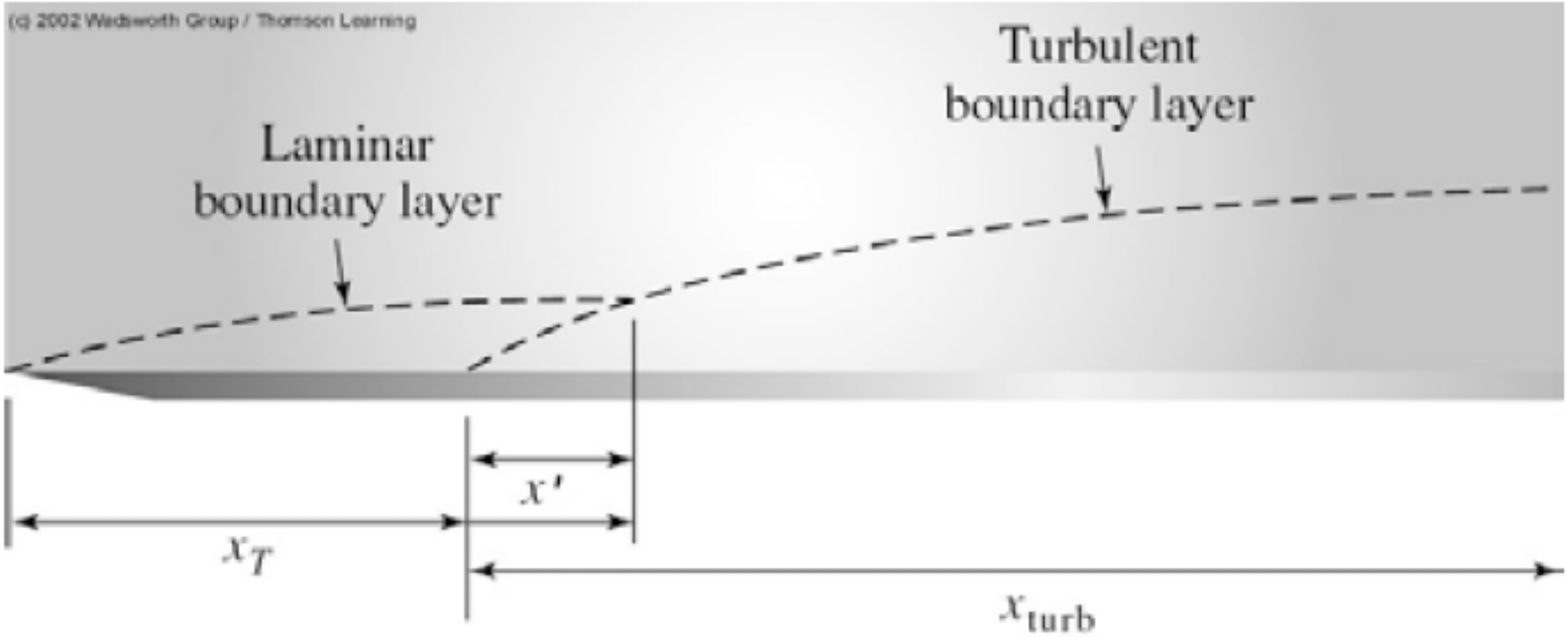
Section 8, Lecture 2

Supersonic Wings with Skin Friction: *Accounting for Compressibility*



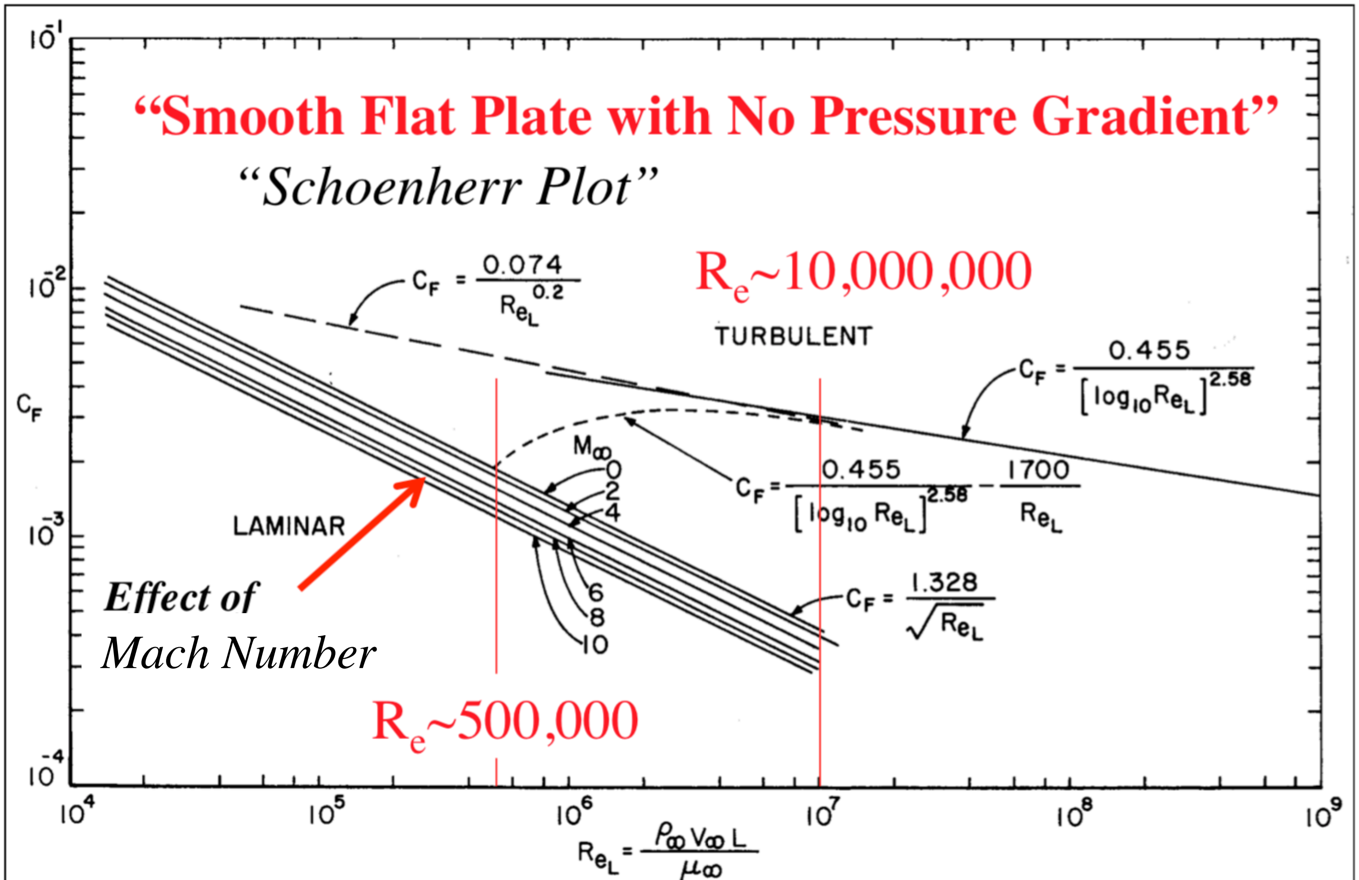
- Not in Anderson

Compressibility Effects on Skin Friction Model

- We derived previous model assuming incompressible flow
- How about the effects of compressibility?
- No Simple Clean Analytical Solution for Turbulent Flow
- One *commonly used method* … evaluate R_e based on averaged conditions within Boundary layer

Effect of Mach Number on Reynolds Number

$$R_e = \frac{\rho \cdot V \cdot c}{\mu} = \left(\frac{p}{R_g \cdot T}\right) \cdot \left(\frac{c \cdot V_\infty}{\mu_M}\right) = \sqrt{\frac{\gamma}{R_g \cdot T}} \cdot \left(\frac{p \cdot c}{\mu}\right) \cdot M$$



- *But this analysis does not take into account the effect of Boundary Layer Heating*

Effect of Compressible Boundary Layer Heating on Reynolds Number

- *Incompressible Boundary Layer*

$$R_{e_\infty} = \frac{\rho_\infty \cdot c \cdot V_\infty}{\mu_\infty}$$

- *Compressible Boundary Layer, Heats Up Density Drops, Viscosity Increases*

$$R_{e_M} = \frac{\rho_M \cdot c \cdot V_\infty}{\mu_M}$$

Effect of Compressible Boundary Layer Heating on Reynolds Number (2)

- *Compressible Boundary Layer*

$$R_{e_M} = \frac{\rho_M \cdot V_\infty \cdot c}{\mu_M} = \left(\frac{p_\infty}{R_g \cdot T_M}\right) \cdot \left(\frac{c \cdot V_\infty}{\mu_M}\right) = \left(\frac{p_\infty}{R_g}\right) \cdot \left(c \cdot V_\infty\right) \cdot \left(\frac{1}{T_M \cdot \mu_M}\right)$$

- From Sutherland's Formula --- μ as a function of T_M

$$\mu_M = \mu_\infty \cdot \left(\frac{T_M}{T_\infty}\right)^{3/2} \cdot \left(\frac{T_\infty + C_s}{T_M + C_s}\right)$$

Effect of Compressible Boundary Layer Heating on Reynolds Number (3)

- *Substitute into Compressible Reynold's Number*

$$R_{e_M} = \left(\frac{p_\infty}{R_g}\right)\cdot\left(\frac{c\cdot V_\infty}{\mu_\infty}\right)\cdot\left(\frac{1}{T_M\left(\frac{T_M}{T_\infty}\right)^{3/2}\cdot\left(\frac{T_\infty + C_s}{T_M + C_s}\right)}\right) = \left(\frac{p_\infty}{R_g\cdot T_\infty}\right)\cdot\left(\frac{c\cdot V_\infty}{\mu_\infty}\right)\cdot\left(\left(\frac{T_\infty}{T_M}\right)\frac{1}{\left(\frac{T_M}{T_\infty}\right)^{3/2}\cdot\left(\frac{T_\infty + C_s}{T_M + C_s}\right)}\right)$$

- *Collecting Terms*

$$R_{e_M} = \left(\frac{p_\infty}{R_g\cdot T_\infty}\right)\cdot\left(\frac{c\cdot V_\infty}{\mu_\infty}\right)\cdot\left(\left(\frac{T_\infty}{T_M}\right)\frac{1}{\left(\frac{T_M}{T_\infty}\right)^{3/2}\cdot\left(\frac{T_\infty + C_s}{T_M + C_s}\right)}\right) = \left(\frac{\rho_\infty\cdot c\cdot V_\infty}{\mu_\infty}\right)\cdot\left(\frac{T_\infty}{T_M}\right)^{5/2}\cdot\left(\frac{T_M + C_s}{T_\infty + C_s}\right) = \left(R_{e_\infty}\right)\cdot\left(\frac{T_\infty}{T_M}\right)^{5/2}\cdot\left(\frac{T_M + C_s}{T_\infty + C_s}\right)$$

Effect of Compressible Boundary Layer Heating on Reynolds Number (4)

- *Substituting*

$$R_{e_M} = \frac{\rho_M \cdot V_\infty \cdot c}{\mu_M}$$

- *Effect of Boundary Layer heating is to Decrease the Effective Reynolds Number from the Freestream Value*

$$\left(\frac{R_{e_M}}{R_{e_\infty}}\right) = \left(\frac{T_\infty}{T_M}\right)^{5/2} \cdot \left(\frac{T_M + C_s}{T_\infty + C_s}\right)$$

Compressibility Effects on Skin Friction Model (cont'd)

- *What is the effect on Skin Friction,* $T_M \sim T_{avg}$ *(average Boundary layer Temperature)*

$$\left[C_{D_{fric}}\right]_{compressible} = \frac{7}{225\left[\mathrm{R_e}\right]^{\frac{1}{7}}} = \frac{7}{225\left[\frac{\rho V c}{\mu}\right]^{\frac{1}{7}}} = \frac{7}{225\left[\frac{\rho(T_{avg}) c V_\infty}{\mu(T_{avg})}\right]^{\frac{1}{7}}}$$

$$\rho(T_{avg}) = \rho_\infty \frac{T_\infty}{T_{avg}} \rightarrow Gas...Law...no...lateral...pressure...gradient$$

$$\rho_{Tavg} = \frac{p_{Tavg}}{R_g \cdot T_{avg}}, \quad \rho_\infty = \frac{p_\infty}{R_g \cdot T_\infty} \rightarrow \begin{matrix} p_{Tavg} = p_\infty \\ \text{no Lateral Pressure Gradient} \end{matrix} \rightarrow \frac{\rho_{Tavg}}{\rho_\infty} = \frac{T_{avg}}{T_\infty}$$

Compressibility Effects on Skin Friction Model (cont'd)

- *What is the effect on Skin Friction,* $T_M \sim T_{avg}$ *(average Boundary layer Temperature)*

$$\left[C_{D_{fric}}\right]_{compressible} = \frac{7}{225\left[\mathrm{R_e}\right]^{\frac{1}{7}}} = \frac{7}{225\left[\dfrac{\rho V c}{\mu}\right]^{\frac{1}{7}}} = \frac{7}{225\left[\dfrac{\rho(T_{avg}) c V_\infty}{\mu(T_{avg})}\right]^{\frac{1}{7}}}$$

- From Sutherland's Formula --- μ as a function of T_{avg}

$$\rightarrow \mu(T_{avg}) = \mu(T_\infty)\left(\frac{T_{avg}}{T_\infty}\right)^{3/2}\left(\frac{T_\infty + C_s}{T_{avg} + C_s}\right)$$

$$\rightarrow \frac{\rho_{Tavg}}{\rho_\infty} = \frac{T_{avg}}{T_\infty}$$

Substitute and Collect terms

Compressibility Effects on Skin Friction Model (revisited)

$$\left[C_{D_{fric}}\right]_{compressible} = \frac{7}{225\left[\dfrac{\rho_\infty \dfrac{T_\infty}{T_{avg}} c V_\infty}{\mu(T_\infty)\left(\dfrac{T_{avg}}{T_\infty}\right)^{3/2}\left(\dfrac{T_\infty + C_s}{T_{avg} + C_s}\right)}\right]^{\frac{1}{7}}} =$$

$$\frac{7}{225\left[\dfrac{\rho_\infty c V_\infty}{\mu(T_\infty)}\left(\dfrac{T_\infty}{T_{avg}}\right)^{5/2}\left(\dfrac{T_{avg} + C_s}{T_\infty + C_s}\right)\right]^{\frac{1}{7}}} = \frac{7}{225\left[\dfrac{\rho_\infty c V_\infty}{\mu(T_\infty)}\right]^{\frac{1}{7}}\left[\left(\dfrac{T_\infty}{T_{avg}}\right)^{5/2}\left(\dfrac{T_{avg} + C_s}{T_\infty + C_s}\right)\right]^{\frac{1}{7}}}$$

$$\rightarrow\rightarrow \left[C_{D_{fric}}\right]_{compressible} = \frac{\left[C_{D_{fric}}\right]_{incompressible}}{\left[\left(\dfrac{T_\infty}{T_{avg}}\right)^{5/2}\left(\dfrac{T_{avg} + C_s}{T_\infty + C_s}\right)\right]^{\frac{1}{7}}}$$

- Plugging into expression for turbulent skin friction

Compressibility Effects on Skin Friction Model (cont'd)

- What is "T_{avg}" in across the depth of boundary layer?

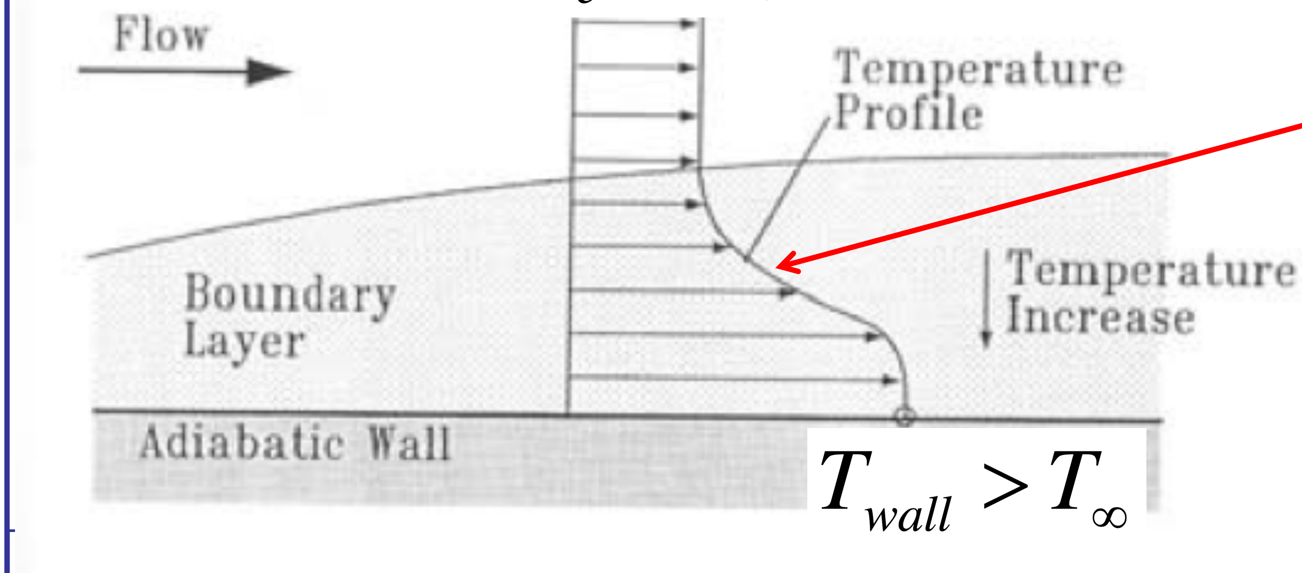
$$\left[C_{D_{fric}}\right]_{compressible} = \frac{\left[C_{D_{fric}}\right]_{incompressible}}{\left[\left(\frac{T_\infty}{T_{avg}}\right)^{5/2}\left(\frac{T_{avg}+C_s}{T_\infty+C_s}\right)\right]^{\frac{1}{7}}}$$

Turbulent Exponent

Compressibility Effects on Skin Friction Model (2)

- *Within boundary layer, gas in contact with the surface is brought to rest as a result of viscosity.*
- *Decrease in velocity results in rise in temperature.*
- *For high speed flows, temperature rise is quite large.*
- *Referred to as the "aerodynamic heating" of a surface.*

$$T_e \approx T_\infty$$



$$T_{wall} > T_\infty$$

what is "average" temperature within boundary layer "slice"

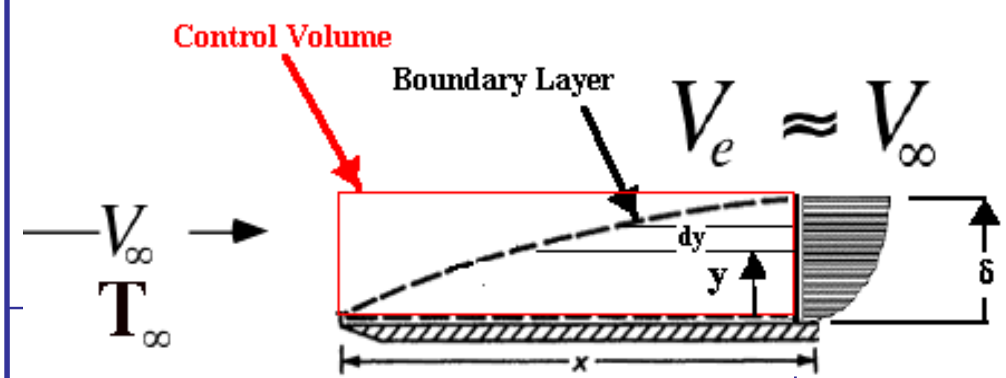
Compressibility Effects on Skin Friction Model (3)

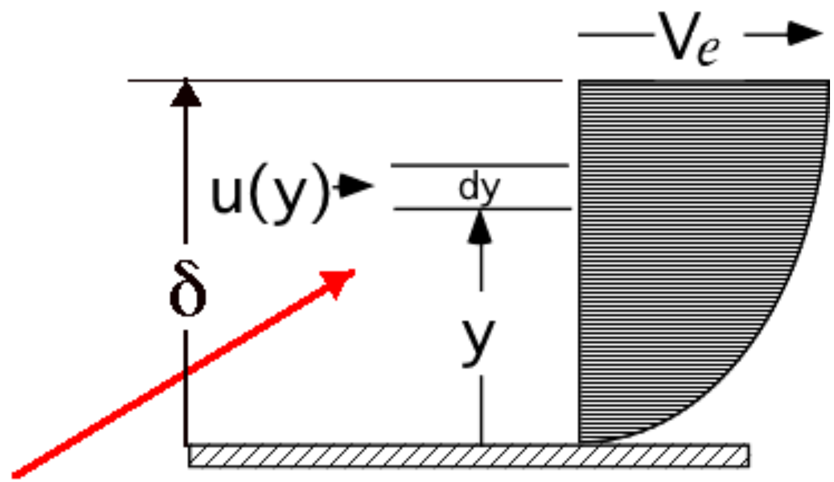
- .. Look at small segment of boundary layer, *dy*, Flow specific enthalpy entering control volume within *dy* segment is

$$h_{0_\infty} = c_p \cdot T_\infty + \frac{V_\infty^2}{2}$$

- Specific enthalpy of flow with boundary layer segment *dy* …

$$h_{0_y} = c_p \cdot T_y + \frac{V_y^2}{2}$$



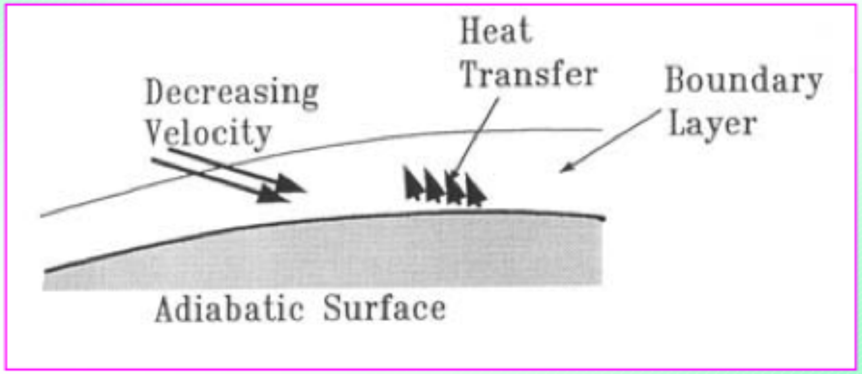


Compressibility Effects on Skin Friction Model (4)

- For completely adiabatic boundary layer flow ..

$$h_{0_\infty} = c_{p_\infty} \cdot T_\infty + \frac{V_\infty^2}{2} = c_{p_y} \cdot T_y + \frac{V_y^2}{2} = h_{0_y}$$

- However, for real high-speed boundary layer flows the actual process between freestream and *x* is not adiabatic.
- *"Dissipation" causes heat transfer towards the colder gas in the freestream flow.*



Heat Transfer in Boundary Layer Away From the Surface

Compressibility Effects on Skin Friction Model (5)

• Heat transfer to external flow accounted for using a "Fudge factor" → $R_f \equiv \text{"Recovery Factor"}$

• ***Recovery Factor -- fraction of kinetic energy of the freestream fluid recovered as thermal energy within boundary layer .. The rest of the heat is "dissipated" to the external flow field***

$$R_f \equiv \frac{T_{aw} - T_\infty}{T_{0_\infty} - T_\infty} \rightarrow T_{aw} = \text{"adiabatic wall temperature"}$$

$$T_{aw} \rightarrow \left| \begin{array}{l} \text{Temperature of the boundarty layer fluid at} \\ \text{the wall when there is no heat transfer from} \\ \text{the boundary layer to the wall} \end{array} \right.$$

Compressibility Effects on Skin Friction Model (6)

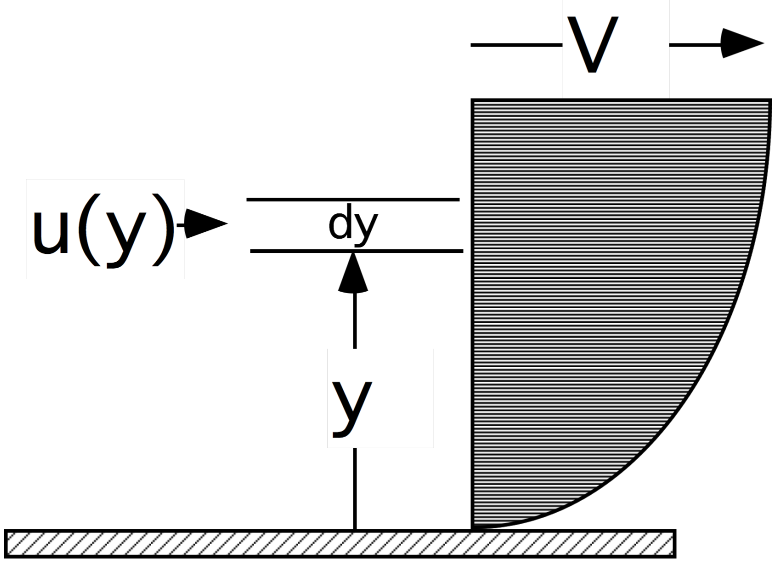
- The modified enthalpy balance across *dy* becomes

-- Assuming constant specific heats

$$T(y) + \frac{u(y)^2}{2C_p} = T_\infty + R_f \frac{V_\infty^2}{2C_p}$$

"fudge factor" … called

recovery factor



- R_f is not a constant … but is a function of the local flow properties

Compressibility Effects on Skin Friction Model (7)

$$T(y) + \frac{u(y)^2}{2C_p} = T_\infty + R_f \frac{V_\infty^2}{2C_p}$$

"fudge factor" … called

recovery factor

R_c = 1 --> Adiabatic flow

R_c = 0 --> Isothermal flow

If flow were completely adiabatic .. At the wall u_{wall} =0 and …. T_{wall} = T_0

$$\left. T(0) + \frac{u(0)^2}{2C_p} = T_\infty + 1 \cdot \frac{V_\infty^2}{2C_p} = T_0 \right|_{R_c=1}$$

Conversely if all energy were dissipated as heat T_{wall} = T_∞

$$\left. T(0) + \frac{u(0)^2}{2C_p} = T_\infty + 0 \cdot \frac{V_\infty^2}{2C_p} = T_\infty \right|_{R_c=0}$$

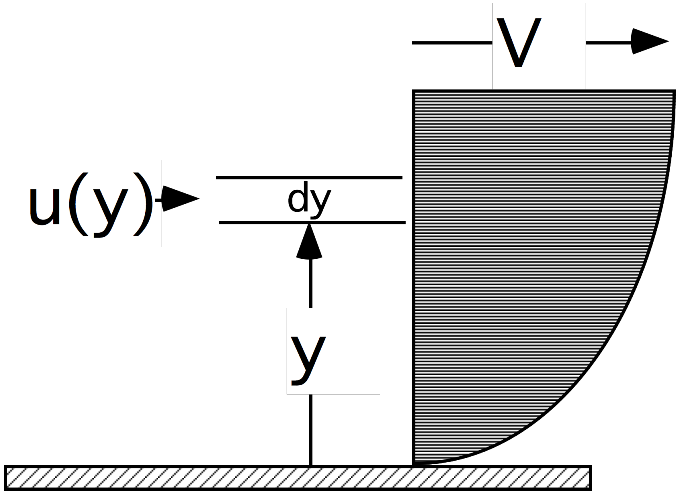
- ***Reality lies in between …. More favored to adiabatic flow***

Compressibility Effects on Skin Friction Model (8)

$$T_{aw} \equiv T_{(y=0)} = T_\infty + R_f \frac{V_\infty^2}{2C_p} =$$

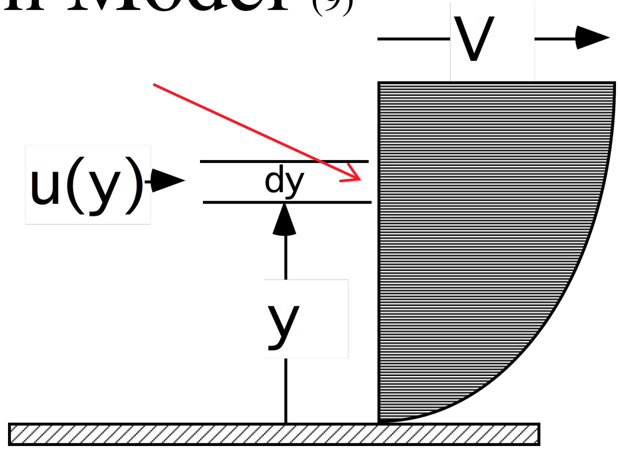
$$T_\infty \left[1 + R_f \frac{\gamma - 1}{2} M_\infty^2 \right] \rightarrow$$

"Adiabatic wall temperature"



- *Temperature at wall in a moving compressible fluid stream* ***when there is no heat transfer between the wall and the fluid stream …***
- ***Sort of a misnomer because boundary layer flow is not adiabatic! … that is why we Need to use the recovery factor … to account for non-adiabaticity***
- "Adiabatic wall" refers to **"no heat transfer"** from Boundary layer to the wall
- Heat transfer to the wall $T(y=0) < T_{aw}$

Compressibility Effects on Skin Friction Model (9)



- Look at B.L. segment *dy*

… Solve for *T(y)*

$$T(y) = T_{aw} - \frac{u(y)^2}{2 \cdot c_p} = T_{aw} - \frac{V_\infty^2}{2 \cdot c_p}\left(\frac{u(y)}{V_\infty}\right)^2$$

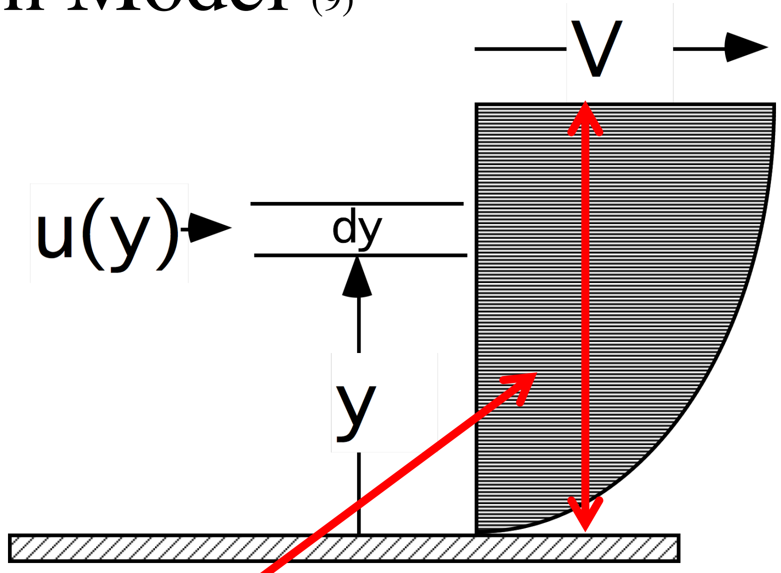
- *Since* $T_{aw} = T_\infty + R_f \frac{V_\infty^2}{2 \cdot c_p}$

$$\rightarrow T(y) = T_\infty + R_f \frac{V_\infty^2}{2 \cdot c_p} - \frac{V_\infty^2}{2 \cdot c_p}\left(\frac{u(y)}{V_\infty}\right)^2 =$$

$$T_\infty + R_f \frac{V_\infty^2}{2 \cdot c_p}\left(1 - \frac{1}{R_f}\left(\frac{u(y)}{V_\infty}\right)^2\right)$$

Compressibility Effects on Skin Friction Model (9)

$$T(y) = T_\infty + R_f \frac{V_\infty^2}{2C_p}\left(1 - \frac{1}{R_f}\left[\frac{u(y)}{V_\infty}\right]^2\right)$$



- Calculate the Average Temperature of boundary layer By Integrating Across Boundary layer And dividing by boundary layer height ***(δ)***

$$T_{avg} \approx \frac{1}{\delta}\int_0^\delta T(y)dy = \frac{1}{\delta}\int_0^\delta T(y)dy = \int_0^1 T\left(\frac{y}{\delta}\right)d\left(\frac{y}{\delta}\right)$$

Compressibility Effects on Skin Friction Model (10)

- Assuming ***R_f ~ constant w.r.t. y***

$$T_{avg} = \frac{1}{\delta}\cdot\int_0^\delta T(y)\cdot dy = \frac{1}{\delta}\cdot\int_0^\delta\left[T_\infty + R_f\frac{V_\infty^2}{2\cdot c_p}\left(1-\frac{1}{R_f}\left(\frac{u(y)}{V_\infty}\right)^2\right)\right]\cdot dy =$$

$$\frac{1}{\delta}\cdot T_\infty\cdot\delta + R_f\frac{V_\infty^2}{2\cdot c_p}\cdot\frac{1}{\delta}\int_0^\delta\left[\left(1-\frac{1}{R_f}\left(\frac{u(y)}{V_\infty}\right)^2\right)\right]\cdot dy = T_\infty + R_f\frac{V_\infty^2}{2\cdot c_p}\cdot\frac{1}{\delta}\int_0^\delta\left[\left(1-\frac{1}{R_f}\left(\frac{u(y)}{V_\infty}\right)^2\right)\right]\cdot dy$$

- ***Assuming "normal" turbulent velocity profile … (n=7)***

$$\frac{u_{(y)}}{V_e} = \left(\frac{y}{\delta}\right)^{\frac{1}{7}}$$

$$T_{avg} = T_\infty + R_f\frac{V_\infty^2}{2\cdot c_p}\cdot\frac{1}{\delta}\int_0^\delta\left[\left(1-\frac{1}{R_f}\left[\left(\frac{y}{\delta}\right)^{1/7}\right]^2\right)\right]\cdot dy$$

Compressibility Effects on Skin Friction Model (11)

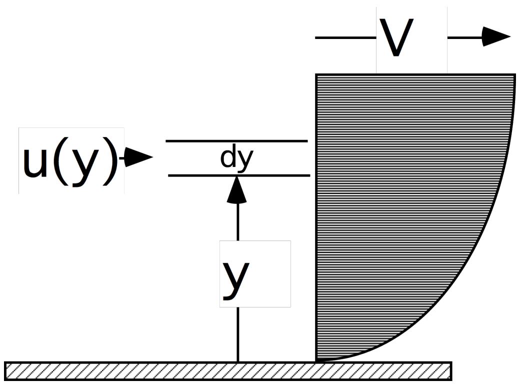
$$\frac{u_{(y)}}{V_e} = \left(\frac{y}{\delta}\right)^{\frac{1}{7}}$$

- Evaluating the Integral

$$T_{avg} = T_\infty + R_f \frac{V_\infty^2}{2 \cdot c_p} \cdot \frac{1}{\delta} \int_0^\delta \left[\left(1 - \frac{1}{R_f}\left[\left(\frac{y}{\delta}\right)^{1/7}\right]^2\right)\right] \cdot dy = T_\infty + R_f \frac{V_\infty^2}{2 \cdot c_p} \cdot \frac{1}{\delta} \int_0^\delta \left[\left(1 - \frac{1}{R_f}\left(\frac{y}{\delta}\right)^{2/7}\right)\right] \cdot dy =$$

$$T_{avg} = T_\infty + R_f \frac{V_\infty^2}{2 \cdot c_p} \cdot \frac{1}{\delta}\left(y - \frac{1}{R_f} \frac{\delta \cdot \left(\frac{y}{\delta}\right)^{9/7}}{9/7}\right)\Big|_0^\delta = T_\infty + R_f \frac{V_\infty^2}{2 \cdot c_p} \cdot \left(1 - \frac{7}{9}\frac{1}{R_f}\right)$$

$$T_{avg} = T_\infty + \frac{V_\infty^2}{2 \cdot c_p} \cdot \left(R_f - \frac{7}{9}\right)$$



- ***Valid for 1/7 power Turbulent Flow***

General Turbulent Flow Compressible Model

- Replace *"7" with 'n"*

$$\frac{u_{(y)}}{V_e} = \left(\frac{y}{\delta}\right)^{\frac{1}{n}}$$

$$\left[C_{D_{fric}}\right]_{compressible} = \frac{\left[C_{D_{fric}}\right]_{incompressible}}{\left[\left(\frac{T_\infty}{T_{avg}}\right)^{5/2}\left(\frac{T_{avg}+C_s}{T_\infty+C_s}\right)\right]^{\frac{1}{n}}}$$

$$T_{avg} \approx T_\infty + \frac{1}{\delta}\frac{V_\infty^{\ 2}}{2c_p}\int_0^\delta\left(1-\left[\frac{u(y)}{V_e}\right]^2\right)dy = T_\infty + \frac{V_\infty^{\ 2}}{2c_p}\int_0^1\left(1-[\xi]^{\frac{2}{n}}\right)dy =$$

$$T_\infty + R_f\frac{V_\infty^2}{2C_p}\left[1-\frac{1}{R_f}\frac{[\xi]^{\frac{2}{n}+1}\Big|_0^1}{\frac{2}{n}+1}\right] = T_\infty + R_f\frac{V_\infty^2}{2C_p}\left[1-\frac{1}{R_f}\frac{n}{n+2}\right] =$$

$$\boxed{T_{avg} \approx T_\infty\left[1+\left(R_f-\frac{n}{n+2}\right)\left(\frac{\gamma-1}{2}M_\infty^{\ 2}\right)\right]}$$

- **Valid for "general" Turbulent Flow**

Discussion on Recovery Factor

- Recall that ***Recovery Factor*** → ***fraction of kinetic energy of the freestream fluid recovered as thermal energy within boundary layer .. The rest of the heat is "dissipated" to the external flow field***
- Not a "constant" but is dependent on the fluid flow properties
- *The value of R_f for a given geometrical flow situation, according to dimensionial analysis is a function of the Reynolds number, the Mach number, and the Prandtl number*

$$R_f \rightarrow f(R_e, M, P_r)$$

Discussion on Recovery Factor (2)

• Experimental studies for flow over a "near flat" surface

→Reynolds number only effects the value of the recovery factor by determining whether the flow in the boundary layer is *laminar* or *turbulent*

→Mach number has a negligible effect on the recovery factor for small surface pressure gradients

→ R_f *primarily* → $f(P_r)$

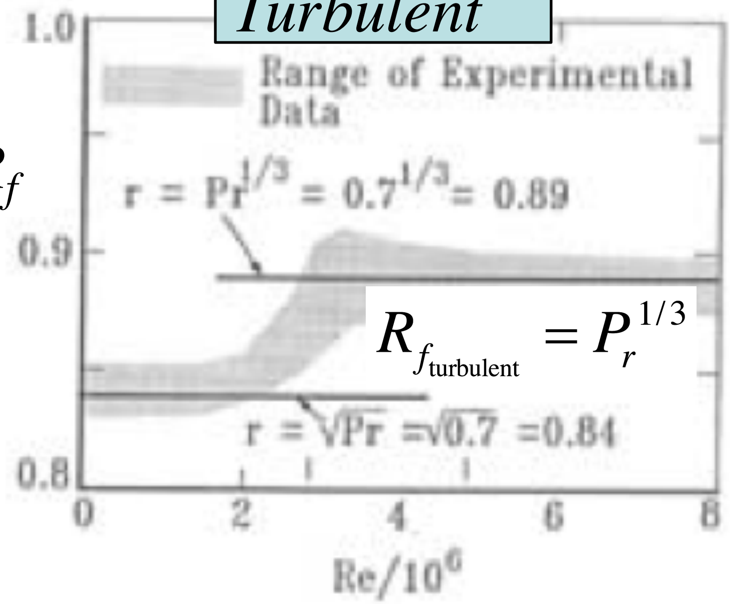
• The Prandtl number is a dimensionless parameter of a convecting system that characterizes the regime of convection. …. (momentum diffusivity) / (thermal diffusivity) }

• P_r measure of relative importance of skin friction and heat transfer in viscous flow.

$$P_r = \frac{c_p \mu}{\kappa} \rightarrow \{\mu\text{=dynamic viscosity}, \kappa\text{=thermal conductivity}\}$$

Discussion on Recovery Factor (3)

Turbulent



R_f

$$R_{f_{\text{turbulent}}} = P_r^{1/3}$$

Laminar

$$R_{f_{\text{laminar}}} = \sqrt{P_r}$$

$$\Pr = \frac{\textit{kinematic viscosity}}{\textit{thermal diffusivity}} \approx \frac{\mu / \rho}{\kappa}$$

μ = absolute or dynamic viscosity (kg/m-s, cP)
c_p = specific heat (J/kg K
k = thermal conductivity (W/m K)

- What values make sense for R_f?

- P_r can vary widely, But for Turbulent Flow is limited to less than 1
- Typically $0.5 < P_r < 1$

Prandtl Number

- *For Turbulent Flows*

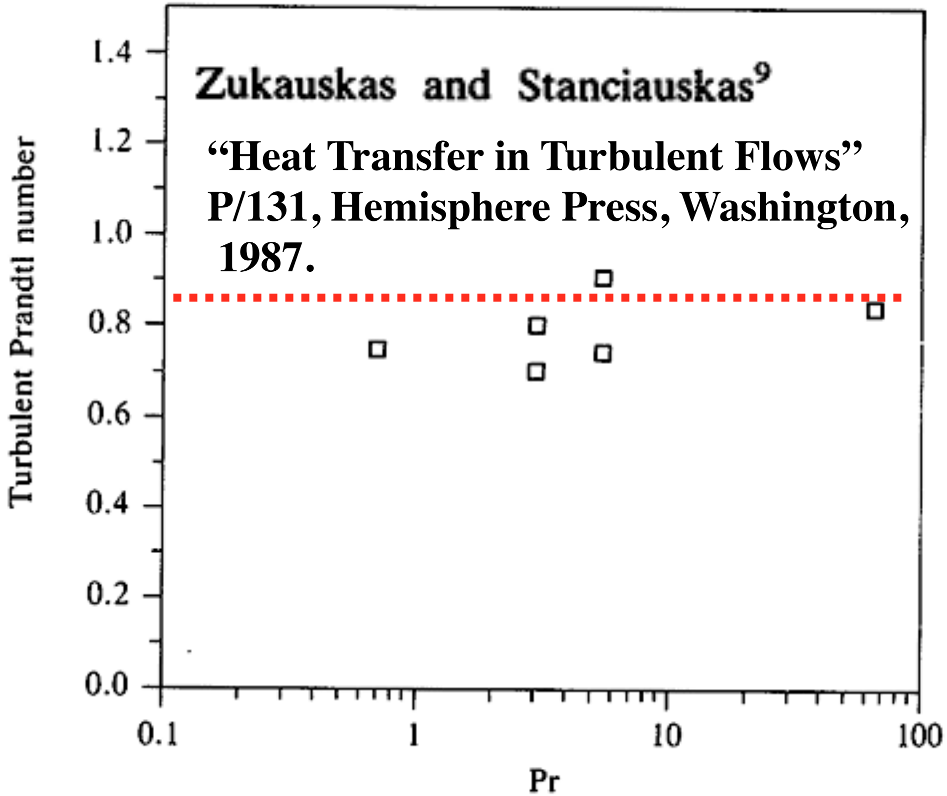
$$0.7^{1/3} \le R_{f_{turbulent}} < 1 \rightarrow 0.89 \le R_{f_{turbulent}} < 1$$

Pr ~ 0.85 for non-dissociated "air"

- R_c turbulent ~ 0.95

- Often the Approximation $R_c \sim 1$ is used

- **"Unity Prandtl Number Flow**



Collected Algorithm (1)

$$\left[C_{D_{fric}}\right]_{incompressible} = \left\{ \frac{7}{225\left[R_{e_\infty}\right]^{\frac{1}{7}}} \rightarrow \boxed{R_{e_\infty} = \frac{\rho_\infty V_\infty c}{\mu_\infty}} \rightarrow \boxed{R_{e_\infty} = \frac{\rho_\infty V_\infty c}{\mu_\infty}} \right\}$$

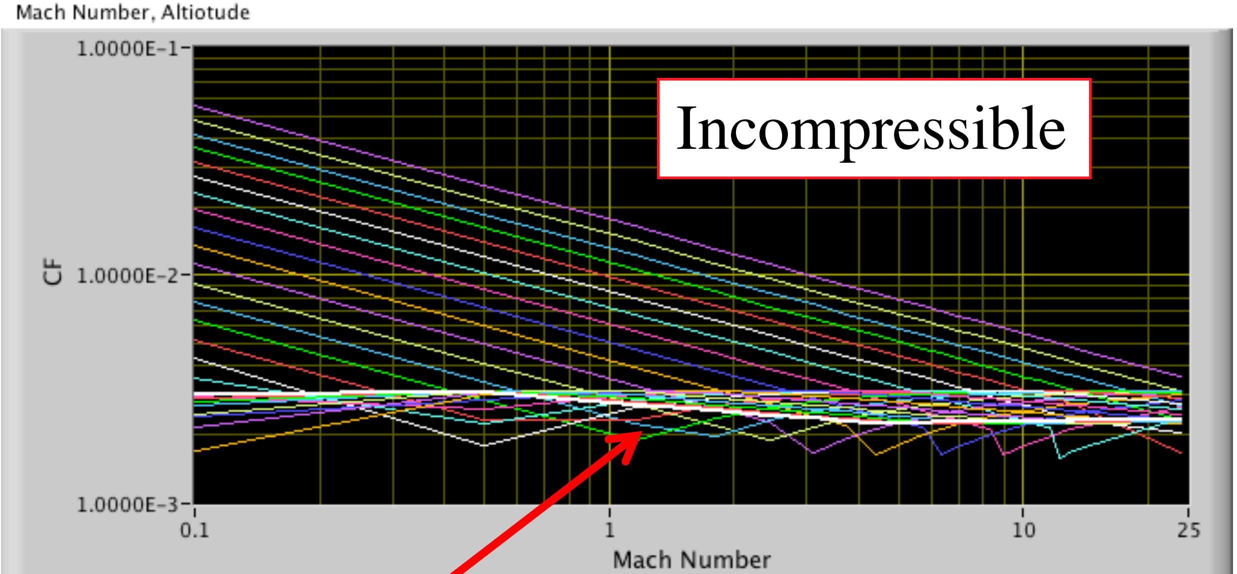
$$\left[C_{D_{fric}}\right]_{compressible} = \frac{\left[C_{D_{fric}}\right]_{incompressible}}{\left[\left(\frac{T_\infty}{T_{avg}}\right)^{5/2}\left(\frac{T_{avg}+C_s}{T_\infty+C_s}\right)\right]^{\frac{1}{7}}} \rightarrow C_s = 120^0 K \ for \ air$$

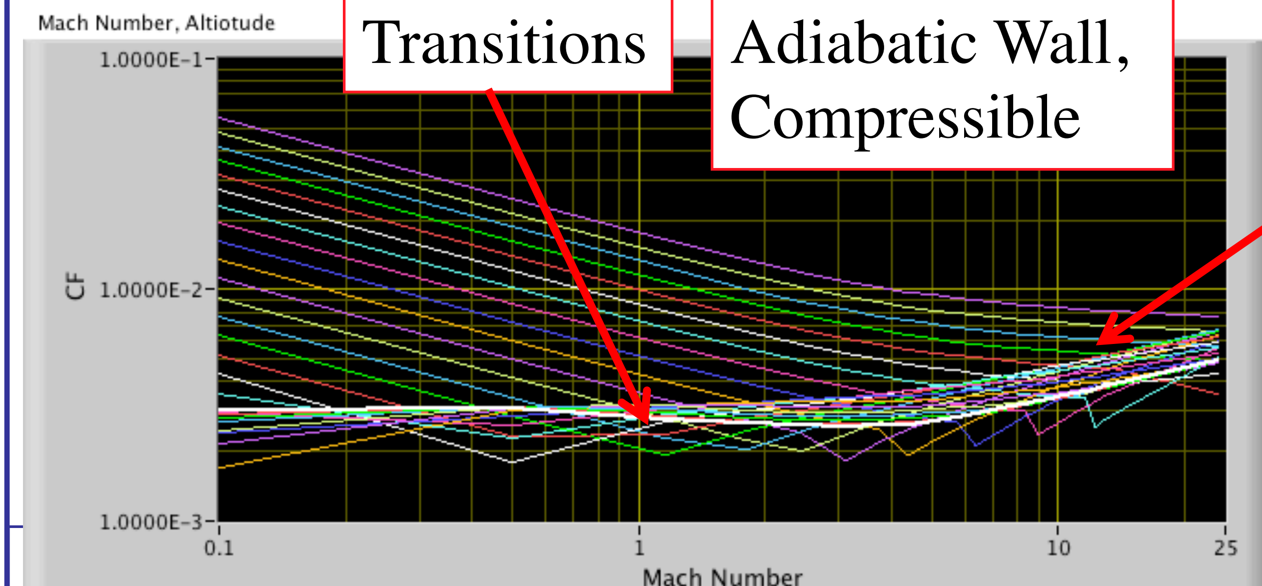
- ***Adiabatic Wall***

$$T_{avg} \approx T_\infty\left[1+\frac{2}{9}\left(\frac{\gamma-1}{2}M_\infty^{\ 2}\right)\right]$$

- **Valid for Turbulent Flow, Adiabatic Wall, Unity Prandtl Number**

Incompressible, Adiabatic Wall Skin Friction Versus Mach Number





Transitions

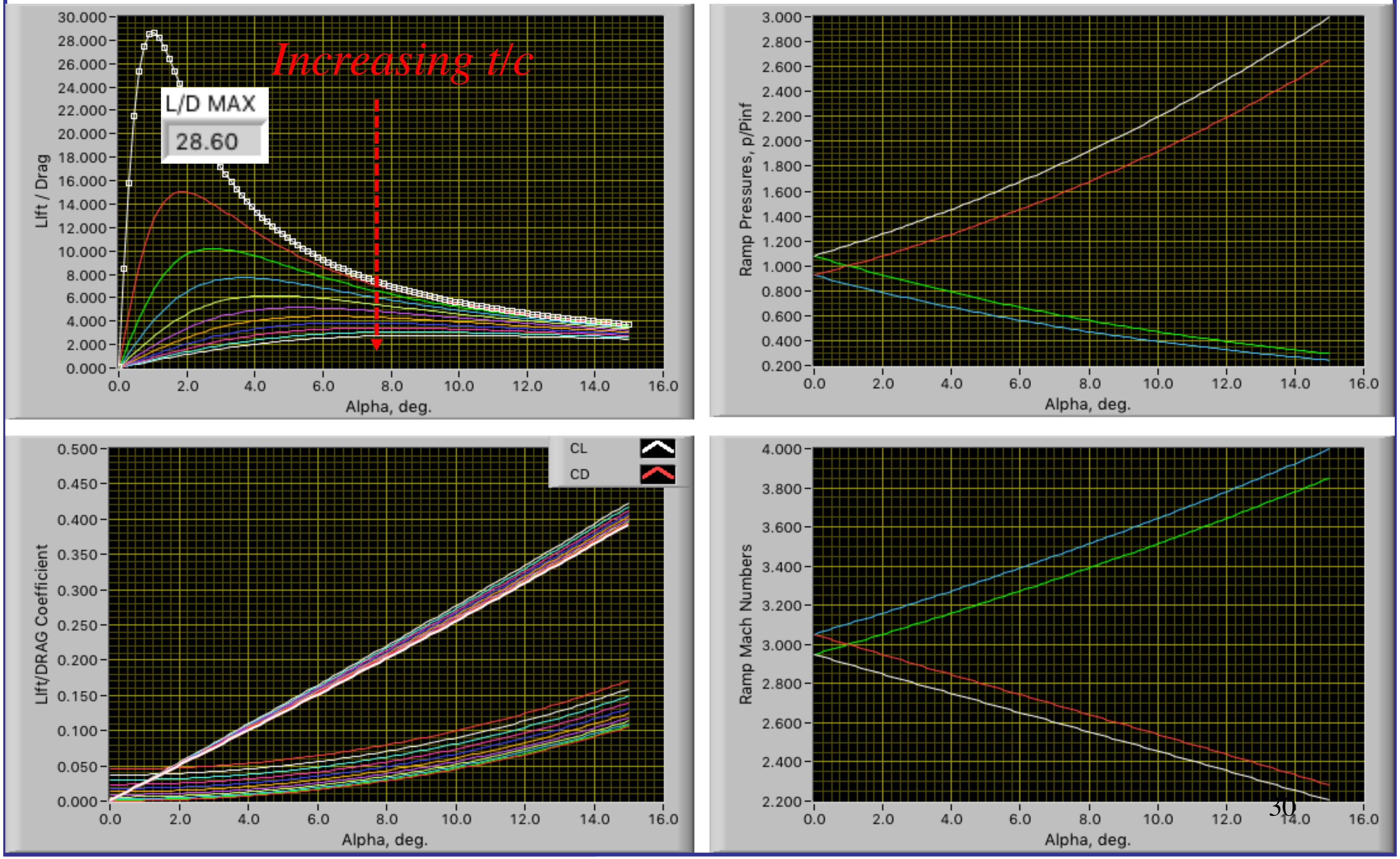
- **Altitude Variation from 0 to 80 km**
- **Mach Variation From 0.1 to 24.0**

Pr =1.0

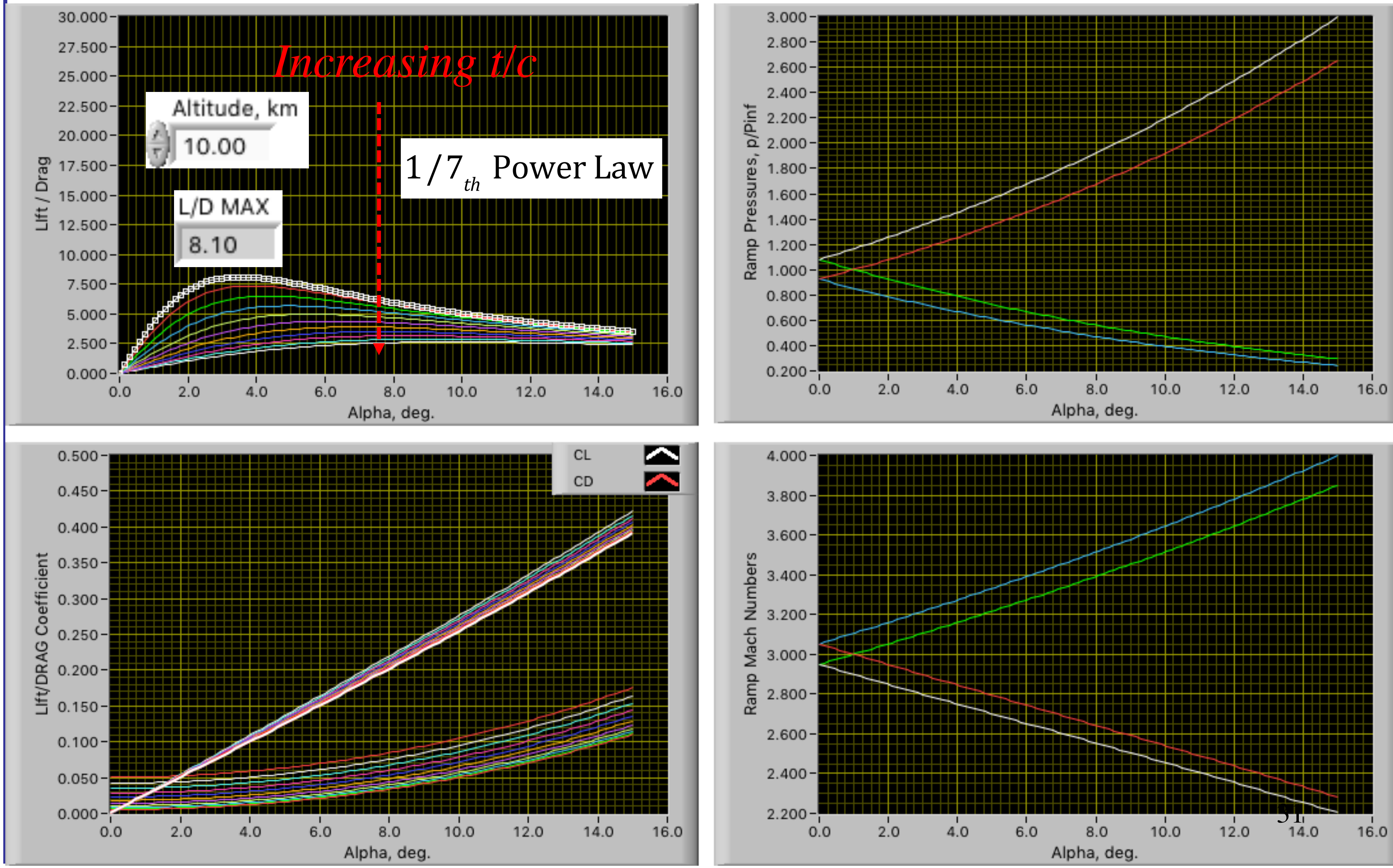
1-meter chord

Significant C_F Growth at Higher Mach Numbers

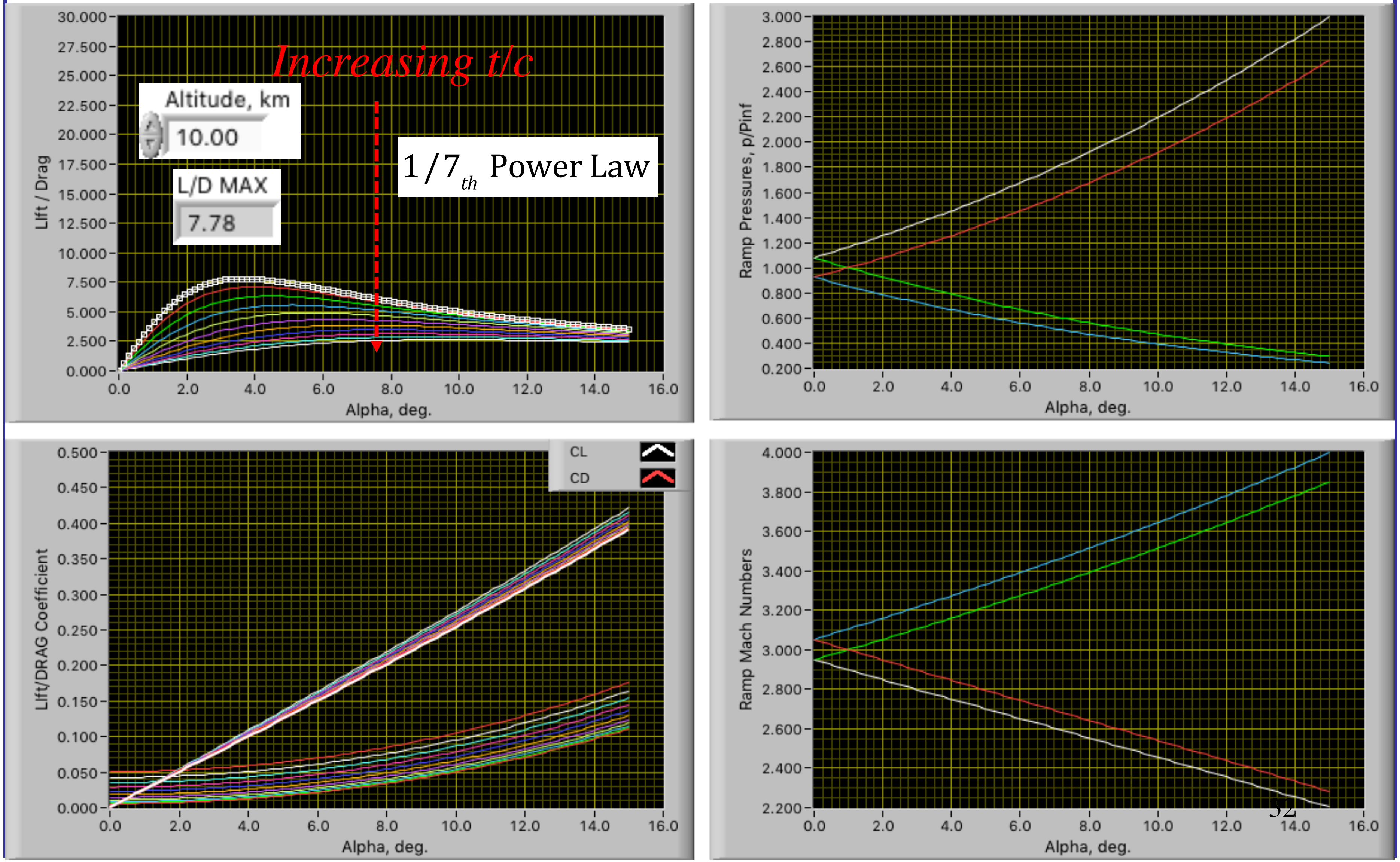
Symmetric Double-wedge Airfoil … L/D @ Mach 3 (Inviscid)



Symmetric Double-wedge Airfoil … L/D @ Mach 3 (Turbulent, Viscous Flow, Incompressible)



Symmetric Double-wedge Airfoil … L/D @ Mach 3 (Turbulent, Viscous Flow, Compressible Correction)



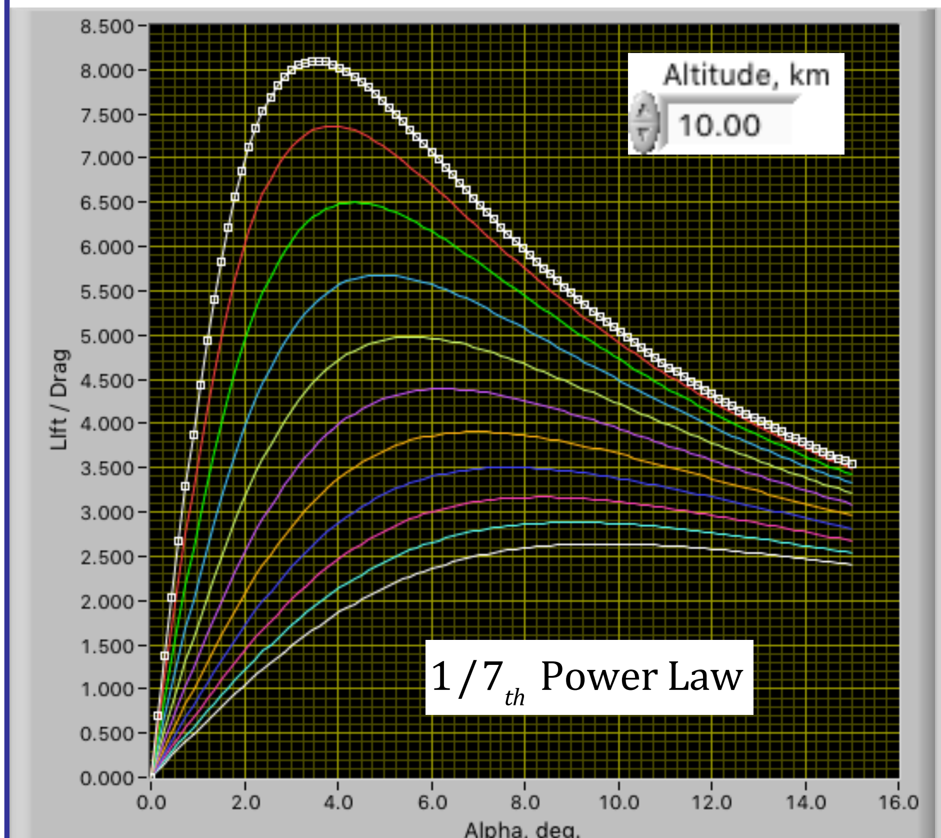
Utah State University

Mechanical & Aerospace Engineering

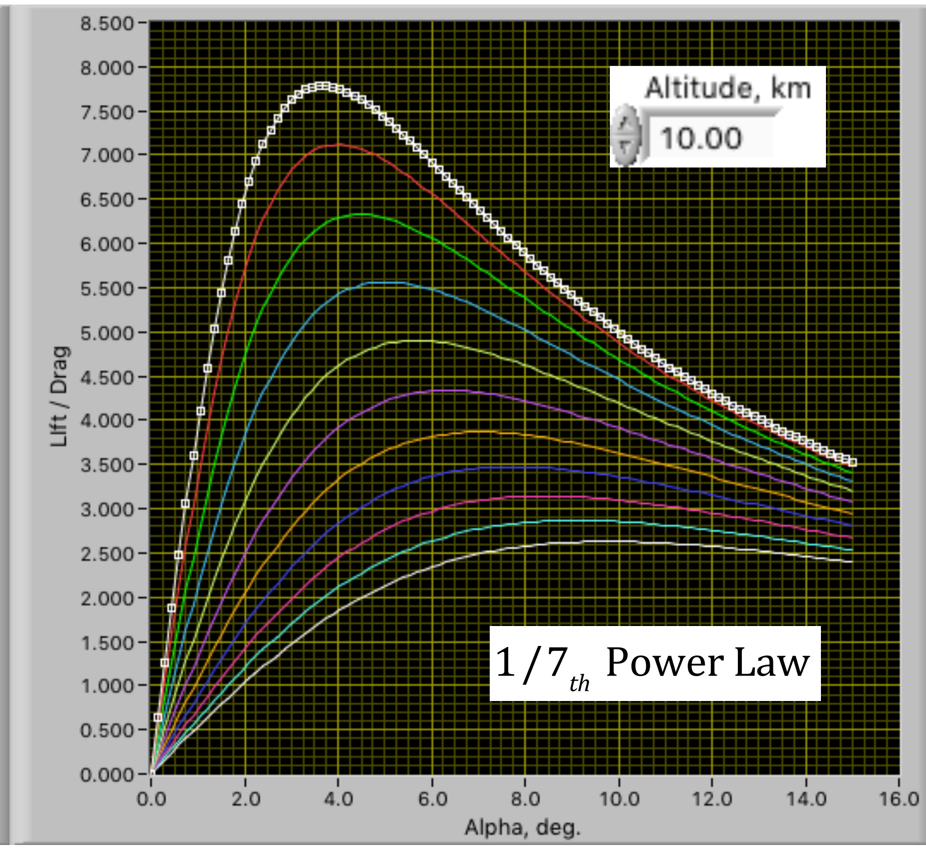
Symmetric Double-wedge Airfoil … L/D (revisited)

- Blow up of Previous pages

M_∞ =3.0, Incompressible

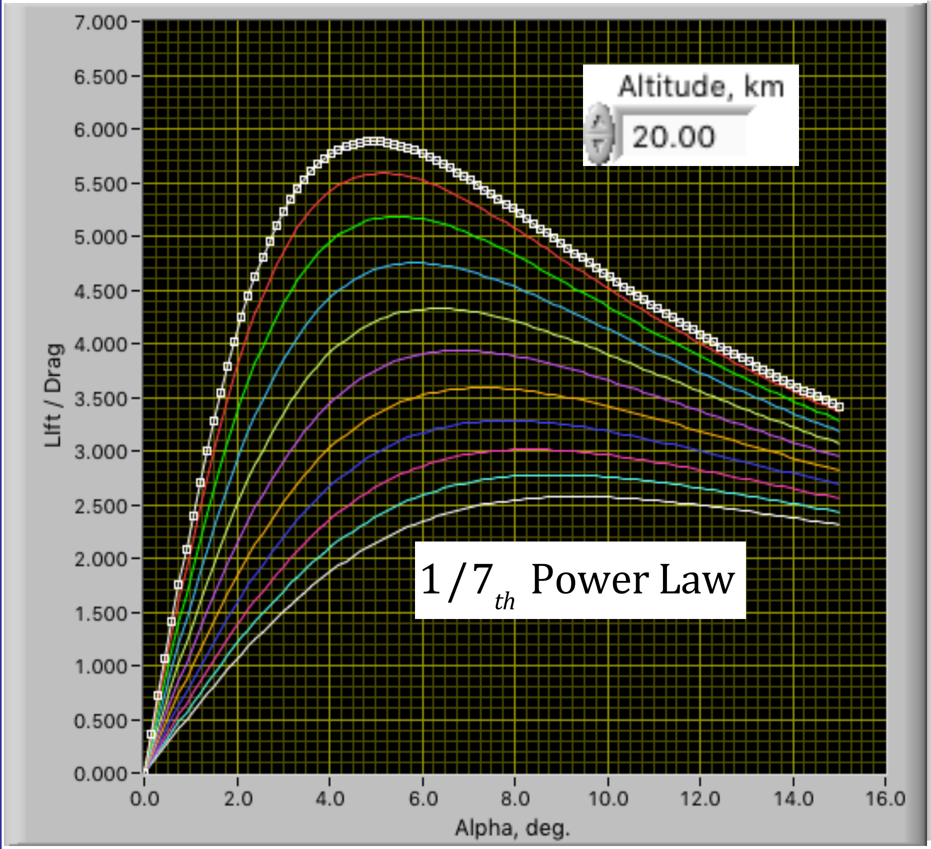


M_∞ =3.0, Compressible

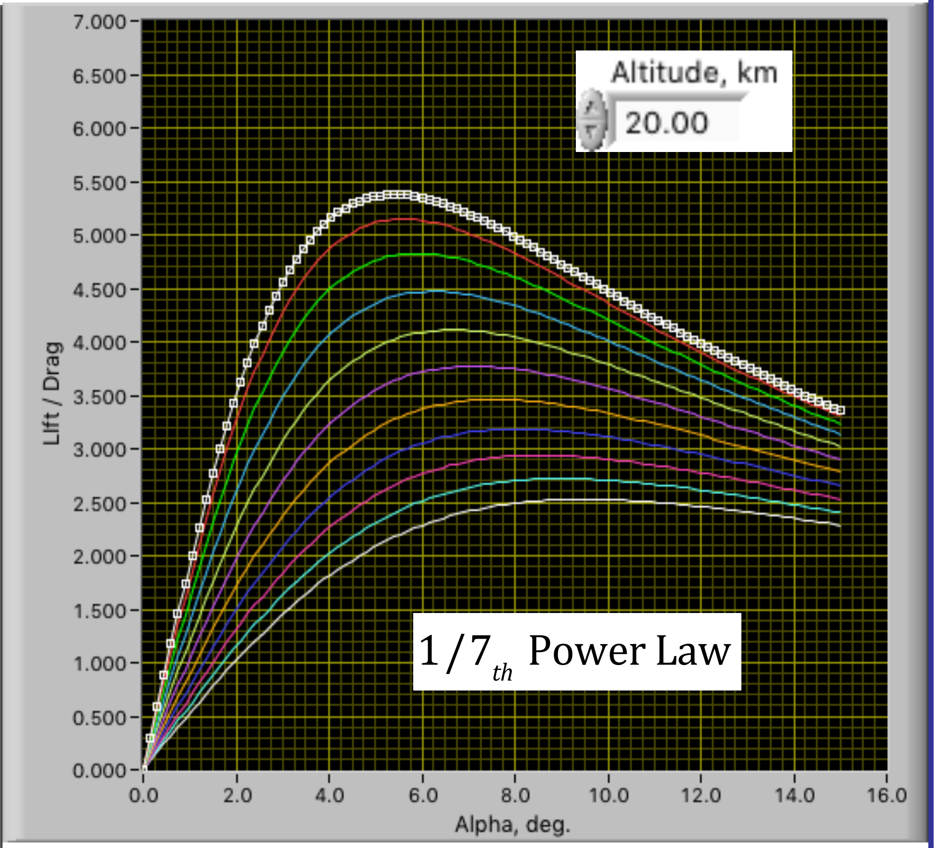


Symmetric Double-wedge Airfoil … L/D (revisited)

$M_\infty = 5.0$, *Incompressible*



$M_\infty = 5.0$, *Compressible*



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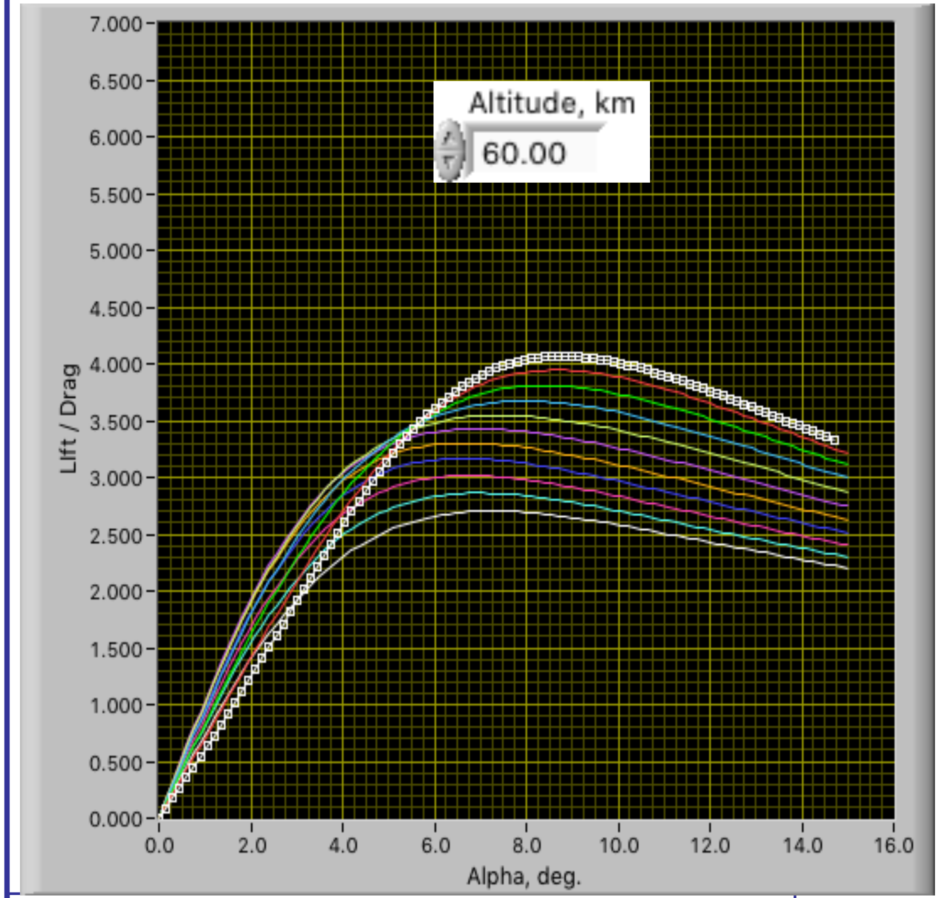
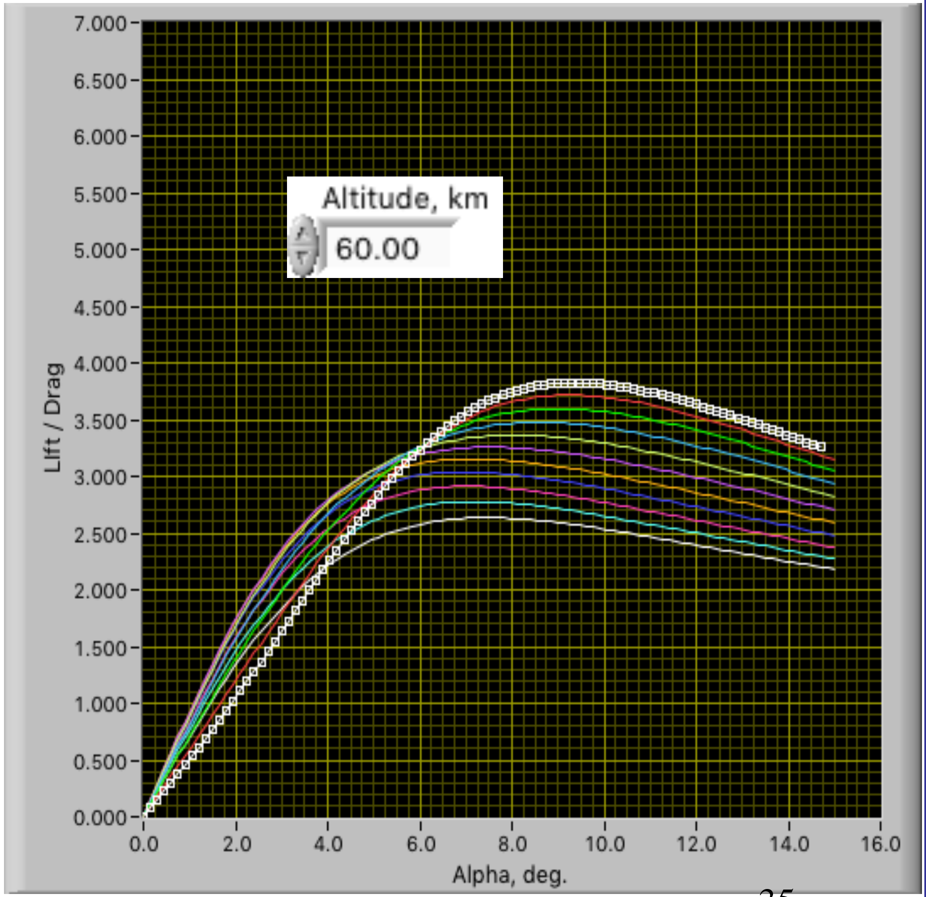
Symmetric Double-wedge Airfoil … L/D (revisited)

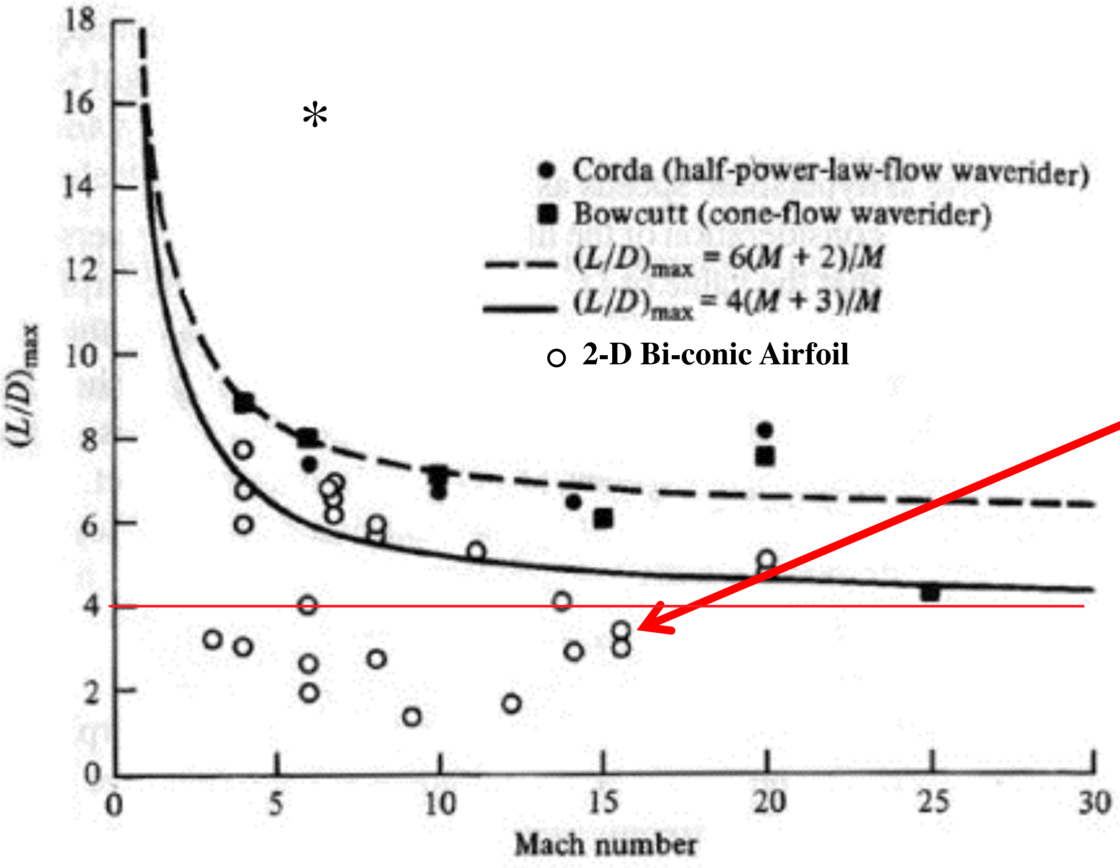
M_∞ =25.0, Incompressible



M_∞ =25.0, Compressible



Symmetric Double-wedge Airfoil … L/D (revisited)



- Hypersonic L/D > 4.0 is approaching theoretical limit For 2-D Flow

Anderson … Hypersonic and High temperature Gas Dynamics

Final Project

- **This a MAJOR Project for this class ….! *(25% of Final Grade)***

- Part 1 →Derive a Compressibility correction for laminar skin friction coefficient Based on parabolic velocity profile model

Due Wednesday December 15, 2021, Dropbox or USU Box

$$\left(C_{D_{fric}}\right)_{compressible} = \frac{1.328}{\left(\dfrac{\rho(T_{avg})\cdot c\cdot V_\infty}{\mu(T_{avg})}\right)^{\frac{1}{2}}}$$

$$\frac{u_{(y/\delta)}}{V_e} = \left[\frac{2y}{\delta} - \left(\frac{y}{\delta}\right)^2\right]$$

$$\rho(T_{avg}) = \rho_\infty \frac{T_\infty}{T_{avg}}$$

$$\mu(T_{avg}) = \mu(T_\infty)\left(\frac{T_{avg}}{T_\infty}\right)^{3/2}\left(\frac{T_\infty + 120^\circ\mathrm{K}}{T_{avg} + 120^\circ\mathrm{K}}\right)$$

- Approximate the Average Temperature Of boundary layer By Integrating Across Boundary layer And dividing by boundary layer height *(δ)*

$$T\left(\frac{y}{\delta}\right) = T_\infty + R_f \frac{V_\infty^2}{2C_p}\left(1 - \frac{1}{R_f}\left[\frac{u(y/\delta)}{V_\infty}\right]^2\right)$$

$$\frac{\rho_\infty \cdot c\cdot V_\infty}{\mu_\infty} < 500{,}000$$

$$T_{avg} \approx \frac{1}{\delta}\int_0^\delta T(y)dy = \frac{1}{\delta}\int_0^\delta T(y)dy = \int_0^1 T\left(\frac{y}{\delta}\right) d\left(\frac{y}{\delta}\right)$$

Question: is R_f=1 a good assumption for laminar flow?

Final Project (cont'd)

• Transitional Model

$$\left(C_{D_{transition}}\right)_{compressible} = \frac{\left(C_{D_{transition}}\right)_{incompressible}}{\left(\left(\frac{T_\infty}{T_{avg}}\right)^{5/2}\left(\frac{T_{avg}+120^{o}\mathrm{K}}{T_\infty+120^{o}\mathrm{K}}\right)\right)^{\frac{1}{7}}}$$

• For Compressible Transitional Model ….. Use turbulent velocity profile, turbulent flow compressibility correction

$$\frac{u_{(y/\delta)}}{V_e} = \left(\frac{y}{\delta}\right)^{\frac{1}{7}}$$

$$500{,}000 \le \frac{\rho_\infty \cdot c \cdot V_\infty}{\mu_\infty} < 10^7$$

Final Project (cont'd)

- Code up "DOUBLE-WEDGE" Aerofoil Model Supersonic flow … (use methods of section 7.2. …)

- Build skin friction model for accounting for Laminar / Transitional and Turbulent conditions, Adiabatic Compressibility

- ***Assume Unity Prandtl number .. We'll find out that this assumption is not very good for laminar flow***

  - Use appropriate Compressibility ***correction for laminar or or turbulent flow (for transitional flow Use Turbulent Correction)***

- Compute L/D_{max} for 2° half angle wing , 2 meter chord
(plot vs alpha pick max)

i) Inviscid flow
ii) Viscous flow, turbulent only, 1/7th power law
iii) Transitional flow (laminar, transitional, and/or turbulent
.. Mach/altitude dependent, turbulent n that varies with Reynolds number)

Final Project (cont'd)

- ***Assume Unity Prandtl number***

- Use appropriate Compressibility correction for laminar or turbulent flow *(transitional flow .. Use turbulent compressibility model)*

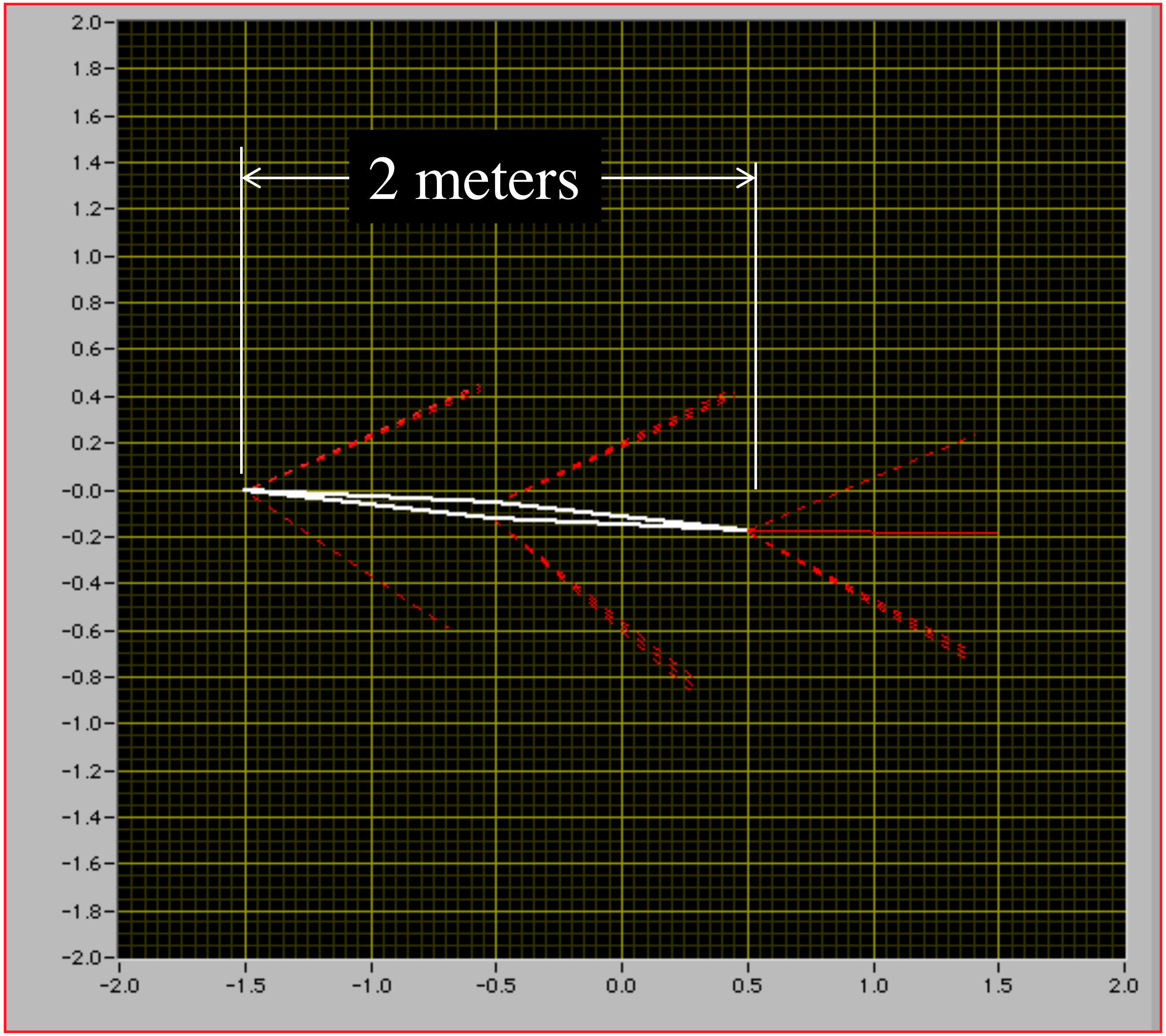
- Plot as function of Mach number at 10 km, 20 km, 30 km, 40km, 50 km altitudes *(ignore exit angle at end of wing … no effect on L/D) …*

2 meter chord to plate …

For ($1.25 < M_\infty < 10$), …. assume normal air properties for γ, R_g, c_p … etc.

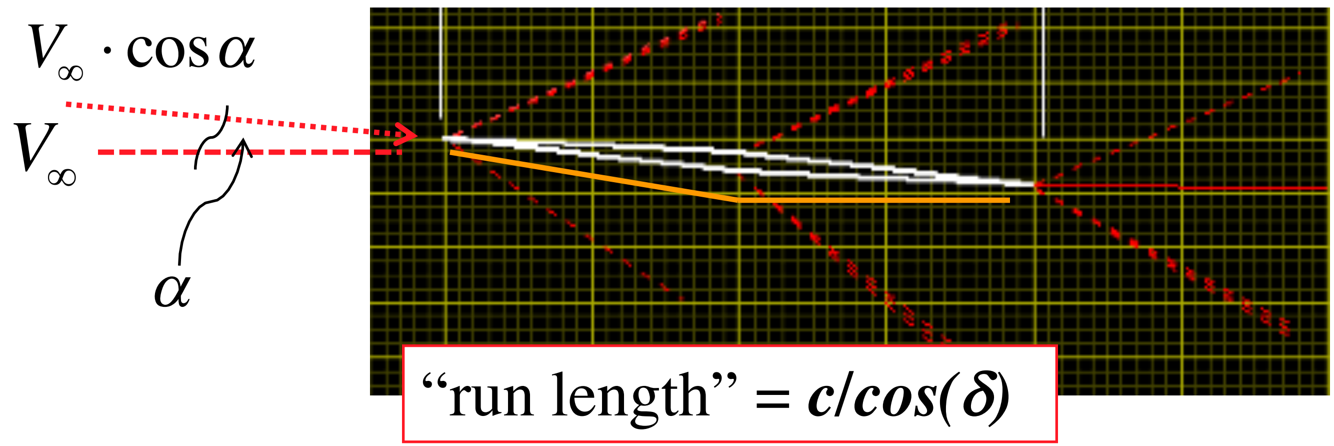
- Identify *laminar, transitional and turbulent regimes* on plot

Final Project 8 (cont'd)



Final Project 8 (cont'd)

$$R_e(\infty) = \frac{\rho_\infty \cdot V_{\infty\|} \cdot C_{\substack{run\\length}}}{\mu_\infty} = \frac{\rho_\infty \cdot (V_\infty \cdot \cos\alpha) \cdot \dfrac{c_{plate}}{\cos\delta}}{\mu_\infty} = \frac{\rho_\infty \cdot V_\infty \cdot \left(c_{plate} \cdot \dfrac{\cos\alpha}{\cos\delta}\right)}{\mu_\infty}$$



Wing has two sides that contribute to skin drag

Final Project 8 (cont'd)

• Things to consider …

-Skin friction coefficient was normalized using the area of one slide of the plate …for our model … ***we have friction on both sides of the wing (approximated as a flat plate)***

… be sure to account for this "two-sidedness" accordingly … i.e. convert to drag first add up terms and then normalize by total planform area of wing … *b x c*

-- Approximate "*c*" in Reynolds number calculations by "run length" along plate in direction of incoming flow and flow parallel to plate axis …..

-- Always use freestream Reynolds number based on flow along wing center axis for inviscid B.L. Calculations

-- Assume no skin drag contribution to lift .

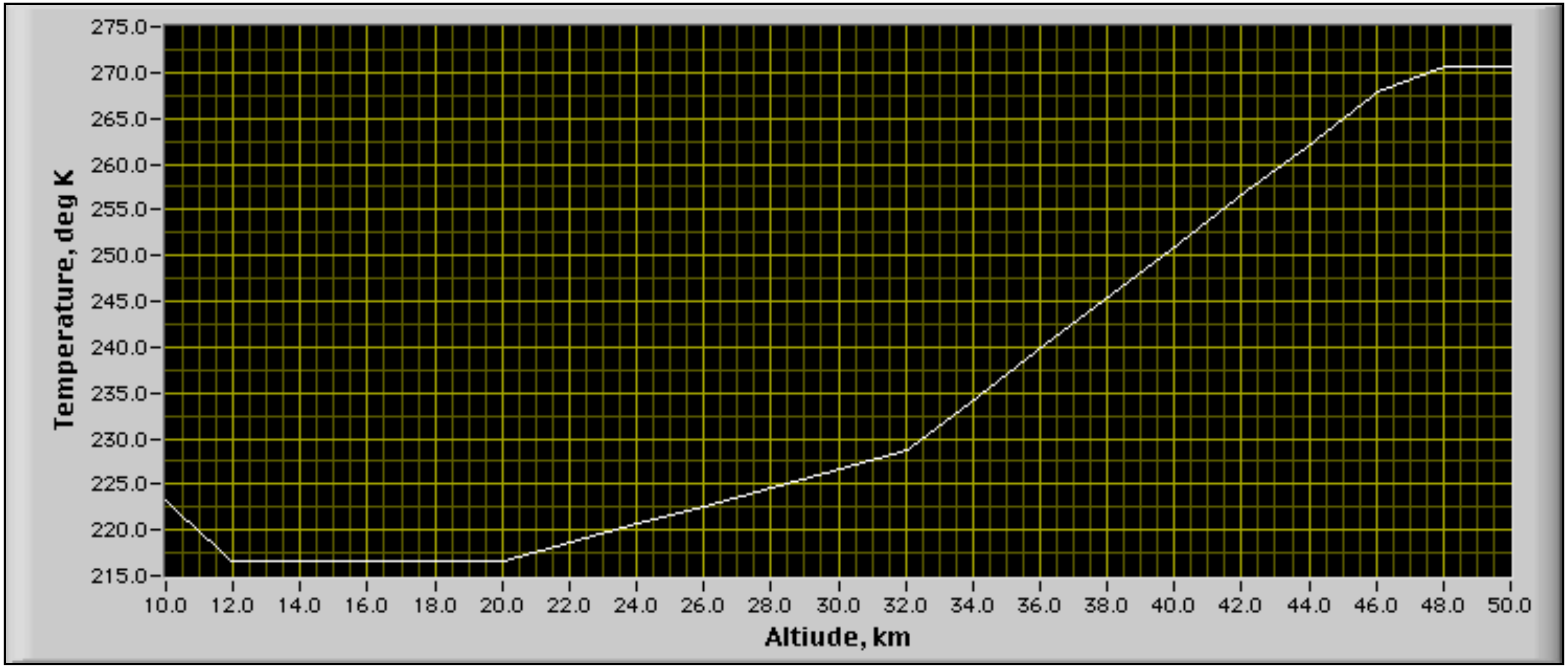
Include an executive summary on your "design philosophy" … why you chose to model

this wing as you did … what were your results, why?

i.e. I want a detailed Report! All plots must have units, axis labels, figure numbers, and a caption

Final Project 8 (cont'd)

See: http://www.neng.usu.edu/classes/mae/5420/Compressible_fluids/section8/StandardAtmosphere.txt



Key data, Ambient Temperature, °K VS ALTITUDE

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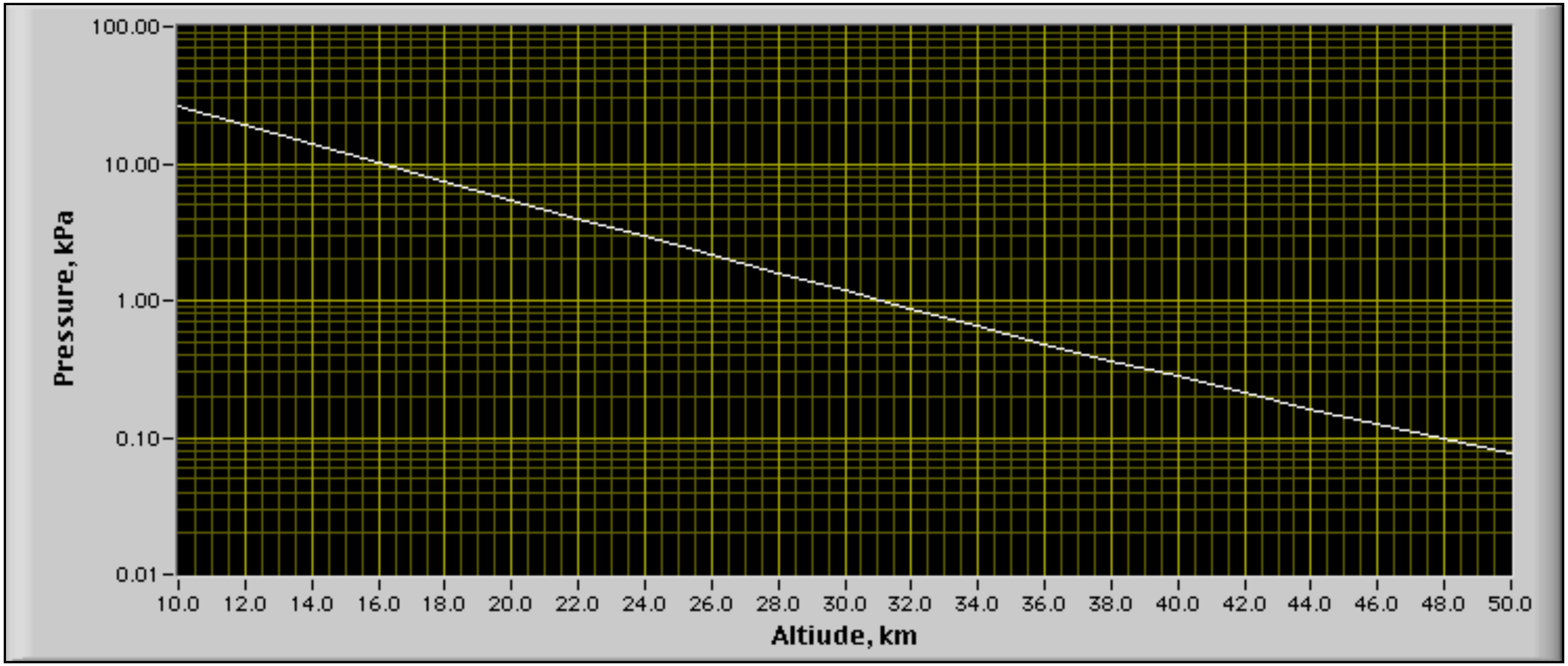
Final Project 8 (cont'd)

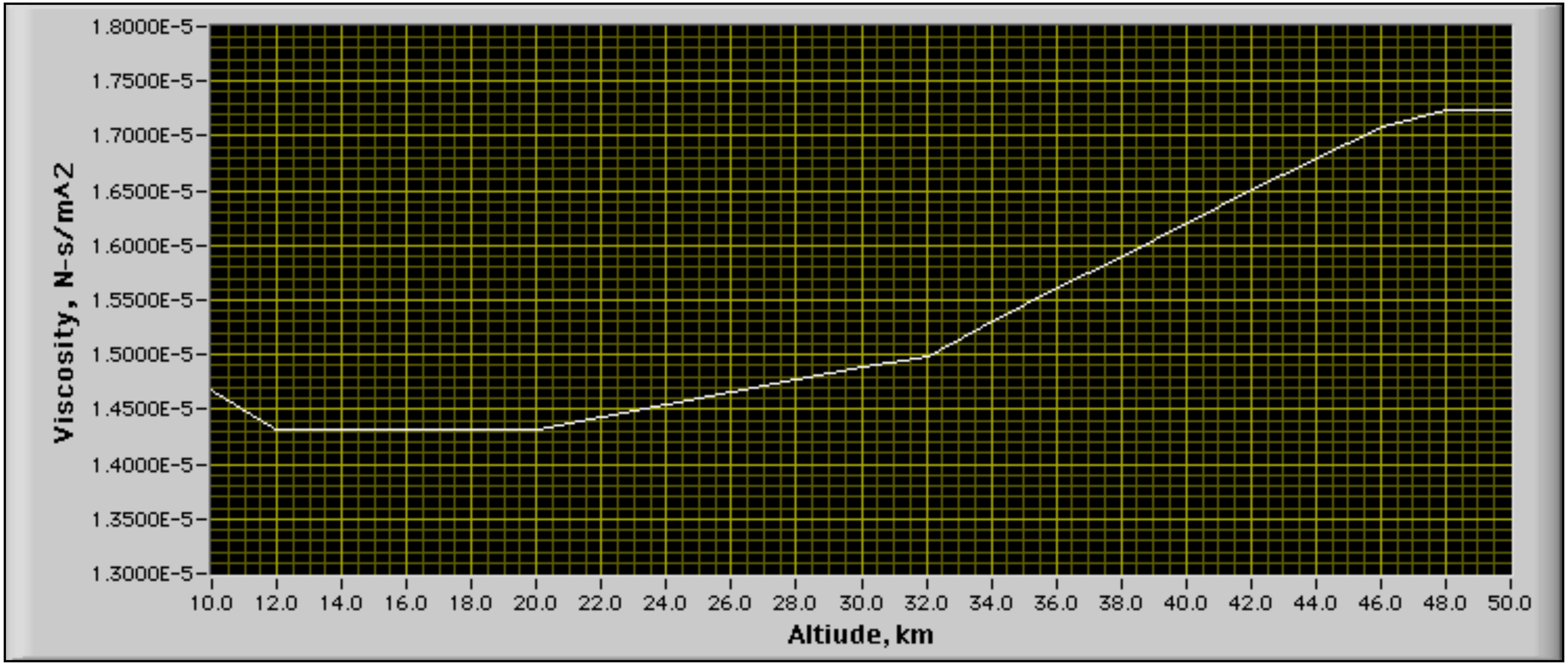
See: http://www.neng.usu.edu/classes/mae/5420/Compressible_fluids/section8/StandardAtmosphere.txt



Key data, Ambient Pressure, kPa VS ALTITUDE

Final Project 8 (cont'd)

See: http://www.neng.usu.edu/classes/mae/5420/Compressible_fluids/section8/StandardAtmosphere.txt



Key data, viscosity (μ), Nt-sec/m^2 VS ALTITUDE

Appendix: Non-Adiabatic wall Corrections

Alternate Methods, non adiabatic wall

- For a non-adiabatic wall … average temperature method is not as accurate …"cold" wall pulls heat away from flow … reducing boundary Layer temperatures

- ***Eckert's Reference temperature (semi-empirical) … for air***

Actual Wall Temperature **Adiabatic Wall Temperature**

$$T_{ref} = T_\infty + 0.5\left(T_{wall} - T_\infty\right) + 0.22\left(T_{aw} - T_\infty\right)$$

Reference temperature

Temperature at edge of Boundary layer

$$T_{aw} \approx T_\infty\left[1 + R_f\left(\frac{\gamma - 1}{2} M_\infty^{\ 2}\right)\right]$$

https://www.sciencedirect.com/science/article/abs/pii/0017931067901597

- ***Boundary layer properties are Evaluated at T_{ref} and Not T_{avg}***

Collected Algorithm (2)

- ***Non-adiabatic wall***

$$\left[C_{D_{fric}}\right]_{compressible} = \frac{\left[C_{D_{fric}}\right]_{incompressible}}{\left[\left(\frac{T_{\infty}}{T_{ref}}\right)^{5/2}\left(\frac{T_{ref}+C_s}{T_{\infty}+C_s}\right)\right]^{\frac{1}{7}}} \rightarrow C_s = 120^0 K \textit{ for air}$$

$$\boxed{T_{ref} = T_{\infty} + 0.5\left(T_{wall} - T_{\infty}\right) + 0.22\left(T_{aw} - T_{\infty}\right)}$$

- **Valid for Turbulent Flow, Non-adiabatic wall ~ unity Prandtl number**

Questions?