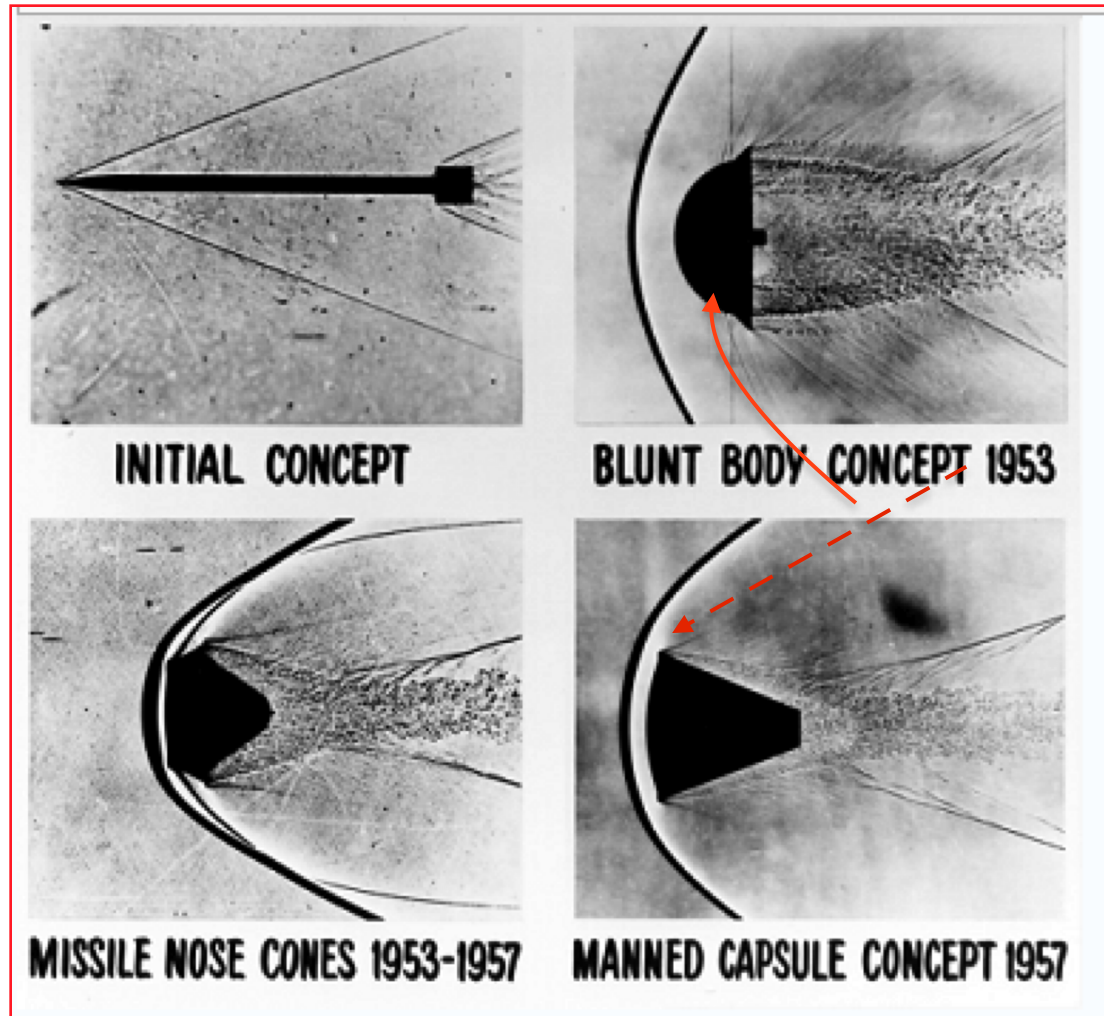
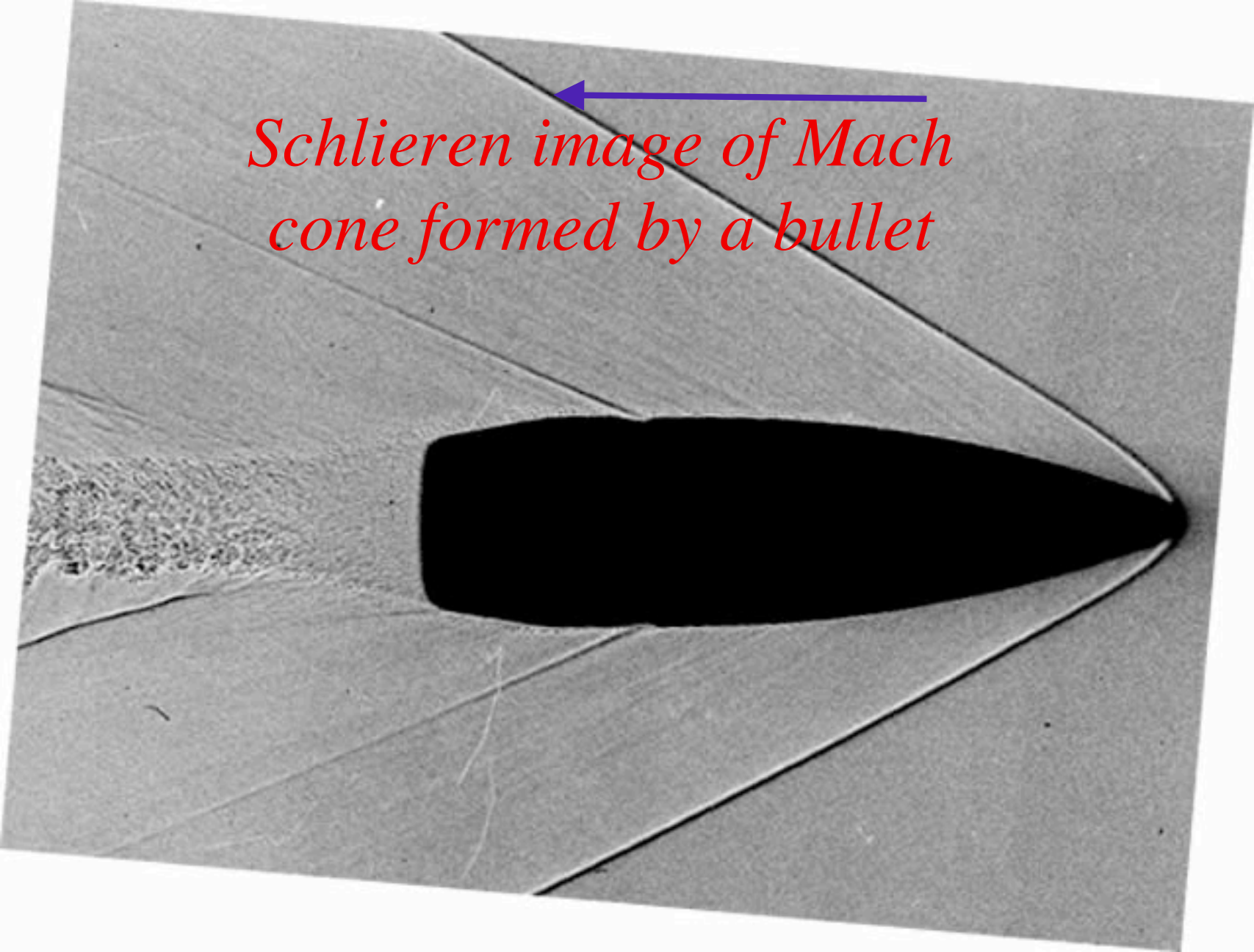


Section 8 Lecture 3: Supersonic Flow Around a Blunt Body

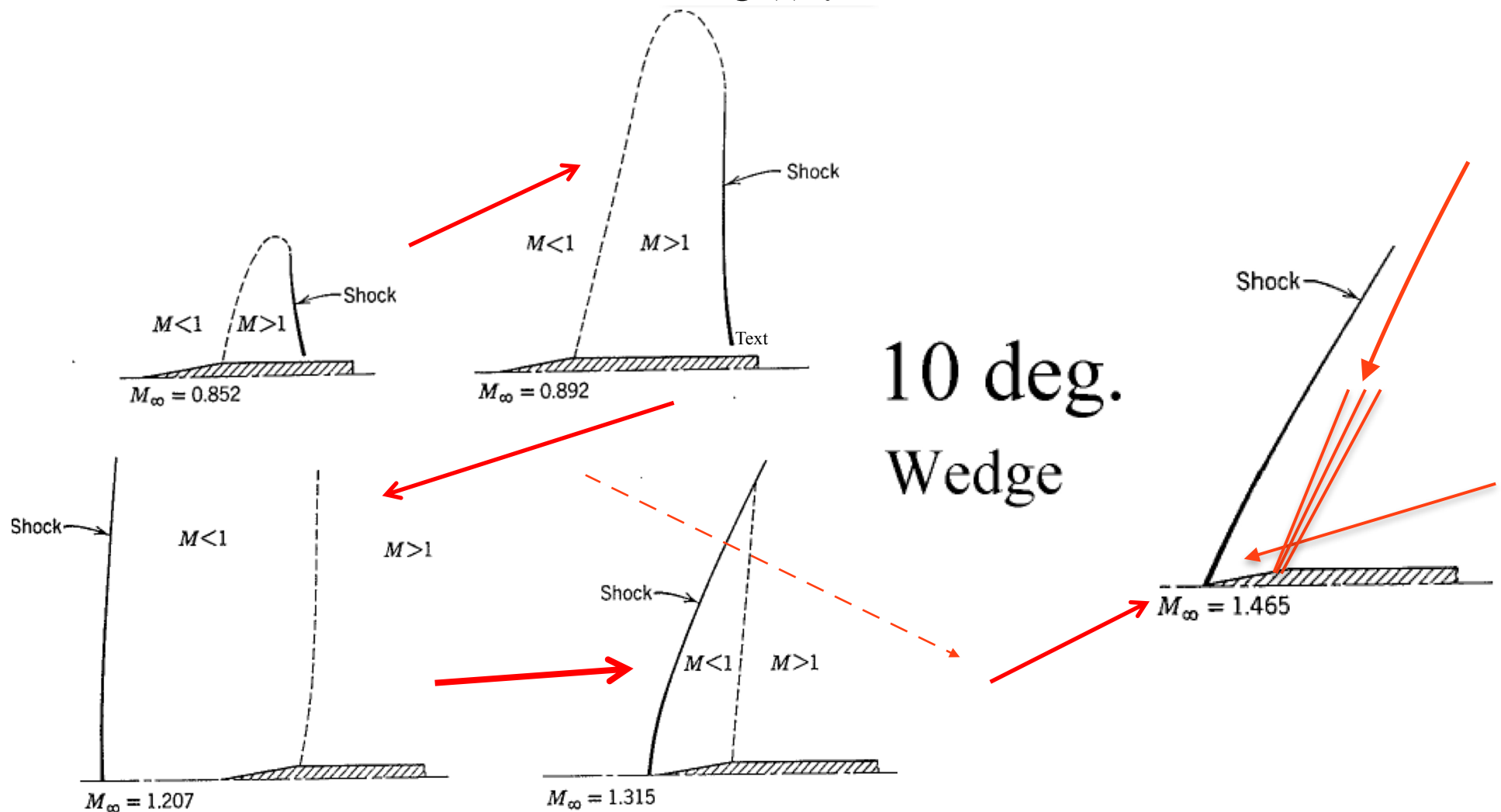


- Anderson,
Chapter 4 pp. 163-167

*Schlieren image of Mach
cone formed by a bullet*



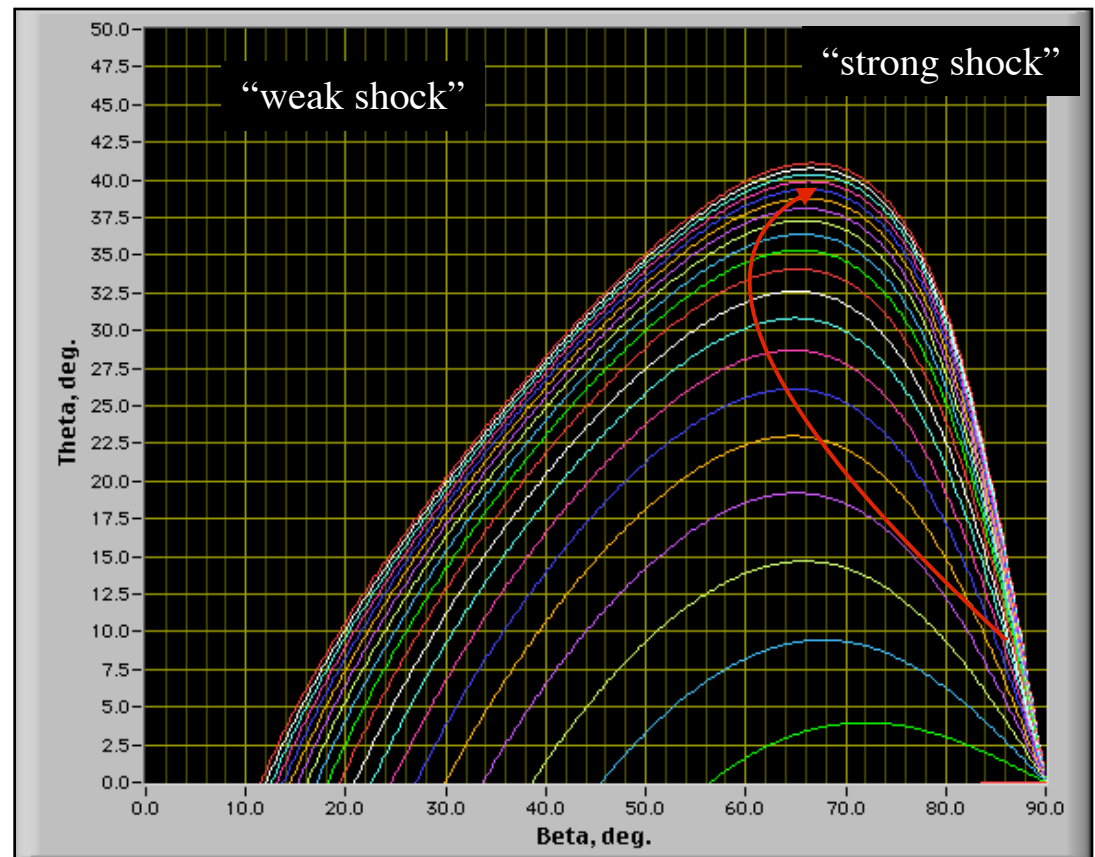
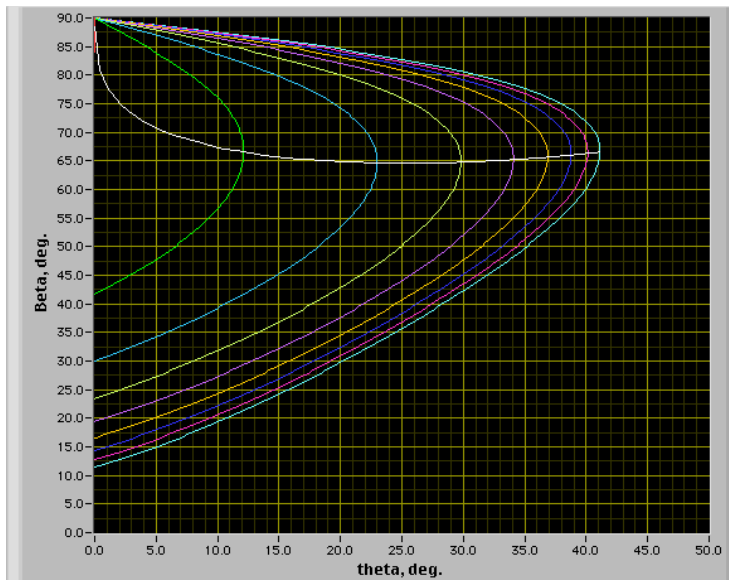
What Happens on Wedge Airfoil in Transonic Flow?



Maximum Wedge Deflection Angle

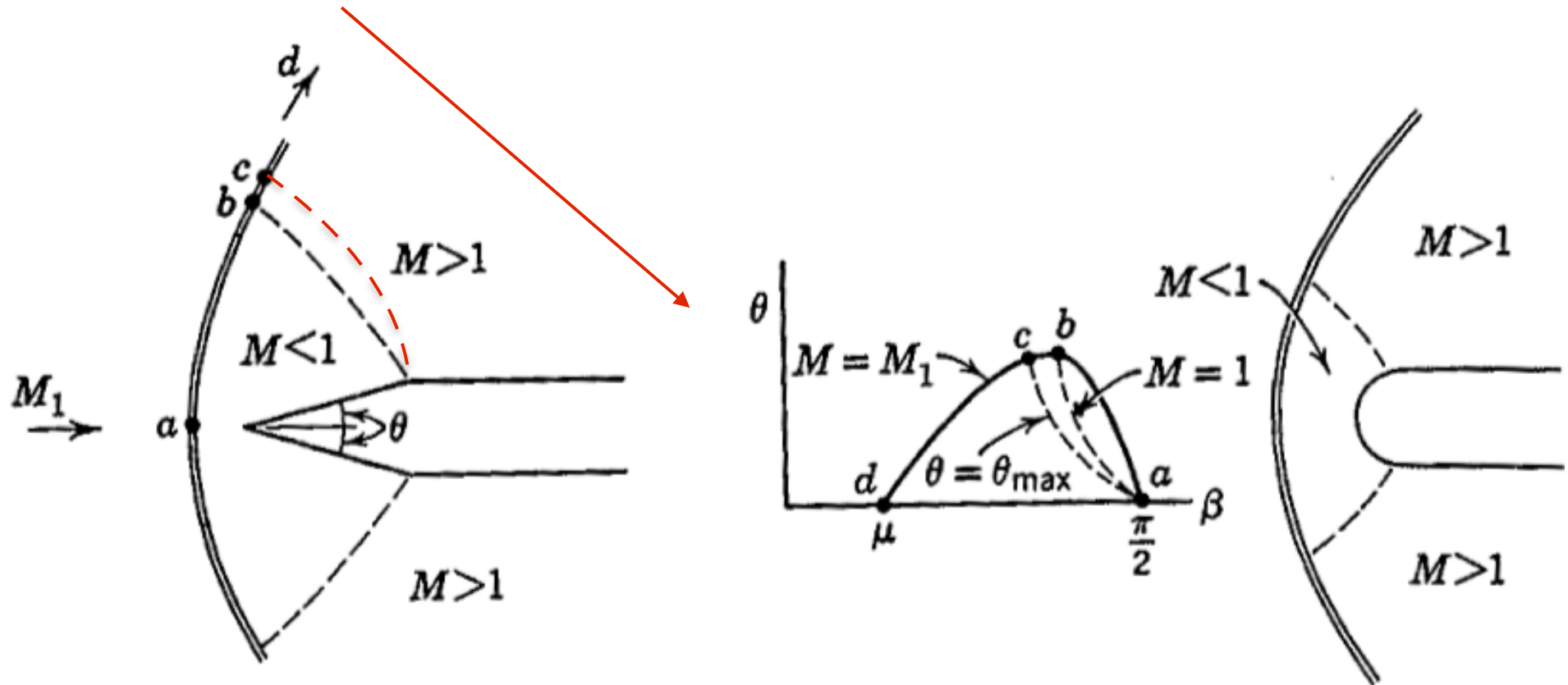
- Swap axes on the Oblique Shock wave Plot
- Clearly a Maximum for θ for each Mach number

θ



β

Maximum Flow Deflection Angle (cont'd)



Max Deflection (“Wedge”) Angle and Detached Shockwaves (cont’d)

- *from Oblique Shock relations*

$$\theta = \tan^{-1} \left[\frac{2 \{ M_1^2 \sin^2(\beta) - 1 \}}{\tan(\beta) [2 + M_1^2 [\gamma + \cos(2\beta)]]} \right]$$

- *Necessary condition for maximum:*

$$\frac{\partial \theta}{\partial \beta} = 0 \Rightarrow \frac{\partial \left(\tan^{-1} \left[\frac{2 \{ M_1^2 \sin^2(\beta) - 1 \}}{\tan(\beta) [2 + M_1^2 [\gamma + \cos(2\beta)]]} \right] \right)}{\partial \beta} = 0$$


Max Deflection (“Wedge”) Angle and Detached Shockwaves (cont’d)

• Chain rule $\frac{\partial}{\partial \beta} [\tan^{-1}(f(\beta))] = \frac{1}{1 + (f(\beta))^2} \left(\frac{\partial f}{\partial \beta} \right)$

$$\frac{1}{1 + (f(\beta))^2} \left(\frac{\partial f}{\partial \beta} \right) =$$

$$\frac{1}{1 + \left[\frac{2 \{ M_1^2 \sin^2(\beta) - 1 \}}{\tan(\beta) [2 + M_1^2 [\gamma + \cos(2\beta)]]} \right]^2} \left(\frac{2 [M_1^4 \sin^2(\beta) [1 + \gamma \cos(2\beta)] + M_1^2 (2 \cos(2\beta) + \gamma - 1) + 2]}{\sin^2(\beta) [2 + M_1^2 [\gamma + \cos(2\beta)]]^2} \right) =$$

$$\frac{2 [M_1^4 \sin^2(\beta) [1 + \gamma \cos(2\beta)] + M_1^2 (2 \cos(2\beta) + \gamma - 1) + 2]}{(\tan(\beta) [2 + M_1^2 [\gamma + \cos(2\beta)]]^2 + (2 \{ M_1^2 \sin^2(\beta) - 1 \})^2} \left(\frac{1}{\cos^2(\beta)} \right)$$

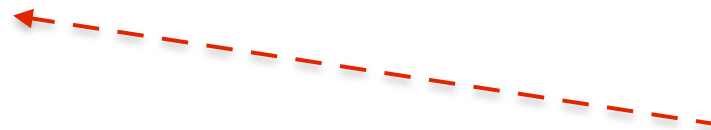
Max Deflection (“Wedge”) Angle and Detached Shockwaves (cont’d)

- Necessary condition for maximum:

$$\frac{2 \left[M_1^4 \sin^2(\beta) [1 + \gamma \cos(2\beta)] + M_1^2 (2 \cos(2\beta) + \gamma - 1) + 2 \right] \left(\frac{1}{\cos^2(\beta)} \right)}{\left(\tan(\beta) [2 + M_1^2 [\gamma + \cos(2\beta)]] \right)^2 + \left(2 \{ M_1^2 \sin^2(\beta) - 1 \} \right)^2} = 0$$

$$\Rightarrow \boxed{2 \left[M_1^4 \sin^2(\beta) [1 + \gamma \cos(2\beta)] + M_1^2 (2 \cos(2\beta) + \gamma - 1) + 2 \right] = 0}$$

- Let $\sin^2(\beta) = \frac{1}{2} [1 - \cos(2\beta)]$



- Plug in and simplify

Max Deflection (“Wedge”) Angle and Detached Shockwaves (cont’d)

- Necessary condition for maximum:

$$-\frac{\gamma}{2}M_1^4 [\cos(2\beta)]^2 + \frac{1}{2}M_1^2 (M_1^2(\gamma - 1) + 4)\cos(2\beta) + [2 + M_1^2(\gamma - 1)] = 0$$

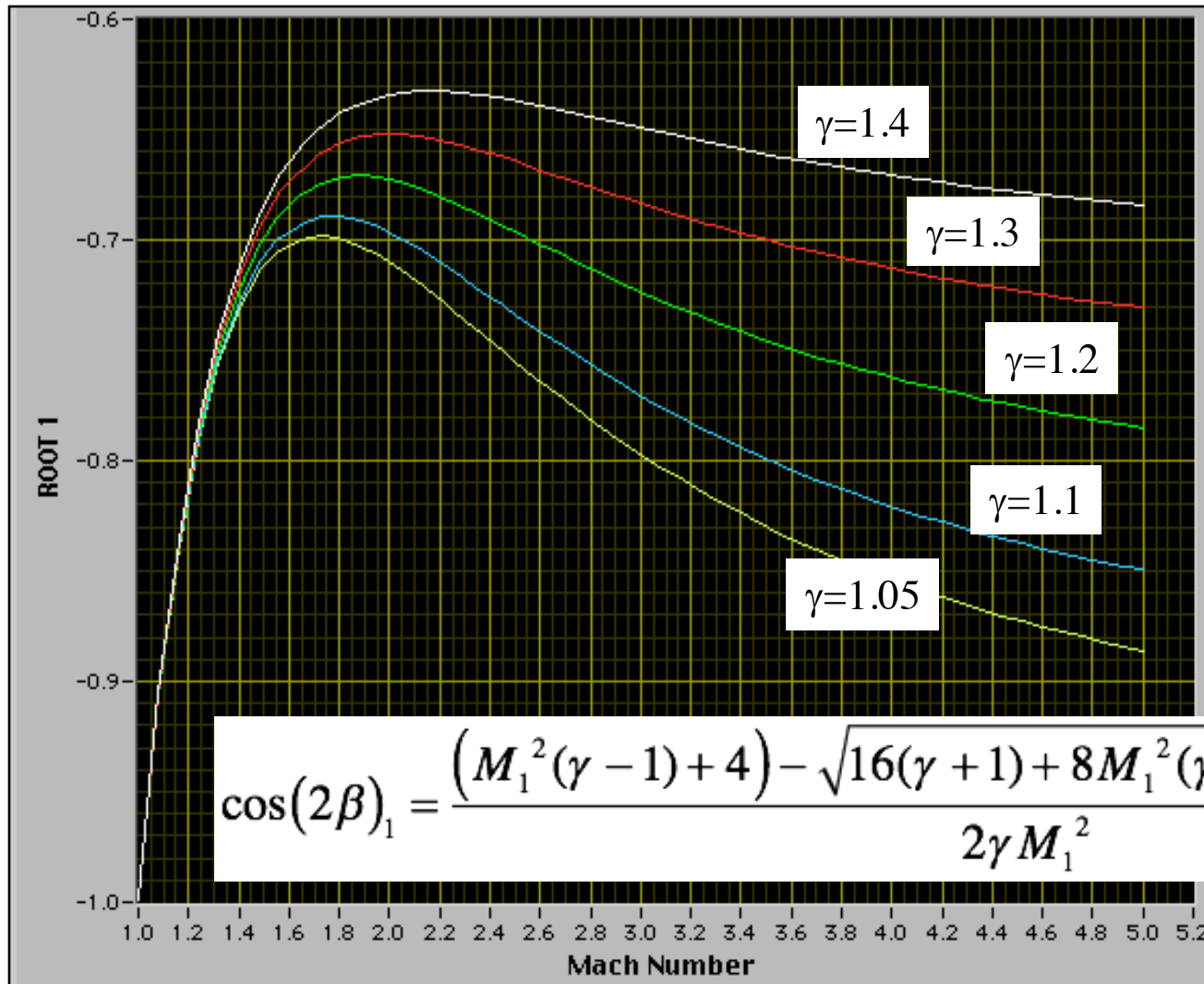
- Second Order Equation Solve for $\cos(2\beta)$

$$\cos(2\beta)_1 = \frac{(M_1^2(\gamma - 1) + 4) - \sqrt{16(\gamma + 1) + 8M_1^2(\gamma^2 - 1) + M_1^4(\gamma + 1)^2}}{2\gamma M_1^2}$$

$$\cos(2\beta)_2 = \frac{(M_1^2(\gamma - 1) + 4) + \sqrt{16(\gamma + 1) + 8M_1^2(\gamma^2 - 1) + M_1^4(\gamma + 1)^2}}{2\gamma M_1^2}$$

Which Root?

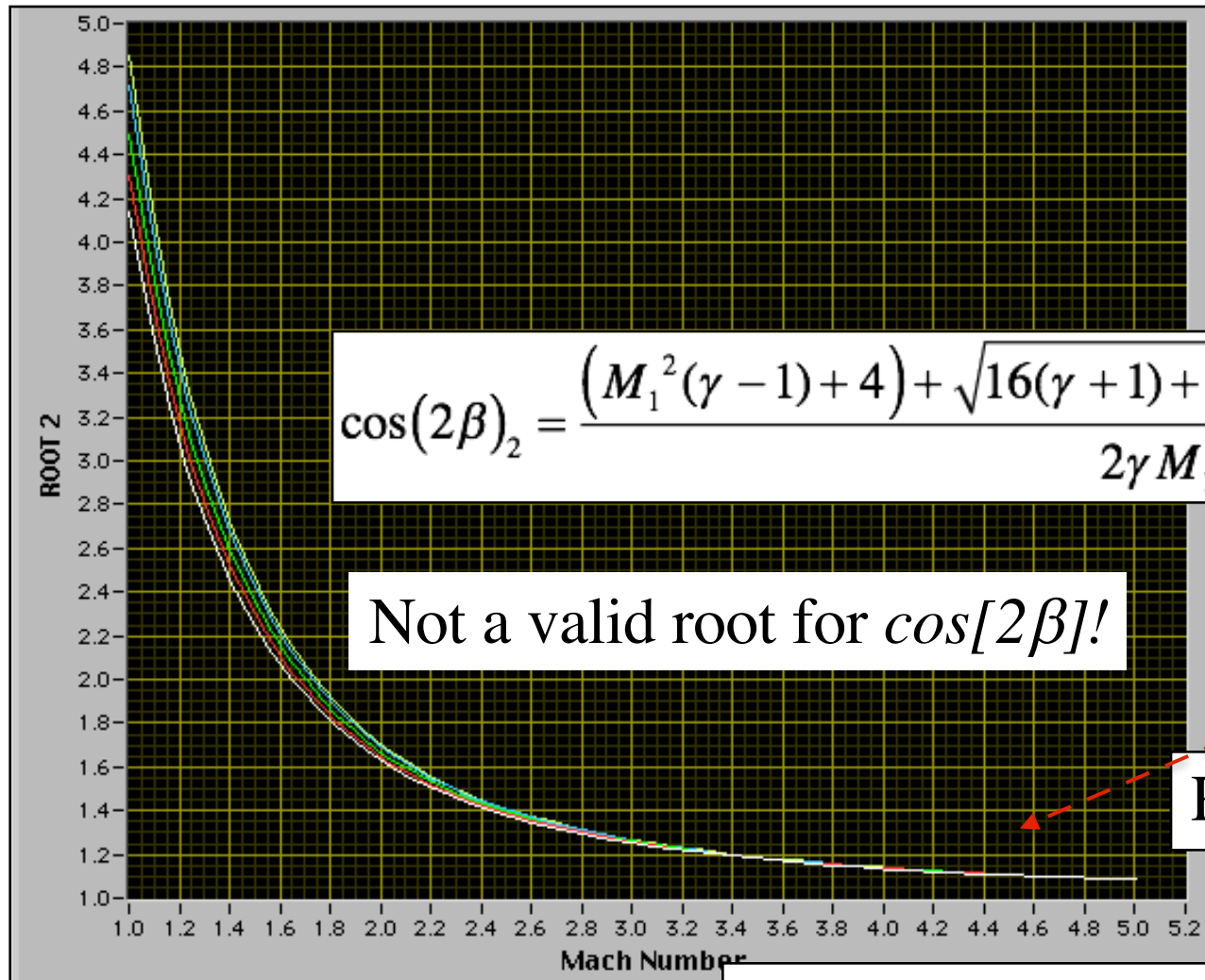
Max Deflection (“Wedge”) Angle and Detached Shockwaves (cont’d)



Root 1

$$\cos(2\beta)_1 = \frac{(M_1^2(\gamma - 1) + 4) - \sqrt{16(\gamma + 1) + 8M_1^2(\gamma^2 - 1) + M_1^4(\gamma + 1)^2}}{2\gamma M_1^2}$$

Max Deflection (“Wedge”) Angle and Detached Shockwaves (cont’d)



Root 2

$$\cos(2\beta)_2 = \frac{(M_1^2(\gamma - 1) + 4) + \sqrt{16(\gamma + 1) + 8M_1^2(\gamma^2 - 1) + M_1^4(\gamma + 1)^2}}{2\gamma M_1^2}$$

Root 1 is Proper root

$\|\cos(2\beta)\|$ must be < 1

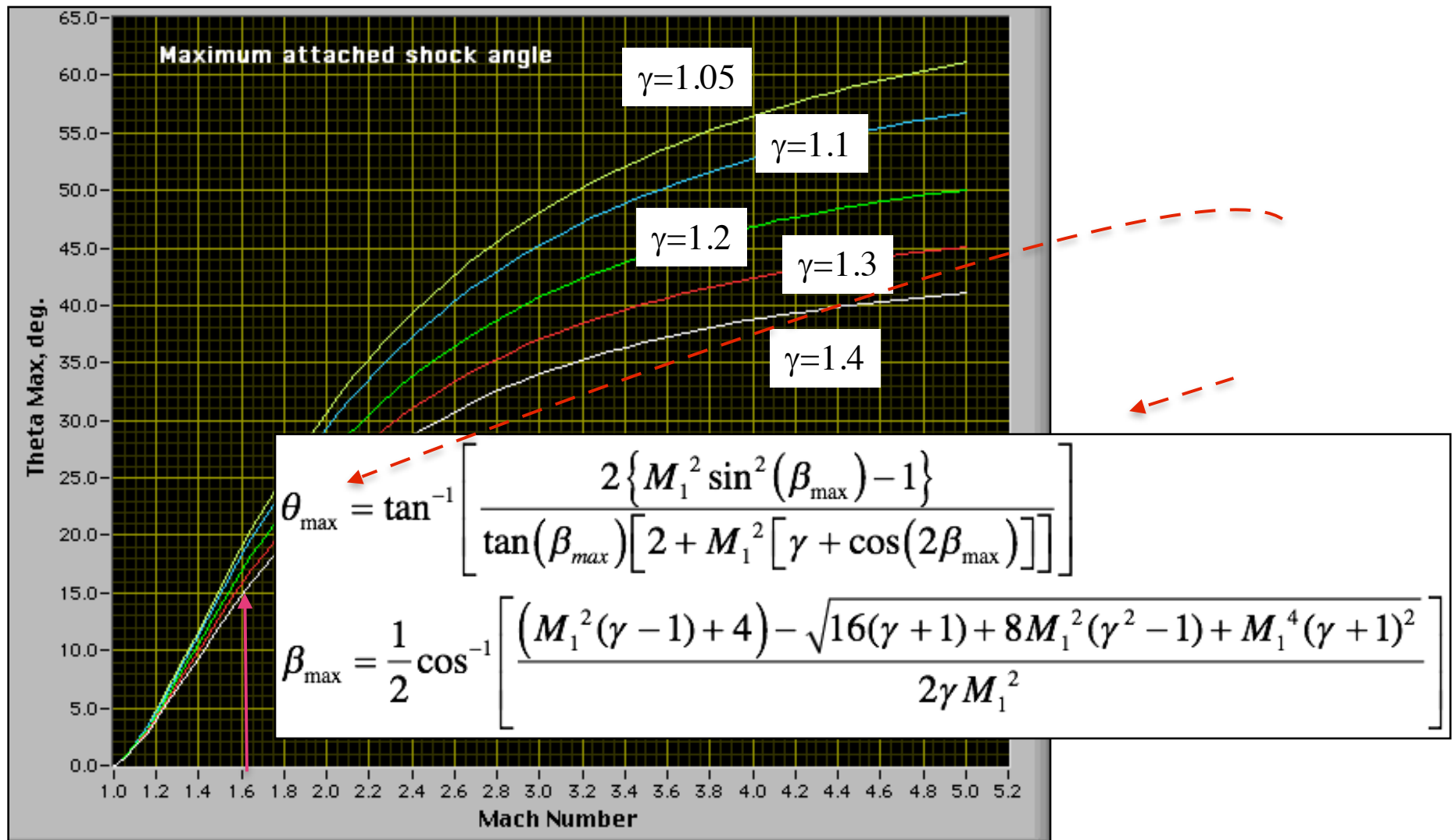
Max Deflection (“Wedge”) Angle and Detached Shockwaves (cont’d)

$$\beta_{\max} = \frac{1}{2} \cos^{-1} \left[\frac{(M_1^2(\gamma - 1) + 4) - \sqrt{16(\gamma + 1) + 8M_1^2(\gamma^2 - 1) + M_1^4(\gamma + 1)^2}}{2\gamma M_1^2} \right]$$

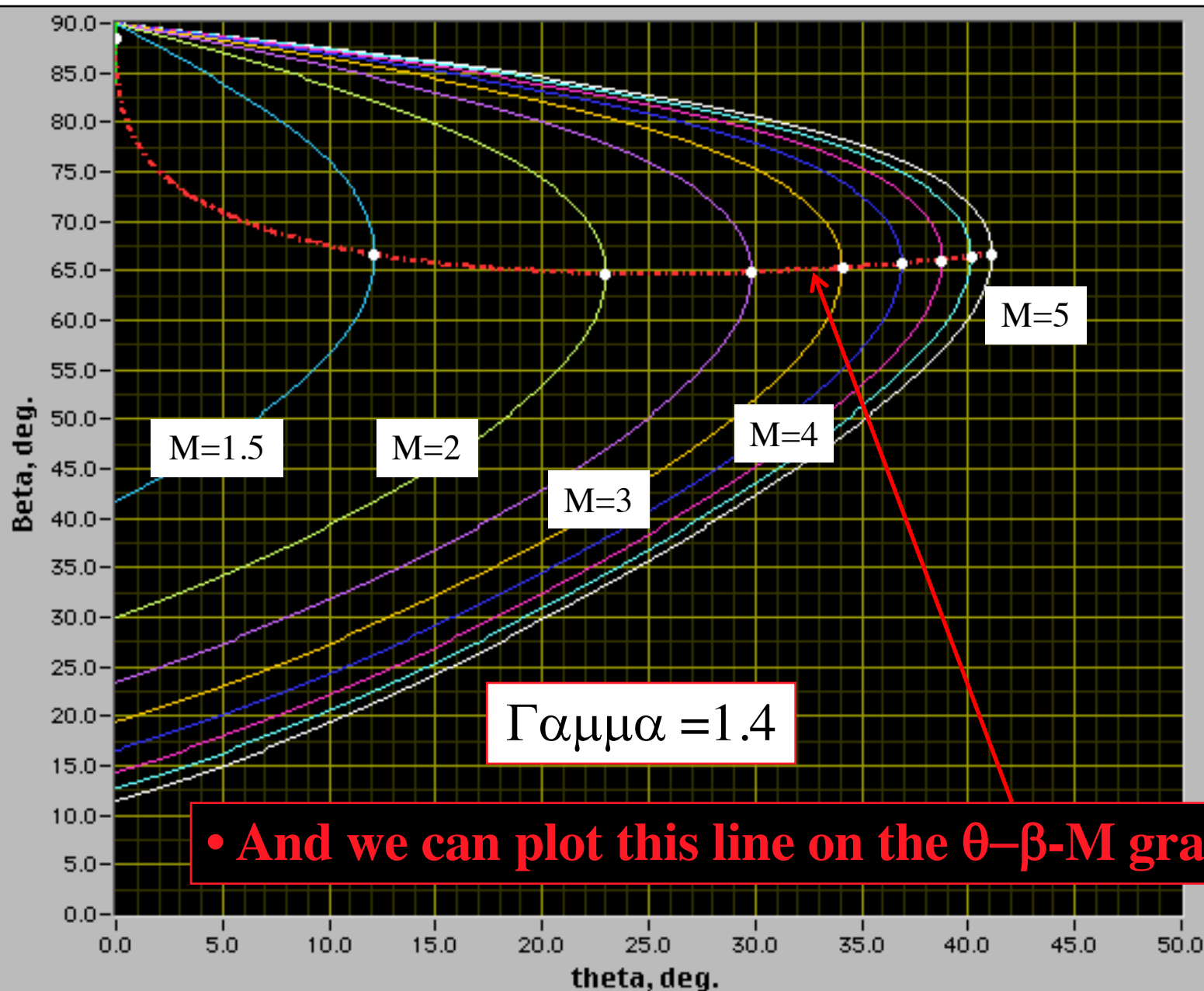
$$\theta_{\max} = \tan^{-1} \left[\frac{2 \{ M_1^2 \sin^2(\beta_{\max}) - 1 \}}{\tan(\beta_{\max}) [2 + M_1^2 [\gamma + \cos(2\beta_{\max})]]} \right]$$

- Explicit Solution for maximum wedge angle

Max Deflection (“Wedge”) Angle and Detached Shockwaves (continued)



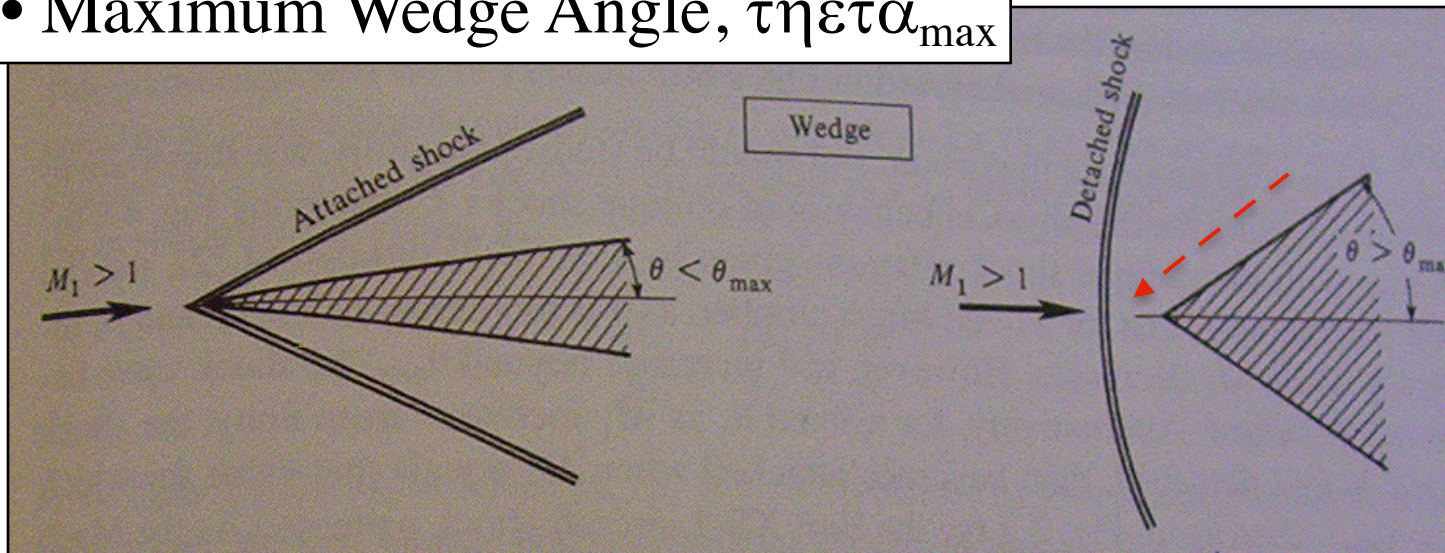
Max Deflection (“Wedge”) Angle (concluded)



• And we can plot this line on the θ - β - M graph

Criterion for Detached Shock Wave

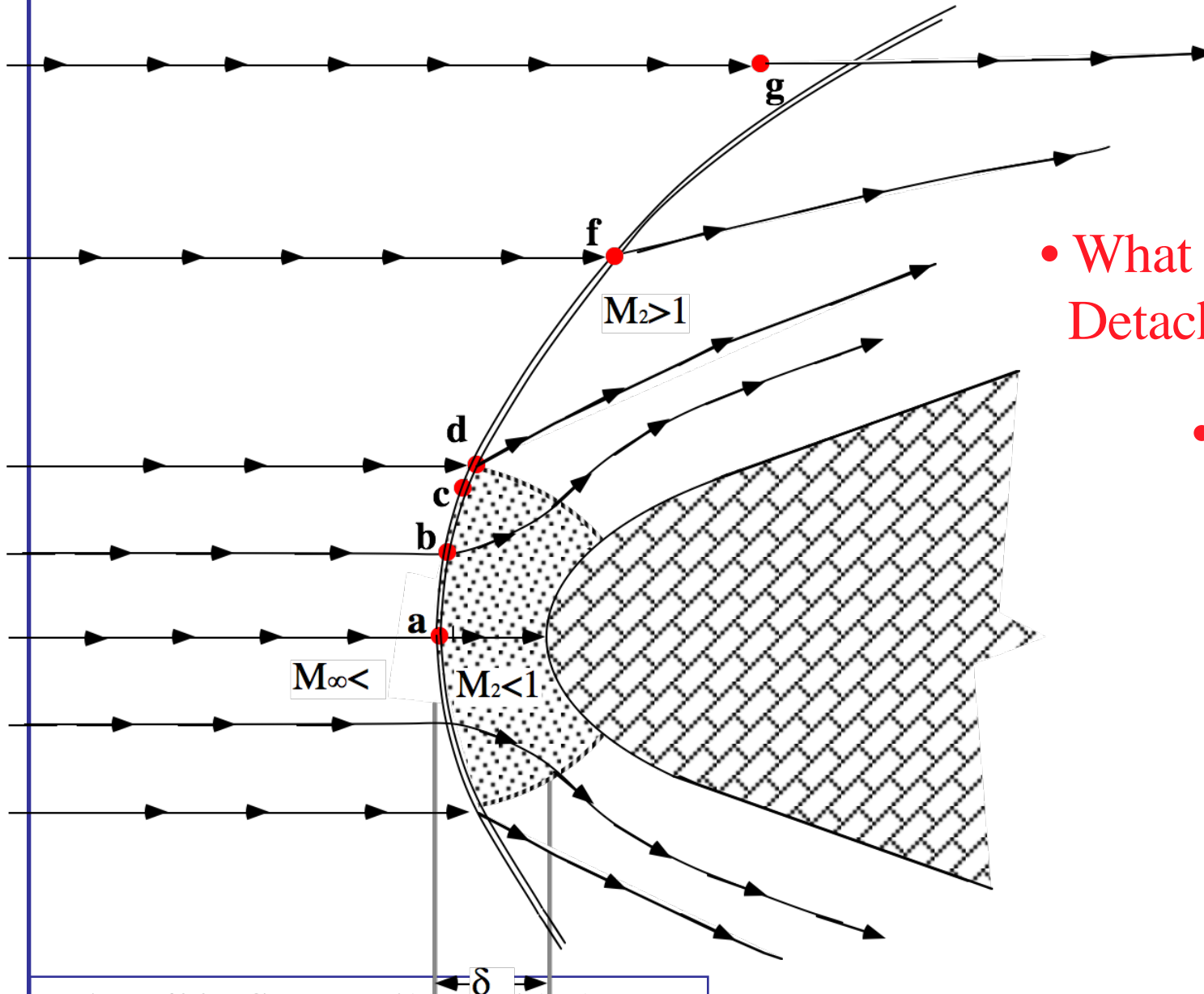
- Maximum Wedge Angle, $\theta_{\max}$



$$\beta_{\max} = \frac{1}{2} \cos^{-1} \left[\frac{(M_1^2(\gamma - 1) + 4) - \sqrt{16(\gamma + 1) + 8M_1^2(\gamma^2 - 1) + M_1^4(\gamma + 1)^2}}{2\gamma M_1^2} \right]$$

$$\theta_{\max} = \tan^{-1} \left[\frac{2 \{ M_1^2 \sin^2(\beta_{\max}) - 1 \}}{\tan(\beta_{\max}) [2 + M_1^2 [\gamma + \cos(2\beta_{\max})]]} \right]$$

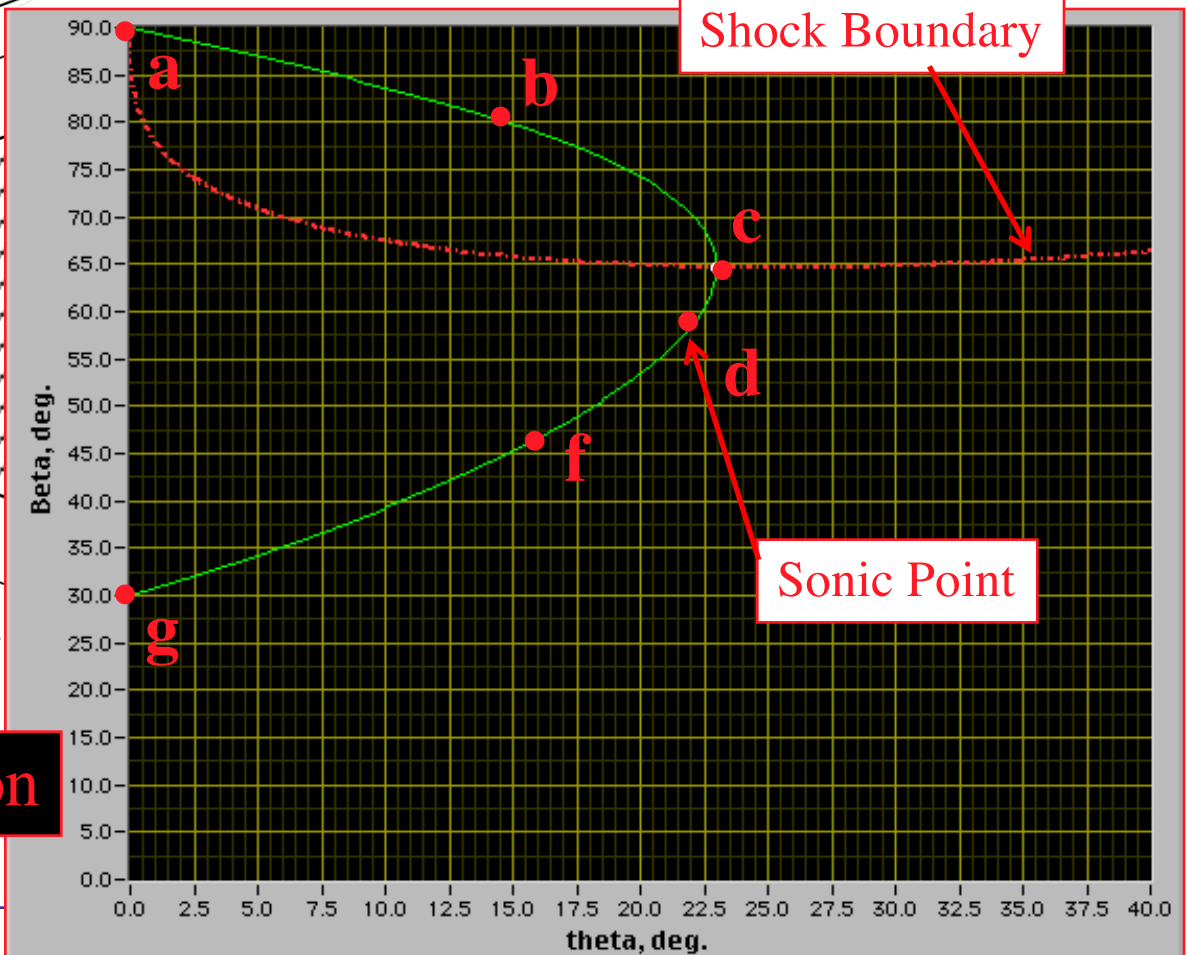
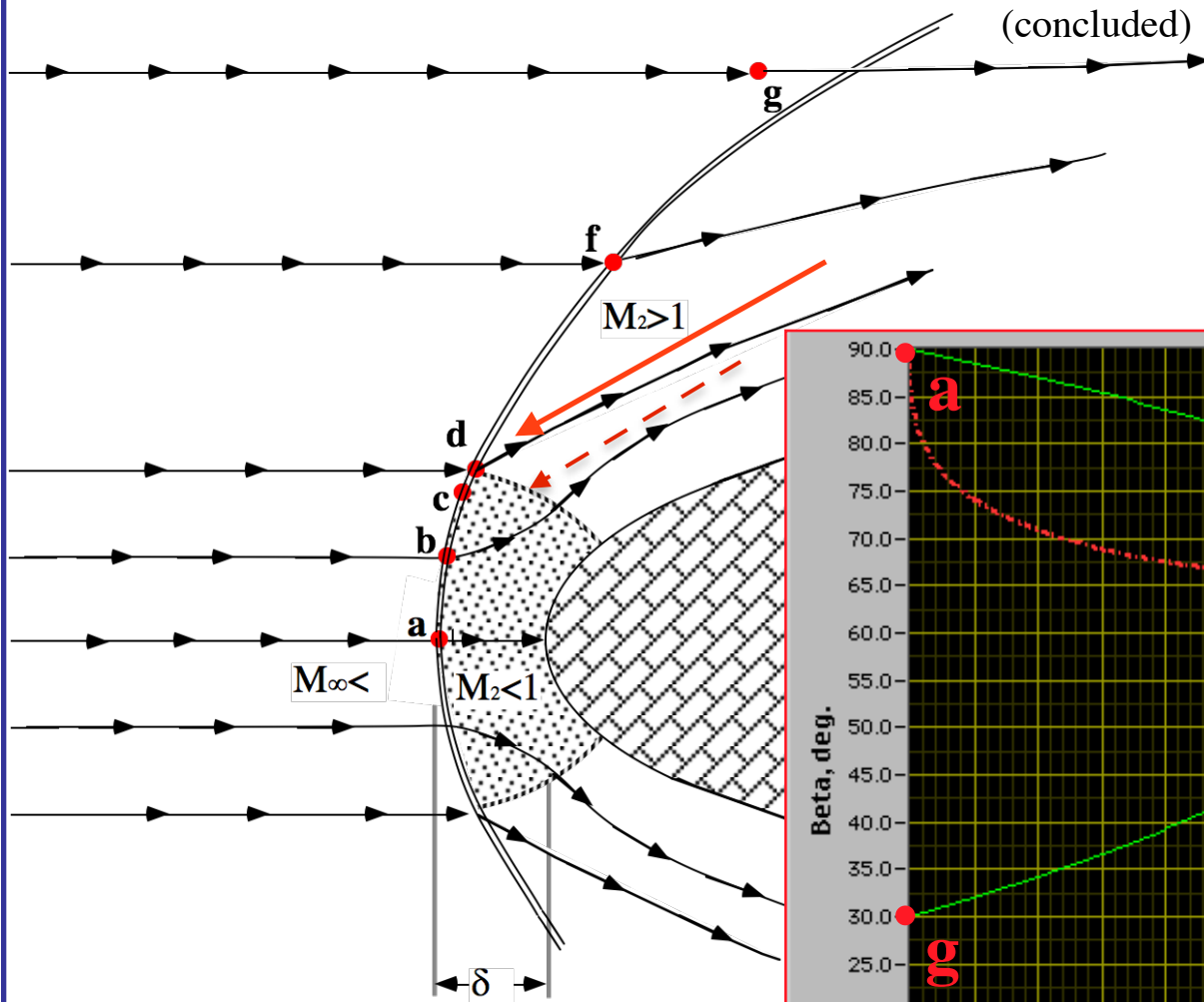
Flow Across Detached Shock Wave



- What Happens Behind Detached Shock Wave

- Mach 2.0

Flow Across Detached Shock Wave



• Find Sonic Line Criterion

Criterion for Sonic Point Behind ~~Detached~~ Shock

- from oblique shock theory

$$M_2 = \frac{M_{n_2}}{\sin(\beta - \theta)} \quad M_{n_2} = \sqrt{\frac{\left(1 + \frac{(\gamma - 1)}{2}(M_1 \sin \beta)^2\right)}{\left(\gamma(M_1 \sin \beta)^2 - \frac{(\gamma - 1)}{2}\right)}}$$

- Along sonic line $M_2=1$

$$\therefore \sin^2(\beta - \theta) = \frac{\left(1 + \frac{(\gamma - 1)}{2}(M_1 \sin \beta)^2\right)}{\left(\gamma(M_1 \sin \beta)^2 - \frac{(\gamma - 1)}{2}\right)}$$

Criterion for Sonic Point Behind Detached Shock (cont'd)

• But from derivations of section 6

$$\frac{\tan(\beta - \theta)}{\tan(\beta)} \nearrow = \frac{\left(2 + (\gamma - 1)[M_1 \sin(\beta)]^2\right)}{(\gamma + 1)[M_1 \sin(\beta)]^2}$$

$$\begin{aligned} \tan(\beta - \theta) &= \frac{\tan(\beta) \left(2 + (\gamma - 1)[M_1 \sin(\beta)]^2\right)}{(\gamma + 1)[M_1 \sin(\beta)]^2} \rightarrow \\ \sin(\beta - \theta) &= \cos(\beta - \theta) \frac{\tan(\beta) \left(2 + (\gamma - 1)[M_1 \sin(\beta)]^2\right)}{(\gamma + 1)[M_1 \sin(\beta)]^2} \end{aligned}$$

Criterion for Sonic Point Behind Detached Shock (cont'd)

- Square both sides

$$\sin^2(\beta - \theta) = \cos^2(\beta - \theta) \left[\frac{\tan(\beta) \left(2 + (\gamma - 1) [M_1 \sin(\beta)]^2 \right)}{(\gamma + 1) [M_1 \sin(\beta)]^2} \right]^2 \rightarrow$$

$$\sin^2(\beta - \theta) = [1 - \sin^2(\beta - \theta)] \left[\frac{\tan(\beta) \left(2 + (\gamma - 1) [M_1 \sin(\beta)]^2 \right)}{(\gamma + 1) [M_1 \sin(\beta)]^2} \right]^2$$

- Solve for $\sin^2(\beta - \theta)$

Criterion for Sonic Point Behind Detached Shock (cont'd)

- Solve for $\sin^2(\beta - \theta)$

$$\left[1 + \frac{\left[\tan(\beta) \left(2 + (\gamma - 1) [M_1 \sin(\beta)]^2 \right) \right]^2}{(\gamma + 1) [M_1 \sin(\beta)]^2} \right] \sin^2(\beta - \theta) = \left[\frac{\left[\tan(\beta) \left(2 + (\gamma - 1) [M_1 \sin(\beta)]^2 \right) \right]^2}{(\gamma + 1) [M_1 \sin(\beta)]^2} \right] \rightarrow$$

$$\left[\left((\gamma + 1) [M_1 \sin(\beta)]^2 \right)^2 + \left[\tan(\beta) \left(2 + (\gamma - 1) [M_1 \sin(\beta)]^2 \right) \right]^2 \right] \sin^2(\beta - \theta) =$$

$$\left[\tan(\beta) \left(2 + (\gamma - 1) [M_1 \sin(\beta)]^2 \right) \right]^2$$

$$\sin^2(\beta - \theta) = \frac{\left[\tan(\beta) \left(2 + (\gamma - 1) [M_1 \sin(\beta)]^2 \right) \right]^2}{\left[\left((\gamma + 1) [M_1 \sin(\beta)]^2 \right)^2 + \left[\tan(\beta) \left(2 + (\gamma - 1) [M_1 \sin(\beta)]^2 \right) \right]^2 \right]}$$

Criterion for Sonic Point Behind Detached Shock (cont'd)

- Compare Equations

Sonic Point

$$\sin^2(\beta - \theta) = \frac{\left(1 + \frac{(\gamma - 1)}{2}(M_1 \sin \beta)^2\right)}{\left(\gamma(M_1 \sin \beta)^2 - \frac{(\gamma - 1)}{2}\right)}$$

General Oblique shock

$$\sin^2(\beta - \theta) = \frac{\left[\tan(\beta)\left(2 + (\gamma - 1)[M_1 \sin(\beta)]^2\right)\right]^2}{\left[\left((\gamma + 1)[M_1 \sin(\beta)]^2\right)^2 + \left[\tan(\beta)\left(2 + (\gamma - 1)[M_1 \sin(\beta)]^2\right)\right]^2\right]}$$

Criterion for Sonic Point Behind Detached Shock (cont'd)

- Eliminate $\sin^2(\beta - \theta)$

$$\frac{\left(1 + \frac{(\gamma - 1)}{2}(M_1 \sin \beta)^2\right)}{\left(\gamma(M_1 \sin \beta)^2 - \frac{(\gamma - 1)}{2}\right)} =$$

- Implicit function for β at “sonic line”

- OK ... LET'S SOLVE FOR β

$$\frac{\left[\tan(\beta)\left(2 + (\gamma - 1)[M_1 \sin(\beta)]^2\right)\right]^2}{\left[\left((\gamma + 1)[M_1 \sin(\beta)]^2\right)^2 + \left[\tan(\beta)\left(2 + (\gamma - 1)[M_1 \sin(\beta)]^2\right)\right]^2\right]}$$

Criterion for Sonic Point Behind Detached Shock (cont'd)

- factor out $\cos(\beta)$ from $\tan(\beta)$

$$\left(1 + \frac{(\gamma - 1)}{2}(M_1 \sin \beta)^2\right) \left[\left((\gamma + 1)[M_1 \sin(\beta)]^2 \right)^2 + \left[\tan(\beta) \left(2 + (\gamma - 1)[M_1 \sin(\beta)]^2 \right) \right]^2 \right] =$$

$$\left(\gamma (M_1 \sin \beta)^2 - \frac{(\gamma - 1)}{2} \right) \left[\tan(\beta) \left(2 + (\gamma - 1)[M_1 \sin(\beta)]^2 \right) \right]^2 \rightarrow$$

$$\left(1 + \frac{(\gamma - 1)}{2}(M_1 \sin \beta)^2\right) \left[\cos^2(\beta) \left((\gamma + 1)[M_1 \sin(\beta)]^2 \right)^2 + \sin^2(\beta) \left[\left(2 + (\gamma - 1)[M_1 \sin(\beta)]^2 \right) \right]^2 \right] =$$

$$\sin^2(\beta) \left(\gamma (M_1 \sin \beta)^2 - \frac{(\gamma - 1)}{2} \right) \left[\left(2 + (\gamma - 1)[M_1 \sin(\beta)]^2 \right) \right]^2$$

- But $\cos^2(\beta) = 1 - \sin^2(\beta)$

Criterion for Sonic Point Behind Detached Shock (cont'd)

- Substituting in $\cos^2(\beta) = 1 - \sin^2(\beta)$

$$\left(1 + \frac{(\gamma - 1)}{2}(M_1 \sin \beta)^2\right) \left[\left[1 - \sin^2(\beta)\right] \left((\gamma + 1)[M_1 \sin(\beta)]^2\right)^2 + \sin^2(\beta) \left[\left(2 + (\gamma - 1)[M_1 \sin(\beta)]^2\right) \right]^2 \right] =$$

$$\sin^2(\beta) \left(\gamma (M_1 \sin \beta)^2 - \frac{(\gamma - 1)}{2} \right) \left[\left(2 + (\gamma - 1)[M_1 \sin(\beta)]^2\right) \right]^2$$

- Expand and Collect terms in $\sin^2(\beta)$

$$-\left\{ \gamma (\gamma^2 - 1) M_1^6 \right\} \left(\sin^2 \beta \right)^3 - \left\{ \left(\frac{\gamma + 1}{2} \right) M_1^4 (3 - 8\gamma + \gamma^2 + (\gamma^2 - 1) M_1^2) \right\} \left(\sin^2 \beta \right)^2 +$$

$$M_1^2 \left\{ 2(\gamma^2 - \gamma - 2) + (\gamma + 1)^2 M_1^2 \right\} \left(\sin^2 \beta \right) + 2(\gamma + 1) = 0$$

- CUBIC Polynomial in $\sin^2(\beta)$

Criterion for Sonic Point Behind Detached Shock (cont'd)

- 3-explicit solutions to polynomial

$$\left\{ \begin{array}{l} \left[\sin^2(\beta) \right]_1 \\ \left[\sin^2(\beta) \right]_2 \\ \left[\sin^2(\beta) \right]_3 \end{array} \right\} = \left\{ \begin{array}{l} -\frac{2}{(\gamma-1)M_1^2} \\ \frac{(\gamma-3)M_1^2 + (\gamma+1)M_1^4 - \sqrt{16\gamma M_1^4 + [(\gamma-3)M_1^2 + (\gamma+1)M_1^4]^2}}{4\gamma M_1^4} \\ \frac{(\gamma-3)M_1^2 + (\gamma+1)M_1^4 + \sqrt{16\gamma M_1^4 + [(\gamma-3)M_1^2 + (\gamma+1)M_1^4]^2}}{4\gamma M_1^4} \end{array} \right\}$$

- Which one is correct?

Criterion for Sonic Point Behind Detached Shock (cont'd)

- Solution 1

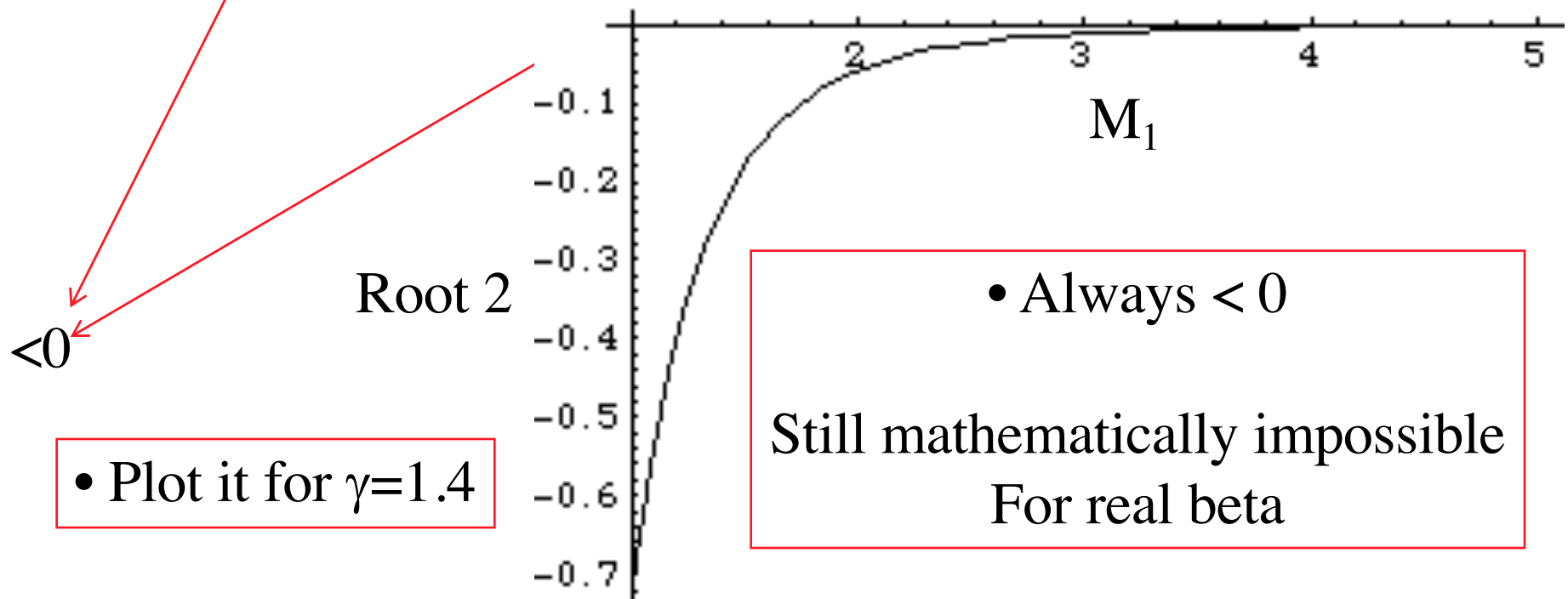
$$\left[\sin^2(\beta) \right]_1 \neq - \frac{2}{(\gamma - 1) M_1^2}$$

- Mathematically impossible for real valued beta!

Criterion for Sonic Point Behind Detached Shock (cont'd)

- Solution 2

$$[\sin^2(\beta)]_2 = \frac{(\gamma - 3)M_1^2 + (\gamma + 1)M_1^4 - \sqrt{16\gamma M_1^4 + [(\gamma - 3)M_1^2 + (\gamma + 1)M_1^4]^2}}{4\gamma M_1^4}$$



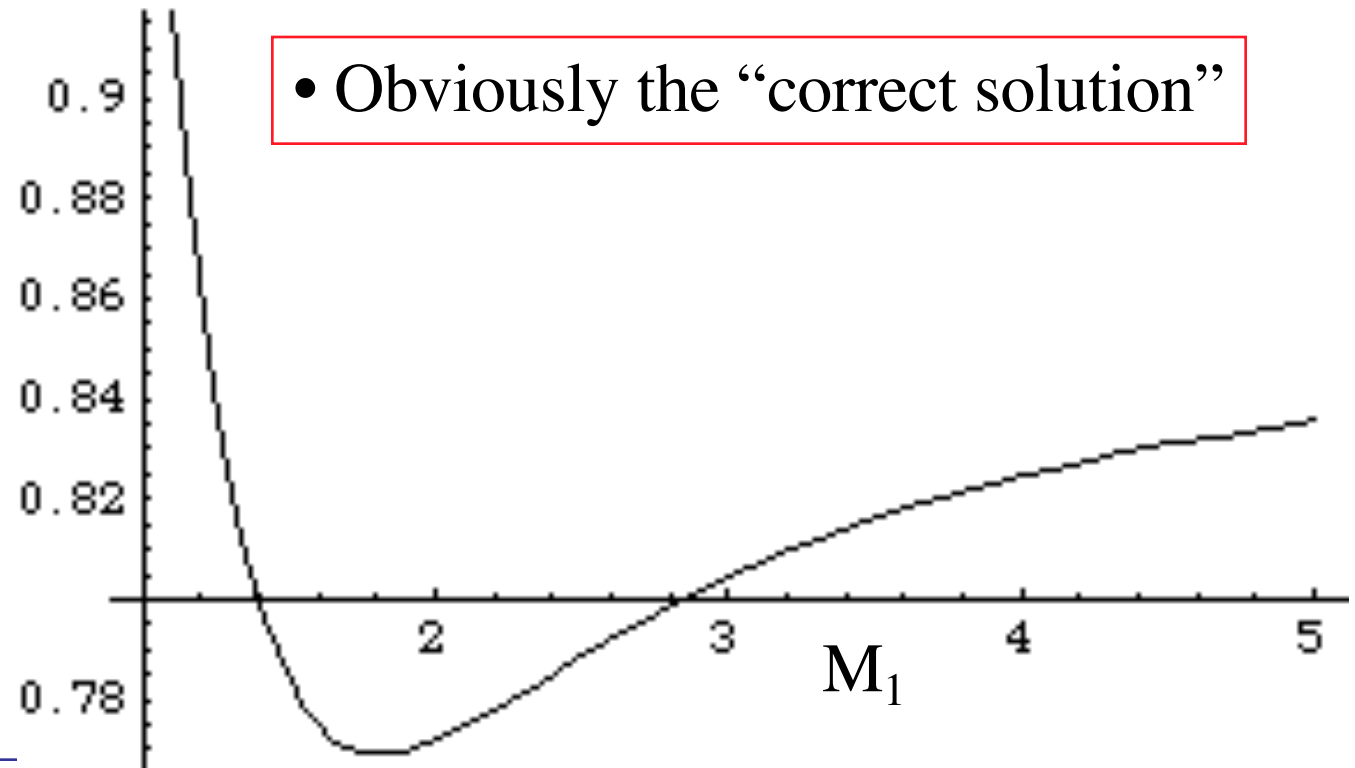
Criterion for Sonic Point Behind Detached Shock (cont'd)

- Solution 3

$$[\sin^2(\beta)]_3 = \frac{(\gamma - 3)M_1^2 + (\gamma + 1)M_1^4 + \sqrt{16\gamma M_1^4 + [(\gamma - 3)M_1^2 + (\gamma + 1)M_1^4]^2}}{4\gamma M_1^4}$$

Root 3

- Plot it for $\gamma=1.4$



Criterion for Sonic Point Behind Detached Shock (cont'd)

- Sonic Line Solution

$$\beta_{sonic} = \frac{180}{\pi} \sin^{-1} \sqrt{\frac{(\gamma - 3)M_1^2 + (\gamma + 1)M_1^4 + \sqrt{16\gamma M_1^4 + [(\gamma - 3)M_1^2 + (\gamma + 1)M_1^4]^2}}{4\gamma M_1^4}}$$

- Check Shock Relations

- Let $\gamma=1.4$, $M_1=2.0$

behind detached shock

$$\beta_{sonic} = \frac{180}{\pi} \sin^{-1} \left(\left(\frac{((1.4 - 3)(2^2) + (1.4 + 1)2^4) + (16 \cdot 1.4 \cdot 2^4 + ((1.4 - 3)(2^2) + (1.4 + 1)2^4)^2)^{0.5}}{4 \cdot 1.4 \cdot 2^4} \right)^{0.5} \right)$$

$$= 61.4854^\circ$$

Criterion for Sonic Point Behind Detached Shock (cont'd)

- Compute Wedge Angle

$$\beta_{sonic} = 61.4854^\circ$$

$$\tan(\theta) = \frac{2 \{ M_1^2 \sin^2(\beta) - 1 \}}{\tan(\beta) [2 + M_1^2 [\gamma + \cos(2\beta)]]}$$

Sonic *Sonic*

$$\frac{180}{\pi} \operatorname{atan} \left(\frac{2 \left(2^2 \left(\sin^2 \left(\frac{\pi}{180} 61.4854 \right) \right) - 1 \right)}{\left(\tan \left(\frac{\pi}{180} 61.4854 \right) \right) \left(2 + 2^2 \left(1.4 + \cos \left(2 \frac{\pi}{180} 61.4854 \right) \right) \right)} \right) = 22.706$$

Criterion for Sonic Point Behind Detached Shock (cont'd)

- Compute M_{n_2}

$$\beta_{sonic} = 61.4854^\circ$$

$$M_{n_2} = \sqrt{\frac{\left(1 + \frac{(\gamma - 1)}{2} (M_1 \sin \beta)^2\right)}{\left(\gamma (M_1 \sin \beta)^2 - \frac{(\gamma - 1)}{2}\right)}}$$

$$= \left(\frac{1 + \frac{1.4 - 1}{2} \left(\left(2 \left(\sin \left(\frac{\pi}{180} 61.4854 \right) \right) \right)^2 \right)}{1.4 \left(\left(2 \left(\sin \left(\frac{\pi}{180} 61.4854 \right) \right) \right)^2 \right) - \frac{(1.4 - 1)}{2}} \right)^{0.5} = 0.6263$$

Criterion for Sonic Point Behind Detached Shock (cont'd)

- Compute M_2 Downstream of Shock

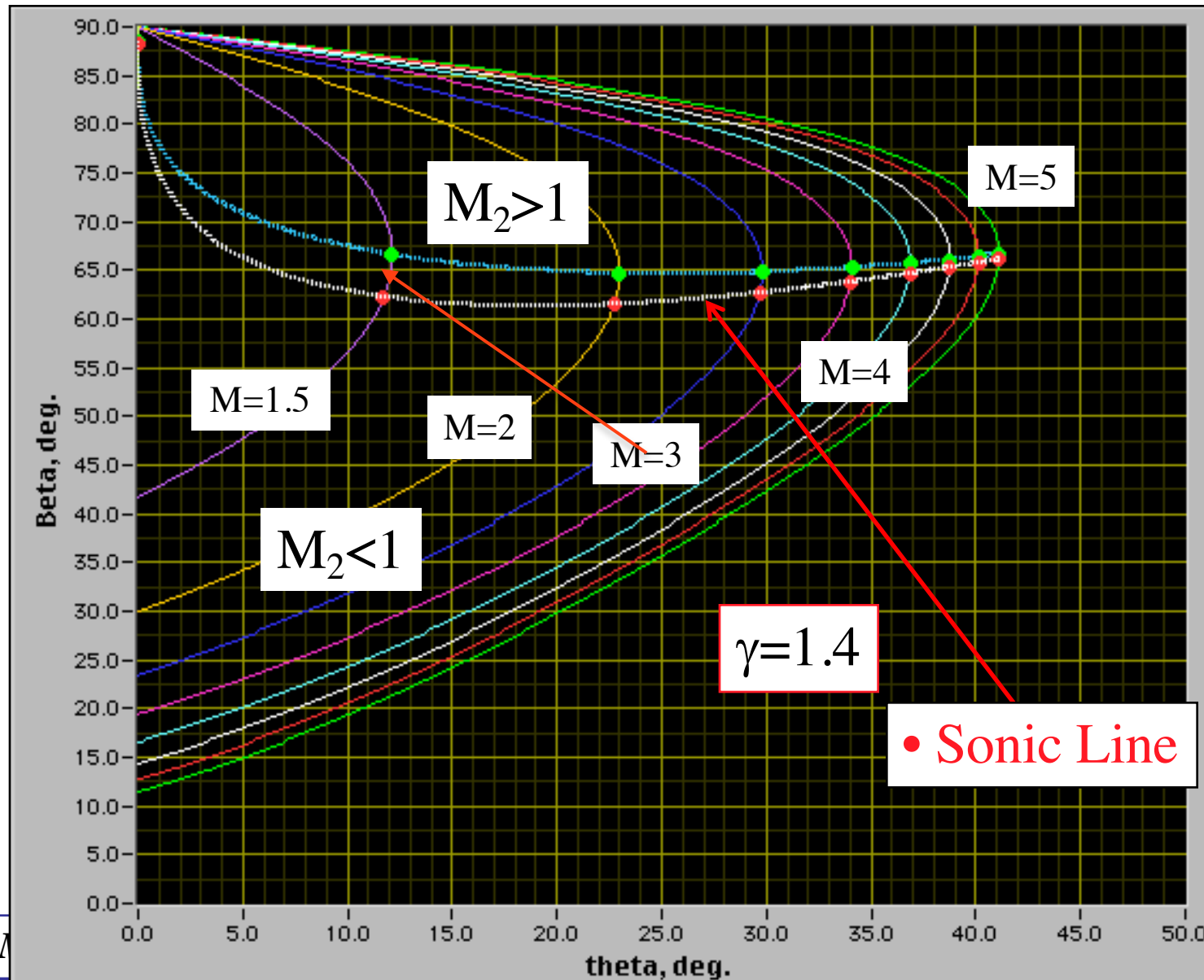
$$\beta_{sonic} = 61.4854^\circ \quad \theta = 22.706^\circ \quad M_{n_2} = 0.6263$$

$$M_2 = \frac{M_{n_2}}{\sin(\beta - \theta)} = \frac{0.62632}{\sin\left(\frac{\pi}{180} (61.4854 - 22.706)\right)}$$

=1.0000 check!

- And we can plot this line on the θ - β - M graph

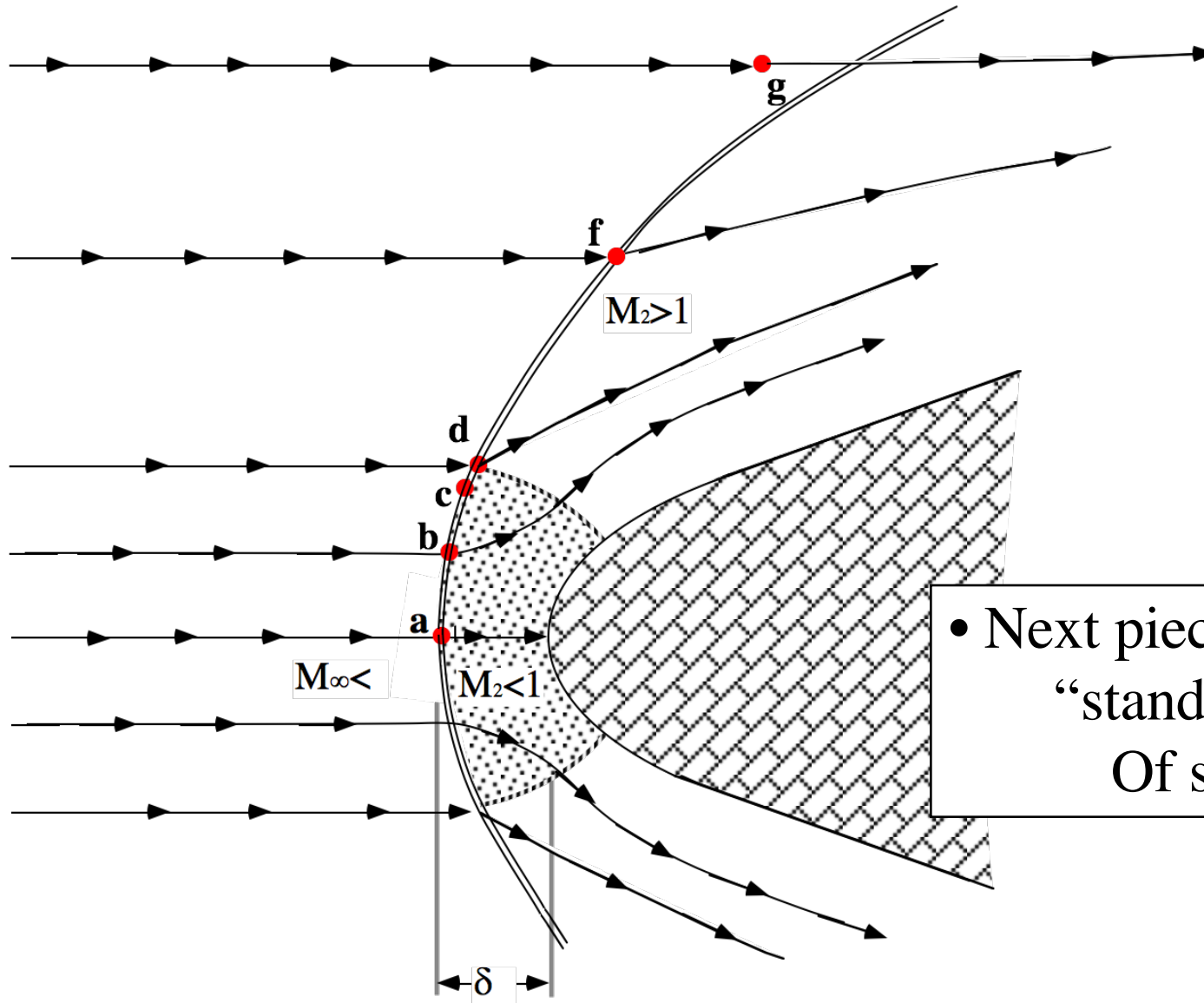
Criterion for Sonic Point Behind Detached Shock (cont'd)



(concluded)



Detached Wave Standoff Distance

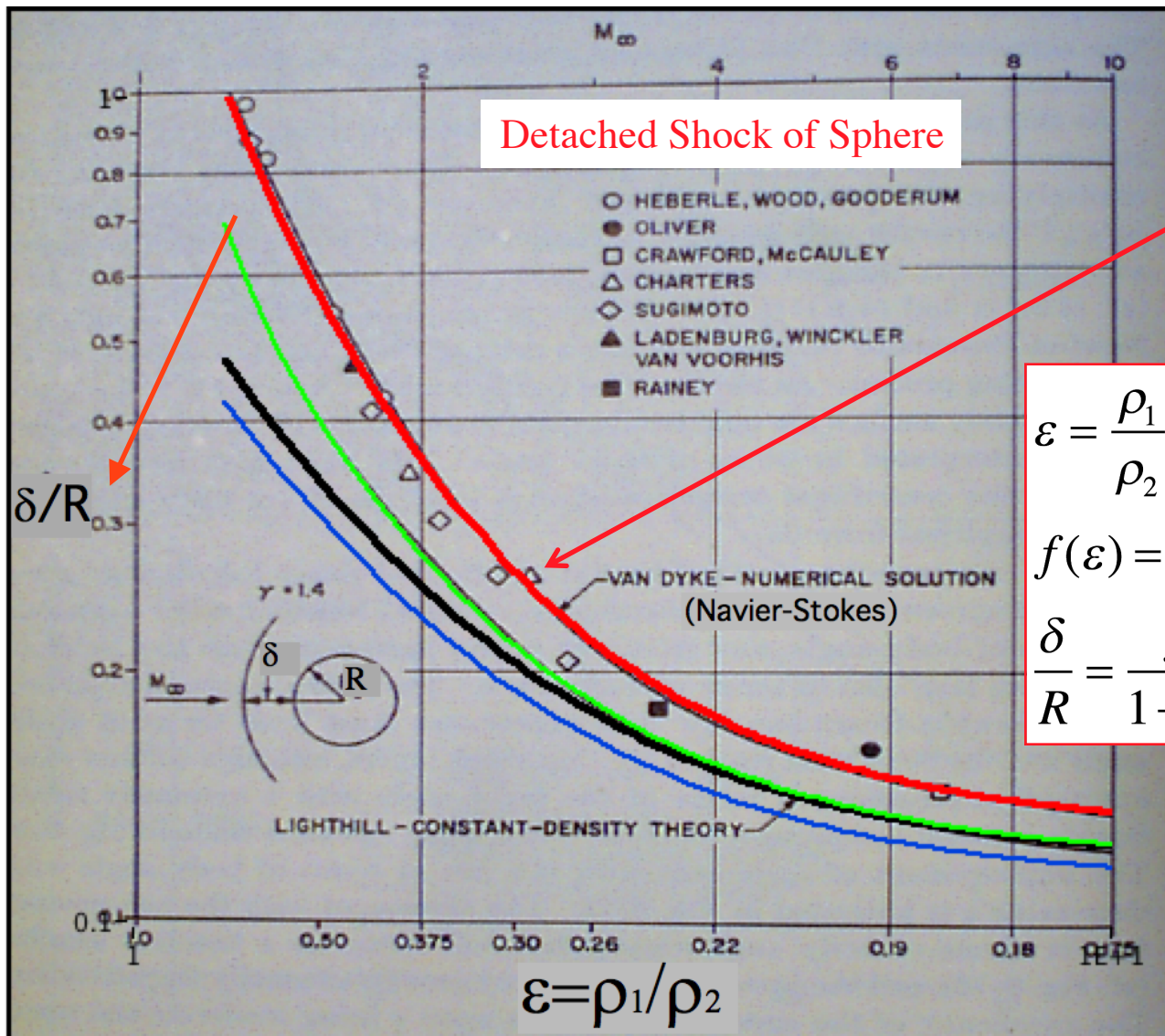


- Next piece of puzzle is the “standoff distance” Of shockwave

Detached Wave Standoff Distance

(cont'd)

- Complex Theory .. But simple approximations are possible



$$\epsilon = \rho_1/\rho_2$$

- Curve fit of N/S equations

$$\epsilon = \frac{\rho_1}{\rho_2} \rightarrow$$

$$f(\epsilon) = \epsilon \left[3\epsilon + 1.6\epsilon^{3/2} - (3\epsilon)^{1/2} + 1 \right]$$

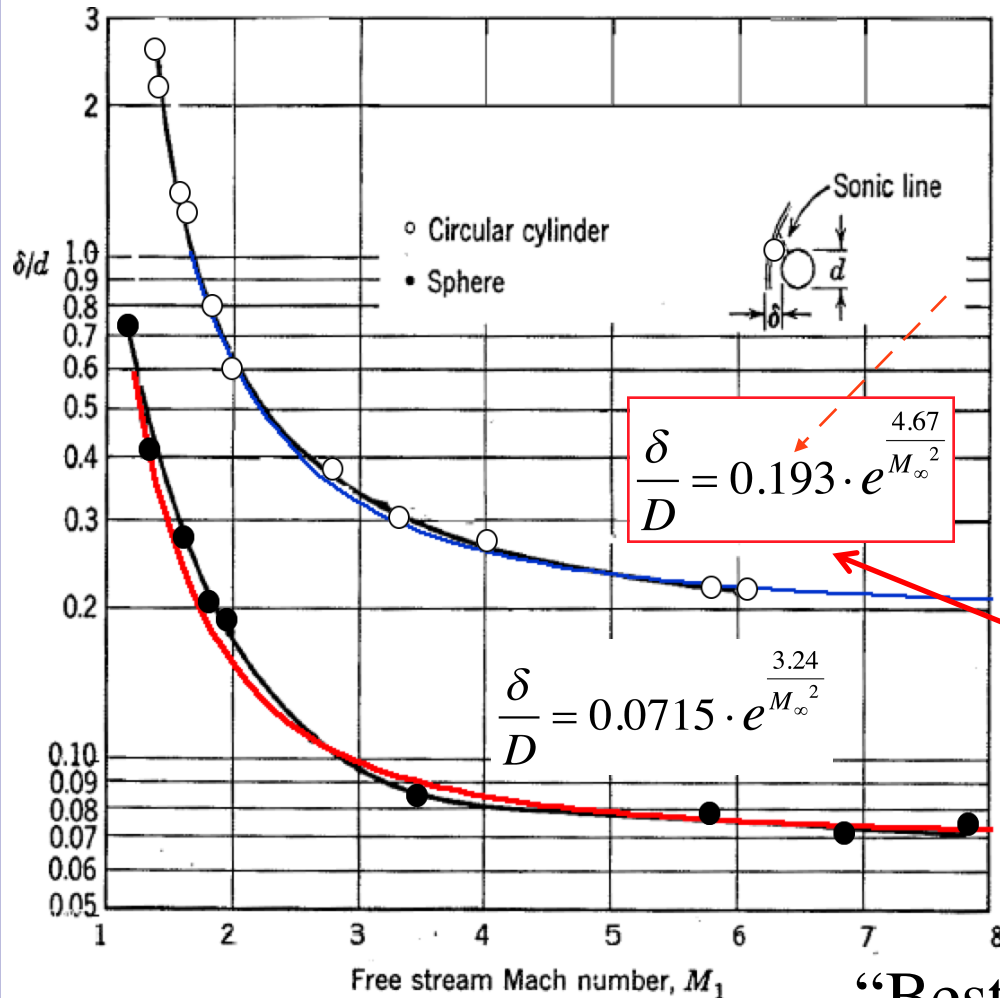
$$\frac{\delta}{R} = \frac{f(\epsilon)}{1 - f(\epsilon)}$$

- Ref: Hypersonic Flow Theory
Hayes and Probstein,
Academic press, 1959

Detached Wave Standoff Distance

(cont'd)

- Complex Theory .. But simple approximations are possible



Data From:

Solomon, G. E., Shock Wave Detachment Distances for Plane and Axially Symmetric Flow, NACA Tech Note 3213 (1952)

Correlations by:

Billig, F., S., "Shock Wave Shapes Around Spherical- and Cylindrical-Nosed Bodies," Journal of Spacecraft and Rockets, Vol 4., No. 6, June 1967, pp 822-823.

Cylinder-Wedge,
Sphere-Cone, air
 $\gamma = 1.4$

“Best simple correlation” available

2-D Shock Wave Shape

- Extending the Correlations by Billig

-- 2-D Shockwave (cylinder-wedge)

Stand - Off - Distance

$$\dots\dots\dots \frac{\delta}{D} = 0.193 \cdot e^{\frac{4.67}{M_\infty^2}}$$

Radius - of - Curvature

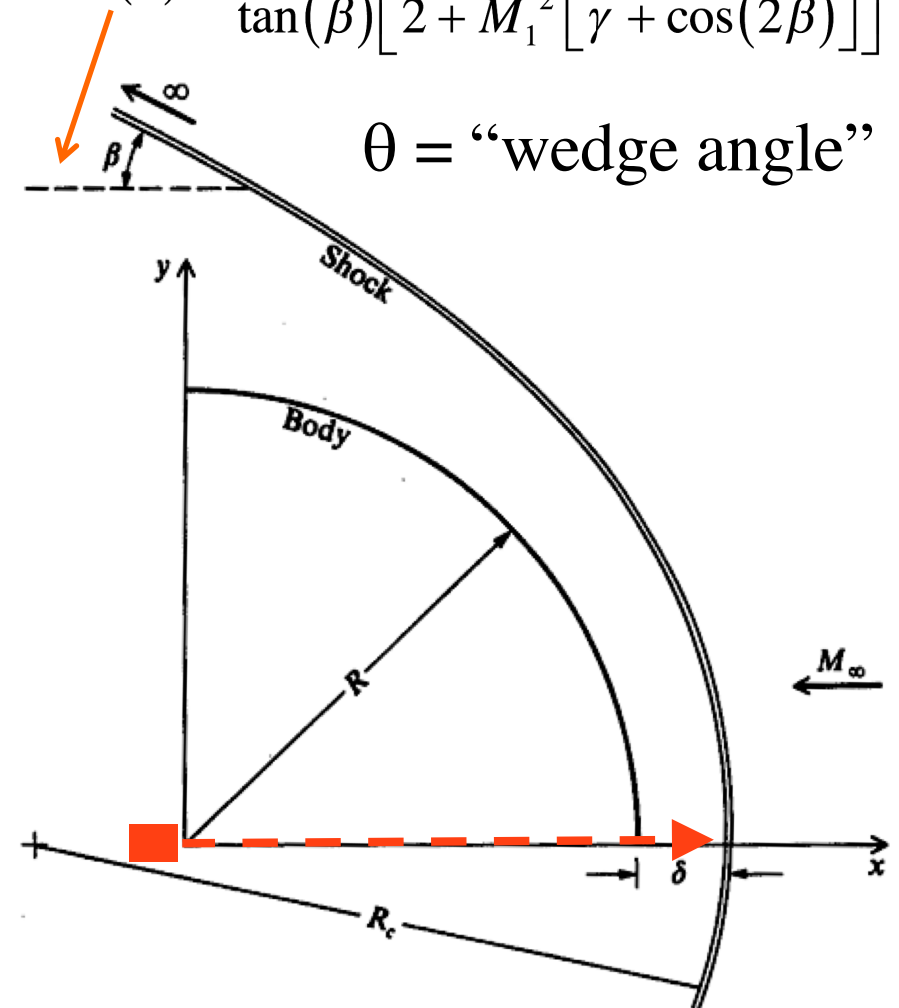
$$\dots\dots\dots \frac{R_c}{D} = 0.693 \cdot e^{\frac{1.8}{(M_\infty - 1)^{0.75}}}$$

Shape

$$\dots\dots\dots x = \frac{1}{2}D + \delta - R_c \frac{\left(\sqrt{1 + \frac{y^2 \tan^2 \beta}{R_c^2}} - 1 \right)}{\tan^2 \beta}$$

$$\tan(\theta) = \frac{2 \{ M_1^2 \sin^2(\beta) - 1 \}}{\tan(\beta) [2 + M_1^2 [\gamma + \cos(2\beta)]]}$$

θ = “wedge angle”



3-D Shock Wave Shape

- Extending the Correlations by Billig
 - 3-D Shockwave (sphere-cone)

We'll defer the discussion on conical shockwave until later

Stand - Off - Distance

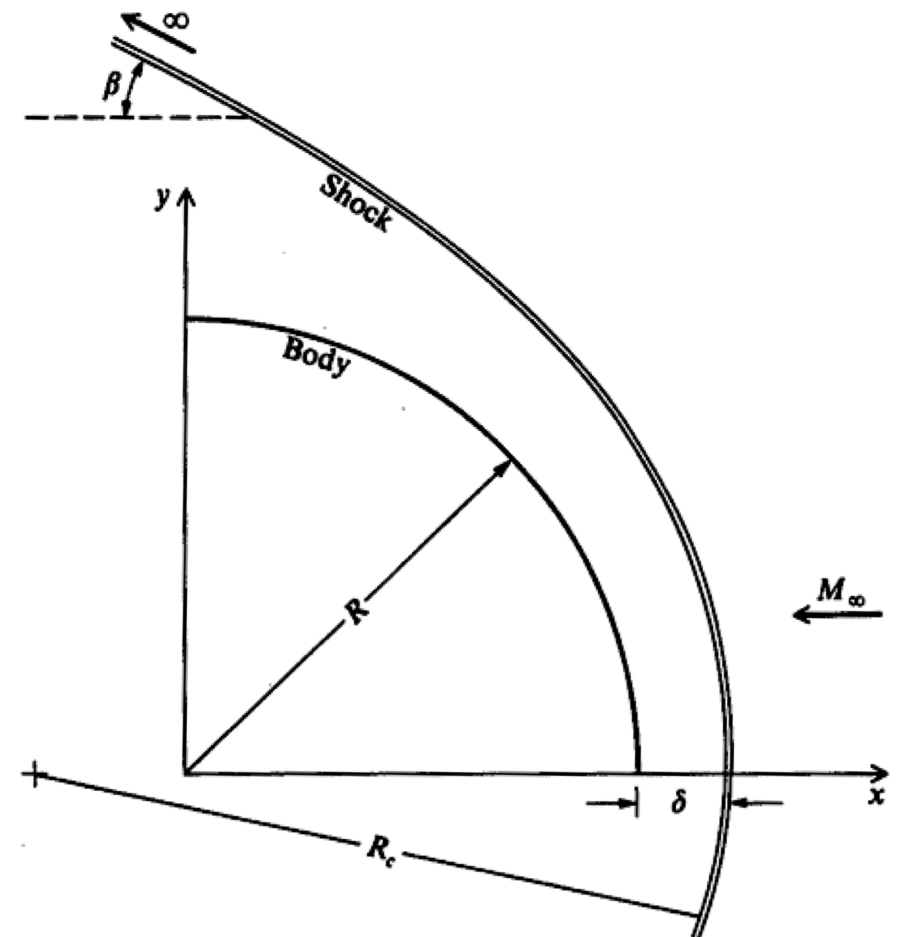
$$\dots\dots\dots \frac{\delta}{D} = 0.0715 \cdot e^{\frac{3.24}{M_\infty^2}}$$

Radius - of - Curvature

$$\dots\dots\dots \frac{R_c}{D} = 0.5715 \cdot e^{\frac{0.54}{(M_\infty - 1)^{1.2}}}$$

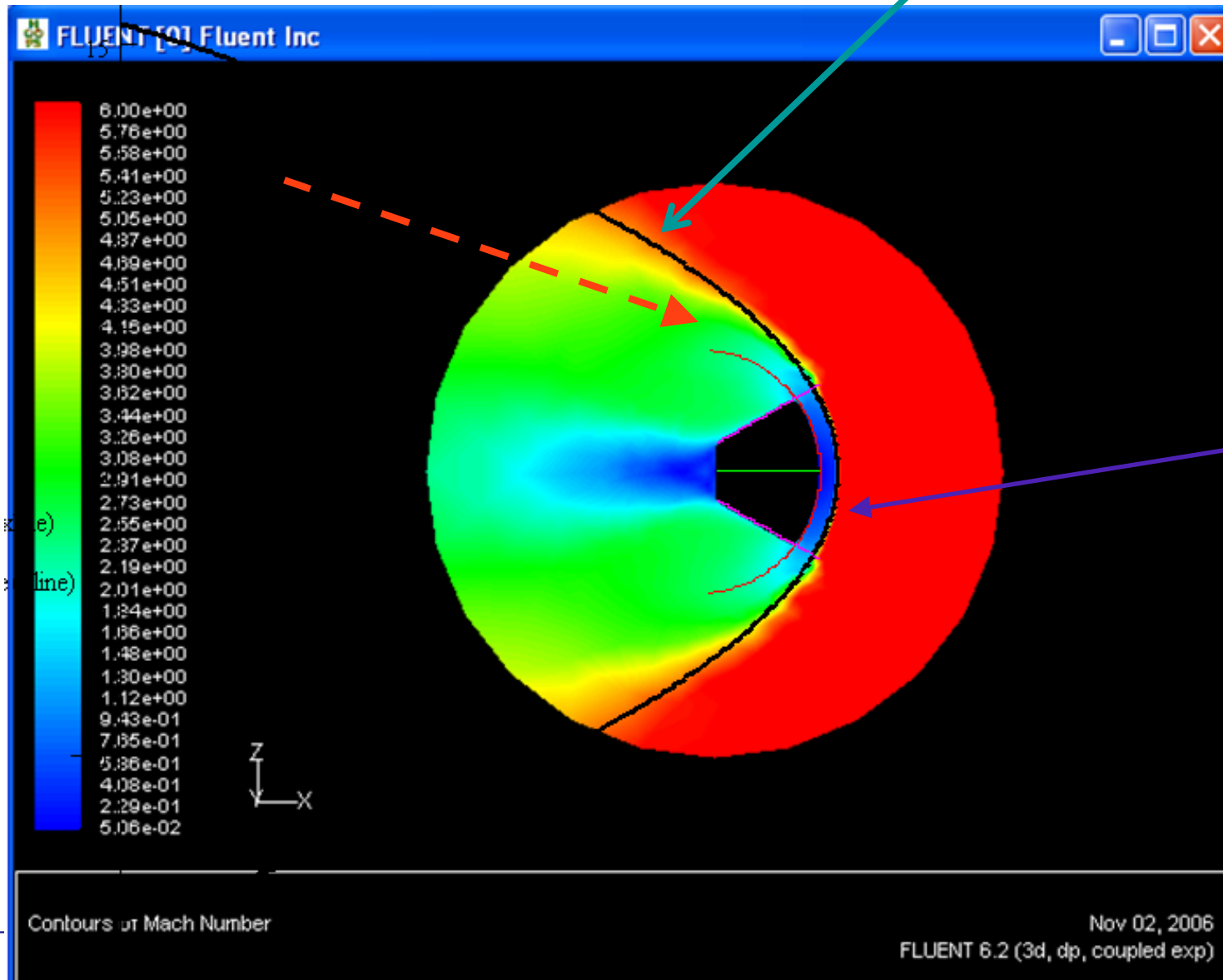
Shape

$$\dots\dots\dots x = \frac{1}{2}D + \delta - R_c \frac{\left(\sqrt{1 + \frac{y^2 \tan^2 \beta}{R_c^2}} - 1 \right)}{\tan^2 \beta}$$



3-D Shock Wave Shape (cont'd)

- 3-D Shockwave (sphere-cone) Correlations by Billig



- CEV Mach 6 Shock wave image

Collected Expressions for Detached Shockwave

- Boundary Between Strong and Weak Shock

$$\beta_{\max} = \frac{1}{2} \cos^{-1} \left[\frac{(M_1^2(\gamma - 1) + 4) - \sqrt{16(\gamma + 1) + 8M_1^2(\gamma^2 - 1) + M_1^4(\gamma + 1)^2}}{2\gamma M_1^2} \right]$$

- Wave Angle at Sonic Line

$$\beta_{\text{sonic}} = \frac{180}{\pi} \sin^{-1} \sqrt{\frac{(\gamma - 3)M_1^2 + (\gamma + 1)M_1^4 + \sqrt{16\gamma M_1^4 + [(\gamma - 3)M_1^2 + (\gamma + 1)M_1^4]^2}}{4\gamma M_1^4}}$$

- Shock Wave Offset Distance

$$\frac{\delta}{D} = 0.193 \cdot e^{\frac{4.67}{M_\infty^2}}$$

2-D

$$\frac{\delta}{D} = 0.0715 \cdot e^{\frac{3.24}{M_\infty^2}}$$

3-D

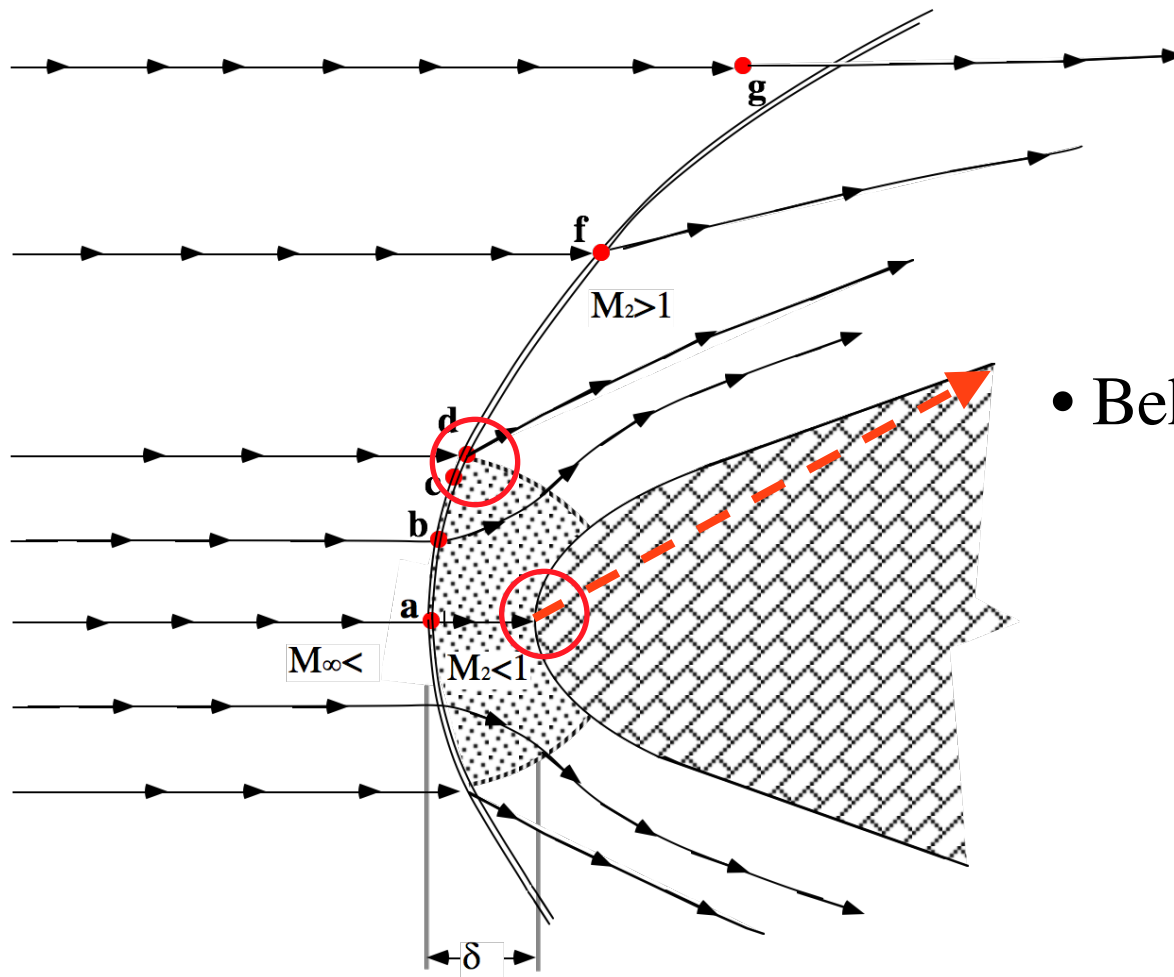
Collected Expressions for Detached Shockwaves (cont'd)

- *from Oblique Shock relations*

$$\theta_{\substack{\text{max} \\ \text{sonic}}} = \tan^{-1} \left[\frac{2 \left\{ M_1^2 \sin^2(\beta_{\substack{\text{max} \\ \text{sonic}}}) - 1 \right\}}{\tan(\beta_{\substack{\text{max} \\ \text{sonic}}}) \left[2 + M_1^2 \left[\gamma + \cos(2\beta_{\substack{\text{max} \\ \text{sonic}}}) \right] \right]} \right]$$

Applies for both Beta_max and Beta_sonic

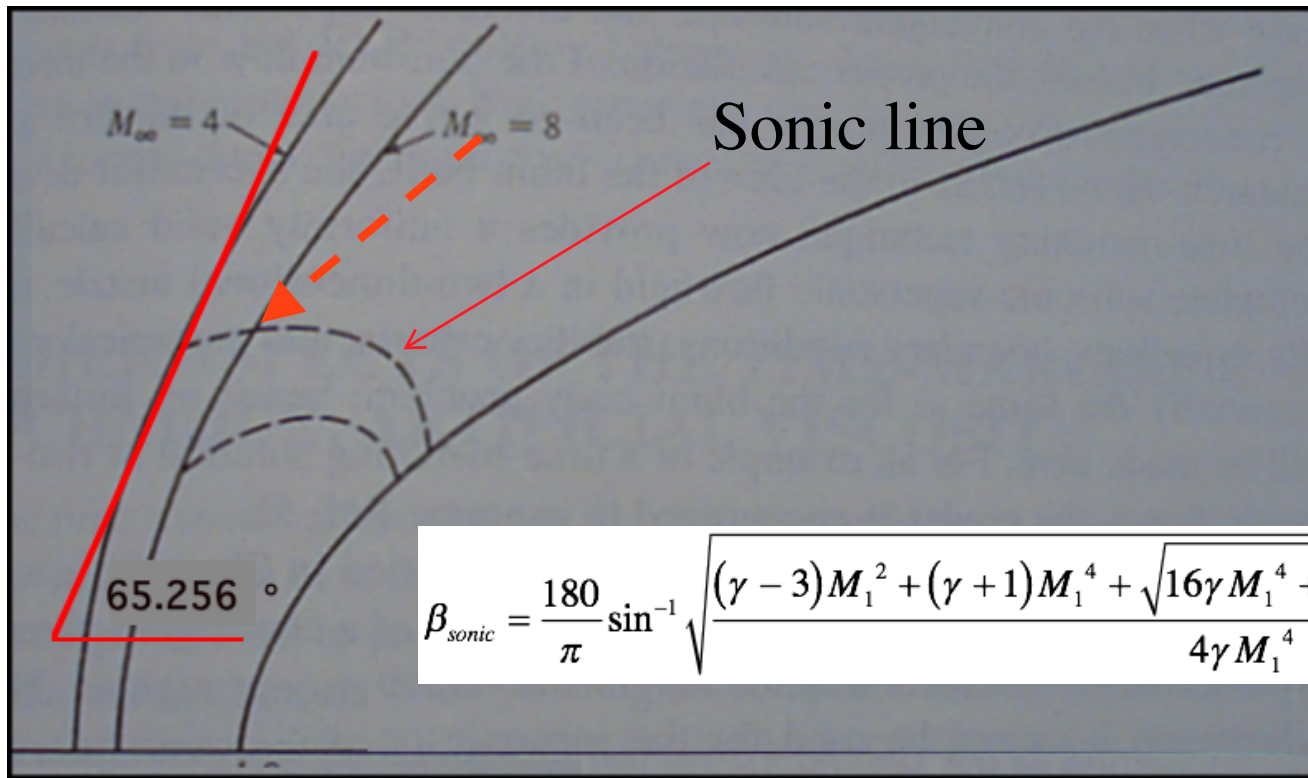
Surface Conditions behind detached shock wave: *sonic point*



- Find Surface Point where $M=1$
- Behind Shock at Sonic Point
- $M=1.0$

Surface Conditions behind detached shock wave: *sonic point* (cont'd)

- Example: Mach 4.0 flow across parabolic cylinder



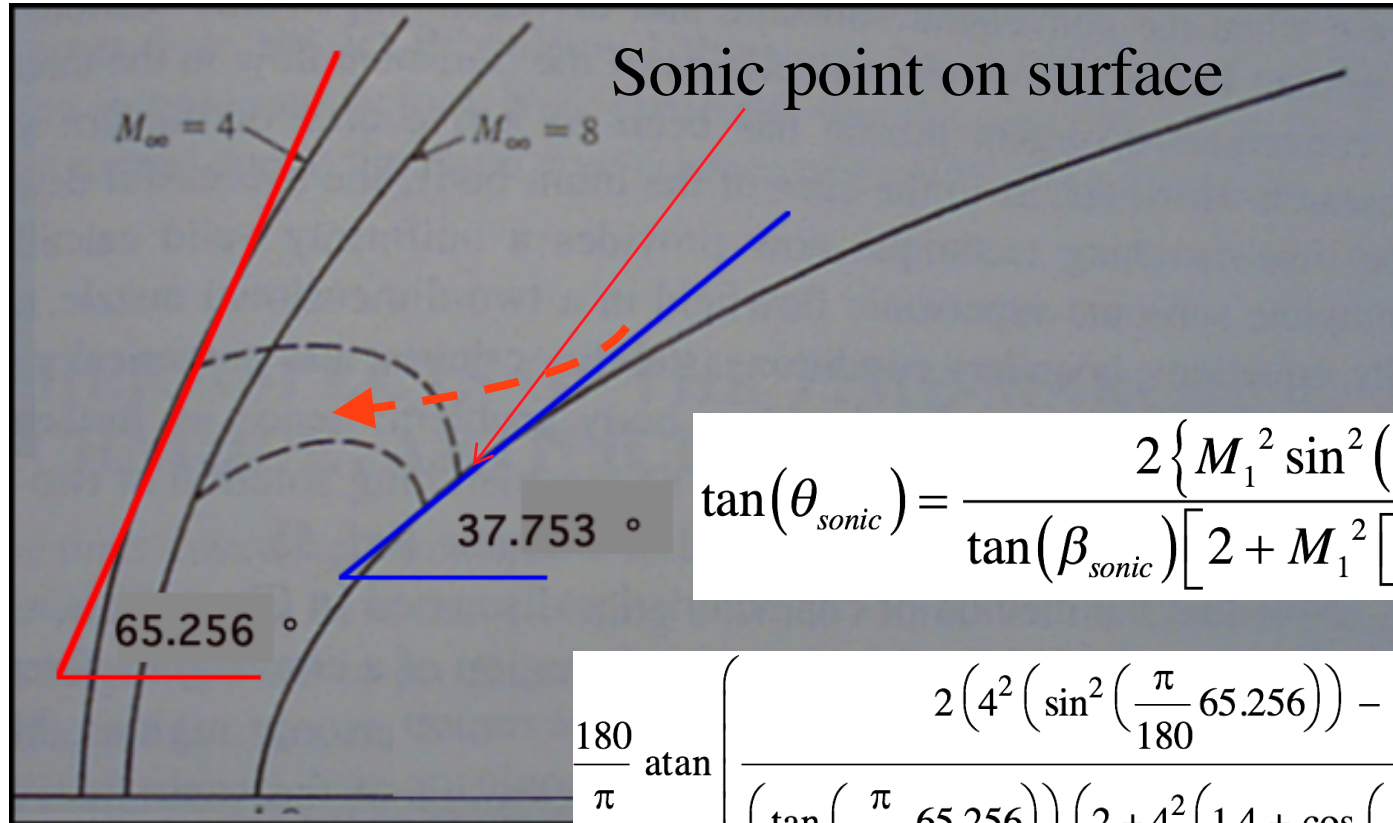
$$\beta_{sonic} = \frac{180}{\pi} \sin^{-1} \sqrt{\frac{(\gamma - 3)M_1^2 + (\gamma + 1)M_1^4 + \sqrt{16\gamma M_1^4 + [(\gamma - 3)M_1^2 + (\gamma + 1)M_1^4]^2}}{4\gamma M_1^4}} =$$

$$\frac{180}{\pi} \sin^{-1} \left(\left((1.4 - 3) (4^2) + (1.4 + 1) 4^4 \right) + \left(16 \cdot 1.4 \cdot 4^4 + \left((1.4 - 3) (4^2) + (1.4 + 1) 4^4 \right)^2 \right)^{0.5} / 4 \cdot 1.4 \cdot 4^4 \right)^{0.5} = 65.256^\circ$$

Surface Conditions behind

detached shock wave: *sonic point* (cont'd)

- Example: Mach 4.0 flow across parabolic cylinder



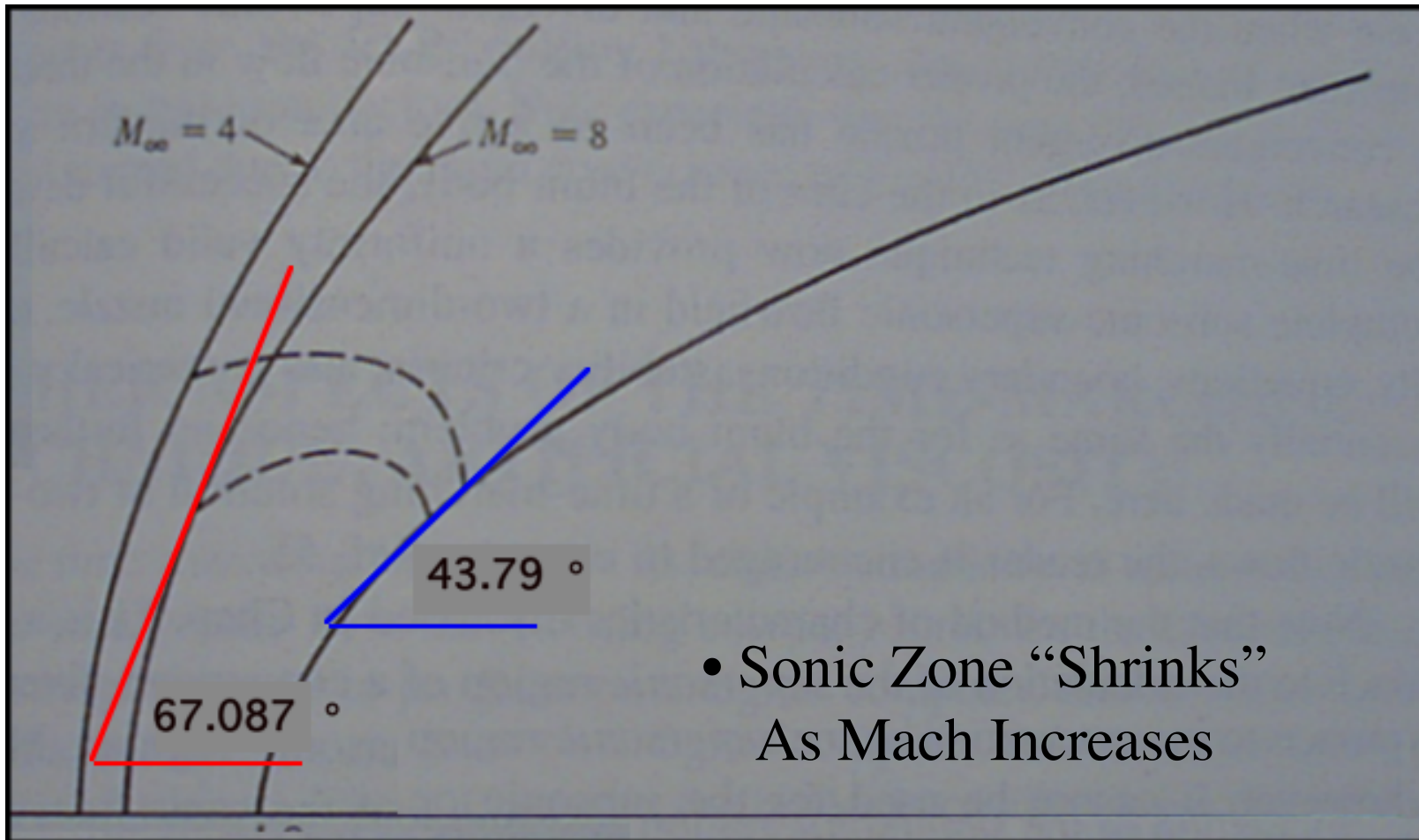
$$\tan(\theta_{sonic}) = \frac{2 \{ M_1^2 \sin^2(\beta_{sonic}) - 1 \}}{\tan(\beta_{sonic}) [2 + M_1^2 [\gamma + \cos(2\beta_{sonic})]]} =$$

$$\frac{180}{\pi} \operatorname{atan} \left(\frac{2 \left(4^2 \left(\sin^2 \left(\frac{\pi}{180} 65.256 \right) \right) - 1 \right)}{\left(\tan \left(\frac{\pi}{180} 65.256 \right) \right) \left(2 + 4^2 \left(1.4 + \cos \left(\frac{2\pi}{180} 65.256 \right) \right) \right)} \right)$$

$$= 37.753^\circ$$

Surface Conditions behind detached shock wave: *sonic point* (cont'd)

- Example: Mach 8.0 flow across parabolic cylinder

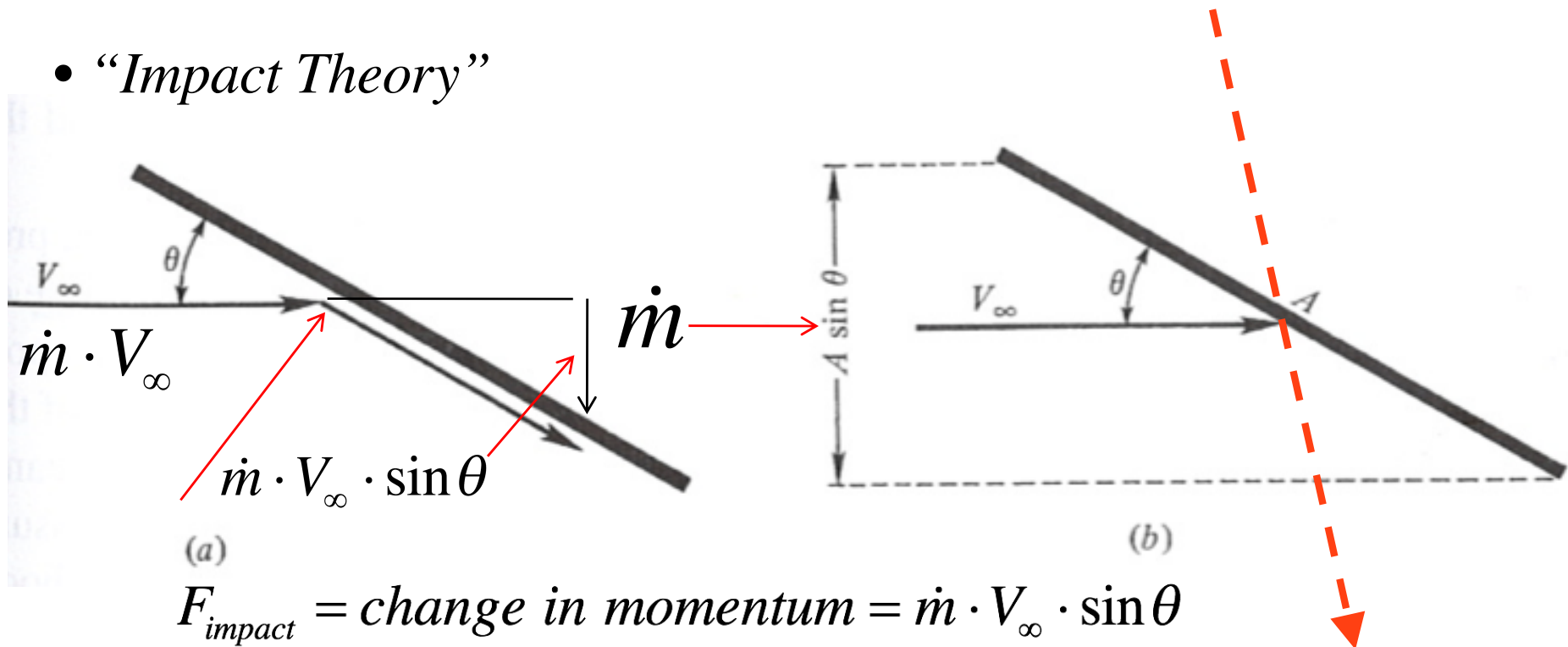


- Sonic Zone “Shrinks”
As Mach Increases

Newtonian Flow Analysis

- Newton had an Often unrecognized original contribution to Fluid Mechanics Propositions 34 and 35 in “*Principia...*”

- “*Impact Theory*”

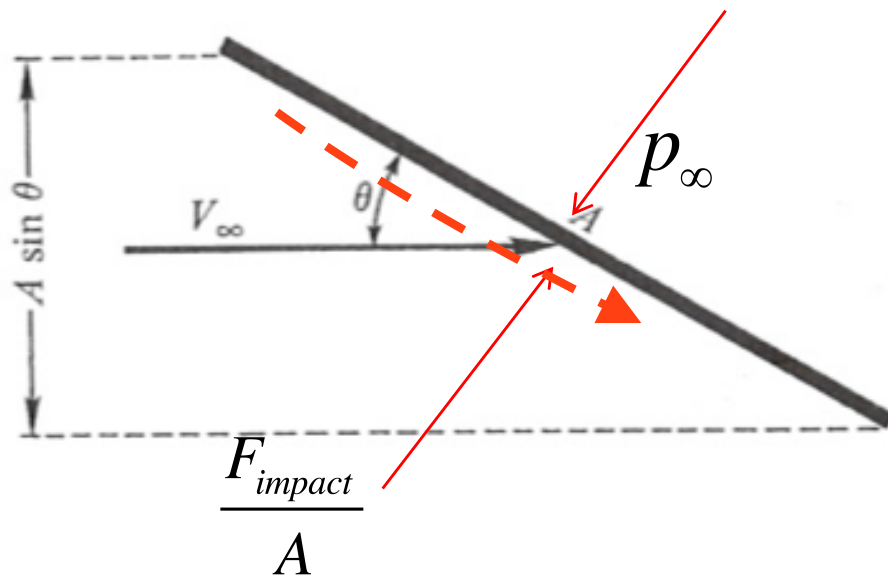


$$F_{\text{impact}} = \text{change in momentum} = \dot{m} \cdot V_\infty \cdot \sin \theta$$

$$\rightarrow F_{\text{impact}} = \rho_\infty \cdot V_\infty \cdot (A \sin \theta) \cdot V_\infty \cdot \sin \theta = \rho_\infty \cdot V_\infty^2 \cdot A \cdot \sin^2 \theta$$

Newtonian Flow Analysis (2)

$$F_{\text{impact}} = \rho_{\infty} \cdot V_{\infty}^2 \cdot A \cdot \sin^2 \theta \rightarrow \boxed{p(\theta) = \frac{F_{\text{impact}}}{A} + p_{\infty} = \rho_{\infty} \cdot V_{\infty}^2 \cdot \sin^2 \theta + p_{\infty}}$$



Newton Ignores the random motion of the molecules and Considers only the linear or translational motion

$$\boxed{C_p(\theta) = \frac{p(\theta) - p_{\infty}}{\frac{1}{2} \rho_{\infty} \cdot V_{\infty}^2} = \frac{\rho_{\infty} \cdot V_{\infty}^2 \cdot \sin^2 \theta + p_{\infty} - p_{\infty}}{\frac{1}{2} \rho_{\infty} \cdot V_{\infty}^2} = 2 \cdot \sin^2 \theta}$$

Newtonian Flow Analysis (3)

$$C_p(\theta) = \frac{p(\theta) - p_\infty}{\frac{1}{2} \rho_\infty \cdot V_\infty^2} = 2 \cdot \sin^2 \theta$$

What happens in Newtonian Model when

$$\theta = 90^\circ \quad ? \quad C_p(90^\circ) = 2 \cdot \sin^2(90^\circ) = 2$$

But for direct impact ...

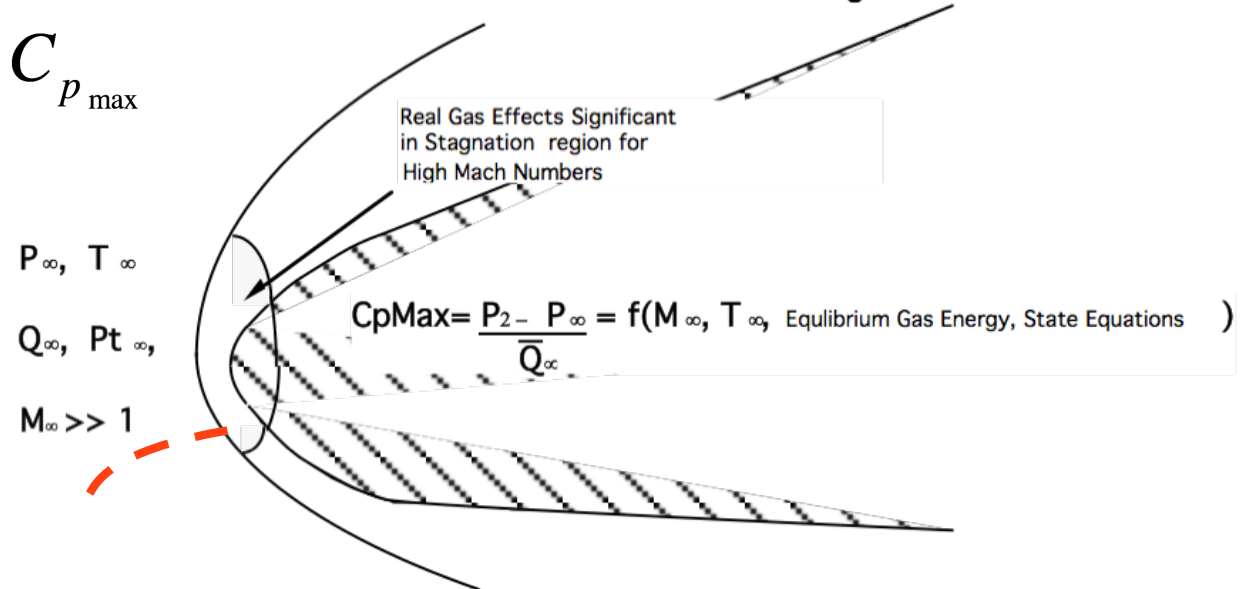
$$p(90^\circ) = P_0 \rightarrow C_p(90^\circ) = \frac{P_0 - p_\infty}{\frac{1}{2} \rho_\infty \cdot V_\infty^2} \equiv C_{p_{\max}} !$$



Newtonian Flow Analysis (4)

$$C_p(90^\circ) = \frac{P_0 - p_\infty}{\frac{1}{2} \rho_\infty \cdot V_\infty^2} \equiv C_{p_{\max}}$$

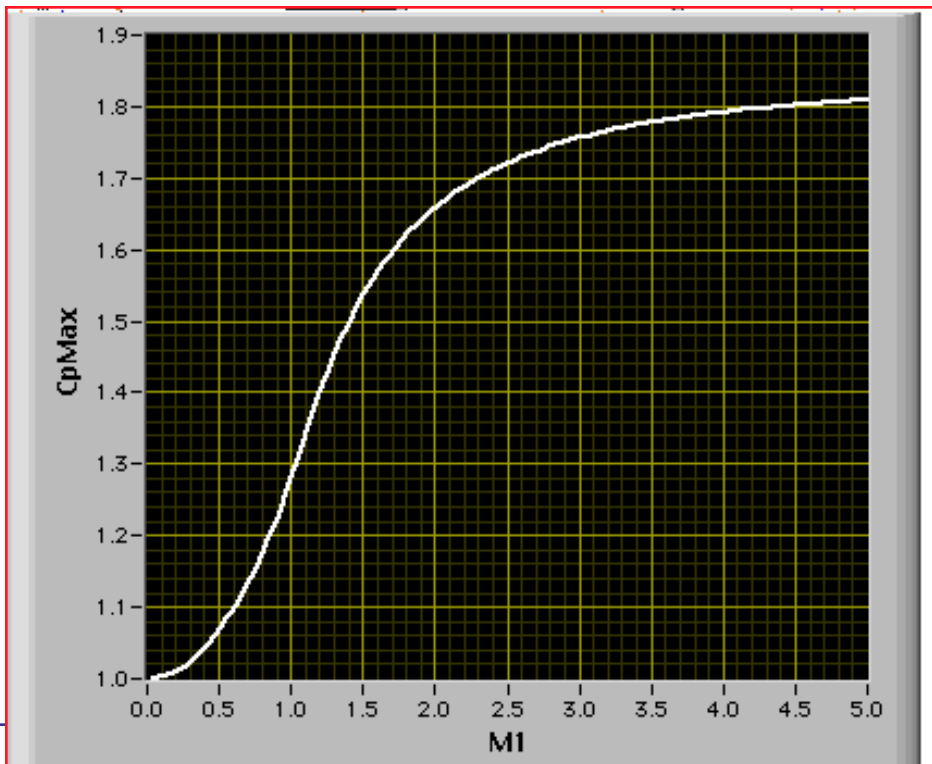
Schematic of the Bow Shock near the Stagnation Point



$$C_{p_{\max}} = \left[\frac{(P_{02} - p_\infty)}{\frac{1}{2} \rho_\infty V_\infty^2} \right] = \frac{(P_{02} - p_\infty)}{\frac{\gamma}{2} p_\infty M_\infty^2} = \frac{1}{\frac{\gamma}{2} M_\infty^2} \left\{ \frac{\left(\frac{\gamma + 1}{2} M_\infty^2 \right)^{\left(\frac{\gamma}{\gamma - 1} \right)}}{\left(\left(\frac{2\gamma}{(\gamma + 1)} M_\infty^2 - \frac{(\gamma - 1)}{(\gamma + 1)} \right)^{\left(\frac{1}{\gamma - 1} \right)} \right)} - 1 \right\}$$

Newtonian Flow Analysis (5)

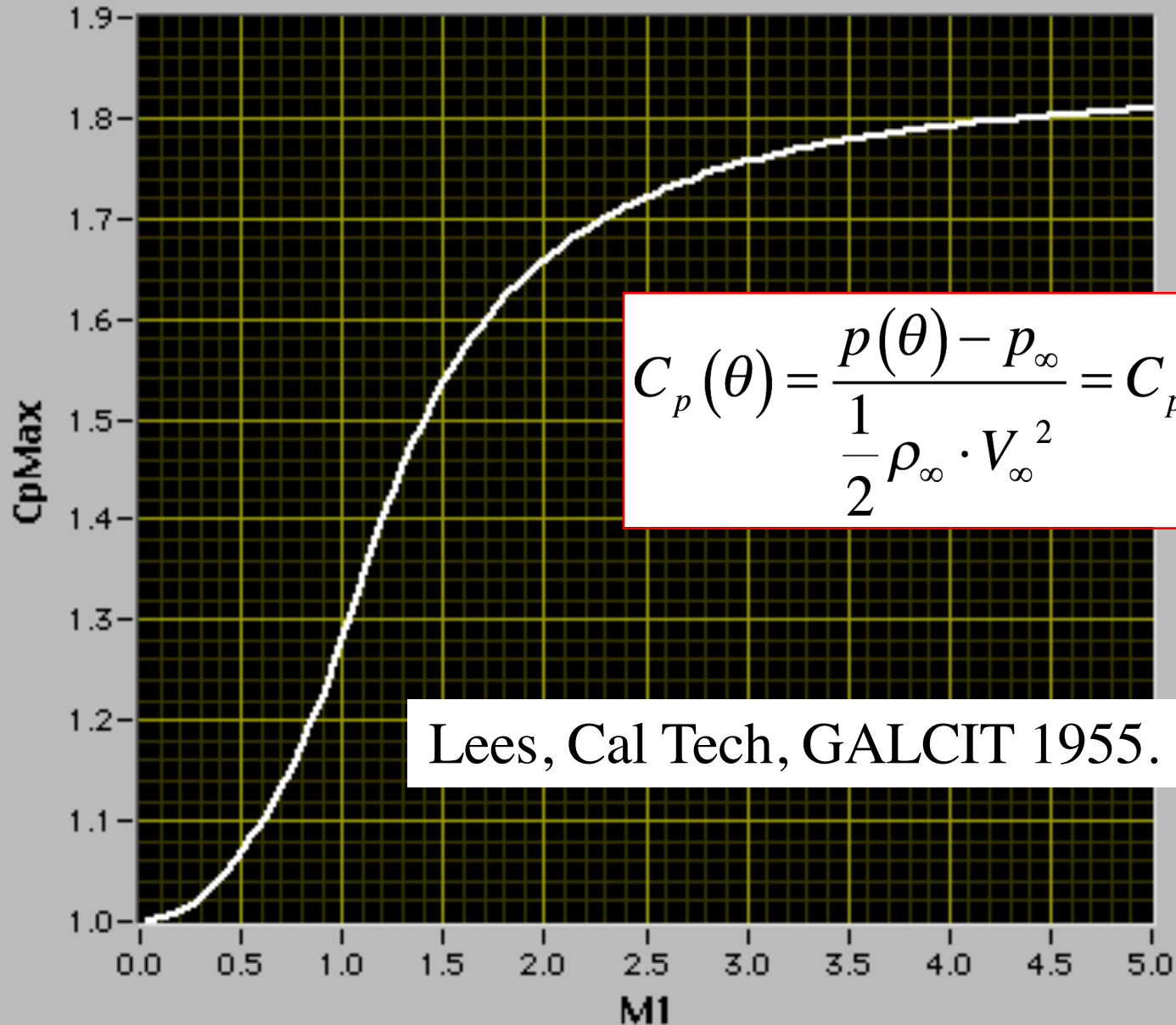
$$C_{p_{\max}} = \frac{1}{\frac{\gamma}{2} M_{\infty}^2} \left\{ \frac{\left(\frac{\gamma+1}{2} M_{\infty}^2 \right)^{\left(\frac{\gamma}{\gamma-1} \right)}}{\left(\frac{2\gamma}{(\gamma+1)} M_{\infty}^2 - \frac{(\gamma-1)}{(\gamma+1)} \right)^{\left(\frac{1}{\gamma-1} \right)}} - 1 \right\}$$



$$C_p(90^\circ) = \frac{P_0 - p_\infty}{\frac{1}{2} \rho_\infty \cdot V_\infty^2} \equiv C_{p_{\max}}$$

< 2 always for $\gamma = 1.4$

“Modified Newtonian Flow”



$$C_p(\theta) = \frac{p(\theta) - p_\infty}{\frac{1}{2} \rho_\infty \cdot V_\infty^2} = C_{p_{max}} \cdot \sin^2 \theta$$

Lees, Cal Tech, GALCIT 1955.

Modified Newtonian Flow (2)

$$C_p(\theta) = \frac{p(\theta) - p_\infty}{\frac{1}{2} \rho_\infty \cdot V_\infty^2} = C_{p_{\max}} \cdot \sin^2 \theta$$

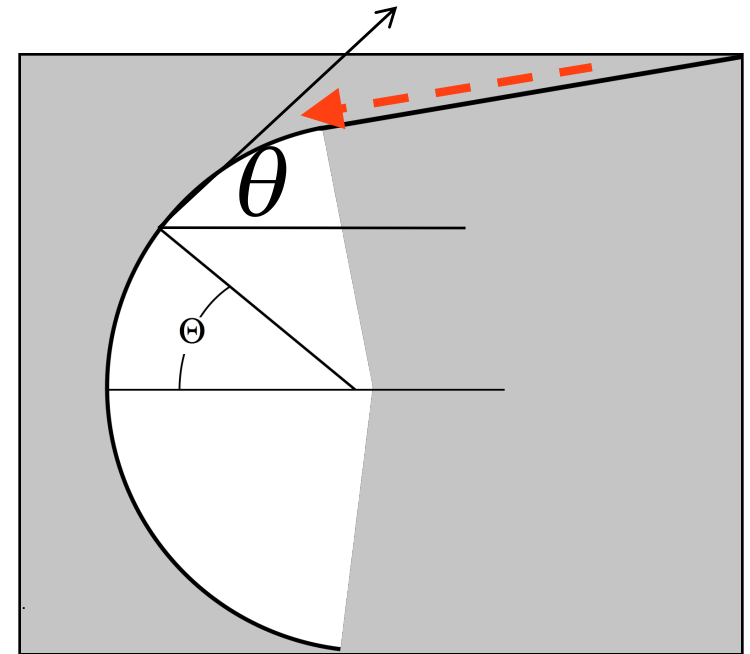
Often convenient to express model in terms of a Polar surface coordinate

$$\Theta = 90^\circ - \theta$$

$$C_p(\Theta) = \frac{p(\Theta) - p_\infty}{\frac{1}{2} \rho_\infty \cdot V_\infty^2} = C_{p_{\max}} \cdot \sin^2(90^\circ - \Theta) =$$

$$C_{p_{\max}} \cdot [\sin(90^\circ) \cdot \cos(\Theta) - \sin(\Theta) \cdot \cos(90^\circ)]^2$$

$$\Rightarrow C_p(\Theta) = \frac{p(\Theta) - p_\infty}{\frac{1}{2} \rho_\infty \cdot V_\infty^2} = C_{p_{\max}} \cdot \cos^2(\Theta)$$

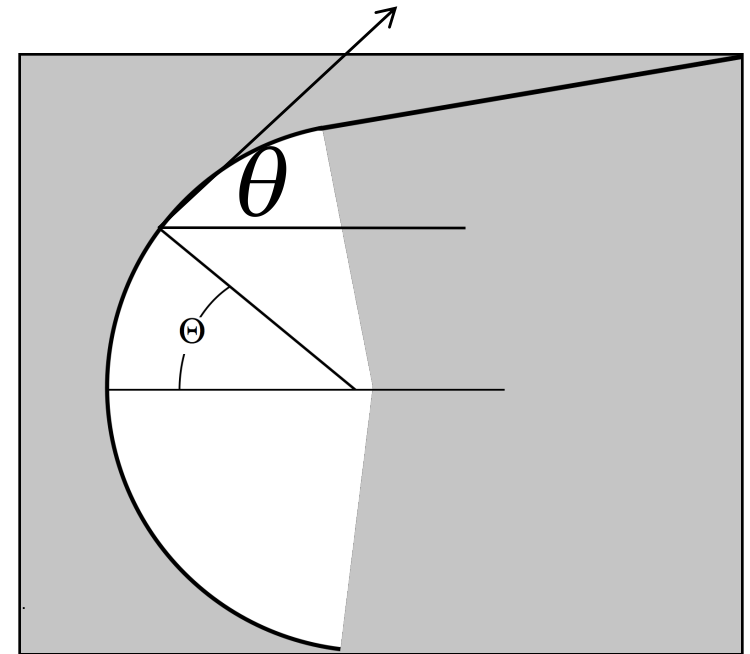


Modified Newtonian Flow (3)

Expressed in terms of pressure ratio

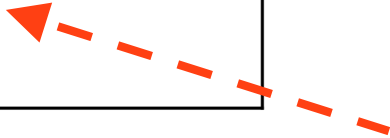
$$C_p(\Theta) = \frac{p(\Theta) - p_\infty}{\frac{1}{2} \rho_\infty \cdot V_\infty^2} = C_{p_{\max}} \cdot \cos^2(\Theta) \Rightarrow \frac{p(\Theta) - p_\infty}{\frac{\gamma}{2} p_\infty \cdot M_\infty^2} = C_{p_{\max}} \cdot \cos^2(\Theta)$$

$$\boxed{\frac{p(\Theta)}{p_\infty} = 1 + \frac{\gamma}{2} \cdot M_\infty^2 \cdot C_{p_{\max}} \cdot \cos^2(\Theta)}$$



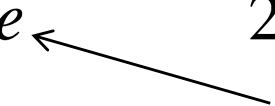
$$\Theta = 90^\circ - \theta$$

Modified Newtonian Flow (4)

$$\Rightarrow C_p(\Theta) = \frac{p(\Theta) - p_\infty}{\frac{1}{2} \rho_\infty \cdot V_\infty^2} = C_{p_{\max}} \cdot \cos^2(\Theta)$$


*“equivalent to infinite
Mach number
assumption”*

- As derived in section 3

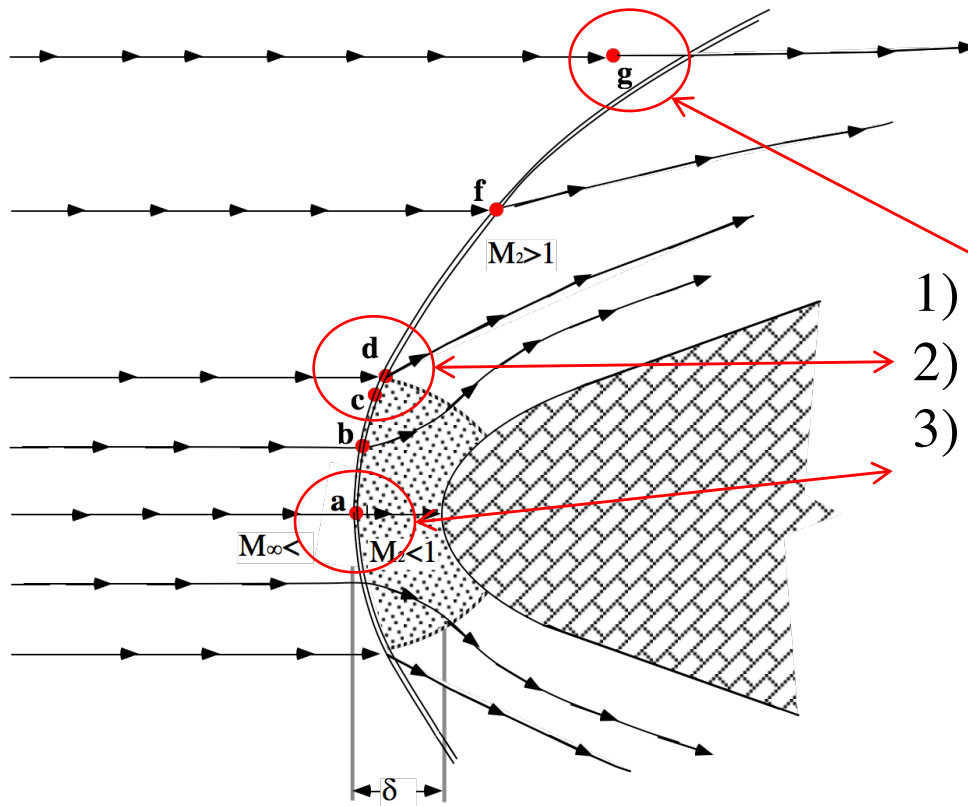
$$\frac{V^2/2}{e} = \frac{\gamma(\gamma-1)}{2} M^2$$


For Newtonian model ... Newton
Ignores the random motion of the
molecules and Considers only the linear
or translational motion

Mach number is a measure of the ratio of the fluid
Kinetic energy to the fluid internal energy (direct
motion To random thermal motion of gas
molecules) -- **Fundamental Parameter of
Compressible Flow** --

Example 3

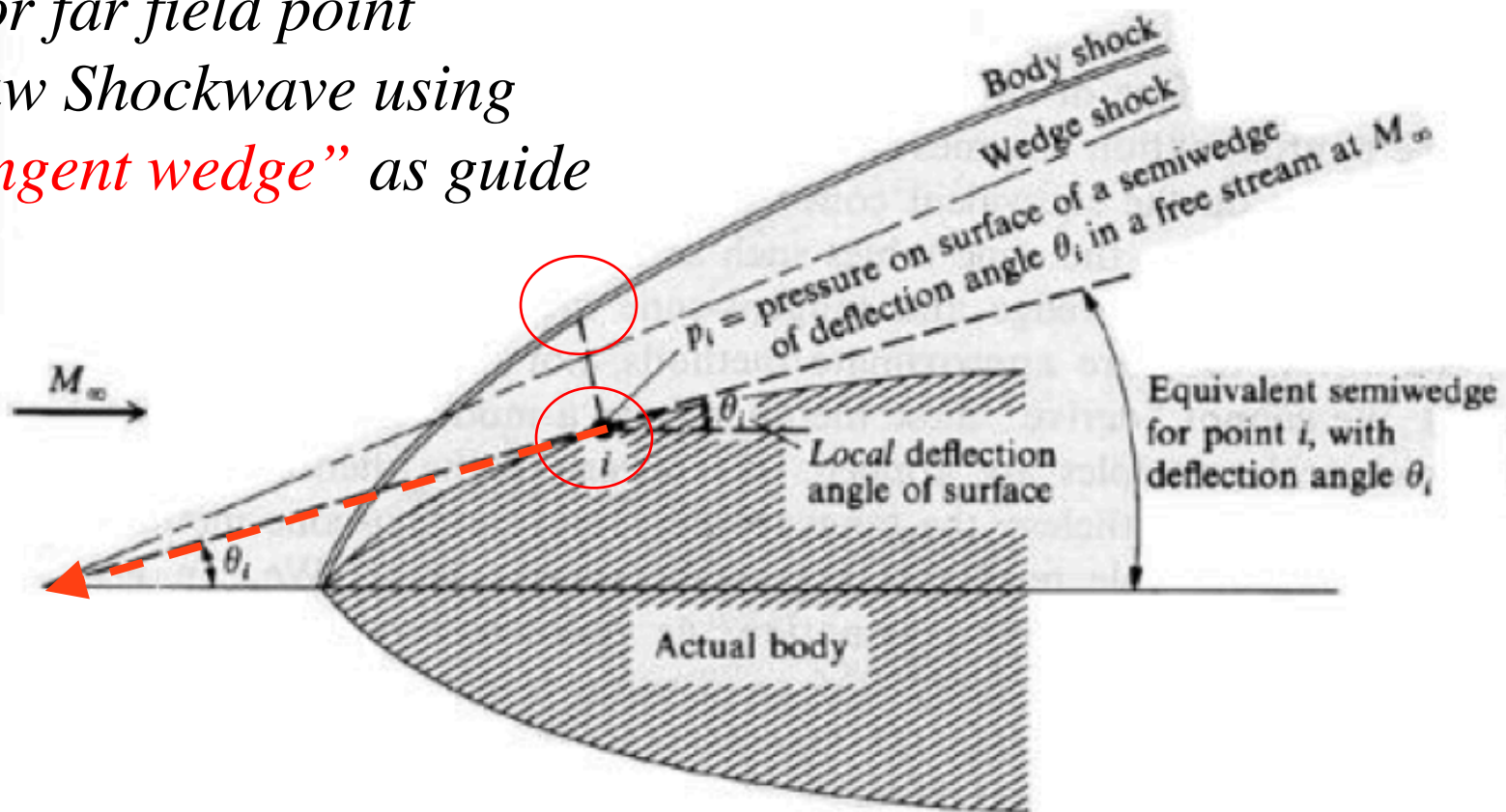
- Example: Cylindrically Blunted 2-D wedge, $M_\infty=10.0$, 1-cm Radius
- *Map conditions at three points along shock*



- 1) Far field (g)... “tangent wedge”
- 2) Sonic Point (d)
- 3) Stagnation Point (a)

Example 3 ₍₂₎

- Example Cylindrically Blunted 2-D wedge, $M_\infty = 10.0$ 1-cm Radius
- For far field point
Draw Shockwave using
“tangent wedge” as guide

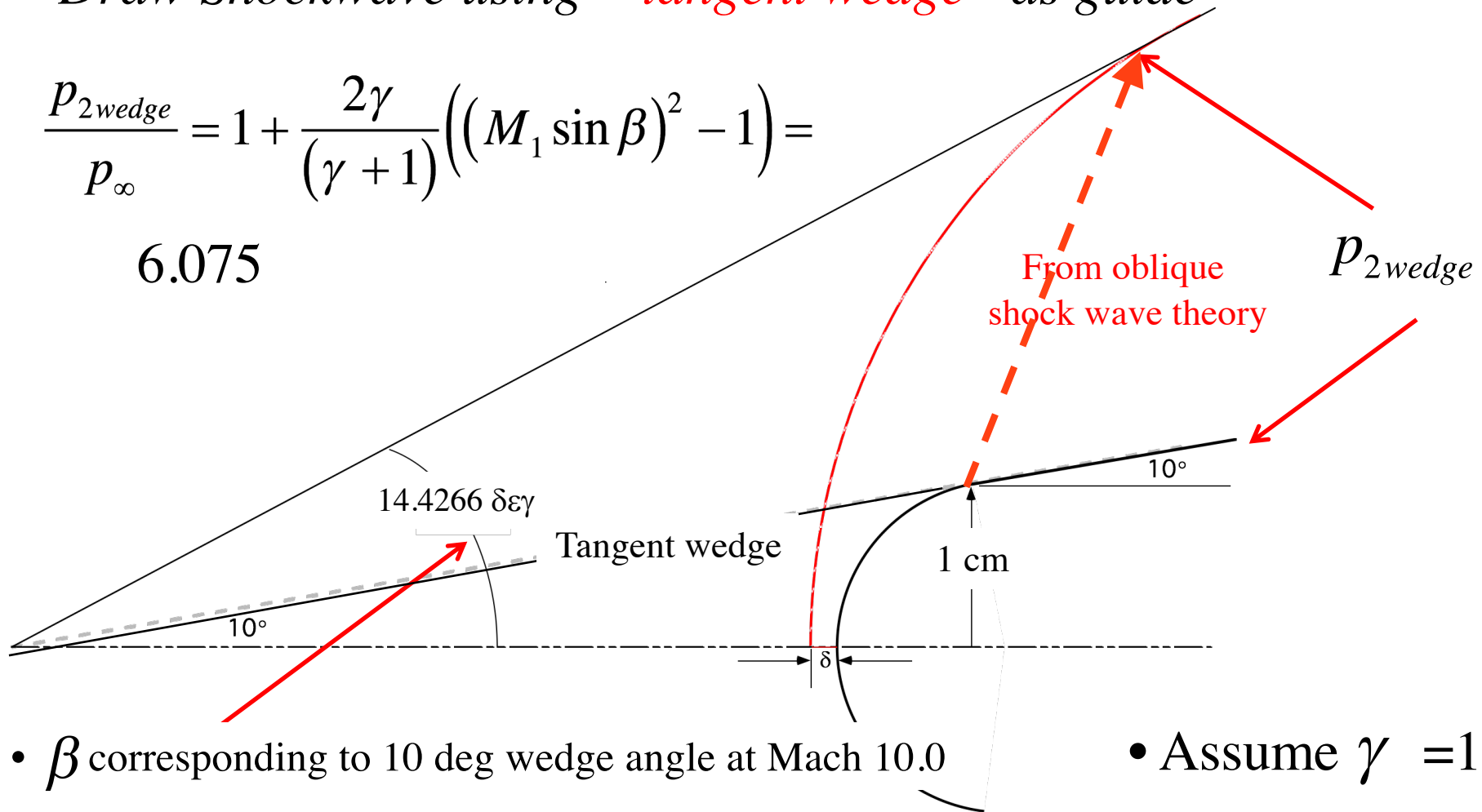


Example 3 (3)

- Example Cylindrically Blunted 2-D wedge, $M_\infty=10.0$ 1-cm Radius
- Draw Shockwave using “*tangent wedge*” as guide

$$\frac{p_{2wedge}}{p_\infty} = 1 + \frac{2\gamma}{(\gamma + 1)} \left((M_1 \sin \beta)^2 - 1 \right) =$$

6.075

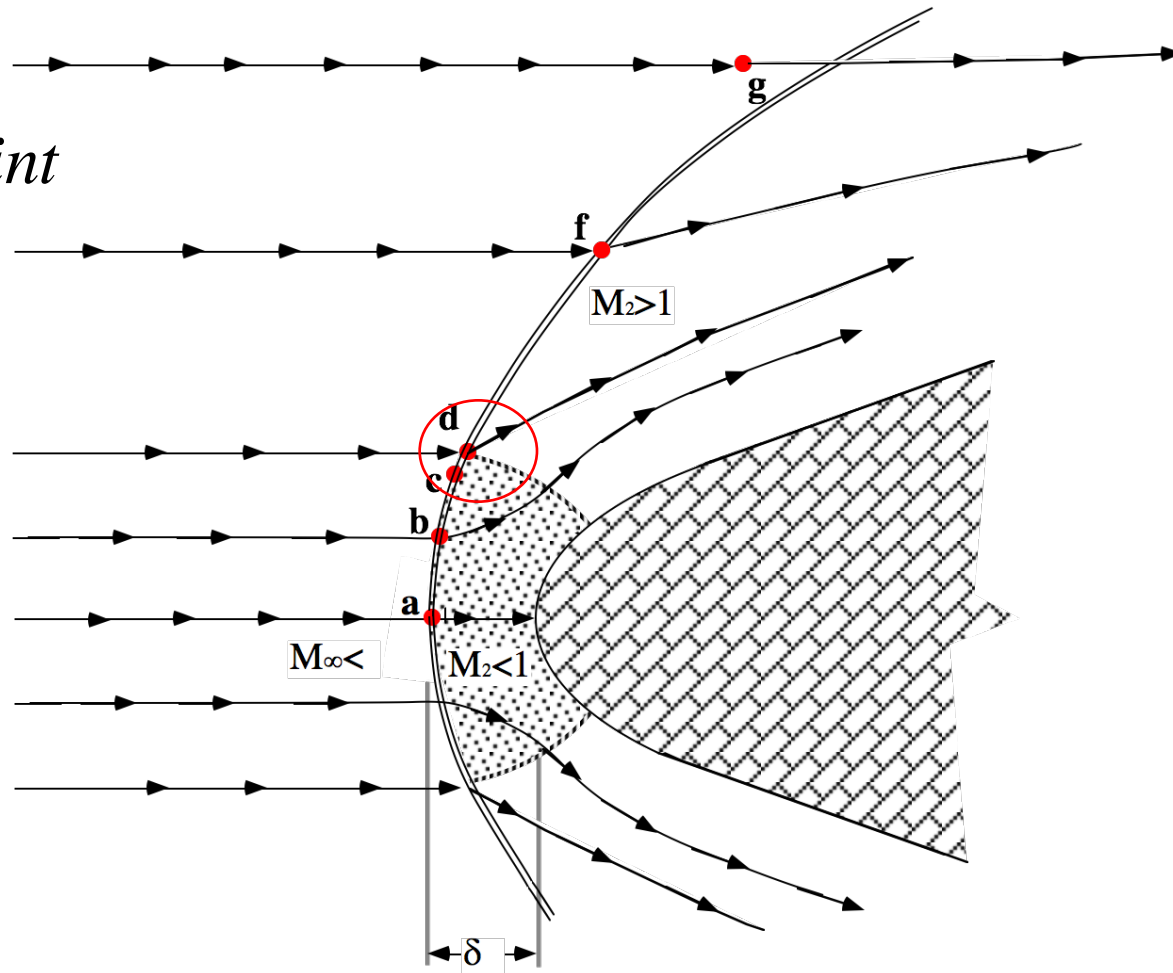


- β corresponding to 10 deg wedge angle at Mach 10.0

- Assume $\gamma = 1.4$

Example 3: Sonic Point Surface Conditions behind detached shock wave: (5)

Sonic Point



Example 3 ₍₆₎

- Compute Standoff Distance

$$\frac{\delta}{D} = 0.193 \cdot e^{\frac{4.67}{M_\infty^2}} = 2 \cdot 1 \cdot 0.193 \exp\left(\frac{4.67}{1.75^2}\right) = 0.4044 \text{ cm}$$

- Compute Sonic Point on Shock

$$\beta_{sonic} = \frac{180}{\pi} \sin^{-1} \sqrt{\frac{(\gamma - 3)M_1^2 + (\gamma + 1)M_1^4 + \sqrt{16\gamma M_1^4 + [(\gamma - 3)M_1^2 + (\gamma + 1)M_1^4]^2}}{4\gamma M_1^4}} = 67.335 \text{ deg.}$$

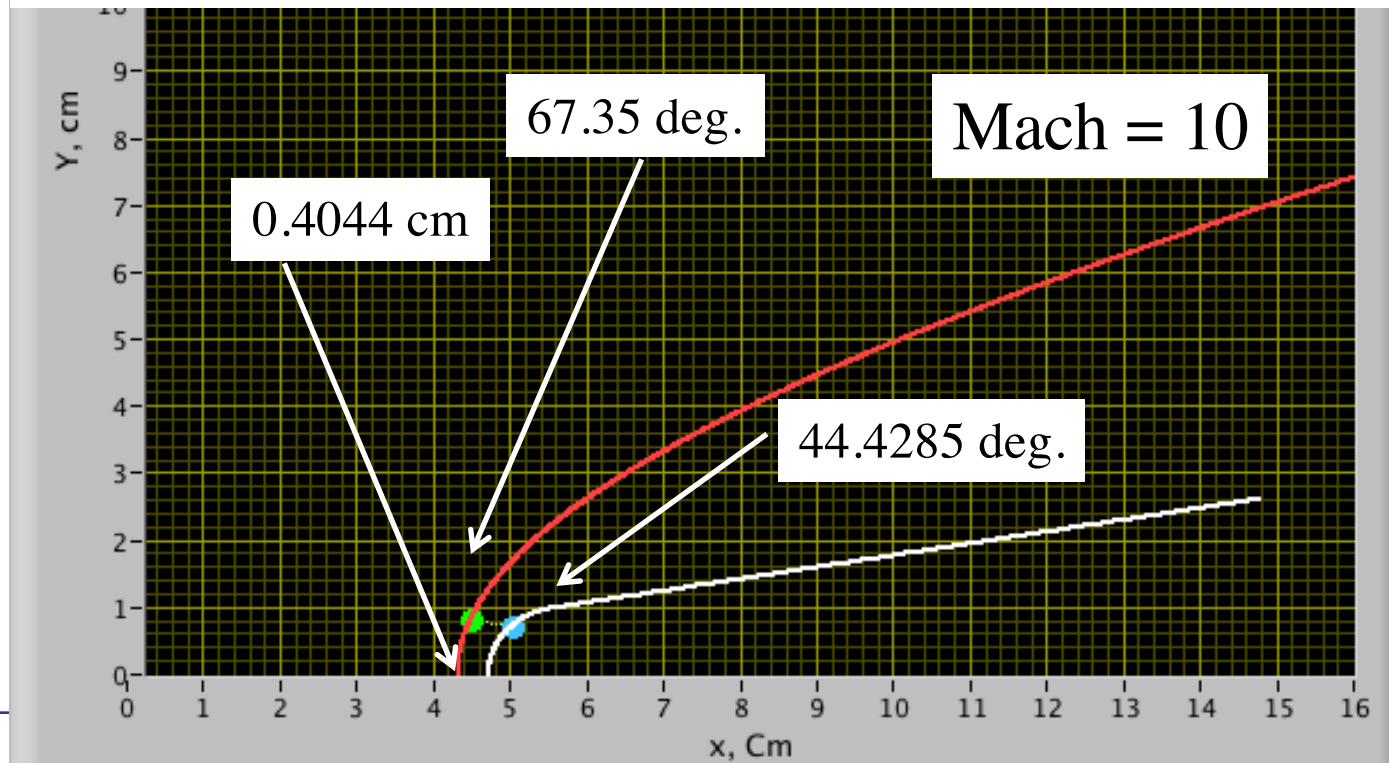
- Compute Sonic Point on Surface

$$\tan(\theta_{sonic}) = \frac{2\{M_1^2 \sin^2(\beta_{sonic}) - 1\}}{\tan(\beta_{sonic})[2 + M_1^2[\gamma + \cos(2\beta_{sonic})]]} = 44.428 \text{ deg.}$$

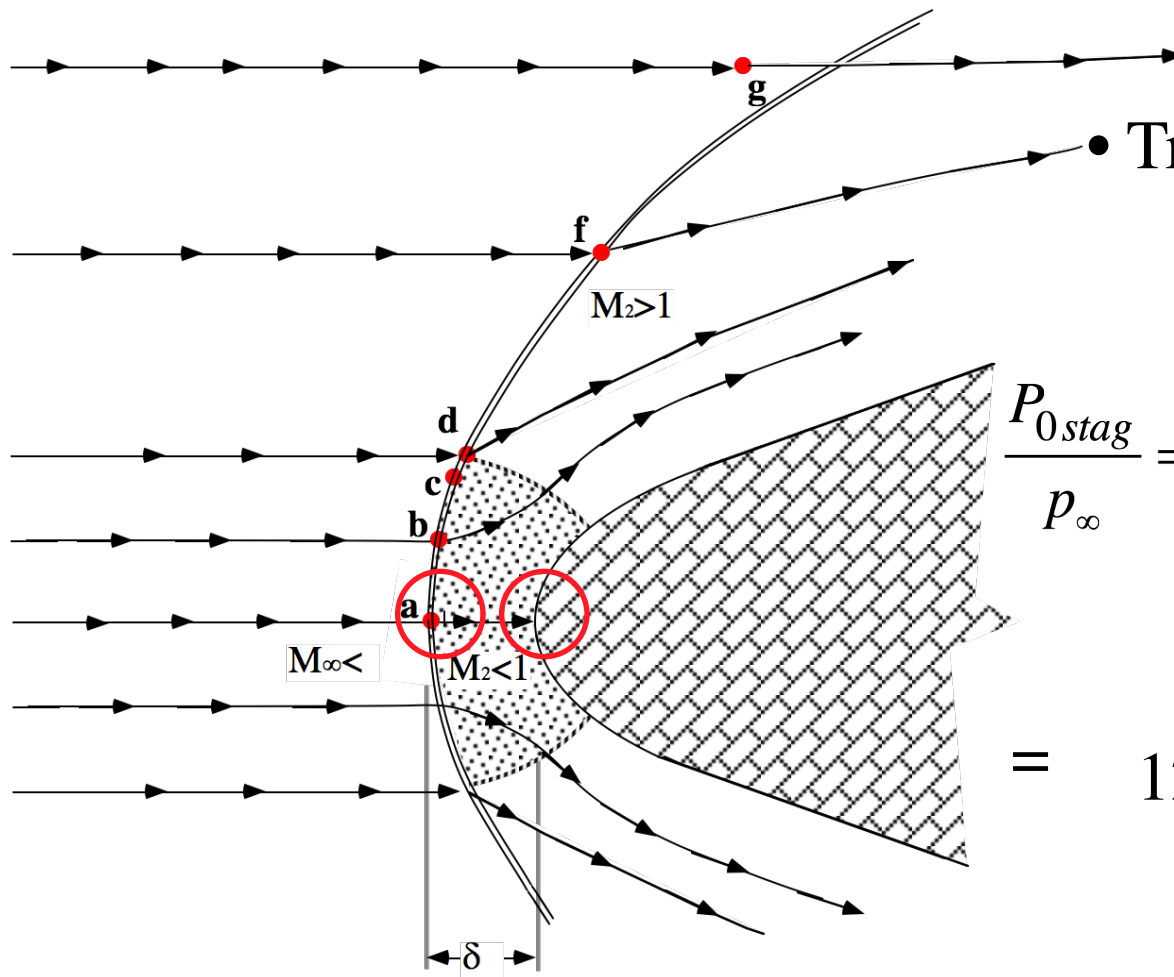
Example 3 (cont'd)

$$\frac{p_{sonic}}{p_{\infty}} = \frac{p_{sonic}}{p_{0sonic}} \cdot \frac{p_{0sonic}}{p_{\infty}} = \frac{1}{\left(1 + \frac{\gamma + 1}{2}\right)^{\left(\frac{\gamma}{\gamma - 1}\right)}} \left\{ \frac{\left(\frac{\gamma + 1}{2} [M_{\infty} \sin(\beta_{sonic})]^2\right)^{\left(\frac{\gamma}{\gamma - 1}\right)}}{\left(\frac{2\gamma}{(\gamma + 1)} [M_{\infty} \sin(\beta_{sonic})]^2 - \frac{(\gamma - 1)}{(\gamma + 1)}\right)^{\left(\frac{1}{\gamma - 1}\right)}} \right\} =$$

$$0.52828 \times 110.098512 = 64.16$$



Surface Conditions behind detached shock wave: *stagnation point* (cont'd)



- Treat as normal shockwave

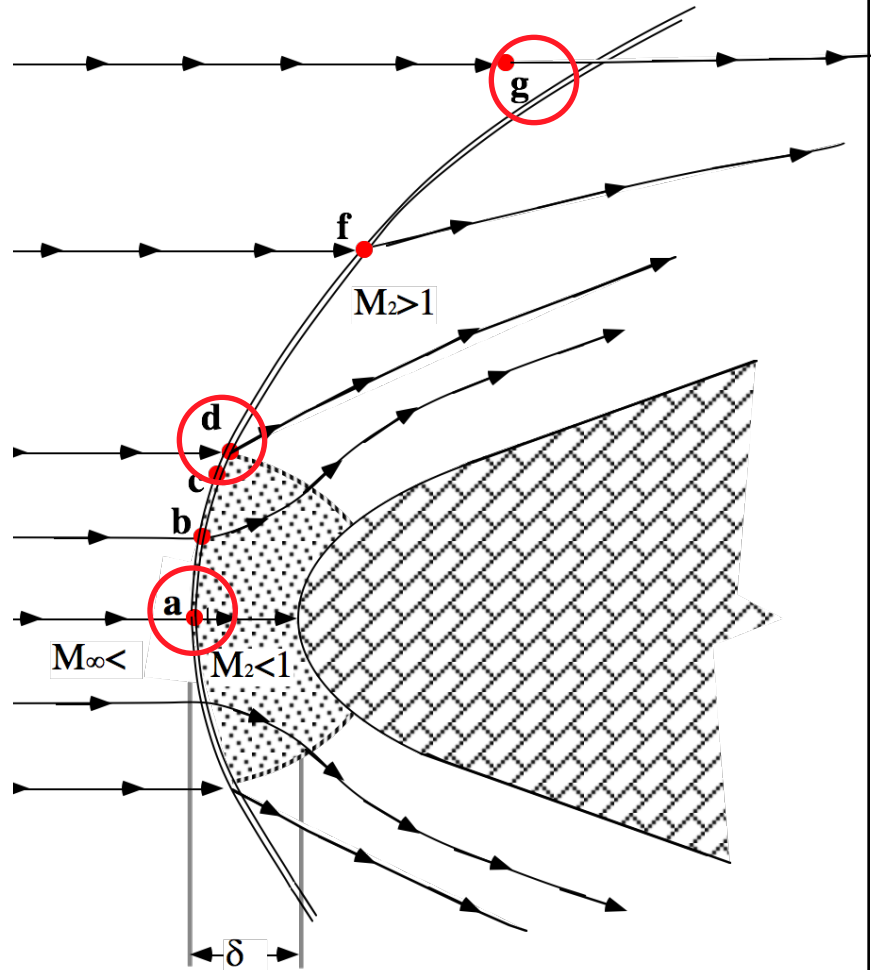
$$\frac{P_{0stag}}{P_\infty} = \left\{ \frac{\left(\frac{\gamma + 1}{2} M_\infty^2 \right)^{\left(\frac{\gamma}{\gamma - 1} \right)}}{\left(\frac{2\gamma}{(\gamma + 1)} M_\infty^2 - \frac{(\gamma - 1)}{(\gamma + 1)} \right)^{\left(\frac{1}{\gamma - 1} \right)}} \right\}$$

$$= 129.21$$

- $M_\infty = 10.0$

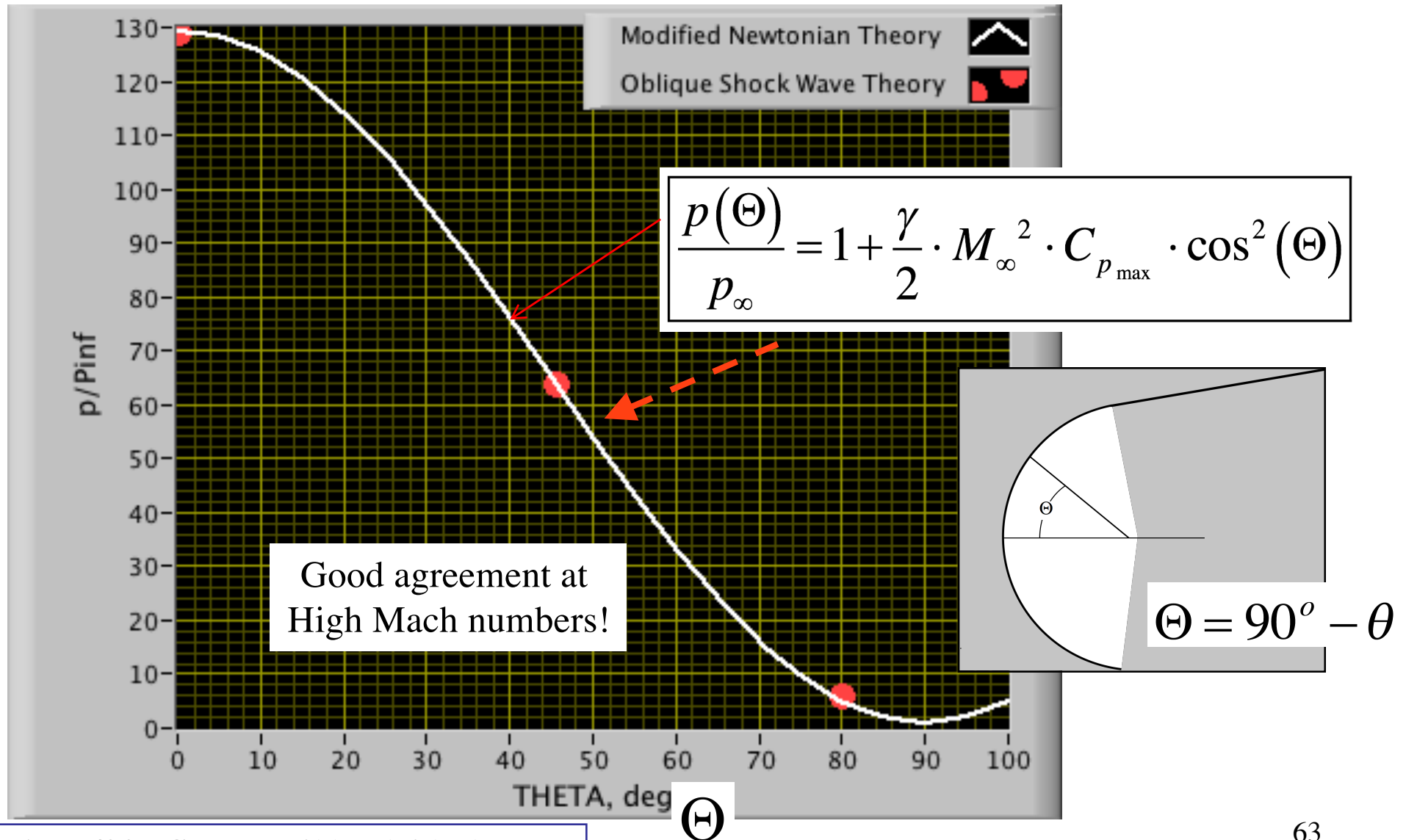
Plot Conditions along detached shock

summary $M_\infty = 10.0$



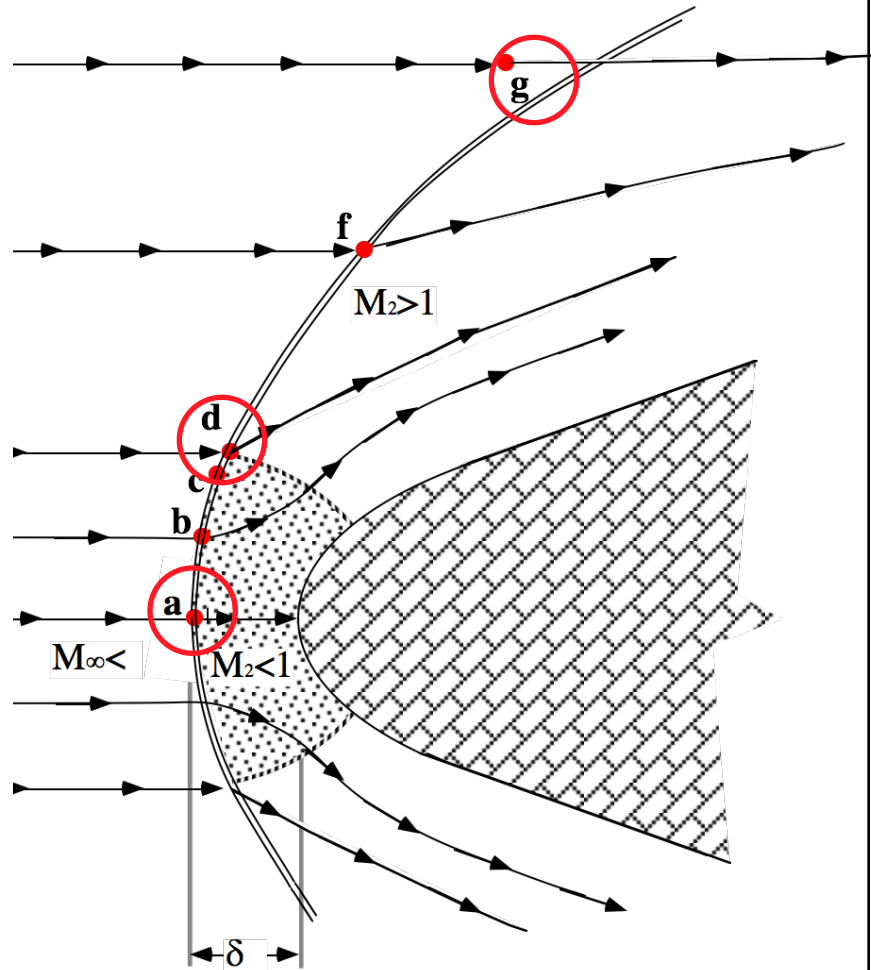
Point on Shock	beta, deg β	theta, deg θ	90°- theta Θ	$\frac{p_\theta}{p_\infty}$
a	90°	90°	0°	129.21
d	67.34°	44.43°	45.57°	64.16
g	14.43°	10°	80°	6.075

Surface Conditions behind detached shock wave: Mach 10.0



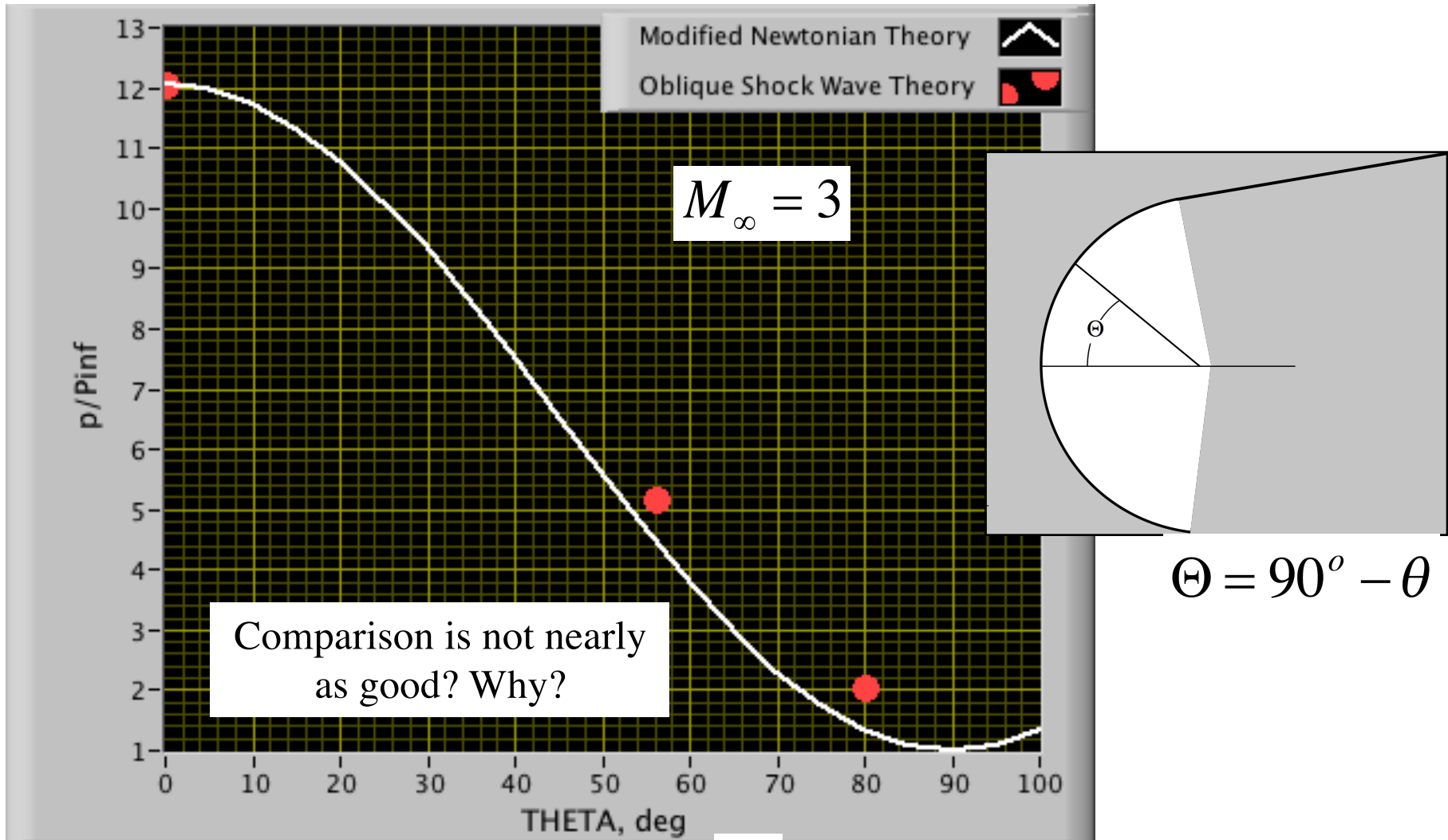
Repeat Oblique Shock Calculations at

$$M_{\infty} = 3$$

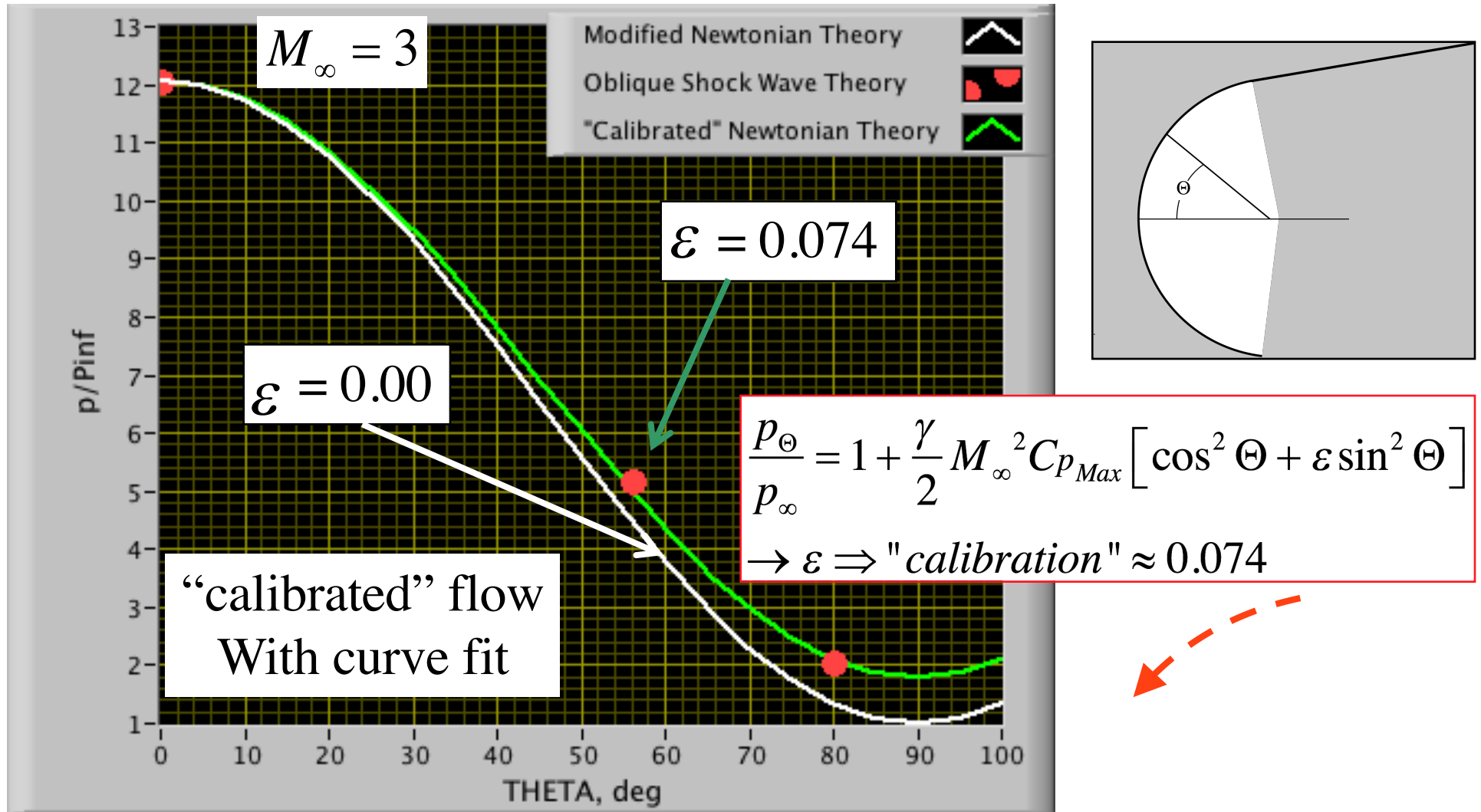


Point on Shock	beta, deg β	theta, deg θ	90°- theta Θ	$\frac{p_{\theta}}{p_{\infty}}$
a	90°	90°	0°	12.06
d	63.77°	34.01°	55.99°	5.18
g	27.38°	10°	80°	2.05

Surface Conditions behind detached shock wave: *Mach 3.0*



Surface Conditions behind detached shock wave: *Mach 3*



Surface Conditions behind detached shock wave: *Surface plot*

- In limit as $M \rightarrow \infty$ Modified Newtonian flow model is
A good descriptor of the local surface pressures

$$\frac{p_{\Theta}}{p_{\infty}} = 1 + \frac{\gamma}{2} M_{\infty}^2 C_{p_{Max}} \cos^2 \Theta$$

- At lower Mach numbers we include a “calibration” term

$$\frac{p(\Theta)}{p_{\infty}} = 1 + \frac{\gamma}{2} \cdot M_{\infty}^2 \cdot C_{p_{max}} \cdot \left[\cos^2(\Theta) + \varepsilon \cdot \sin^2(\Theta) \right]$$

Modified Newtonian Flow, Concluded

- Writing in terms of pressure coefficient

$$C_p(\Theta) = \frac{p(\Theta) - p_\infty}{\frac{\gamma}{2} \cdot p_\infty \cdot M_\infty^2} = C_{p_{\max}} \cdot \left[\cos^2(\Theta) + \varepsilon \cdot \sin^2(\Theta) \right]$$

$$C_{p_{\max}} \cdot \left[\cos^2 \Theta + \varepsilon \cdot (1 - \cos^2 \Theta) \right] = C_{p_{\max}} \cdot \left[(1 - \varepsilon) \cdot \cos^2 \Theta + \varepsilon \right]$$

or...

$$C_p(\theta) = \frac{p(\theta) - p_\infty}{\frac{\gamma}{2} \cdot p_\infty \cdot M_\infty^2} = C_{p_{\max}} \cdot \left[\sin^2(\theta) + \varepsilon \cdot \cos^2(\theta) \right] = C_{p_{\max}} \cdot \left[(1 - \varepsilon) \cdot \sin^2(\theta) + \varepsilon \right]$$

- “Calibrated Newtonian Flow”

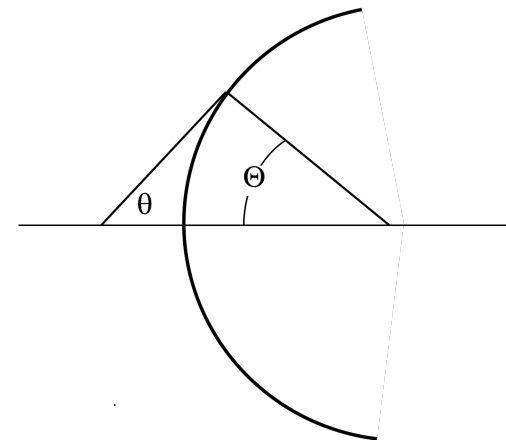
Semi-empirical model

Valid for very high speeds, 3-D

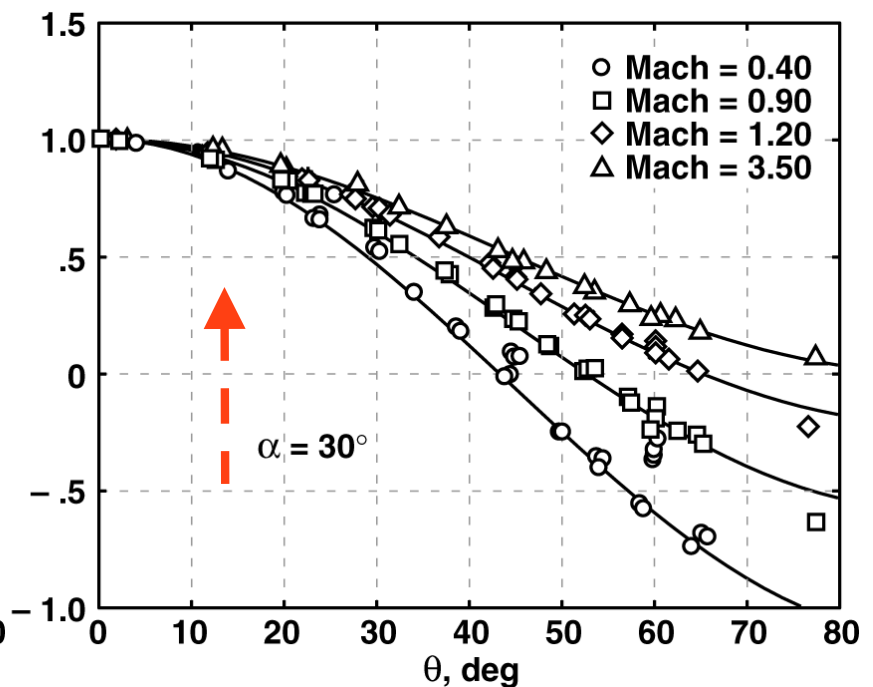
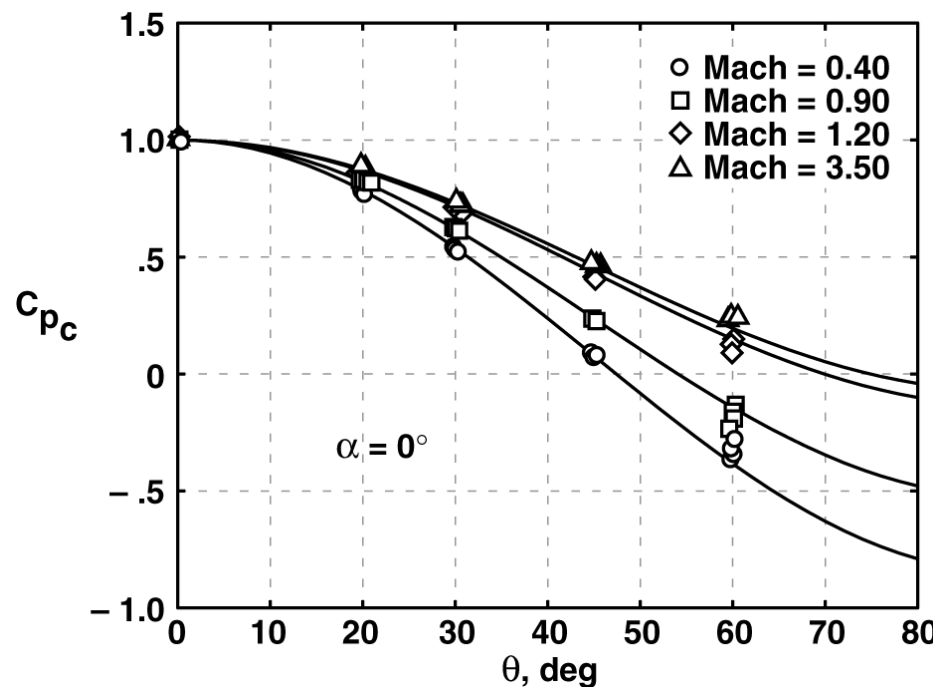
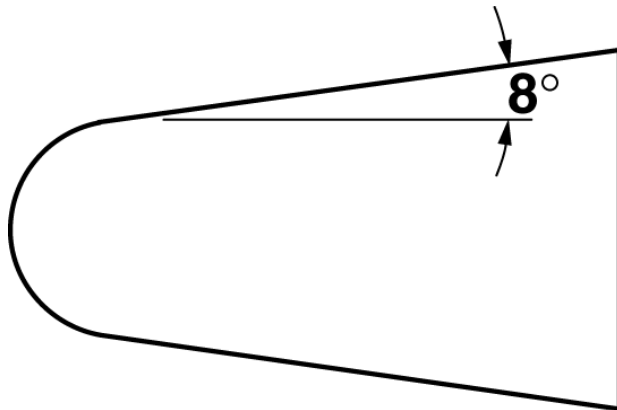
And 2-D blunt bodies ... accurate

For both sonic and supersonic regions

Behind detached shock wave



Calibration Examples: 8-deg Cone



More Calibration Examples

Analytical

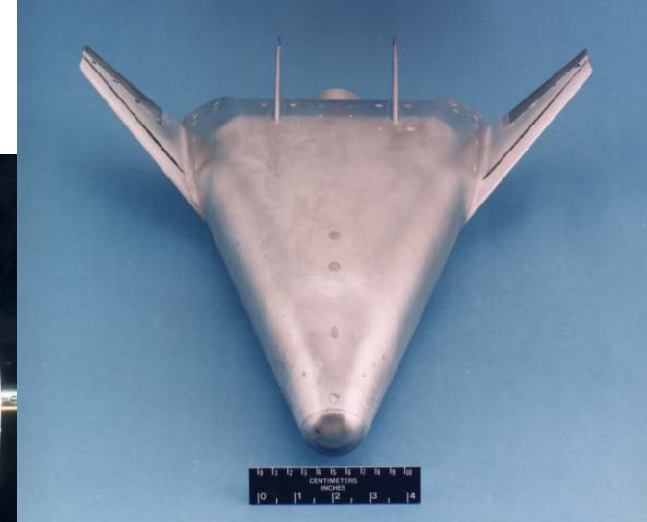
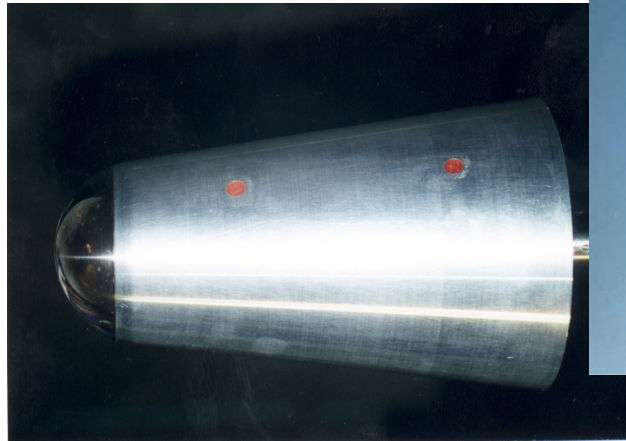
- Potential solutions: sphere, cylinder, arbitrary ellipsoid

Wind Tunnel

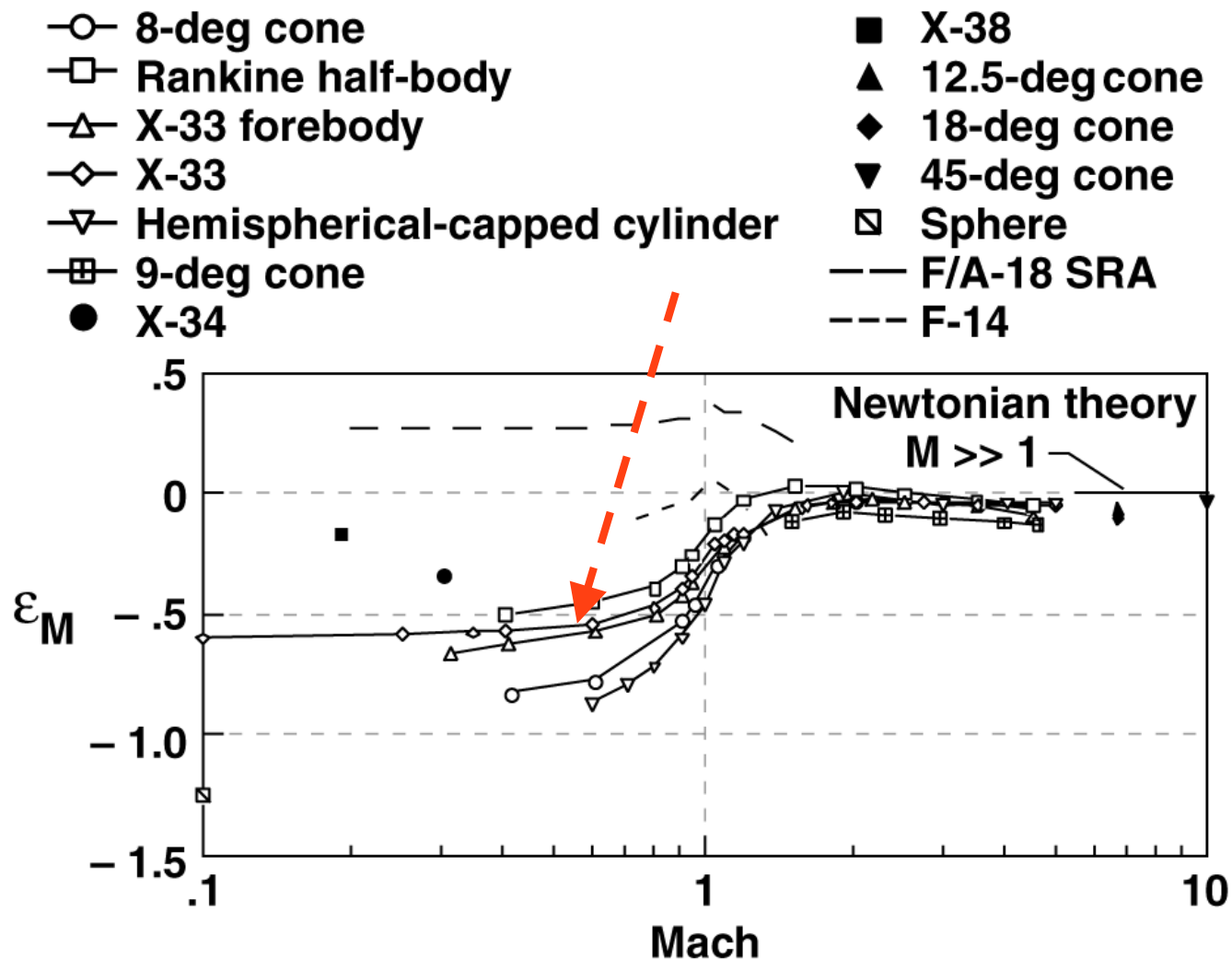
- 8-deg sphere-cone, Rankine half-body, X-33 forebody ($M=.3 - 4.75$)
- 9, 12.5, 18, and 45-deg sphere cones
- X-33
- X-34
- X-38
- Sphere

Flight Data

- F/A-18 SRA
- F-14



Mach Effect on Calibration (ϵ):

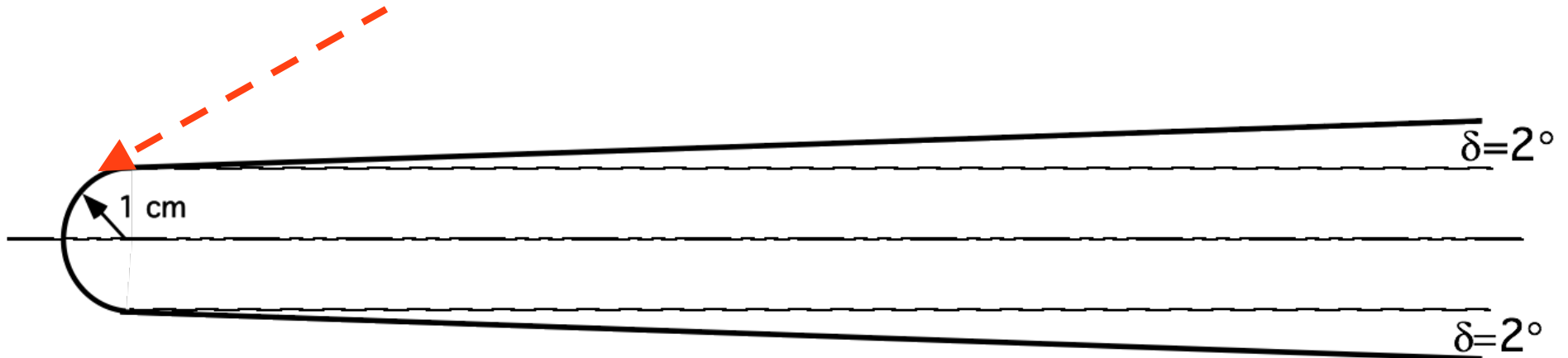


Bottom Line

- Even though flow across detached shock wave is extremely complex and is difficult to model theoretically, it turns out that simple engineering models do a good job with the pressure distribution
- Allow the drag due to detached shock wave to be estimated using very simple tools

Example Calculation, Blunt Edge Drag

- Consider a 2° angle (δ) diamond-wedge wing, 2 meter chord (c),
With A spherically blunted leading edge with a 1 cm radius (R)



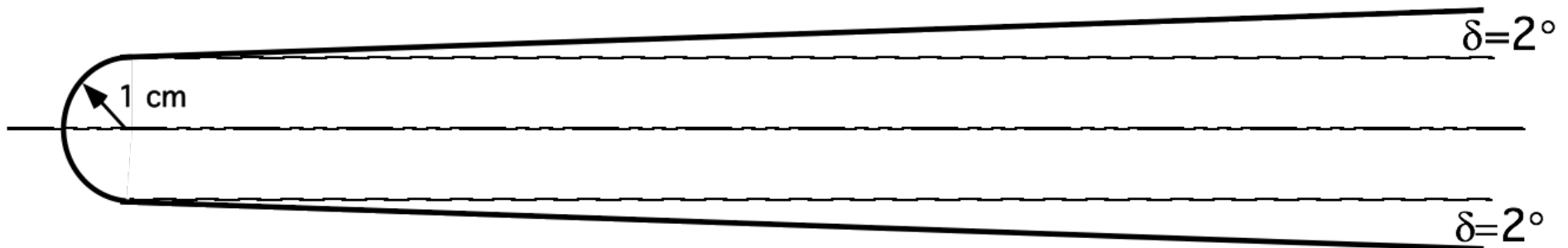
- Show that in general, the drag coefficient due to the the blunt leading edge is:

$$C_{D_{LE}} = \int_{-\left(\frac{\pi}{2}-\delta\right)}^{\left(\frac{\pi}{2}-\delta\right)} Cp(\Theta) \cos(\Theta) \frac{R}{c} d\Theta$$

Example Calculation, Blunt Edge Drag (cont'd)

- Use the calibrated Newtonian Model

$$C_p(\Theta) = C_{p_{\max}} \left[\cos^2(\Theta) + \varepsilon \sin^2(\Theta) \right] = C_{p_{\max}} \cdot \left[(1 - \varepsilon) \cdot \cos^2 \Theta + \varepsilon \right]$$

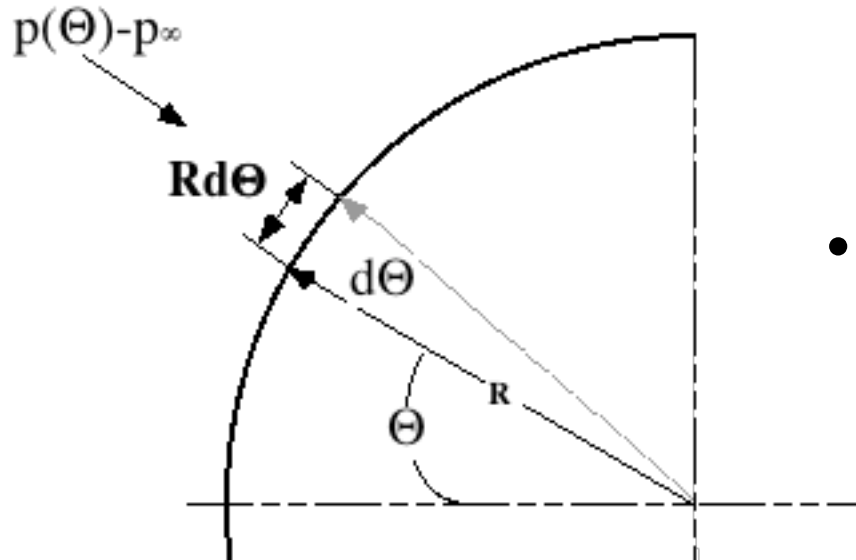


... calculate the leading edge drag coefficient
at $M_\infty = 3.0$ ($\varepsilon \approx 0.074$) AND $M_\infty = 10.0$ ($\varepsilon \approx 0.00$)

- Compare to wave drag coefficient and skin drag coefficient (referenced to plan area) for the same conditions at 25 km altitude

EXAMPLE Solution (cont'd)

- Drag Pressure drag on small increment of blunted nose



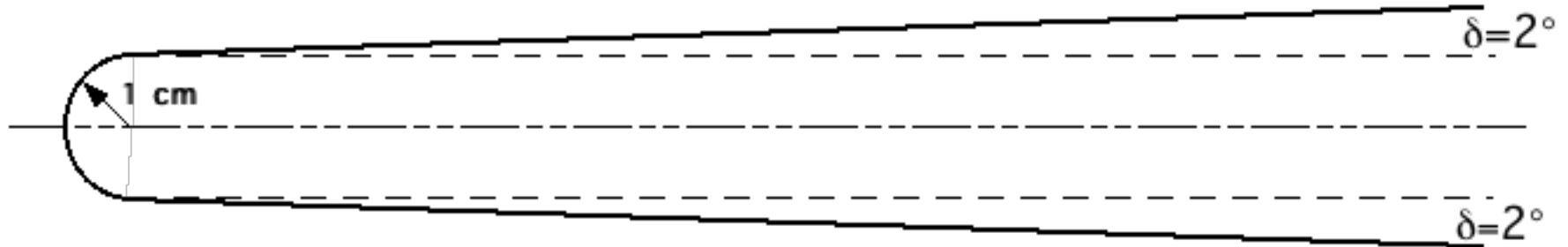
- Axial component of pressure force on surface increment

$$\partial D_{LE} = (p(\Theta) - p_{\infty}) \times \cos(\Theta) \times [b \times R d\Theta] \rightarrow$$

$$\partial C_{D_{LE}} = \frac{\partial D_{LE}}{\bar{q}_{\infty} \times (b \times c)} = \frac{(p(\Theta) - p_{\infty}) \times \cos(\Theta) \times [b \times R d\Theta]}{\bar{q}_{\infty} \times (b \times c)} = C_p(\Theta) \cos(\Theta) \frac{R}{c} d\Theta$$

EXAMPLE Solution (cont'd)

- Integrate from intersection of bottom surface to intersection of top surface



$$C_{D_{LE}} = \int_{-\left(\frac{\pi}{2}-\delta\right)}^{\left(\frac{\pi}{2}-\delta\right)} Cp(\Theta) \cos(\Theta) \frac{R}{c} d\Theta$$

EXAMPLE Solution (cont'd)

- Plug in calibrated Newtonian Model

$$C_{D_{LE}} = \int_{-\left(\frac{\pi}{2}-\delta\right)}^{\left(\frac{\pi}{2}-\delta\right)} C_{p_{\max}} \left[\cos^2(\Theta) + \varepsilon \sin^2(\Theta) \right] \cos(\Theta) \frac{R}{c} d\Theta =$$

$$C_{p_{\max}} \frac{R}{c} \int_{-\left(\frac{\pi}{2}-\delta\right)}^{\left(\frac{\pi}{2}-\delta\right)} \left[\cos^2(\Theta) + \varepsilon (1 - \cos^2(\Theta)) \right] \cos(\Theta) d\Theta =$$

$$C_{p_{\max}} \frac{R}{c} \int_{-\left(\frac{\pi}{2}-\delta\right)}^{\left(\frac{\pi}{2}-\delta\right)} \left[(1 - \varepsilon) \cos^3(\Theta) + \varepsilon \cos(\Theta) \right] d\Theta$$

EXAMPLE Solution (cont'd)

- Evaluate integrals

$$\int_{-\left(\frac{\pi}{2}-\delta\right)}^{\left(\frac{\pi}{2}-\delta\right)} \cos(\Theta) d\Theta = 2 \sin\left(\frac{\pi}{2} - \delta\right) = 2 \cos(\delta)$$

$$\int_{-\left(\frac{\pi}{2}-\delta\right)}^{\left(\frac{\pi}{2}-\delta\right)} \cos^3(\Theta) d\Theta = 2 \left[\frac{3}{4} \sin\left(\frac{\pi}{2} - \delta\right) + \frac{1}{12} \sin\left(3\left(\frac{\pi}{2} - \delta\right)\right) \right] = \frac{3}{2} \cos(\delta) - \frac{1}{6} \cos(3\delta)$$

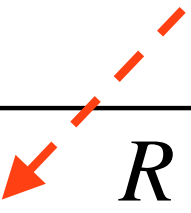
- Substitute into original equation

EXAMPLE Solution (cont'd)

- Simplify

$$C_{D_{LE}} = C_{p_{\max}} \frac{R}{c} \int_{-\left(\frac{\pi}{2}-\delta\right)}^{\left(\frac{\pi}{2}-\delta\right)} [(1-\varepsilon)\cos^3(\Theta) + \varepsilon\cos(\Theta)] d\Theta =$$

$$C_{p_{\max}} \frac{R}{c} (1-\varepsilon) \left[\frac{3}{2} \cos(\delta) - \frac{1}{6} \cos(3\delta) \right] + 2\varepsilon \cos(\delta) = C_{p_{\max}} \frac{R}{c} \left[\frac{3+\varepsilon}{2} \cos(\delta) + \frac{\varepsilon-1}{6} \cos(3\delta) \right]$$



$$C_{D_{LE}} = C_{p_{\max}} \frac{R}{c} \left[\frac{3+\varepsilon}{2} \cos(\delta) + \frac{\varepsilon-1}{6} \cos(3\delta) \right]$$

EXAMPLE Solution (cont'd)

- Compute Leading edge drag at Mach 3.0
- $C_{p_{\text{Max}}} = 1.75571$, $\varepsilon = 0.074$, $\delta = 2^\circ$, $P = 0.01$ m, $c = 2.0$ m

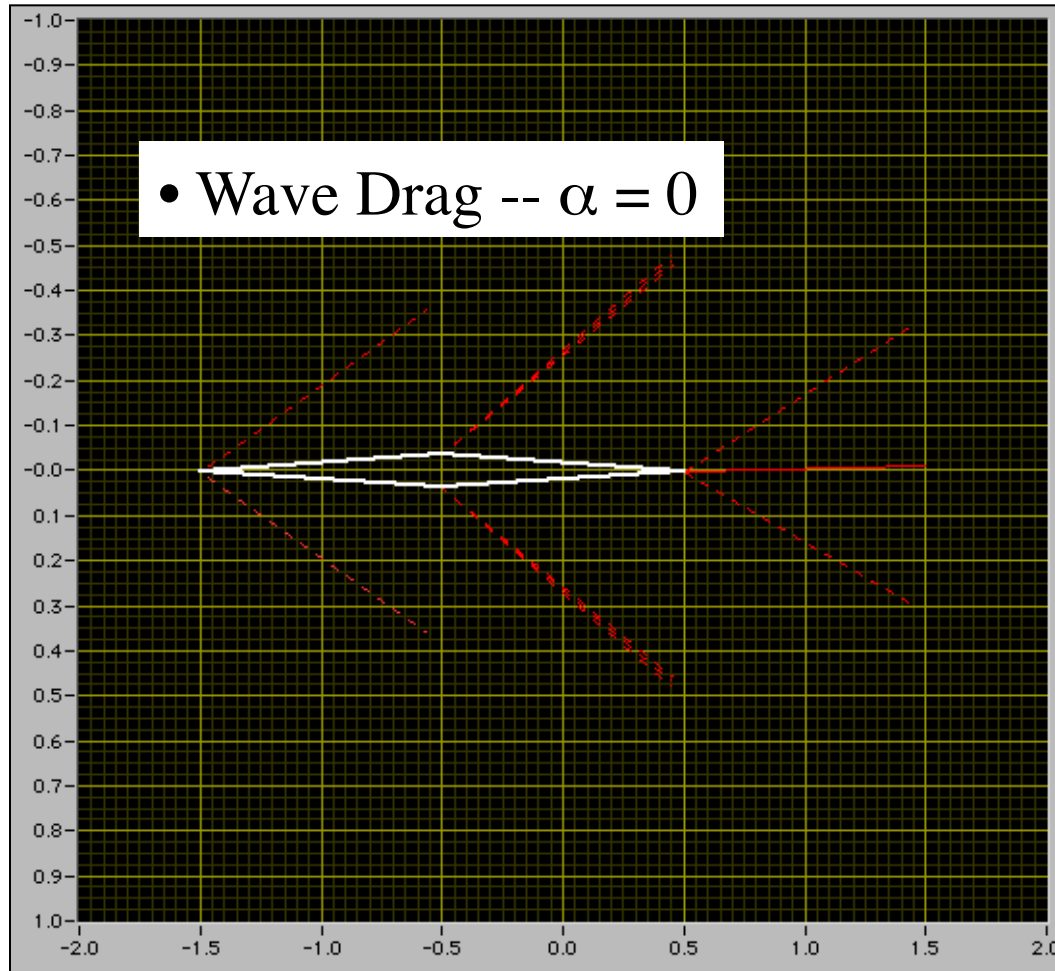
$$C_{D_{LE}} = C_{p_{\text{max}}} \frac{R}{c} \left[\frac{3 + \varepsilon}{2} \cos(\delta) + \frac{\varepsilon - 1}{6} \cos(3\delta) \right]$$

$$1.75571 \left(\frac{0.01}{2} \right) \left(\frac{(3 + 0.074)}{2} \cos \left(\frac{\pi}{180} 2 \right) + \frac{0.074 - 1}{6} \cos \left(\frac{\pi}{180} 3 \cdot 2 \right) \right)$$

$$= 0.012137$$

EXAMPLE Solution (cont'd)

- Compute Wave Coefficient at Mach 3.0



Total CL	-0.00000
Total CD	0.00173
Total L/D	-0.00014
Lref	1.9988

- Wave $C_D = 0.00173$

EXAMPLE Solution (cont'd)

- Compute Skin Drag Coefficient at Mach 3.0

Reference Conditions

Temperature, deg. K	310.30
Density (kg/m ³)	2.818999E-2
Viscosity (N-sec/m ²)	0.00002
Gamma	1.399211
Sonic Velocity (M/sec)	353.13

Velocity, Reynolds Number, CF	
U, m/sec	8.95365E+2
ReL	2.62815E+6
CF	3.76550E-3

$$0.0037655 \cdot 2$$

$$= 0.007531$$

EXAMPLE Solution (cont'd)

- Compute Leading edge drag at Mach 10.0
- $C_{p_{\text{Max}}} = 1.83167$, $\varepsilon = 0.000$, $\delta = 2^\circ$, $P = 0.01$ m, $c = 2.0$ m

$$C_{D_{LE}} = C_{p_{\text{max}}} \frac{R}{c} \left[\frac{3 + \varepsilon}{2} \cos(\delta) + \frac{\varepsilon - 1}{6} \cos(3\delta) \right]$$

$$1.83167 \left(\frac{0.01}{2} \right) \left(\frac{(3 + 0.000)}{2} \cos \left(\frac{\pi}{180} 2 \right) + \frac{0.000 - 1}{6} \cos \left(\frac{\pi}{180} 3 \cdot 2 \right) \right)$$

$$= 0.012211$$

EXAMPLE Solution (cont'd)

- Comparison

• Mach 3.0

$$\begin{aligned} C_{D_{LE}} &= 0.012137 \longrightarrow 56.7\% \\ C_{D_{wave}} &= 0.001730 \\ C_{D_{fric}} &= 0.007531 \end{aligned}$$

• Mach 10.0

$$\begin{aligned} C_{D_{LE}} &= 0.012211 \longrightarrow 56.9\% \\ C_{D_{wave}} &= 0.000500 \\ C_{D_{fric}} &= 0.008769 \end{aligned}$$

“not negligible”