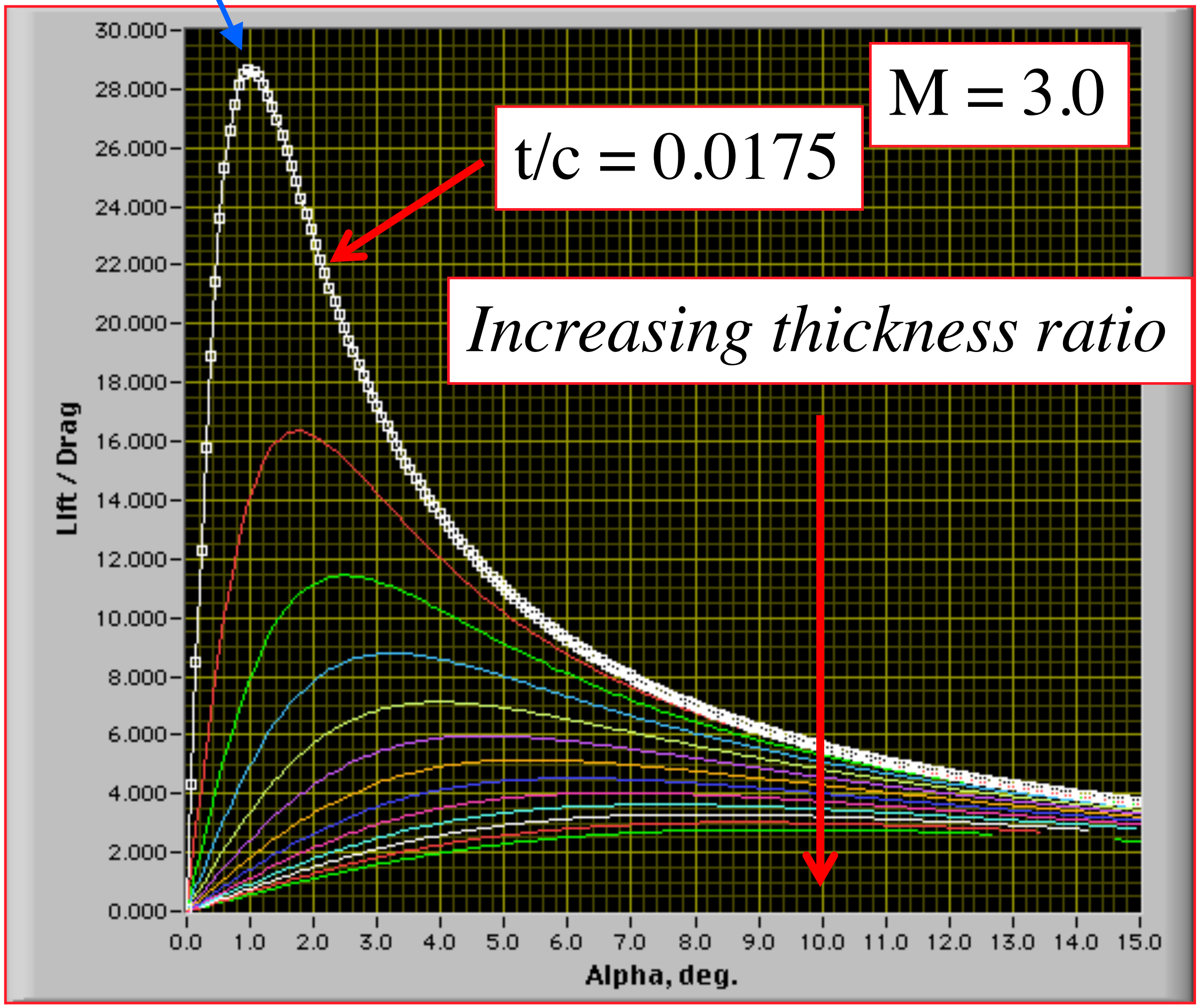
Supersonic Wings with Skin Friction:

A Primer on Boundary Layers and Skin Friction



- Not in Anderson

Lift/Drag Ratio on Symmetric Double-Wedge Airfoil



- For Inviscid flow Supersonic Lift to drag ratio almost infinite for very thin airfoil

- But airfoils do not fly in inviscid flows

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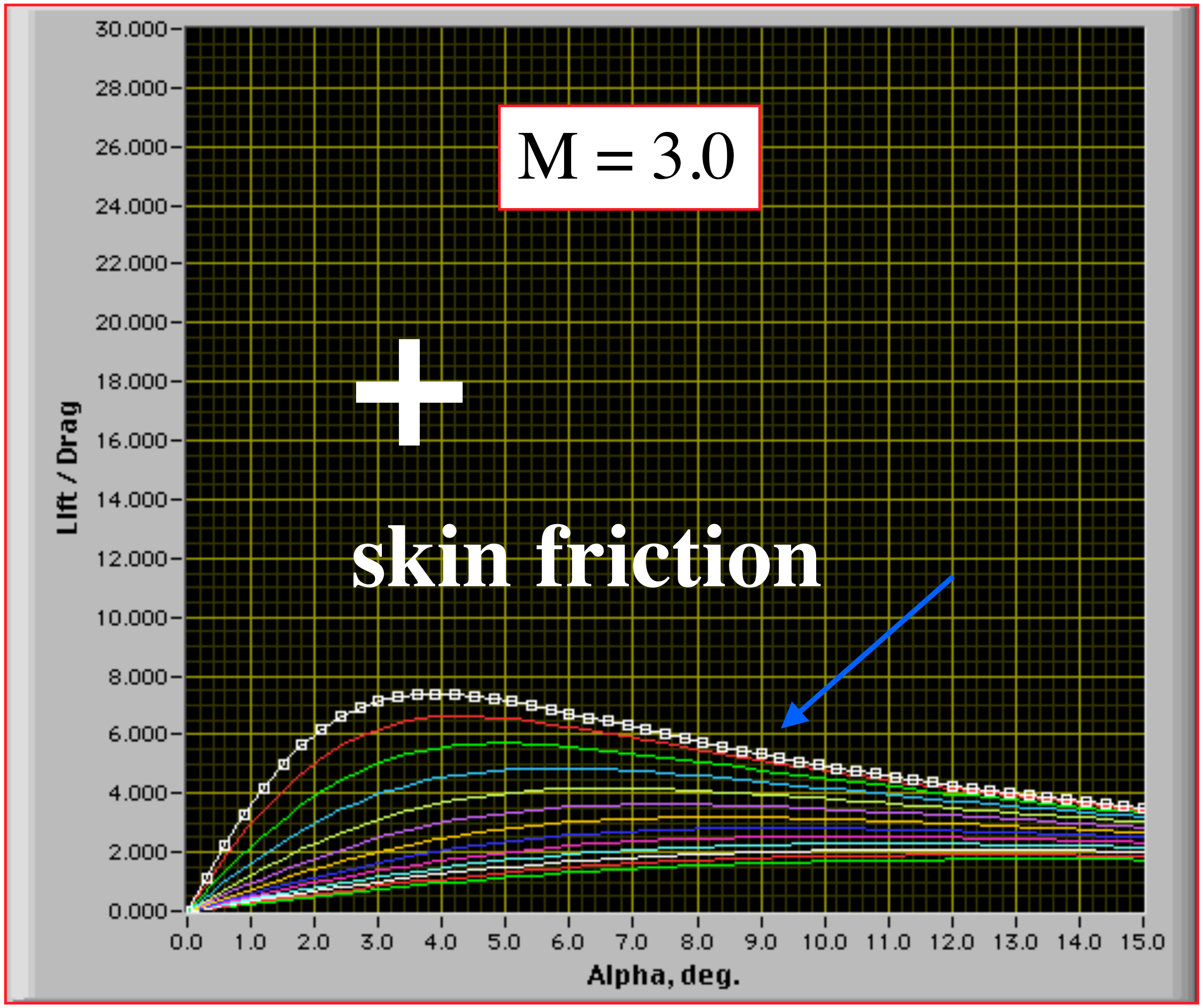
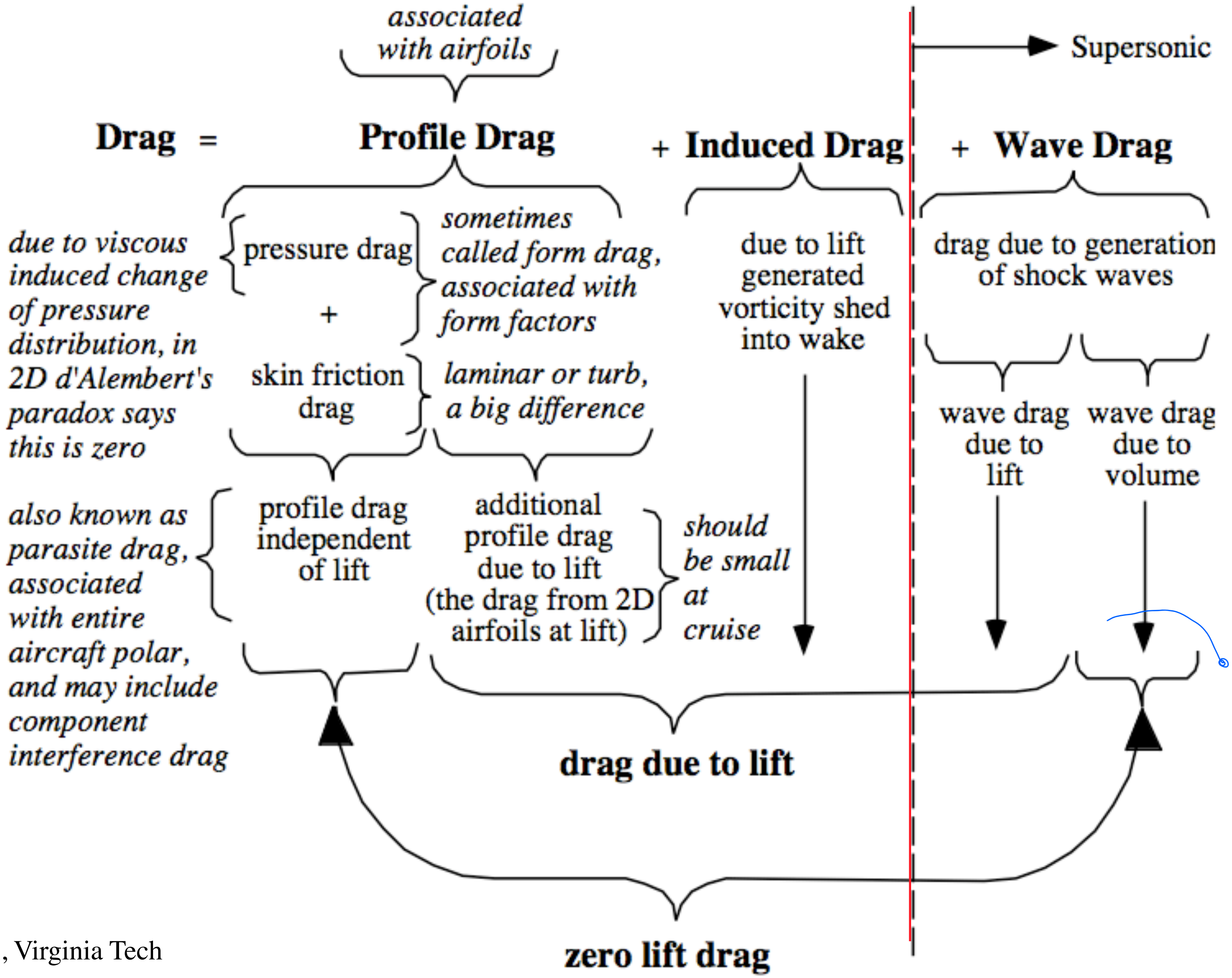
Example: Symmetric Double-wedge Airfoil … L/D (cont'd)



- Friction effects have small effect on Nozzle flow or flow in "large "ducts"

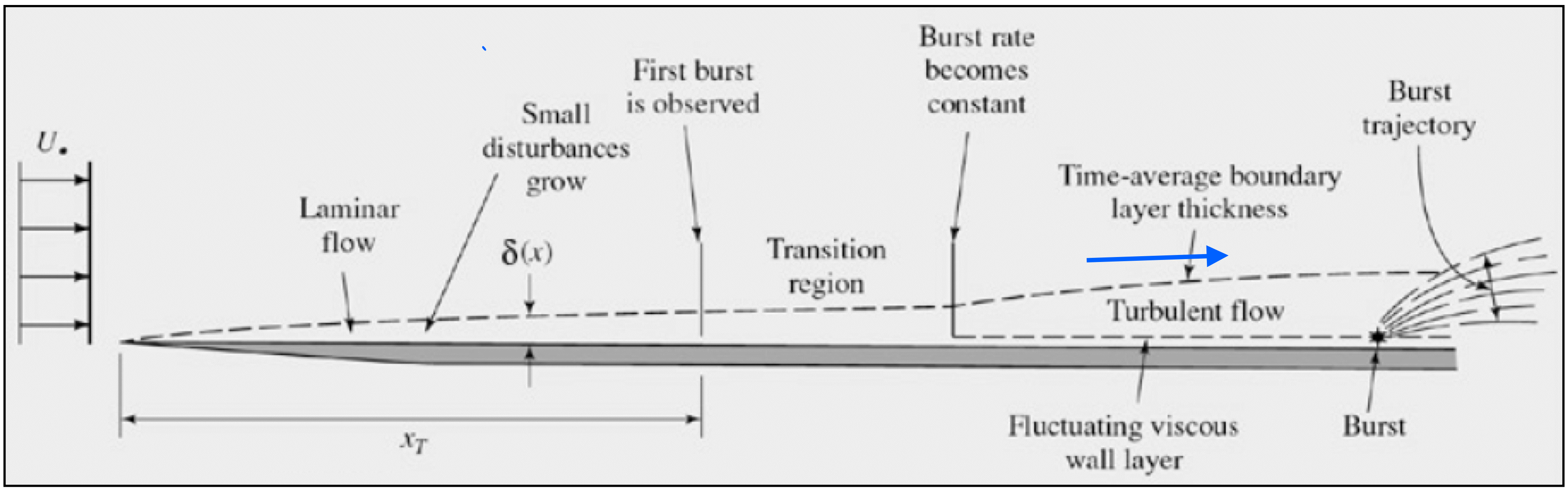
- But contribute significantly to reduce the performance of supersonic wings

Types of Drag



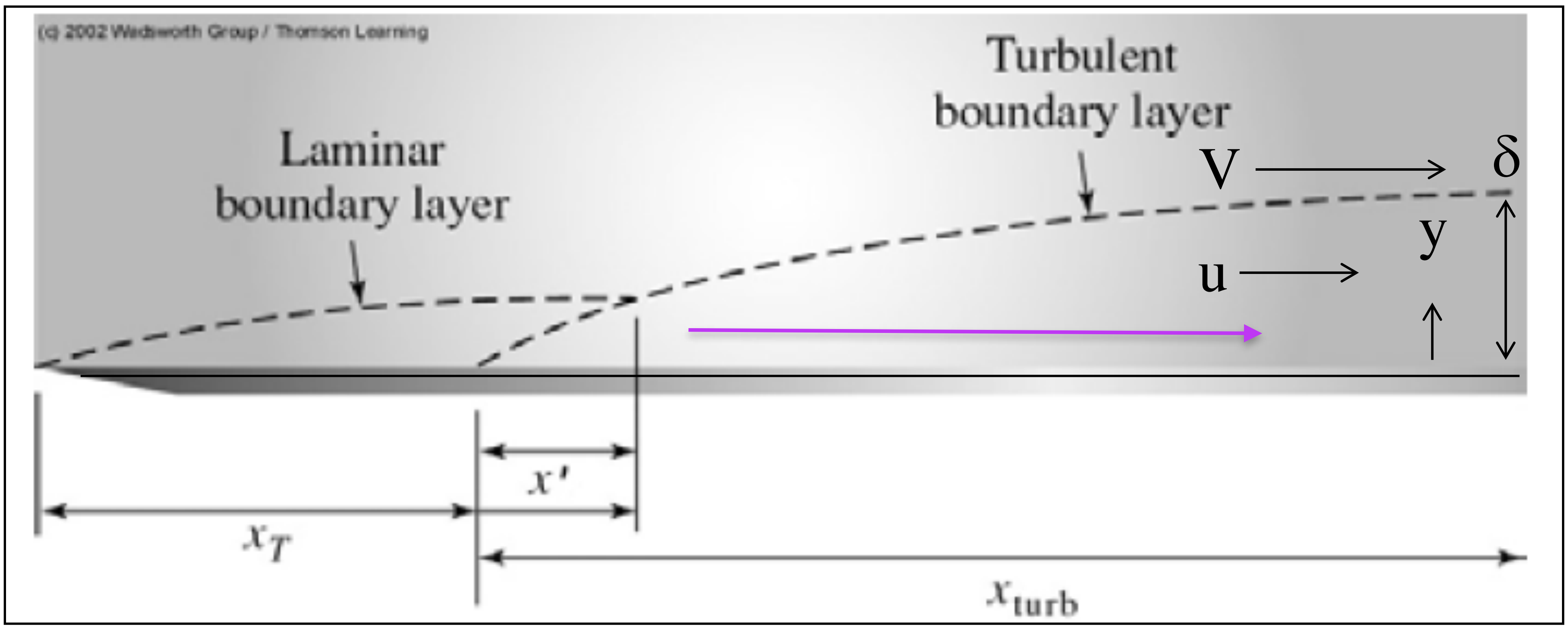
What causes "Skin Friction"

- Gases are not completely non -viscous … when gas flows over a solid boundary, Molecules "stick" to the surface and impede the flow of other molecules … eventually creating an uneven velocity distribution for some small but finite layer … the "boundary layer"



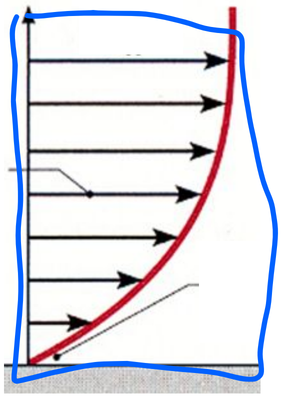
What causes "Skin Friction" (cont'd)

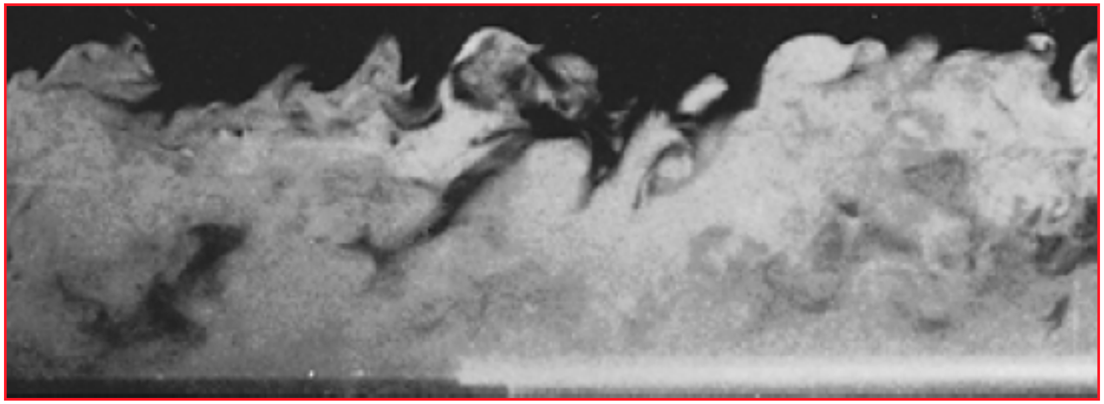
- The "momentum defect" within the boundary layer manifests itself as a "drag" on the surface

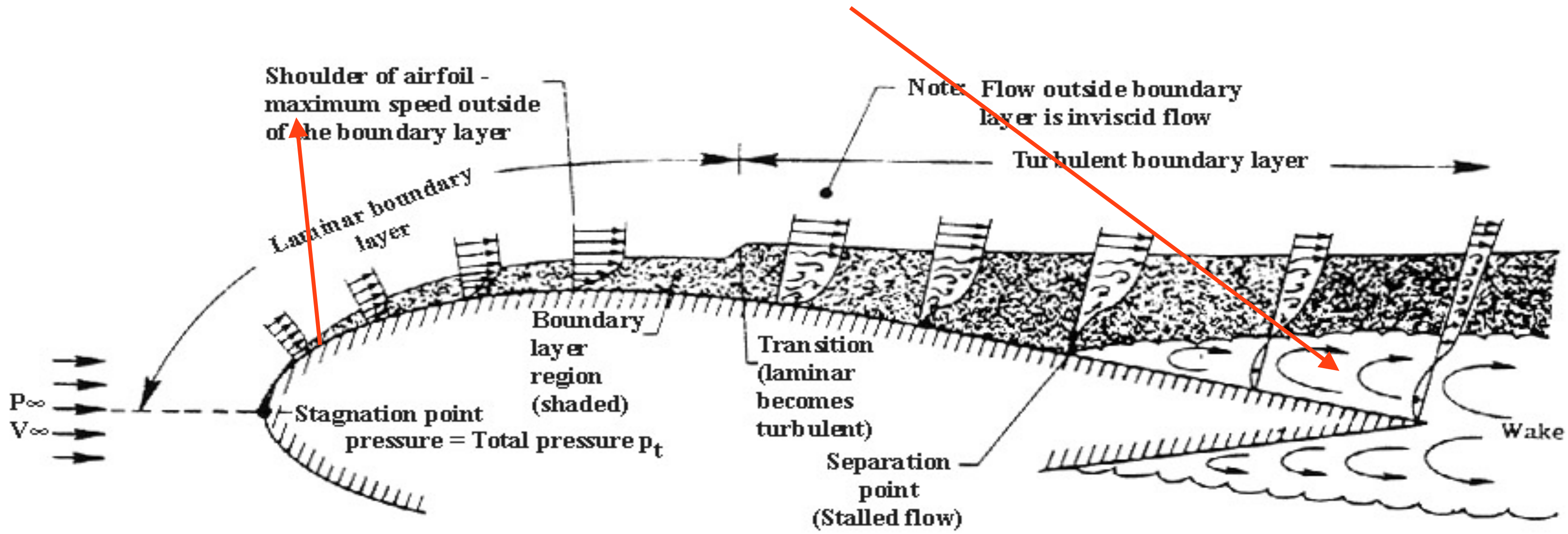


Two Types of "Boundary Layers"

- **Laminar**
- **Turbulent**

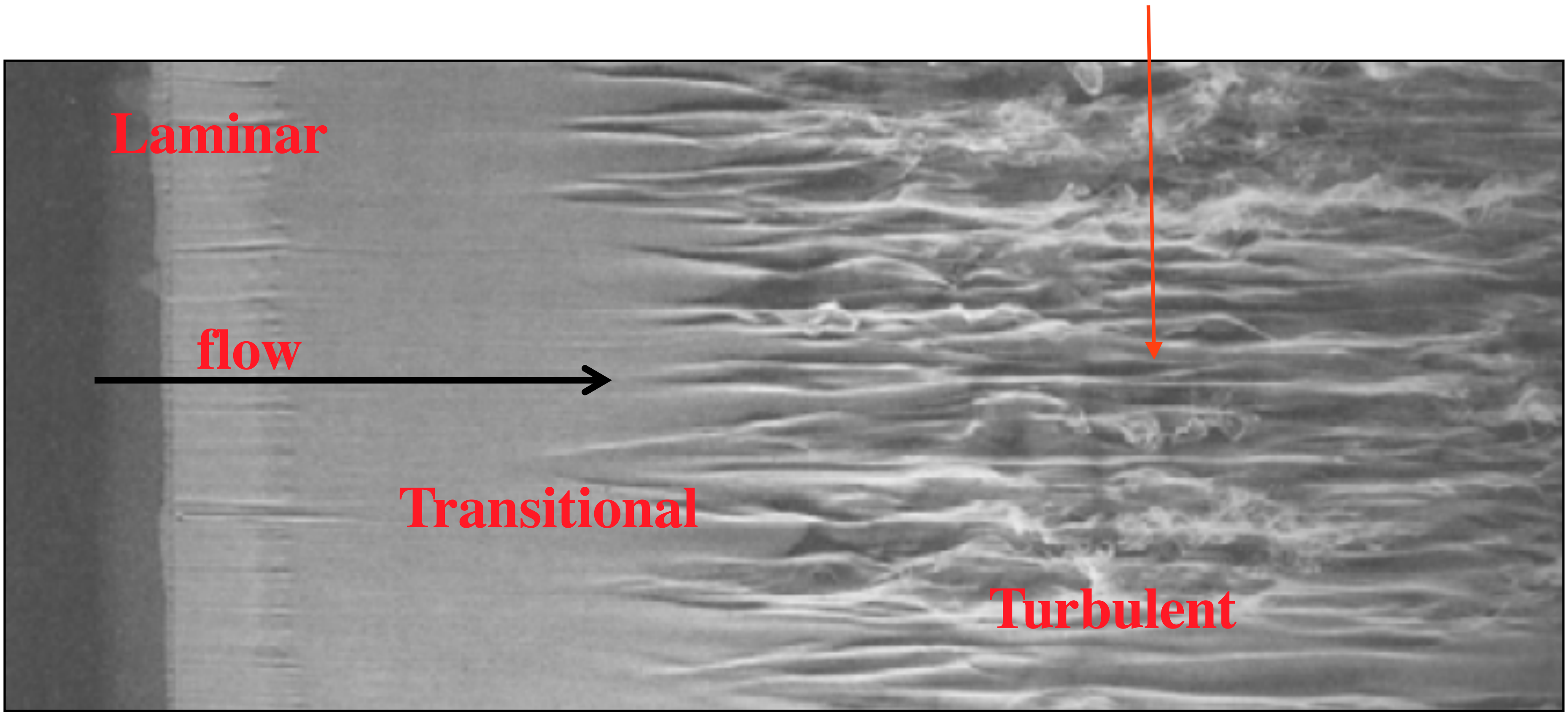






Boundary Layer (continued)

- Two Types of Boundary Layers



- Laminar --> *orderly*
- Turbulent --> *chaotic*

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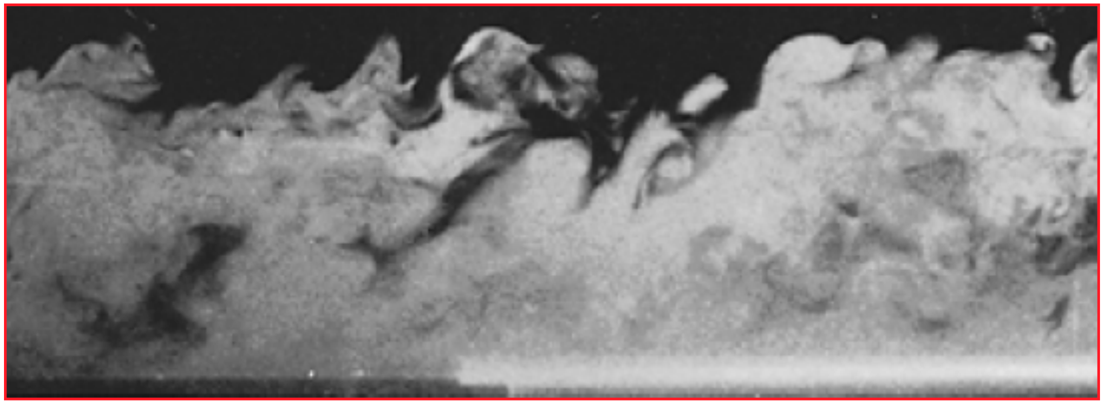
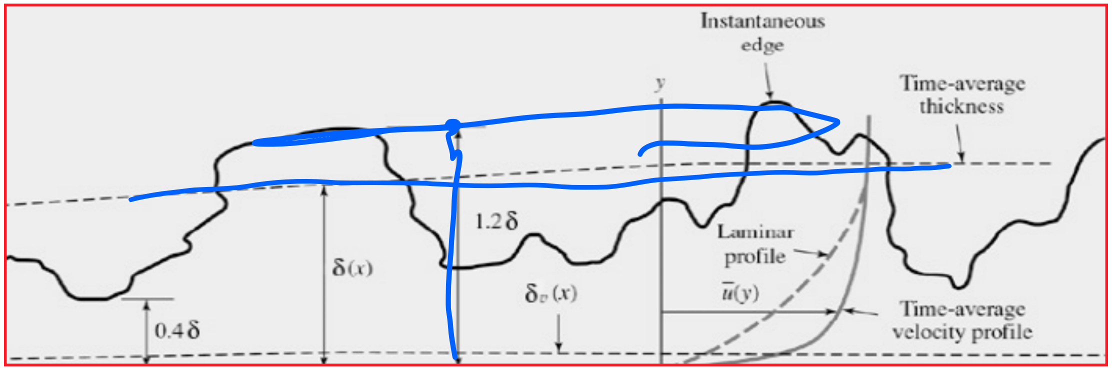
~~Laminar~~ Boundary Layer Model

Turbulent





"Skin Friction" Momentum Defect Model

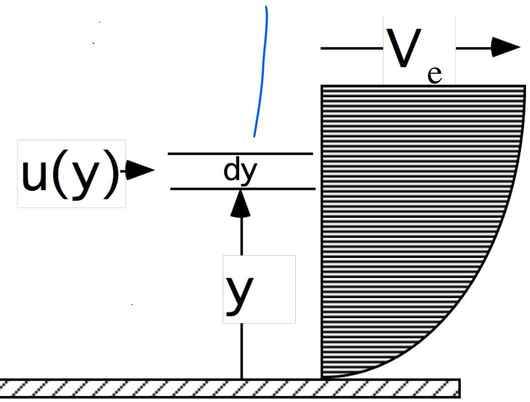
Start with incompressible flow for now (ρ = const)

- Look at the flow near the aft end of a flat plate with span, *b*

- Mass flow into the small segment *dy*:

$$\dot{m}(y) = \rho(y)u(y)b\partial y$$

$$\partial M_{omentum} = \dot{m}(y)\left[V_e - u(y)\right]$$



- Momentum defect from freestream across segment *dy:*

- Assuming no pressure gradient along plate

Drag = loss in momentum $F = \frac{\partial}{\partial t}\left[M_{omentum}\right]$

$$\rightarrow \partial D = \dot{m}\left[V_e - u(y)\right] = \rho\left[b\partial y\right]u(y)\left[V_e - u(y)\right]$$

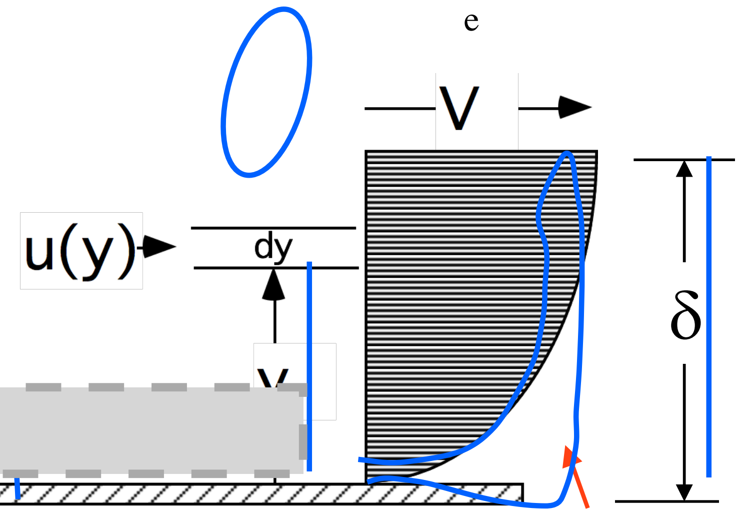
Simple "Skin Friction" Model (cont'd)

- Integrating across the depth of the boundary layer

$$D = \rho V_e^{\,2} b \int_0^{\delta} \frac{u(y)}{V_e}\left[1 - \frac{u(y)}{V_e}\right] dy \Rightarrow$$

$$let \Rightarrow \xi = \frac{y}{\delta} \Rightarrow d\xi = \frac{dy}{\delta} \Rightarrow$$

$$\int_0^{\delta} \frac{u(y)}{V_e}\left[1 - \frac{u(y)}{V_e}\right] dy = \delta \int_0^{1} \frac{u(\xi)}{V_e}\left[1 - \frac{u(\xi)}{V_e}\right] d\xi$$



$$D = \rho V_e^{\,2} b \delta \int_0^{1} \frac{u(\xi)}{V_e}\left[1 - \frac{u(\xi)}{V_e}\right] d\xi$$

$$\Theta = \delta \int_0^{1} \frac{u(\xi)}{V_e}\left[1 - \frac{u(\xi)}{V_e}\right] d\xi$$

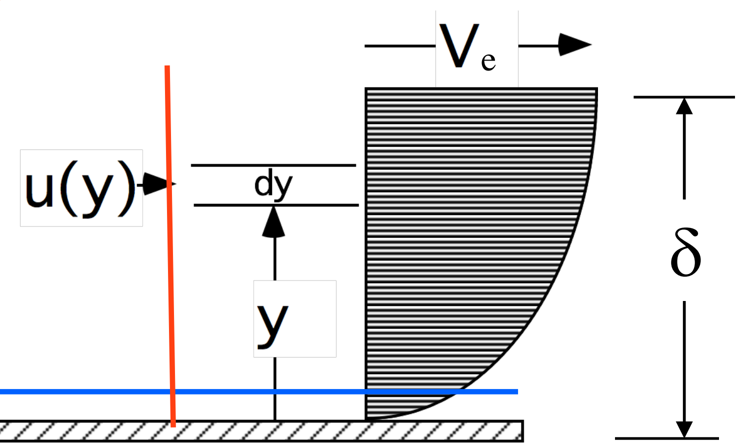
"momentum thickness"

Simple "Skin Friction" Model (concluded)

- Normalize Drag by plate area *(bc)* and dynamic pressure, $\frac{1}{2}\rho V_e^{\,2}$

$$\frac{D}{\frac{1}{2}\rho V_e^{\,2} bc} = C_{D_{fric}} =$$

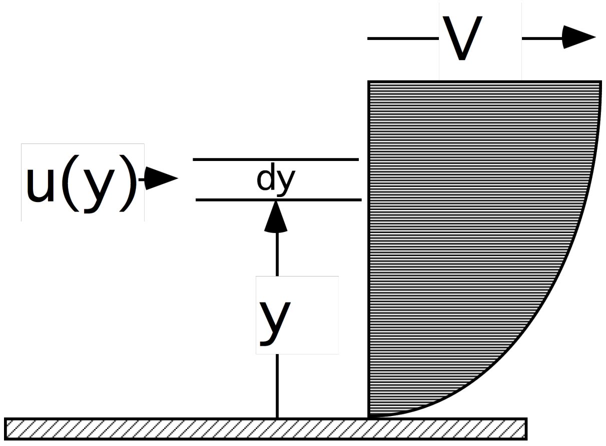
$$2\frac{\delta}{c}\int_0^1 \frac{u(\xi)}{V_e}\left[1 - \frac{u(\xi)}{V_e}\right] d\xi = 2\frac{\Theta}{c}$$



Θ → "momentum thickness"

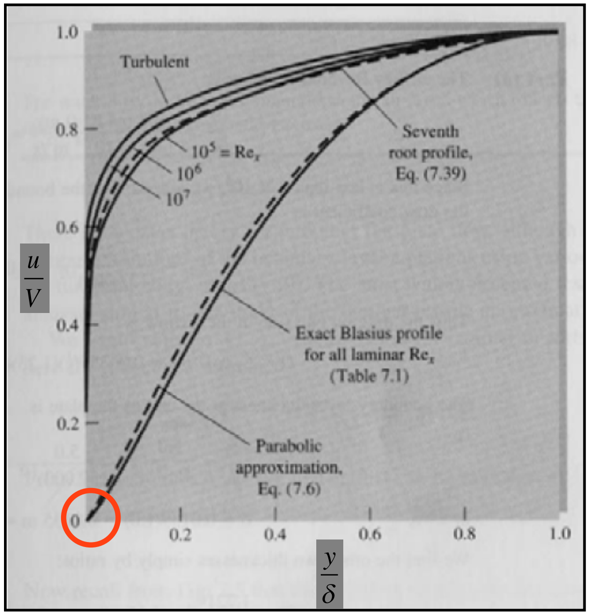
Velocity Profile in Laminar Boundary Layer

- Simple Velocity Profile Model



Laminar (parabolic)

$$\frac{u_{(y)}}{V_e} = \left[2 \cdot a \left(\frac{y}{\delta} \right) - b \cdot \left(\frac{y}{\delta} \right)^2 \right]$$



Pick {a, b} to Best Match Theoretical Model from Navier Stokes Equations (Blasius Model)

Fitting with "near" Velocity Profile Allows Use of Integral Methods

Theoretical Profile: *http://mae-nas.eng.usu.edu/MAE_5420_Web/section8/Blasius_files/Blasius.html*

Calculate Skin Friction Coefficient for Laminar Boundary Layer, Integral Method

$$\frac{u_{(y)}}{V_e} = \left[2\cdot a\left(\frac{y}{\delta}\right) - b\cdot\left(\frac{y}{\delta}\right)^2\right] \ldots \boxed{\frac{y}{\delta} = 1 \rightarrow \frac{u_{(\delta)}}{V_e} = 1} \rightarrow 2\cdot a - b = 1 \rightarrow b = 2\cdot a - 1$$

$$C_{D_{fric}} = 2\cdot\frac{\delta}{c}\cdot\int_0^1 \frac{u}{V_e}\cdot\left(1 - \frac{u}{V_e}\right)\cdot\frac{dy}{\delta} = 2\cdot\frac{\delta}{c}\cdot\int_0^1\left[2\cdot a\left(\frac{y}{\delta}\right) - b\cdot\left(\frac{y}{\delta}\right)^2\right]\cdot\left(1 - \left[2\cdot a\left(\frac{y}{\delta}\right) - b\cdot\left(\frac{y}{\delta}\right)^2\right]\right)\cdot\frac{dy}{\delta}$$

Let …

$$\rightarrow \frac{y}{\delta} \equiv \xi \rightarrow C_{D_{fric}} = 2\cdot\frac{\delta}{c}\cdot\int_0^1\left[2\cdot a\cdot\xi - b\cdot\xi^2\right] - \left[2\cdot a\cdot\xi - b\cdot\xi^2\right]^2\cdot d\xi$$

Expand and Integrate

$$C_{D_{fric}} = 2\cdot\frac{\delta}{c}\cdot\int_0^1\left[2\cdot a\cdot\xi - \left(4\cdot a^2 + b\right)\cdot\xi^2 + 4\cdot a\cdot b\cdot\xi^3 - b^2\cdot\xi^4\right]\cdot d\xi =$$

$$2\cdot\frac{\delta}{c}\cdot\left[2\cdot a\cdot\frac{\xi^2}{2} - \left(4\cdot a^2 + b\right)\cdot\frac{\xi^3}{3} + 4\cdot a\cdot b\cdot\frac{\xi^4}{4} - b^2\cdot\frac{\xi^5}{5}\right]$$

$$\rightarrow \boxed{C_{D_{fric}} = 2\cdot\left[a - \frac{4\cdot a^2 + b}{3} + a\cdot b - \frac{b^2}{5}\right]\cdot\frac{\delta}{c}}$$

Simple Skin Friction Model for Laminar Boundary Layers

$$\rightarrow \boxed{C_{D_{fric}} = 2 \cdot \left[a - \frac{4 \cdot a^2 + b}{3} + a \cdot b - \frac{b^2}{5} \right] \cdot \frac{\delta}{c}}$$

- But what is δ?

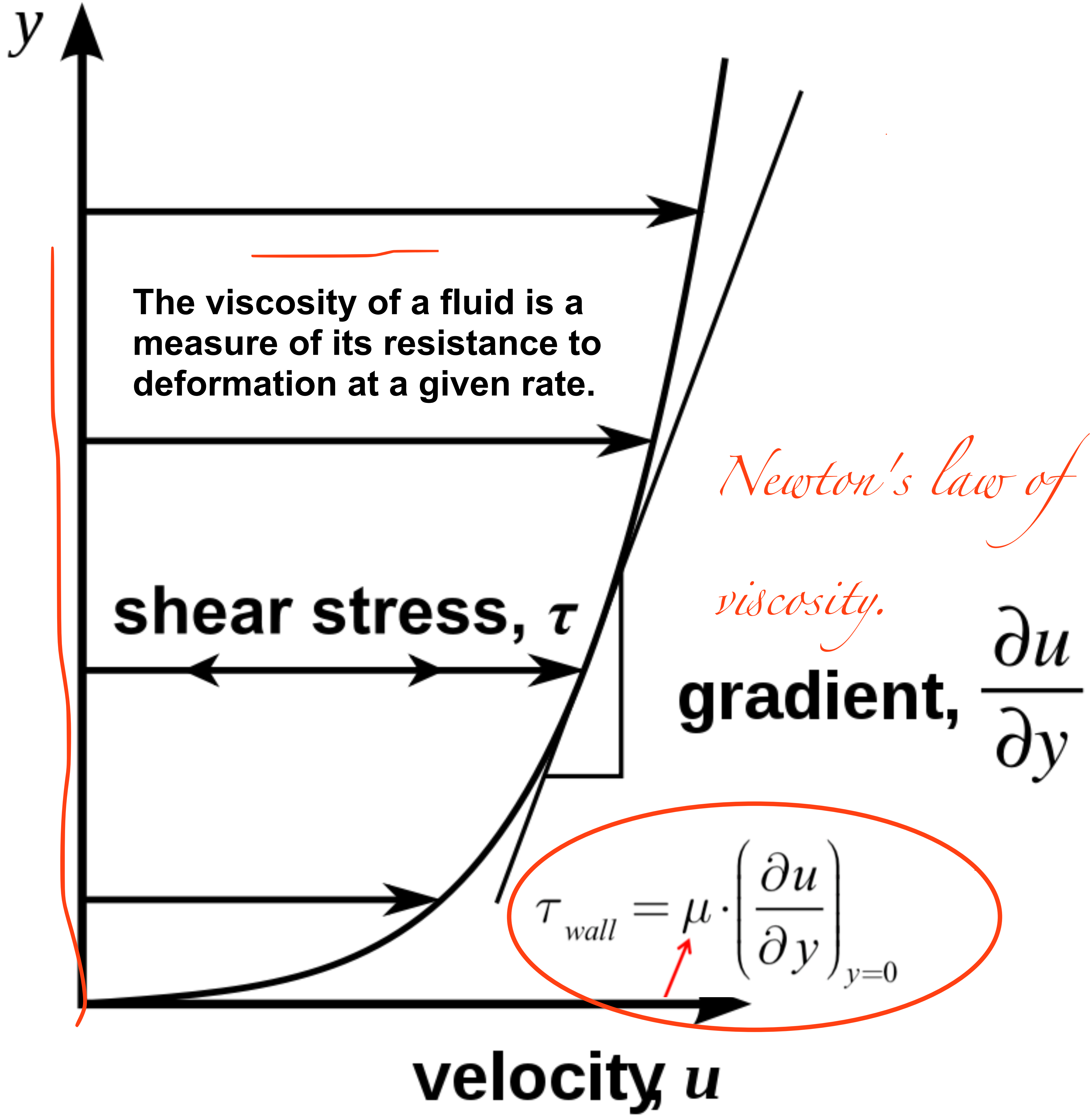
- Define BL thickness as the distance from the plate where local fluid velocity reaches free-stream velocity.

$$\ldots \boxed{\frac{y}{\delta} = 1 \rightarrow \frac{u_{(\delta)}}{V_e} = 1}$$

- *Start with Wall Shear Stress .. Shearing force per unit area*

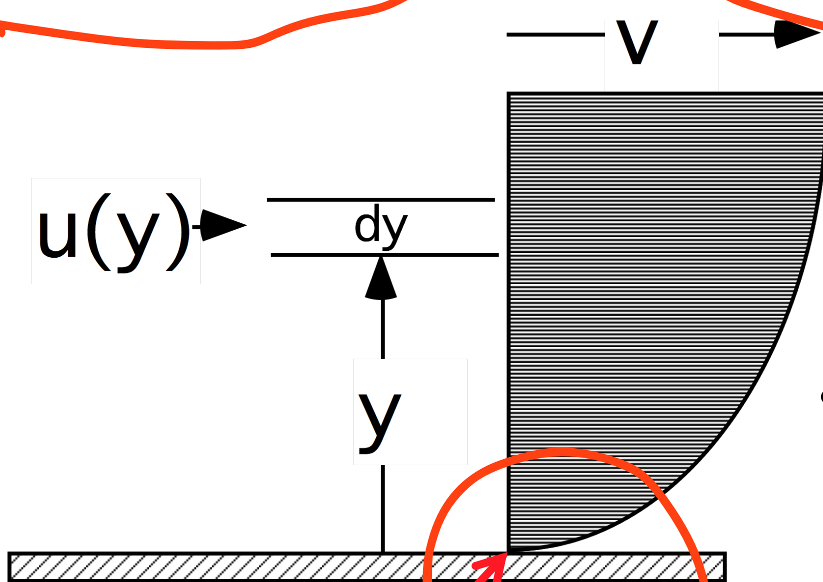
$$\tau_{wall} = \frac{1}{b} \frac{\partial D_{wall\ fric}}{\partial x} \ldots \wp_{roportional} \rightarrow \frac{\partial u(y)}{\partial y} \quad \rightarrow \tau_{wall} = \mu \cdot \left(\frac{\partial u}{\partial y} \right)_{y=0}$$

Constant of Proportionality
"*dynamic viscosity*"


y
The viscosity of a fluid is a measure of its resistance to deformation at a given rate.
shear stress, τ
Newton's law of viscosity.
gradient, ∂u/∂y
τ_wall = μ · (∂u/∂y)_{y=0}
velocity, u

Boundary Layer Thickness

$$\tau_{wall} = \frac{1}{b}\frac{\partial D_{wall_{fric}}}{\partial x} \rightarrow \tau_{wall} = \mu \cdot \left(\frac{\partial u}{\partial y}\right)_{y=0}$$



$$C_{D_{fric}} \equiv \frac{\int_0^c \delta D_{wall_{fric}}}{\frac{1}{2}\rho V_e^2 \cdot b \cdot c} = \frac{\int_0^c \tau_{wall} \cdot b \cdot dx}{\frac{1}{2}\rho V_e^2 \cdot b \cdot c} = \frac{\int_0^c \tau_{wall} \cdot dx}{\frac{1}{2}\rho V_e^2 \cdot c}$$

- From Earlier derivation of momentum thickness

$$\boxed{C_{D_{fric}} = 2\frac{\Theta}{c}}$$

$$\tau_{wall} = \frac{1}{b}\frac{\partial D_{wall_{fric}}}{\partial x}$$

$$C_{D_{fric}} = 2\frac{\Theta}{c} = \frac{\int_0^c \tau_{wall} \cdot dx}{\frac{1}{2}\rho V_e^2 \cdot c}$$

Boundary Layer Thickness (cont'd)

$$C_{D_{fric}} = 2\frac{\Theta}{c} = \frac{\int_0^c \tau_{wall} \cdot dx}{\frac{1}{2}\rho V_e^2 \cdot c} \rightarrow \Theta \cdot \rho V_e^2 = \int_0^c \tau_{wall} \cdot dx$$

Take derivative of above w.r.t. x →

$$\boxed{\tau_w = \rho V_e^2 \frac{\partial \Theta}{\partial x}}$$

• Classical boundary layer equation ..

Von Karman Momentum law for laminar boundary layer

$$b \cdot \frac{\tau_w}{\frac{1}{2}\rho V_e^2} = b \cdot 2\frac{\partial \Theta}{\partial x} \rightarrow c_{f_x} = \frac{\tau_w}{\frac{1}{2}\rho V_e^2} = 2\frac{\partial \Theta}{\partial x}$$

$$c_{f_x} \rightarrow \text{"}\textit{local skin friction coefficient}\text{"}$$

Boundary Layer Thickness (cont'd)

• Now Calculate Laminar Boundary Layer Shear Stress Using Velocity Distribution Model

$$\tau_{wall} = \mu \cdot \left(\frac{\partial u}{\partial y}\right)_{y=0} = \mu \cdot V_e \frac{\partial}{\partial y}\left[2 \cdot a\left(\frac{y}{\delta}\right) - b \cdot \left(\frac{y}{\delta}\right)^2\right]_{y=0} = \frac{2 \cdot a}{\delta} \cdot \mu \cdot V_e$$

• Two ways of calculating τ_{wall}

$$\tau_{wall} = \frac{2 \cdot a}{\delta} \cdot \mu \cdot V_e \quad \ldots \quad \tau_{wall} = \rho \cdot V_e^{\,2} \cdot \frac{\partial \Theta}{\partial x}$$

• *Equating terms …*

$$\rho \cdot V_e^{\,2} \cdot \frac{\partial \Theta}{\partial x} = \frac{2 \cdot a}{\delta} \cdot \mu \cdot V_e$$

$$\rho \cdot V_e^{\,2} \cdot \frac{\partial \Theta}{\partial x} = \frac{2 \cdot a}{\delta} \cdot \mu \cdot V_e \quad \rightarrow \quad \boxed{\frac{\rho \cdot V_e}{\mu} \cdot \frac{\partial \Theta}{\partial x} = \frac{2 \cdot a}{\delta}}$$

Laminar Boundary Layer Thickness (cont'd)

- From earlier momentum analysis

$$\Theta = \frac{c}{2} \cdot C_{D_{fric}} \rightarrow \frac{\partial \Theta}{\partial x} = \frac{\partial}{\partial x}\left(\frac{c}{2} \cdot C_{D_{fric}}\right) = \left[a - \frac{4 \cdot a^2 + b}{3} + a \cdot b - \frac{b^2}{5}\right] \cdot \frac{\partial \delta}{\partial x}$$

- From Previous page $\rightarrow$ $\dfrac{\rho \cdot V_e}{\mu} \cdot \dfrac{\partial \Theta}{\partial x} = \dfrac{2 \cdot a}{\delta}$

$$\frac{\rho \cdot V_e}{\mu} \cdot \left[a - \frac{4 \cdot a^2 + b}{3} + a \cdot b - \frac{b^2}{5}\right] \cdot \frac{\partial \delta}{\partial x} = \frac{2 \cdot a}{\delta}$$

- Separating Variables

$$\frac{\rho \cdot V_e}{\mu} \cdot \left[a - \frac{4 \cdot a^2 + b}{3} + a \cdot b - \frac{b^2}{5}\right] \cdot \delta \cdot \partial \delta = 2 \cdot a \cdot \partial x$$

Laminar Boundary Layer Thickness (cont'd)

- From earlier analysis

$$\frac{\rho \cdot V_e}{\mu} \cdot \left[a - \frac{4 \cdot a^2 + b}{3} + a \cdot b - \frac{b^2}{5}\right] \cdot \delta \cdot \partial\delta = 2 \cdot a \cdot \partial x$$

- Integrate from {0,c}

$$\frac{\rho \cdot V_e}{\mu} \cdot \left[a - \frac{4 \cdot a^2 + b}{3} + a \cdot b - \frac{b^2}{5}\right] \cdot \frac{\delta^2}{2} = 2 \cdot a \cdot c$$

- Solve for δ

$$\delta_c = \frac{c}{\left(\frac{\rho \cdot V_e}{\mu} \cdot c\right)^{1/2}} \sqrt{\frac{4a}{\left[a - \frac{4 \cdot a^2 + b}{3} + a \cdot b - \frac{b^2}{5}\right]}} = \frac{c}{\left(R_e\right)^{1/2}} \sqrt{\frac{4a}{\left[a - \frac{4 \cdot a^2 + b}{3} + a \cdot b - \frac{b^2}{5}\right]}}$$

- Define

$$\boxed{R_e = \frac{\rho V_e c}{\mu}}$$

- "Reynolds Number"

Laminar Boundary Layer Thickness (cont'd)

• **Sub into Skin Friction Model**

$$\rightarrow \boxed{C_{D_{fric}} = 2\cdot\left[a - \frac{4\cdot a^2+b}{3} + a\cdot b - \frac{b^2}{5}\right]\cdot\frac{\delta}{c}}$$

$$\boxed{\frac{\delta}{c} = \frac{1}{\left(R_e\right)^{1/2}}\sqrt{\frac{4a}{\left[a - \frac{4\cdot a^2+b}{3} + a\cdot b - \frac{b^2}{5}\right]}}}$$

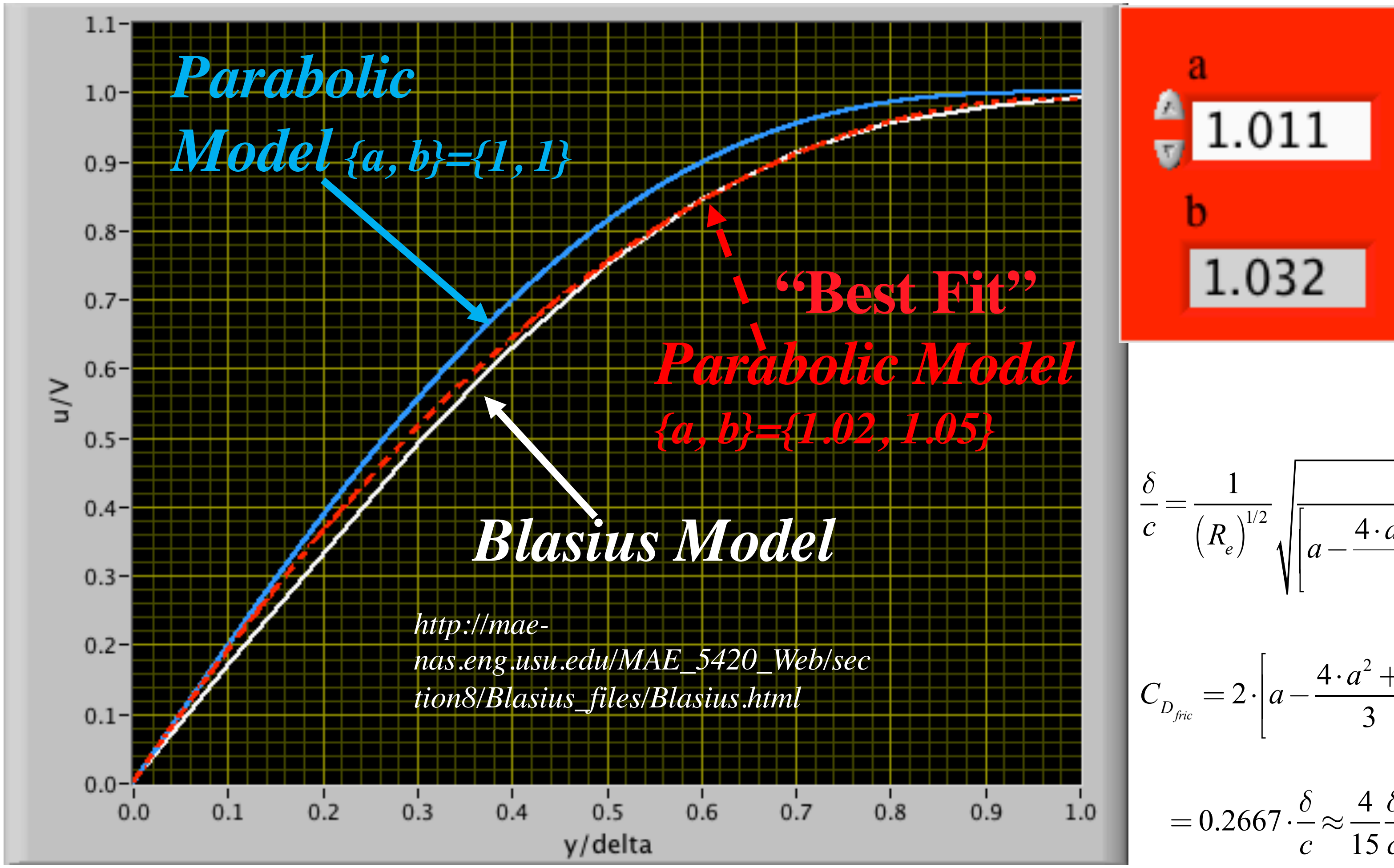
$$C_{D_{fric}} = \frac{2\cdot\left[a - \frac{4\cdot a^2+b}{3} + a\cdot b - \frac{b^2}{5}\right]\cdot\sqrt{\frac{4a}{\left[a - \frac{4\cdot a^2+b}{3} + a\cdot b - \frac{b^2}{5}\right]}}}{\left(R_e\right)^{1/2}} = \frac{4\sqrt{a\cdot\left[a - \frac{4\cdot a^2+b}{3} + a\cdot b - \frac{b^2}{5}\right]}}{\left(R_e\right)^{1/2}}$$

If … {a,b}={1,1}

$$\frac{\delta}{c} = \frac{1}{\left(R_e\right)^{1/2}}\sqrt{\frac{4(1)}{\left[(1) - \frac{4\cdot(1)^2+(1)}{3} + (1)\cdot(1) - \frac{(1)^2}{5}\right]}} = \frac{5.477}{\left(R_e\right)^{1/2}}$$

$$C_{D_{fric}} = \frac{4\sqrt{(1)\cdot\left[(1) - \frac{4\cdot(1)^2+(1)}{3} + (1)\cdot(1) - \frac{(1)^2}{5}\right]}}{\left(R_e\right)^{1/2}} = \frac{1.461}{\left(R_e\right)^{1/2}}$$

"Best Match" Velocity Distribution



"Best Fit" Shape Parameters

$$\frac{\delta}{c} = \frac{1}{\left(R_e\right)^{1/2}} \sqrt{\frac{4a}{\left[a - \frac{4 \cdot a^2 + b}{3} + a \cdot b - \frac{b^2}{5}\right]}} = \frac{4.98}{\left(\mathrm{R}_{e_c}\right)^{1/2}}$$

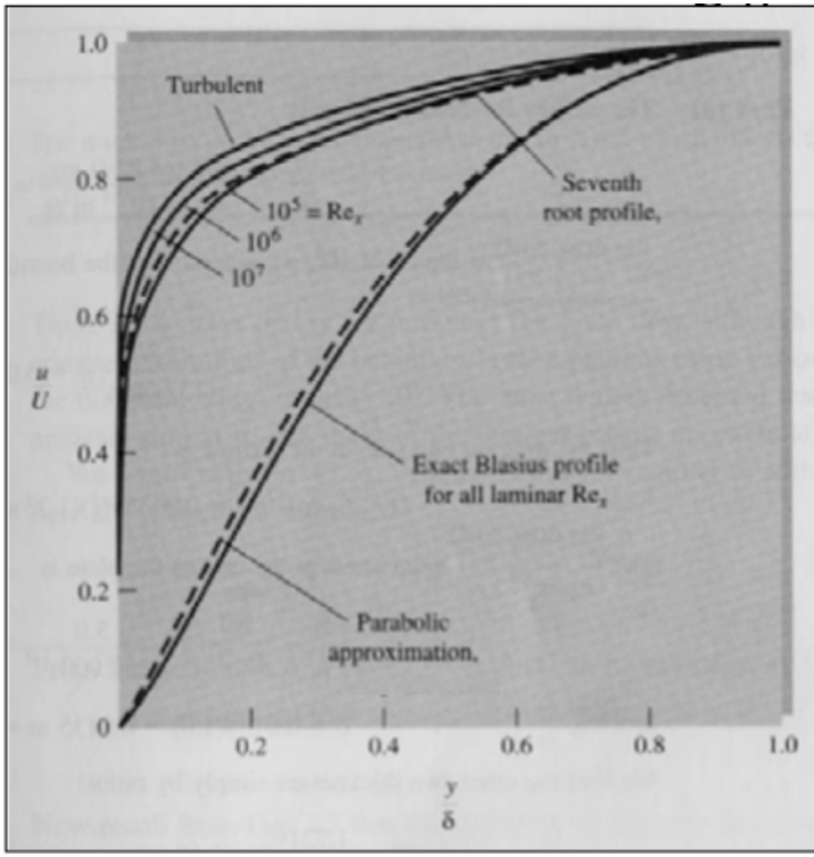
$$C_{D_{fric}} = 2 \cdot \left[a - \frac{4 \cdot a^2 + b}{3} + a \cdot b - \frac{b^2}{5}\right] \cdot \frac{\delta}{c}$$

$$= 0.2667 \cdot \frac{\delta}{c} \approx \frac{4}{15}\frac{\delta}{c} = \frac{\frac{4}{15} \cdot 4.98}{\left(\mathrm{R}_{e_x}\right)^{1/2}} = \frac{1.328}{\left(\mathrm{R}_{e_c}\right)^{1/2}}$$

Laminar Boundary Layer (cont'd)

• Approximate Drag Coefficient Model based on "Best Fit" Parabolic Velocity Profile

$$\frac{\delta}{c} = \frac{4.98}{\left(\mathrm{R}_{e_c}\right)^{1/2}} \qquad C_{D_{fric}} = \frac{1.328}{\left(\mathrm{R}_{e_x}\right)^{1/2}}$$



• *Compares well with Exact "Blasius Solution" … slightly under predicts boundary layer height*

$$\left(\frac{\delta}{c}\right)_{Blasius} = \frac{5}{\left(\mathrm{R}_{e_c}\right)^{1/2}}$$

$$\left(C_{D_{fric}}\right)_{Blasius} = \frac{1.328}{\left(\mathrm{R}_{e_c}\right)^{1/2}}$$

https://www.sciencedirect.com/topics/engineering/blasius-solution

• *Blasius method described in Appendix to section 8 … MAE 5420 students not responsible for exact method*

Laminar Boundary Layer (Concluded)

Blasius Laminar Boundary Layer Solution Summary

Disturbance Thickness: $$\frac{\delta}{x} = \frac{5}{\left(\mathrm{R}_{e_x}\right)^{1/2}}$$

Displacement Thickness: $$\frac{\delta^*}{x} = \left(\frac{\delta^*}{\delta}\right)\left(\frac{\delta}{x}\right) = \frac{1.72}{\sqrt{Re_x}}$$

l=c

Momentum Thickness: $$\frac{\theta}{x} = \left(\frac{\theta}{\delta}\right)\left(\frac{\delta}{x}\right) = \frac{0.665}{\sqrt{Re_x}}$$

Wall Shear Stress: $$\tau_{wall} = \mu \cdot \frac{\partial u}{\partial y}\Big|_{y=0} = \frac{0.332 \cdot \rho \cdot V_e}{\left(\mathrm{R}_{e_x}\right)^{1/2}}$$

Friction Coefficient: $$c_f = \frac{\tau_w}{\frac{1}{2}\rho U^2} = \frac{0.664}{\sqrt{Re_x}}$$

Drag Coefficient for Friction: $$C_{D_{fric}} = \frac{1}{A}\int c_f\, dA = \frac{1}{\ell}\int_{x=0}^{\ell} \frac{0.664}{\sqrt{Re_x}}\, dx = \frac{1.328}{\sqrt{Re_\ell}}$$

Comparison of Reynolds Number and Mach Number

- As derived in section 3

$$\frac{V^2/2}{e} = \frac{\gamma(\gamma-1)}{2} M^2$$

Mach number is a measure of the ratio of the fluid Kinetic energy to the fluid internal energy *(direct motion To random thermal motion of gas molecules)*
-- Fundamental Parameter of Compressible Flow --

- $$R_e = \frac{\rho V_e c}{\mu} = \frac{\rho V_e^2 c}{\mu V_e} = \frac{\frac{2}{\delta}\rho V_e^2 c}{\frac{2}{\delta}\mu V_e} = \frac{4}{\rho/c}\cdot\frac{\frac{1}{2}\rho\cdot V_e^2}{\tau_{wall}} \propto \frac{Inertial - Force/\ Area}{Viscous - Force/\ Area}$$

$$\boxed{\tau_w = \frac{2}{\delta}\mu V_e}$$

Reynold's number is a measure of the *ratio of Inertial Forces acting on the fluid -- to -- Viscous Forces Acting on the fluid*
-- Fundamental Parameter of Viscous Fluid Flow --

What are the Units of μ, R_e?

- *From Definition for Viscosity* →

$$\mu = \frac{\tau_{wall}}{\frac{\partial u}{\partial y}} \rightarrow units \approx \frac{Nt}{m^2} \frac{1}{\frac{m/s}{m}} = \frac{Nt-s}{m^2} = Pa-s$$

$$\frac{Nt-s}{m^2} = \frac{\left(\frac{kg-m}{s^2}\right)-s}{m^2} = \frac{kg}{m-s} = \frac{1000g}{100cm-s} = 10 \cdot \frac{g}{cm-s}$$

$$\boxed{\begin{array}{l} \rightarrow "poise" \equiv 1 \cdot \mathbb{P} = 1 \cdot \frac{g}{cm-s} \rightarrow 0.10 \cdot Pa-s \\ \rightarrow 1 \cdot Pa-s = 10 \cdot \mathbb{P} \\ \rightarrow "centipoise" \equiv 1 \cdot c\mathbb{P} = 0.001 \cdot Pa-s = 0.01 \cdot \mathbb{P} \end{array}}$$

$$\left.\begin{array}{l} R_{e_x} = \frac{\rho \cdot V \cdot x}{\mu} \\ R_{e_{\bar{c}}} = \frac{\rho \cdot V \cdot \bar{c}}{\mu} \end{array}\right| \rightarrow$$

$$units? \approx \frac{\frac{kg}{m^3} \cdot \frac{m}{s} \cdot m}{\frac{Nt-s}{m^2}} = \frac{\frac{kg}{m-s}}{\frac{kg}{m-s}} = dimensionless!$$

1000*Centipoise= Pa-sec

- Most common table lookup form for viscosity data

What are the Units of μ? (2)

- *YOU've Used Viscosity Units Before Motor Oil?* →

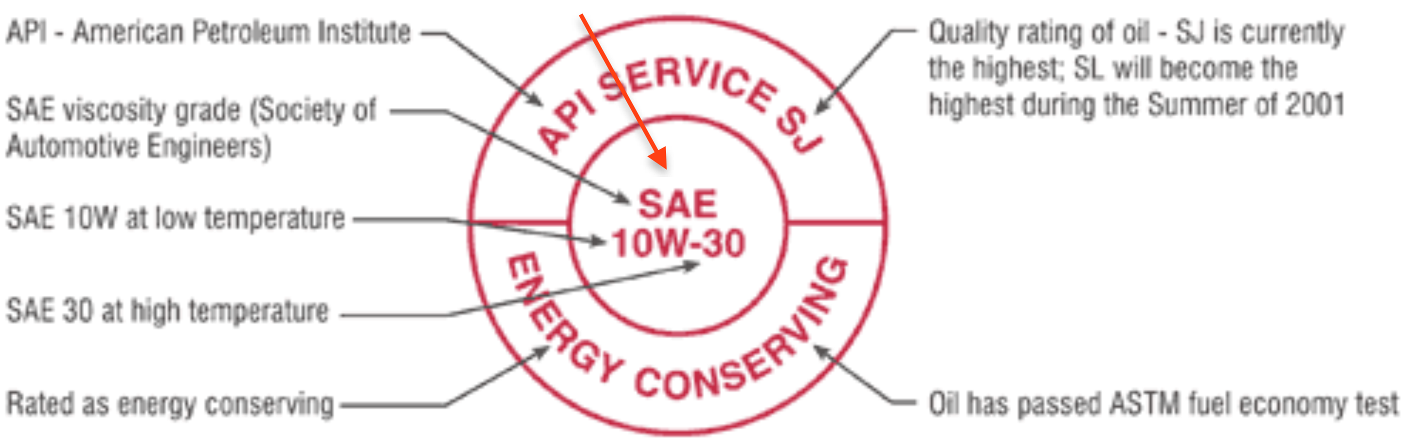
5W-30 Oil Rating?

SAW J300 Oil Classification

"W" is for Winter!

A 5W-30 oil is a mult-viscosity oil that behaves as 5-weight oil at low temperatures but gives the protection of 30-weight oil at the high engine operating temperatures.

Viscosity is defined as oil's resistance to flow and shear and is expressed as ***centipoise (cP).***



Gas Viscosity Model

- Sutherland's Formula

Result from kinetic theory that expresses viscosity as a function of temperature

$$\boxed{\mu(T) = \mu(T_s)\left(\frac{T}{T_s}\right)^{3/2}\left(\frac{T_s + C_s}{T + C_s}\right)}$$

- μ = viscosity in (Pa s) at input temperature T
- μ_S = reference viscosity in (Pa s) at reference temperature T_S
- T = input temperature in degrees Kelvin
- T_S = reference temperature in degrees Kelvin
- C_S = Sutherland's constant

- Write $\boldsymbol{\mu}$ in terms of freestream temperature Outside of Boundary layer

$$\frac{\mu(T)}{\mu(T_\infty)} = \frac{\mu_s\left(\frac{T}{T_s}\right)^{3/2}\left(\frac{T_s + C_s}{T + C_s}\right)}{\mu_s\left(\frac{T_\infty}{T_0}\right)^{3/2}\left(\frac{T_s + C_s}{T_\infty + C_s}\right)}$$

$$\rightarrow \mu(T) = \mu(T_\infty)\left(\frac{T}{T_\infty}\right)^{3/2}\left(\frac{T_\infty + C_s}{T + C_s}\right)$$

Sutherland's constant & reference temperature for some gases

Gas	C_S	T_S	μ_S
	-	K	Pa s
air	120	291.15	0.00001827
nitrogen	111	300.55	0.00001781
oxygen	127	292.25	0.00002018
carbon dioxide	240	293.15	0.0000148
carbon monoxide	118	288.15	0.0000172
hydrogen	72	293.85	0.00000876
ammonia	370	293.15	0.00000982
sulphur dioxide	416	293.65	0.00001254

Sutherland, W. (1893), "The viscosity of gases and molecular force", Philosophical Magazine, S. 5, 36, pp. 507-531 (1893). Australian physicist who studied the temperature-dependence of ideal gases. In 1893, he developed an empirical-theoretical relationship between the temperature and viscosity of an ideal gas.

Reynolds Number Scales?

• High-Speed and Large Configurations Operate with Mostly Turbulent Boundary Layers

• Aircraft Carrier – $R_{eL} \simeq 10^9$

• F-18 – 25 x 10^6

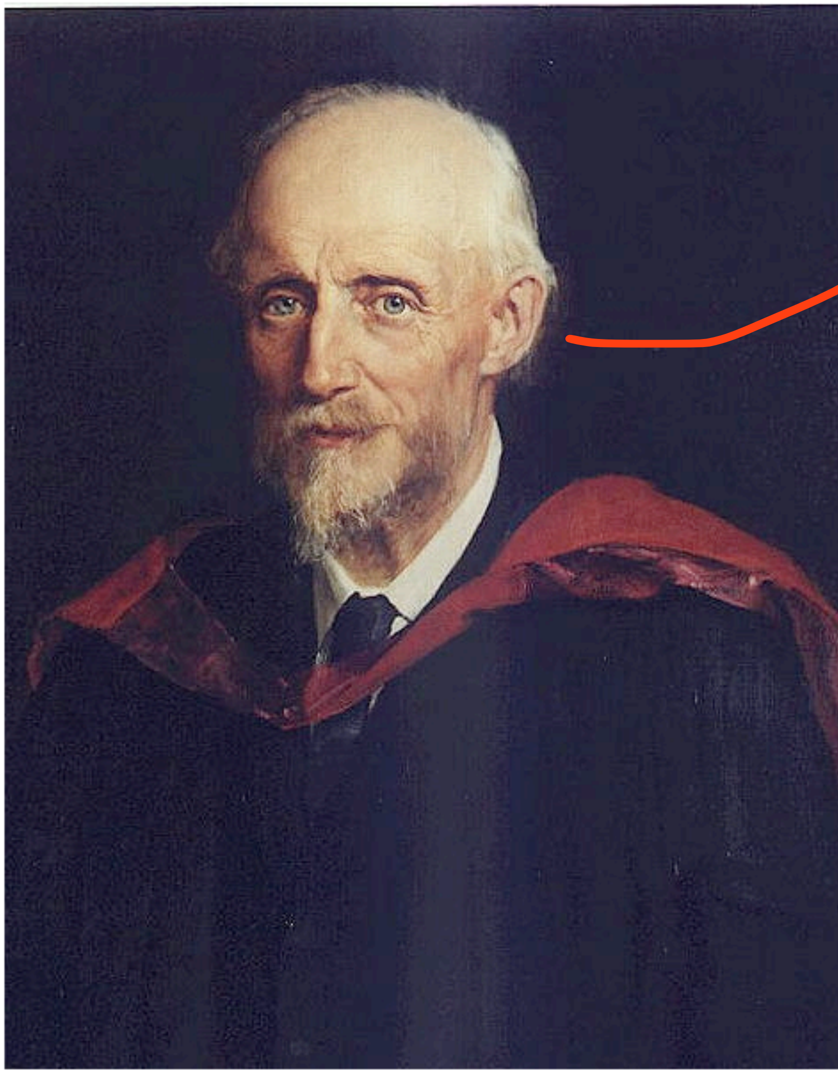
• Bumble Bee – $R_{eL} \simeq 10^2$

• Bacterium – $R_{eL} \simeq 10^{-5}$

• Low-Speed and Small Configurations Operate with Mostly Laminar Boundary Layers

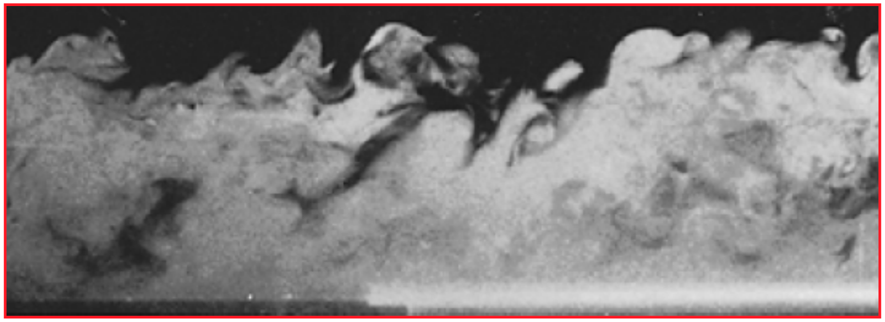
Reynolds most famously studied the conditions in which the flow of fluid in pipes transitioned from laminar flow to turbulent flow. In 1883 Reynolds demonstrated the transition to turbulent flow in a classic experiment in which he examined the behavior of water flow under different flow rates using a small jet of dyed water introduced into the centre of flow in a larger pipe.

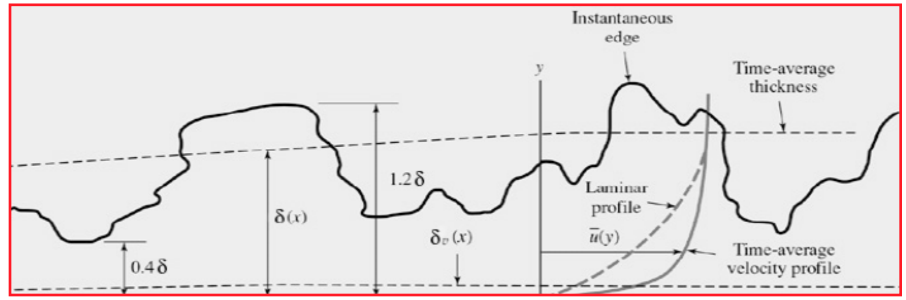
The larger pipe was glass so the behavior of the layer of dyed flow could be observed, and at the end of this pipe there was a flow control valve used to vary the water velocity inside the tube. When the velocity was low, the dyed layer remained distinct through the entire length of the large tube. When the velocity was increased, the layer broke up at a given point and diffused throughout the fluid's cross-section. The point at which this happened was the transition point from laminar to turbulent flow.



Osbourne Reynolds

Turbulent Boundary Layer Model

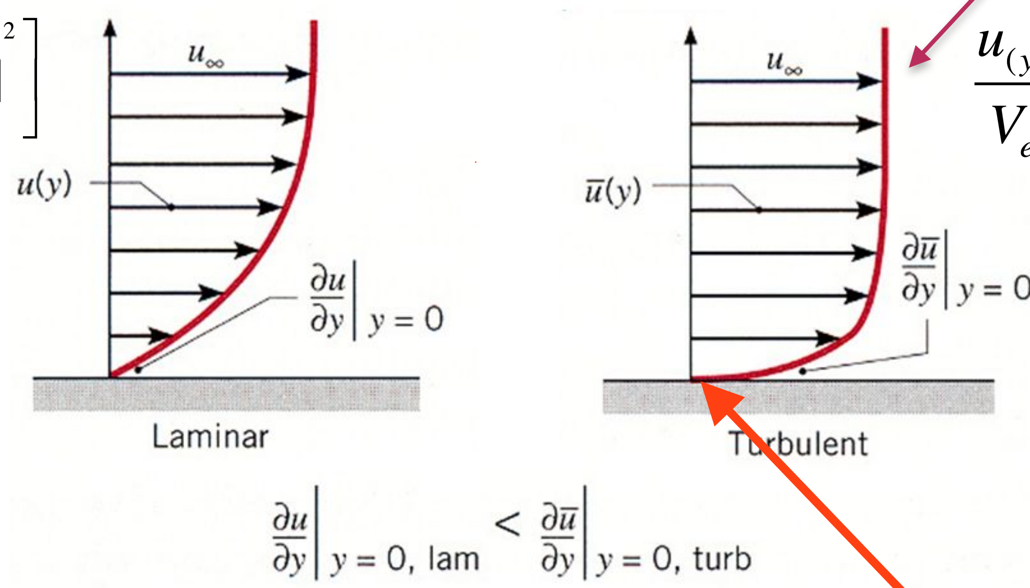




Comparison of laminar and turbulent velocity profile in the boundary layer

$$\frac{u_{(y)}}{V_e} = \left[\frac{2 \cdot y}{\delta} - a \cdot \left(\frac{y}{\delta}\right)^2\right]$$

$$\frac{u_{(y)}}{V_e} = \left(\frac{y}{\delta}\right)^{\frac{1}{n}}$$



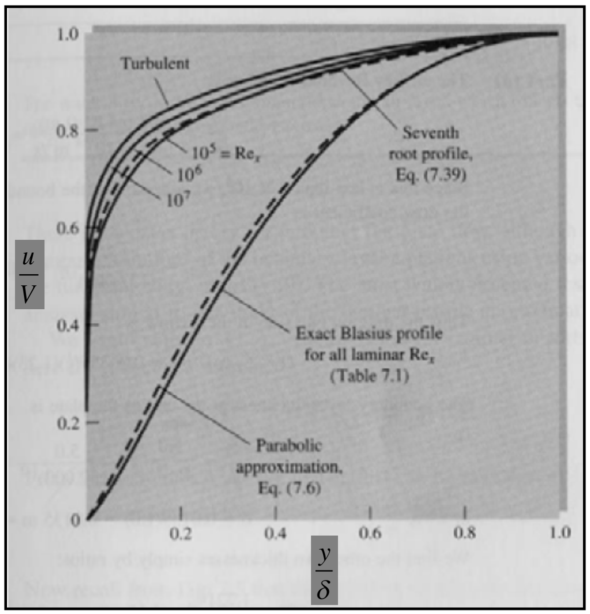
Velocity Profile in Turbulent Boundary Layer

- Power-Law Velocity Profile Models

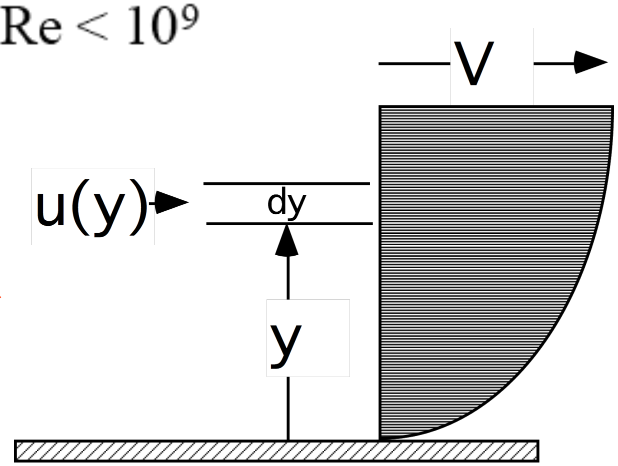
Turbulent (exponential curve fit)

$$\frac{u_{(y)}}{V_e} = \left(\frac{y}{\delta}\right)^{\frac{1}{n}}$$

We'll find out what n is later



$n = 7$	$Re < 10^7$
$n = 8$	$10^7 < Re < 10^8$
$n = 9$	$10^8 < Re < 10^9$



Calculate Skin Friction Coefficient for Turbulent Boundary Layer

- **Turbulent Boundary Layer**

$$C_{D_{fric}} = 2\frac{\delta}{c}\int_0^1 \frac{u(\frac{y}{\delta})}{V_e}\left[1-\frac{u(\frac{y}{\delta})}{V_e}\right]d\left[\frac{y}{\delta}\right] = 2\frac{\delta}{c}\int_0^1\left[\left(\frac{y}{\delta}\right)^{\frac{1}{n}}\right]\left[1-\left(\frac{y}{\delta}\right)^{\frac{1}{n}}\right]d\left[\frac{y}{\delta}\right] =$$

$$2\frac{\delta}{c}\int_0^1\left[\left(\frac{y}{\delta}\right)^{\frac{1}{n}}\right]\left[1-\left(\frac{y}{\delta}\right)^{\frac{1}{n}}\right]d\left[\frac{y}{\delta}\right] = 2\frac{\delta}{c}\int_0^1\left[\left(\frac{y}{\delta}\right)^{\frac{1}{n}}-\left(\frac{y}{\delta}\right)^{\frac{2}{n}}\right]d\left[\frac{y}{\delta}\right] =$$

$$2\frac{\delta}{c}\left[\frac{\left(\frac{y}{\delta}\right)^{\frac{1}{n}+1}}{\frac{1}{n}+1}-\frac{\left(\frac{y}{\delta}\right)^{\frac{2}{n}+1}}{\frac{2}{n}+1}\right]_0^1 = 2\frac{\delta}{c}\left[\frac{1}{\frac{1}{n}+1}-\frac{1}{\frac{2}{n}+1}\right] = 2\frac{\delta}{c}\left[\frac{n}{n+1}-\frac{n}{n+2}\right] =$$

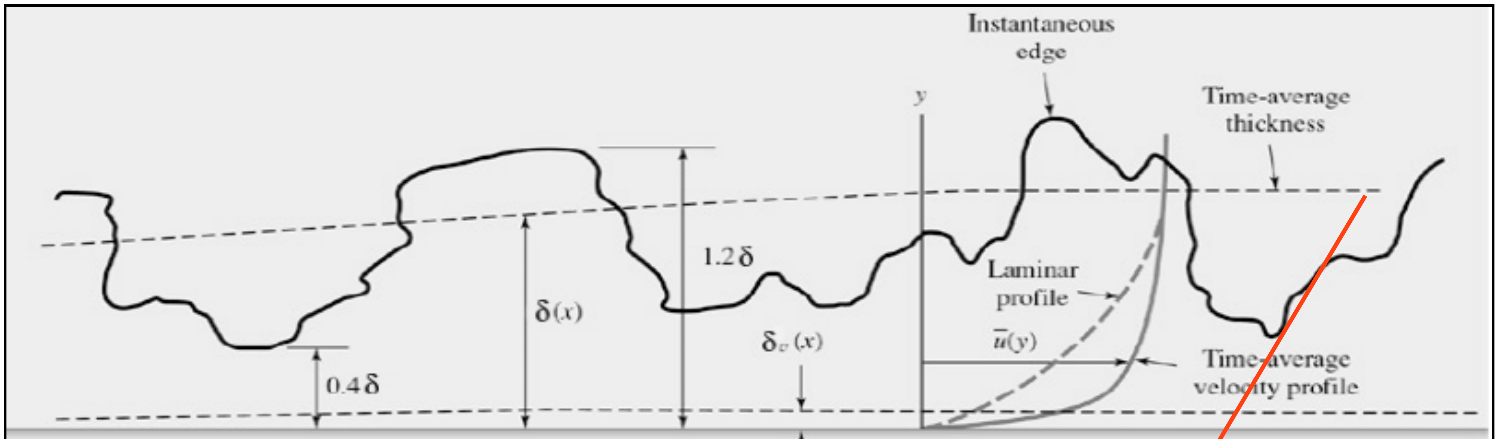
$$2\frac{\delta}{c}\left[\frac{n^2+2n-n^2-n}{(n+1)(n+2)}\right] = 2\frac{\delta}{c}\left[\frac{n}{(n+1)(n+2)}\right]$$

$$\boxed{\rightarrow\rightarrow\rightarrow C_{D_{fric}} = 2\frac{\delta}{c}\left[\frac{n}{(n+1)(n+2)}\right]_{turbulent}}$$

Turbulent Skin Friction Revisited

- **Turbulent Boundary Layer**

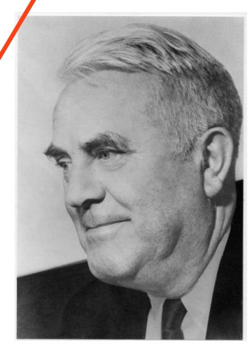
- No Theoretical Prediction for Boundary Layer Thickness for Turbulent Boundary layer $\left(\frac{\partial u}{\partial y}\right)_{y=0} \neq 0$



- **"Time averaged" Statistical Empirical Correlation**

… Herrman Schlichting

$$\delta = \frac{0.16c}{\left[\mathrm{R_e}\right]^{\frac{1}{n}}}$$



Skin Friction Revisited

(Turbulent Boundary Layer) (concluded)

- **Turbulent Boundary Layer**

$$\delta = \frac{0.16c}{\left[\mathrm{R_e}\right]^{\frac{1}{n}}}$$

$$C_{D_{fric}} = \frac{\delta}{c}\left[\frac{2n}{(n+1)(n+2)}\right]$$

$$C_{D_{fric}} = \frac{\frac{0.16c}{\left[\mathrm{R_e}\right]^{\frac{1}{n}}}}{c}\left[\frac{2n}{(n+1)(n+2)}\right] = \boxed{\frac{\left[\frac{0.32n}{(n+1)(n+2)}\right]}{\left[\mathrm{R_e}\right]^{\frac{1}{n}}}}$$

Compare Laminar and Turbulent Skin Friction Models

$$\frac{\delta}{c} = \frac{4.98}{\left(\mathrm{R}_{e_c}\right)^{1/2}} \qquad C_{D_{fric}} = \frac{1.328}{\left(\mathrm{R}_{e_x}\right)^{1/2}}$$

Laminar

x

$$\frac{\delta}{c} = \frac{0.16}{\left(R_{e_c}\right)^{1/n}} \qquad C_{D_{fric}} = \frac{\dfrac{0.32n}{(n+1)(n+2)}}{\left[\mathrm{R}_{e_x}\right]^{\frac{1}{n}}}$$

Turbulent

Compare Laminar and Turbulent Skin Friction Coefficients with Wave Drag Coefficient

$$\rightarrow\rightarrow\rightarrow C_{D_{fric}} = \frac{1.328}{\left(\mathrm{R}_{e_x}\right)^{1/2}}$$

laminar

$$\rightarrow\rightarrow\rightarrow C_{D_{fric}} = \frac{\left[\frac{0.32n}{(n+1)(n+2)}\right]}{\left[\mathrm{R_e}\right]^{\frac{1}{n}}}$$

turbulent

Flat Plate with Chord "c"

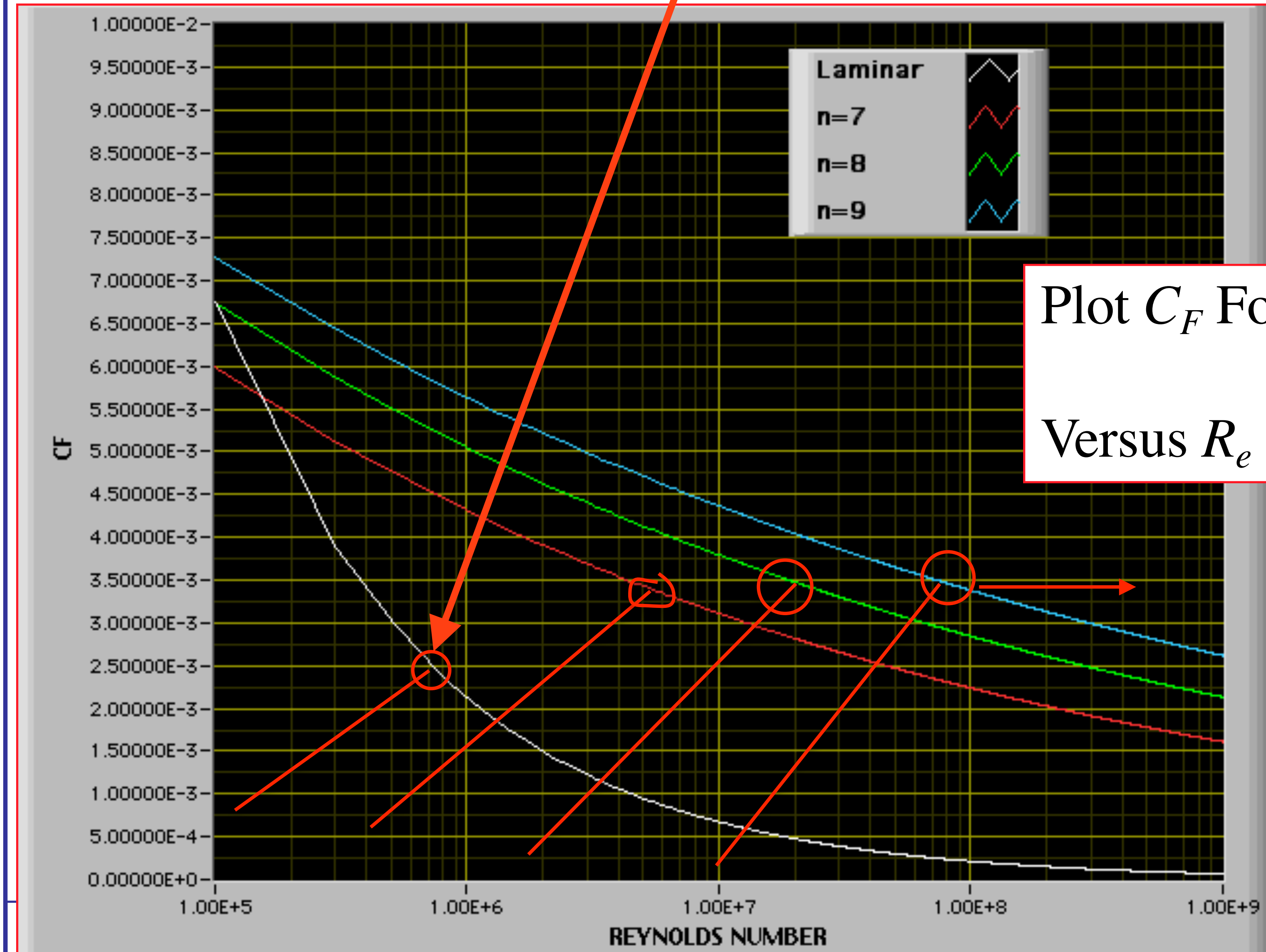
$$\mathrm{R_e} = \frac{\rho V_e c}{\mu}$$

Not! Independent of size

Independent of size

$$C_{D_{wave}} = \frac{D_{rag}}{bc\bar{q}} = \frac{\left[p_2 - p_3\right]}{\frac{\gamma}{2} p_\infty M^2}\left(\frac{t}{c}\right) = \frac{p_2 - p_3}{\frac{\gamma}{2} p_\infty M_\infty^2} \cdot \tan\delta$$

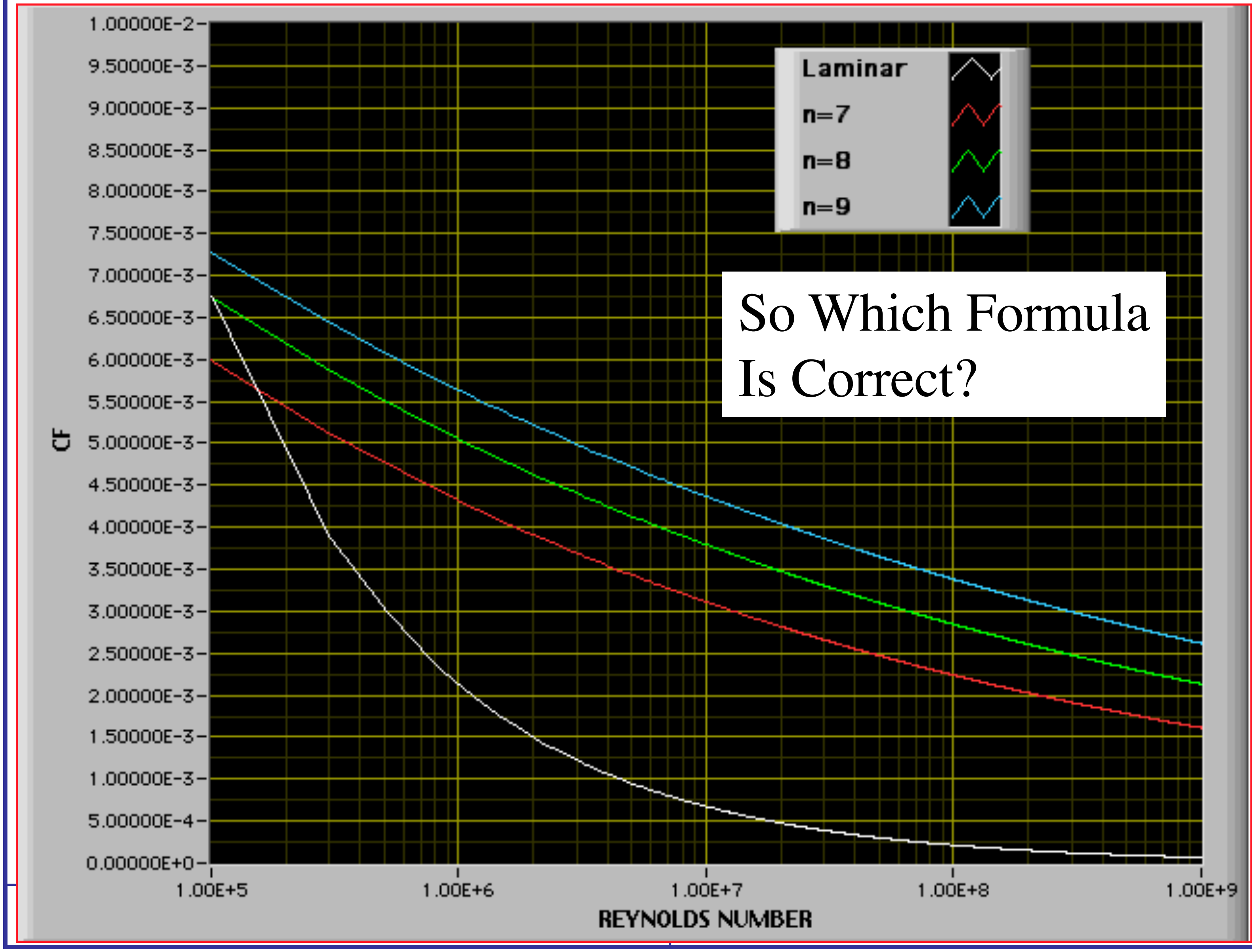
Compare Laminar and Turbulent Skin Friction (cont'd)



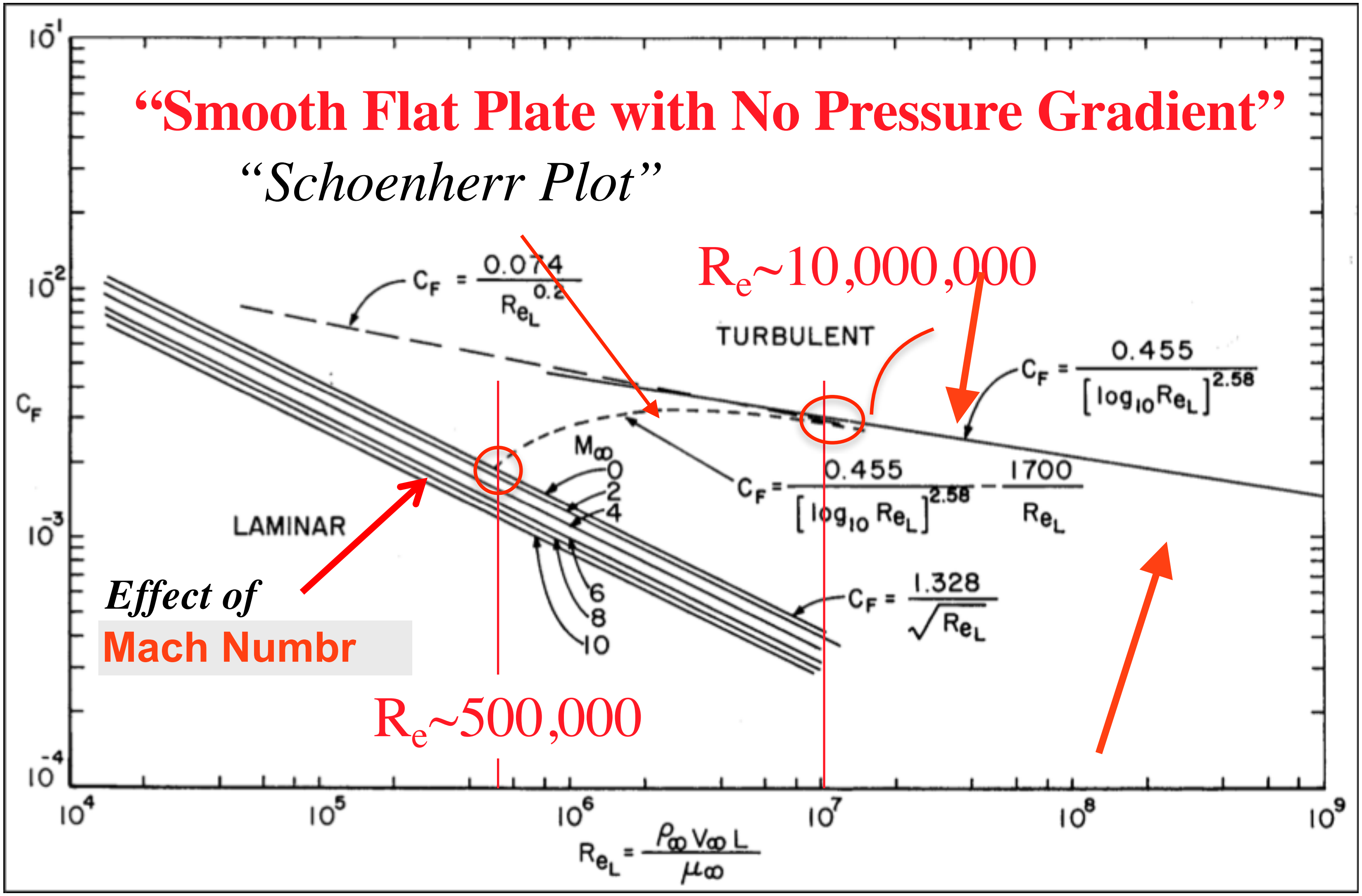
Plot C_F Formulae

Versus R_e

Compare Laminar and Turbulent Skin Friction (cont'd)

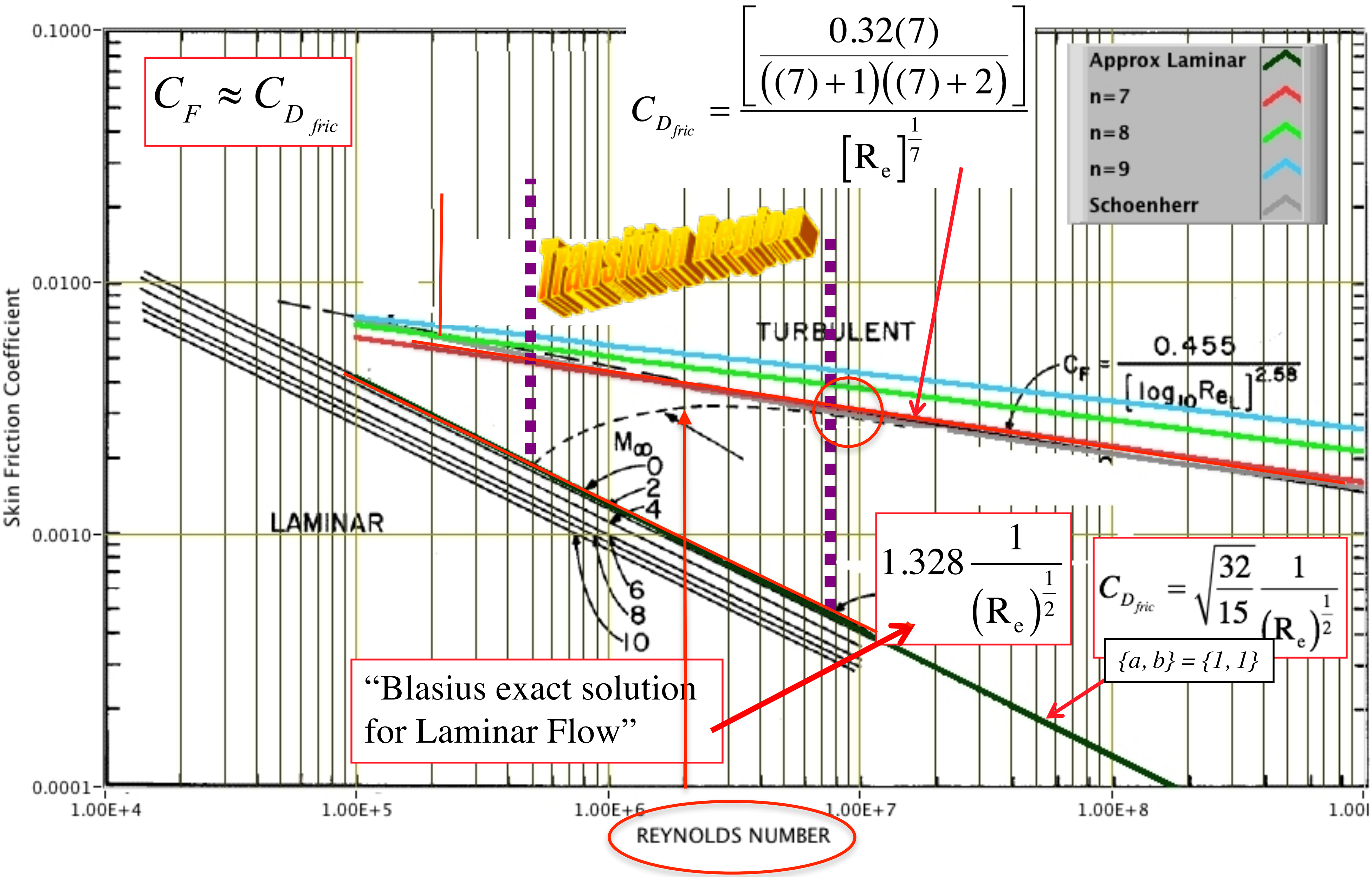


Classical Skin Friction Correlations

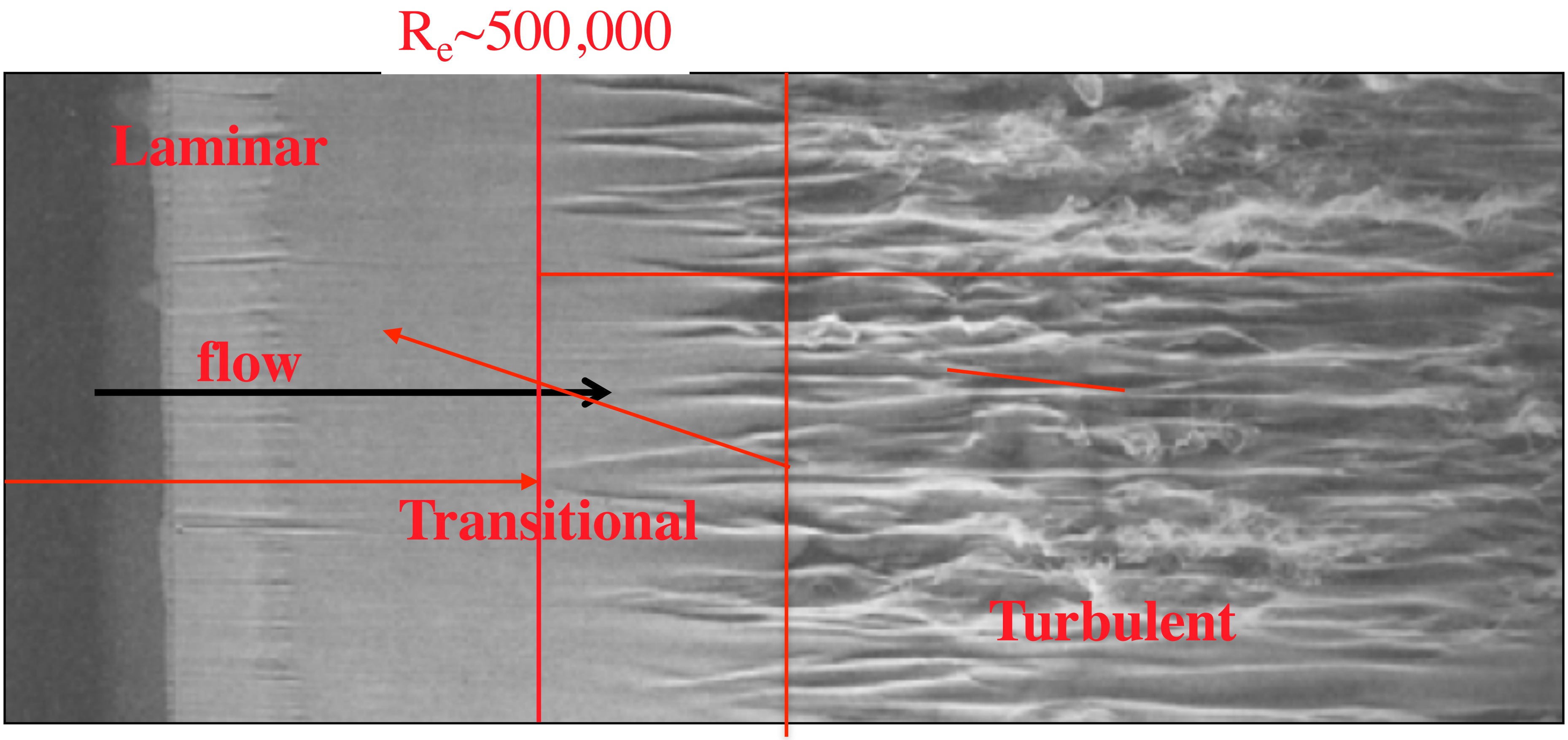


Prof. K.E. Schoenherr, Dean, School of Engineering, Notre Dame University, South Bend, In (1945)

Plot the C_F Laws

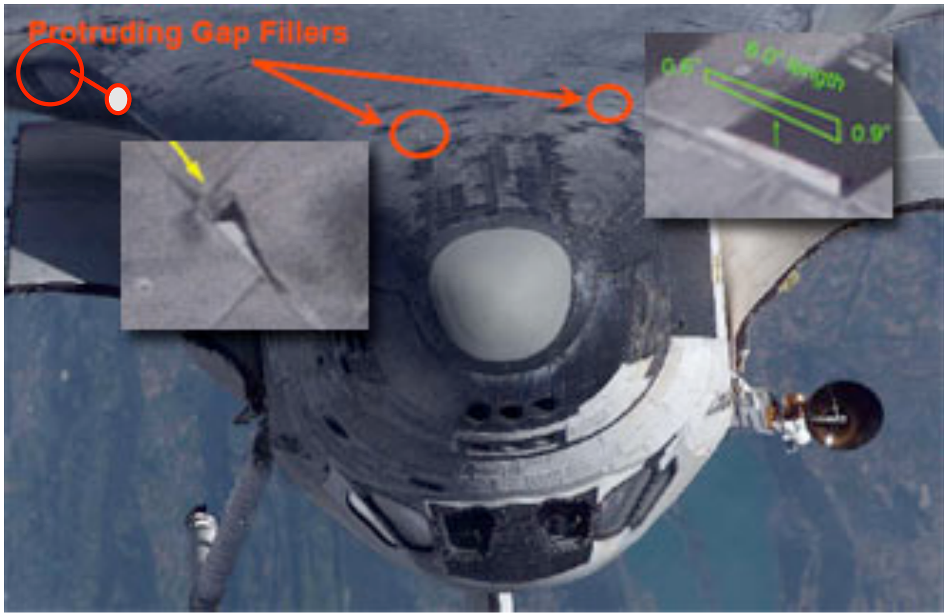


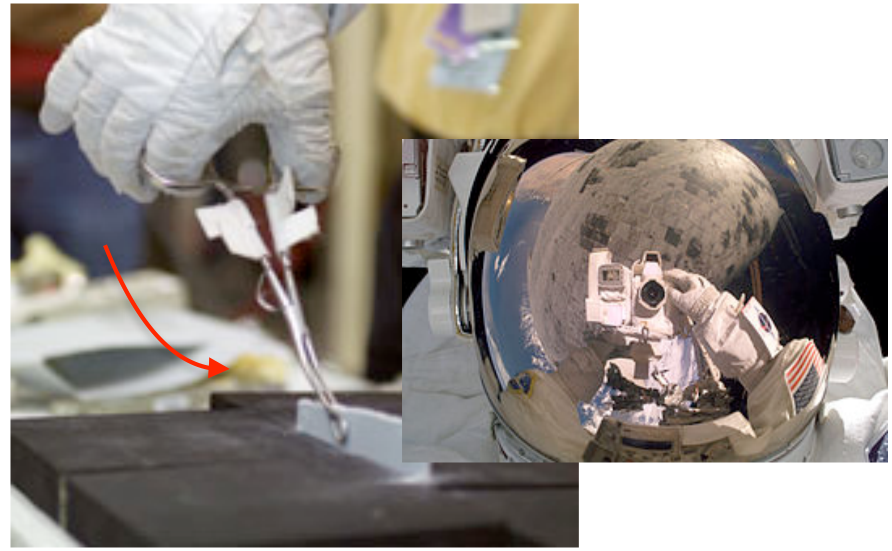
Boundary Layer Transition



- Laminar --> *orderly*
- Turbulent --> *chaotic*

Bypass Vs. Natural Transition?

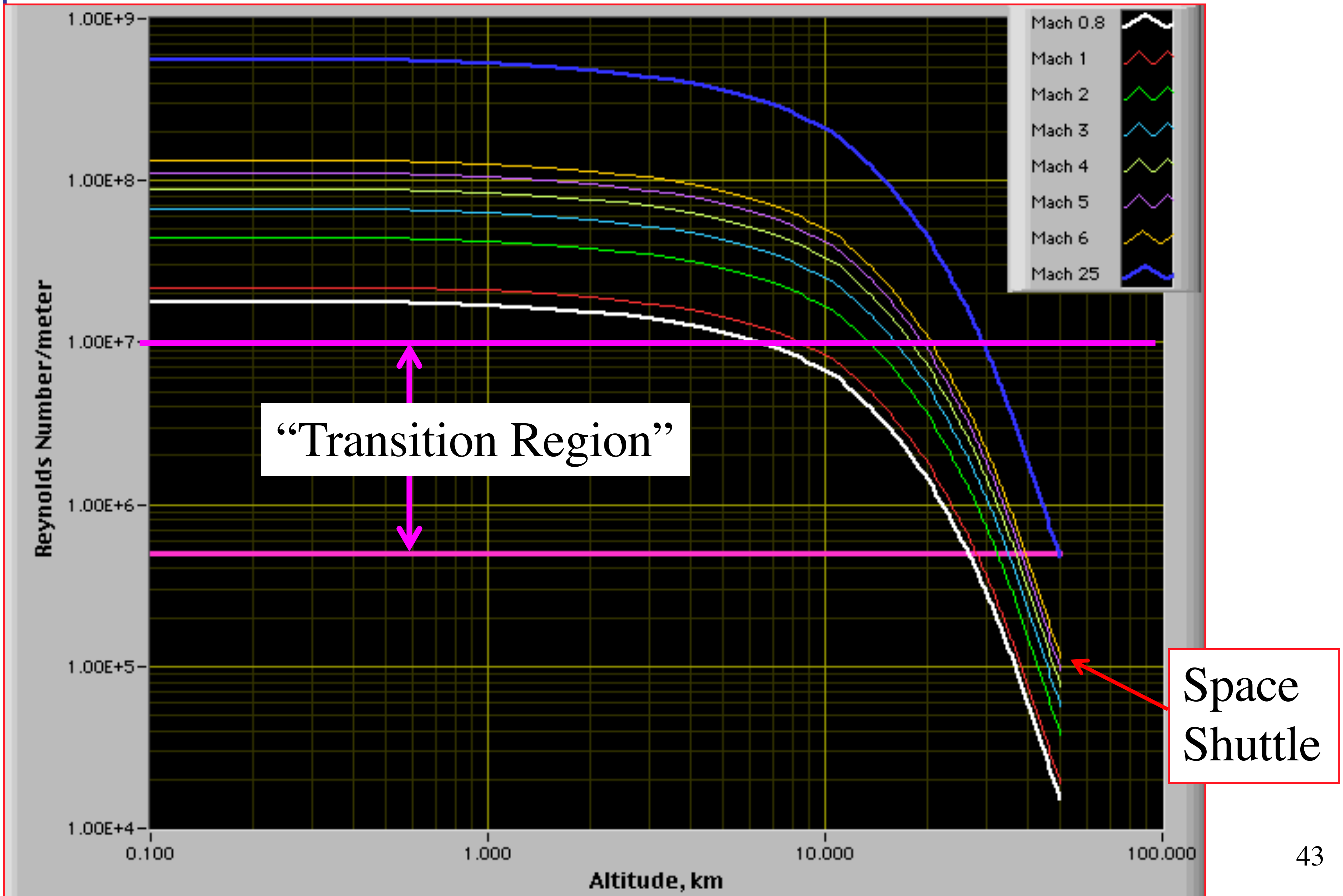




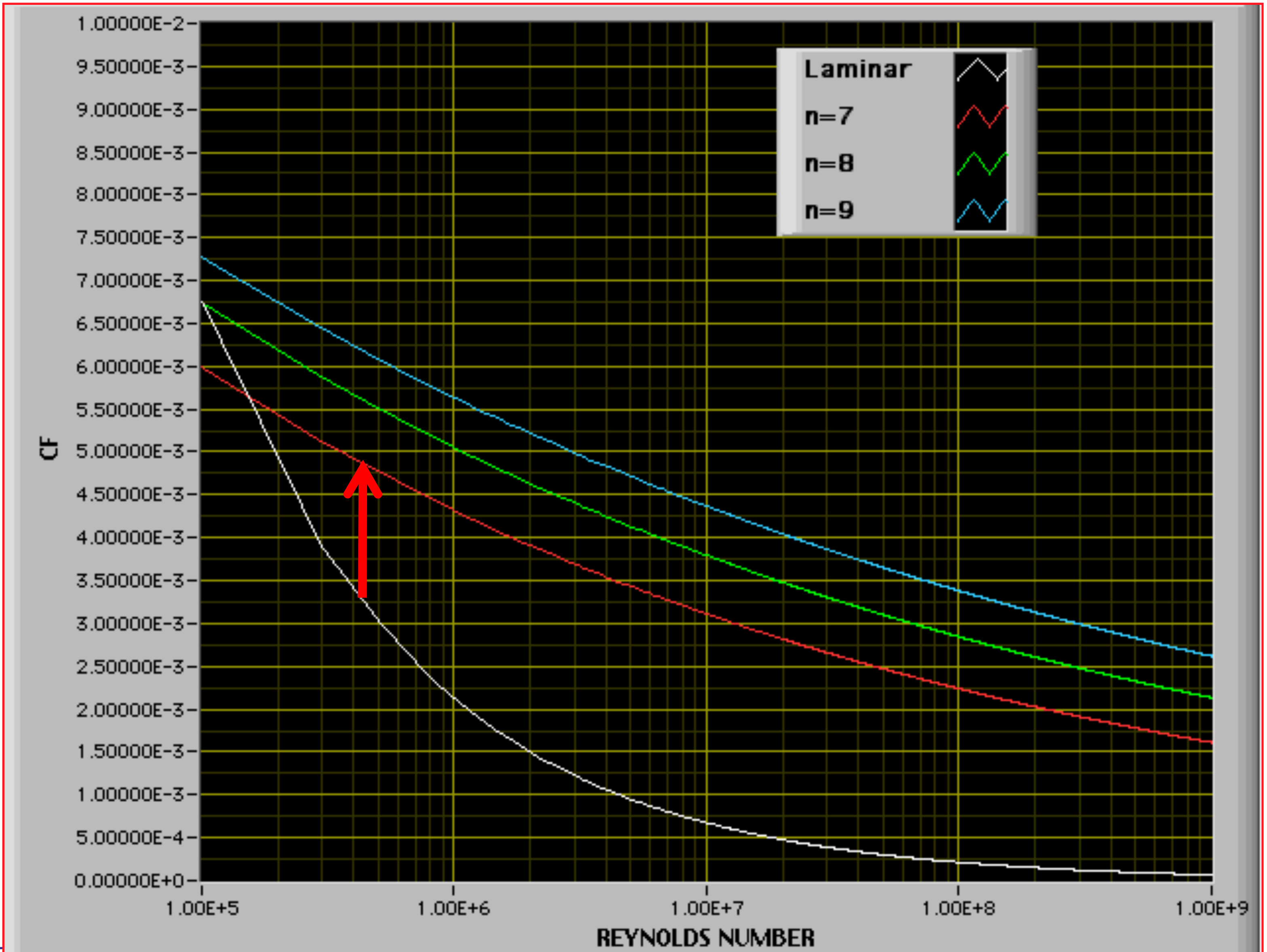
- *Why did NASA Risk a Space Walk on STS-114 with Steve Robinson to remove Shuttle Tile Gap Filler?*

- *To Avoid Premature Bypass Boundary Layer Transition! During Reentry*

Bypass Vs. Natural Transition? (2)



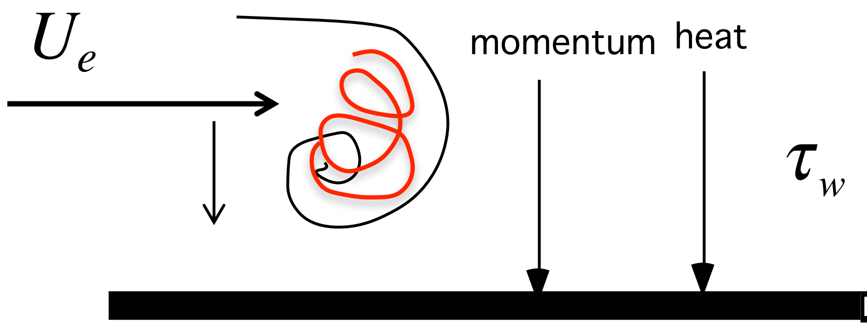
Bypass Vs. Natural Transition? (3)

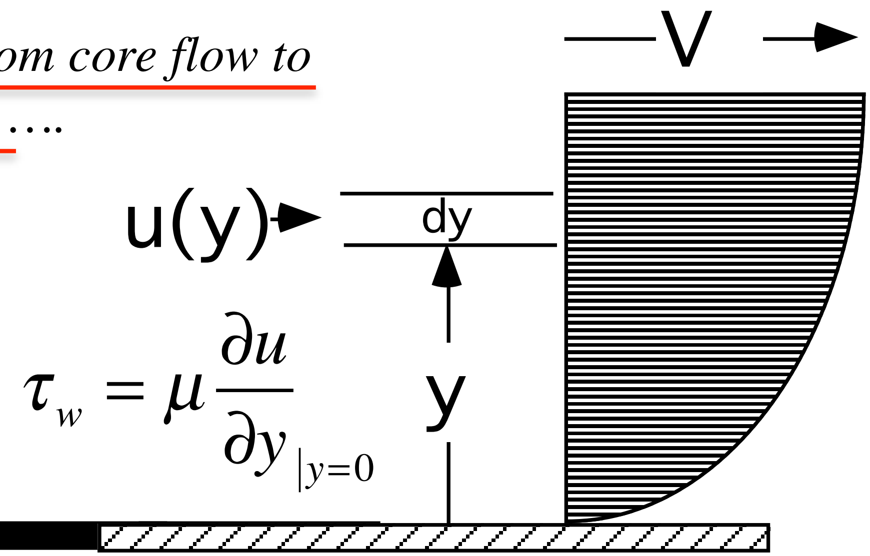


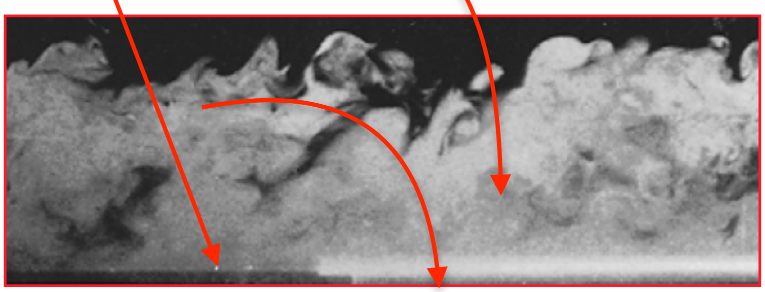
Bypass Vs. Natural Transition? (3)

- *Reynold's Analogy … correlation of heat transfer to skin friction*

- *Turbulent eddy transports momentum from core flow to fluid near wall .. It also transports heat ….*

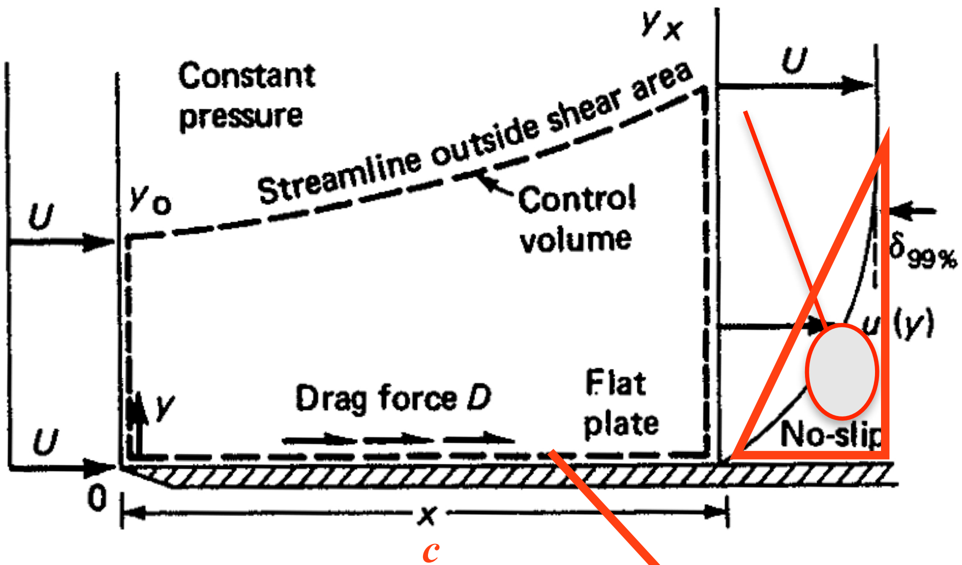






- *Bypass transition dramatically Increases heat transfer to the skin surface!*

Skin Friction Drag on Transitional Boundary Layer



• Accumulated Skin Drag Proportional to Boundary Layer Thickness at Location *X*

	Laminar $R_e \le 5\times10^5$	*Turbulent*$_{(n=7)}$ $R_e \ge 10^7 < 10^8$
δ	$\frac{4.98c}{(R_e)^{\frac{1}{2}}} \rightarrow$	$\frac{0.16c}{[R_e]^{\frac{1}{7}}}$
$C_{D\ friction}$	$\frac{1.328}{(R_e)^{\frac{1}{2}}}$	$\frac{7}{225[R_e]^{\frac{1}{7}}}$

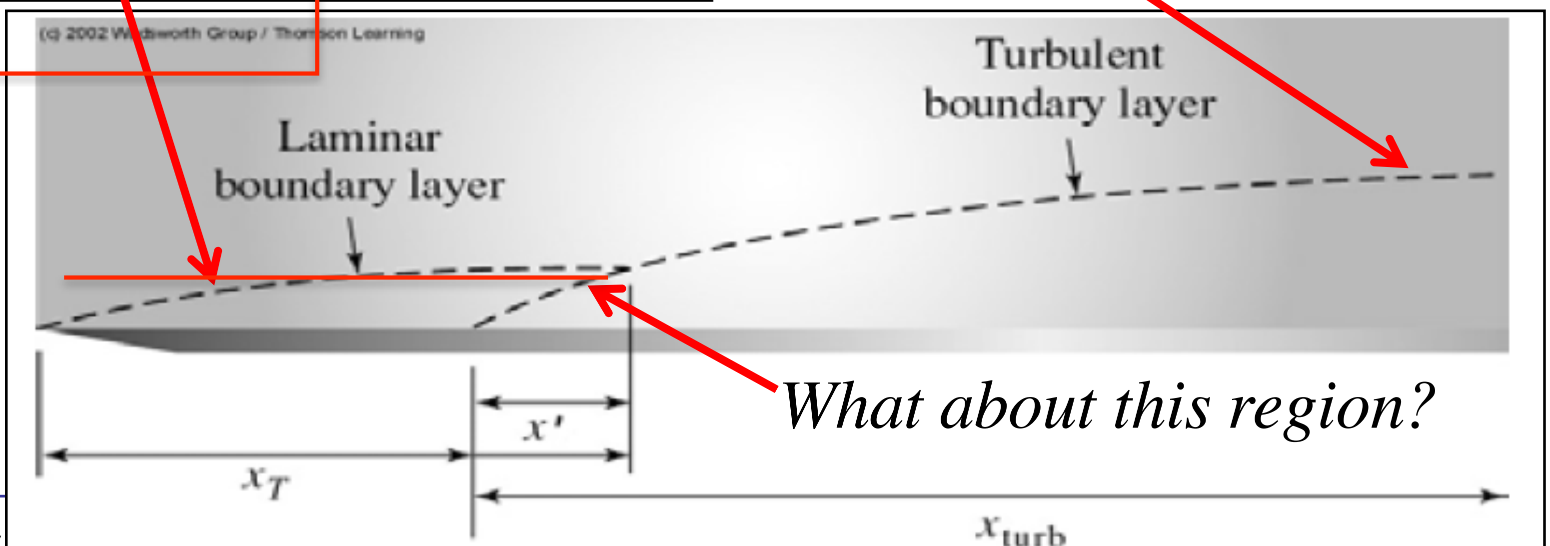
Skin Friction Drag on Transitional Boundary Layer (2)

	Laminar $R_e \leq 5 \times 10^5$	*Turbulent*$_{(n=7)}$ $R_e \geq 10^7 < 10^8$
δ	$\dfrac{4.98c}{(R_e)^{\frac{1}{2}}} \rightarrow$	$\dfrac{0.16c}{[R_e]^{\frac{1}{7}}}$
$C_{D_{friction}}$	$\dfrac{1.328}{(R_e)^{\frac{1}{2}}}$	$\dfrac{-\ 7}{225[R_e]^{\frac{1}{7}}}$

Laminar: $$C_{D_{fric}} = \frac{4}{15}\frac{\delta}{c}$$

Turbulent: $R_e \geq 10^7 < 10^8$ $$C_{D_{fric}} = \frac{\delta}{c}\left[\frac{7}{36}\right]$$

- Empirical turbulent correlations include effects of laminar region on accumulated drag, no need for correction



Skin Friction Drag on Transitional Boundary Layer (3)

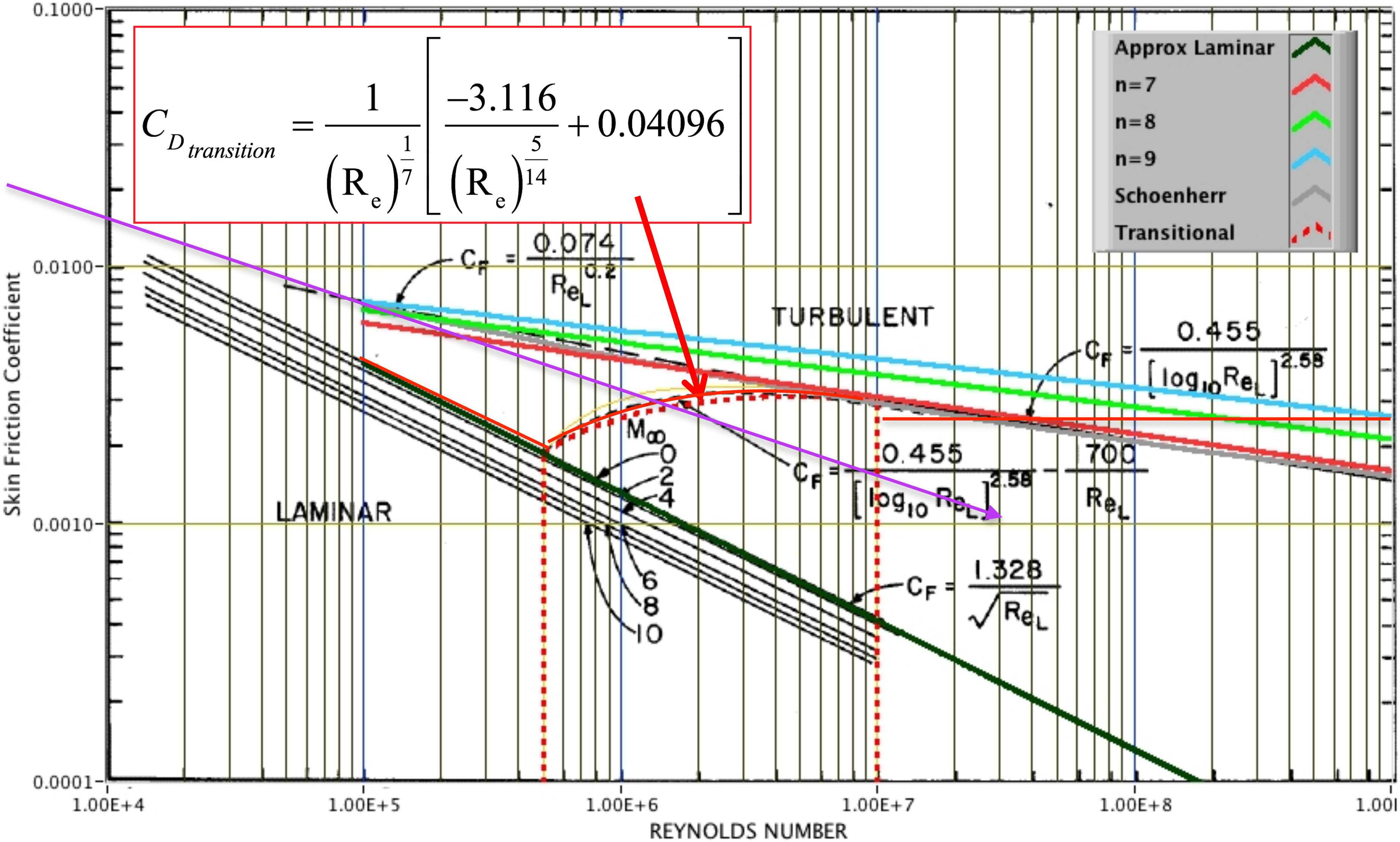
- "Splice" two curves together for transition region

$$C_{D_{transition}} = \frac{a}{\left(R_e\right)^{\frac{1}{2}}} - \frac{b}{\left(R_e\right)^{\frac{1}{7}}} \rightarrow \begin{bmatrix} \dfrac{1}{\left(5\times10^5\right)^{\frac{1}{2}}} & \dfrac{1}{\left(5\times10^5\right)^{\frac{1}{7}}} \\ \dfrac{1}{\left(1\times10^7\right)^{\frac{1}{2}}} & \dfrac{1}{\left(1\times10^7\right)^{\frac{1}{7}}} \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} \dfrac{1.328}{\left(5\times10^5\right)^{\frac{1}{2}}} \\ \dfrac{7}{225\left[1\times10^7\right]^{\frac{1}{7}}} \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} -3.116 \\ -0.4096 \end{bmatrix} \rightarrow C_{D_{transition}} = \frac{-3.116}{\left(R_e\right)^{\frac{1}{2}}} - \frac{-0.04096}{\left(R_e\right)^{\frac{1}{7}}} = \frac{1}{\left(R_e\right)^{\frac{1}{7}}} \left[\frac{-3.116}{\left(R_e\right)^{\frac{5}{14}}} + 0.04096\right]$$

Skin Friction Laws Revisited

$$C_{D\,transition} = \frac{1}{\left(R_e\right)^{\frac{1}{7}}}\left[\frac{-3.116}{\left(R_e\right)^{\frac{5}{14}}} + 0.04096\right]$$



Collected Simple Skin Friction Model *(1)*

- So For Our Purposes … we'll use the "1/nth power" Boundary layer law … and the *Exact Laminar Solution + Transitional Splice*

$R_e < 500{,}000$

$$\rightarrow\rightarrow\rightarrow C_{D_{fric}} = 1.328\frac{1}{\left(R_e\right)^{\frac{1}{2}}}$$

laminar

- See appendix on Section 8 web link for exact solution

Exact Blasius Solution

$500{,}000 \leq R_e < 10{,}000{,}000$

$$C_{D\,fric} = \frac{1}{\left(R_e\right)^{\frac{1}{7}}}\left[\frac{-3.116}{\left(R_e\right)^{\frac{5}{14}}} + 0.04096\right]$$

transitional

"Splice" of Laminar Solution and 1/7th power law Model

Collected Simple Skin Friction Model *(2)*

$10^7 < R_e < 10^8$ *-> n=7*

$$\rightarrow\rightarrow\rightarrow C_{D_{fric}} = \frac{\left[\frac{0.32\cdot 7}{(7+1)(7+2)}\right]}{\left[R_e\right]^{\frac{1}{7}}} = \frac{7}{225\left[R_e\right]^{\frac{1}{7}}}$$

turbulent

"transitions" ~ continuous

… for most turbulent flows 1/7th power law is sufficiently accurate

$10^8 < R_e < 10^9$ *-> $n = Log_{10}[R_e]$ -1*

$$\rightarrow\rightarrow\rightarrow C_{D_{fric}} = \frac{\left[\frac{0.32\cdot n}{(n+1)(n+2)}\right]}{\left[R_e\right]^{\frac{1}{n}}} \rightarrow n = Log_{10}\left[R_e\right]-1$$

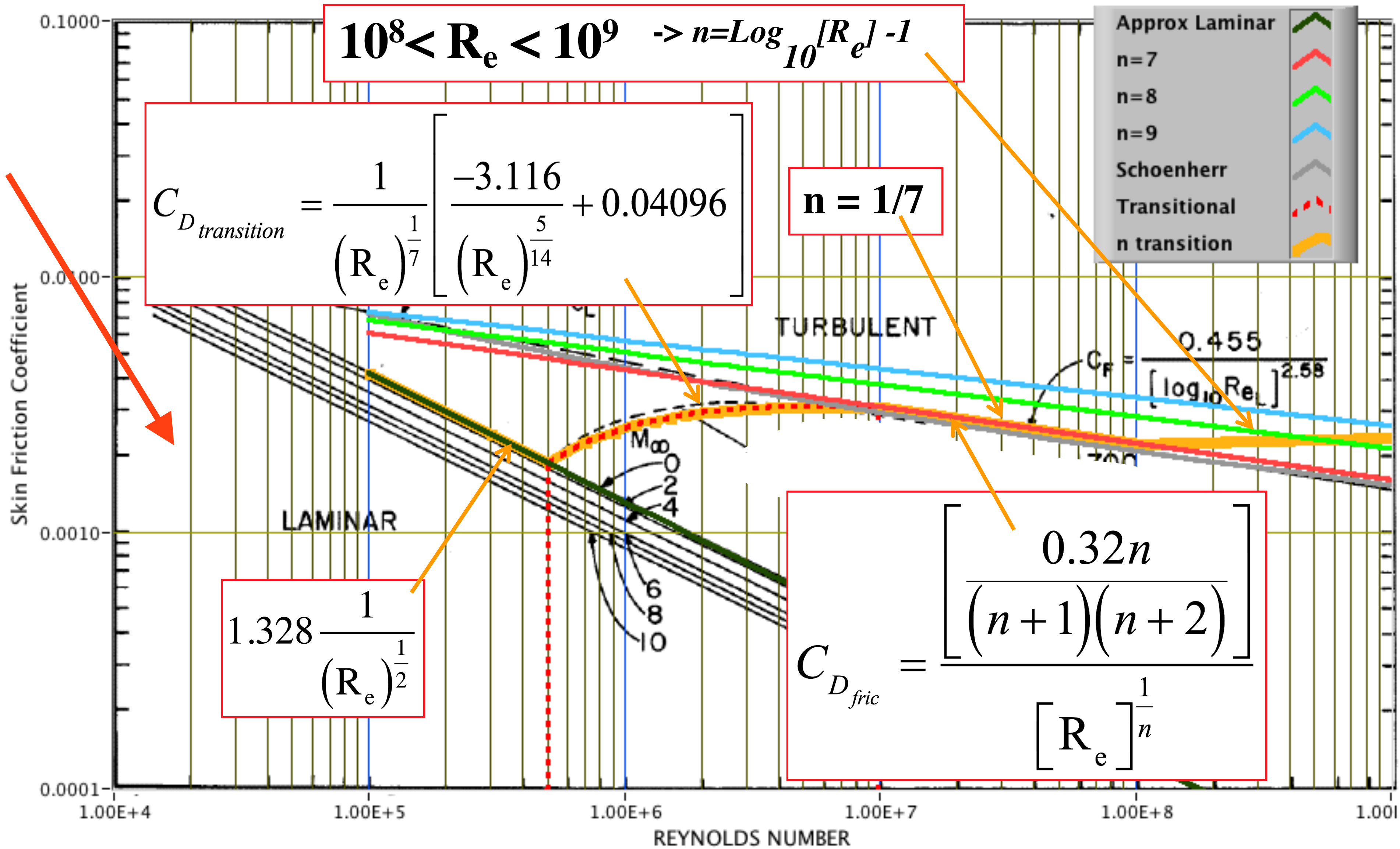
turbulent

Skin Friction Laws, The Full Monty

$10^8 < R_e < 10^9$ -> $n = Log_{10}[R_e] - 1$

$$C_{D_{transition}} = \frac{1}{(R_e)^{\frac{1}{7}}}\left[\frac{-3.116}{(R_e)^{\frac{5}{14}}} + 0.04096\right]$$

n = 1/7



$$1.328\frac{1}{(R_e)^{\frac{1}{2}}}$$

$$C_{D_{fric}} = \frac{\left[\frac{0.32n}{(n+1)(n+2)}\right]}{[R_e]^{\frac{1}{n}}}$$

How About Three Dimensional Configurations?

NACA

REPORT 1161

AVERAGE SKIN-FRICTION DRAG COEFFICIENTS FROM TANK TESTS OF A PARABOLIC BODY OF REVOLUTION (NACA RM-10)[1]

By Elmo J. Mottard and J. Dan Loposer

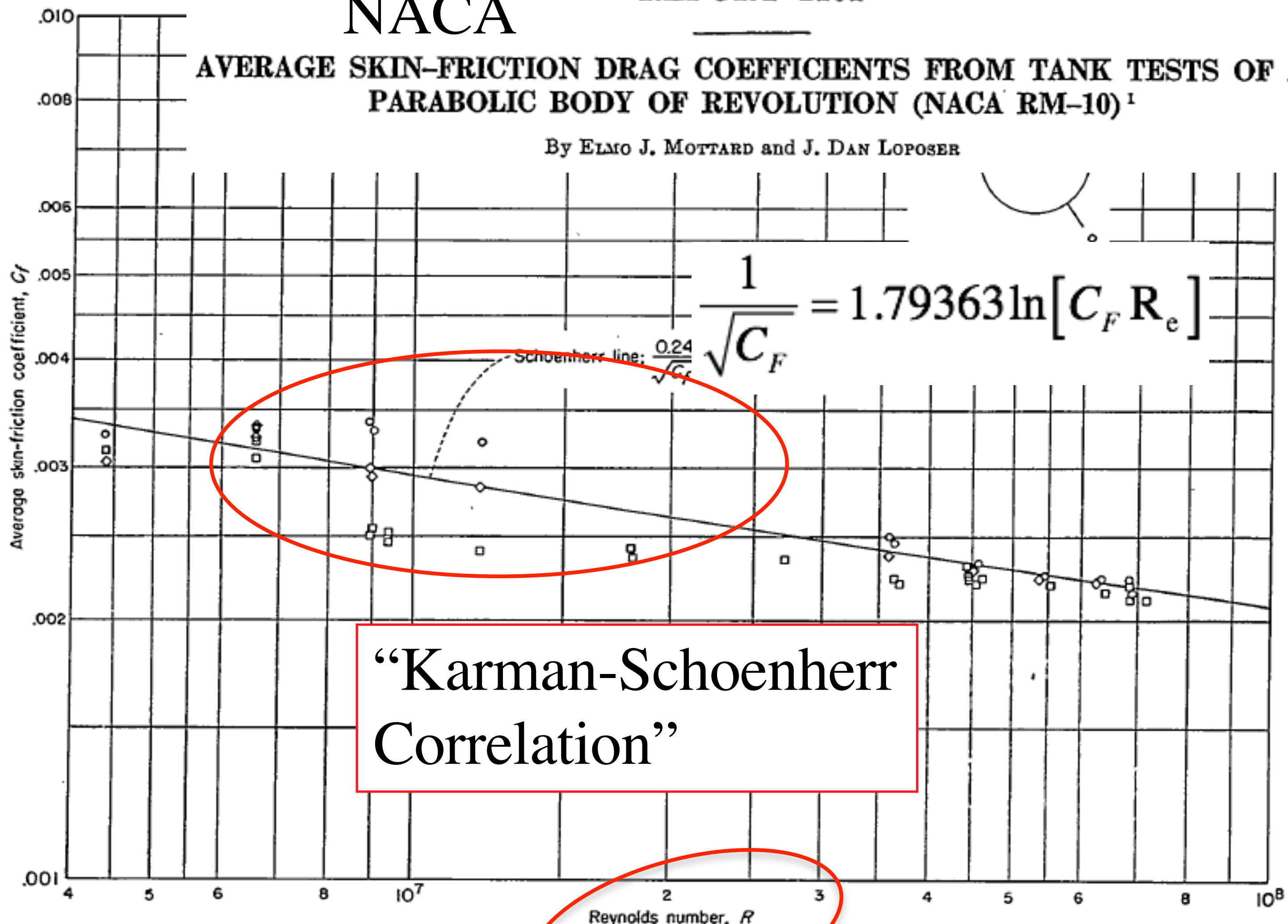


Figure 6.—Variation of average skin-friction coefficient, at the three measurement stations, with Reynolds number.

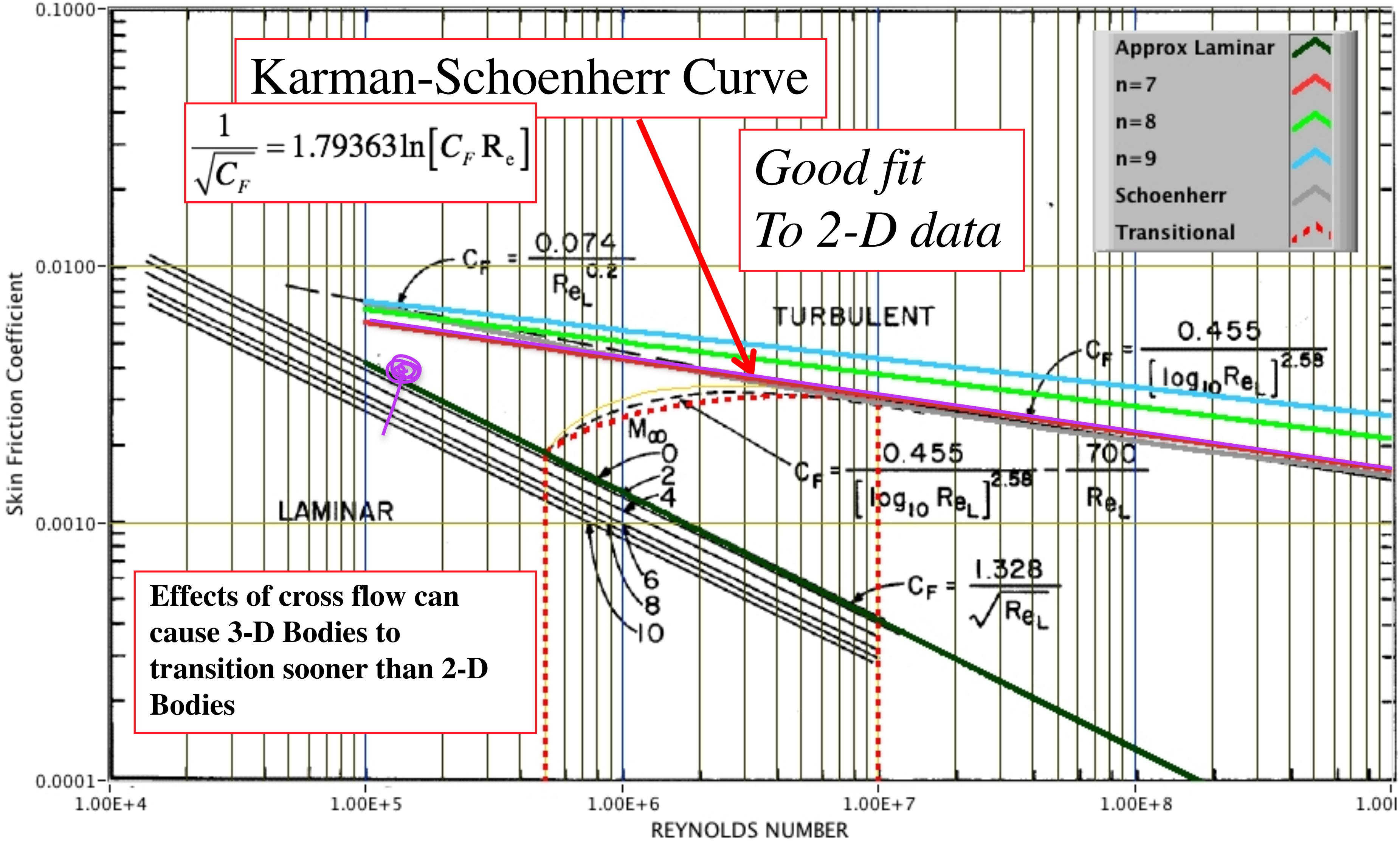
Skin Friction Laws Revisited

Karman-Schoenherr Curve

$$\frac{1}{\sqrt{C_F}} = 1.79363 \ln\left[C_F \mathrm{R_e}\right]$$

Good fit
To 2-D data

Effects of cross flow can cause 3-D Bodies to transition sooner than 2-D Bodies



Questions?