Rocket Science 101:
Basic Concepts and Definitions

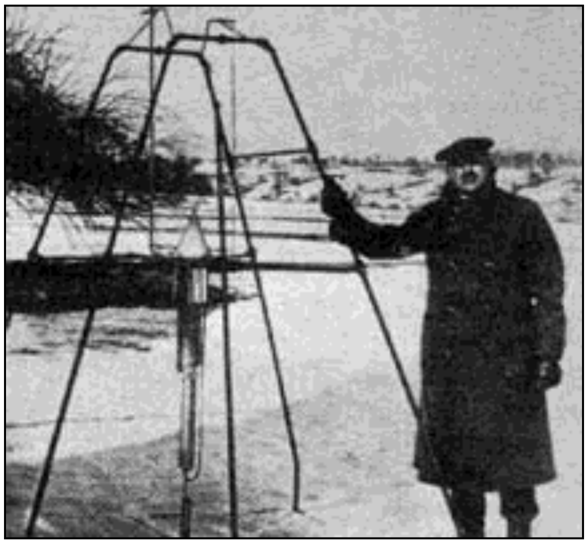
Newton's Laws as Applied to "Rocket Science"

... its not just a job ... its an adventure

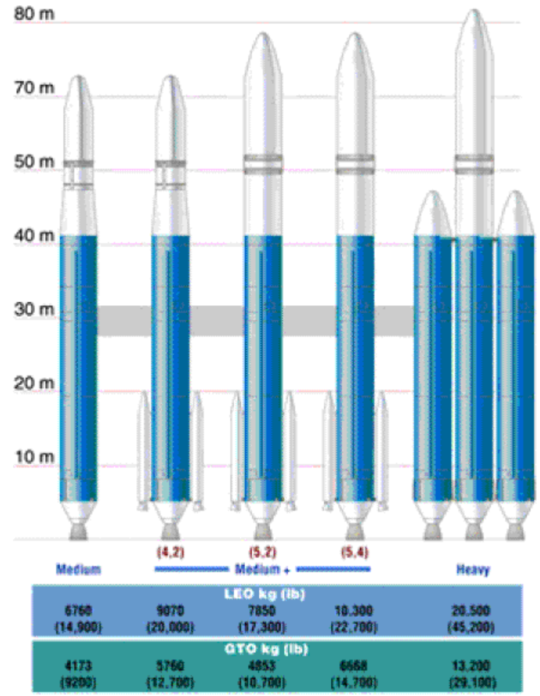
• How Does a Rocket Work?

Taylor, Chapter 3.

Rockets: Past, Present, and Future



Robert Goddard With his Original Rocket system



Delta IV … biggest commercial Rocket system currently in US arsenal

Material from <u>Rockets into Space</u> by Frank H. Winter, ISBN 0-674-77660-7

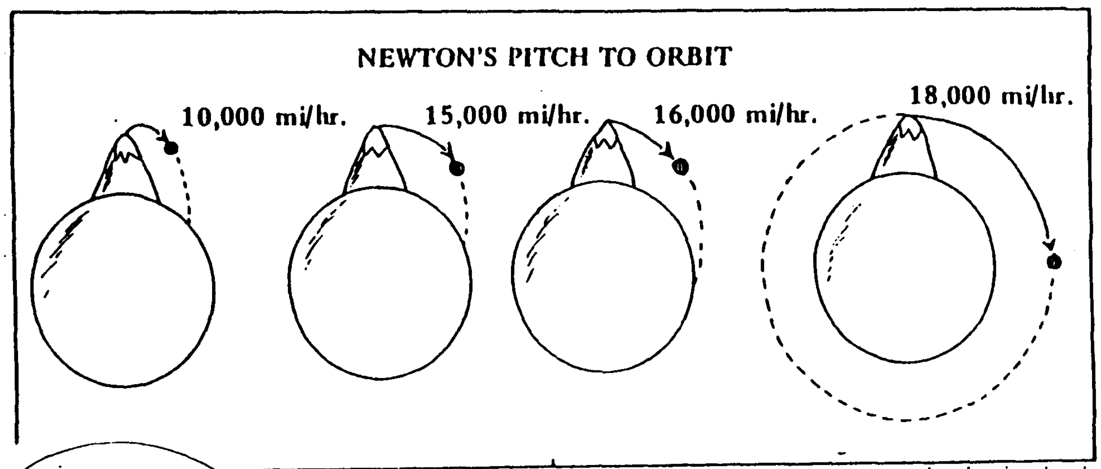
Earliest Rockets as weapons

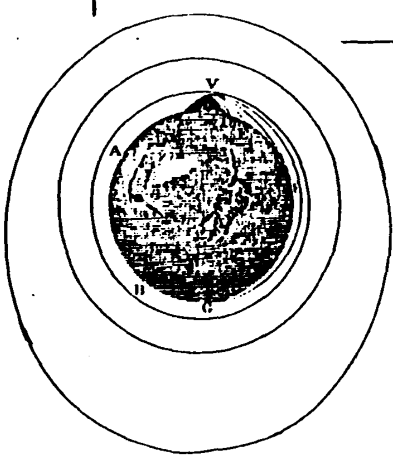
- Chinese development, Sung dynasty (A.D. 960-1279)
  - Primarily psychological
- William Congreve, England, 1804
  - thus “the rockets red glare” during the war of 1812.
  - 1.5 mile range, very poor accuracy.
- V2 in WWII

First Principle of Rocket Flight

- "For every action there is an equal and opposite reaction." Isaac Newton, 1687, following Archytas of Tarentum, 360 BC, and Hero of Alexandria, circa 50 AD.
- "Rockets move because the flame pushes against the surrounding air." Edme Mariotte, 1717
- *Which one is correct?*

Isaac Newton explains how to launch a Satellite





ACTUAL ILLUSTRATION FROM ISAAC NEWTON'S BOOK "SYSTEM OF THE WORLD", PUBLISHED IN 1687.

HIGHER SPEED CARRIES A BODY FURTHER BEFORE IT FALLS TO THE GROUND. IF THE SPEED IS GREAT ENOUGH, IT WILL NOT FALL AT ALL.

The Three Amigos of Spaceflight Theory

- Konstantin Tsiolkovsky
- Hermann Oberth
- Robert Goddard
- Independent and parallel development of Rocket theory

Three Amigos

•Tsiolkovsky

•Goddard

•Oberth

Konstantin Tsiolkovsky
1857 - 1935

- Deaf Russian School Teacher - fascinated with space flight, started by writing Science Fiction Novels
- Discovered that practical space flight depended on liquid fuel rockets in the 1890's, and developed the fundamental Rocket equation in 1897.
- Calculated escape velocity, minimum orbital velocity, benefit of equatorial launch, and benefit of multi-stage rockets
- Excellent theory, Not well published, not as important as he could have been.
- Famous for development of "Rocket Equation"

Robert H. Goddard
1882 - 1945

- Also a loner, developed rocket theory in 1909-1910,
- Forte was as an experimenter, actually building and testing liquid fuel rockets (first flight in 1926.)
- In a report to his sponsors (Smithsonian Institute) in 1920, he described a rocket trip to the moon. This subjected him to ridicule since the common belief was still that a rocket needed air to push against.
- Goddard ended with 214 patents covering details of rocket design

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Goddard and his Rocket

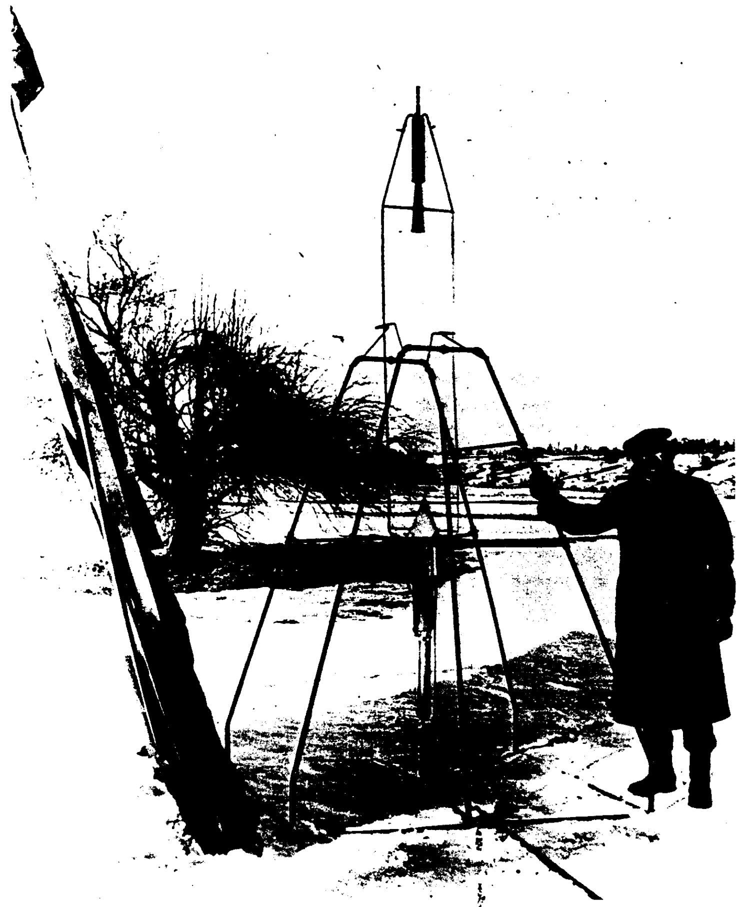


Figure 4. Robert Goddard standing beside the world's first successfully flown liquid-fuel rocket, which was launched on March 16, 1926. Note the rocket nozzle on the top. The asbestos-covered cone on the bottom directed exhaust gases away from the propellant tanks below. Goddard found this "nose-driven" design unstable and later shifted the rocket motor to the bottom. The "tail-driven" configuration was more stable and became standard in all rockets.

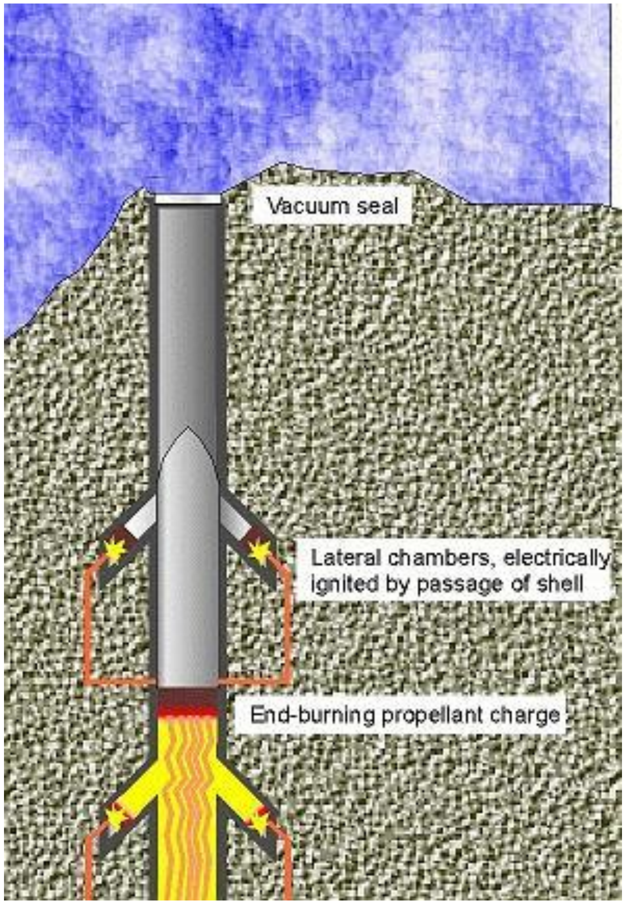
Hermann Oberth
1894 - 1989

- His 1923 book: Die Rakete zu den Planetenraumen (The Rocket into Planetary Space) covered the entire spectrum of manned and unmanned rocket flight.

- Because it was published and widely read, he had more influence on the growth of rocket concepts then either of the others. His book spawned several rocket societies in Germany, significantly the German Rocket society, out of which the German army recruited Werner Von Braun in 1932 and started the project which produced the V2.

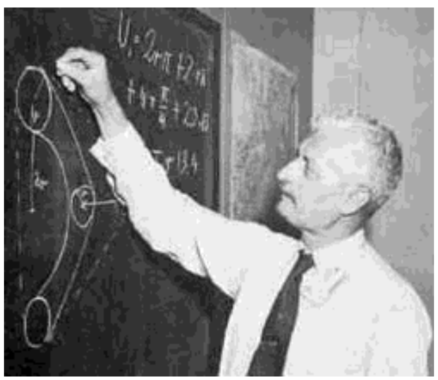
Valier-Oberth Moon Gun *Nation*: Germany.

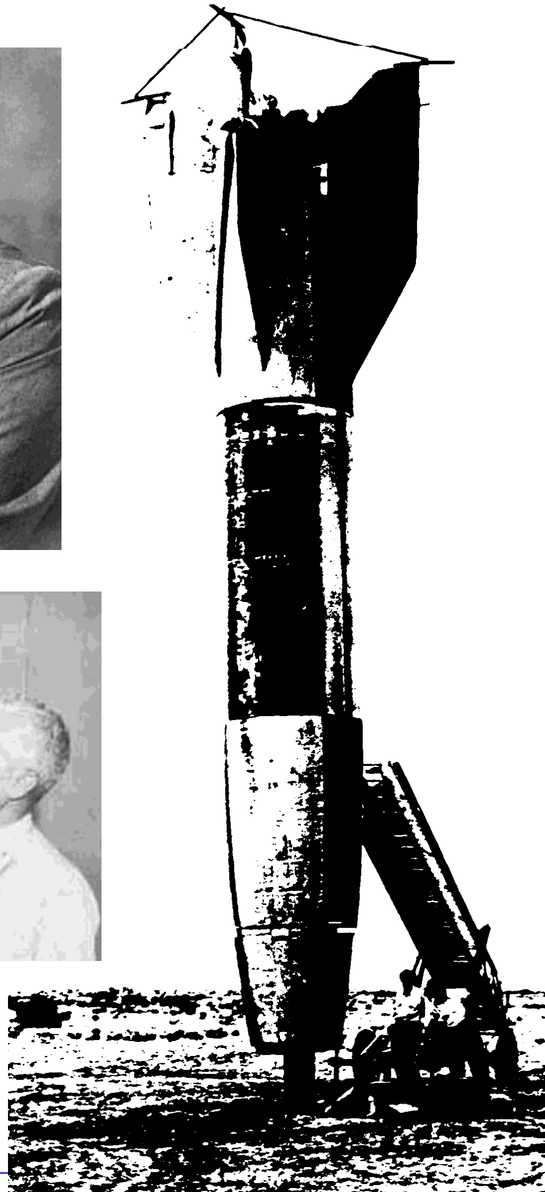
In the 1920's members of the German VfR (Society for Space Travel) amused themselves by redesigning Verne's moon gun. In 1926 rocket pioneers Max Valier and Hermann Oberth designed a gun that would rectify Verne's technical mistakes and be actually capable of firing a projectile to the moon.



The V2

- Challenge was to deliver a one ton warhead, 180 nm range.
- Final design: 2300 lb warhead, 190 nm range. 47 ft long, 5.4 ft diameter, 28,229 lb takeoff weight. 59,500 lb thrust for 68 seconds.
- 6400 weapon launches
- The Americans got Von Braun and 117 other scientists, and about 100 rockets. The Soviets got the facilities and about the same number of rockets.
- 60 plus V2's and V2 mods were launched in the late 40's in US. All were sub-orbital, highest altitude was 244 miles





- V-2 Rocket
First Operational
System

Interest of Entities

Military Space Activities

- Communications
- Missile Warning
- Launch Operations
- Meteorology & Geodesy
- Navigation
- Imaging & Signal Intelligence
- Satellite Tracking
- Anti-Satellite Weapons
- Wide Area/Ocean Surveillance

Civil Space Activities

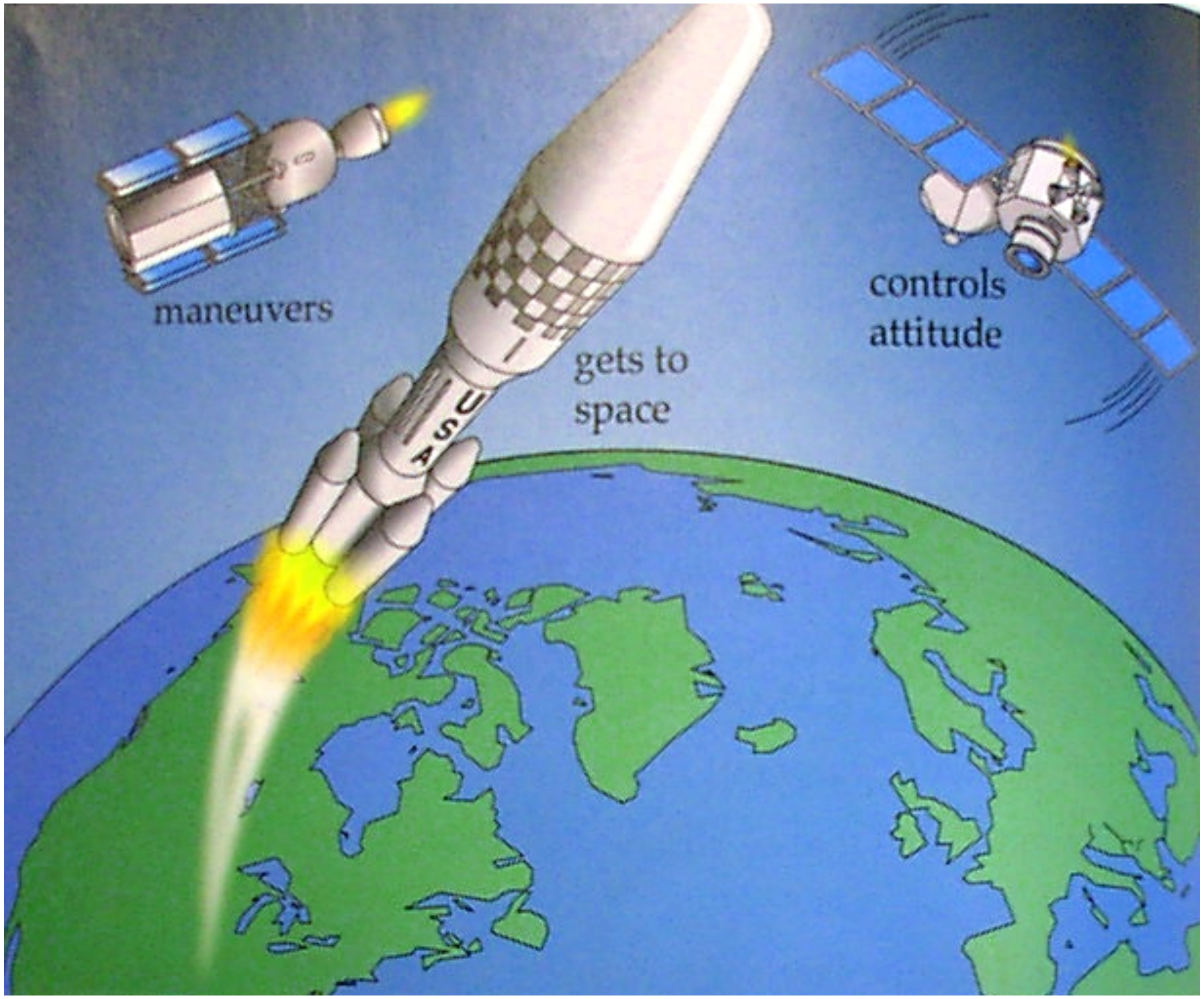
- Science
- Launch Operations
- Disaster Relief/Monitoring
- Astrophysics
- Human Space Flight
- Meteorology
- Microgravity Research
- Environmental Modeling

Commercial Space Activities

- Design, Development, and Operation of Launch Vehicles/Facilities, Satellites/ Spacecraft, Ground Stations, and Sensors
- Telecomm. (including Personal Communications, Television/Cable, Radio, etc.)
- Support Services (including standards/allocations, insurance, consulting, etc.)
- Emerging Applications & Technologies (including remote sensing, geodesy, navigation, microgravity, broadband, etc.)

Space Tourism

What does a rocket "do"?

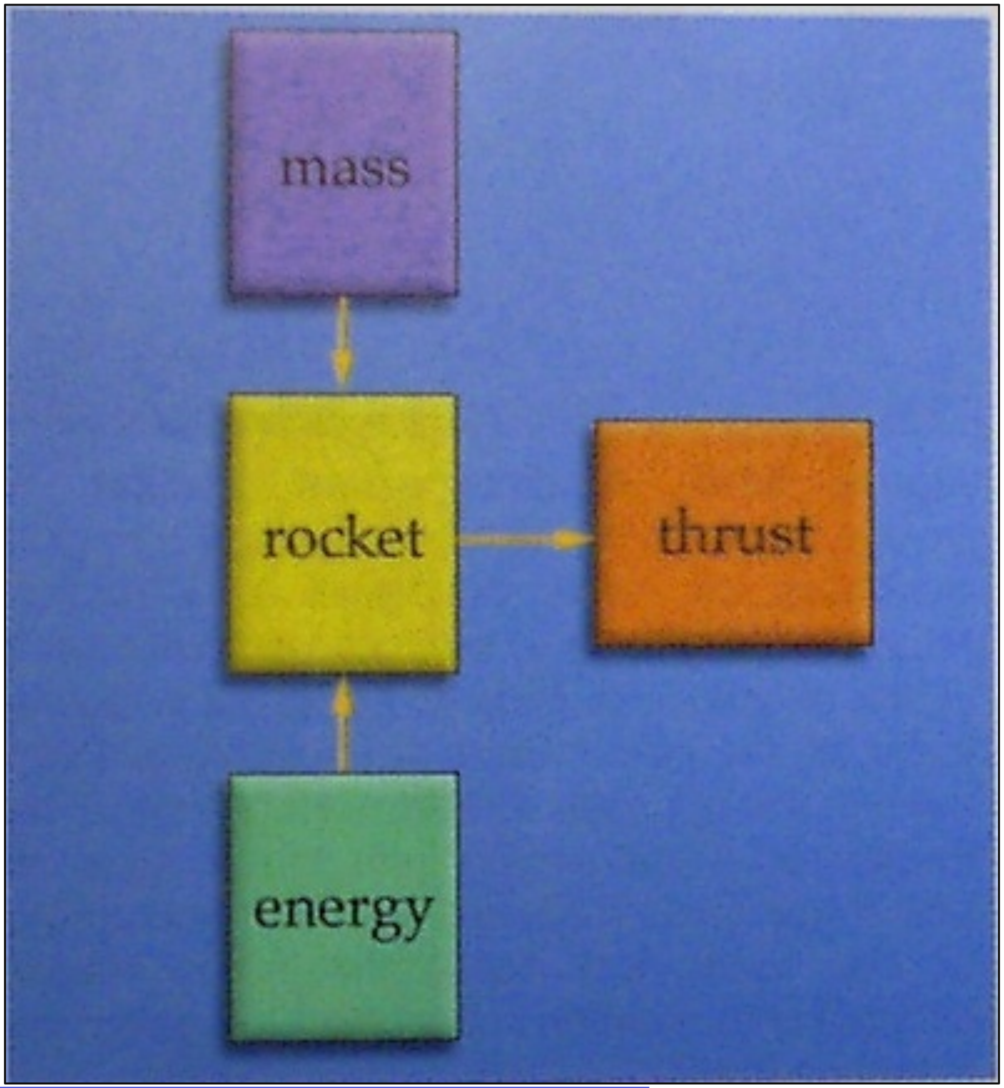


Rockets take spacecraft to orbit

Move them around in space, and

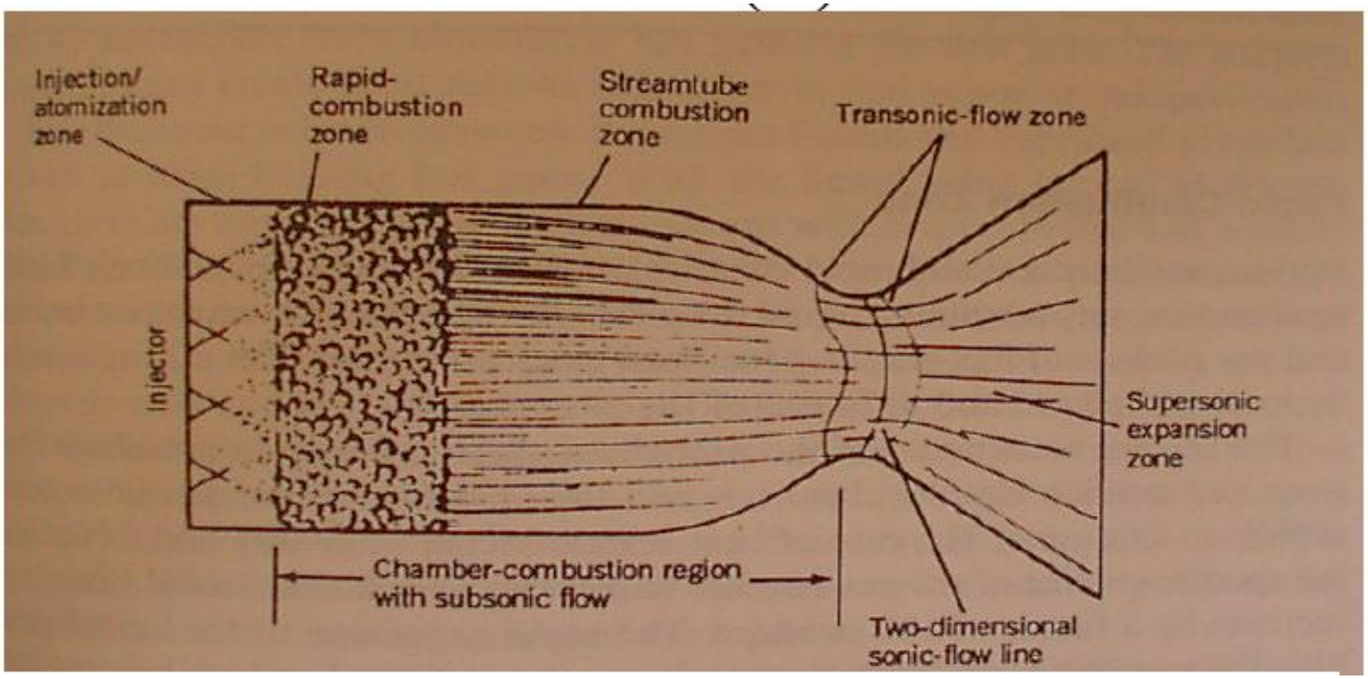
Slow them down for atmospheric reentry

Rocketry Basics



Rocket's basic function is to take mass, add energy, and convert that to thrust.

Rocketry Basics (2)



Combustion is an exothermic chemical reaction. Often an external heat source is required (igniter) to supply the necessary energy to a threshold level where combustion is self sustaining

Propellants that combust spontaneously are referred to as *Hypergolic*

Rocketry Basics (2)

- Combustion Produces High temperature gaseous By-products

- Gases Escape Through Nozzle Throat

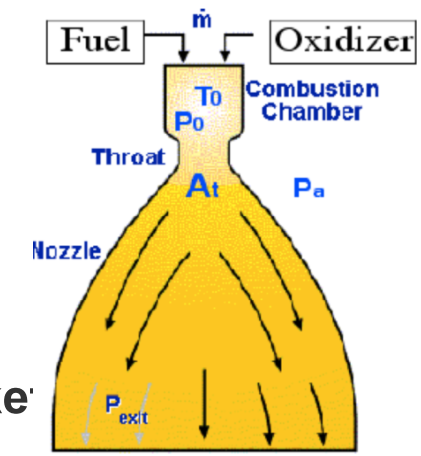
- Nozzle Throat Chokes (maximum mass flow)

- Since Gases cannot escape as fast as they are produced
  … Pressure builds up

- As Pressure Builds .. Choking mass flow grows

- Eventually Steady State Condition is reached

What is a NOZZLE? (1)



• FUNCTION of rocket nozzle is to convert thermal energy in propellants into kinetic energy as efficiently as possible

• Nozzle is substantial part of the total engine mass.

• Many of the historical data suggest that 50% of solid rocket stemmed from nozzle problems.

The design of the nozzle must trade off:

1. Nozzle size (needed to get better performance) against nozzle weight penalty.

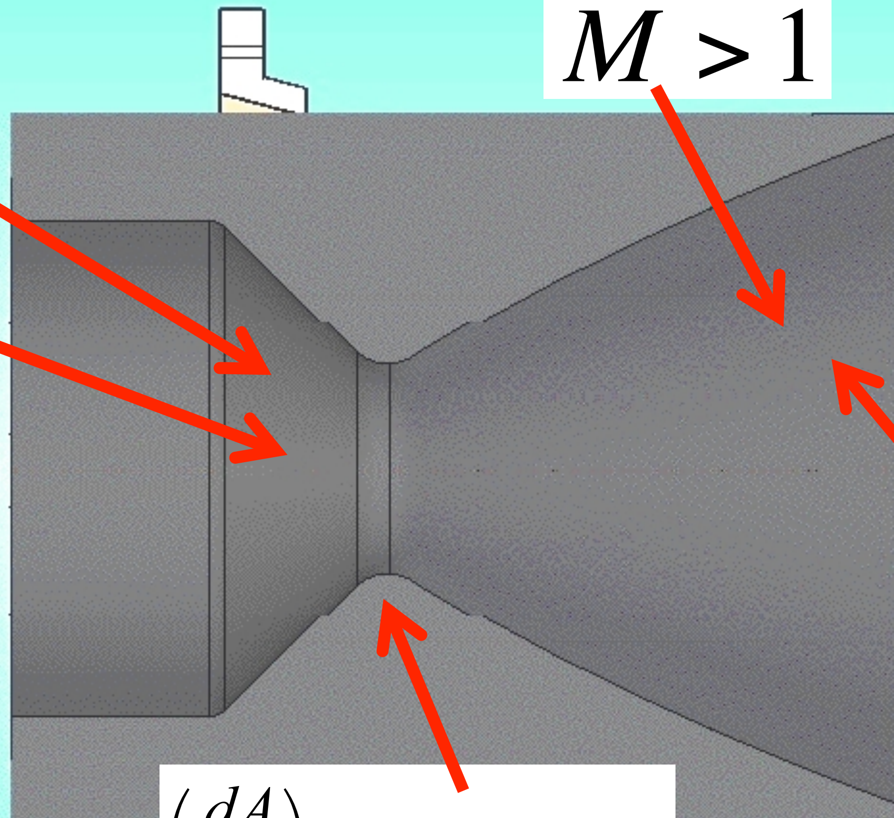
2. Complexity of the shape for shock-free performance vs. cost of fabrication

Why does a rocket nozzle look like this?

$$M < 1$$

$$\left(\frac{dA}{A}\right) < 0$$

$$\left(\frac{dV}{V}\right) > 0$$

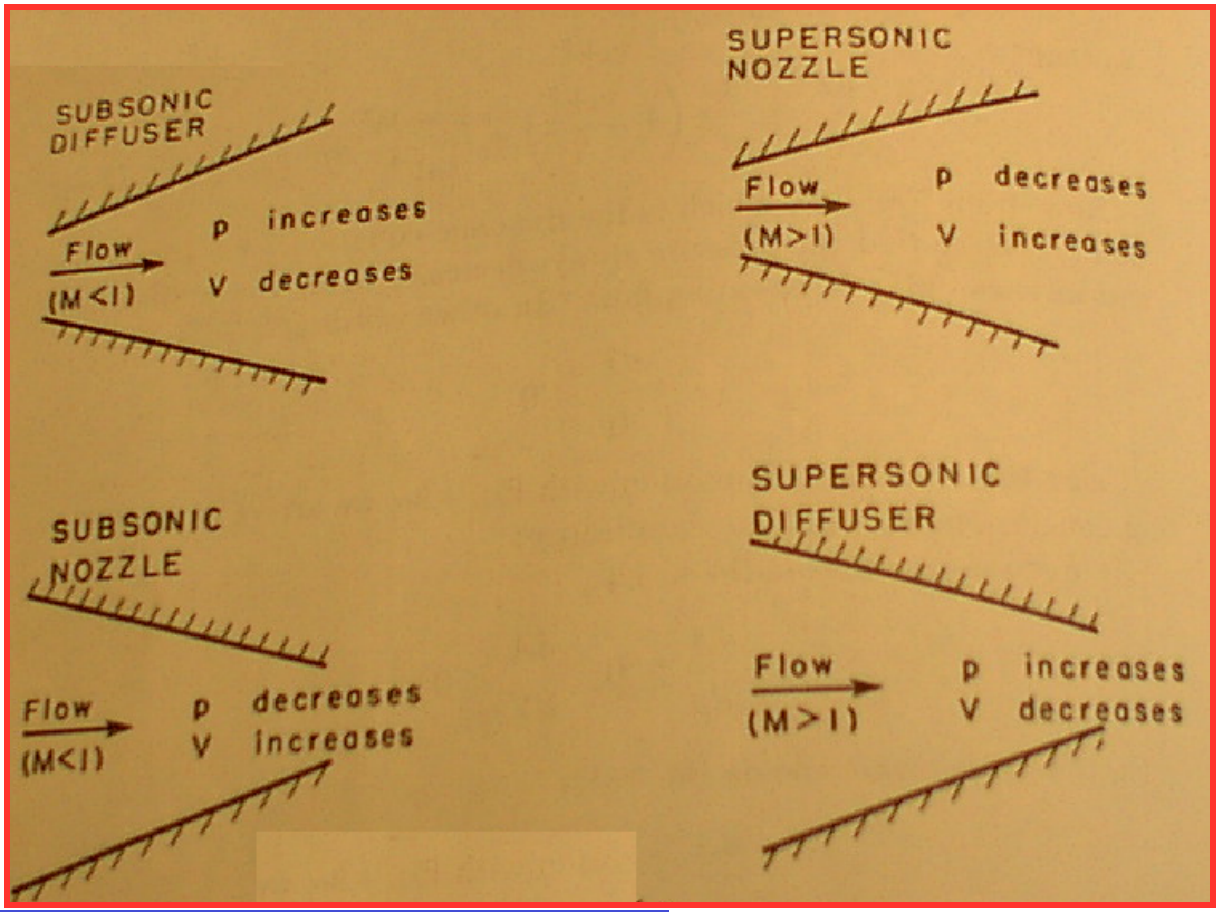


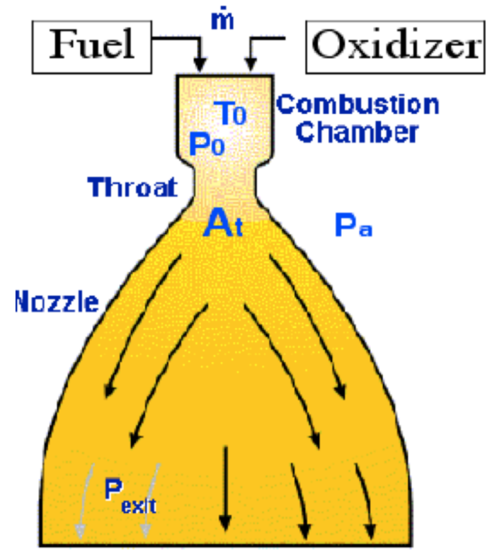
$$M > 1$$

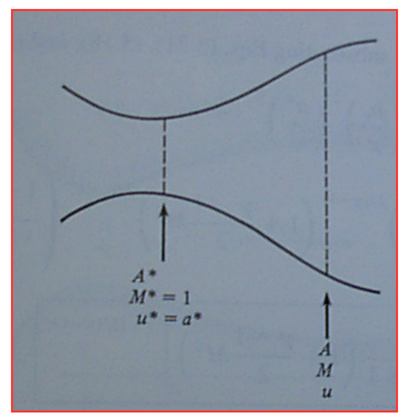
$$\left(\frac{dV}{V}\right) > 0$$

$$\left(\frac{dA}{A}\right) = 0 \;\; .... \;\; M = 1$$

Fundamental Properties of Supersonic and Supersonic Flow



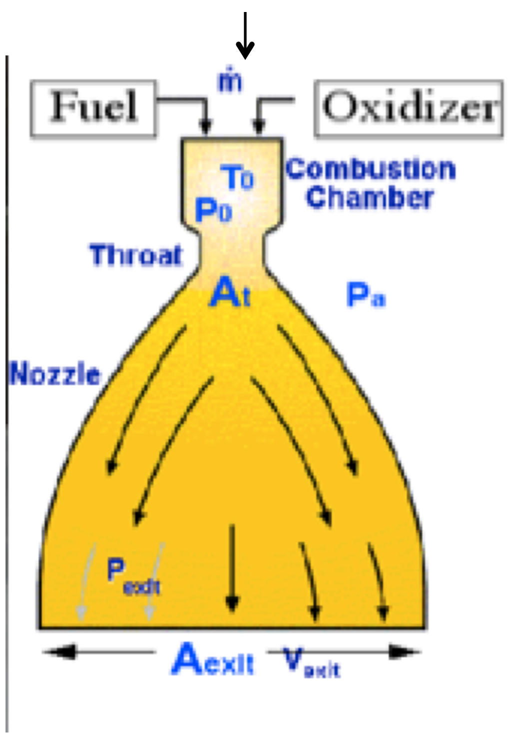




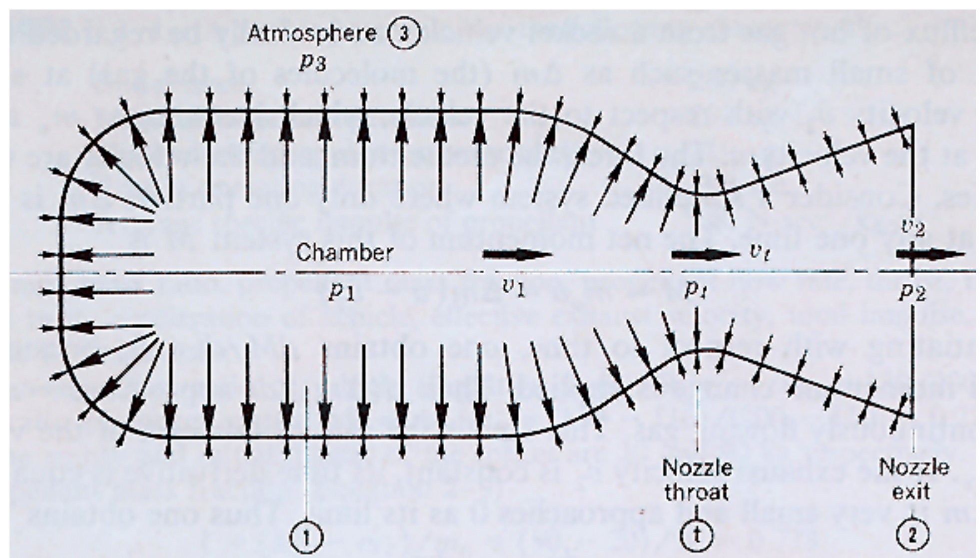
Rocket Thrust Equation

$$F = \dot{m}_e V_e + \left(p_e A_e - p_\infty A_e\right)$$

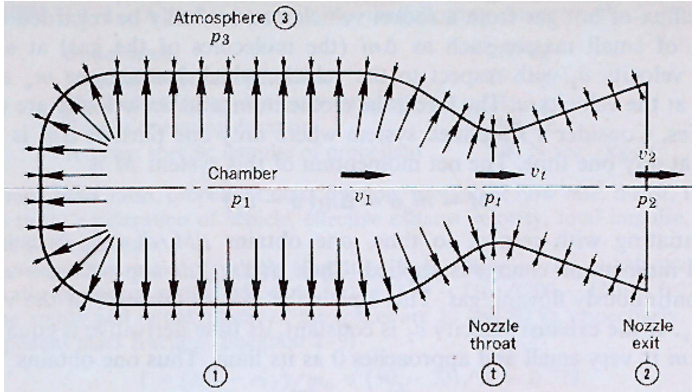
$$\dot{m}_i = 0$$



• Thrust + Oxidizer enters combustion Chamber at ~0 velocity, combustion Adds energy … High Chamber pressure Accelerates flow through Nozzle *Resultant pressure forces produce thrust*



Rocket Thrust Equation (cont'd)



What Produces Thrust?

a) Increase in momentum of the propellant fluid (momentum thrust)

b) Pressure at the exit plane being higher than the outside pressure (pressure thrust).

Where does the thrust act?

In the rocket engine, the force is felt on the nozzle and the combustor walls, and is transmitted through the engine mountings to the rest of the vehicle.

OK … derive the forces acting on the rocket in another way

- *Using Newton’s laws*

Newton

Newton's First Law

• An object at rest stays at rest unless acted on by an external force

Concept of *inertia* ... the resistance to changes in motion

"No forces here!"

• An object in motion, stays in motion in a straight line unless acted on by an external force

"No forces here!"


Newton

Newton's Second Law

"The acceleration produced by a force is directly proportional to the force and inversely proportional to the mass which is being accelerated"

$$\text{Newton's Second law} \qquad \bar{F} = m\,\bar{a} = m\frac{d\bar{V}}{dt}$$

- **But what happens when the mass is no longer constant?**
- **Newton recognized that the early formulation of second law was incomplete and modified the formulation accordingly**

$$\boxed{\bar{F} = \frac{d\left[m\bar{V}\right]}{dt} = \frac{d\left[\bar{P}\right]}{dt}} \Rightarrow \bar{P} \equiv m\bar{V} \left[\text{"momentum vector"}\right]$$

Newton

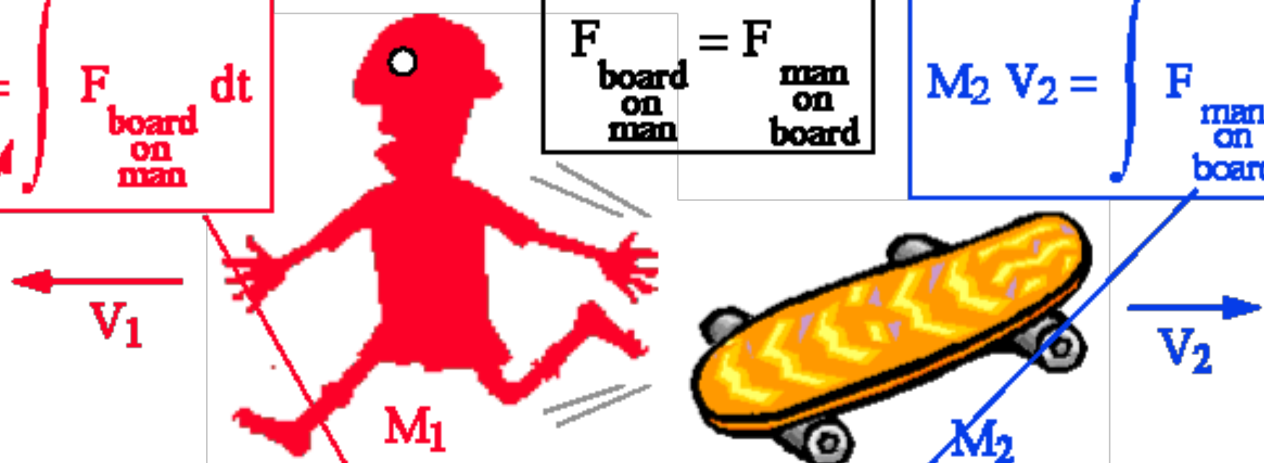
Newton's Third Law = Conservation of momentum

- *For every action, there is an equal and oppposite RE-action*

$$F_{\substack{\text{board}\\ \text{on}\\ \text{man}}} = F_{\substack{\text{man}\\ \text{on}\\ \text{board}}}$$

$$M_1 V_1 = \int F_{\substack{\text{board}\\ \text{on}\\ \text{man}}} dt$$

$$M_2\, V_2 = \int F_{\substack{\text{man}\\ \text{on}\\ \text{board}}} dt$$



Man jumps off skate board ... Initially at rest

$$\overline{F} = \frac{d\left[m\overline{V}\right]}{dt}$$

Newton II

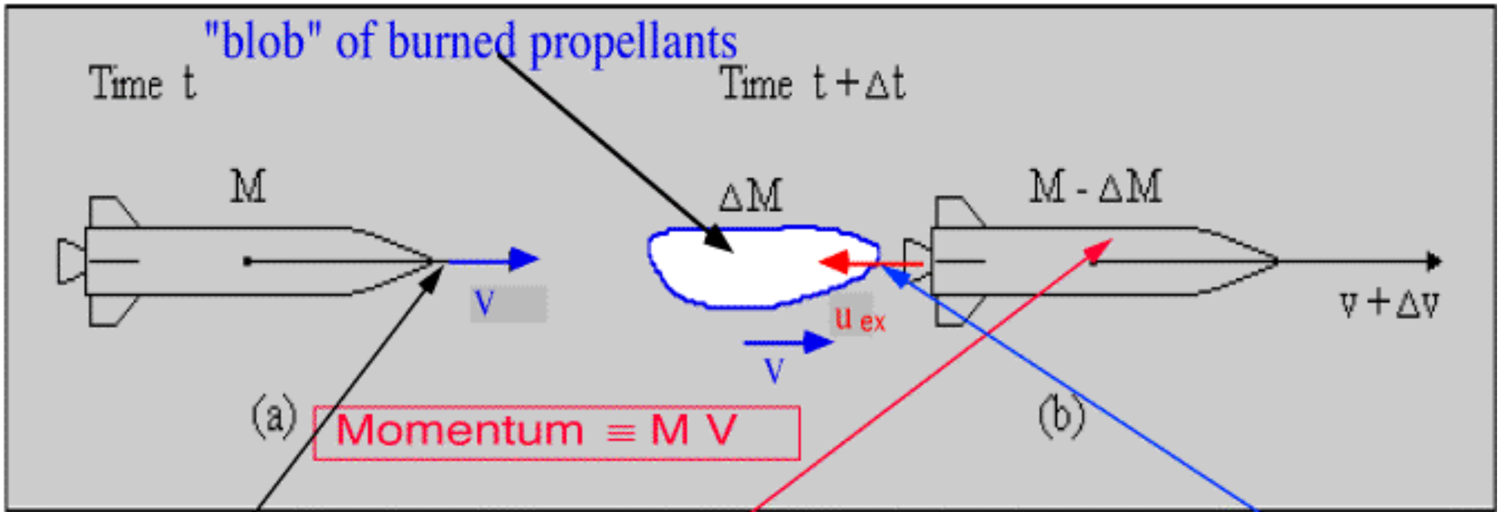
$$M_1 V_1 = M_2 V_2$$

Newton

Newton's Third Law =

Conservation of momentum

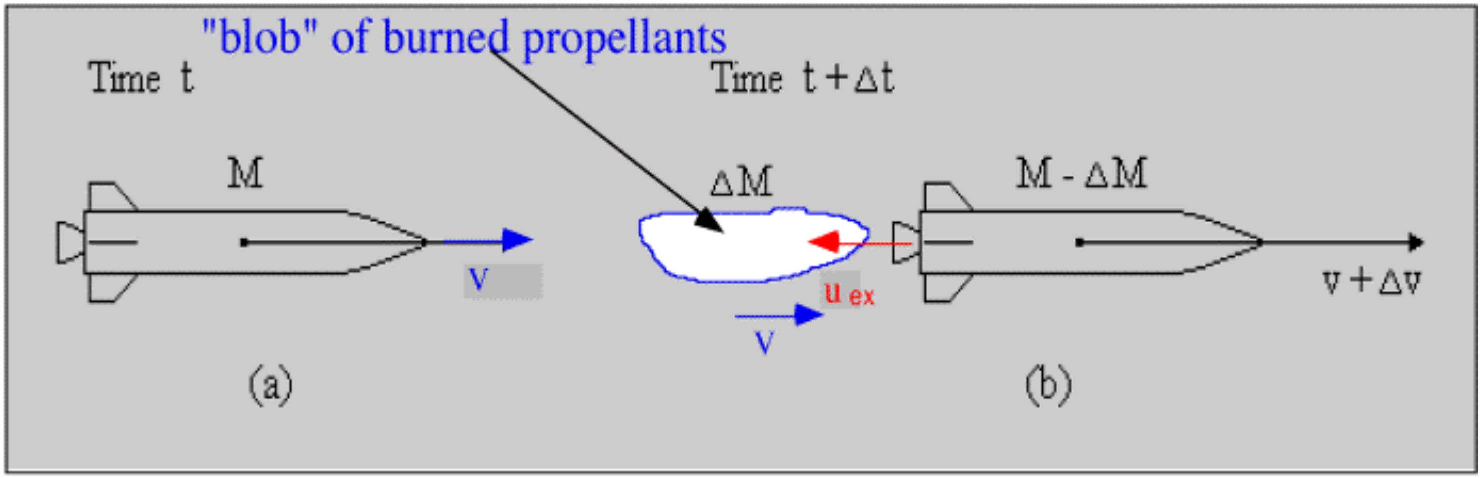
Look at a rocket in horizontal flight



$$M V = [M - \Delta M][V + \Delta V] + \Delta M [V - U_{ex}]$$

$$\sum \overline{F}_{external} = \frac{d\overline{P}}{dt} = \frac{d[M\overline{V}]}{dt}$$

Newton's Third Law (cont'd)



$$M\,V = [M - \Delta M]\,[V + \Delta V] + \Delta M\,[V - U_{ex}]$$

$$\Downarrow$$

$$\underline{M\,V} = \underline{M\,V} - \underline{\Delta M\,V} + M\,\Delta V - \underline{\Delta M\,\Delta V} + \underline{\Delta M\,V} - \Delta M\,U_{ex}$$

$$\Downarrow$$

$$M\,\Delta V = \Delta M\,U_{ex} + \Delta M\,\Delta V$$

Newton's Third Law (cont'd)

- Dividing by Δt and evaluating limit $\{\Delta M , \Delta V , \Delta t\} \to 0$

$$\left[\frac{M \frac{\Delta V}{\Delta t} = \frac{\Delta M}{\Delta t} U_{ex} + \Delta M \frac{\Delta V}{\Delta t}}{\lim \{ \Delta M, \Delta V, \Delta t \} \Rightarrow 0} \right] =$$

We shrink time
As small as possible

$$\Downarrow$$

Engine massflow

$$F = M \frac{dV}{dt} = \frac{dM}{dt} U_{ex}$$

Reaction Force on Rocket

Engine thrust equation

Compare the results

Using Newton's laws

$$F = M\frac{dV}{dt} = \frac{dM}{dt}U_{ex}$$

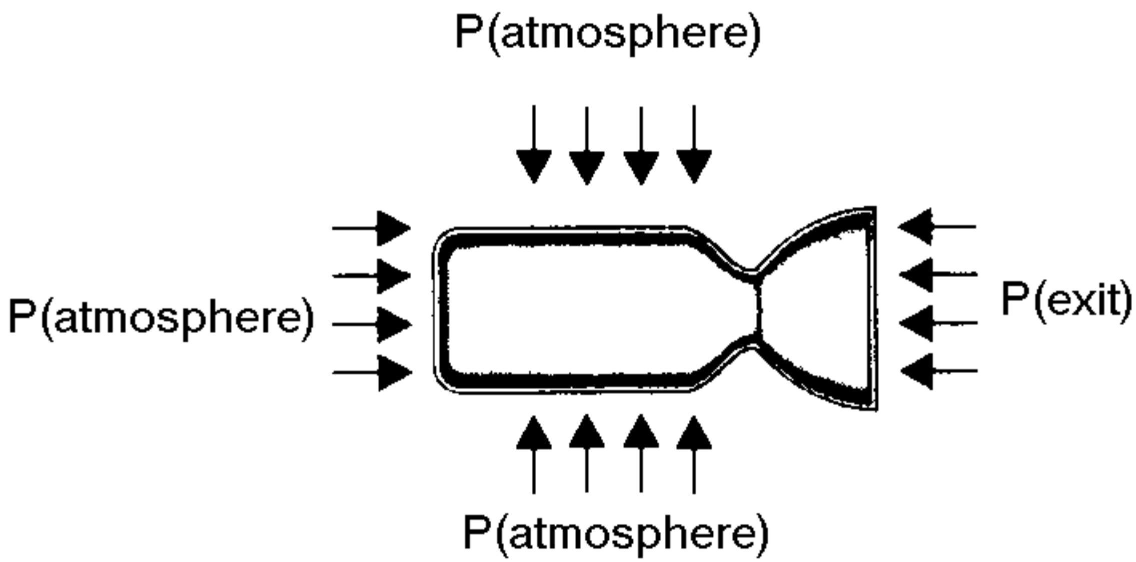
Using Fluid Conservation laws

$$\boxed{F = \dot{m}_e V_e + \left(p_e A_e - p_\infty A_e\right)}$$

$$U_{ex} = V_e + \frac{p_e A_e - p_\infty A_e}{\dot{m}_e} \equiv C_e$$

• *Effective Exhaust Velocity*

Pressure Thrust



- Pressure is identical from all directions except for the Area of the exit nozzle. This pressure difference produces a thrust (which may be negative or positive.)

Specific Impulse

• Specific Impulse is a scalable characterization of a rocket's Ability to deliver a certain *(specific)* impulse for a given weight of propellant

$$I_{sp} = \frac{\mathrm{I}_{mpulse}}{g_0 M_{propellant}} = \frac{\int_0^t F_{thrust} dt}{g_0 \int_0^t \dot{m}_{propellant}\, dt}$$

$$\rightarrow g_0 = 9.806 \frac{m}{\sec^2} (mks)$$

Mean specific impulse

Specific Impulse (cont'd)

- At a constant altitude, with
Constant mass flow through engine

$$I_{sp} = \frac{\mathrm{I}_{mpulse}}{g_0 M_{propellant}} = \frac{\int_0^t F_{thrust} dt}{g_0 \int_0^t \dot{m}_{propellant} \, dt} = \frac{F_{thrust}}{g_0 \, \dot{m}_{propellant}}$$

- *Instantaneous specific impulse*

Specific Impulse (cont'd)

- Historically, I_{sp} was measured in units of *seconds*

$$I_{sp} = \frac{|\bar{F}|}{\dot{m}_p} \Rightarrow \text{(English Units)} \frac{\cancel{\text{lbf}}}{\cancel{\text{lbm}}/\text{sec}} \approx \text{seconds, right?}$$

Wrong! *lbm*s are not a fundamental unit for mass

(Slugs are the fundamental english unit of mass)

$$I_{sp} = \frac{|\bar{F}|}{\dot{m}_p} \Rightarrow \text{(MKS units)} \frac{\text{Nt}}{\text{kg/sec}} \approx \frac{\text{kg-m/sec}^2}{\text{kg/sec}} \approx \frac{\text{m}}{\text{sec}}$$

- Since most engine manufacturers still give I_{sp} in *seconds* -- we correct for this by letting

$$\boxed{I_{sp} \equiv \frac{|\bar{F}|}{g_0\, \dot{m}_p}} \Rightarrow g_0 \approx 9.81 \frac{\text{m}}{\text{sec}^2} \text{ [acceleration of gravity at sea level]}$$

English Units -- use *slugs* not *lbms* !

$$\text{(MKS units)} \quad \frac{\dfrac{\text{Nt}}{\text{kg/sec}}}{\dfrac{\text{m}}{\text{sec}^2}} \approx \frac{\dfrac{\text{kg-m/sec}^2}{\text{kg/sec}}}{\dfrac{\text{m}}{\text{sec}^2}} \approx \text{sec}$$

Specific Impulse *(cont'd)*

$$I_{sp} = \frac{1}{g_0} \frac{F_{thrust}}{\dot{m}_{propellant}} \rightarrow \dot{m}_e \equiv \dot{m}_{propellant} \rightarrow$$

$$I_{sp} = \frac{1}{g_0} \left[V_e + \frac{p_e A_e - p_\infty A_e}{\dot{m}_e} \right] \equiv \frac{C_e}{g_0}$$

"Units ~ seconds"

- *Effective Exhaust Velocity*

Specific Impulse (cont'd)

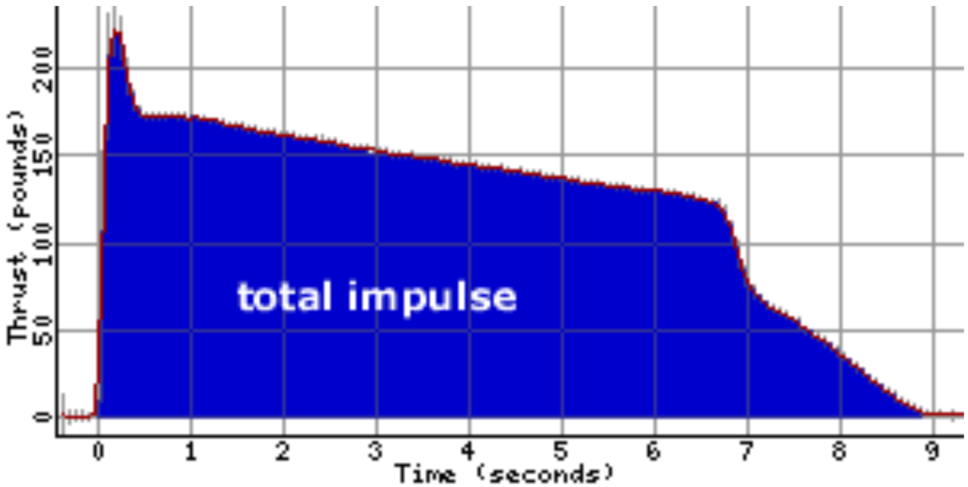
- Example

- **Aparticular engine with a specific impulse of 300 sec. will produce one pound (force!) of thrust for 300 seconds -- or**

- **Another engine with a specific impulse of 300 sec. may produce 300 pounds (force!) of thrust for 1 second**

Specific Impulse (cont'd)

- Look at total impulse for a rocket

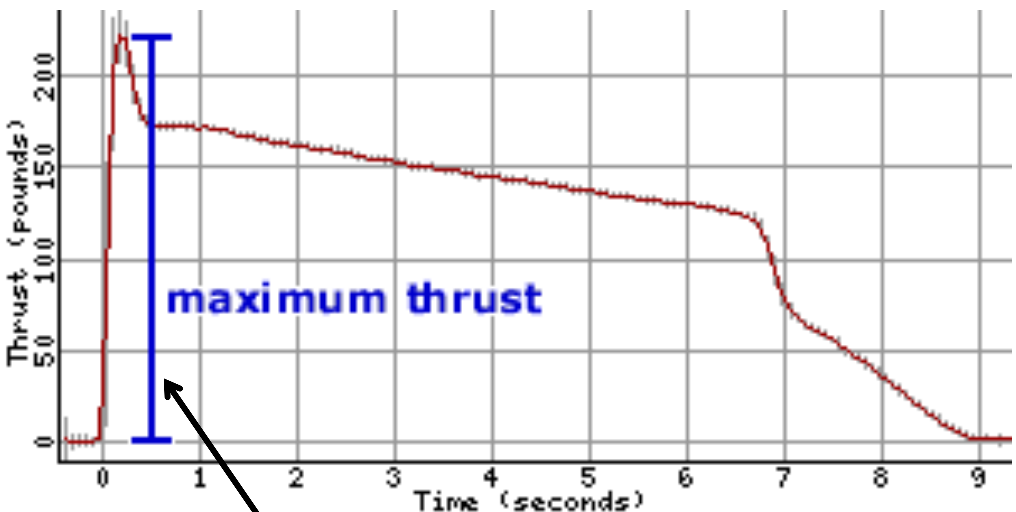


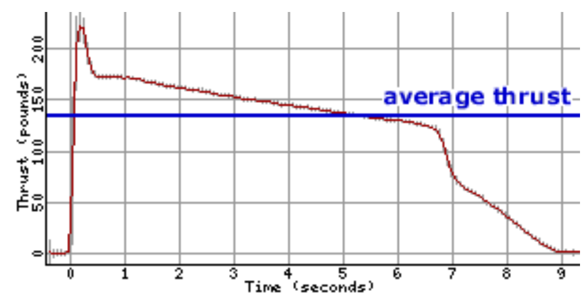
- Mean I_{sp}

$$I_{sp} = \frac{\mathrm{I}_{mpulse}}{g_0 M_{propellant}} = \frac{\int_0^t F_{thrust}\,dt}{g_0 \int_0^t \dot{m}_{propellant}\,dt}$$

Specific Impulse (cont'd)

- Look at instantaneous impulse for a rocket





Instantaneous $\dot{m}_{propellant}$

$$I_{sp} = \frac{1}{g_0}\frac{F_{thrust}}{\dot{m}_{propellant}}$$

- *Not necessarily the same*

Example 2

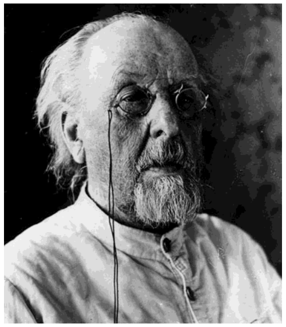
- A man is sitting in a rowboat throwing bricks over the stern. Each brick weighs 5 lbs, he is throwing six bricks per minute, at a velocity of 32 fps. What is his thrust and I_{sp}?

$$F = \dot{m}_{propellant} C_e = \frac{6_{bricks}}{1\,\mathrm{min}} \times 5 \frac{lbm}{brick} \times 32 \frac{ft}{\sec} \times \frac{1\,\mathrm{min}}{60\,\sec} =$$

$$\frac{6 \times 5 \times 32}{60} \frac{lbm - ft}{\sec^2} \; \underline{...ooops...need....g_c}$$

$$\boxed{F = \frac{6 \times 5 \times 32}{60} \frac{lbm - ft}{\sec^2} \times \frac{1}{32.1742 \dfrac{lbm - ft}{lbf - \sec^2}} =}$$

$$\boxed{I_{sp} = \frac{F}{\dot{m}_{propellant} g_0} = \frac{0.497\,lbf \times 32.1742 \dfrac{ft}{\sec^2}}{\dfrac{6_{bricks}}{1\,\mathrm{min}} \times 5 \dfrac{lbm}{brick} \times \dfrac{1\,\mathrm{min}}{60\,\sec} \times 32.1742 \dfrac{lbm - ft}{lbf - \sec^2}} = 0.994\,\sec}$$



How Much Fuel? "The Rocket Equation"

Conservation of momentum leads to the so-called rocket equation, which trades off exhaust velocity with payload fraction. Based on the assumption of short impulses with coast phases between them, it applies to chemical and nuclear-thermal rockets. First derived by Konstantin Tsiolkowsky in 1895 for straight-line rocket motion with constant exhaust velocity, it is also valid for elliptical trajectories with only initial and final impulses.

Re-visit Newton's Third Law (cont'd)

- Dividing by Δt and evaluating limit $\{\Delta M , \Delta V , \Delta t\}$ -> 0

$$\left[\frac{M \frac{\Delta V}{\Delta t} = \frac{\Delta M}{\Delta t} U_{ex} + \Delta M \frac{\Delta V}{\Delta t}}{\lim \{ \Delta M, \Delta V, \Delta t \} \Rightarrow 0} \right] =$$

$$\Downarrow$$

$$M \frac{dV}{dt} = \frac{dM}{dt} U_{ex} = \dot{m}_{propellant} C_e$$

Rocket Equation (cont'd)

$$M\frac{dV}{dt} = \dot{m}_{propellant} C_e = g_0 I_{sp} \dot{m}_{propellant} \rightarrow \dot{m}_{propellant} = -\frac{dM}{dt}$$

$$\frac{dV}{dt} = g_0 I_{sp} \frac{-\frac{dM}{dt}}{M} \rightarrow \boxed{dV = -g_0 I_{sp} \frac{dM}{M}}$$

M -> rocket mass

- Assuming constant I_{sp} and burn rate …. integrating over a burn time t_{burn}

$$V_{final} - V_0 = -g_0 I_{sp} \ln\left[M_{final}\right] + g_0 I_{sp} \ln\left[M_0\right] = g_0 I_{sp} \ln\left[\frac{M_0}{M_{final}}\right]$$

$$V_{final} = V_0 + g_0 I_{sp} \ln\left[\frac{M_0}{M_{final}}\right]$$

Anatomy of the Rocket Equation

- Consider a rocket burn of duration t_{burn}

Initial Mass

$$V_{final} = V_0 + g_0 I_{sp} \ln\left[\frac{M_0}{M_{final}}\right]$$

Initial Velocity

Final Mass

Final Velocity

$$M_{final} = M_0 - \underline{\dot{m}_{propellant} \times t_{burn}}$$

Consumed propellant

Anatomy of the Rocket Equation (cont'd)

- Or rewriting

$$\Delta V = V_{final} - V_0 \qquad M_0 = M_{final} + m_{propellant}$$

$$\Delta V = g_0 I_{sp} \ln\left[1 + \frac{m_{propellant}}{M_{final}}\right] = g_0 I_{sp} \ln\left[1 + P_{mf}\right]$$

$$P_{mf} = \text{"propellant mass fraction"}$$

- Sometimes $\dfrac{m_{propellant}}{M_{final} + m_{propellant}}$ Is also called propellant mass Fraction or "load mass fraction"

"Propellant Mass Fraction"

- **How do we compute the amount of propellant required?**

$$\frac{M_0}{M_{final}} = \frac{M_{dry} + M_{payload} + M_{\substack{fuel \\ + oxidizer}}}{M_{dry} + M_{payload}} = 1 + P_{mf}$$

$$\Downarrow$$

Load mass fraction

$$L_{mf} \equiv \frac{M_{\substack{fuel \\ + oxidizer}}}{M_{dry} + M_{payload} + M_{\substack{fuel \\ + oxidizer}}} = \frac{P_{mf}}{1 + P_{mf}}$$

Better to work with

Propellant mass fraction

$$P_{mf} \equiv \frac{M_{\substack{fuel \\ + oxidizer}}}{M_{dry} + M_{payload}}$$

$$\Delta V_{burn} = g_0 \, I_{sp} \, ln\left[\left(\frac{M_{initial}}{M_{final}}\right)_{burn}\right] = g_0 \, I_{sp} \, ln\left[\left(1 + P_{mf}\right)_{burn}\right]$$

Relating Delta V delivered by a rocket burn to propellant Mass fraction

"Propellant Mass Fraction"

Propellant Budgeting Equation

- **Solving for P_{mf}**

$$\left(P_{mf}\right)_{burn} = e^{\left[\frac{\Delta V_{burn}}{g_0 I_{sp}}\right]} - 1$$

- **Mass of Fuel and oxidizer required for a burn to give a specified ΔV**

$$M_{\substack{fuel \\ + oxidizer}} = \left[M_{dry} + M_{payload}\right] \left[e^{\left[\frac{\Delta V_{burn}}{g_0 I_{sp}}\right]} - 1 \right]$$

Ramifications of "the Rocket Equation"

- **Any increase in ΔV must come from increasing I_{sp} or P_{mf}**

  -- First case (I_{sp}) requires adopting a more efficient propulsion system

  --Second case (mass fraction) requires reduction of the structural mass or reduced payload (for same vehicle weight)

  -- Can't just add more propellant -- because that means bigger tanks and the dry weight rises proportionately

- **Reducing payload to obtain more ΔV is a bad-tradeoff**

Ramifications of "the Rocket Equation" (cont'd)

- **Reducing Structural weight to increase P_{mf} is a viable option** -- but it comes at a high price (adds inherent risks)

  -- lighter vehicle tend to damage more easily
  -- reduced redundancy in critical sub-systems
  -- there are limits as to how light a vehicle can be

- **Best Option is to increase efficiency of the propulsion system (increase I_{sp})**

  -- "easier said than done" -- requires significant advances in propulsion technology

Ramifications of "the Rocket Equation"(cont'd)

Specific Impulse (revisited)

- For chemical Rockets, I_{sp} depends on the type of fuel/oxydizer used

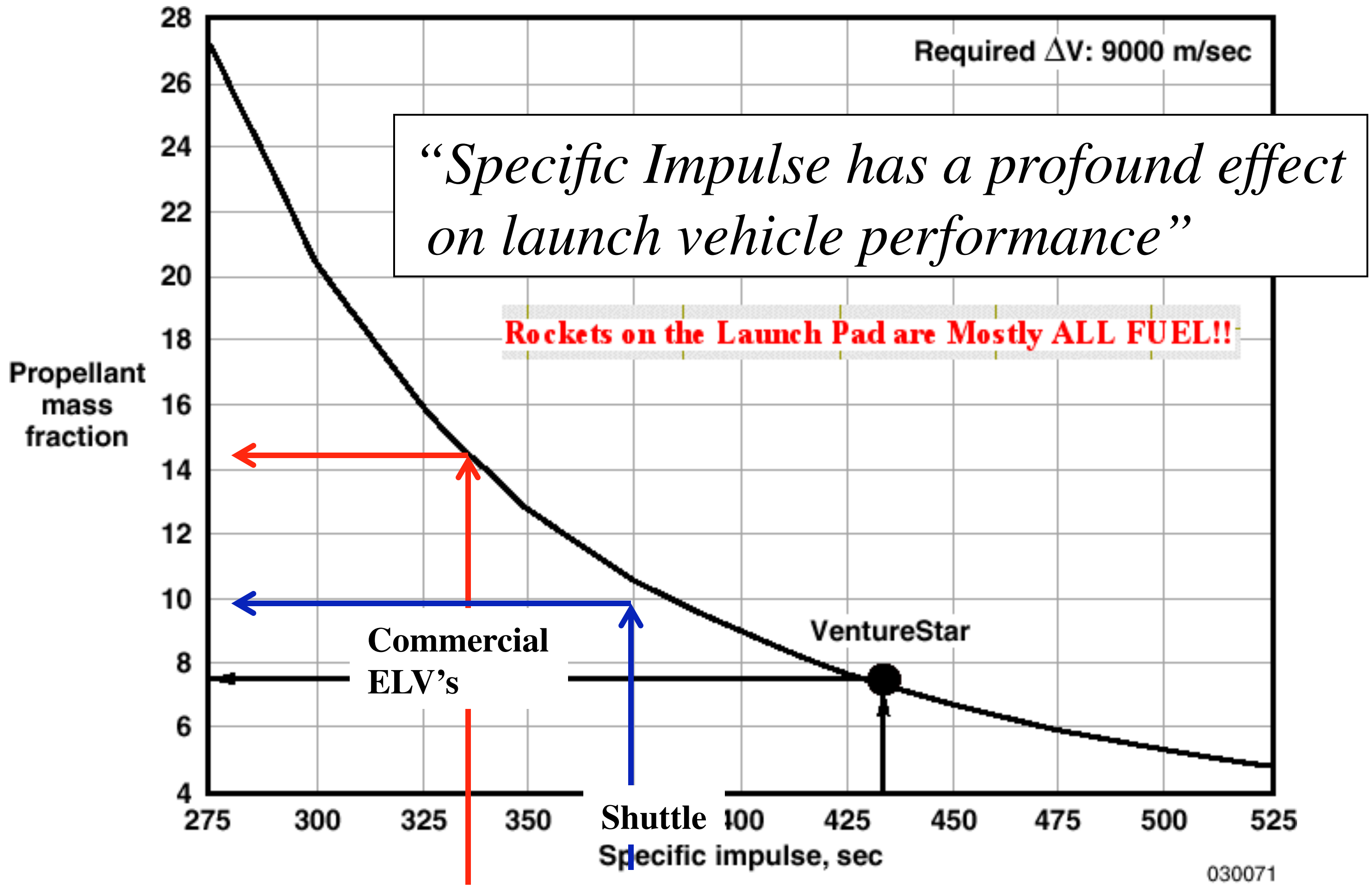
Vacuum ISP

Fuel	Oxidizer	Isp (s)
Liquid propellants		
Hydrogen (LH2)	Oxygen (LOX)	450
Kerosene (RP-4)	Oxygen (LOX)	260
Monomethyl hydrazine	Nitrogen Tetraoxide	310
Solid propellants		
Powered Al	Ammonium Perchlorate	270

Specific Impulse (revisited)

450 sec is "best you can get" with chemical rockets

"Propellant Mass Fraction"Ramifications of the Rocket Equation



Approximate SSTO propellant mass fraction required for low earth orbit.

ΔV for a Vertically Accelerating Vehicle

- **Rocket Equation originally derived for straight and level travel**
- **What happens for vertically climbing rocket ?**

For example .. Look at a hovering vehicle … Lunar Lander
During hover, change in velocity is zero .. So according to …

$$\Delta V = g_0 I_{sp} \cdot \ln\left[1 + \frac{m_{propellant}}{M_{dry}}\right]$$

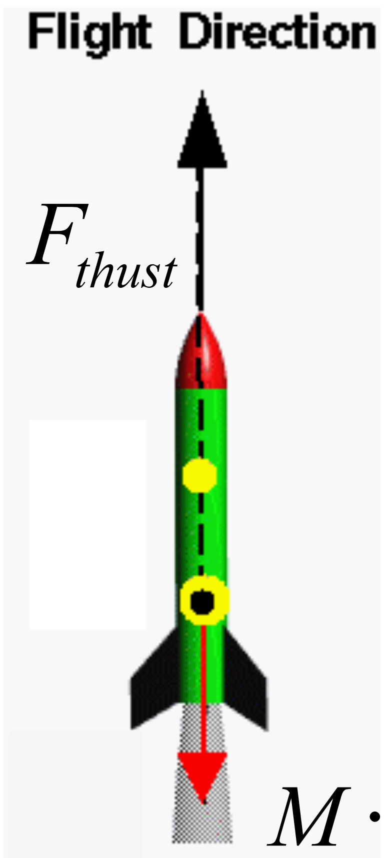
$$\rightarrow \left| \begin{array}{l} \Delta V = 0 \\ \rightarrow \ln\left[1 + \dfrac{m_{propellant}}{M_{dry}}\right] \end{array} \right. = 0 \rightarrow \boxed{m_{propellant} = 0}$$

We burn no gas! Of course this result is absurd! Need to account for ***"gravity losses"***

ΔV for a Vertically Accelerating Vehicle

•Look at 1-D equations of motion (2) for vertically accelerating rocket

•Ignore Aerodynamic Drag, assume constant g_0



$M \cdot g$

Instantaneously :

$$m\frac{dV}{dt} = F_{thrust} - m \cdot g_0 \rightarrow \frac{dV}{dt} = \frac{F_{thrust}}{m} - g_0$$

$$for\ constant\ thrust,\ m(t) = M_{initial} - \dot{m}\, t$$

$$\rightarrow \frac{dV}{dt} = \frac{F_{thrust}}{\left(M_{initial} - \dot{m} \cdot t\right)} - g_0$$

$$\rightarrow V_{t_{burn}} = V_0 + \int_0^{t_{burn}} \frac{F_{thrust}}{\left(M_{initial} - \dot{m} \cdot \tau\right)} d\tau - g_0 \cdot t_{burn}$$

Velocity @ Motor Burnout

ΔV for a Vertically Accelerating Vehicle (3)

$$\int_0^{t_{burn}} \frac{d\tau}{\left(M_{initial} - \dot{m}\cdot\tau\right)} = \frac{1}{\dot{m}} \int_{t=0}^{t=t_{burn}} \frac{-du}{u} = \frac{1}{\dot{m}}\left[\ln\left(M_0\right) - \ln\left(M_0 - \dot{m}\cdot t_{burn}\right)\right]$$

$$\rightarrow \boxed{\begin{array}{c} M_0 = M_{initial} \\ M_{initial} - \dot{m}\cdot t_{burn} = M_{final} \end{array}}$$

•Evaluate Integral

$$\int_0^{t_{burn}} \frac{d\tau}{\left(M_{initial} - \dot{m}\cdot\tau\right)} = \frac{1}{\dot{m}}\left[\ln\left(M_{initial}\right) - \ln\left(M_{final}\right)\right] = \frac{1}{\dot{m}}\left[\ln\left(\frac{M_{initial}}{M_{final}}\right)\right]$$

$\rightarrow$ *Substitute back in*

$$\boxed{\Delta V_{t_{burn}} = V_{t_{burn}} - V_0 = \frac{F_{thrust}}{\dot{m}}\left[\ln\left(\frac{M_{initial}}{M_{final}}\right)\right] - g_0\cdot t_{burn}}$$

ΔV for a Vertically Accelerating Vehicle (4)

•Apply earlier fundamental definitions

$$\Delta V_{t_{burn}} = \frac{F_{thrust}}{\dot{m}}\left[\ln\left(\frac{M_{initial}}{M_{final}}\right)\right] - g_0 \cdot t_{burn}$$

$$\frac{M_{initial}}{M_{final}} = \frac{M_{final} + m_{propellant}}{M_{final}} = 1 + P_{mf}$$

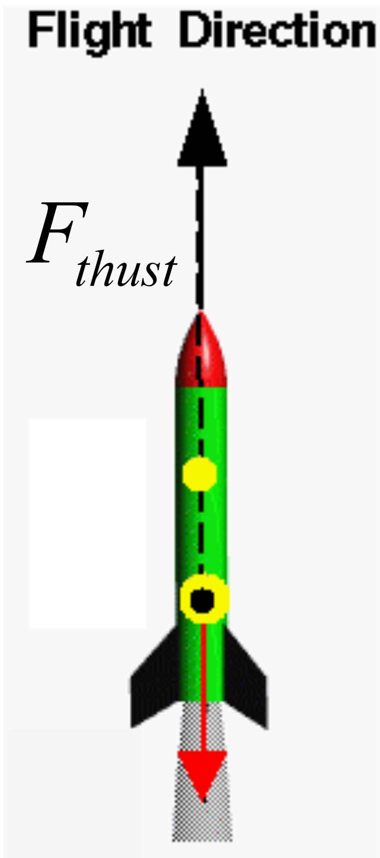
$$\frac{F_{thrust}}{\dot{m}} = g_0 \cdot I_{sp} \qquad t_{burn} \approx \frac{g_0 \cdot I_{sp} \cdot m_{propellant}}{F_{thrust}}$$

$$\boxed{\Delta V_{t_{burn}} = g_0 \cdot I_{sp} \cdot \left[\ln\left(1 + P_{mf}\right)\right] - g_0 \cdot t_{burn}}$$

"gravity loss" for Vertical acceleration

ΔV for a Vertically Accelerating Vehicle (7)

•*Summary*



$M \cdot g$

$$\boxed{\Delta V_{t_{burn}} = g_0 \cdot I_{sp} \left[\ln\left(1 + P_{mf}\right)\right] - g_0 \cdot t_{burn}}$$

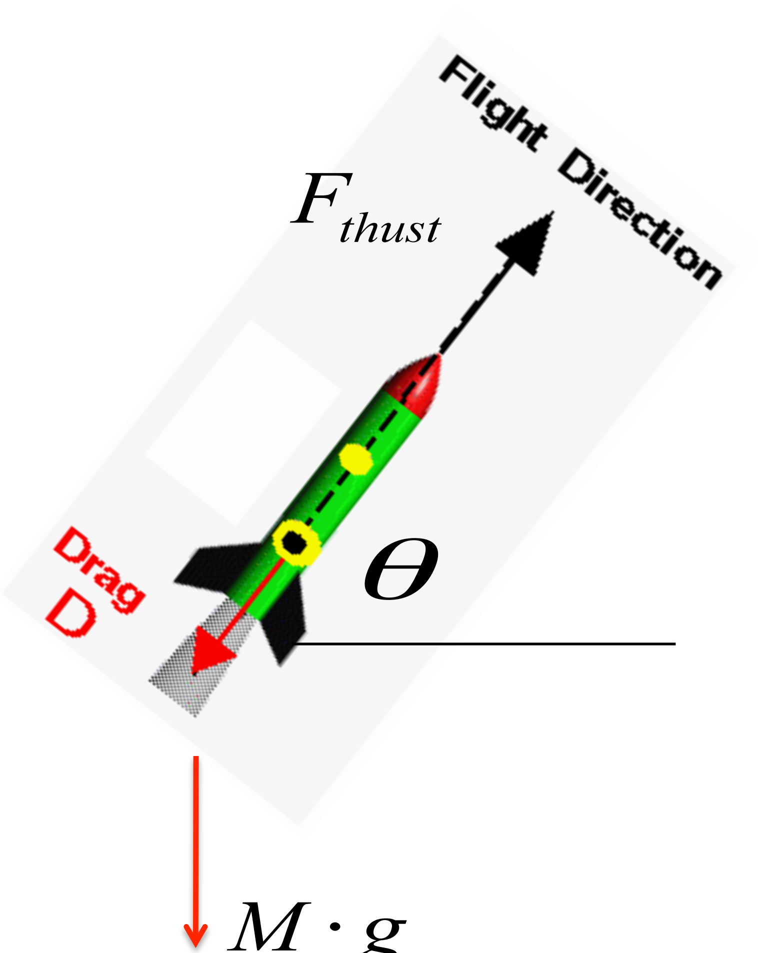
$\rightarrow$

$$P_{mf} = \frac{M_{propellant}}{M_{final}}$$

$$t_{burn} = \frac{g_0 \cdot I_{sp} \cdot m_{propellamt}}{F_{thrust}}$$

Or More Generally …..

$$\theta \rightarrow \text{"}Pitch\ Angle\text{"}$$



$$(\Delta V)_{t_{burn}} = g_0 \cdot I_{sp} \cdot \ln\left(1 + P_{mf}\right) - \int_0^{T_{burn}} g(t) \cdot \sin\theta \cdot dt$$

Gravity Losses

Available ΔV … for given mass fraction

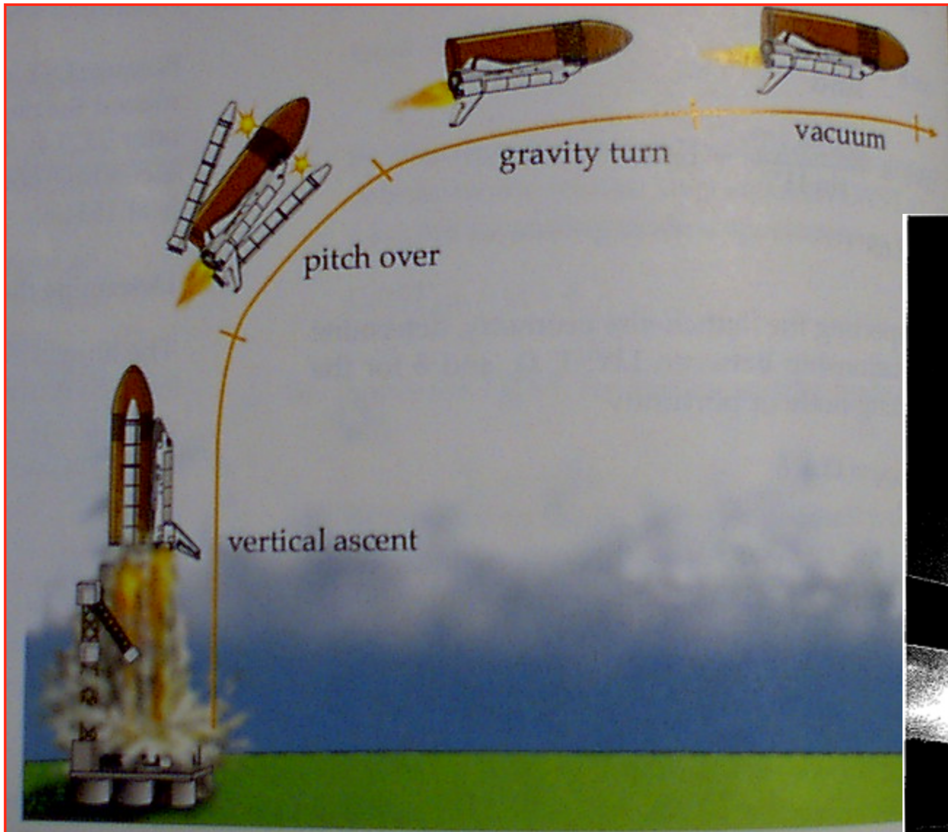
$$\left(\Delta V\right)_{t_{burn}} =$$

$$g_0 \cdot I_{sp} \cdot \ln\left(1 + P_{mf}\right) -$$

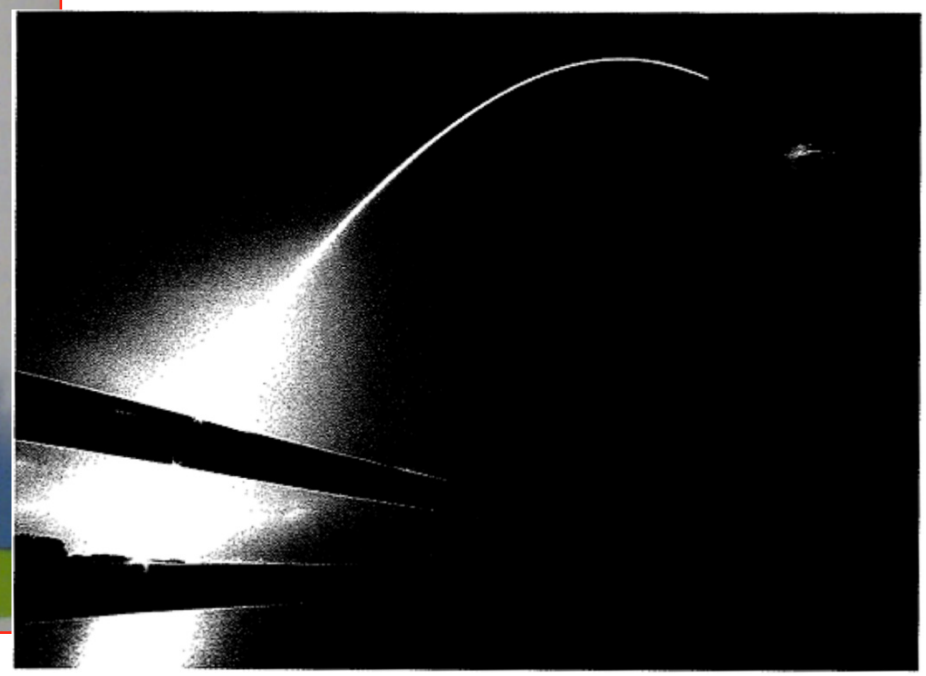
$$\int_0^{T_{burn}} g(t) \cdot \sin\theta \cdot dt \rightarrow \boxed{\begin{array}{l} Path\ dependent \\ Velocity Loss \end{array}}$$

$$\boxed{P_{mf} = \frac{m_{propellant}}{m_{final}}}$$

Why Does this “Turn” Occur at Launch?



Phases of Launch Vehicle Ascent. During ascent a launch vehicle goes through four phases—vertical ascent, pitch over, gravity turn, and vacuum.



Gravity-turn maneuver of an ascending Delta II rocket with Messenger spacecraft on August 3, 2004.


Questions??