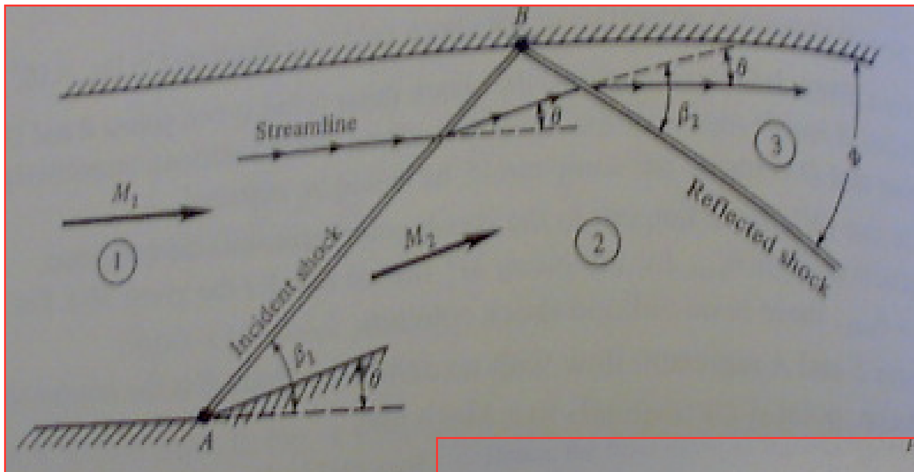
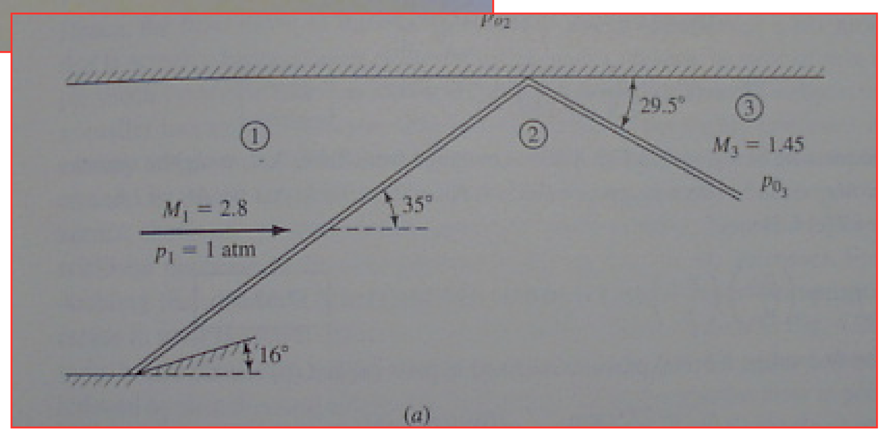
Section 9 Lecture 1: Supersonic Flow Through Systems of Multiple Shockwave



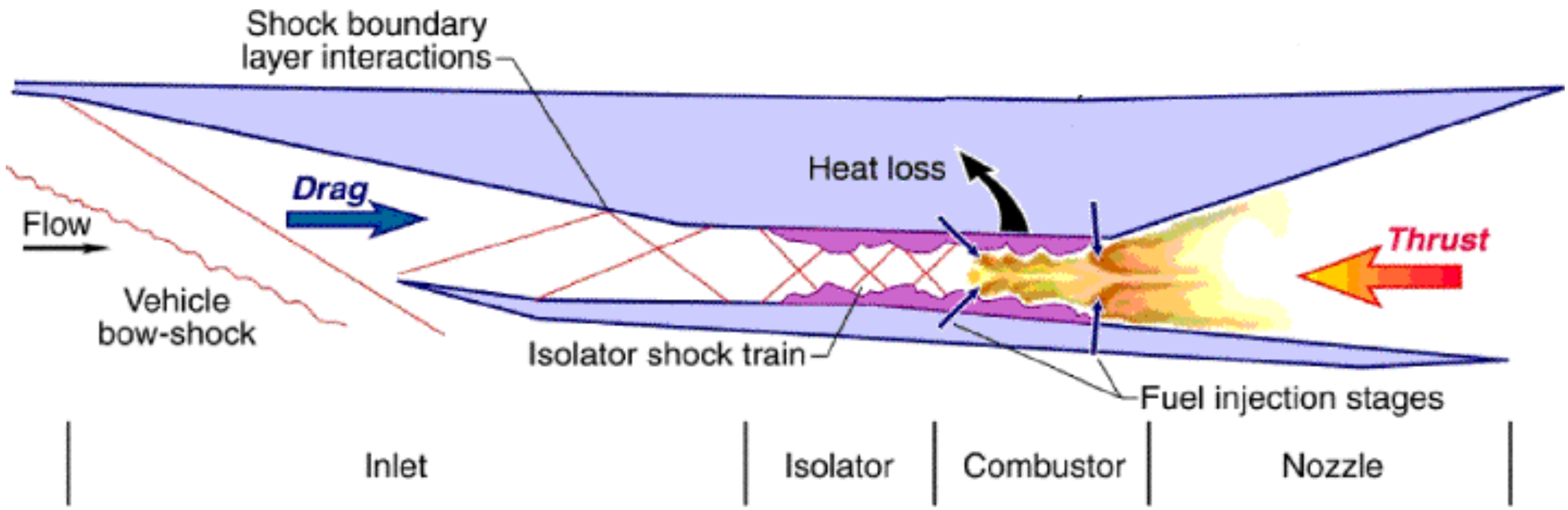


- Anderson, Chapter 4 pp. 152-164

ScrRamjet Design Issues, I

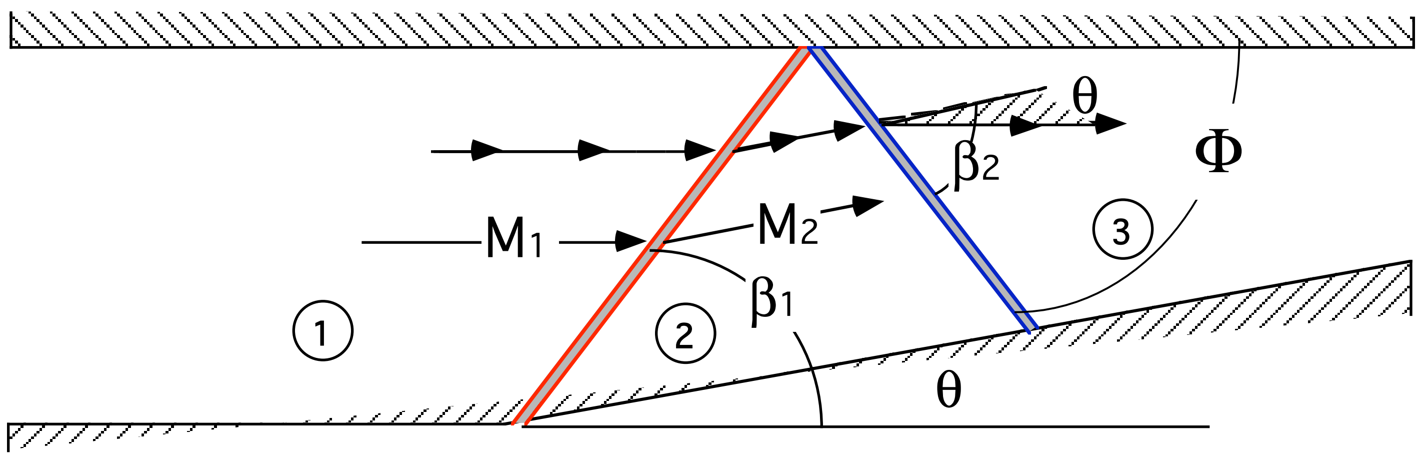
- What if we keep engine flow path supersonic to minimize stagnation pressure loss?

- How do we keep the Inflow supersonic?



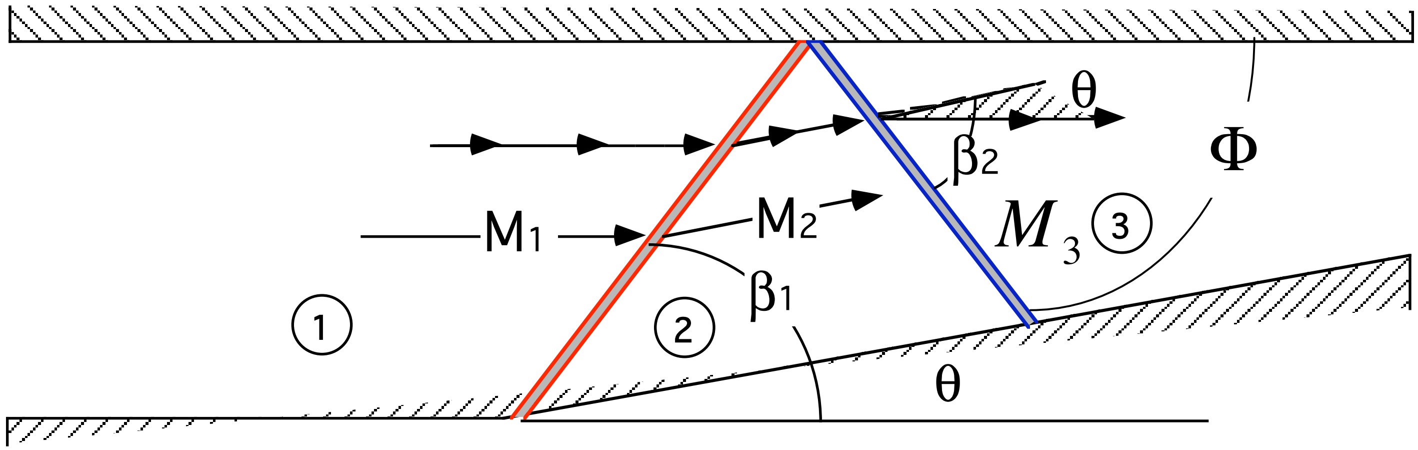
- Series of very weak (highly oblique) shockwaves and expansion shocks keep the flow supersonic throughout the engine

Geometry of a Shock Wave: Reflected From a Solid Boundary



$\beta_1 \neq \beta_2$ • Not Billiards … Why?

Example 1



- Find β_2, Φ, M_3

- Example Calculation

$$M_1 = 3.6$$

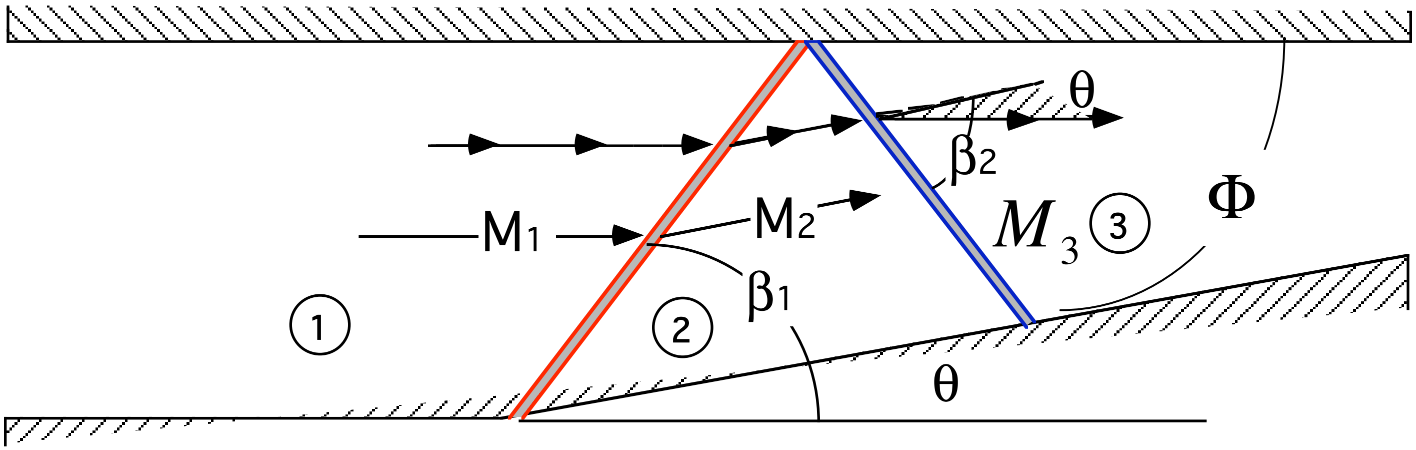
$$\tan(\theta) = \frac{2\left\{M_1^{\ 2}\sin^2(\beta) - 1\right\}}{\tan(\beta)\left[2 + M_1^{\ 2}\left[\gamma + \cos(2\beta)\right]\right]} \rightarrow$$

$$\theta = 20^o$$

$$\gamma = 1.4$$

$$\frac{180}{\pi}\,\mathrm{atan}\left(\frac{2\left(3.6^2\sin^2\left(\frac{\pi}{180}34.1102\right) - 1\right)}{\left(\tan\left(\frac{\pi}{180}34.1102\right)\right)\left(2 + 3.6^2\left(1.4 + \cos\left(\frac{\pi}{180}2\cdot 34.1102\right)\right)\right)}\right) = 20°$$

Example 1 (cont'd)



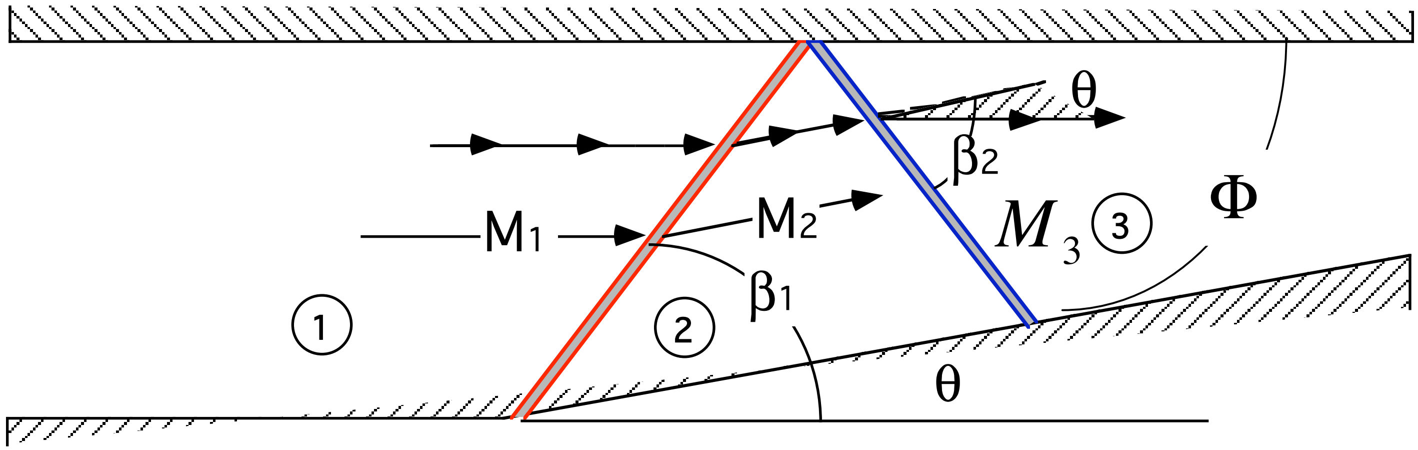
- Solve for β_1

$$\tan(\theta) = \frac{2\left\{M_1^{\,2}\sin^2(\beta)-1\right\}}{\tan(\beta)\left[2+M_1^{\,2}\left[\gamma+\cos(2\beta)\right]\right]} \rightarrow$$

$$\frac{180}{\pi}\,\mathrm{atan}\left(\frac{2\left(3.6^2\sin^2\left(\frac{\pi}{180}34.1102\right)-1\right)}{\left(\tan\left(\frac{\pi}{180}34.1102\right)\right)\left(2+3.6^2\left(1.4+\cos\left(\frac{\pi}{180}2\cdot 34.1102\right)\right)\right)}\right) = 20^\circ$$

$$\boxed{\beta_1 = 34.1102^\circ}$$

Example 1 (cont'd)

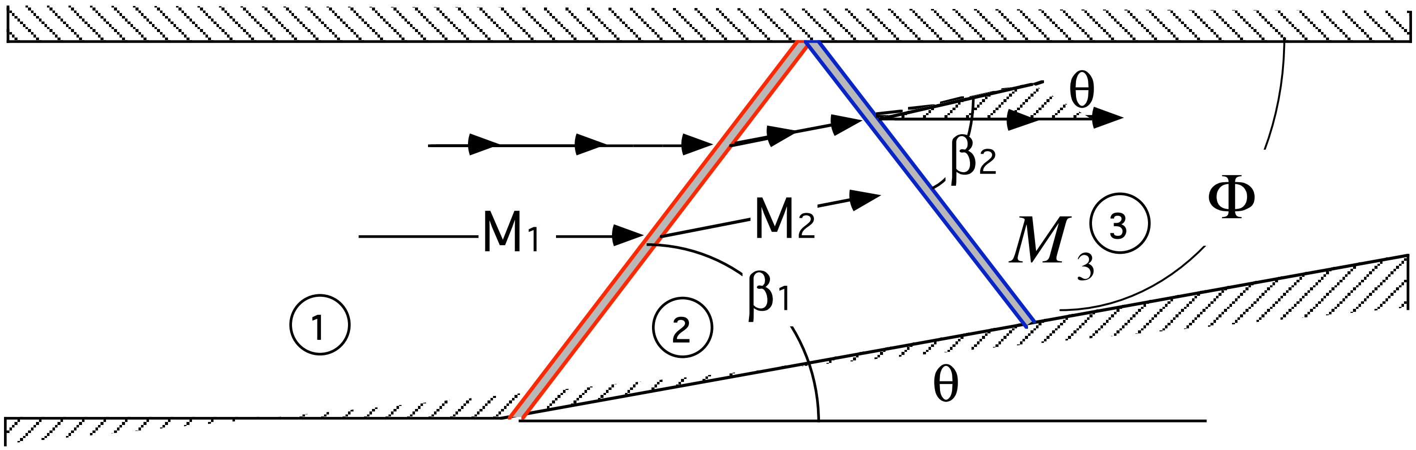


- $M_{1n} = M_1 \sin \beta_1 = 3.6 \sin\left(\frac{\pi}{180} 34.1102\right) = 2.0188$

- Normal Shock Solver-->

$$M_2 n = 0.574168 \rightarrow M_2 = \frac{M_2 n}{\sin(\beta_1 - \theta)} = \frac{0.574168}{\sin\left(\frac{\pi}{180}(34.1102 - 20)\right)} = 2.3552$$

Example 1 (cont'd)



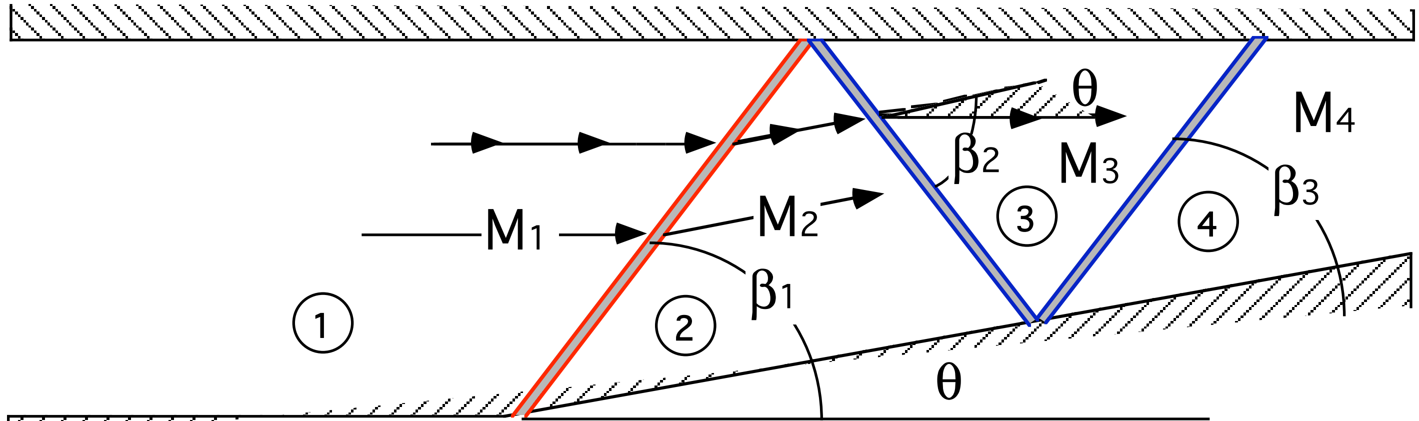
θ-β-M solver, for $M_2 = 2.3552$ and $\theta_2 = 20°$,

---> $\beta_2 = 45.0534° \longrightarrow \Phi = 45.0534° - 20° = 25.0534°$

$$\left(M_2 n\right)_{\beta_2} = 2.3552 \sin(45.0534) = 0.649303$$

$$\rightarrow M_3 = \frac{\left(M_2 n\right)_{\beta_2}}{\sin(\beta_2 - \theta)} = \frac{0.649303}{\sin\left(\frac{\pi}{180}(45.0534 - 20)\right)} = 1.5333$$

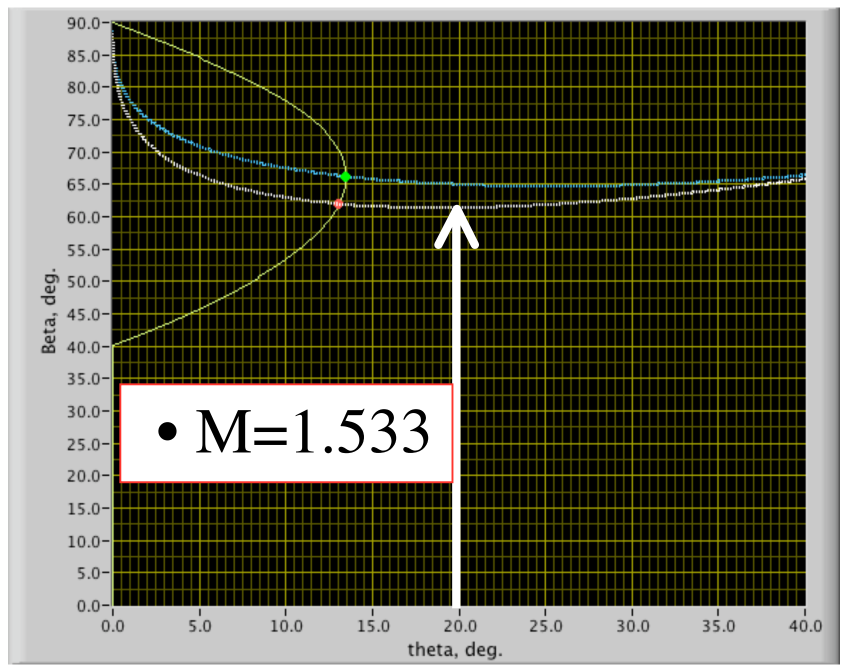
Example 1 (cont'd)



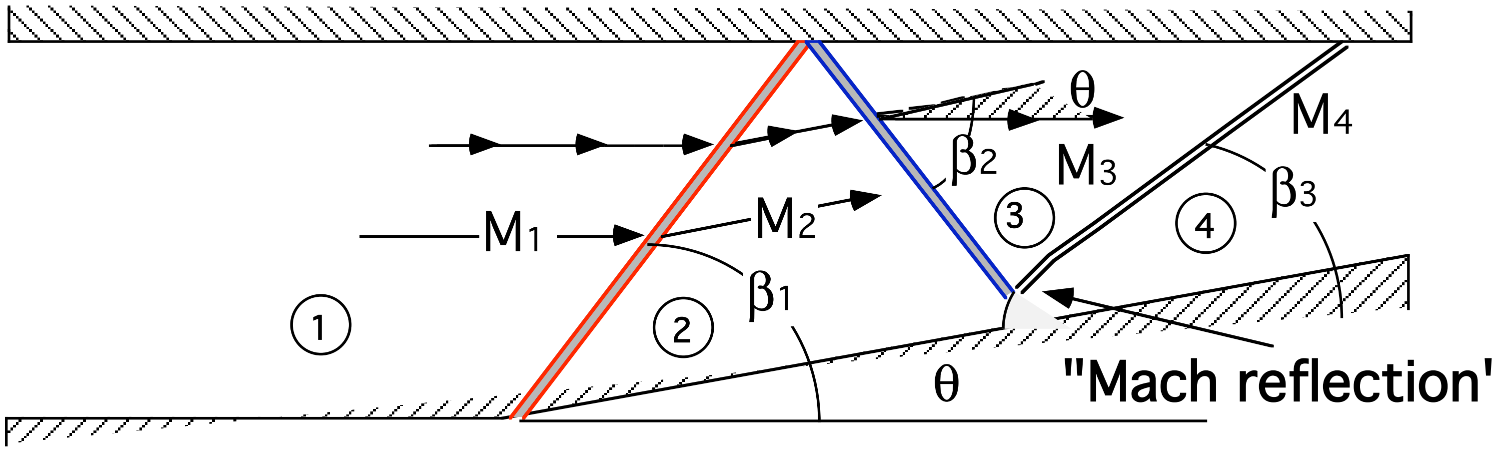
θ-β-M solver, for M_2 = 2.3552 and θ_2 = 20°,

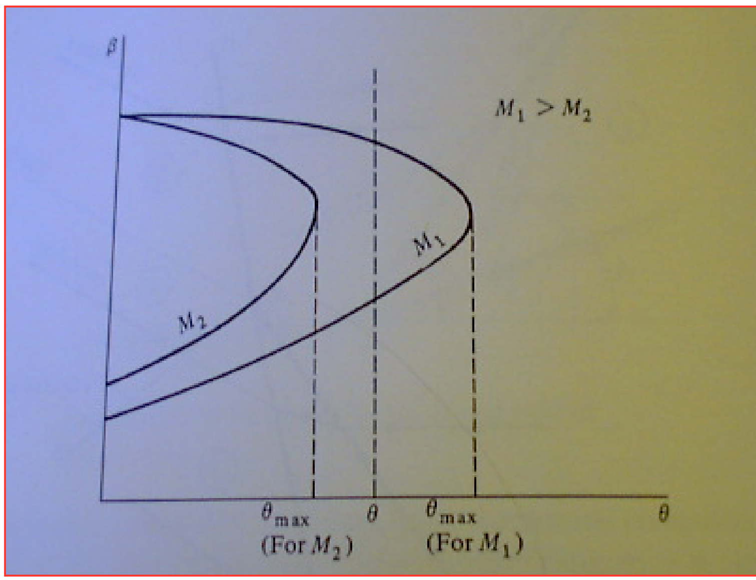
- 20° >θ_{max}

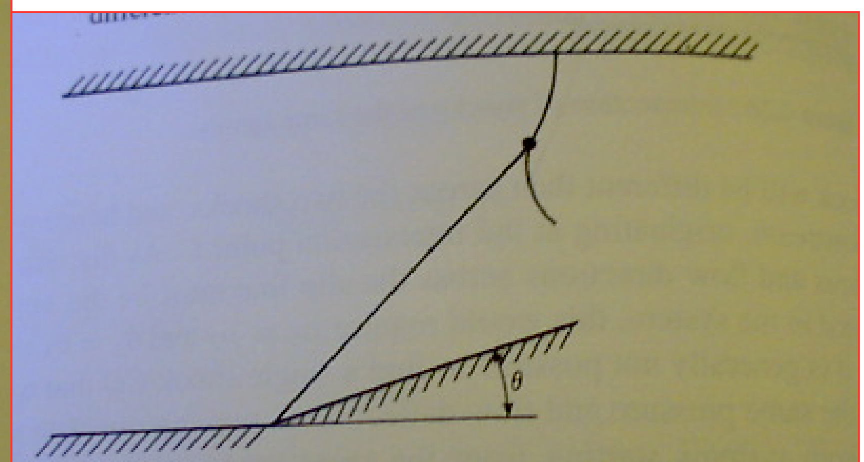
Detached Shockwave!



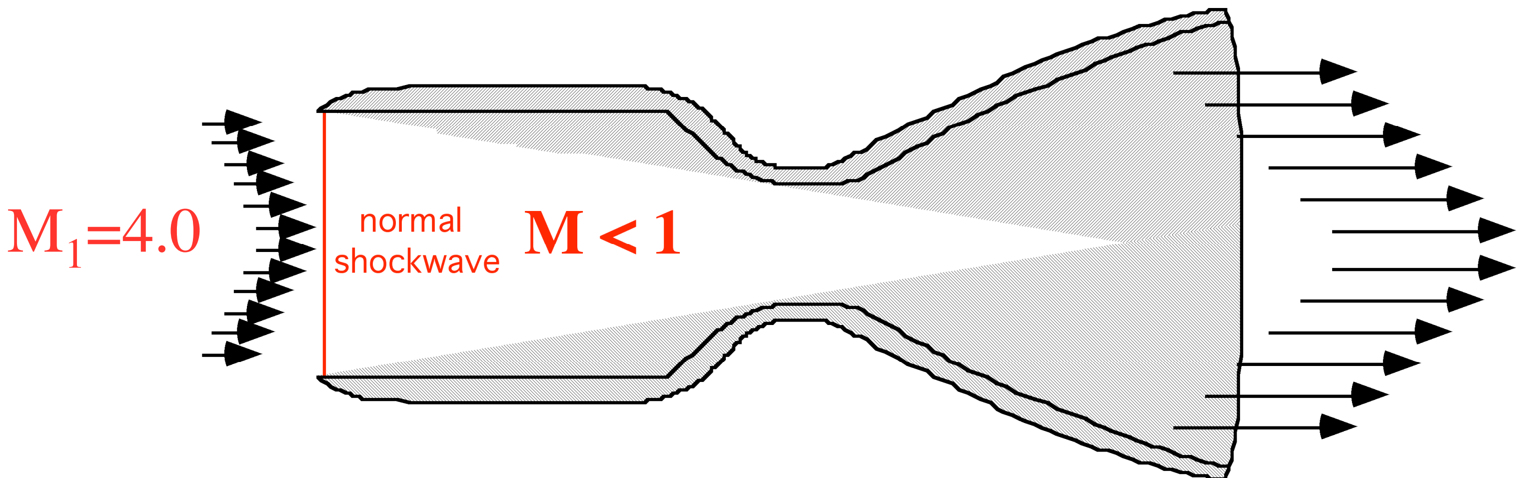
Example 1 (cont'd)



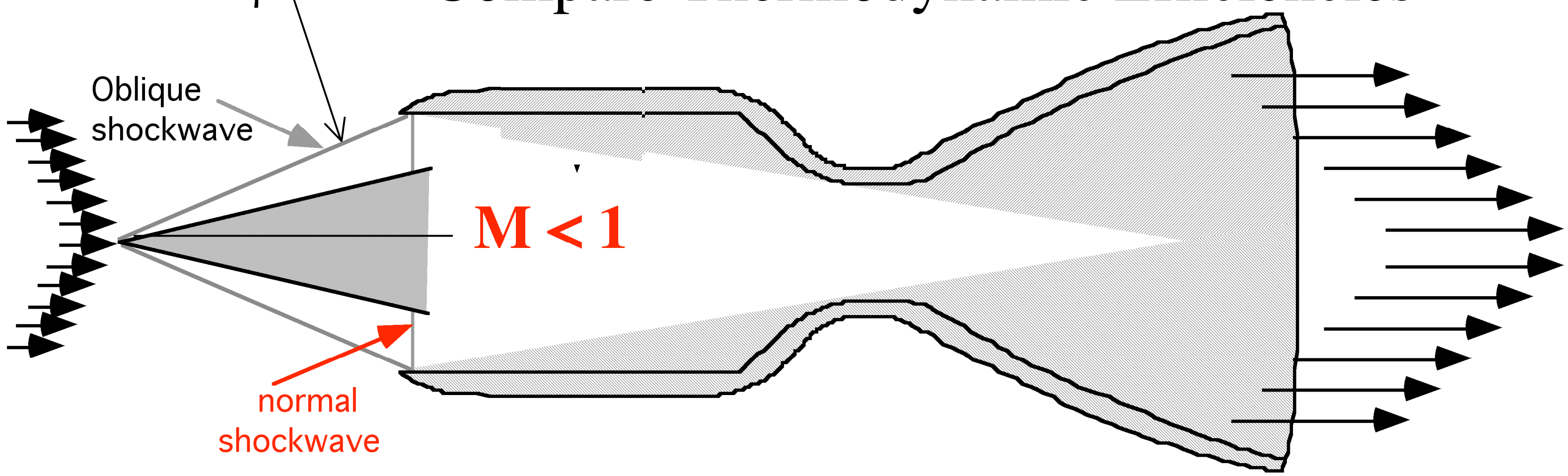




2-D Ramjet Inlet Example



- Compare Thermodynamic Efficiencies



Ramjet design

- So … we put a spike in front of the inlet

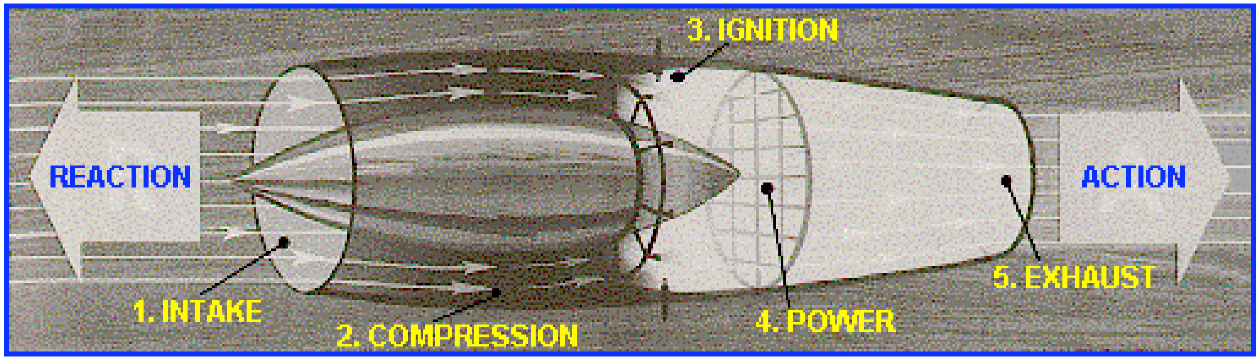


Figure 6-10 Ramjet engine.

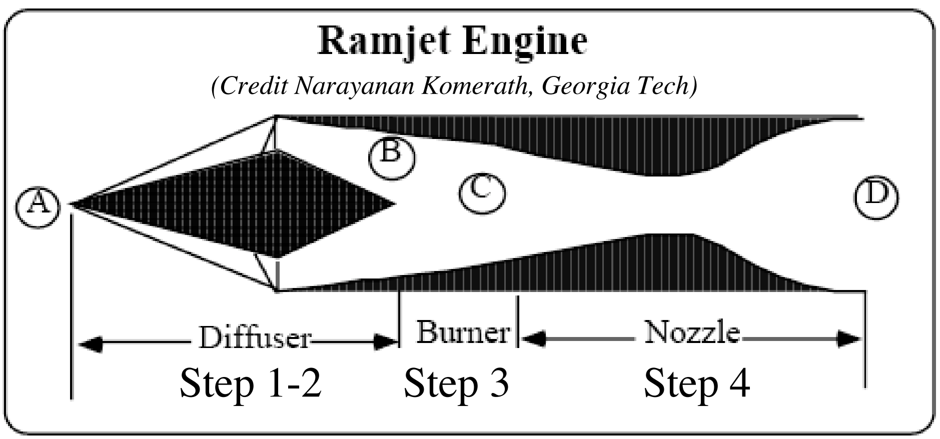
- How does this spike Help?
- By forming an Oblique Shock wave ahead of the inlet

**Actually the J-85 engine on the SR-71 is a turbojet … but you get the picture

Thermodynamic Efficiency of Ideal Ramjet

- Net Work Available --> work perform by system in step 4 minus work required for step 1-2

- Net heat input --> heat input during step 3 (combustion) - heat lost in exhaust plume

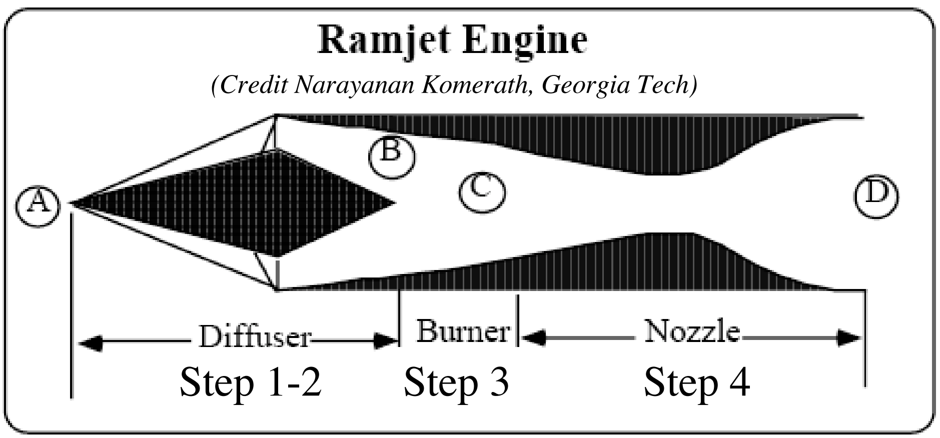


Thermodynamic Efficiency of Ideal Ramjet (cont'd)

- Ideal Cycle Efficiency(η) = (Net work output)/(Net heat input}

$$\frac{Net\ Work}{\dot{m}} = (h_A - h_B) + (h_C - h_D)$$

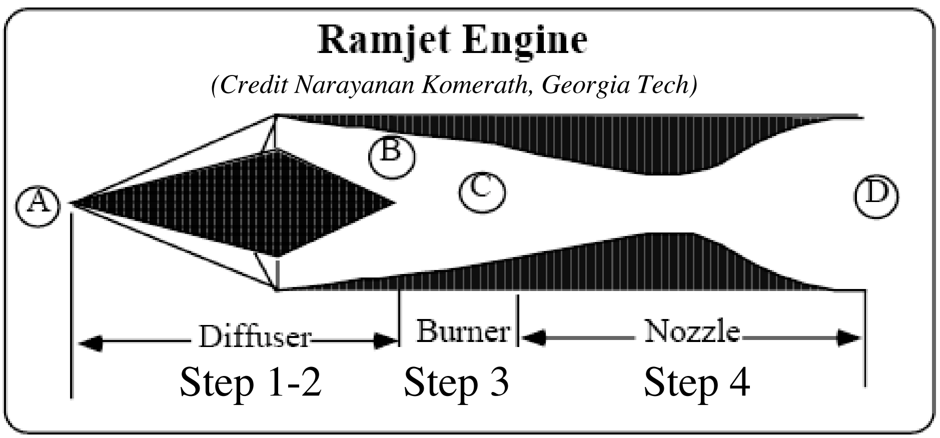
$$\frac{Net\ Heat\ Input}{\dot{m}} = (h_C - h_B)$$



Thermodynamic Efficiency of Ideal Ramjet (cont'd)

- Ideal Cycle Efficiency = (Net work output)/(Net heat input}

$$\eta = \frac{\left(\dfrac{Net\ Work}{\dot{m}}\right)}{\left(\dfrac{Net\ Heat\ Input}{\dot{m}}\right)} = \frac{(h_A - h_B) + (h_C - h_D)}{(h_C - h_B)} = 1 - \frac{(h_D - h_A)}{(h_C - h_B)}$$



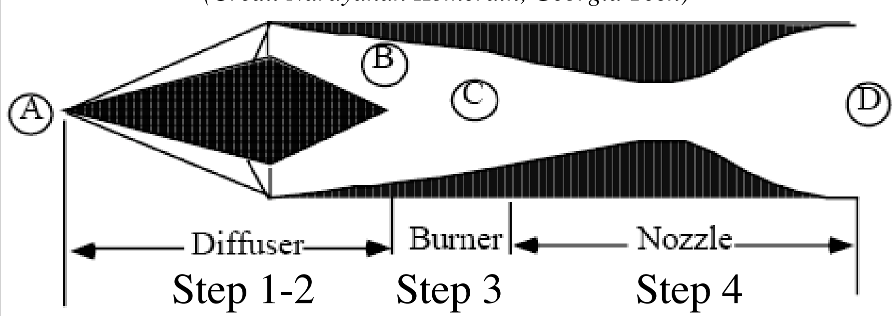
Thermodynamic Efficiency of Ideal Ramjet (cont'd)

- Assume … $Cp_{air} \sim Cp_{products}$

$$\eta = 1 - \frac{\left(C_{p\ products} T_D - C_{p\ air} T_A\right)}{\left(C_{p\ air} T_C - C_{p\ products} T_B\right)} = 1 - \frac{\left(T_D - \dfrac{C_{p\ air}}{C_{p\ products}} T_A\right)}{\left(\dfrac{C_{p\ air}}{C_{p\ products}} T_C - T_B\right)}$$

Ramjet Engine

(Credit Narayanan Komerath, Georgia Tech)



Thermodynamic Efficiency of Ideal Ramjet (cont'd)

- For simplicity … let … $Cp_{air} \sim Cp_{products}$

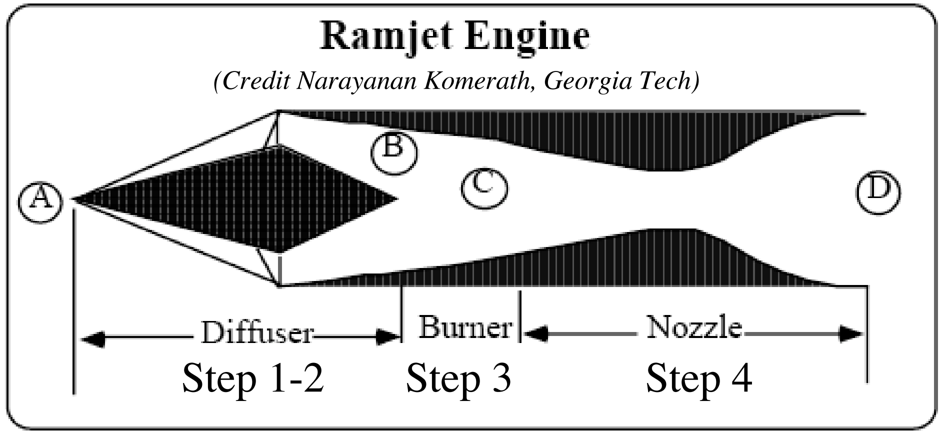
$$\eta \approx 1 - \frac{\left(T_D - \frac{C_{p\,air}}{C_{p\,products}} T_A\right)}{\left(\frac{C_{p\,air}}{C_{p\,products}} T_C - T_B\right)} = 1 - \frac{\left(T_D - T_A\right)}{\left(T_C - T_B\right)}$$

$$= 1 - \frac{\left(\frac{T_D}{T_C} - \frac{T_A}{T_C}\right)}{\left(1 - \frac{T_B}{T_C}\right)} = 1 - \frac{\left(\frac{T_D}{T_C} - \frac{T_A}{T_B}\frac{T_B}{T_C}\right)}{\left(1 - \frac{T_B}{T_C}\right)}$$

Thermodynamic Efficiency of Ideal Ramjet (cont'd)

- From C-->D flow is isentropic …

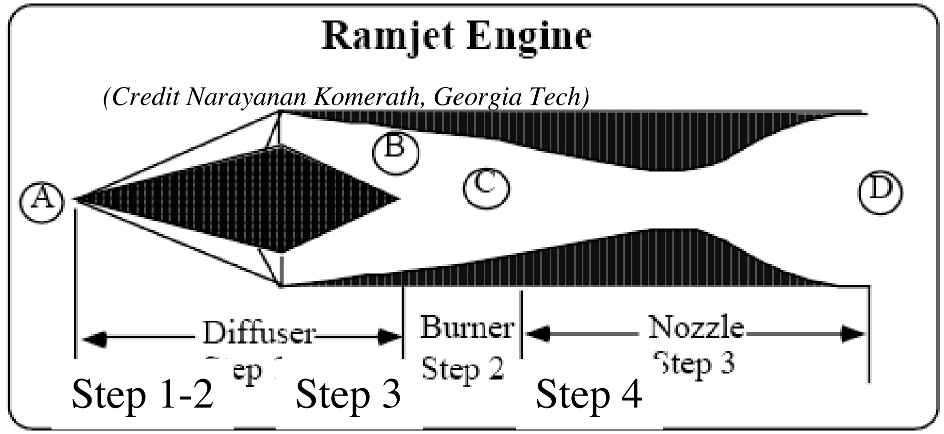
$$\rightarrow \frac{T_D}{T_C} = \left(\frac{P_D}{P_C}\right)^{\frac{\gamma-1}{\gamma}}$$



$$\eta = 1 - \frac{\left(\frac{T_D}{T_C} - \frac{T_A}{T_B}\frac{T_B}{T_C}\right)}{\left(1 - \frac{T_B}{T_C}\right)} = 1 - \frac{\left(\left(\frac{P_D}{P_C}\right)^{\frac{\gamma-1}{\gamma}} - \frac{T_A}{T_B}\frac{T_B}{T_C}\right)}{\left(1 - \frac{T_B}{T_C}\right)}$$

Thermodynamic Efficiency of Ideal Ramjet (cont'd)

- Adiabatic compression across diffuser

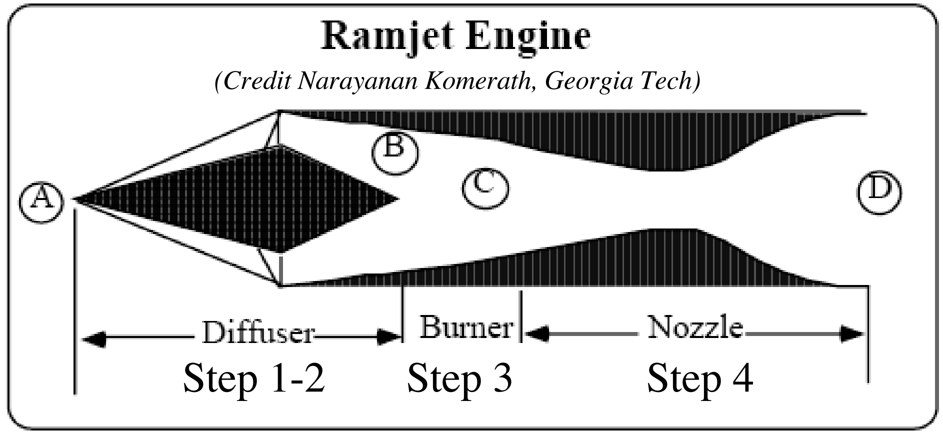


$$\left(\frac{P_A}{P_B}\right) = \left(\frac{P_A}{P_{0_A}} \times \frac{P_{0_A}}{P_{0_B}} \times \frac{P_{0_B}}{P_B}\right) =$$

$$\left(\frac{T_A}{T_{0_A}}\right)^{\frac{\gamma}{\gamma-1}} \left(\frac{T_{0_B}}{T_B}\right)^{\frac{\gamma}{\gamma-1}} \frac{P_{0_A}}{P_{0_B}} \rightarrow T_{0_A} = T_{0_B} \rightarrow \frac{T_A}{T_B} = \left(\frac{P_A}{P_B}\right)^{\frac{\gamma-1}{\gamma}} \left(\frac{P_{0_B}}{P_{0_A}}\right)^{\frac{\gamma-1}{\gamma}}$$

Thermodynamic Efficiency of Ideal Ramjet (cont'd)

- Sub into efficiency equation

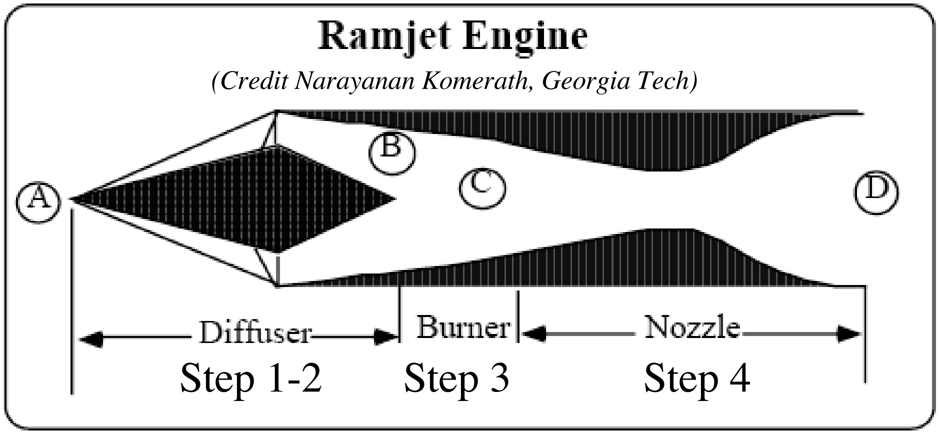


$$\eta = 1 - \frac{\left(\left(\frac{P_D}{P_C}\right)^{\frac{\gamma-1}{\gamma}} - \left(\frac{P_A}{P_B}\right)^{\frac{\gamma-1}{\gamma}} \left(\frac{P_{0_B}}{P_{0_A}}\right)^{\frac{\gamma-1}{\gamma}} \frac{T_B}{T_C} \right)}{\left(1 - \frac{T_B}{T_C}\right)}$$

Thermodynamic Efficiency of Ideal Ramjet (cont'd)

$$ideal\ nozzle \rightarrow P_A = P_D$$

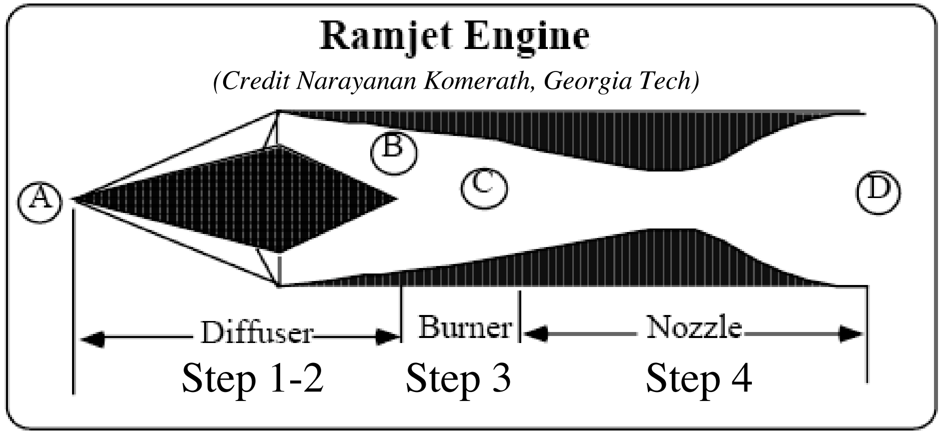
$$ideal\ burner \rightarrow P_B = P_C$$



$$\eta = 1 - \left(\frac{P_A}{P_B}\right)^{\frac{\gamma-1}{\gamma}} \frac{\left(1 - \left(\frac{P_{0_B}}{P_{0_A}}\right)^{\frac{\gamma-1}{\gamma}} \frac{T_B}{T_C}\right)}{\left(1 - \frac{T_B}{T_C}\right)} = 1 - \left(\frac{P_A}{P_B}\right)^{\frac{\gamma-1}{\gamma}} \frac{\left(T_C - \left(\frac{P_{0_B}}{P_{0_A}}\right)^{\frac{\gamma-1}{\gamma}} T_B\right)}{\left(T_C - T_B\right)}$$

Thermodynamic Efficiency of Ideal Ramjet (cont'd)

$$\eta = 1 - \left(\frac{P_A}{P_B}\right)^{\frac{\gamma-1}{\gamma}} \frac{\left(T_C - \left(\frac{P_{0_B}}{P_{0_A}}\right)^{\frac{\gamma-1}{\gamma}} T_B\right)}{\left(T_C - T_B\right)}$$

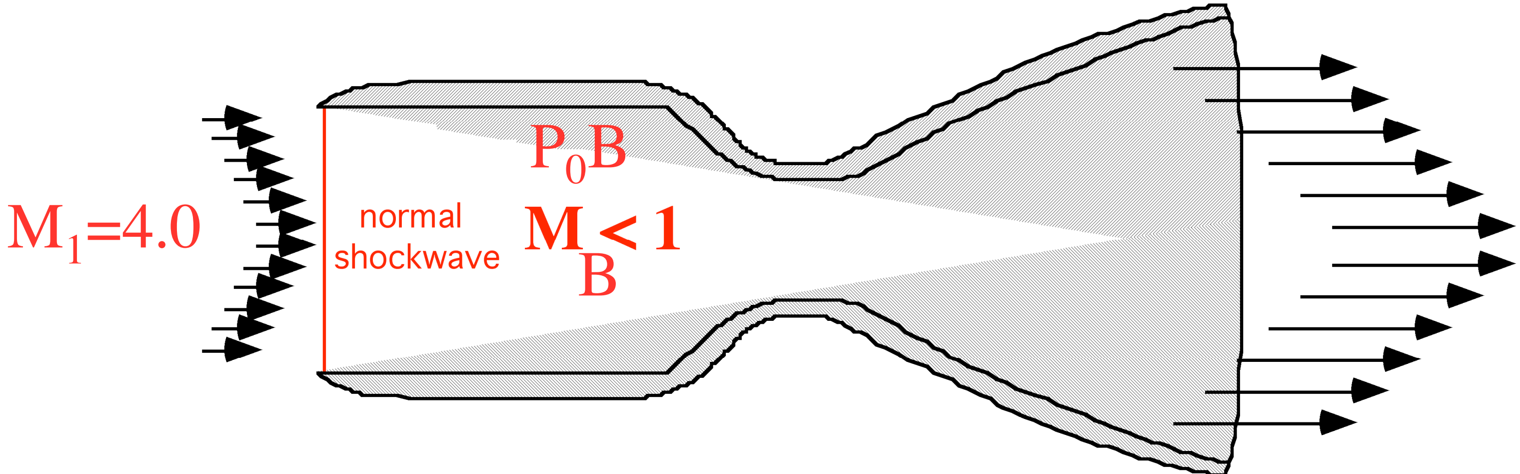


i) As engine pressure ratio, P_B/P_A, goes up … η *goes up*

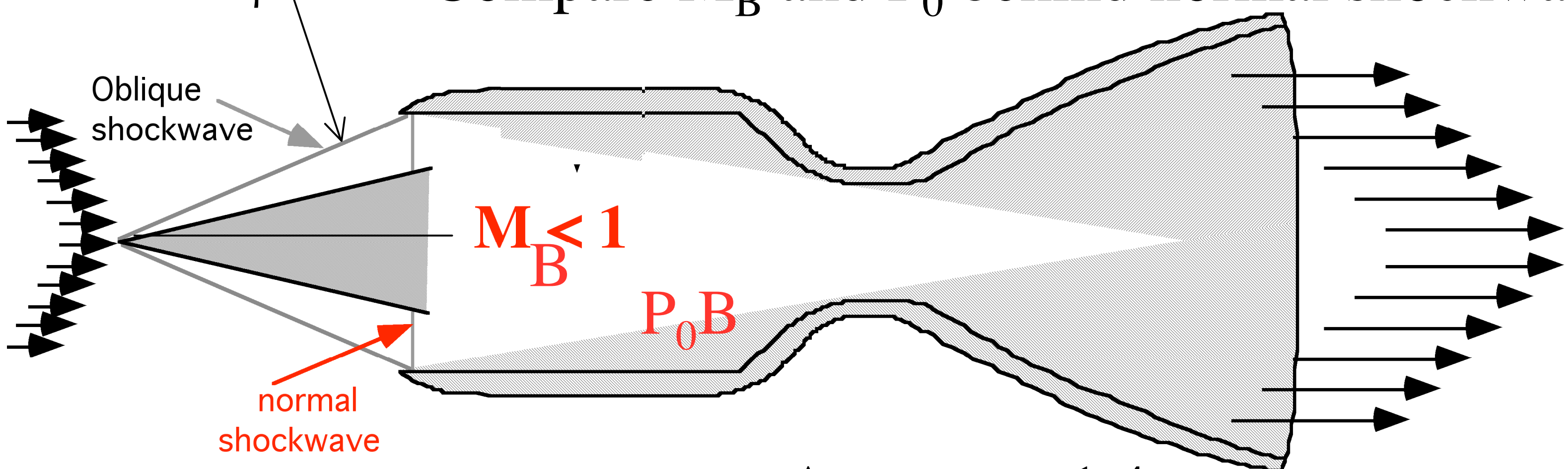
ii) As combustor temperature difference T_C-T_B goes up … η *goes up*

iii) As inlet total pressure ratio (P_{0_B}/P_{0_A}) goes down … (stagnation pressure loss goes up) … η *goes down*

2-D Ramjet Inlet Example

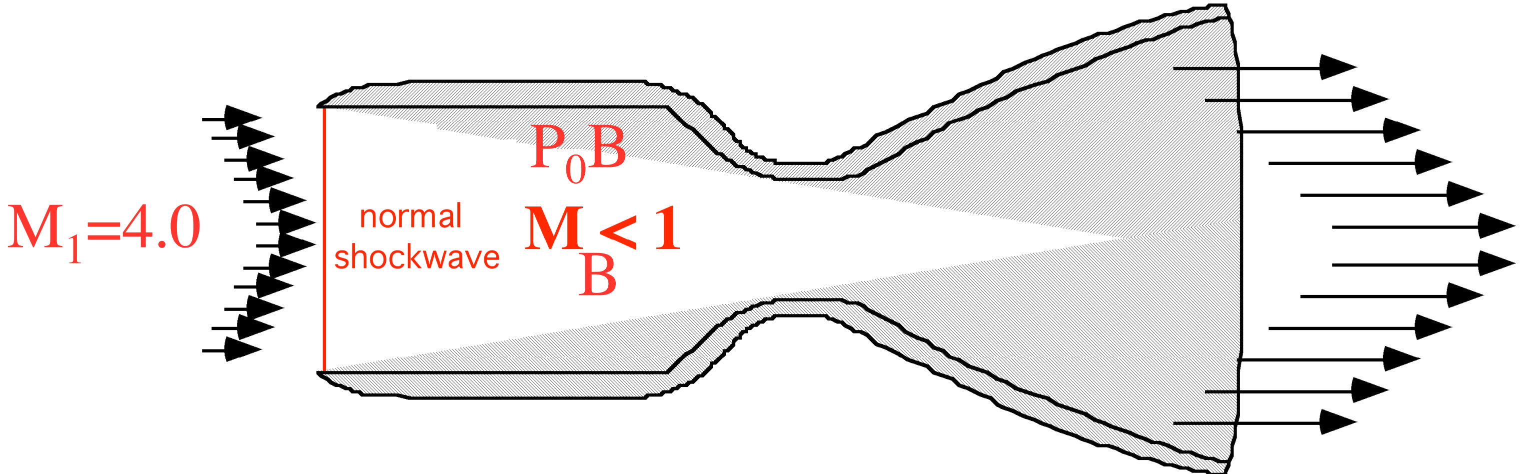


- Compare M_B and P_0 behind normal shockwaves



Assume γ=1.4

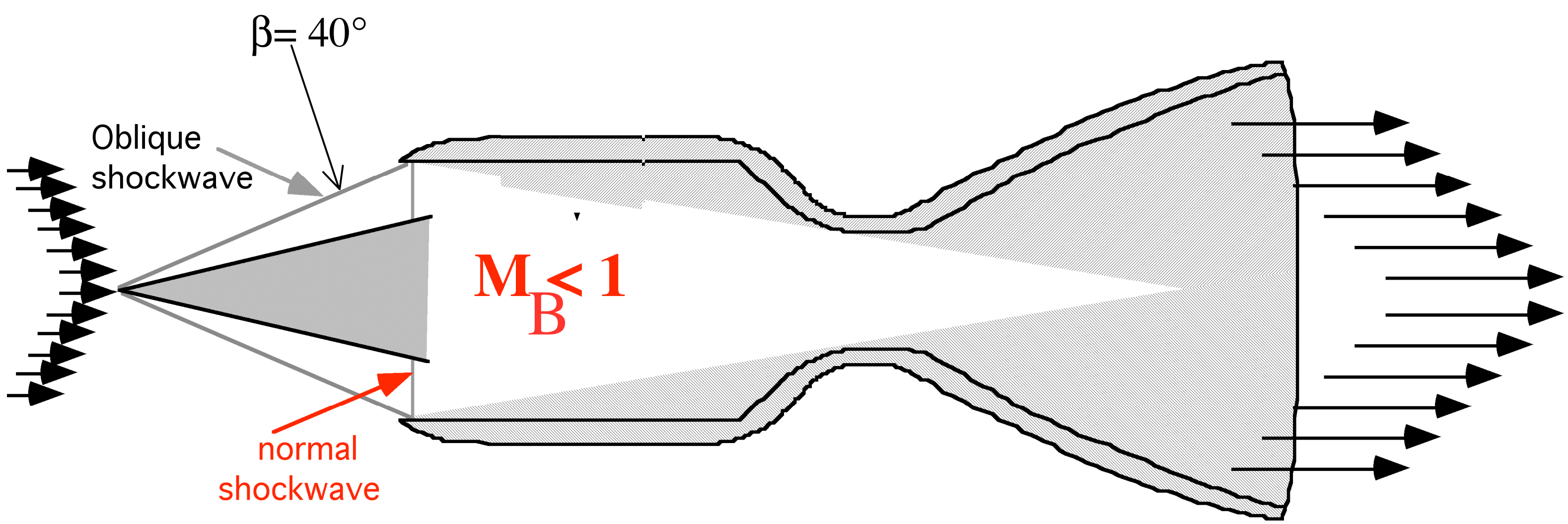
2-D Ramjet Inlet Example (cont'd)



- From Normal Shock wave solver

$$M_\infty \xrightarrow{normal..shock} M_B = 0.434959 \Rightarrow \begin{bmatrix} \dfrac{P_{0_B}}{P_{0_\infty}} = 0.1388 \\ \\ \dfrac{p_B}{p_\infty} = 18.5 \\ \\ \end{bmatrix}$$

2-D Ramjet Inlet Example (cont'd)



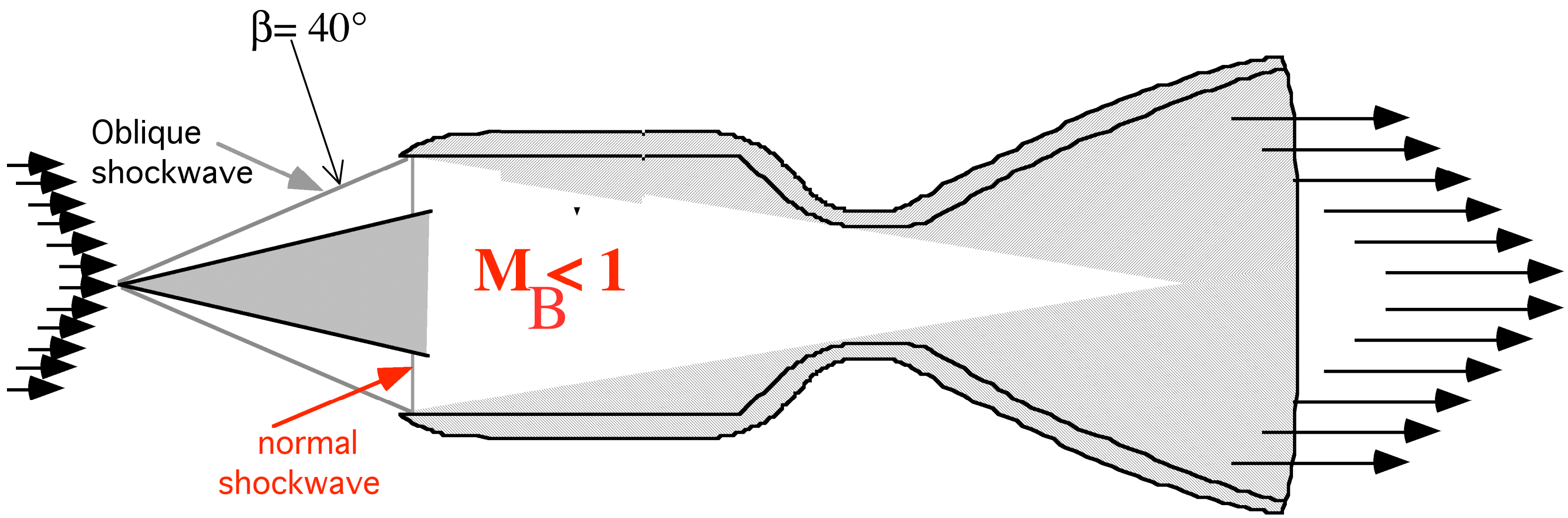
- Across Oblique Shock wave

- $M_{1n} = M_1 \sin\beta_1 = 4\sin\left(\frac{\pi}{180}40\right)$ =2.571 $\longrightarrow$ M_2n=0.5064

$$\tan(\theta) = \frac{2\left\{M_1^{\,2}\sin^2(\beta)-1\right\}}{\tan(\beta)\left[2+M_1^{\,2}\left[\gamma+\cos(2\beta)\right]\right]} \rightarrow \frac{180}{\pi}\,\mathrm{atan}\left(\frac{2\left(4^2\sin^2\left(\frac{\pi}{180}40\right)-1\right)}{\left(\tan\left(\frac{\pi}{180}40\right)\right)\left(2+4^2\left(1.4+\cos\left(\frac{\pi}{180}2\cdot 40\right)\right)\right)}\right)$$

$$\boxed{\theta = 26.2^\circ}$$

2-D Ramjet Inlet Example (cont'd)

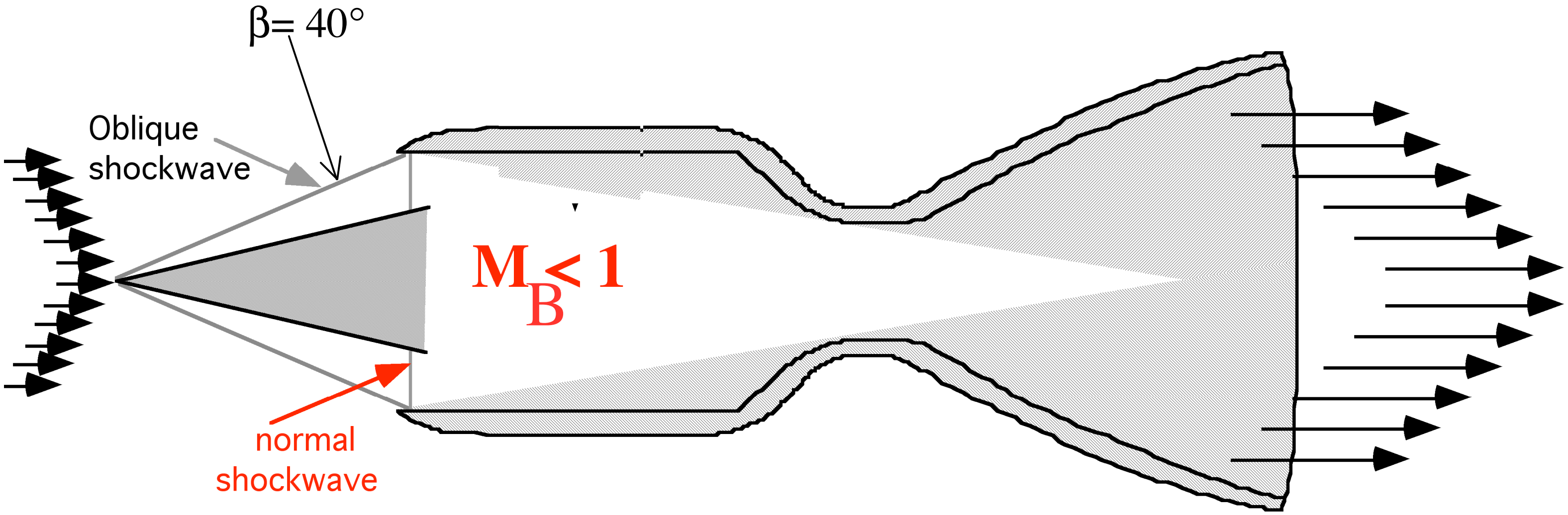


- Across Oblique Shock wave

$$M_2 n = 0.5064 \rightarrow \boxed{M_2 = \frac{M_2 n}{\sin(\beta_1 - \theta)} = \frac{0.5064}{\sin\left(\frac{\pi}{180}(40 - 26.2)\right)} = 2.123}$$

$$P_{0_2}/P_{0_1} = 0.4711$$

2-D Ramjet Inlet Example (cont'd)



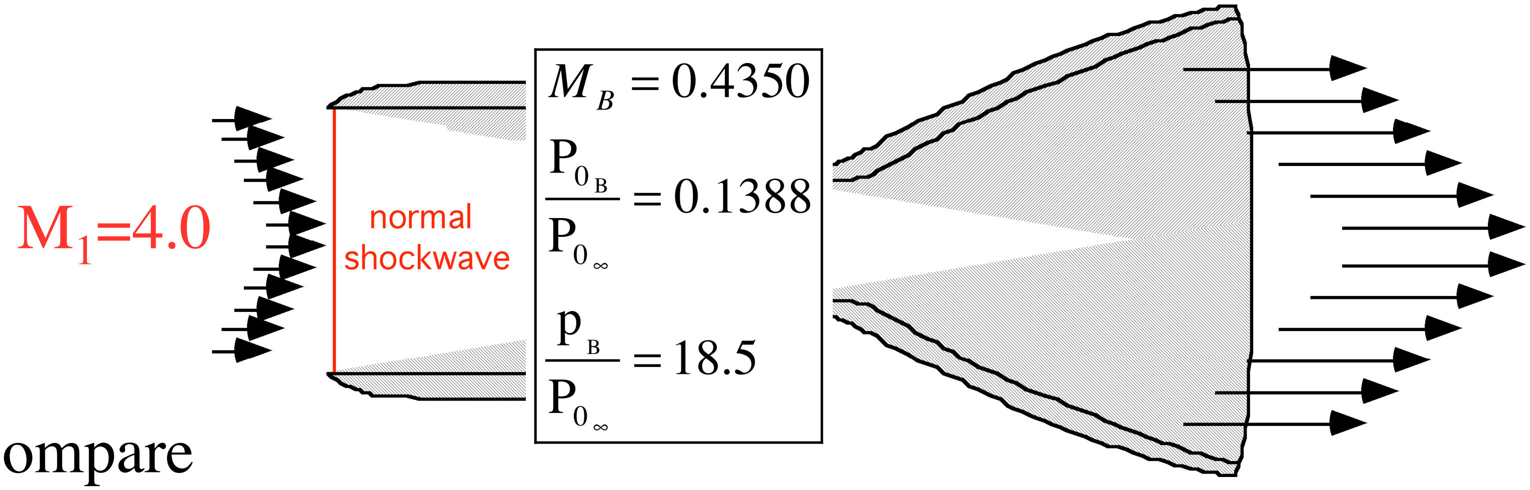
- Across Normal Shock wave *(behind oblique Shock)*

$$M_2 = 2.123 \xrightarrow{normal..shock} M_B = 0.557853 \Rightarrow$$

$$\frac{P_{0_B}}{P_{0_2}} = 0.663531 \Rightarrow \frac{P_{0_B}}{P_{0_\infty}} = \frac{P_{0_B}}{P_{0_2}} \frac{P_{0_2}}{P_{0_\infty}} = (0.663531)(0.4711) = 0.3126$$

$$\frac{p_B}{p_\infty} = \frac{p_B}{P_{0_B}} \times \frac{P_{0_B}}{P_{0_\infty}} \times \frac{P_{0_\infty}}{p_\infty} = \frac{0.3126\left(\left(1+\frac{1.4-1}{2}4^2\right)^{\frac{1.4}{(1.4-1)}}\right)}{\left(\left(1+\frac{1.4-1}{2}0.557853^2\right)^{\frac{1.4}{(1.4-1)}}\right)} = 38.422$$

2-D Ramjet Inlet Example (cont'd)

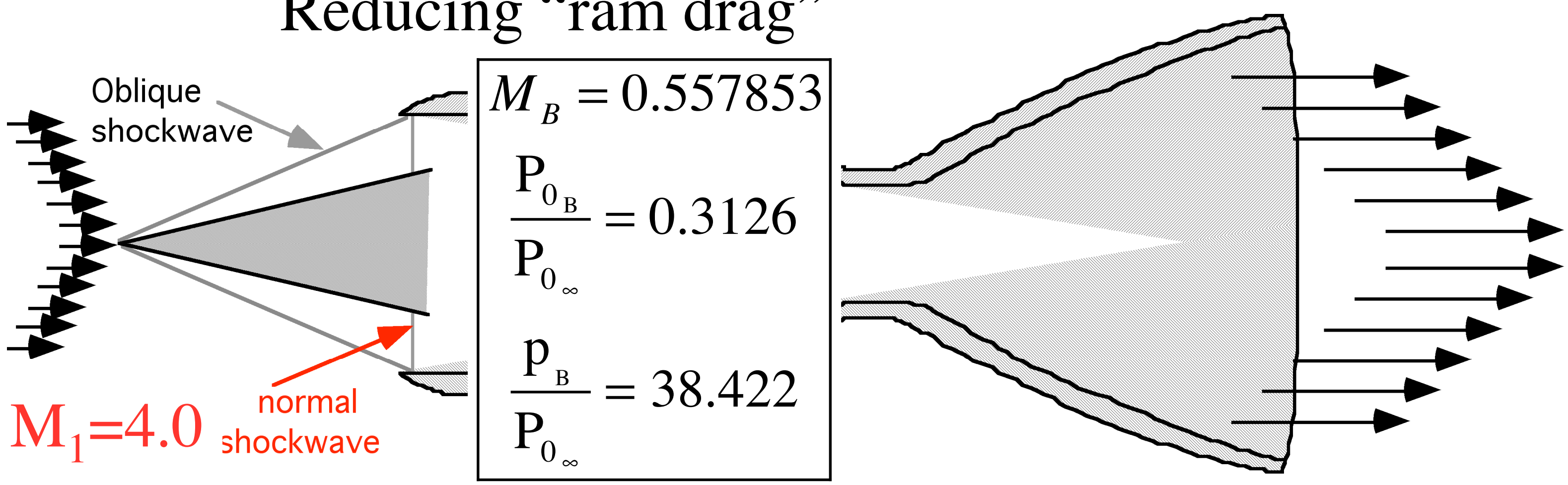


$$M_B = 0.4350$$

$$\frac{P_{0_B}}{P_{0_\infty}} = 0.1388$$

$$\frac{p_B}{P_{0_\infty}} = 18.5$$

- Compare
  - Spike aids in increasing Total Pressure recovery Reducing "ram drag"



$$M_B = 0.557853$$

$$\frac{P_{0_B}}{P_{0_\infty}} = 0.3126$$

$$\frac{p_B}{P_{0_\infty}} = 38.422$$

2-D Ramjet Inlet Example (cont'd)

- … Continuing example … **Incoming Air to Ramjet**

- Molecular weight = 28.96443 kg/kg-mole
- γ = 1.40
- R_g = 287.056 J/°K-(kg)
- T_∞ = 216.65 °K
- p_∞ = 19.330 kPa
- Combustor $\dot{q} = \dot{q}/\dot{m} =$ 500 kJ/kg

- *Assume that mass of added fuel is negligible, exhaust and γ, R_g are the same*

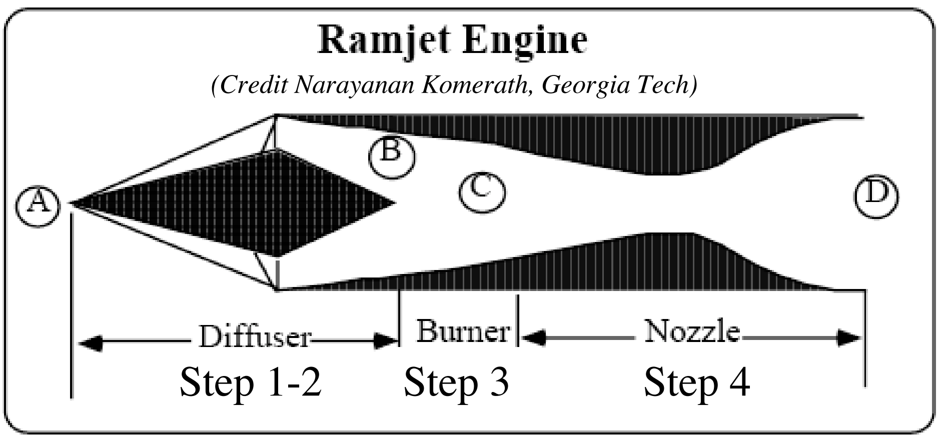
2-D Ramjet Inlet Example (cont'd)

- Compute free stream stagnation temperature

$$T_{0_\infty} = T_\infty\left[1+\frac{\gamma-1}{2}M_\infty{}^2\right] = 216.65\left(1+\frac{1.4-1}{2}4^2\right) = 909.93^\circ\mathrm{K}$$

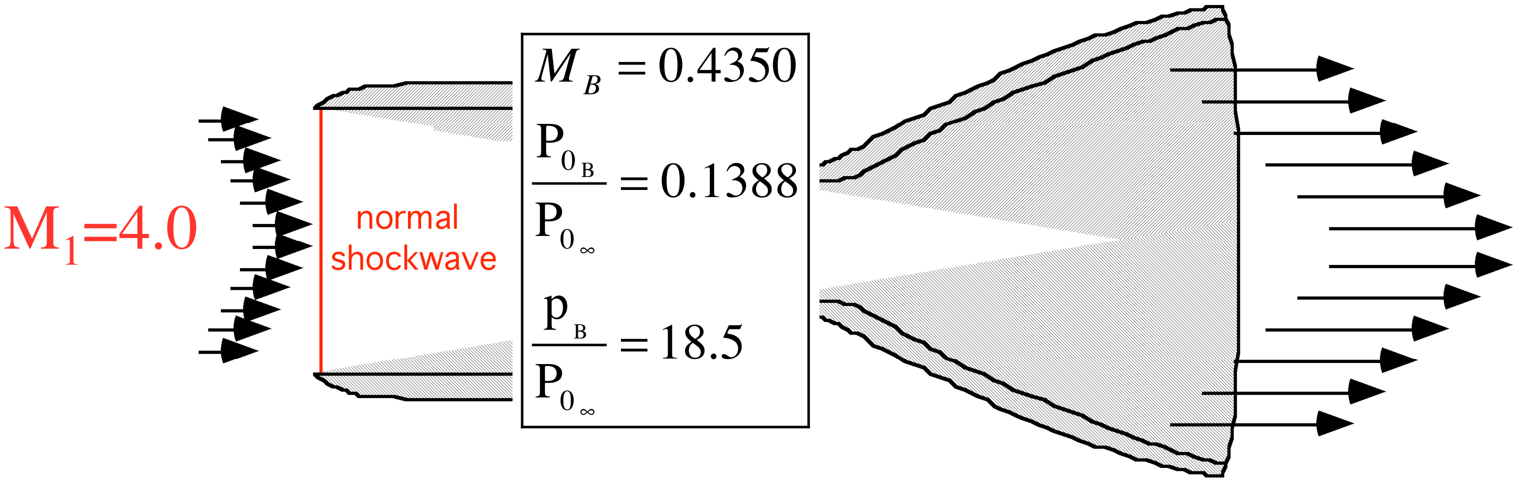
- Compute BURNER stagnation temperature

$$T_{0_C} = \frac{q + c_p T_{0_\infty}}{c_p} = \frac{500\cdot 10^3 + 1004.696\,(909.93)}{1004.696} = 1407.6\ ^\circ\mathrm{K}$$



2-D Ramjet Inlet Example (cont'd)

- Compute efficiency …. Normal shock inlet



$$M_B = 0.4350$$

$$\frac{P_{0_B}}{P_{0_\infty}} = 0.1388$$

$$\frac{p_B}{P_{0_\infty}} = 18.5$$

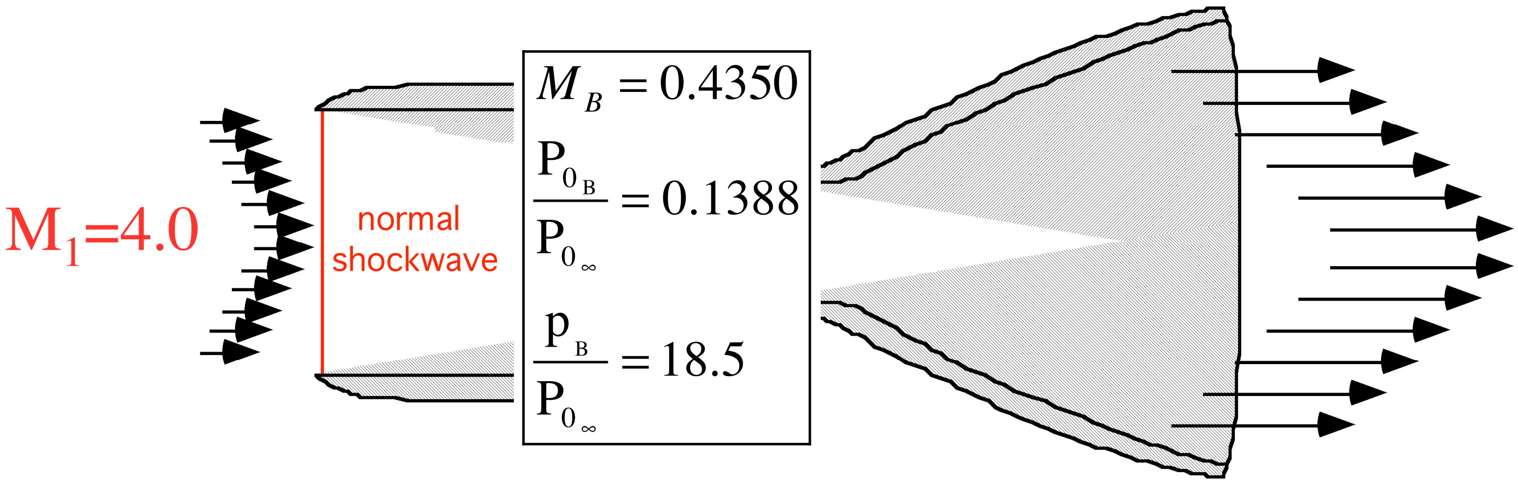
- Compute T_C, T_B

$$T_C = \frac{909.93}{\left(1 + \frac{1.4 - 1}{2} 0.435^2\right)} = 867.75°K$$

$$T_B = \frac{1407.6}{\left(1 + \frac{1.4 - 1}{2} 0.435^2\right)} = 1356.3°K$$

2-D Ramjet Inlet Example (cont'd)

- Compute efficiency …. Normal shock inlet



- Compute T_C, T_B

T_C = 867.75°K

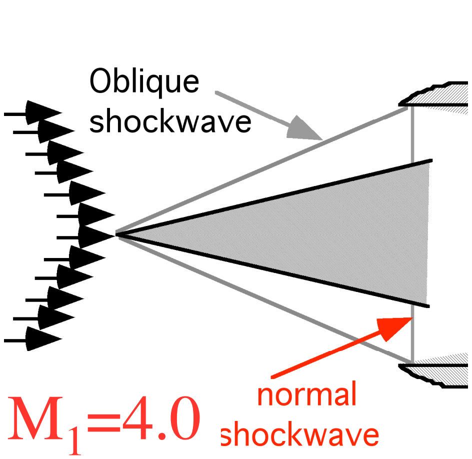
T_B = 1356.3°K

$$\eta = 1 - \left(\frac{P_A}{P_B}\right)^{\frac{\gamma-1}{\gamma}} \frac{\left(T_C - \left(\frac{P_{0_B}}{P_{0_A}}\right)^{\frac{\gamma-1}{\gamma}} T_B\right)}{(T_C - T_B)} =$$

$$1 - \frac{18.5^{\frac{-(1.4-1)}{1.4}}\left(1356.3 - \left(0.1388^{\frac{(1.4-1)}{1.4}}\right) 867.75\right)}{(1356.3 - 867.75)} = 0.2328$$

2-D Ramjet Inlet Example (cont'd)

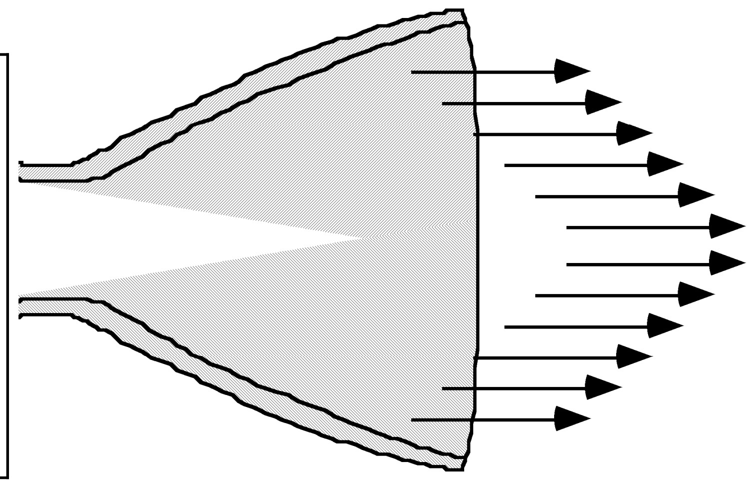
- Compute efficiency …. Oblique shock inlet



$$M_B = 0.557853$$

$$\frac{P_{0_B}}{P_{0_\infty}} = 0.3126$$

$$\frac{p_B}{P_{0_\infty}} = 38.422$$



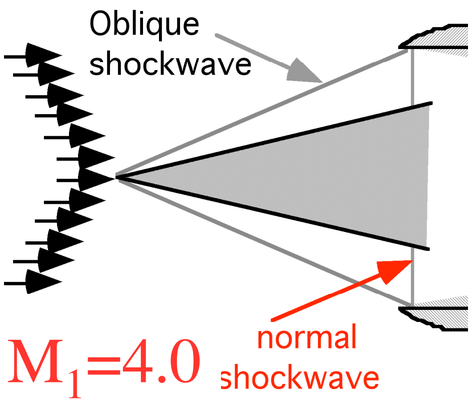
- Compute T_C, T_B

$$T_C = \frac{909.93}{\left(1 + \frac{1.4-1}{2} 0.557853^2\right)} = 856.61°K$$

$$T_B = \frac{1407.6}{\left(1 + \frac{1.4-1}{2} 0.557853^2\right)} = 1325.1°K$$

2-D Ramjet Inlet Example (cont'd)

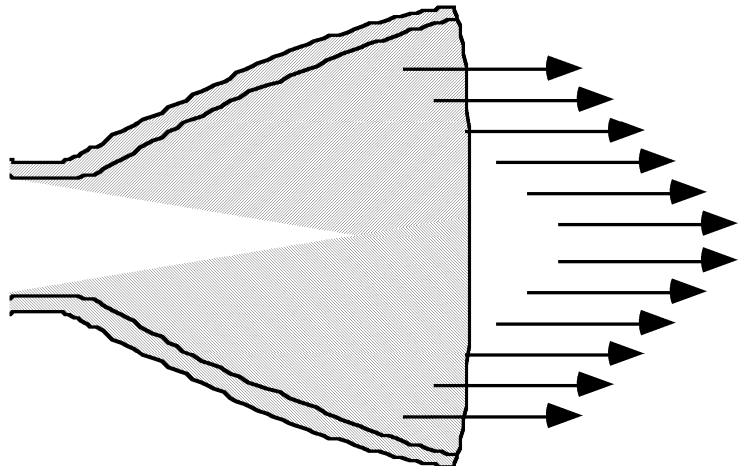
- Compute efficiency …. Oblique shock inlet



$$M_B = 0.557853$$

$$\frac{P_{0_B}}{P_{0_\infty}} = 0.3126$$

$$\frac{p_B}{P_{0_\infty}} = 38.422$$



- Compute T_C, T_B

$T_C = 856.61°K$

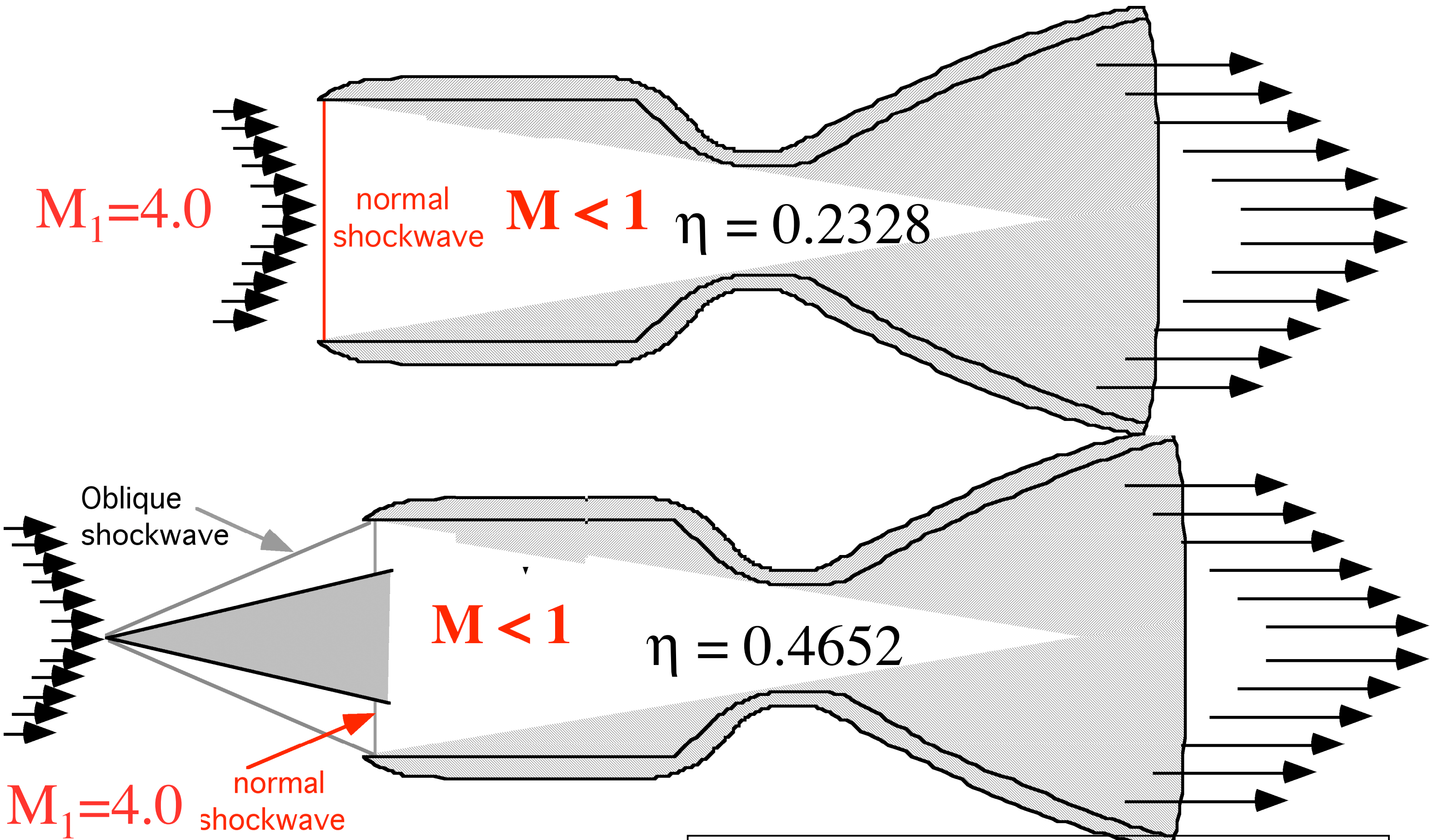
$T_B = 1325.1°K$

$$\eta = 1 - \left(\frac{P_A}{P_B}\right)^{\frac{\gamma-1}{\gamma}} \frac{\left(T_C - \left(\frac{P_{0_B}}{P_{0_A}}\right)^{\frac{\gamma-1}{\gamma}} T_B\right)}{(T_C - T_B)} =$$

$$1 - \frac{38.422^{\frac{-(1.4-1)}{1.4}}\left(1325.1 - \left(0.3126^{\frac{(1.4-1)}{1.4}}\right)856.61\right)}{(1325.1 - 856.61)} = 0.4652$$

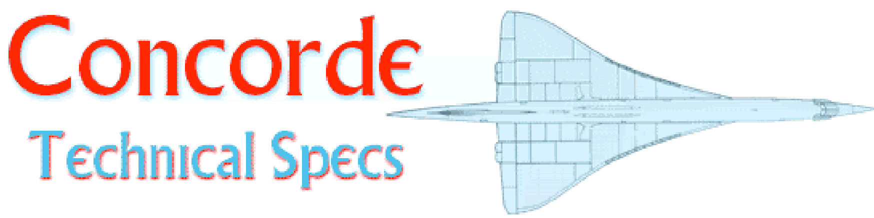
2-D Ramjet Inlet Example (cont'd)

- Compute efficiency …. Oblique shock inlet

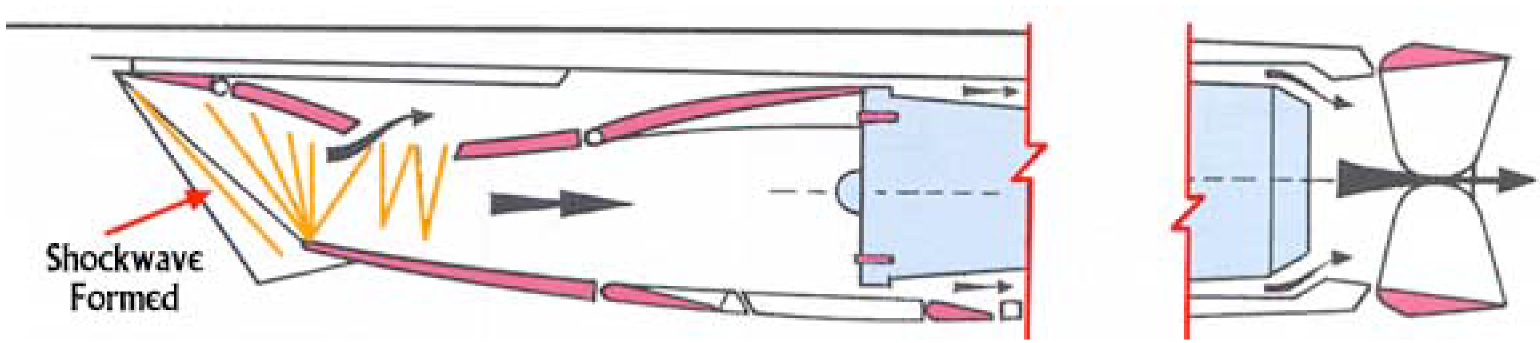


200% increase in efficiency!

Supersonic Inlet: Condorde Inlet Design



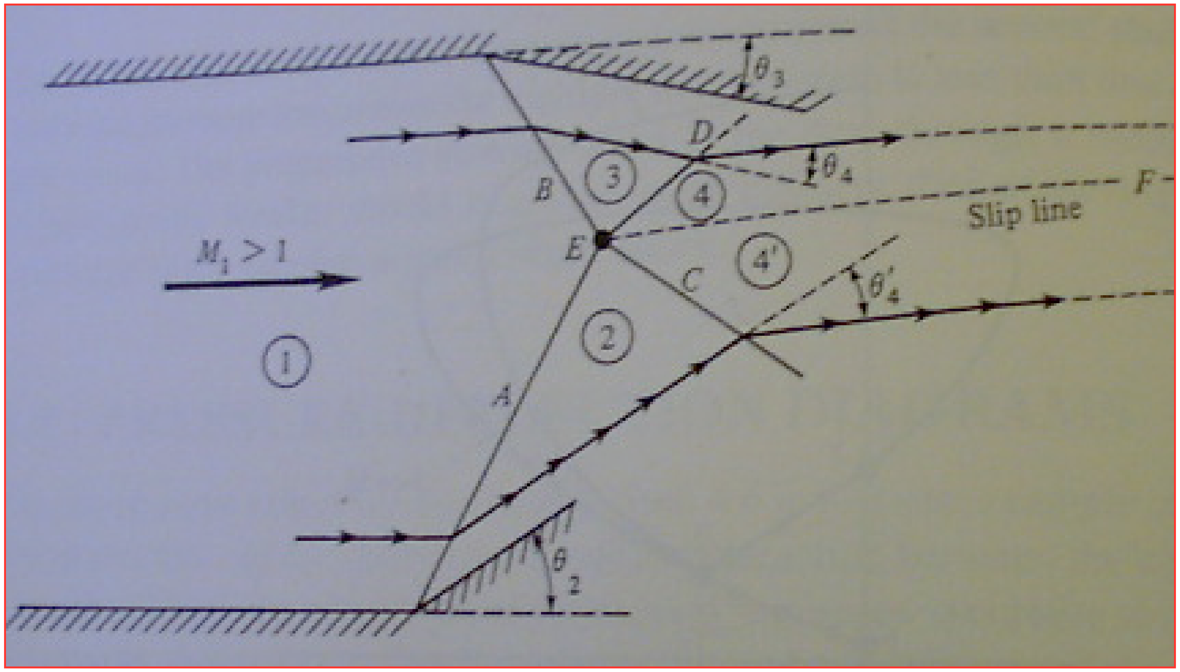
- Mach 2 Cruise



Credit:
http://www.concordesst.com/powerplant.html

Conical versus 2-D Inlets

Geometry of a Shock Wave: Reflected From another Shock wave

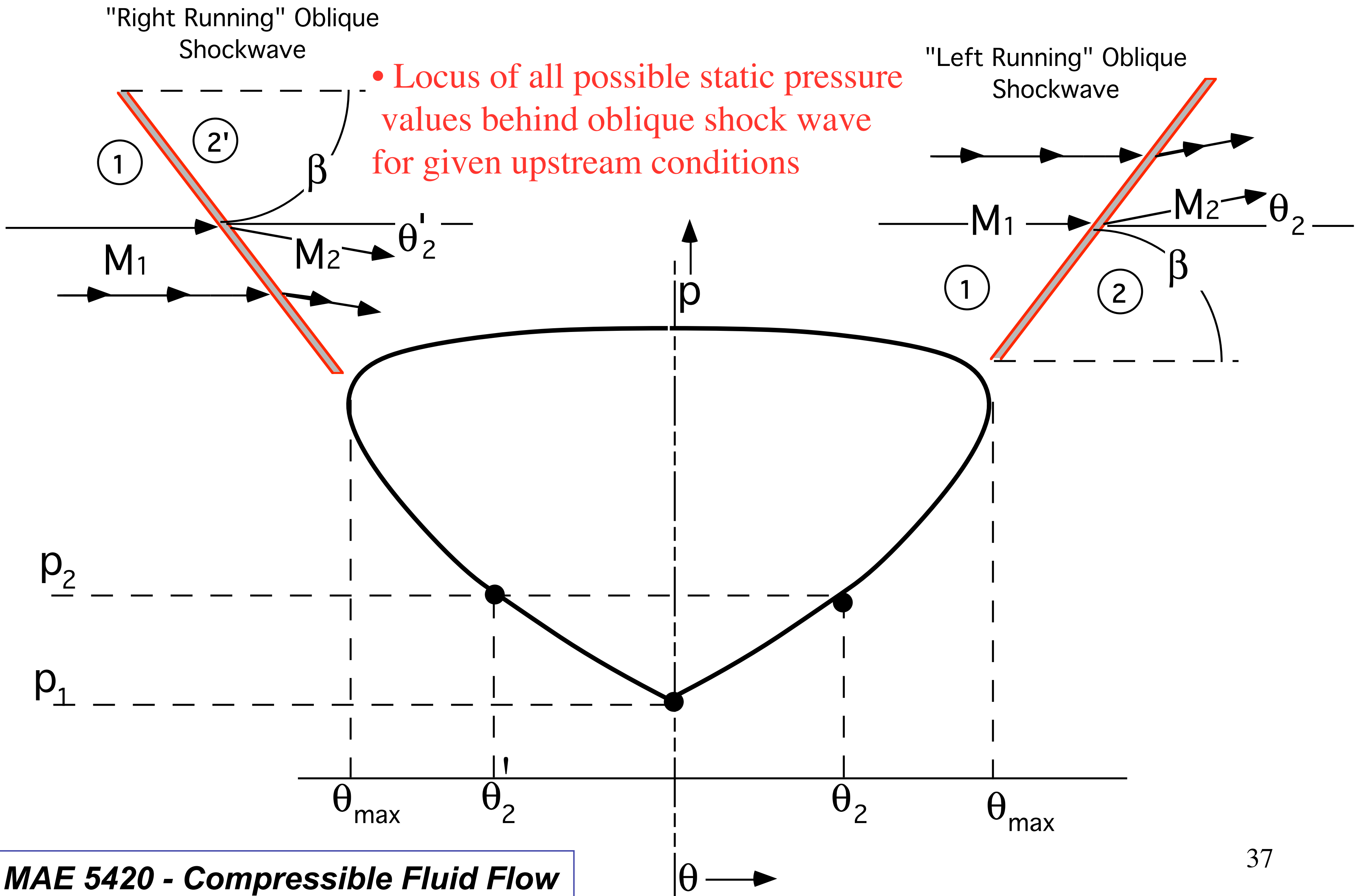


- Shock waves not only reflect from solid boundaries, they also reflect from each other

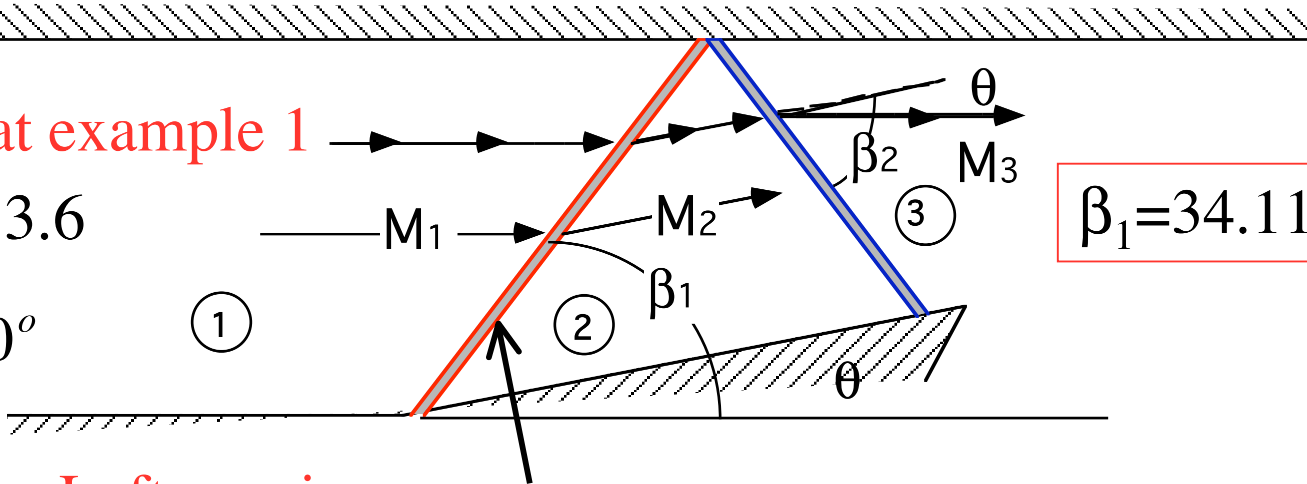
- *Need "more tools" to analyze problem completely*

Pressure-Deflection Diagrams

- Locus of all possible static pressure values behind oblique shock wave for given upstream conditions



Pressure-Deflection Diagrams (cont'd)



- Look at example 1

$M_1 = 3.6$

$\theta = 20^o$

$\gamma=1.4$

- Left running wave

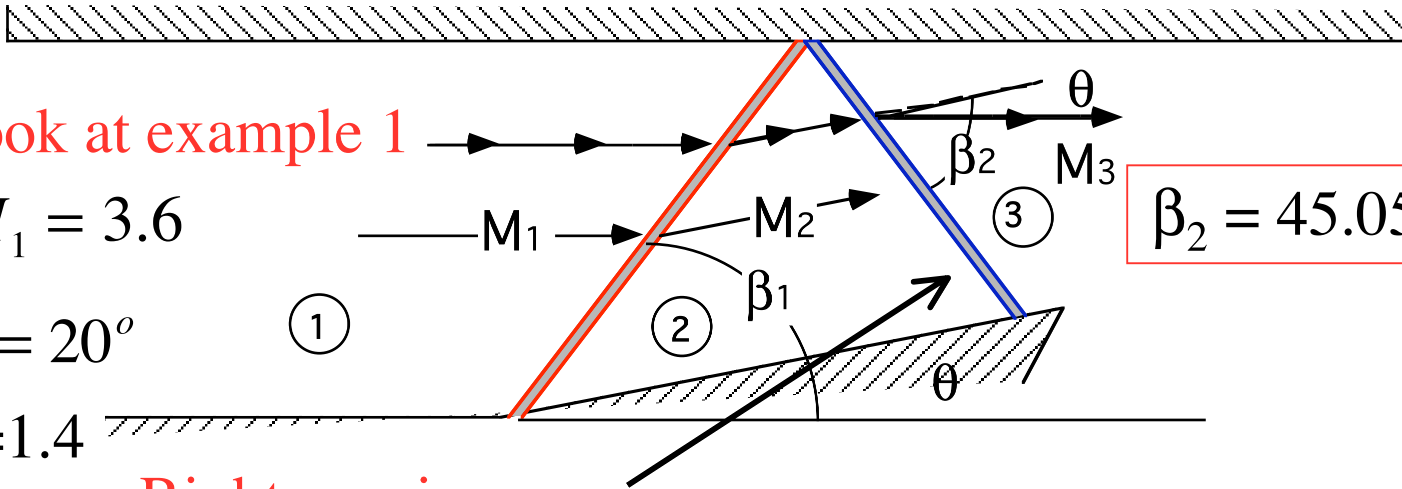
$\boxed{\beta_1=34.1102^\circ}$

- $M_{1n}= M_1 \sin \beta_1 = 3.6 \sin\left(\frac{\pi}{180} 34.1102\right) = 2.0188$
- Normal Shock Solver-->

$$M_2 n = 0.574168 \rightarrow M_2 = \frac{M_2 n}{\sin(\beta_1 - \theta)} = \frac{0.574168}{\sin\left(\frac{\pi}{180}(34.1102-20)\right)} = 2.3552$$

$\boxed{p_2/p_1 = 4.588283}$

Pressure-Deflection Diagrams (cont'd)



- Look at example 1

$M_1 = 3.6$

$\theta = 20^o$

γ=1.4

- Right running wave

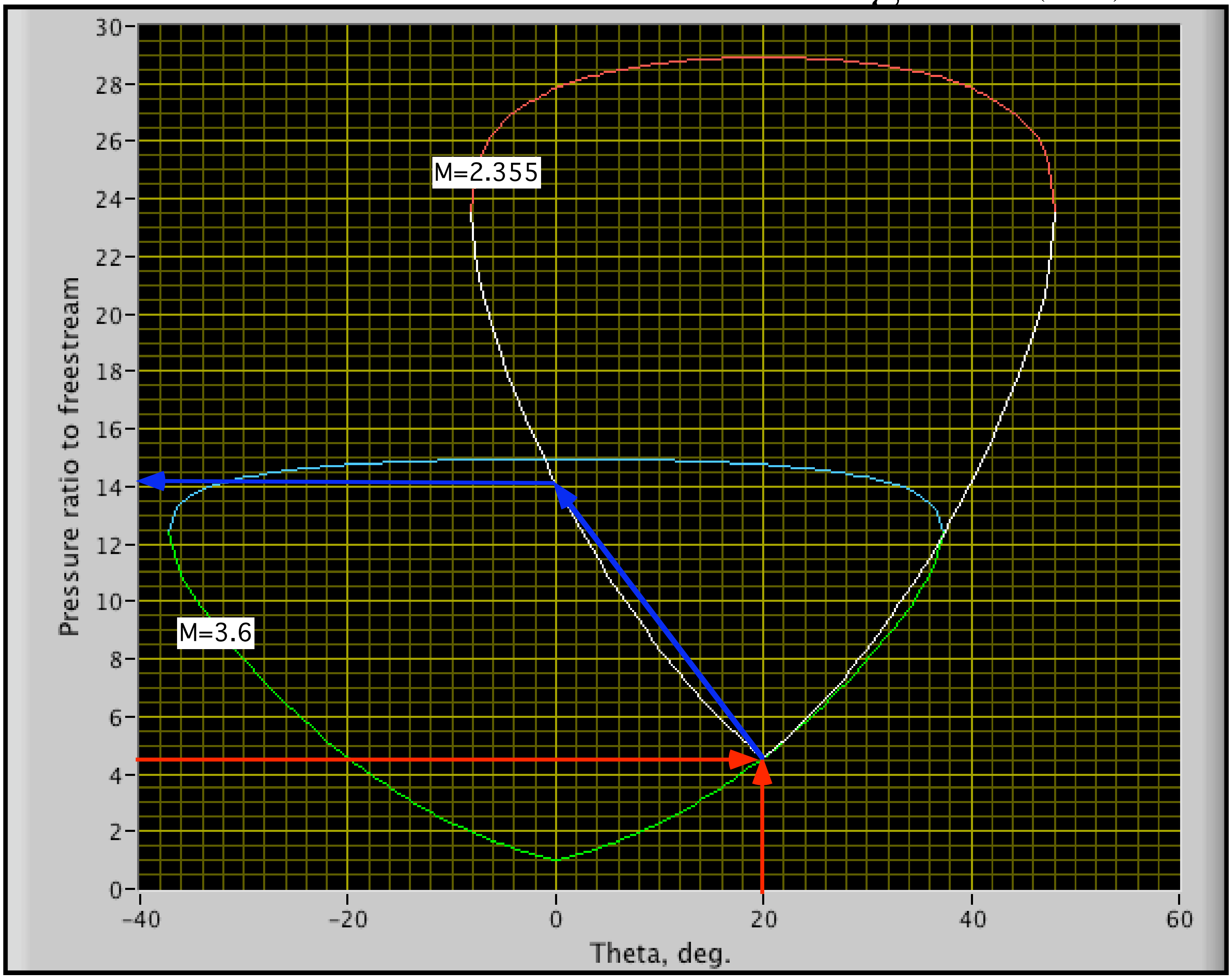
$\beta_2 = 45.0534°$

$$\left(M_2 n\right)_{\beta_2} = 2.3552\sin(45.0534) = 0.649303$$

$$\rightarrow \mathrm{M}_3 = \frac{\left(\mathrm{M}_2\mathrm{n}\right)_{\beta_2}}{\sin(\beta_2 - \theta)} = \frac{0.649303}{\sin\left(\frac{\pi}{180}(45.0534 - 20)\right)} = 1.5333$$

$p_3/p_2 = 3.075094$ --->
$p_3/p_1 = (p_3/p_2)(p_2/p_1) = (3.075094)(4.588283) = 14.109$

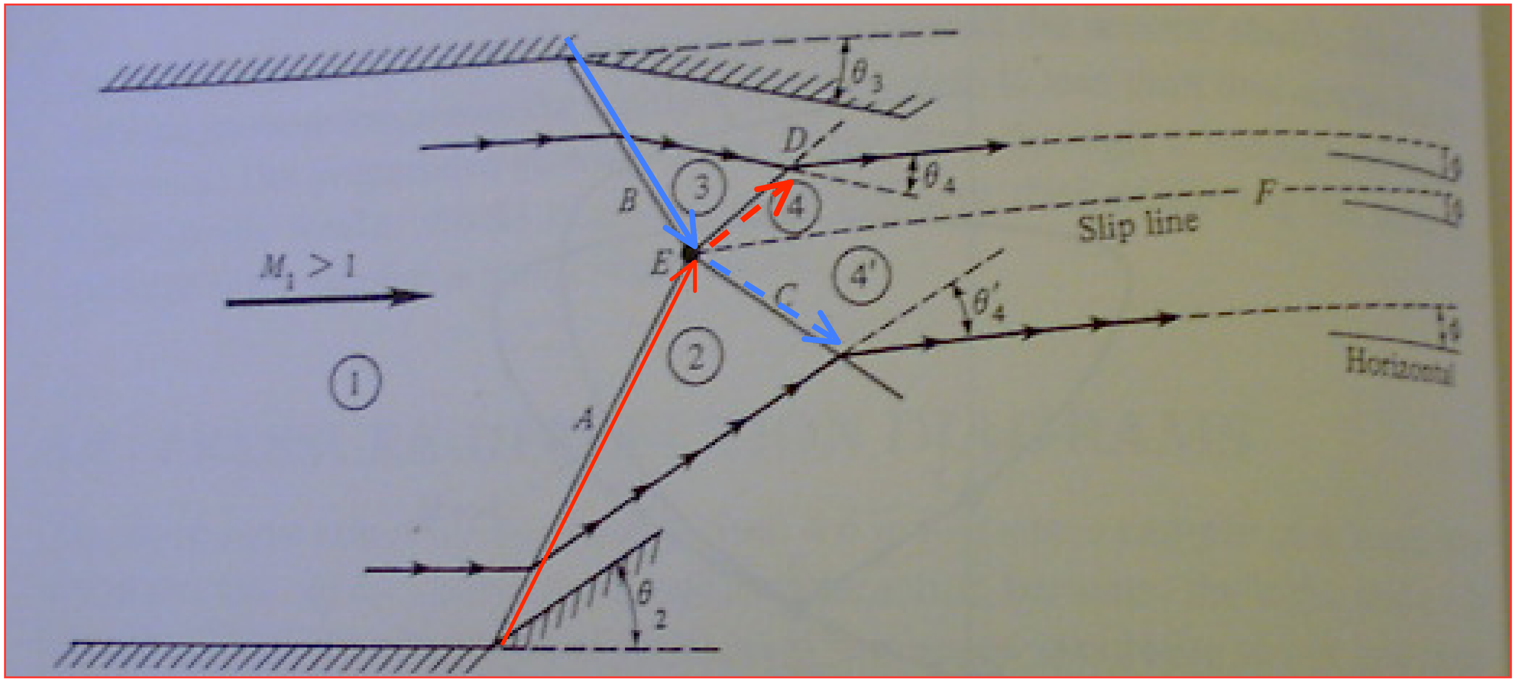
Pressure-Deflection Diagrams (cont'd)



Intersection of Shocks of Opposite Families

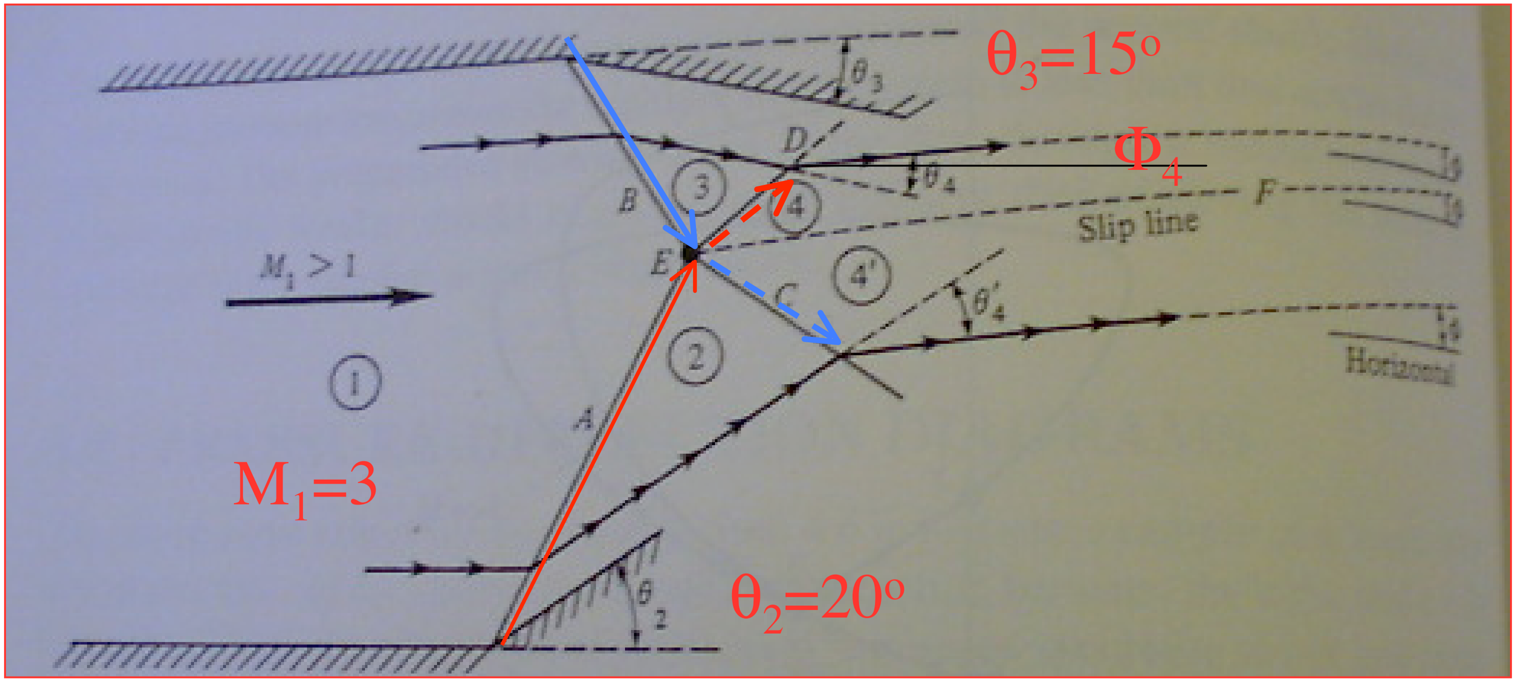
Intersection of “left-running” and “right running” Shock waves

Intersection of Shocks of Opposite Families (cont'd)



- Shock emanating from {A,B} intersect and continue as refracted shocks {C,D} .. At intersection point E, $p_4 = p'_4$

Intersection of Shocks of Opposite Families, Example



- Example: M_1=3, p_1=*1atm*, θ_2=20°, θ_3=15°
- Plot p,θ diagram, p_4, and flow direction Φ_4

Numerical Calculations for Example

From the θ-β-M solver: For $\theta_2 = 20°$ and $M_1 = 3$, $\beta = 37.8°$.

$$M_{n_1} = M_1 = \sin\beta = (3\ )\sin(37.8) = 1.839;\ \frac{p_2}{p_1} = 3.783$$

$$M_{n_2} = 0.6078;\ M_2 = \frac{M_{n_2}}{\mathrm{Sin}(\beta - \theta)} = \frac{0.6078}{\mathrm{Sin}(37.8 - 20)} = 1.99$$

For $\theta_3 = 15°$ and $M_1 = 3$, $\beta = 32.2$

$$M_{n_1} = M_1 \sin\beta = (3)\sin(32.2) = 1.60;\ \frac{p_3}{p_1} = 2.82$$

$$M_{n_3} = 0.6684;\ M_3 = \frac{M_{n_3}}{Sin(\beta - \theta)} = \frac{0.6684}{\mathrm{Sin}(32.2 - 15)} = 2.26$$

Numerical Calculations for Example

(cont'd)

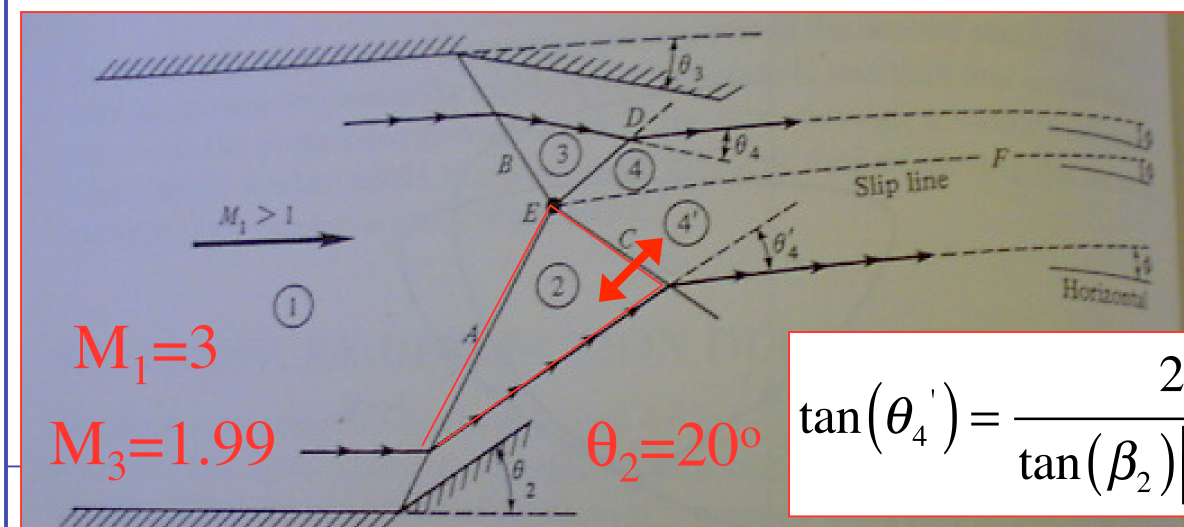
For the upstream flow represented by region 2, plot a pressure deflection diagram from the following calculations:

β_2	θ_4'	$M_{n_2} = M_2 \sin\beta$	$\frac{p_4'}{p_2}$	$\frac{p_4'}{p_1} = \frac{p_4'}{p_2}\frac{p_2}{p_1}$	$\Delta\theta = \theta_2 - \theta_4'$
30	0	1	1	3.783	20
33.3	4	1.09	1.219	4.61	16
37.2	8	1.20	1.513	5.72	12
41.6	12	1.32	1.866	7.06	8
46.7	16	1.45	2.286	8.65	4
53.5	20	1.60	2.820	10.67	0

Pick these values

calculate these values

$$\frac{p_2}{p_1} = 3.783$$



$$\tan\left(\theta_4'\right) = \frac{2\left\{M_2^{\,2}\sin^2\left(\beta_2\right) - 1\right\}}{\tan\left(\beta_2\right)\left[2 + M_2^{\,2}\left[\gamma + \cos\left(2\beta_2\right)\right]\right]}$$

Numerical Calculations for Example

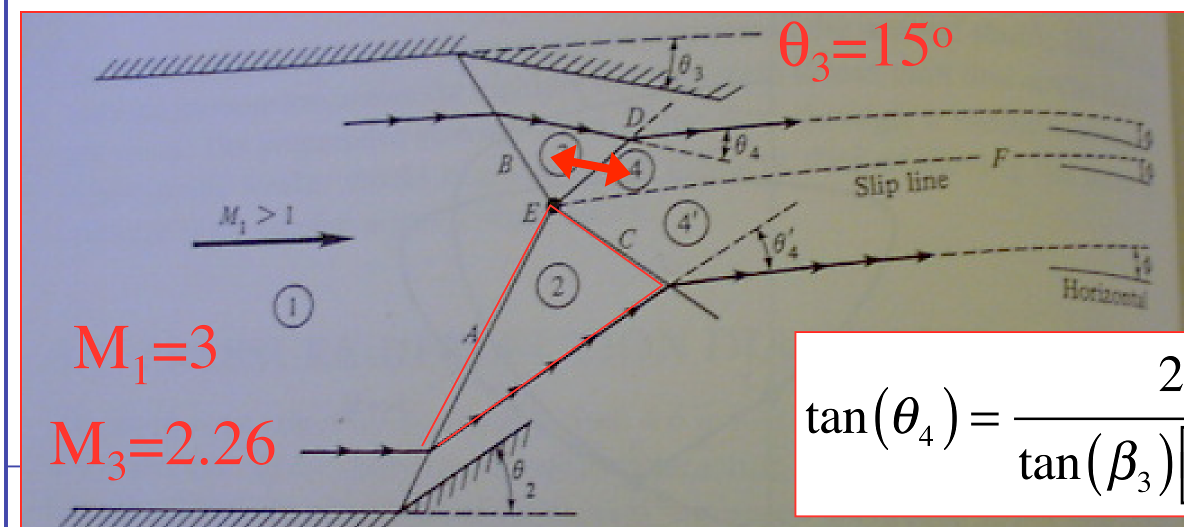
(cont'd)

For the upstream flow represented by region 3, plot a pressure deflection diagram from the following calculations:

Pick these values | ***calculate these values***

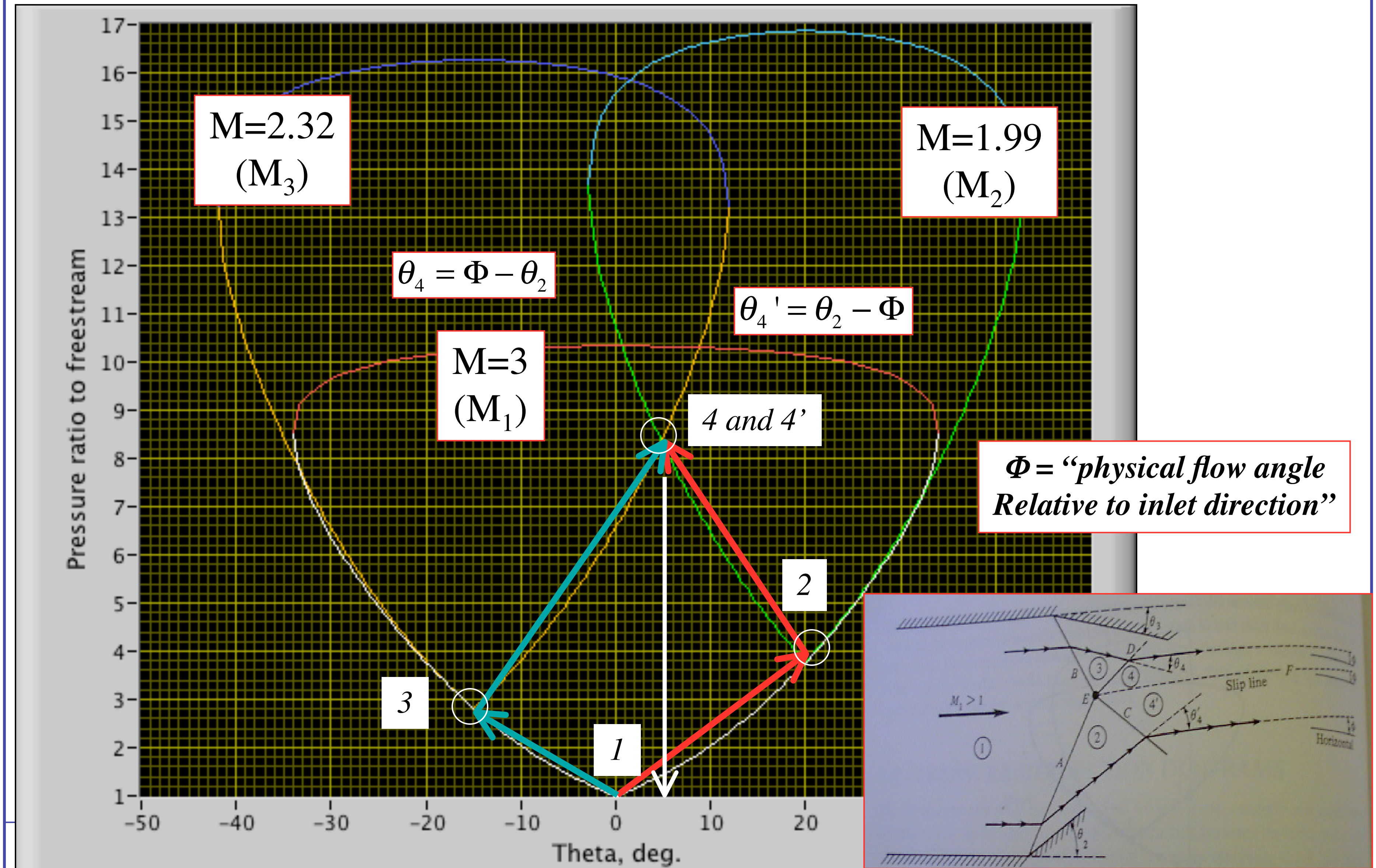
β_3	θ_4	$M_{n_3} = M_3 \sin\beta_3$	$\frac{p_4}{p_2}$	$\frac{p_4}{p_1} = \frac{p_4}{p_3}\frac{p_3}{p_1}$	$\Delta\theta = \theta_4 - \theta_3$
26	0	1	1	2.82	-15
29	4	1.096	1.23	3.47	-11
33	8	1.23	1.598	4.51	- 7
36.8	12	1.35	1.96	5.53	- 3
41.5	16	1.50	2.458	6.93	1
46.8	20	1.65	3.01	8.49	5
53.7	24	1.82	3.698	10.43	9

$$\frac{p_3}{p_1} = 2.82$$



$$\tan(\theta_4) = \frac{2\left\{M_3^{\,2}\sin^2(\beta_3) - 1\right\}}{\tan(\beta_3)\left[2 + M_3^{\,2}\left[\gamma + \cos(2\beta_3)\right]\right]}$$

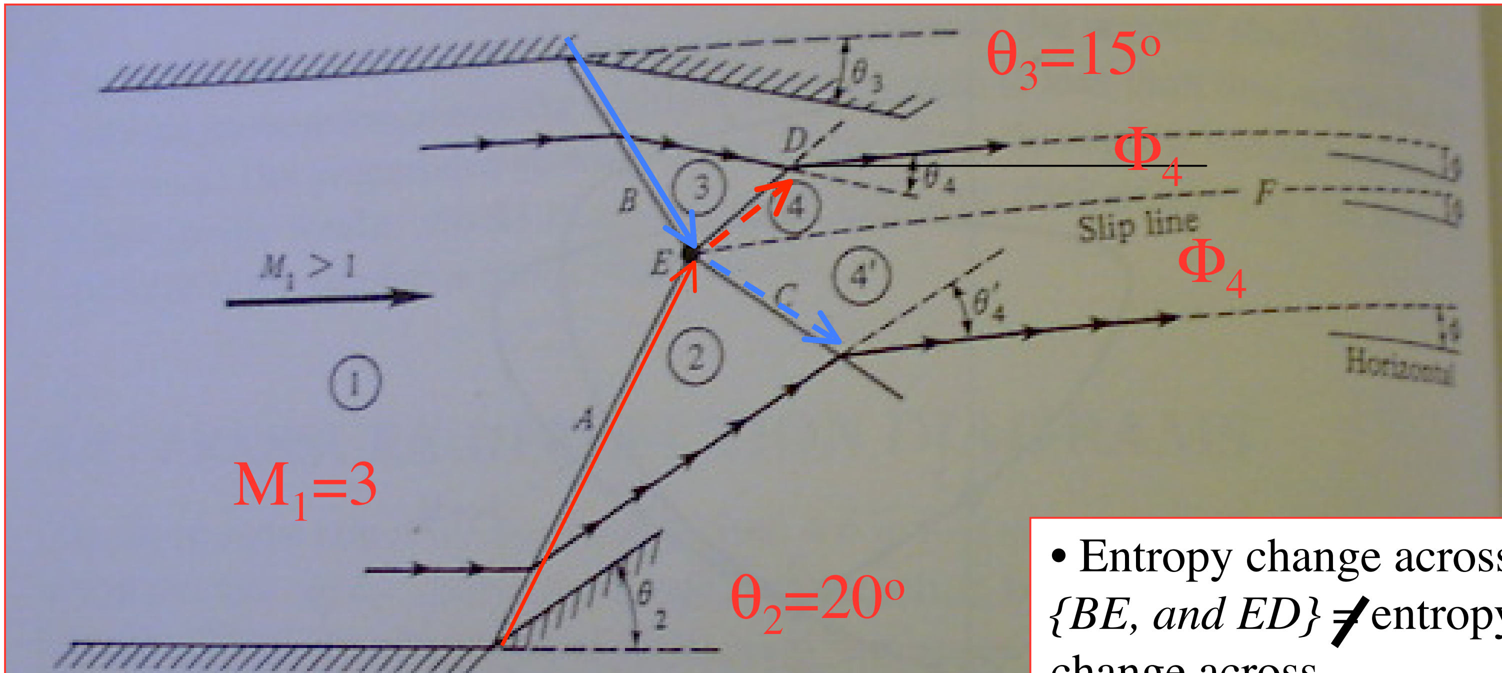
Pressure-deflection-diagram for Example



Φ = "physical flow angle Relative to inlet direction"

Numerical Calculations for Example

(cont'd)



From the graph on previous page

$$p_4 = p_4' = \frac{p_4}{p_1} p_1 = (8.3)(1) = \boxed{8.3 \text{ atm}}$$

$$\boxed{\Phi = 4.5°}$$

- Entropy change across *{BE, and ED}* ≠ entropy change across Shocks *{AE, and EC}* ….

Creates "slip line" emanating From E between regions 4 and 4'

Intersection of Shocks of The Same Family

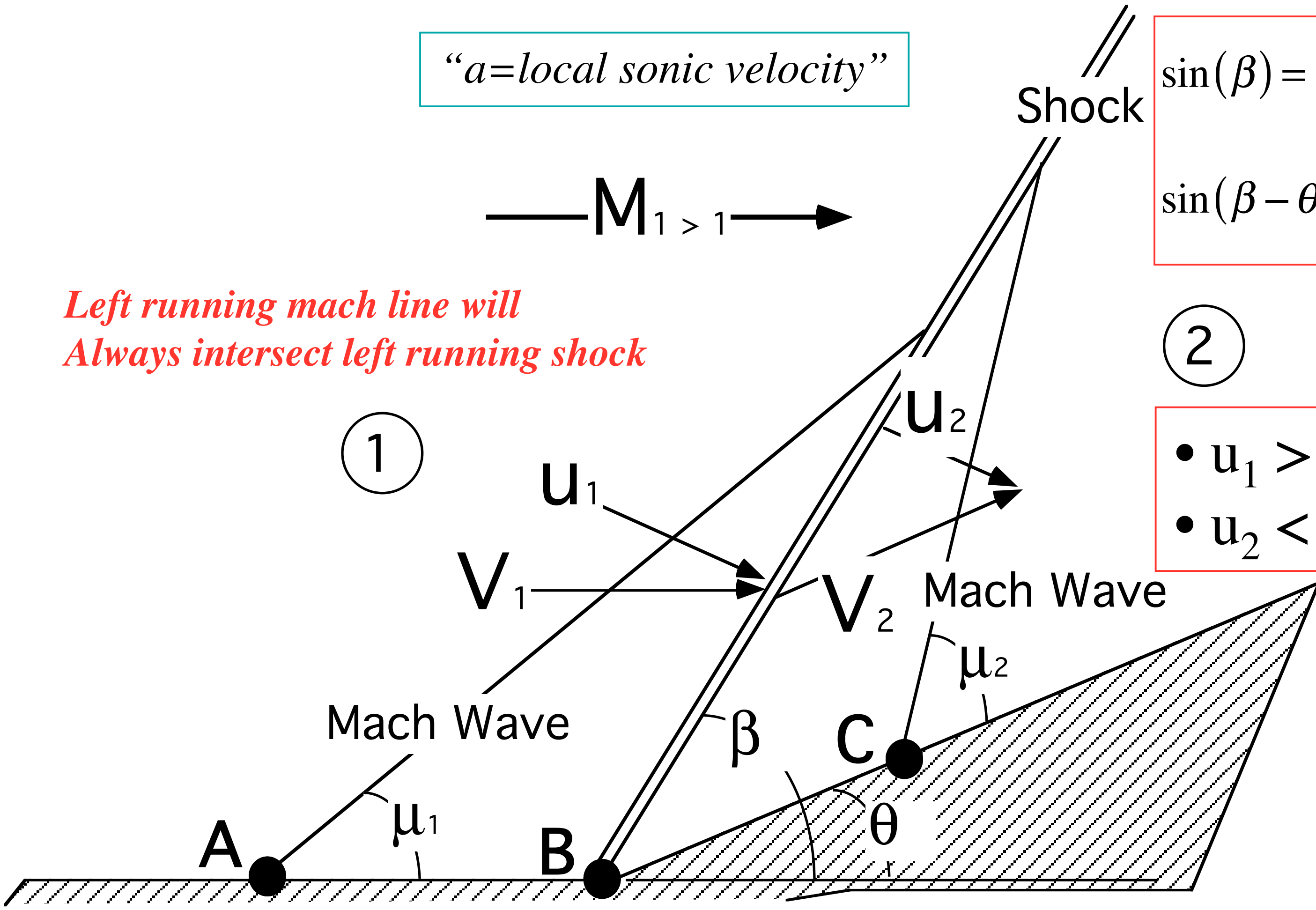
- Will Mach waves intersect shock wave?

"a=local sonic velocity"

$$\sin(\beta) = \frac{u_1}{V_1} \rightarrow \sin(\mu_1) = \frac{a_1}{V_1}$$

$$\sin(\beta - \theta) = \frac{u_2}{V_2} \rightarrow \sin(\mu_2) = \frac{a_2}{V_2}$$

Left running mach line will Always intersect left running shock



- $u_1 > a_1 \dashrightarrow \mu_1 < \beta$
- $u_2 < a_2 \dashrightarrow \mu_2 > \beta - \theta$

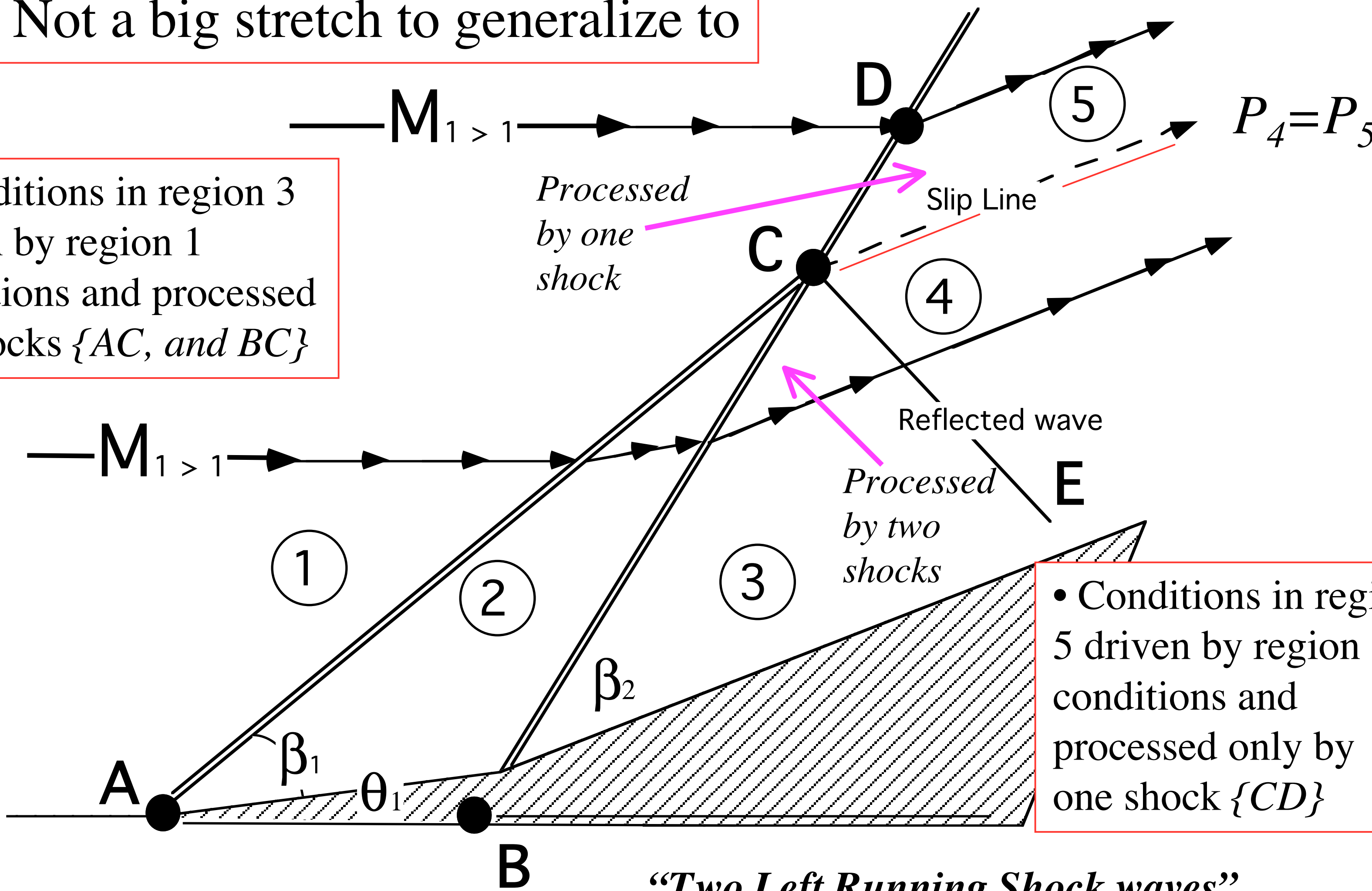
- **yes!**

Intersection of Shocks of The Same Family

(cont'd)

- Not a big stretch to generalize to

- Conditions in region 3 driven by region 1 conditions and processed by shocks *{AC, and BC}*



- Conditions in region 5 driven by region 1 conditions and processed only by one shock *{CD}*

"Two Left Running Shock waves"

Intersection of Shocks of The Same Family

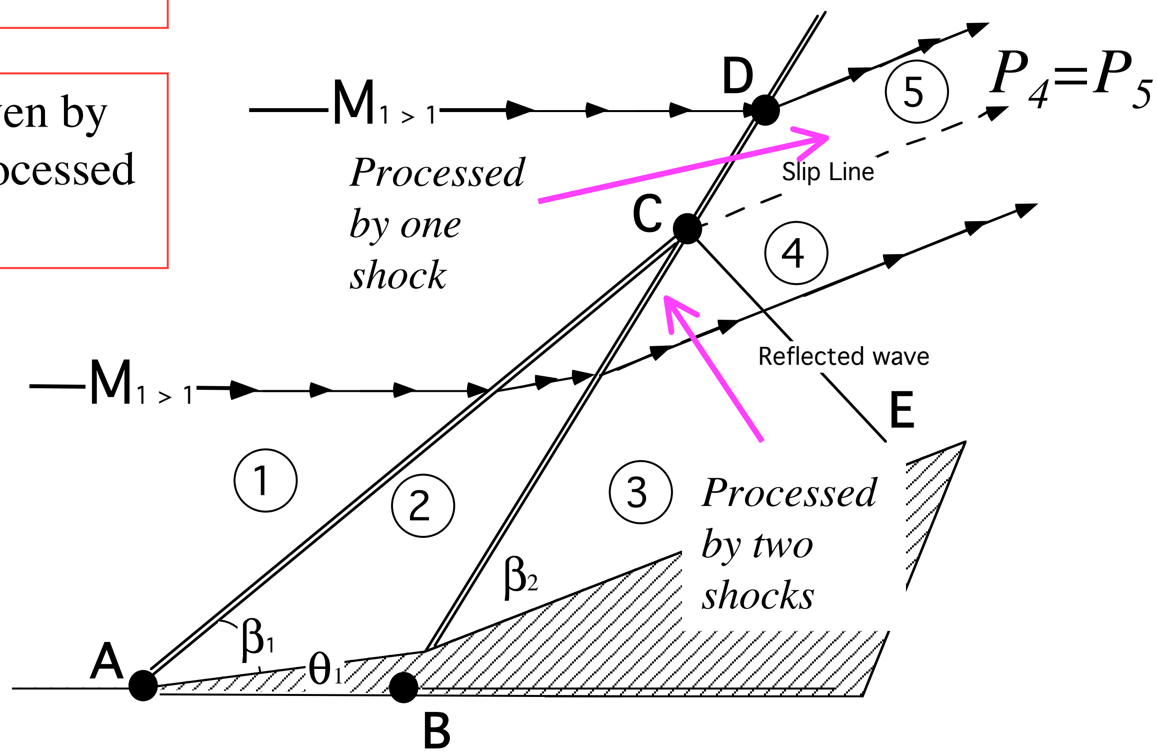
(cont'd)

- Conditions in region 3 driven by Upstream conditions and processed by shocks *{AC, and BC}*

- Conditions in region 5 driven by Upstream conditions and processed Only by shock *{CD}*

- Entropy change across *{CD}* ≠ entropy change across Shocks *{AC, and BC}* ….

Creates "slip line" emanating From C between regions 4 and 5



"Two Left Running Shock waves"

Intersection of Shocks of The Same Family

(cont'd)

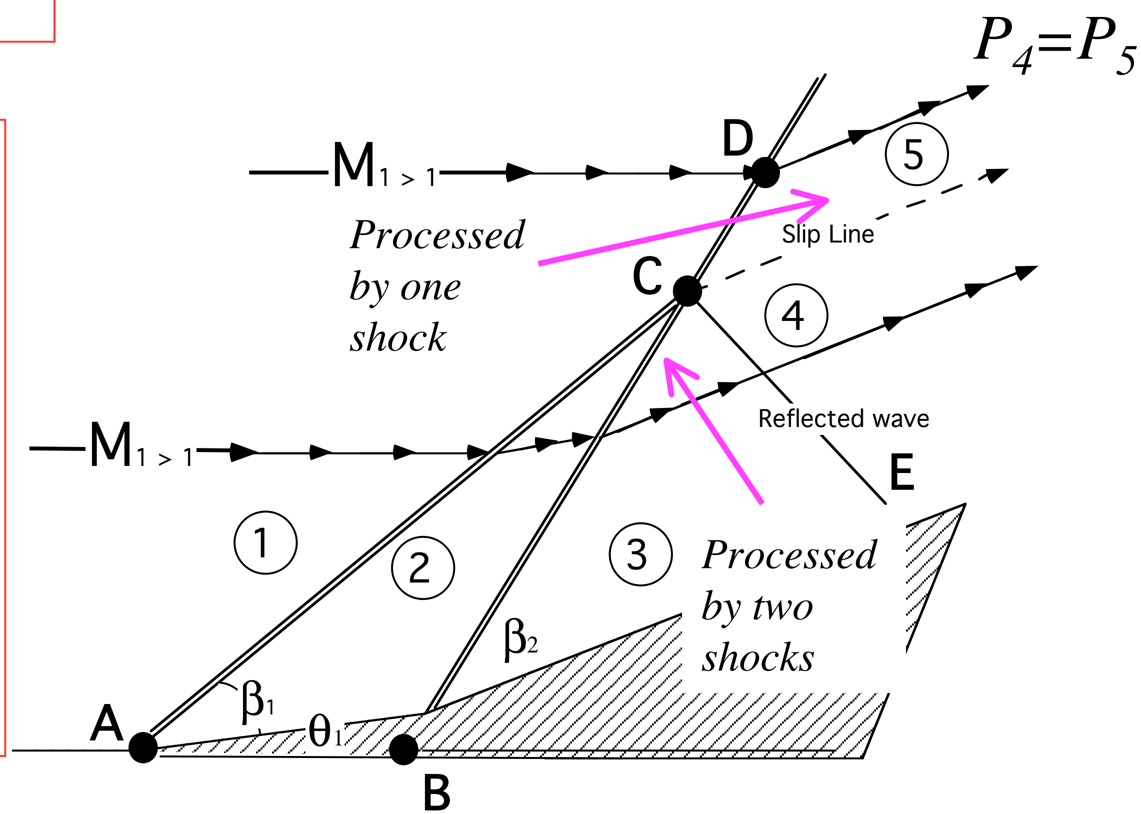
• within slip zone $P_4=P_5$ *and* $\theta_4 = \theta_5,$

• in general for arbitrary Shock waves … $P_3 \neq P_5$

But .. $P_4=P_5$

… How?…Reflected wave E...

… weak shock or Expansion wave depending On relationship of P_3 to P_5



"Two Left Running Shock waves"

Intersection of Shocks of The Same Family

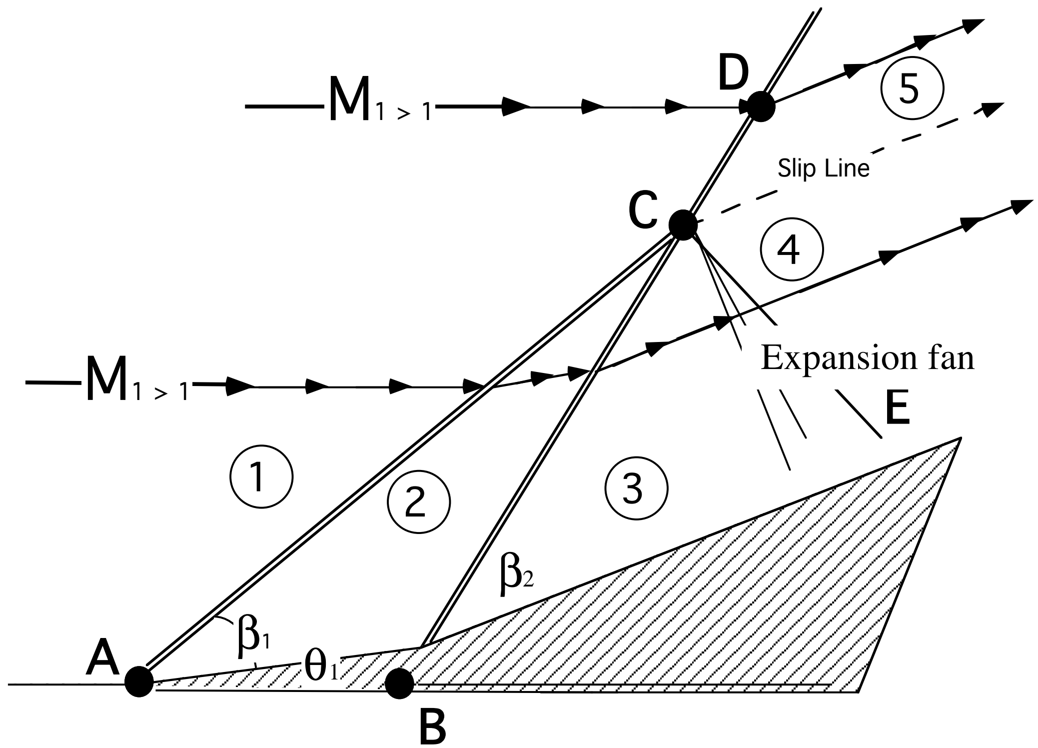
(cont'd)

• within slip zone $P_4 = P_5$ *and* $\theta_4 = \theta_{5}$, *also* $\theta_3 = \theta_5$

$P_3 > P_5$

… expansion fan drops

P_3 to P_3 ($= P_5$)



"Two Left Running Shock waves"

Intersection of Shocks of The Same Family

(cont'd)

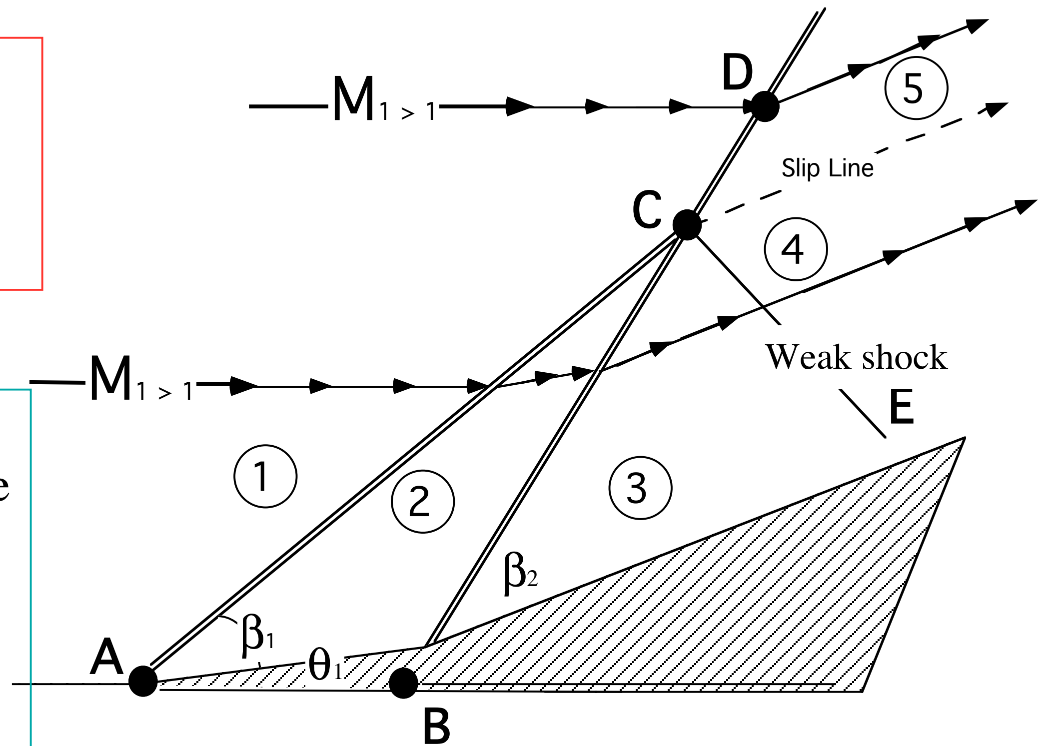
- within slip zone $P_4 = P_5$ *and* $\theta_4 = \theta_{5,\ also}\ \theta_3 = \theta_5$

$P_3 < P_5$

… weak shock wave compresses P_3 to P_4 ($= P_5$)

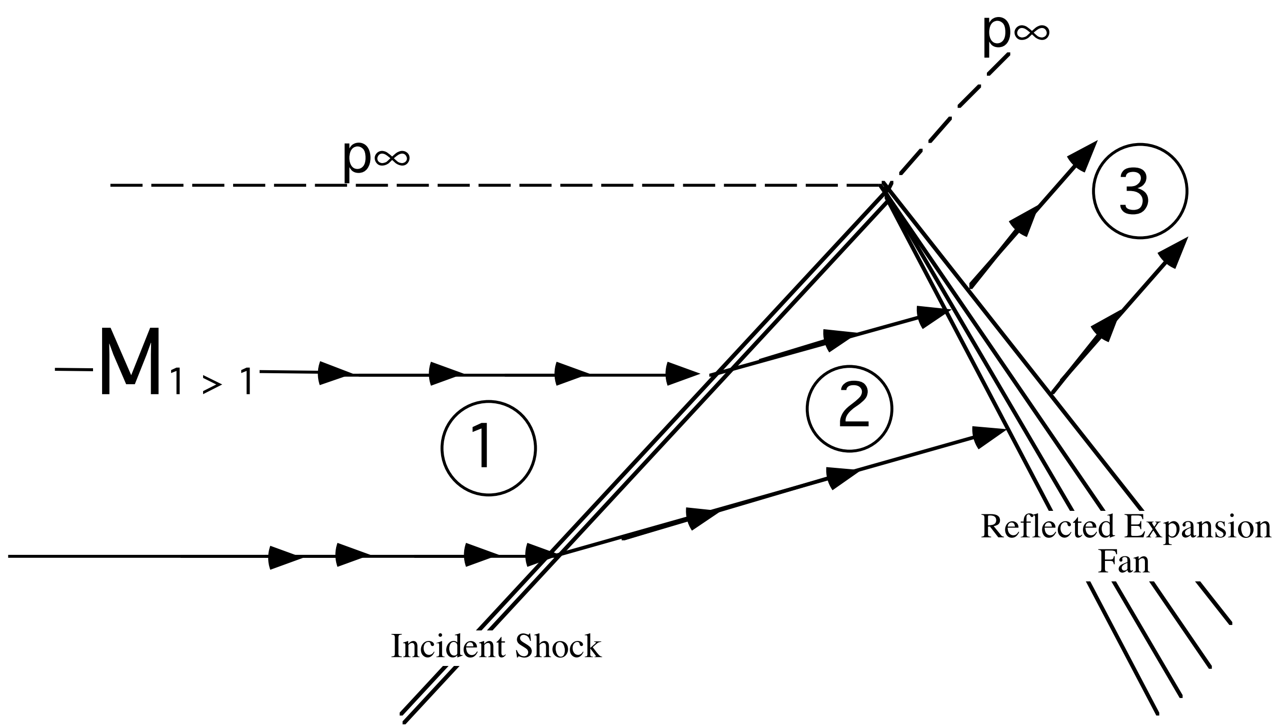
- Model must be smart enough To accommodate this difference

Must iteratively adjust Strength of waves CD and CE Until $P_4 = P_5$ on output side



"Two Left Running Shock waves"

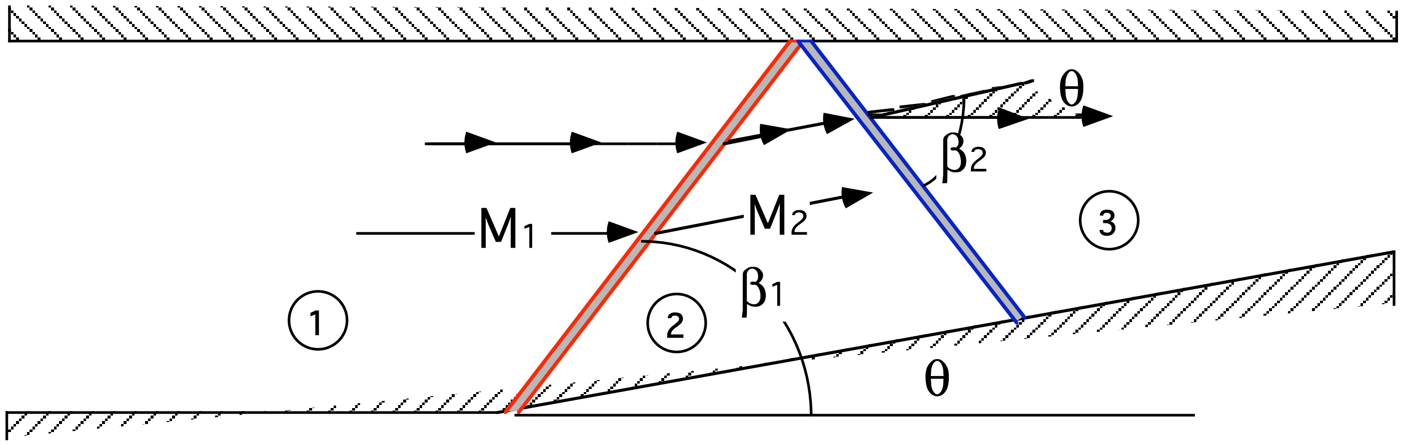
Wave reflections from a free boundary



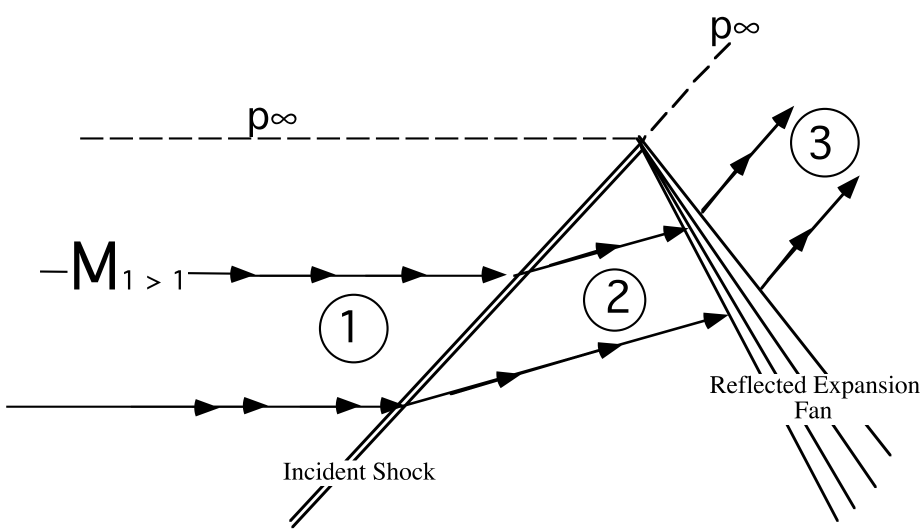
Shock Wave Incident on Constant Pressure Boundary

Wave reflections from a free boundary

(cont'd)



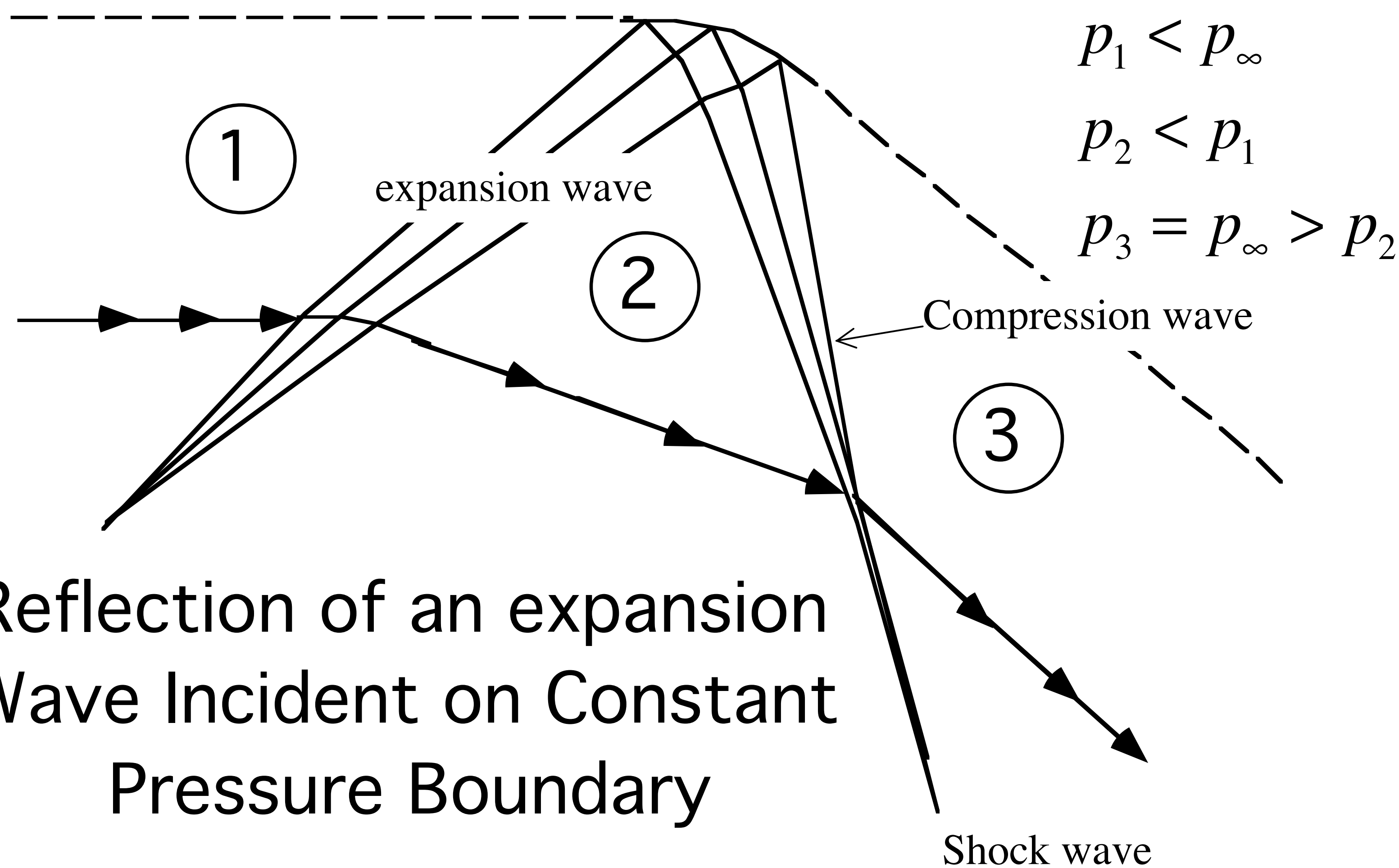
• Compare to Solid Boundary



Shock Wave Incident on Constant Pressure Boundary

Wave reflections from a free boundary

(cont'd)

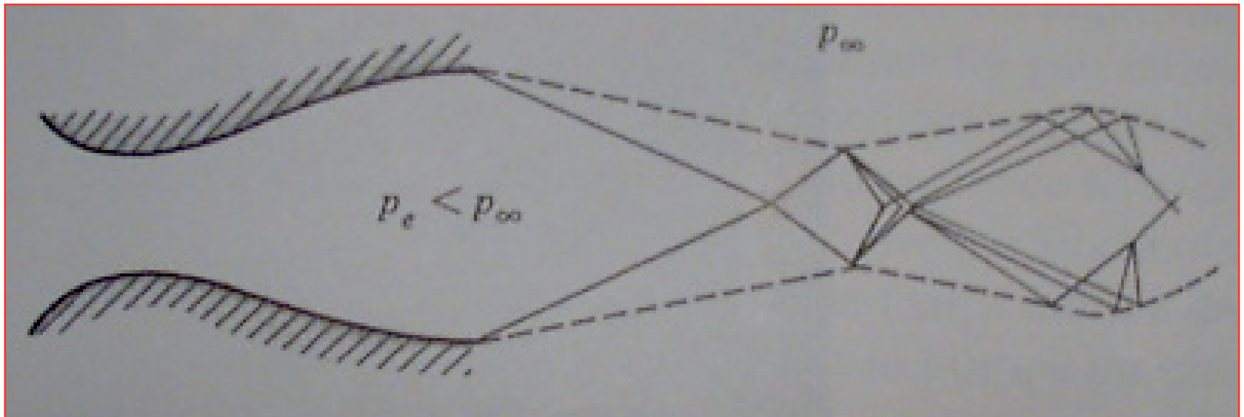


Reflection of an expansion Wave Incident on Constant Pressure Boundary

Rules for Solid and Free Boundaries

1. Waves Incident on a Solid Boundary reflect in a Like manner;
   Compression wave reflects as compression wave, expansion wave reflects as expansion wave

2. Waves Incident on a Free Boundary reflect in an Opposite manner;
   Compression wave reflects as expansion wave, expansion wave reflects as compression wave

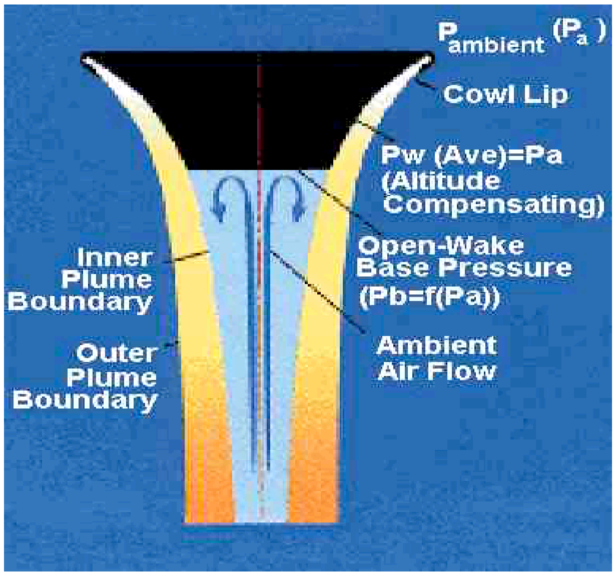


Schematic of the diamond wave pattern in the exhaust from a supersonic nozzle.

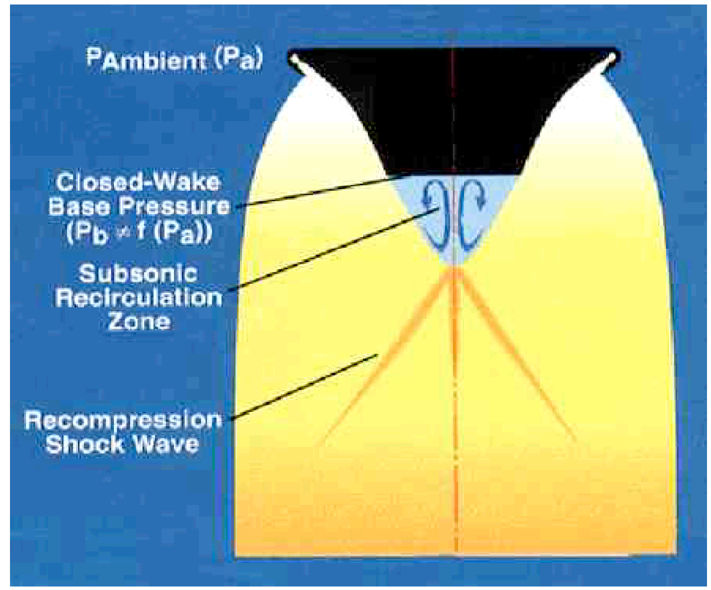
Linear Aerospike Rocket Engine

Nozzle has same effect as telescope nozzle

Lift off



Vacuum (Space)



$$F = F_{Thruster} + F_{Ramp} + F_{Base}$$

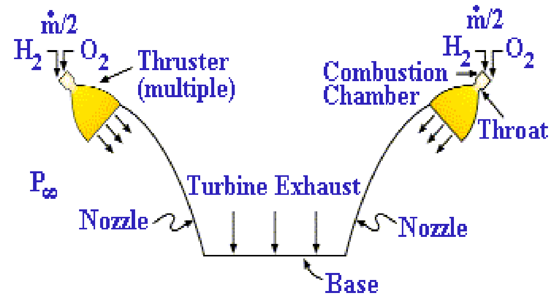
$$F_{Thruster} = \cos\theta\,(\dot{m}V_{exit} + A_{exit}(P_{exit} - P_{\infty}))$$

$$F_{Ramp} = \int^{A_{Ramp}} (P_{Ramp} - P_{\infty})\,dA$$

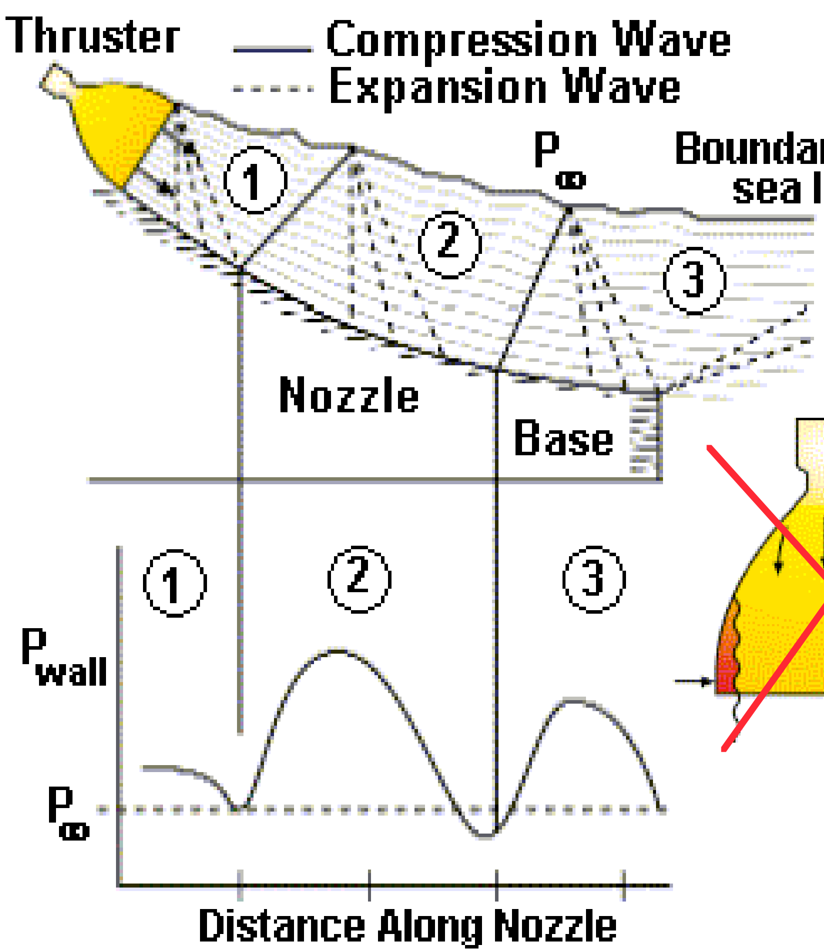
$$F_{Base} = A_{Base}\,(P_{Base} - P_{\infty})$$

- **Aerospike's flow unconstrained, allows best performance**

Linear Aerospike Rocket Engine
(cont'd)



Low Altitude Aerodynamics

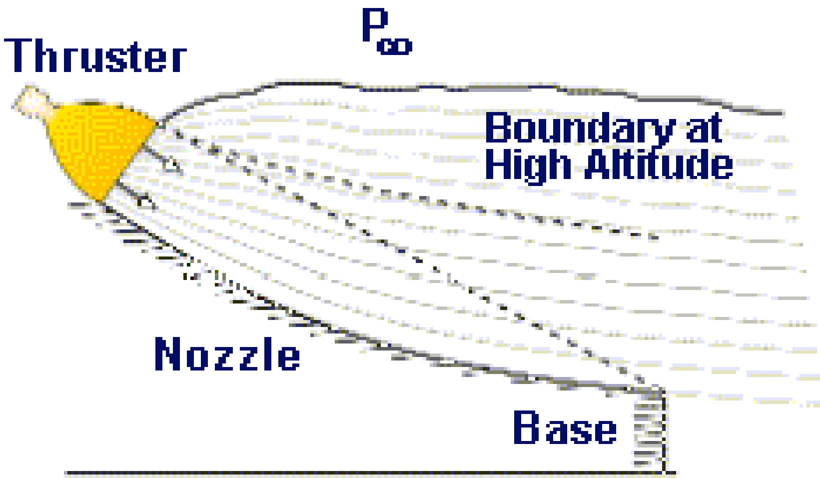


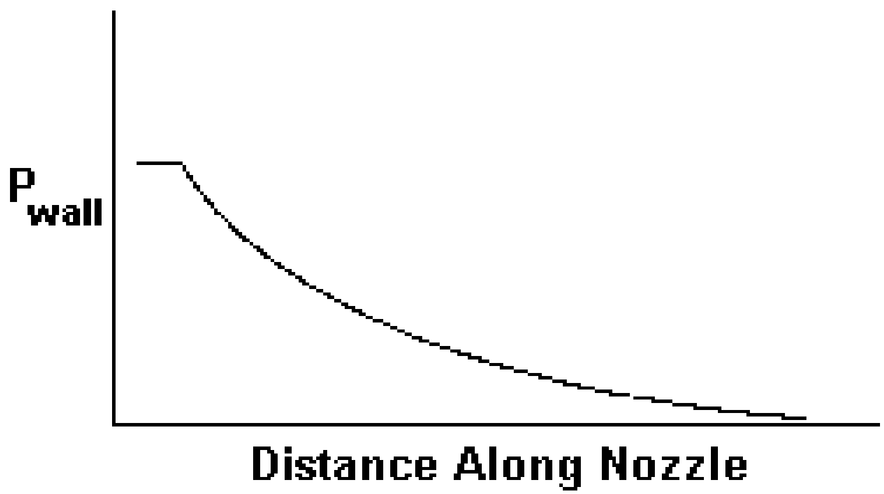
- Thruster flow discharges to ramp
- Expansion waves turn flow axially
- Ramp curves, turns flow axially (at low altitudes)
- Turning causes compression wave from (1) to (2) - nozzle pressure increases
- Compression wave reflects off boundary causing expansion waves
- Flow crosses expansion waves in (2) - nozzle pressure decreases
- Ramp continues to curve and turn flow
- Process repeats (2) to (3)

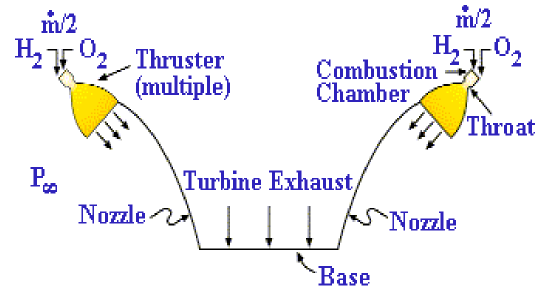
Average nozzle pressure > P, therefore no losses or separation, therefore large area ratio nozzle can be used, enabling SSTO

Linear Aerospike Rocket Engine
(cont'd)

High Altitude Aerodynamics





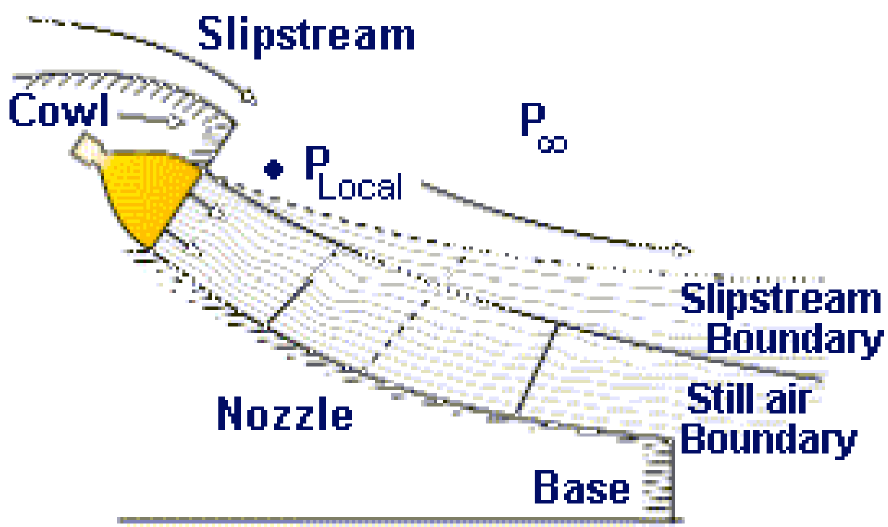


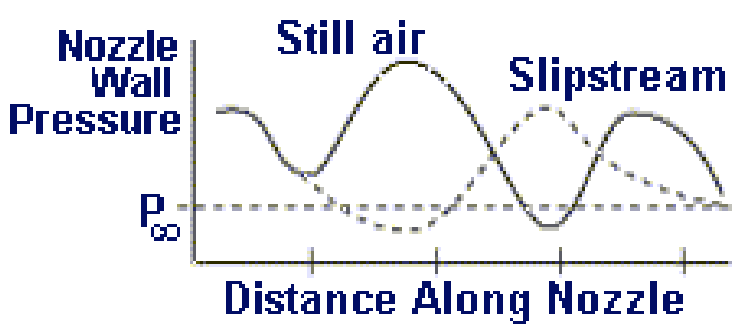
- Thruster flow discharges to ramp
- Expansion waves turn flow axially
- No compression waves exist - all flow turning done by expansion waves
- Nozzle behaves likes a bell

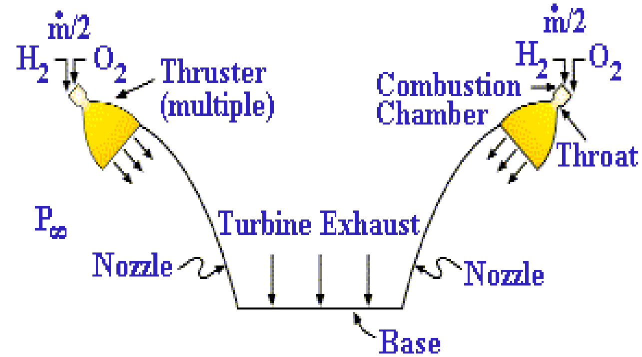
Linear Aerospike Rocket Engine

(concluded)

SlipStream effects





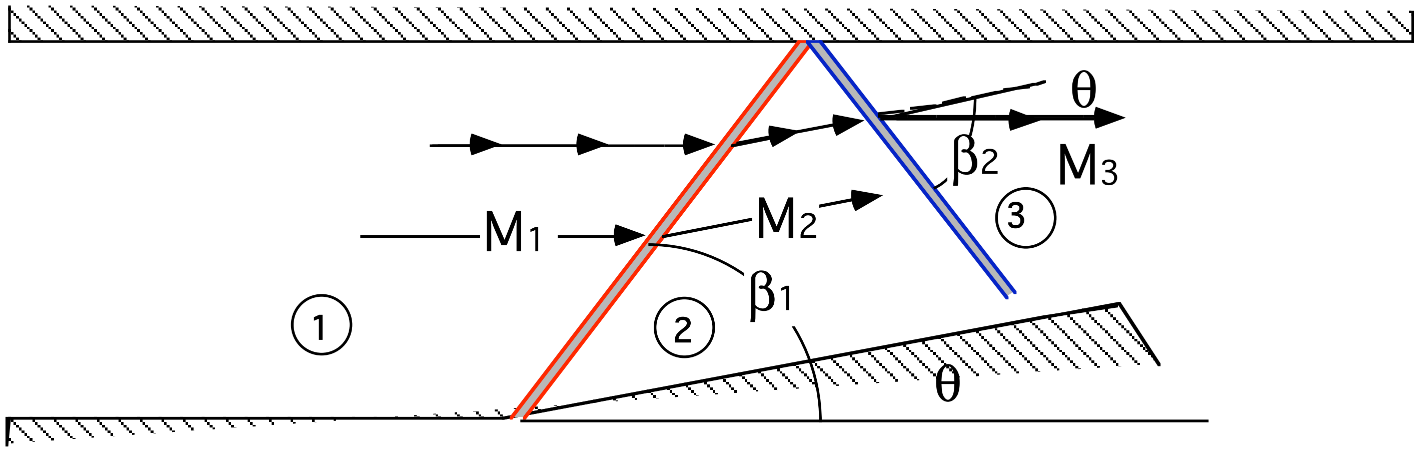


- Air streaming over cowl lowers local pressure - $P_{Local} < P_{infinity}$
- Exhaust plume expands beyond still air case
- Expansion and compression wave systems move aft from still air case

- Resulting recompression Delays Nozzle separation

Bottom Line is that the Linear Aerospike engine realizes about 50% of thetheoretical Isp gains offeredby the Telescoping nozzle

Homework 10



- Calculate

  β_2

  M_3

  p_3

  T_3

$\beta_1 = 30°$

$M_1 = 2.8$

$p_1 = 1$ atm

$T_1 = 300°K$

- Assume $\gamma = 1.4$