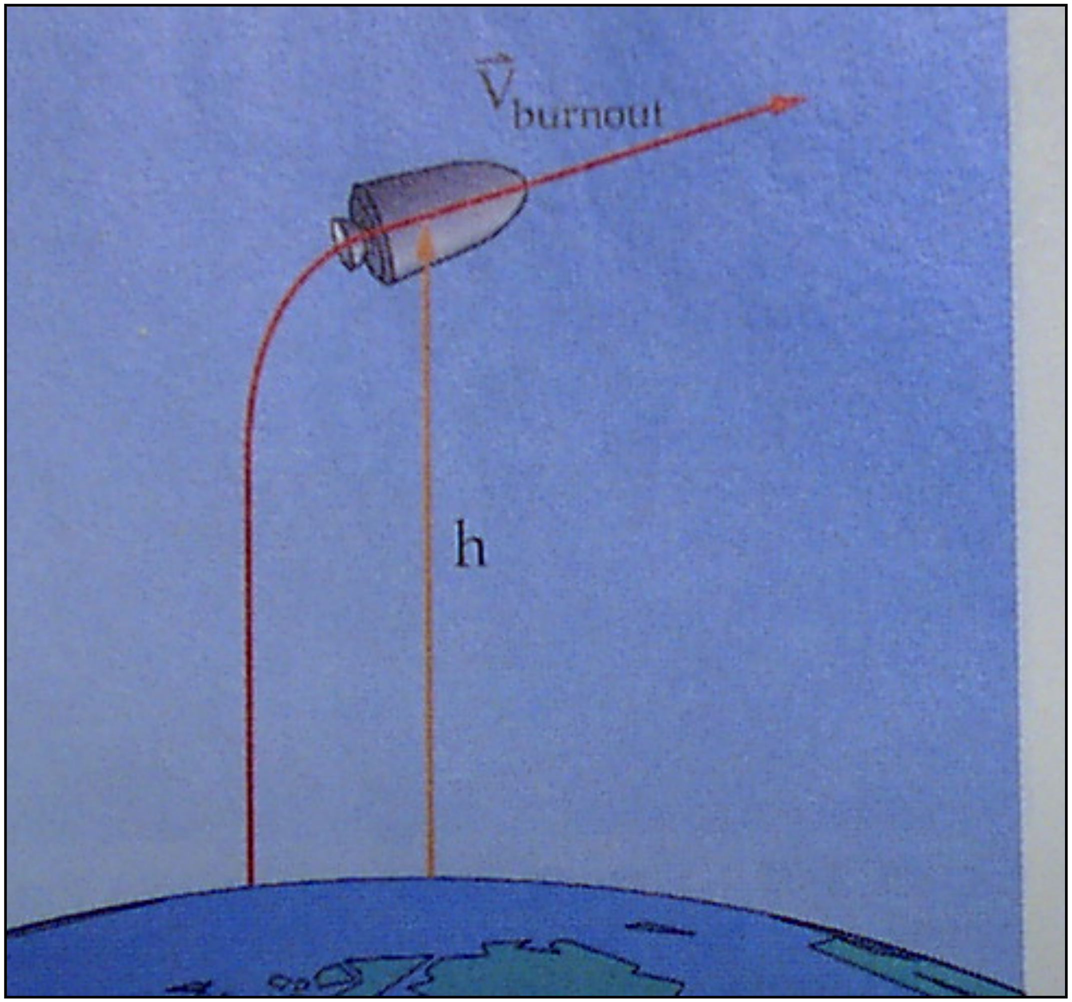
Rocket Science 102 : Energy Analysis, *Available vs Required* ΔV



• Not in Taylor

Available ΔV

Ignoring Aerodynamic Drag …. The "*available Delta V*" for a Given rocket burn/propellant load is

$$V_{t_{burn}} = g_0 \; I_{sp} \; \ln\left(1 + P_{mf}\right)$$

General expression for Rocket accelerating along a Horizontal path

$$P_{mf} = \frac{m_{propellant}}{m_{final}}$$

- Derived using simple assumptions But application is very general
- Ignored gravity and drag losses

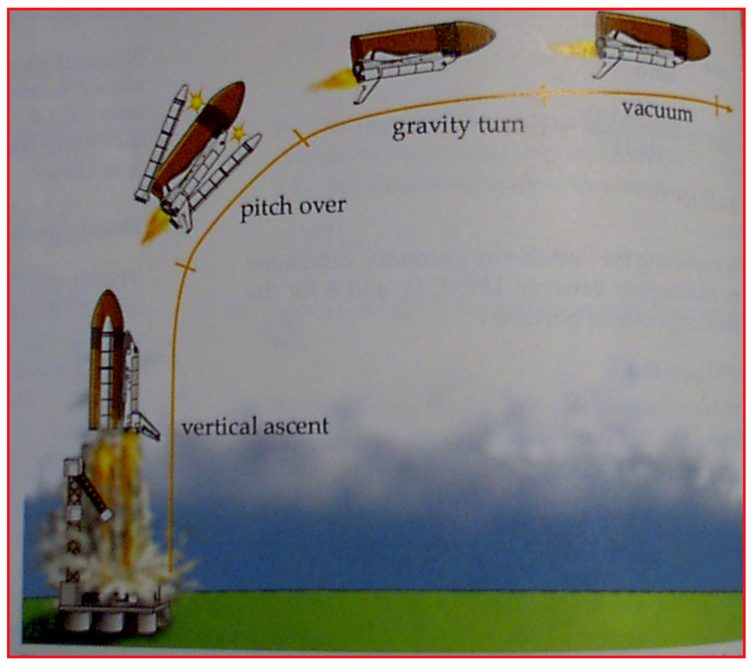
"Gravity Losses"

Propulsive ΔV loss from acting against gravity....

$$\left(\Delta V\right)_{\substack{gravity \\ loss}} = \int_0^{T_{burn}} g(t)\cdot\sin\theta\cdot dt$$

Applies for rocket accelerating along an ARBITRARY path

From previous lecture

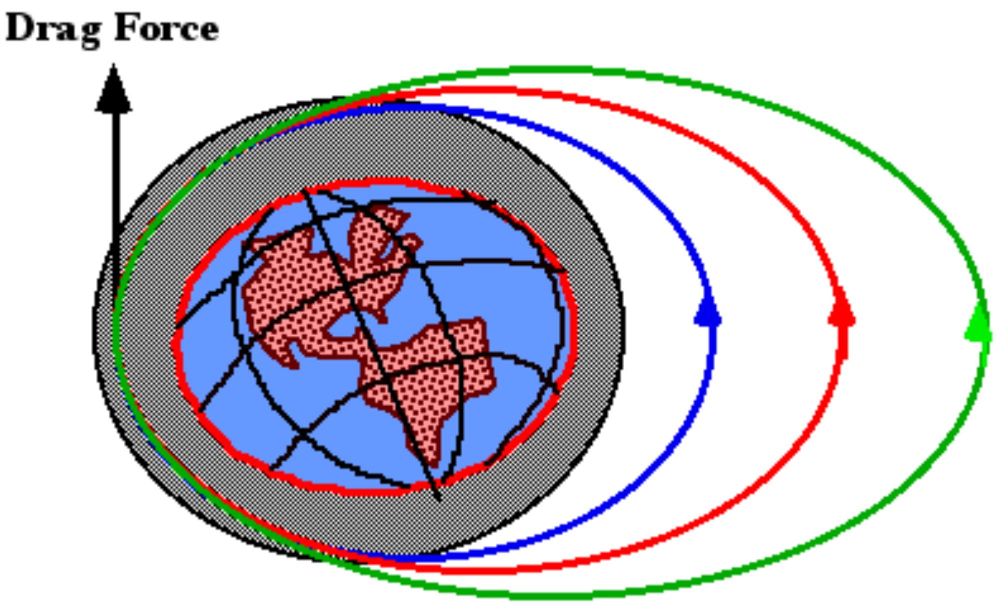


Drag Losses

- Any Orbiting Object with a perigee altitude less than 600 km willl experience the effects of the Earths' outer atmosphere

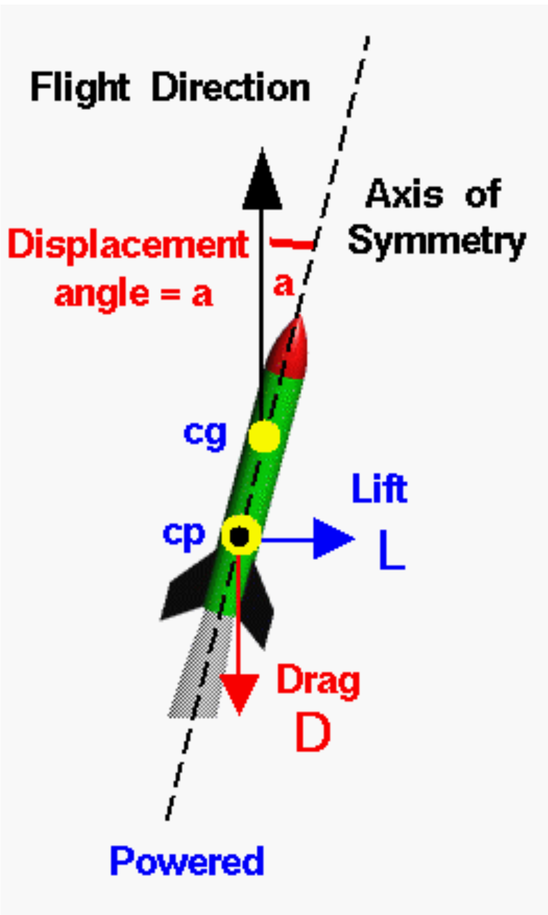
- Resulting *Drag* is a *non-conservative* force, and as such will remove energy from the orbit

- Energy Loss will cause orbit to decay



… and This decay Also applies To launch trajectory

Aerodynamic Forces Acting on Rocket



- **Lift – acts perpendicular to flight path**
- **Drag – acts along flight path**
- **Thrust – acts along longitudinal axis of rocket**

$$C_L = \frac{L_{ift}}{\frac{1}{2} \cdot \rho_{(h)} \cdot V^2 A_{ref}}$$

$$\rightarrow \frac{1}{2} \cdot \rho_{(h)} \cdot V^2 = \bar{q} \rightarrow \text{"DyamicPressure"}$$

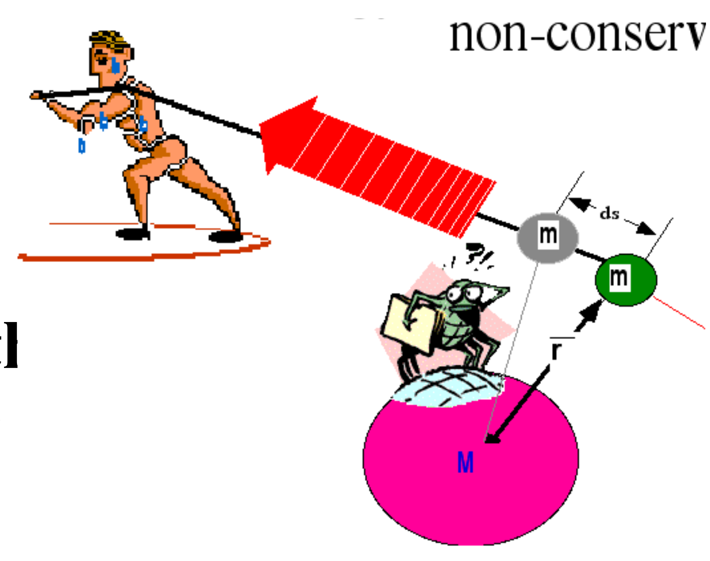
$$C_D = \frac{D_{rag}}{\frac{1}{2} \cdot \rho_{(h)} \cdot V^2 A_{ref}}$$

$$A_{ref} \equiv reference\ area \rightarrow \text{Typically based on planform or maximum cross section}$$

Drag Losses (2)

$$\Delta E_{\substack{non-\\conservative}} = \int_{path} \vec{F}_{\substack{non\\-conservative}} \cdot d\vec{s} = \int_{path} \vec{F}_{\substack{non\\-conservative}} \cdot \frac{d\vec{s}}{dt} \cdot dt$$

$$\int_{t} \vec{F}_{\substack{non\\-conservative}} \cdot \vec{V} \cdot dt \approx \frac{\Delta V_{loss}^{\;2}}{2} \cdot M$$



- **Lift – acts perpendicular to flight path**
  .. *Cannot effect energy level of rocket*

- **Gravity – acts downward *(conservative)***
  … *cannot effect energy level of rocket*

Drag Losses (3)

$$\frac{\Delta E_{drag}}{M} = \frac{1}{2}\cdot\left(\Delta V_{drag}\right)^2 = \int_0^t \frac{D_{rag}\cdot V}{M}dt$$

$\rightarrow$ *equivalent specific energy loss due to drag*

$$\rightarrow \Delta V_{drag} = \sqrt{2\cdot\int_0^t \frac{D_{rag}\cdot V}{M}dt}$$

For constant C_D, M

$$\rightarrow \Delta V_{drag} = \sqrt{2\cdot\int_0^t \frac{1}{2}\frac{C_D\cdot A_{ref}\cdot\rho\cdot V^3}{M}dt} = \sqrt{\frac{C_D\cdot A_{ref}}{M}\int_0^t \rho\cdot V^3\,dt}$$

$$\beta = \frac{M}{C_D\cdot A_{ref}} \rightarrow \text{"Ballistic Coefficient"}$$

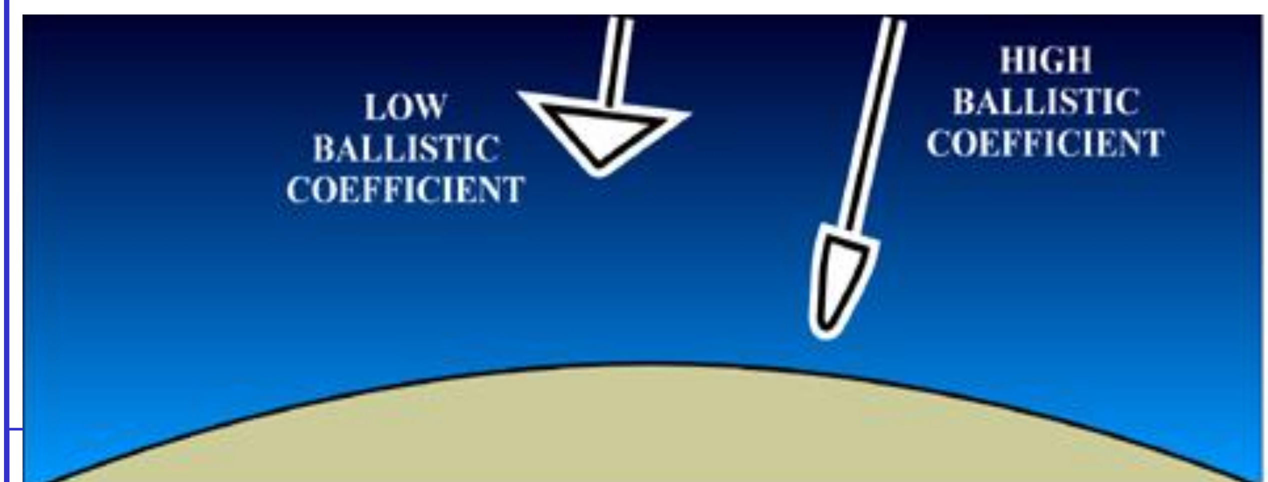
Drag Losses (4)

$$D_{rag} = C_D A_{ref} \frac{1}{2} \rho V^2 \rightarrow \Delta V_{drag} = \sqrt{A_{ref} \int_0^t \frac{C_D \rho V^3}{m} dt} = \sqrt{\int_0^t \frac{\rho V^3}{\beta} dt}$$

• Aerodynamic drag/mass inertial effects can be incorporated into a single parameter …. ***Ballistic Coefficient (β)***

• ….. measure of a projectile's ability to coast. … $\beta = M/C_D A_{ref}$
… M is the projectile's mass and … $C_D A_{ref}$ is the drag *form factor*.

• At given velocity and air density drag deceleration inversely proportional to β



Low Ballistic Coefficients
Dissipate More Energy
Due to drag

Available *Delta V* Summary

$$\Delta V_{t_{burn}} = g_0 \cdot I_{sp}\left[\ln\left(1 + P_{mf}\right)\right] -$$

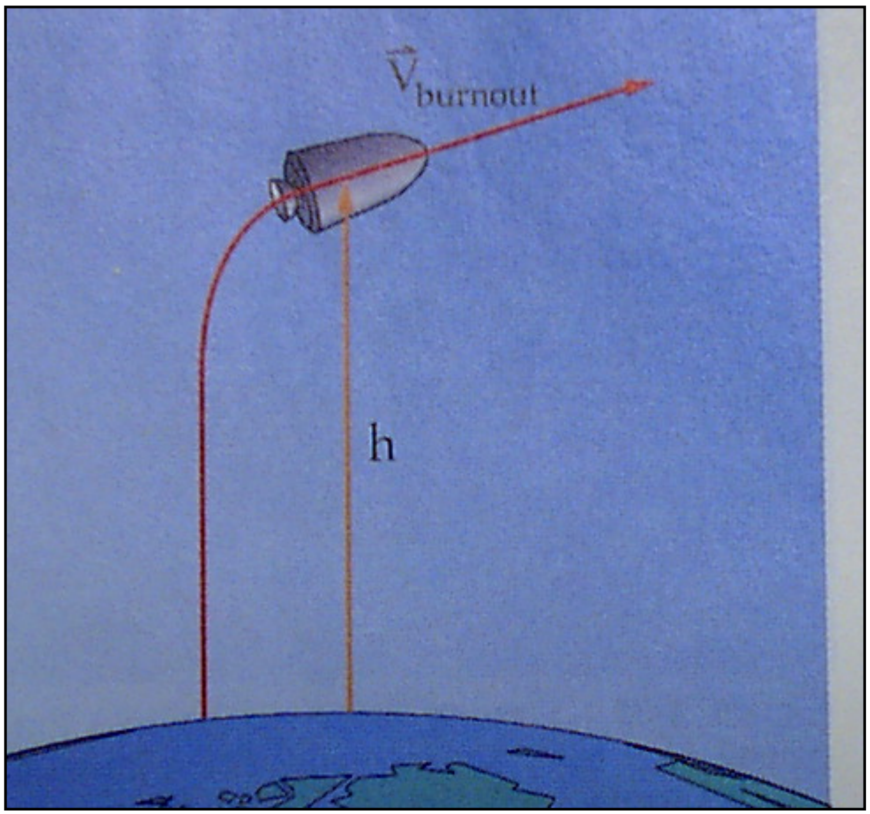
$$\text{"combustion } \Delta V\text{"}$$

$$\int^{t_{burn}} g_{(t)} \cdot t_{burn} \cdot \sin\theta_{(t)} dt - \sqrt{\int_0^{t_{burn}} \frac{\rho V^3}{\beta} dt}$$

"*gravity loss*" "*drag loss*"

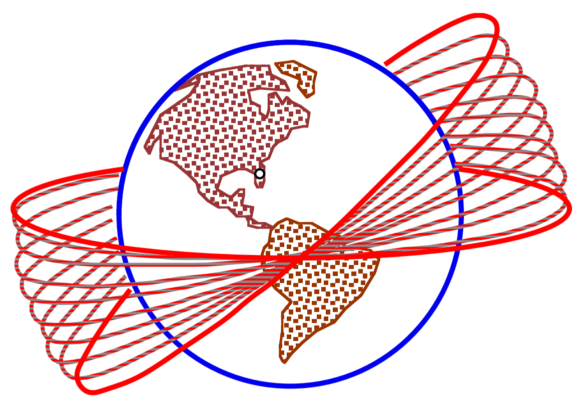
Required ΔV

- Need to accelerate from "standing still" on the ground to orbital velocity, while lifting to orbital altitude, and overcoming gravity and drag losses and insert into proper orbit inclination

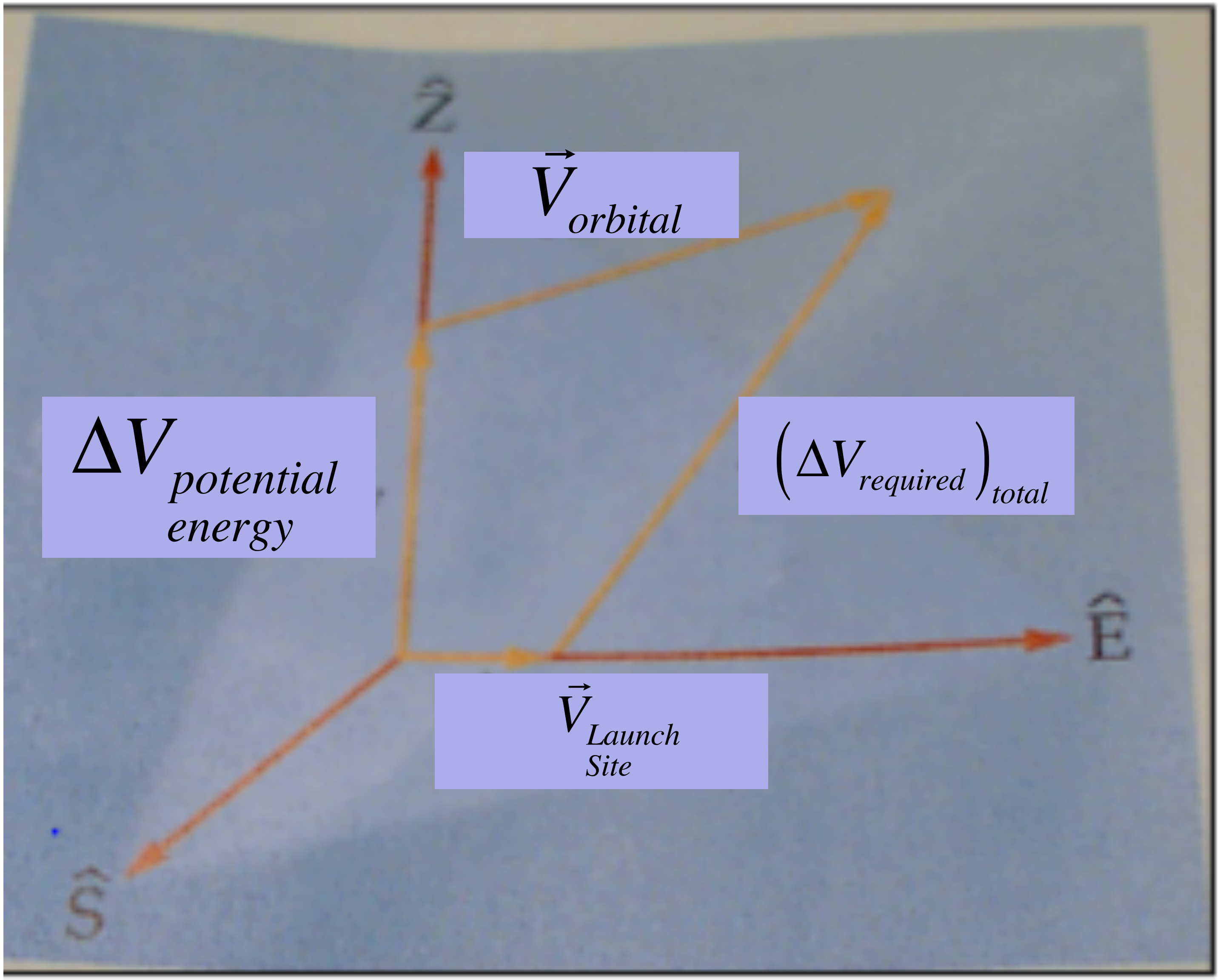


•Factors that Effect Delta V Requirements

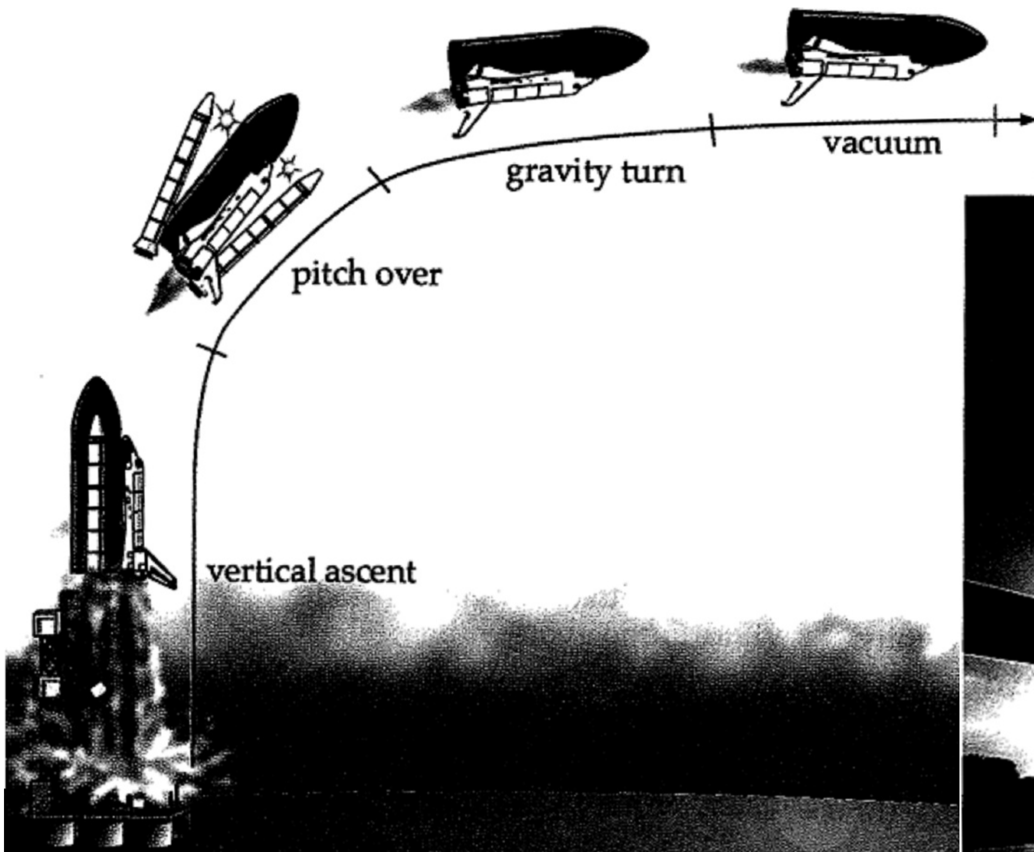
-Required Final Velocity
-Rotational Velocity of Earth
-Required Final Altitude
- Orbit Inclination Angle



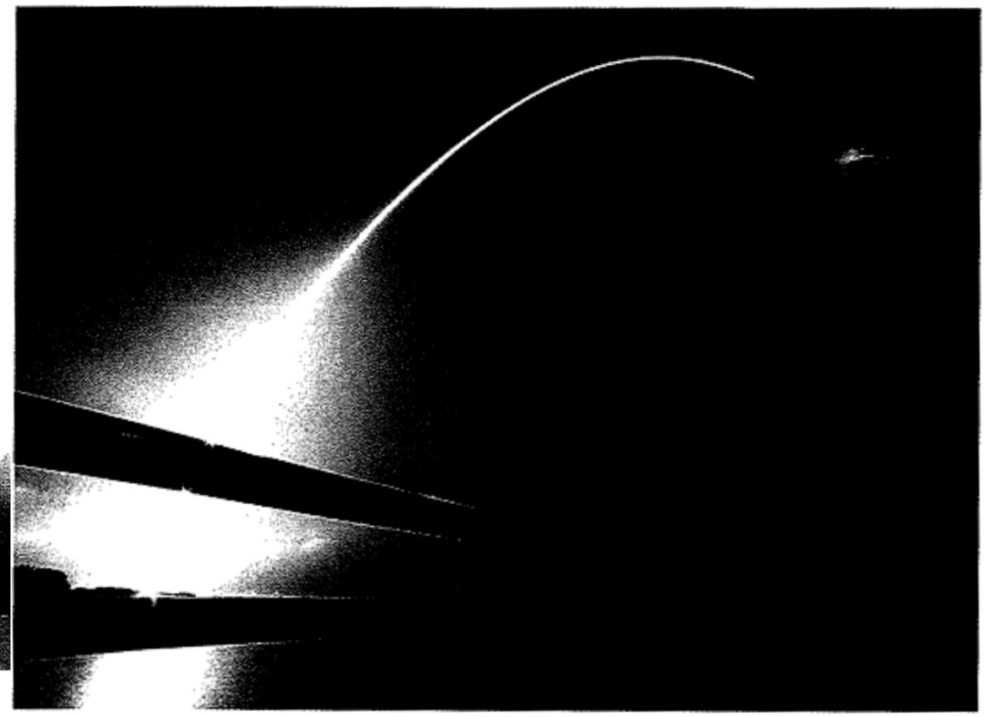
Required ΔV (2)



What Happens at Launch?



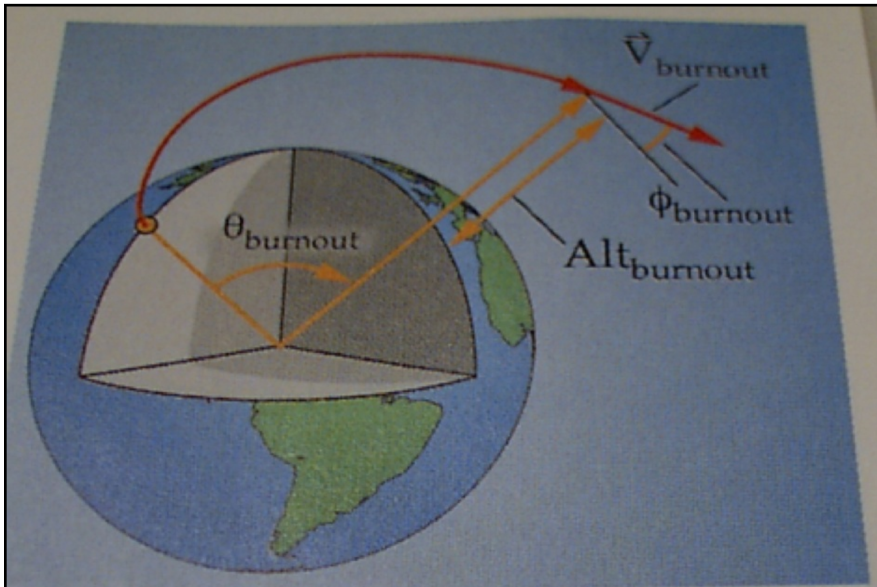
Phases of Launch Vehicle Ascent. During ascent a launch vehicle goes through four phases—vertical ascent, pitch over, gravity turn, and vacuum.

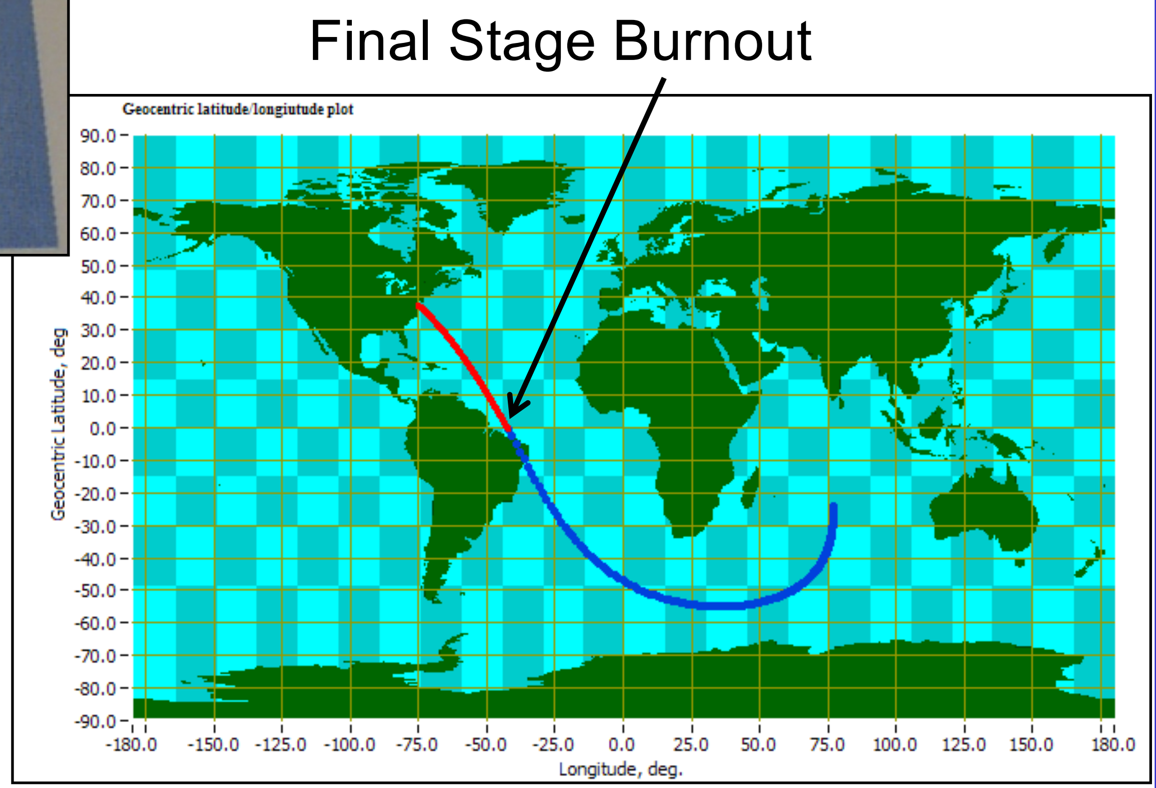


Gravity-turn maneuver of an ascending Delta II rocket with Messenger spacecraft on August 3, 2004.

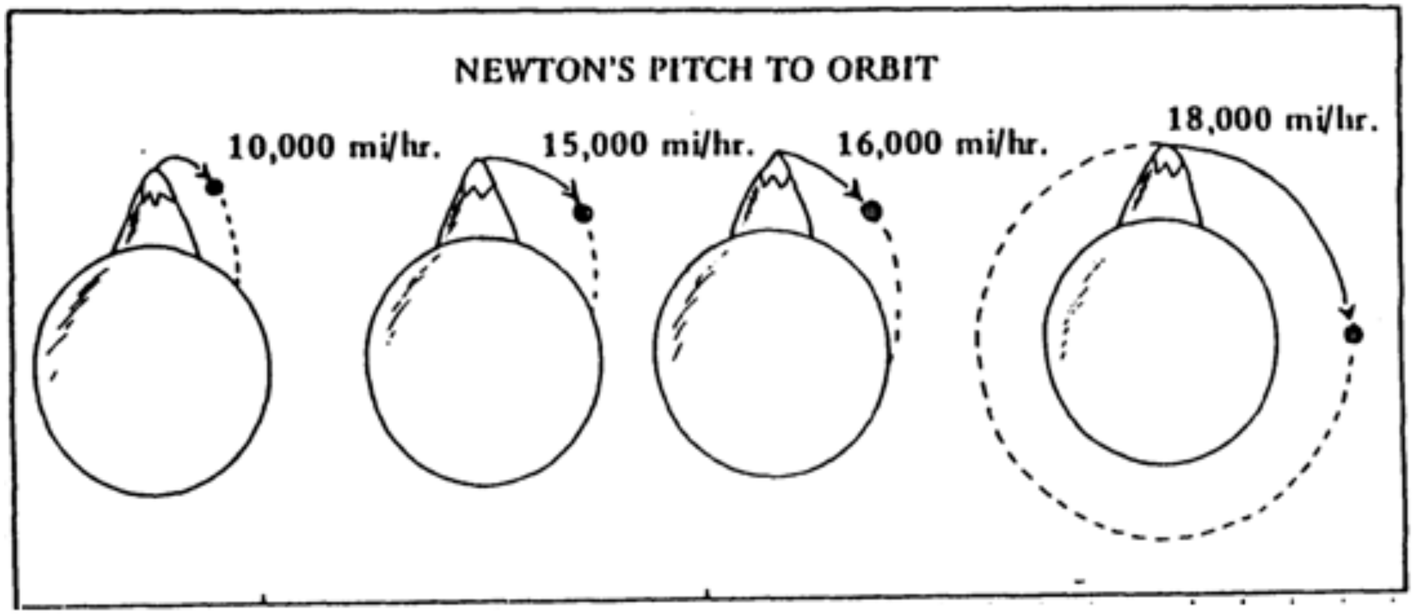
What Happens at Launch? (2)

- Velocity and Position at Burnout Determine Final Orbit





Example 1: Orbital Velocity
Isaac Newton explains how to launch a Satellite



- **Object in orbit is actually in "free-fall"**
  that is ... the object is literally falling around the Earth (or Planet)

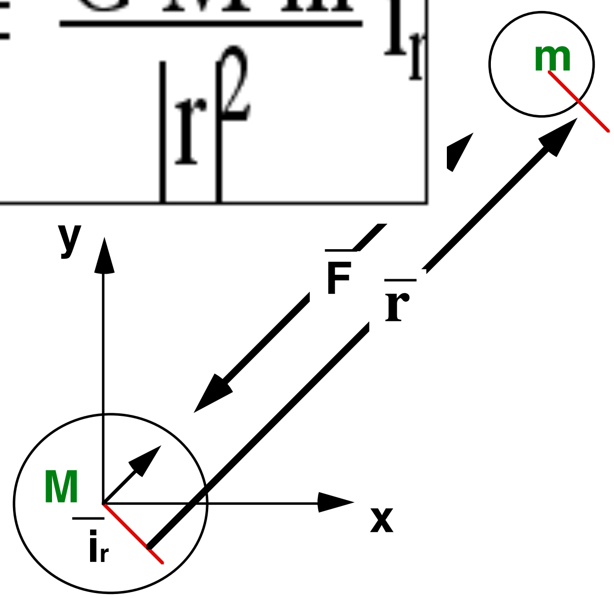
- **When the *Centrifugal Force* of the "free-fall"**
  counters the Gravitational Force ... the object is said to have achieved *Orbital Velocity*

Gravitational Physics

- Now by introducing a bit of *"gravitational physics"* we can unify the entire mathematical analysis

$$\overline{F}_{Mm} = \frac{G\,M\,m}{\left|r\right|^2}\,\overline{i}_r$$

"Inverse-square" law "potential" field



Isaac Newton, (1642-1727)

You've seen it before …..

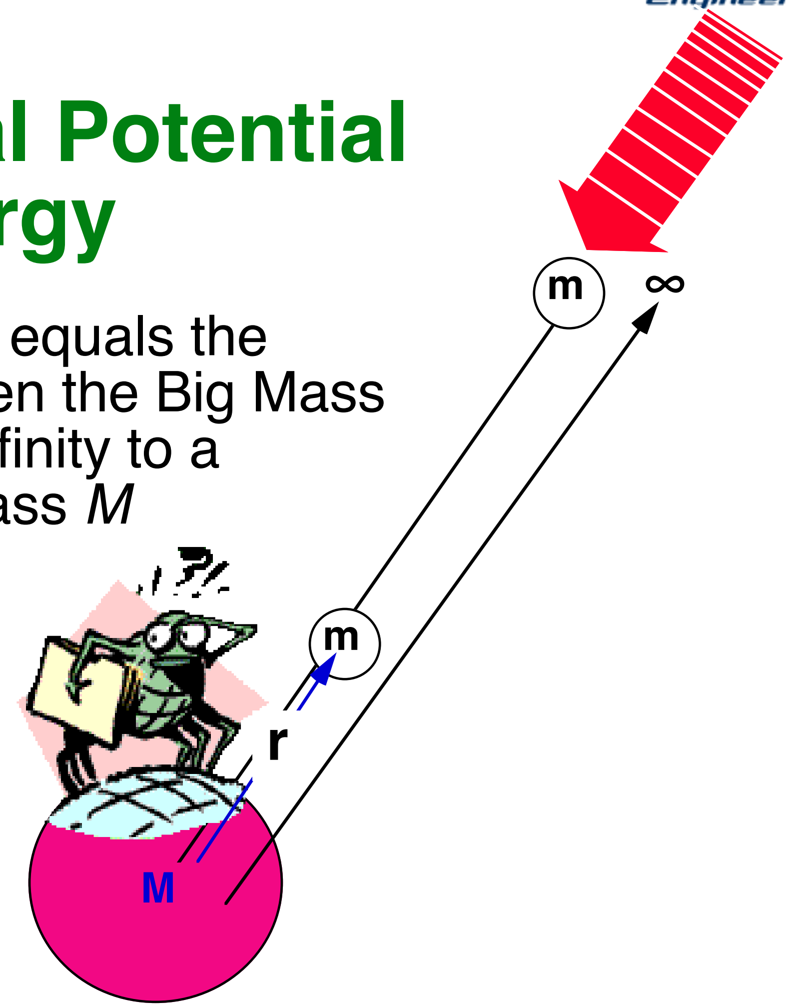
Gravitational Physics
(cont'd)

- Constant *G* appearing in Newton's law of gravitation, known as the ***universal gravitational constant***.

- Numerical value of *G*

$$G = 6.672 \times 10^{-11} \frac{Nt\text{-}m^2}{kg^2} = 3.325 \times 10^{-11} \frac{lbf\text{-}ft^2}{lbm^2}$$

Gravitational Potential Energy



- **G***ravitational potential energy* equals the amount of energy released when the Big Mass M pulls the small mass *m* at infinity to a location *r* in the vicinity of a mass *M*

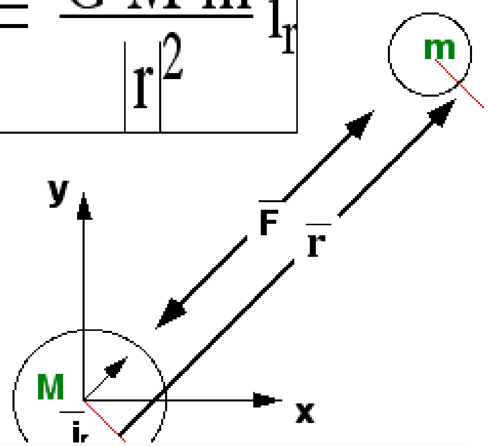
- **Energy of position**

$$P_{E_{grav}} \equiv E_{released} = \int_{\infty}^{\mathbf{r}} \mathbf{F} \cdot d\mathbf{r} =$$

$$\int_{\infty}^{\mathbf{r}} \frac{G\,M\,m}{r^2}\,dr = -G\,M\,m\left[\frac{1}{\mathbf{r}} - \frac{1}{\infty}\right] = \boxed{-\frac{G\,M\,m}{\mathbf{r}}}$$

Orbital Velocity (2)

Gravity $\overline{F}_{Mm} = \frac{G M m}{|r|^2} \overline{i}_r$



"Inverse-square" law "potential" field

Centrifugal Force

$$m \cdot \omega^2 \cdot r = m \cdot \frac{V^2}{r}$$

Ingnoring Drag ... for a Circular orbit

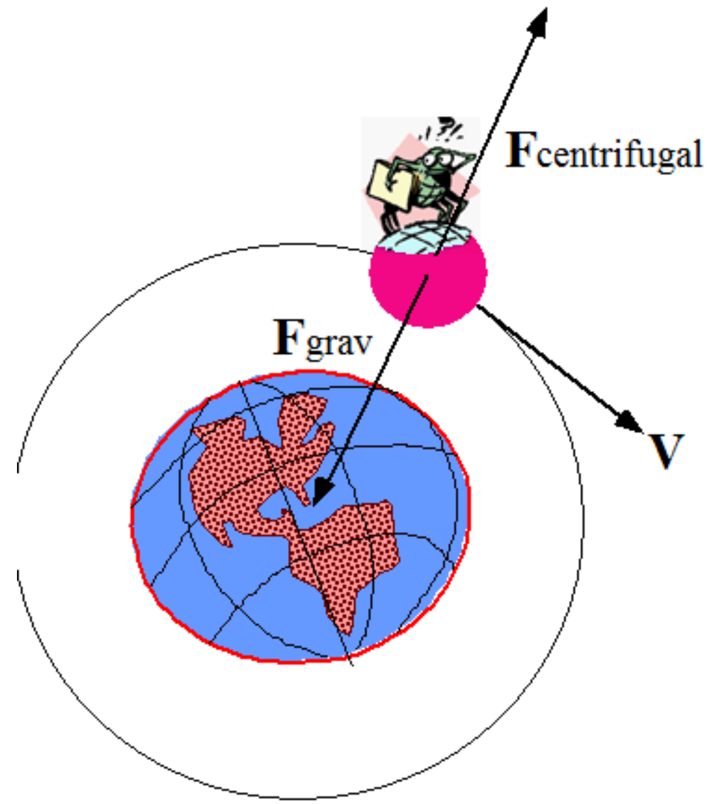
$$\overline{F}_{grav} = \overline{F}_{centrifugal}$$

$$\Downarrow$$

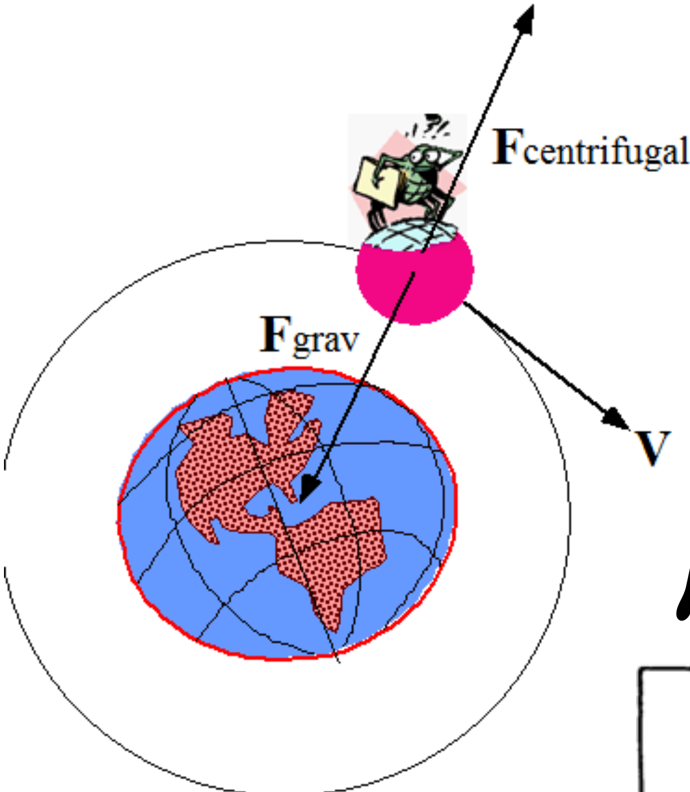
$$\frac{G M m}{|r|^2} = m\,\omega^2\,|r| = m\left[\frac{V}{|r|}\right]^2 |r|$$

$$\Downarrow$$

$$V = \sqrt{\frac{G M}{|r|}}$$



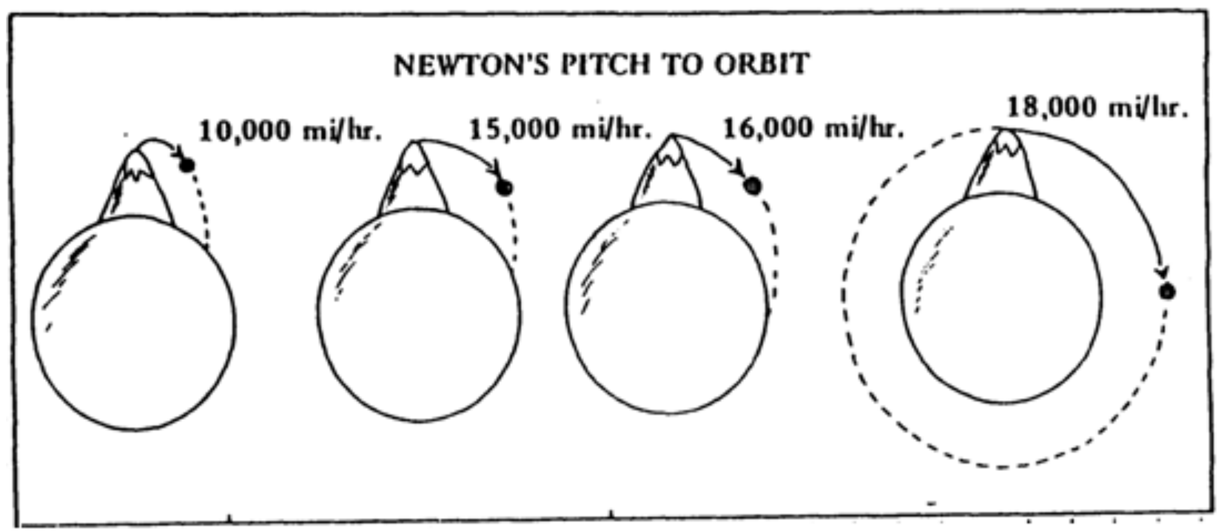
Orbital Velocity (3)



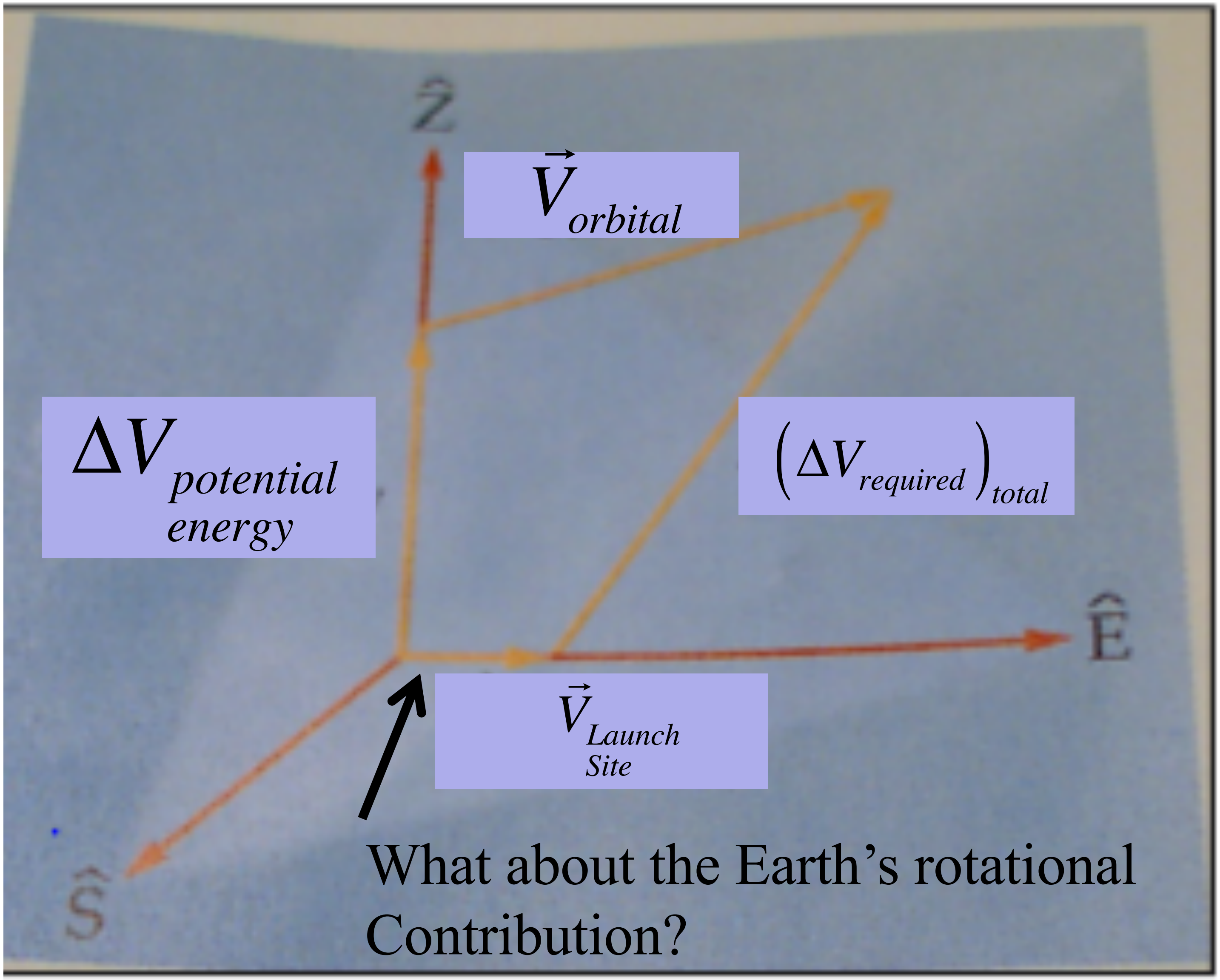
$$V_{orbital} \approx \sqrt{\frac{G \cdot M}{r}} \rightarrow G \cdot M \approx \mu$$

$$V_{orbital} \approx \sqrt{\frac{\mu}{r}} \rightarrow \mu = 3.9860044 \times 10^{5} \ \frac{km^3}{\sec^2}$$

μ **~ planetary gravitational constant**



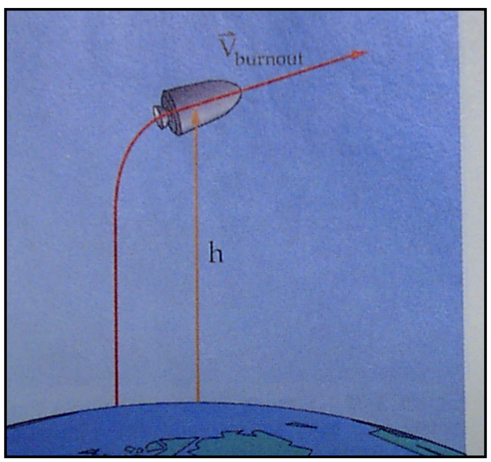
Required ΔV (2)

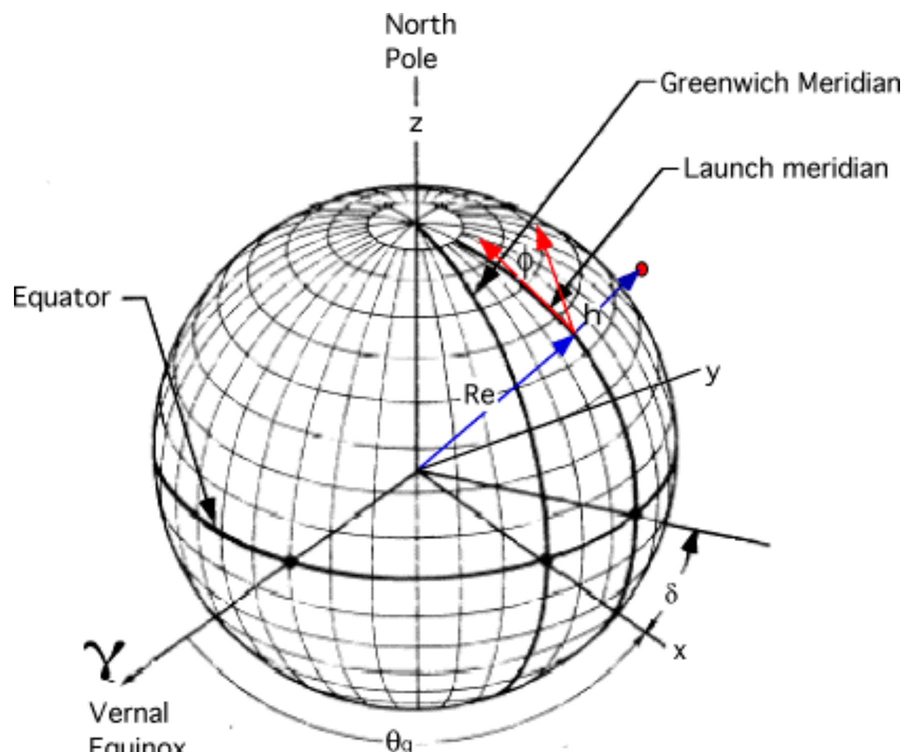


"Earth Delta V Boost"

- Need to accelerate from "standing still" on the ground to orbital velocity, while lifting to orbital altitude, and overcoming drag losses and insert into proper orbit inclination

- But are we really "standing still" on ground? No! The earth is rotating





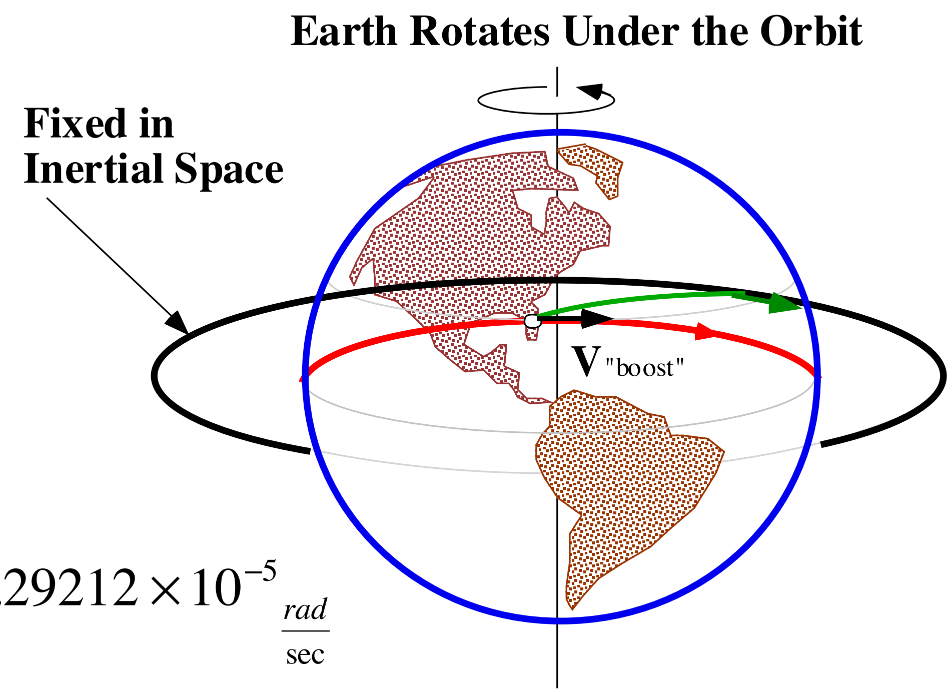
• Earth "Boost" (2)

$\lambda_0 \equiv$ *geocentric latitude*

$$V_{boost}^{(earth)} = \left[R_{earth} + h_0\right]_{launch} \left[\Omega_{earth} \cos[\lambda_0]\right]$$

• Launch Initial Conditions

$V_{boost}^{(earth)}$ acts due east



$$\Omega_{\oplus} = \frac{2 \cdot \pi}{23_{hrs} \cdot 3600 + 60_{mins} \cdot 60 + 4.1_{sec}} = 7.29212 \times 10^{-5} \frac{rad}{sec}$$

What is the tangential velocity of the earth?

$$\text{Velocity} = \text{Distance}/\text{Time}$$

Equatorial Radius

$$R_e = 6378km$$

$$V_{equator} = \frac{2\pi \cdot 6378_{km}}{23_{hrs} \cdot 3600 + 56_{\min} \cdot 60 + 4.1_{\sec}} = 0.4651_{km/\sec}$$

"inertial" equatorial velocity …. Actually exceeds the speed Of Sound! … why no shockwaves?

Angular Velocity of the Earth

- 1 Solar Day = 23 hrs 56 min 4.1 seconds = 86164.1 seconds
- $\Omega_{earth} = \frac{360°}{86164.1 \text{ seconds}} \times \frac{\pi}{180°} = .00007292115 \frac{\text{rad}}{\text{sec}}$

Earth Radius vs Geocentric Latitude

$$\frac{R_{earth(\lambda)}}{R_{eq}} = \sqrt{\frac{1 - e^2_{Earth}}{1 - e^2_{Earth}\cos^2[\lambda]}}$$

Polar Radius: 6356.75170 km

Equatorial Radius: 6378.13649 km

$$e_{Earth} = \sqrt{1 - \left[\frac{b}{a}\right]^2} = \sqrt{\frac{a^2 - b^2}{a^2}} =$$

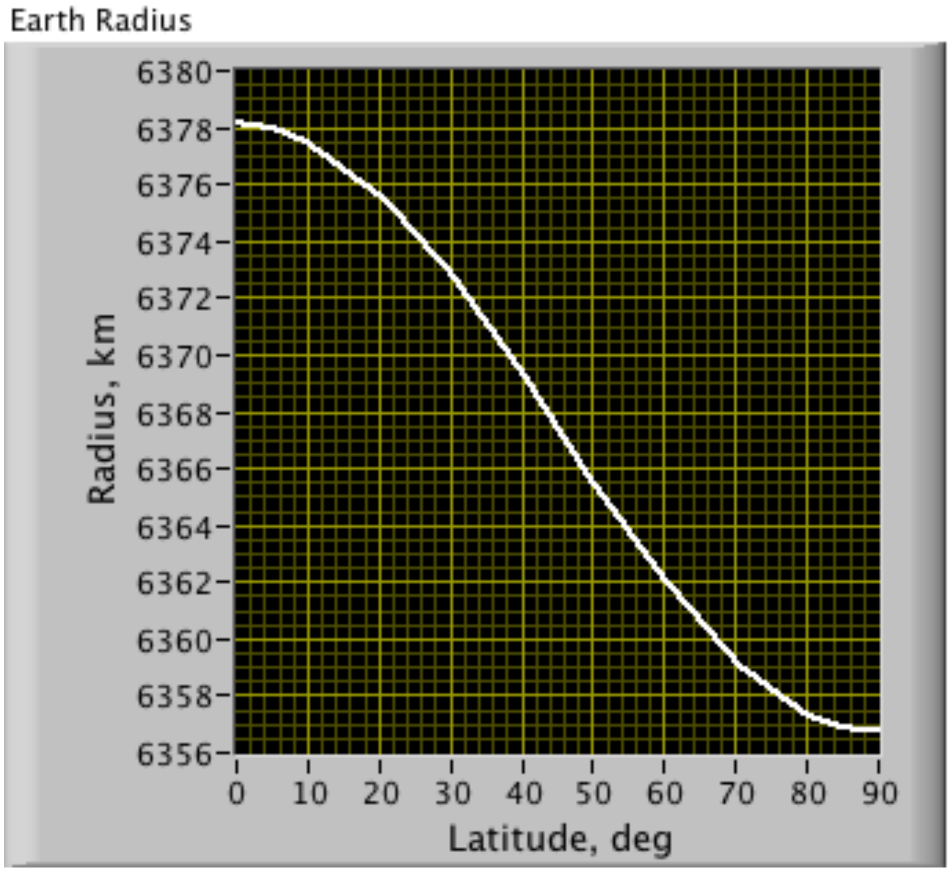
$$\frac{\sqrt{[6378.13649]^2 - 6378.13649^2}}{[6378.13649]} = 0.08181939$$

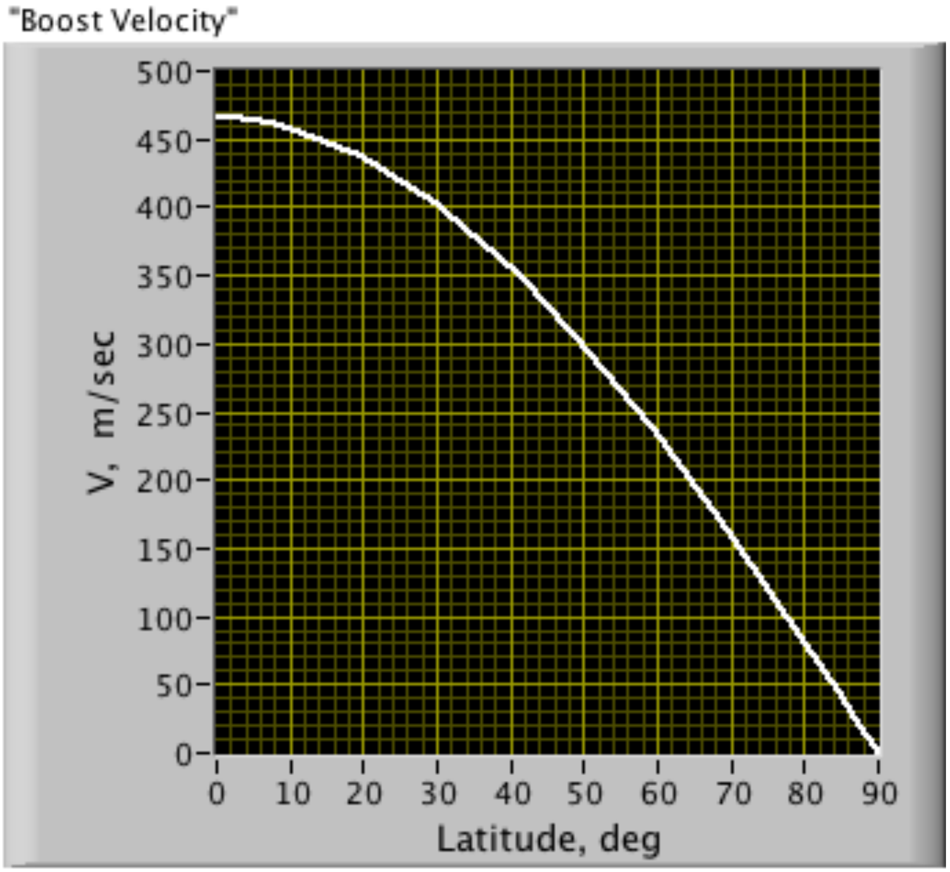
Tangential Velocity at various latitudes

Latitude	cos(lat)	velocity (km/sec)	velocity (ft/sec)
0	1	0.4638	1521
10	0.98481	0.45675	1497.89259
20	0.93969	0.43583	1429.27248
30	0.86603	0.40166	1317.22464
40	0.76604	0.35529	1165.15360
50	0.64279	0.29812	977.67995
60	0.50000	0.23190	760.50000
70	0.34202	0.15863	520.21264
80	0.17365	0.08054	264.11888
90	0.00000	0.00000	0.00000

$$V_{"boost"} = \left(R_{\oplus} + h_{launch}\right) \cdot \Omega_{\oplus} \cdot \cos\lambda$$

What is the tangential velocity of the earth? (2)

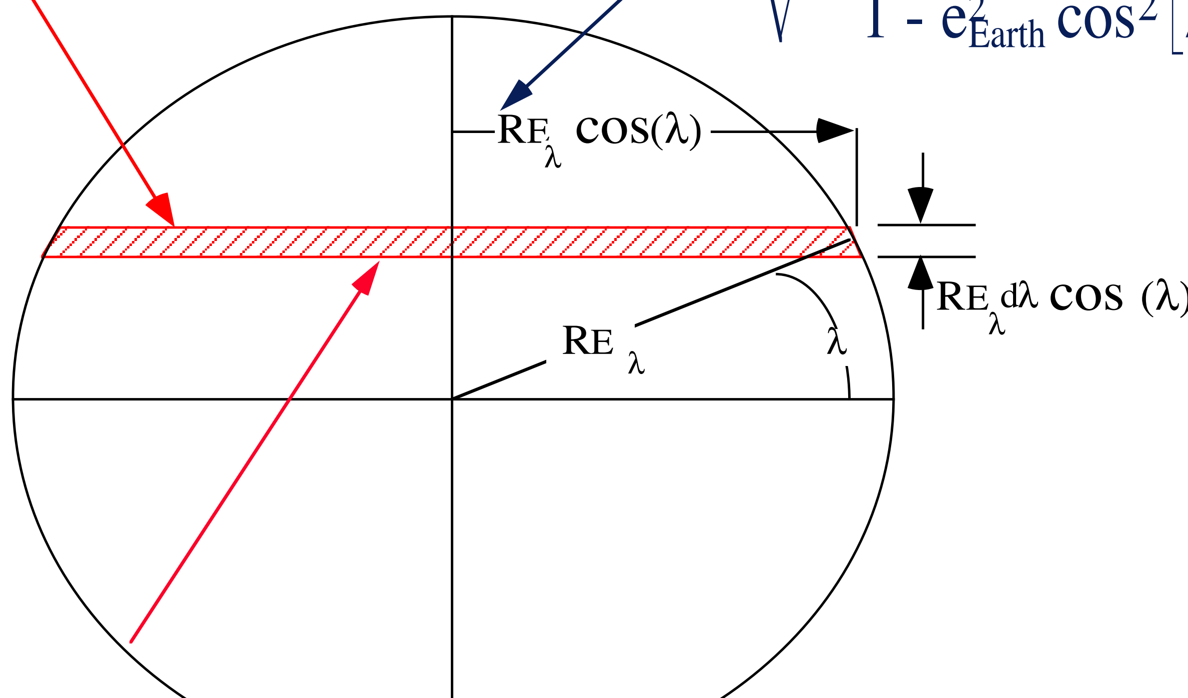




What is the mean radius of the earth?

$$dA = \pi \left[R_{E_\lambda} \cos^2(\lambda) \right]$$

$$R_{E_\lambda} = R_{eq} \sqrt{\frac{1 - e^2_{Earth}}{1 - e^2_{Earth} \cos^2[\lambda]}}$$



- **IAU Convention: Based on Earth's Volume**

Sphere Volume:

$$\frac{4\pi}{3} R^3_{E_{mean}} = V_E$$

$$dV = \pi \left[R_E \cos(\lambda)^2 \right] \times R_{E_\lambda} d\lambda \cos(\lambda)$$

What is the Earth's Mean Radius?

(continued)

- **Based on Volume**

$$\text{Ellipsoid Volume: } \frac{4\pi}{3}\sqrt{1-e^2}\; R_{eq}^3$$

$$\text{Sphere Volume: } \frac{4\pi}{3}\; R_{sphere}^3$$

$$\Downarrow$$

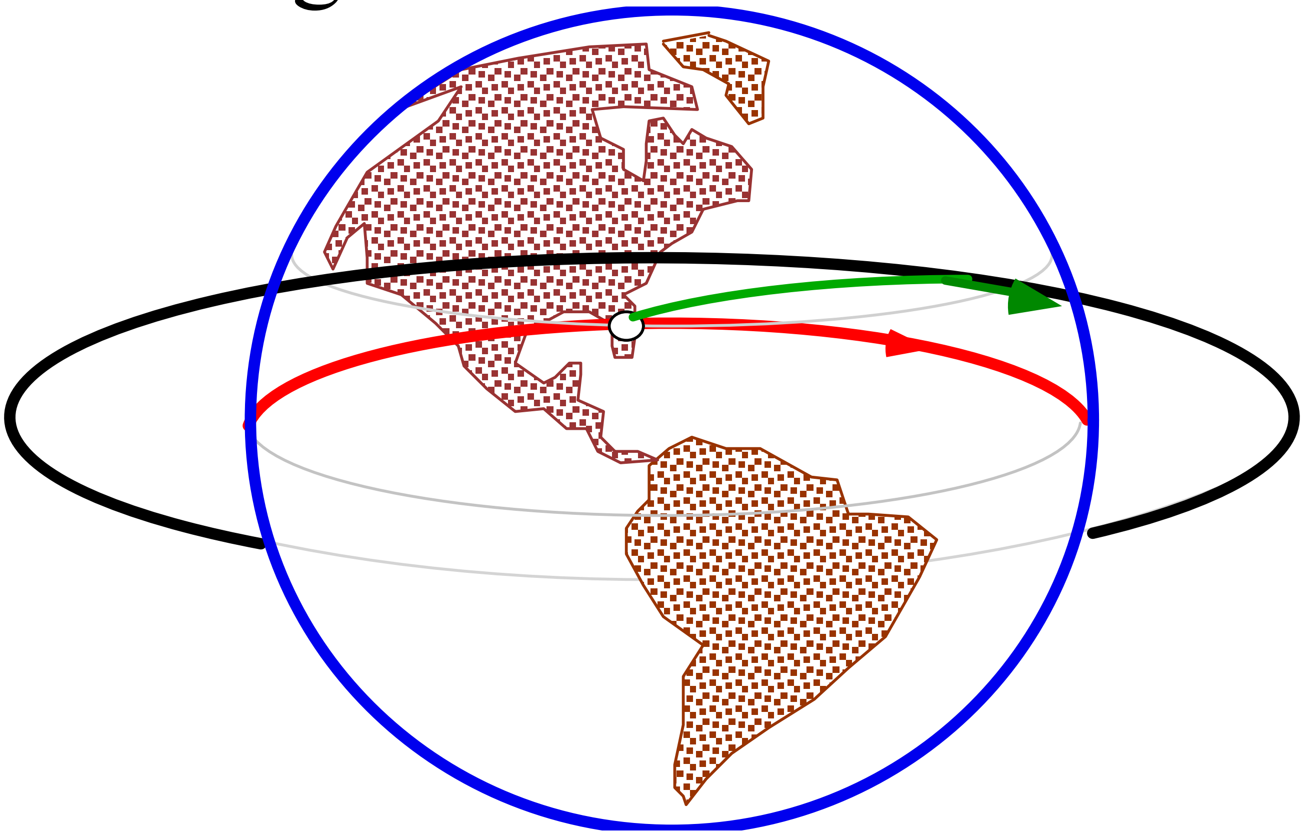
$$R_{sphere} \approx R_{mean} = \left[1 - 0.08181939^2\right]^{1/6} 6378.13649 = \boxed{6371.0002 \text{ km}}$$

- ***Mean Radius*** **we have been using is for a Sphere with same volume as the Earth**

$$M_E = \rho_E V_E$$

"gravitational radius"

High Inclination Launch



• *Physically Impossible* to Launch Directly into an orbit with a *Lower* inclination Angle than the Launch latitude

•Physically Possible to launch directly into any orbit with an inclination angle *greater than* or equal to launch latitude

Example Launch Delta V Calculation

KSC Latitude ~ 28.5^0 to 200 km Orbit .. ***Due east launch***

$$V_{earth} = 0.4638 \cos \frac{}{180} 28.5 = 0.4076 \text{ km/sec}$$

$$V_{orbit} = \sqrt{\frac{3.986 \cdot 10^5 \, km^3 / \sec^2}{r}}$$

Altitude (km)	Radius (km)	Velocity (km/sec)
200	6578	7.7843
600	6978	7.5579
1000	7378	7.3502
20000	26378	3.8873
35768	42146	3.0753

Delta $V_{orbit} = V_{orbit} - V_{earth} =$

7.7843 - 0.4076 =
7.7377 km/sec

High Inclination Launch (2)

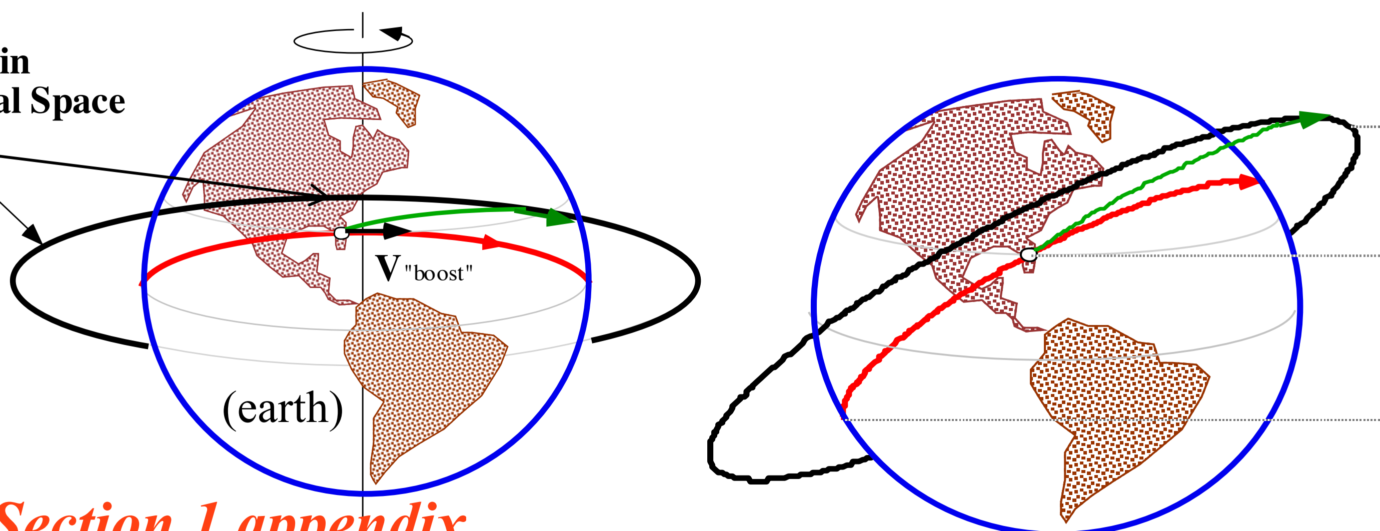
KSC Latitude ~ 28.5^0 to 200 km Orbit .. ***Due east launch***

$$V_{earth} = 0.4638\cos\left(\frac{\pi}{180}28.5\right) = 0.4076 \text{ km/sec}$$

But for a high inclination launch.. We don't get all of the "boost"

V_{boost} Earth Rotational Velocity

V_{boost} acts due east



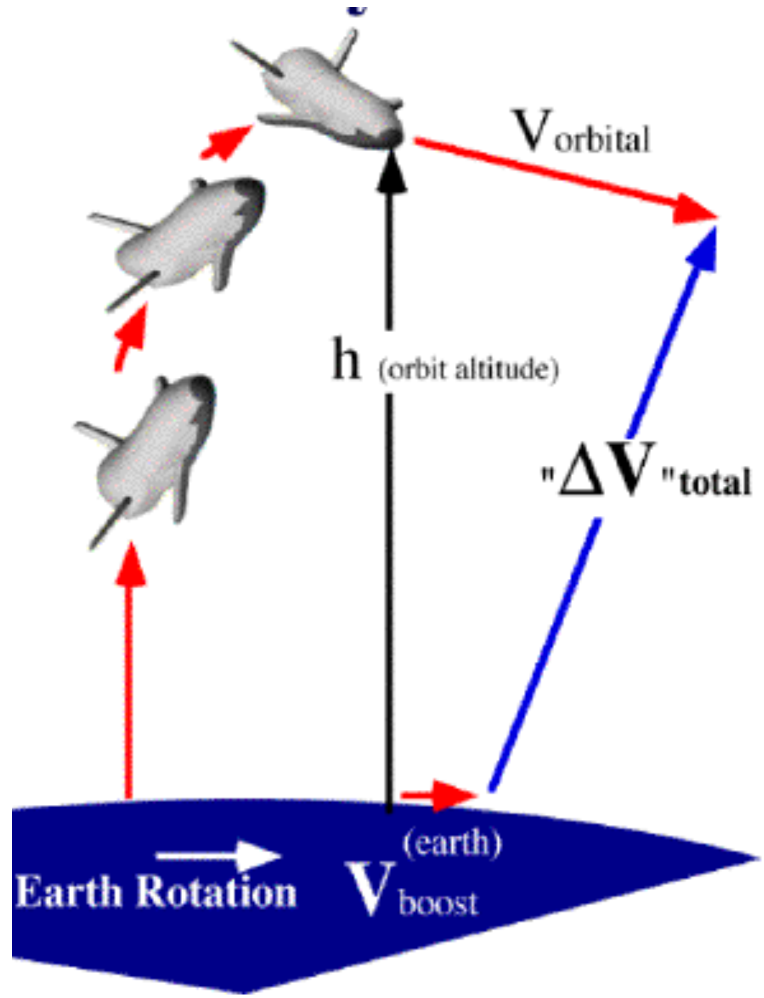
See Derivation in Section 1 appendix

$$V_{"boost"} = \left(R_\oplus + h_{launch}\right)\cdot\Omega_\oplus\cdot\cos\lambda\cdot\sin Az_{launch} = \left(R_\oplus + h_{launch}\right)\cdot\Omega_\oplus\cdot\cos i$$

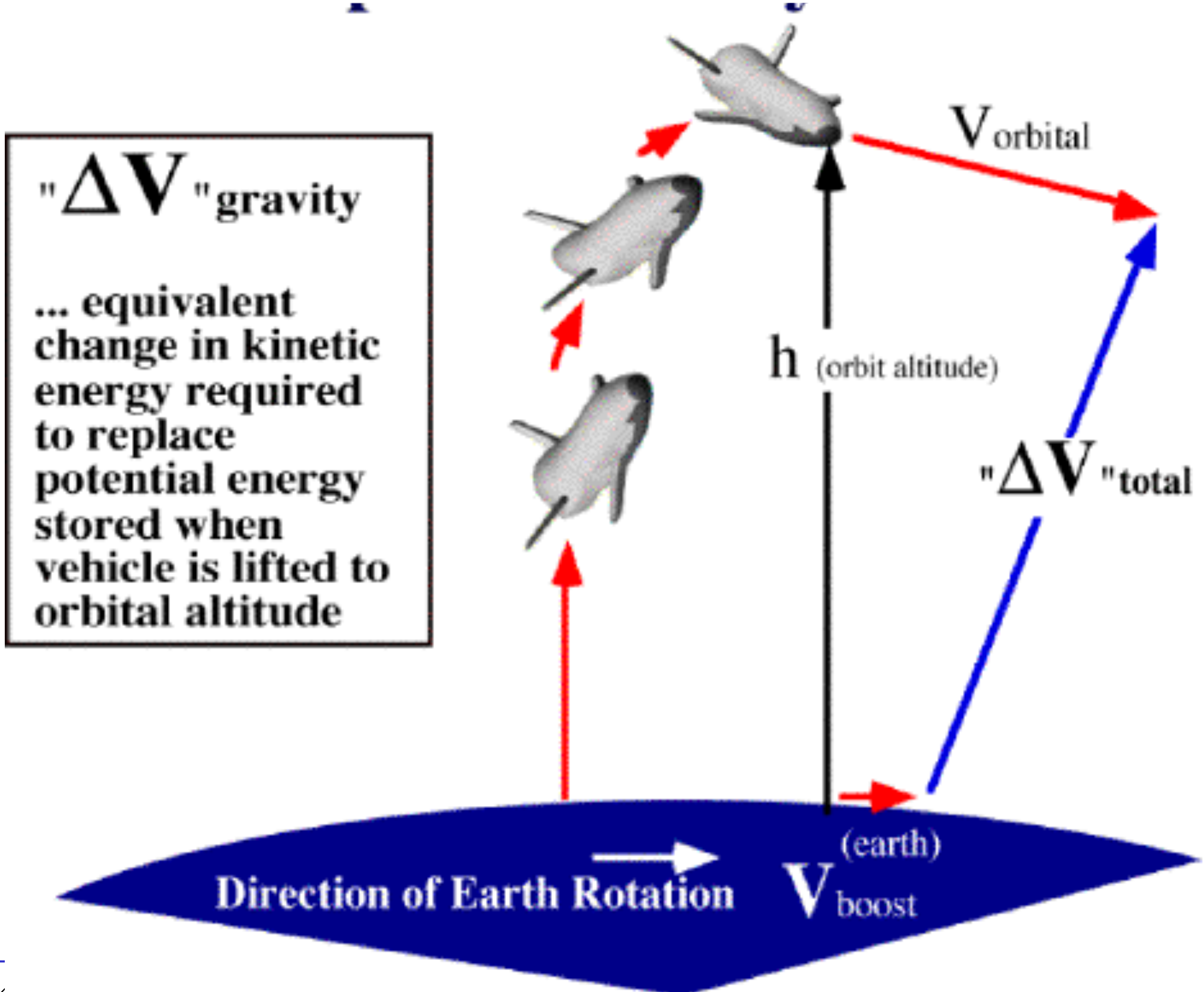
50^o Inclination launch from KSC ~ V_{boost} ~ $0.4638\cos\left(\frac{\pi}{180}55\right)$

$= 0.2660$ km/sec

How do we account for the change in potential energy due to lifting the vehicle 200 km?



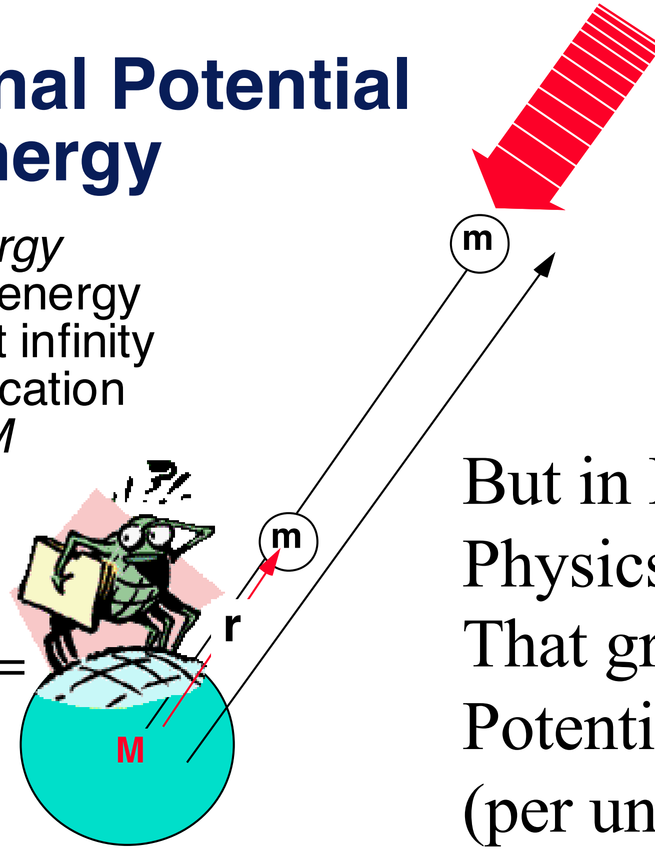
Accounting for Potential Energy



Potential Energy Revisited

Gravitational Potential Energy

•*Gravitational potential energy* equals the amount of work energy released when a mass *m* at infinity is pulled by gravity to the location *r* in the vicinity of a mass *M*



• **Energy of position**

$$U_{grav} \quad W_{performed\ on\ m} = \int^{\mathbf{r}} \mathbf{F} \ dr =$$

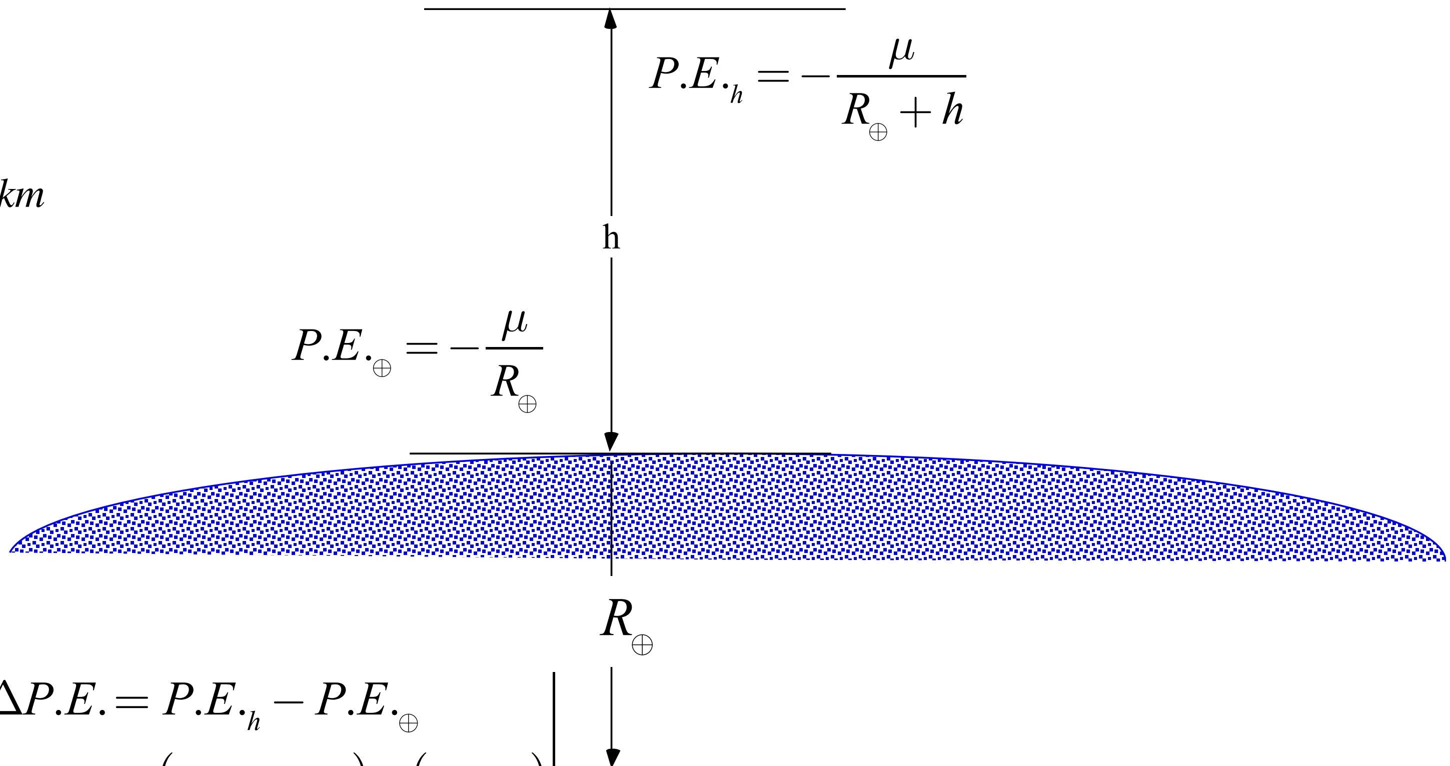
$$\int^{\mathbf{r}} \frac{G\,M\,m}{r^2} dr = -G\,M\,m\left[\frac{1}{\mathbf{r}} - \frac{1}{\ }\right] = \boxed{-\frac{G\,M\,m}{\mathbf{r}}}$$

But in High School Physics you learned That gravitational Potential energy was (per unit mass) was Just …… ***g·h***

… how do these models reconcile?

Potential Energy Revisited (cont'd)

$$R_{\oplus} = r_e \approx 6371_{km}$$



$$P.E._{h} = -\frac{\mu}{R_{\oplus} + h}$$

$$P.E._{\oplus} = -\frac{\mu}{R_{\oplus}}$$

$$P.E._{h} = -\frac{\mu}{R_{\oplus} + h}$$

$$P.E._{\oplus} = -\frac{\mu}{R_{\oplus}}$$

$$\rightarrow \quad \Delta P.E. = P.E._{h} - P.E._{\oplus}$$

$$\Delta P.E. = \left(-\frac{\mu}{R_{\oplus} + h}\right) - \left(-\frac{\mu}{R_{\oplus}}\right)$$

$$\rightarrow \Delta P.E. = \left(-\frac{\mu \cdot R_{\oplus}}{\left(R_{\oplus} + h\right) \cdot R_{\oplus}}\right) - \left(-\frac{\mu \cdot \left(\left(R_{\oplus} + h\right)\right)}{R_{\oplus} \cdot \left(R_{\oplus} + h\right)}\right) = \left[\frac{\mu}{R_{\oplus} \cdot \left(R_{\oplus} + h\right)}\right] \cdot h$$

Potential Energy Revisited (cont'd)

Check acceleration of gravity

$$F_{grav} = \frac{mMG}{r^2}\bar{i}_r \rightarrow \left|\frac{F_{grav}}{m}\right| = g(r) = \frac{\mu}{r^2}$$

$$F_{grav} = \frac{M\cdot m\cdot H}{r^2}\cdot\vec{i}_r \rightarrow \boxed{g(r) \equiv \frac{F_{grav}}{m} = \frac{M\cdot G}{r^2}\cdot\vec{i}_r = \frac{\mu}{r^2}\cdot\vec{i}_r}$$

$$\bar{g} = \frac{1}{h}\int_{R_\oplus}^{R_\oplus+h} g(r)\cdot dr = \frac{1}{h}\int_{R_\oplus}^{R_\oplus+h}\frac{\mu}{r^2}\cdot dr = -\frac{1}{h}\frac{\mu}{r}\Big|_{R_\oplus}^{R_\oplus+h} = -\frac{1}{h}\left[\frac{\mu}{(R_\oplus+h)} - \frac{\mu}{(R_\oplus)}\right] = \frac{\mu}{(R_\oplus+h)\cdot R_\oplus}$$

$$\rightarrow \Delta P.E. = \left[\frac{\mu}{R_\oplus\cdot(R_\oplus+h)}\right]\cdot h$$

$$\rightarrow \bar{g} = \frac{\mu}{(R_\oplus+h)\cdot R_\oplus}$$

$$\rightarrow \boxed{\Delta P.E._{h} = \bar{g}\cdot h}$$

Just like you learned in 12th grade physics!

UtahState UNIVERSITY

Mechanical & Aerospace Engineering

Delta V "gravity"

Potential Energy at Earth's Surface

Potential Energy at Orbital Altitude

equivalent kinetic energy required to overcome gravity

$$\left[\frac{\Delta V^2}{2}\right]_{gravity} = \frac{\mu}{r_e} - \frac{\mu}{r_e+h} =$$

$$\frac{\mu\,(r_e+h) - \mu\, r_e}{r_e\,(r_e+h)} = \frac{\mu\, h}{r_e\,(r_e+h)}$$

r_e = launch altitude

$$\Downarrow$$

$$\Delta V_{gravity} = \sqrt{2\,\frac{\mu\, h}{r_e\,(r_e+h)}}$$

200 km orbit from KSC … $r_e \sim 6371$ km

$$\Delta V_{gravity} \quad \left(\frac{2\,(3.9860044\times10^{5})\,200}{6371\,(6371+200)}\right)^{0.5} = 1.9516 \text{ km/sec}$$

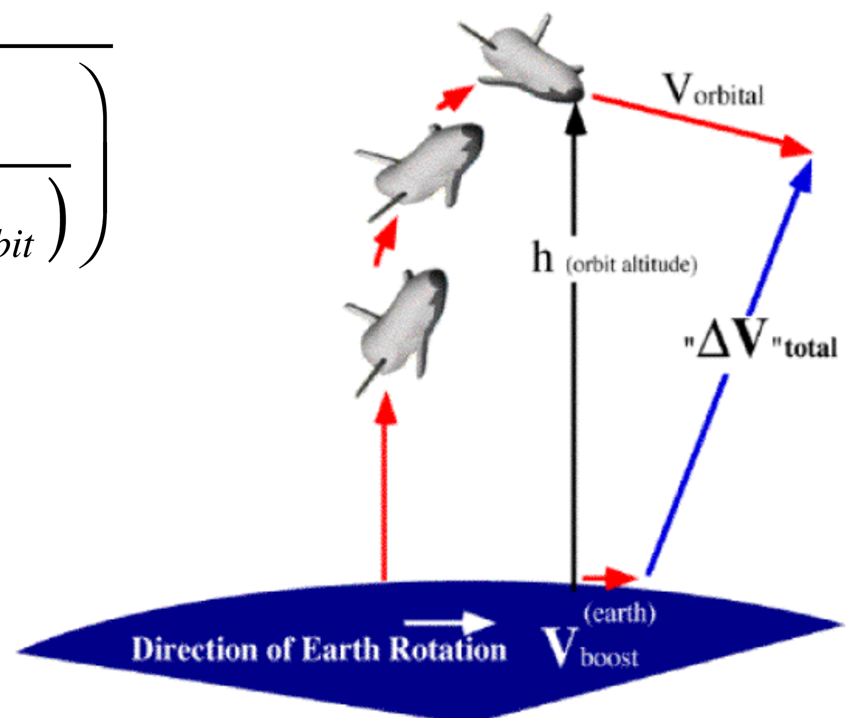
Total Delta V Required (cont'd)

- Root Sum Square of Required Kinetic Energy (Horizontal) + Potential Energy (Vertical)

$$\left(\Delta V_{required}\right)_{total} = \sqrt{\left(V_{orbital} - V_{\substack{"boost"\\ earth}}\right)^2 + \Delta V_{gravity}{}^2} =$$

$$\sqrt{\left(V_{orbital} - V_{\substack{"boost"\\ earth}}\right)^2 + \left(\frac{2\cdot\mu\cdot h_{orbit}}{R_\oplus\cdot\left(R_\oplus + h_{orbit}\right)}\right)}$$

$$V_{"boost"} = \left(R_\oplus + h_{launch}\right)\cdot\Omega_\oplus\cdot\cos i$$



Energy Summary

"Available ΔV" … Path Dependent

Propulsive Energy — gravity loss — energy dissipation

$$\Delta V_{t_{burn}} = g_0 \cdot I_{sp} \cdot \ln\left(1 + P_{mf}\right) - \oint_{t_{burn}} g_{(t)} \cdot t \cdot \sin\left(\theta_{(t)}\right) \cdot dt - \sqrt{\oint_{t_{burn}} \frac{1}{\beta_{(t)}} \cdot \rho_{(t)} \cdot V^3_{(t)} \cdot dt}$$

$$1 + P_{mf} = 1 + \frac{M_{propellant}}{M_{dry}} = \frac{M_{dry}}{M_{dry}} + \frac{M_{propellant}}{M_{dry}} = \frac{M_{initial}}{M_{final}}$$

$$g(t) = \frac{\mu}{R_{\oplus} + h(t)}$$

$$\beta_{(t)} = \frac{m_{(t)}}{C_{D(t)} \cdot A_{ref}}$$

"Required ΔV" … Path Independent

$$\Delta V_{required} = \sqrt{\left(V_{orbit} - V_{boost}\right)^2 + \Delta V^2_{gravity}}$$

→

$$V_{orbit} \approx \sqrt{\frac{\mu}{R_{\oplus} + h}}$$

$$V_{boost} = \left(R_{\oplus} + h_{launch}\right) \cdot \Omega_{\oplus} \cdot \cos\left(i_{orbit}\right)$$

$$\Delta P.E. = \frac{\Delta V^2_{gravity}}{2} = \frac{\mu \cdot h}{\left[R_{\oplus} + h(t)\right] R_{\oplus}} = \bar{g} \cdot h$$

Homework 1

- Space Shuttle has the following mass fraction characteristics

```
Weight (lb)
 Gross lift-off . . . . . . . .  4,500,000
 External Tank (full) . . . .  1,655,600
 External Tank (Inert) . . . .     66,000
 SRBs (2) each at launch . . . 1,292,000
 SRB inert weight, each . . .  . 192,000
```

- ***1) Calculate the actual propellant mass fraction as the shuttle sits on the pad***

$$P_{mf} = \frac{M_{propellant}}{M_{"dry"} + M_{payload}}$$

$$P_{mf} = \frac{M_{initial}}{M_{final}} - 1$$

Homework 1 (cont'd)

- Assume that Shuttle is being launched on a Mission to the International Space Station (ISS)

- ISS orbit altitude is approximately 375 km above Mean sea level (MSL), assume that Shuttle Pad 41A altitude approximately Sea level, *Latitude is 28.5 deg. , ISS Orbit Inclination is 51.6 deg*.

- Assume that the Earth is a perfect sphere with a radius of 6371 km

$$\mu_{\oplus} = M_{\oplus} \cdot G = 3.9860044 \times 10^5 \ \frac{km^3}{\sec^2}$$

- Calculate

1) The required Orbital Velocity
2) The "Boost Velocity" of the Earth at the Pad 41A launch site (along direction of inclination)
3) Equivalent "Delta V" required to lift the shuttle to altitude
4) Total "Delta V" required to reach the ISS orbit

Homework 1 (cont'd)

- The *2* SRB's each burn for approximately 123 seconds and produce 2,650,000 lbf thrust at sea level

- The *3* SSME engines each burn for ~509.5 seconds and each produces 454,000 lbf thrust at sea level

- ***Each*** of the SSME's consume 1040 lbm/sec of propellant

6) Calculate the average specific impulses of the SRB's, the SSME's, and the Effective specific impulse of the Shuttle Launch System as a whole during the First 123 seconds of flight (ignore altitude effects)

Hint: $$I_{sp} = \frac{\int_0^{T_{burn}} F_{thrust} \cdot dt}{g_0 \int_0^{T_{burn}} \dot{m}_{propellant} \cdot dt} = \frac{(\mathrm{I}mpulse)_{total}}{g_0 M_{propellant}}$$

Homework 1 (cont'd)

7) Based on the calculated "Delta V" requirements for the mission, what would be the required propellant mass fraction For the space shuttle to reach orbit in a single stage assuming the mean launch specific impulse?

-- base this calculation on the mean I_{sp} for the system during the first 123 seconds after launch

8) How does the shuttle manage to reach orbit? …. ?

Homework 1 (cont'd)

…. Next evaluate estimate launch conditions by breaking calculation into two "stages" .. That is

i) Stage 1 … first 123 seconds … SRB's and SSME's burning
ii) Stage 2 … after SRB's jettisoned .. Only SSME's burning

"stage 1"

"stage 2":

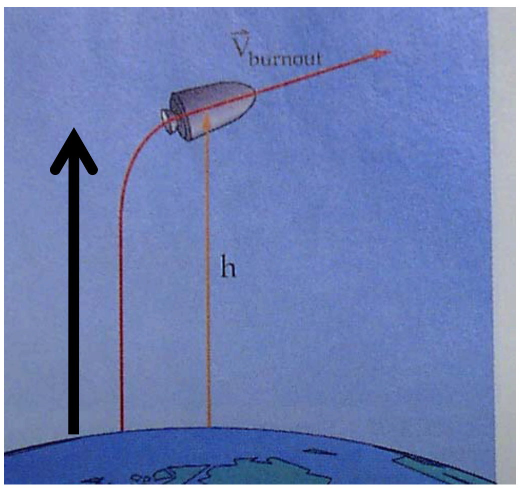
$$\Delta V_{total} = \Delta V_{stage1} + \Delta V_{stage2} + \Delta V_{stage3} \ldots = \sum_{i=1}^{\#\, of stages} \Delta V_{stage\_i}$$

Homework 1 (cont'd)

i) Stage 1 … first 123 seconds … SRB's and SSME's burning

-- Assume shuttle flys ~ "vertically" during Stage 1 flight. …

"stage 1"
Flight is vertical



9) Calculate "Available Delta V" for "stage 1" Based On Mean I_{sp}, and P_{mf} (ignore altitude effects)

-- Include "gravity losses" and assume an 8% drag loss in the available propulsive "Delta V" ... assume g(t) ~ g_0 = 9.8067m/sec^2

$$(\Delta V)_{available} = g_0 \cdot I_{sp} \cdot \ln\left(\frac{M_{initial}}{M_{final}}\right) - (\Delta V)_{\substack{gravity \\ loss}} - (\Delta V)_{drag}$$

$$(\Delta V)_{drag} \approx 0.10 \times g_0 \cdot I_{sp} \cdot \ln\left(\frac{M_{initial}}{M_{final}}\right)$$

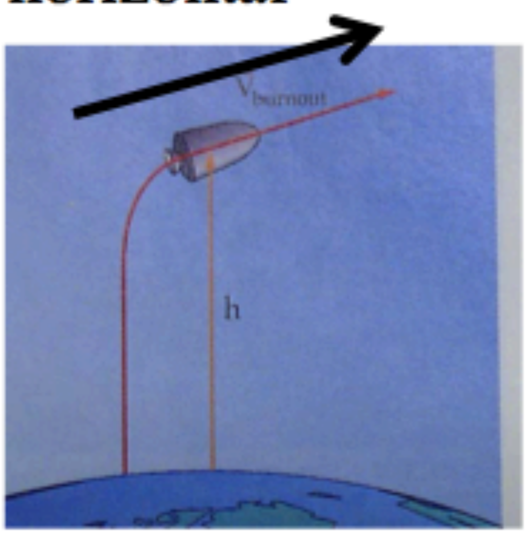
Homework 1 (cont'd)

… Break Calculation into two "stages" .. That is

ii) Stage 2 … flight time from 123 seconds to SSME burnout

-- Assume shuttle flys ~ "horizontally" during Stage 2 flight. …

"stage 2" flight is horizontal



-- 10) Calculate "Available Delta V" Based On SSME I_{sp}, and remaining P_{mf} after the SRB's Have been Jettisoned

-- Assume no drag losses for stage 2 burn

-- 11) Compute total available delta V .. Compare to mission requirements


Questions??