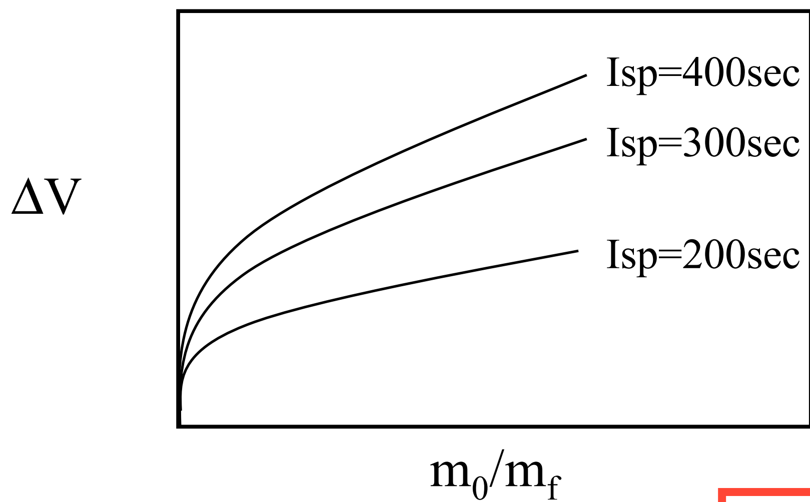
Introduction to Orbital Mechanics: ΔV, the Conic Sections, and Kepler's First Law

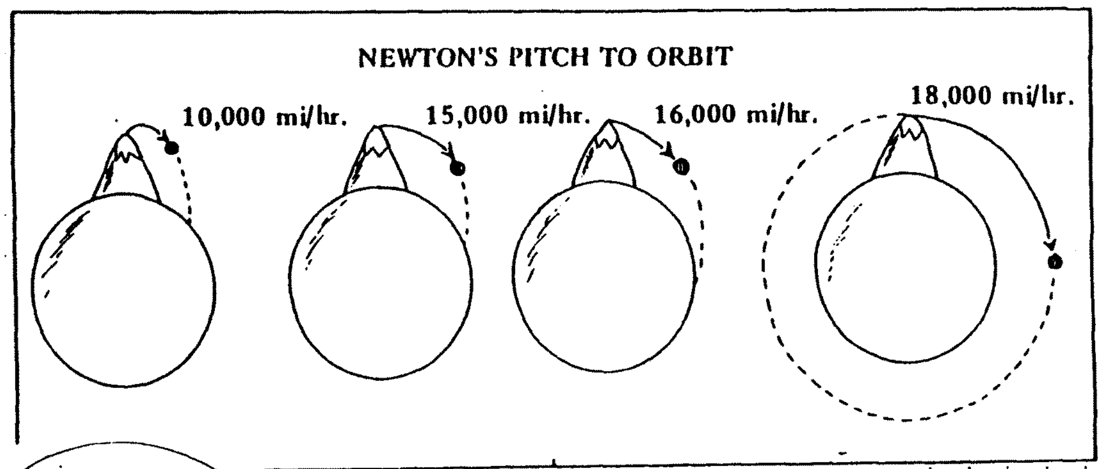
" its where you are AND how fast you are going"

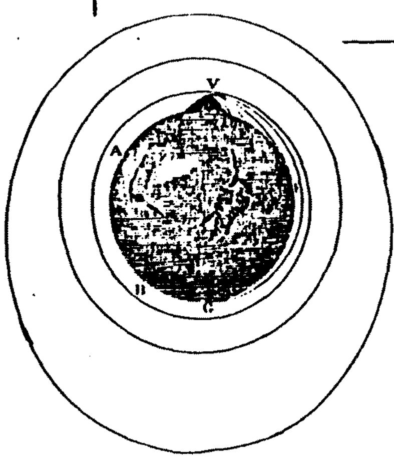


Sutton and Biblarz, Chapter 4

Taylor Chapter 2

Example 1:
Isaac Newton explains how to launch a Satellite

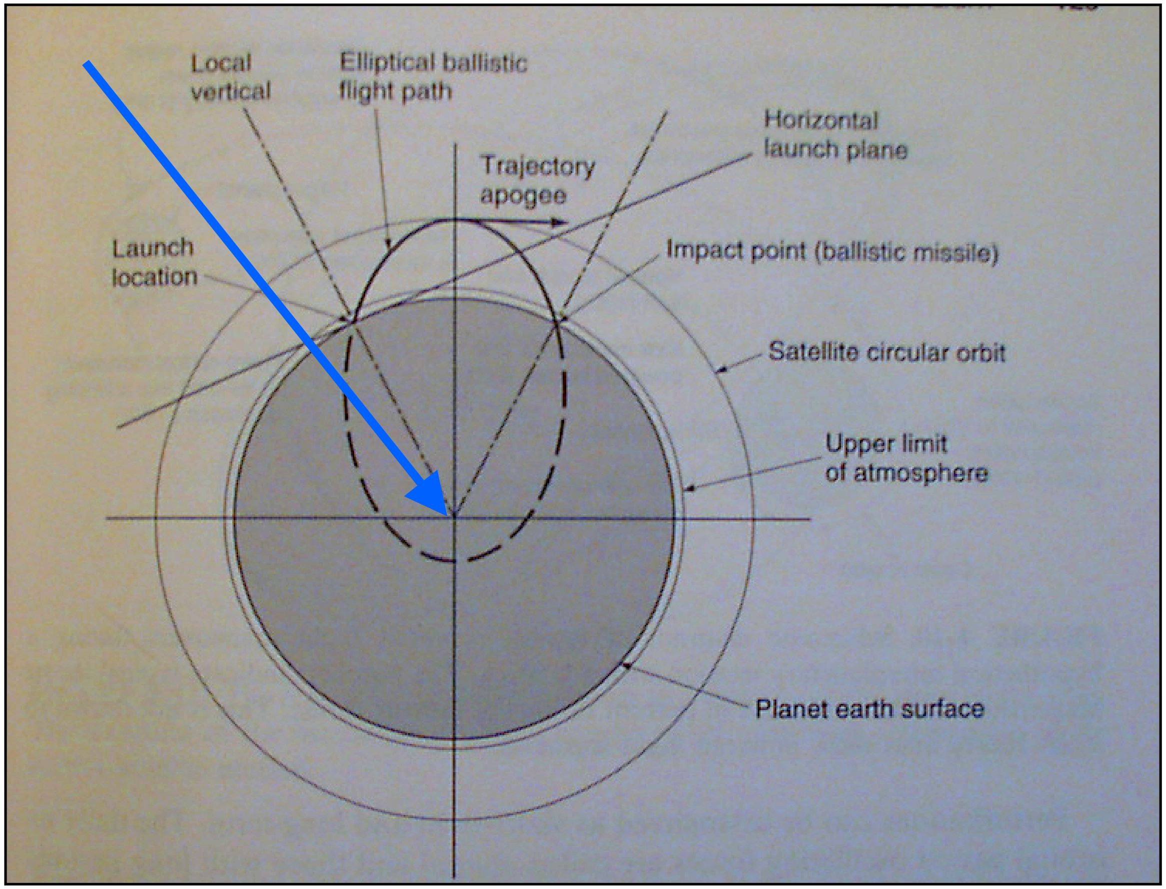




ACTUAL ILLUSTRATION FROM ISAAC NEWTON'S BOOK "SYSTEM OF THE WORLD", PUBLISHED IN 1687.

HIGHER SPEED CARRIES A BODY FURTHER BEFORE IT FALLS TO THE GROUND. IF THE SPEED IS GREAT ENOUGH, IT WILL NOT FALL AT ALL.

Example 2:
Sub Orbital Launch

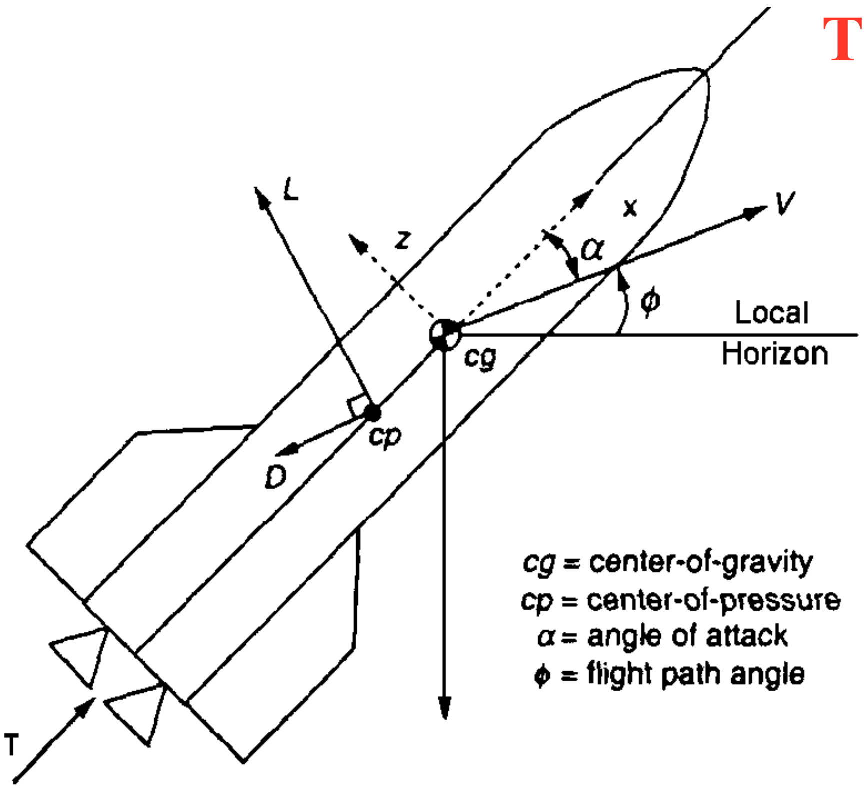


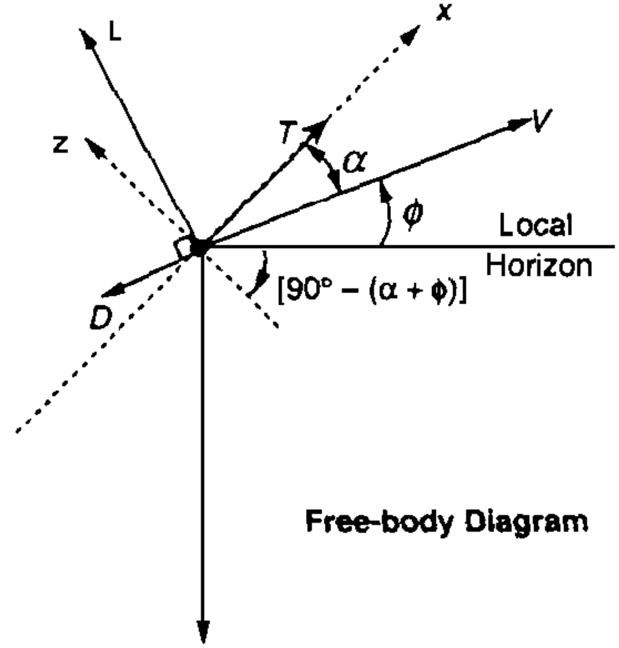
- Still "in orbit" Around earth center
- What is velocity At apogee is Zero?
- Do we still need ΔV?

How Much Delta-V do you need?

$$\Delta V_{required} = \Delta V_{Orbit} + \Delta V_{gravity} + \Delta V_{drag}$$

Thrust, Weight, Lift, Drag





Yes!

$$M_{fuel + oxidizer} = \left[M_{dry} + M_{payload}\right]\left[e^{\left[\frac{\Delta V_{burn}}{g_0 I_{sp}}\right]} - 1\right]$$

Rocket Design 101

…. How Much ΔV do you need to accomplish the mission?

$$\Delta V_{required} = \Delta V_{Orbit} + \Delta V_{gravity} + \Delta V_{drag}$$

$$M_{fuel + oxidizer} = \left[M_{dry} + M_{payload}\right]\left[e^{\left[\frac{\Delta V_{burn}}{g_0 I_{sp}}\right]} - 1\right]$$

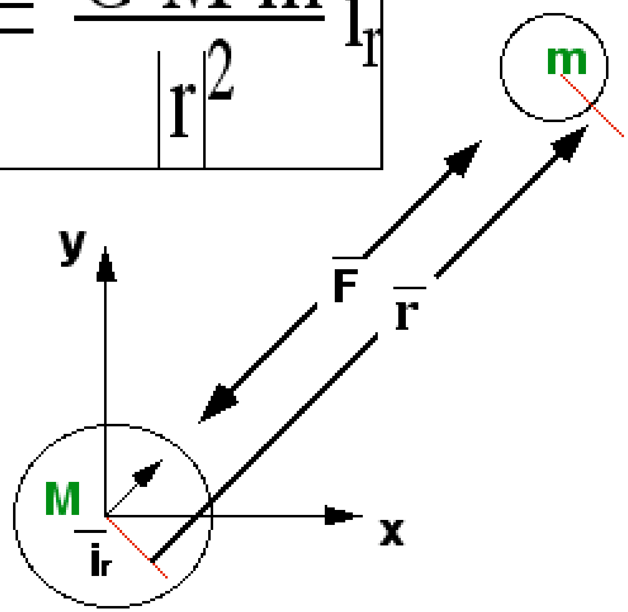
- Obviously we have a LOT! To learn about ΔV!

Gravitational Physics

- **Now a bit of *"gravitational physics"***

$$\overline{F}_{Mm} = \frac{G\,M\,m}{|r|^2}\,\overline{i}_r$$

"Inverse-square" law "potential" field



Isaac Newton, (1642-1727)

Orbital Velocity

- **Object in orbit is actually in "free-fall"**
  that is ... the object is literally falling around the Earth (or Planet)

- **When the *Centrifugal Force* of the "free-fall"** counters the Gravitational Force ... the object is said to have achieved *Orbital Velocity*

Ingnoring Drag ... for a Circular orbit

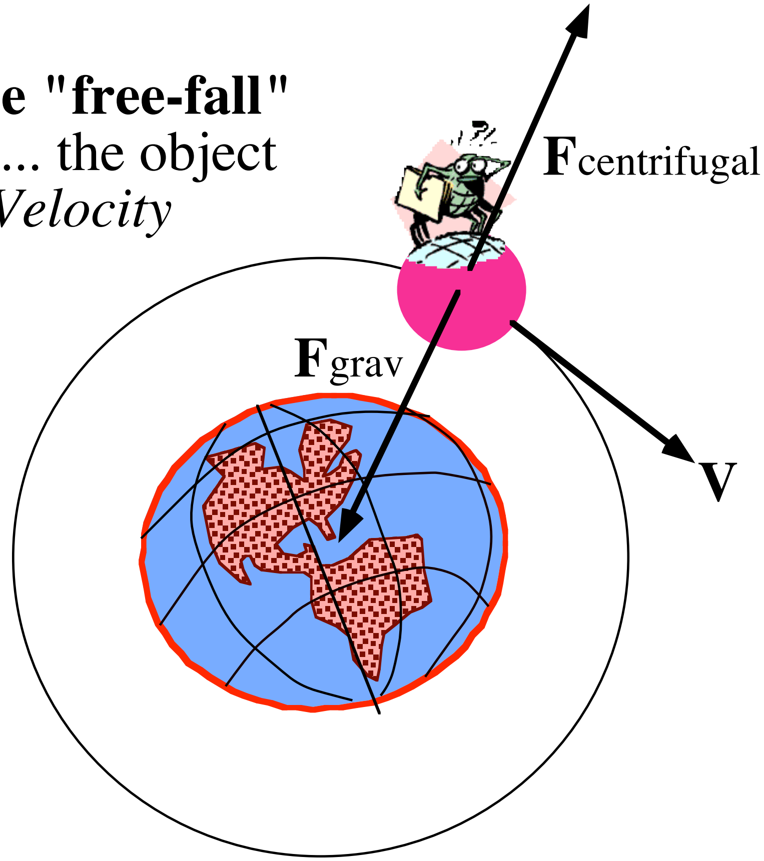
$$\overline{F}_{grav} = \overline{F}_{centrifugal}$$

$$\Downarrow$$

$$\frac{G\,M\,m}{|\mathbf{r}|^2} = m\,\omega^2\,|\mathbf{r}| = m\left[\frac{V}{|\mathbf{r}|}\right]^2 |\mathbf{r}|$$

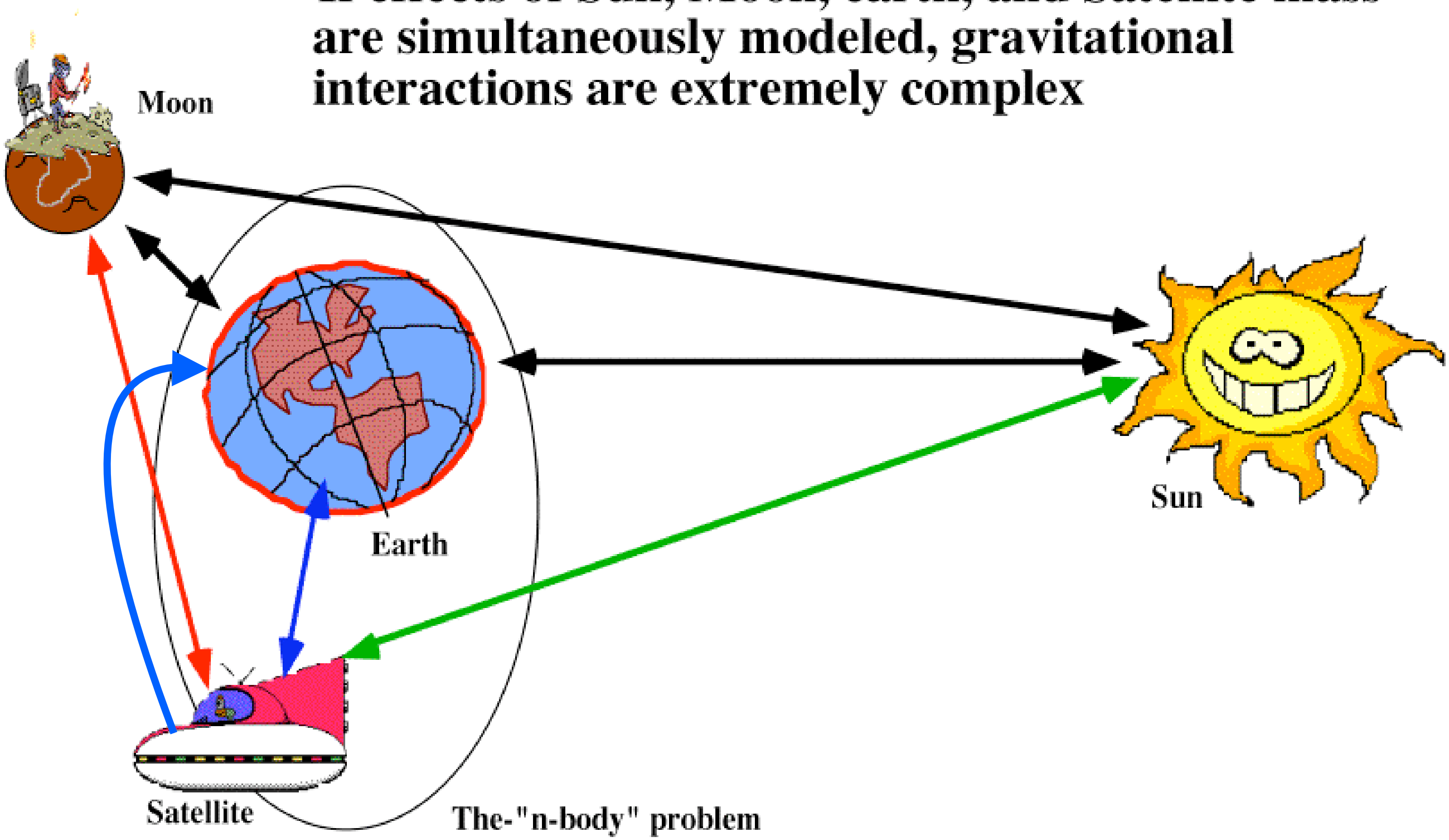
$$\Downarrow$$

$$V = \sqrt{\frac{G\,M}{|\mathbf{r}|}}$$

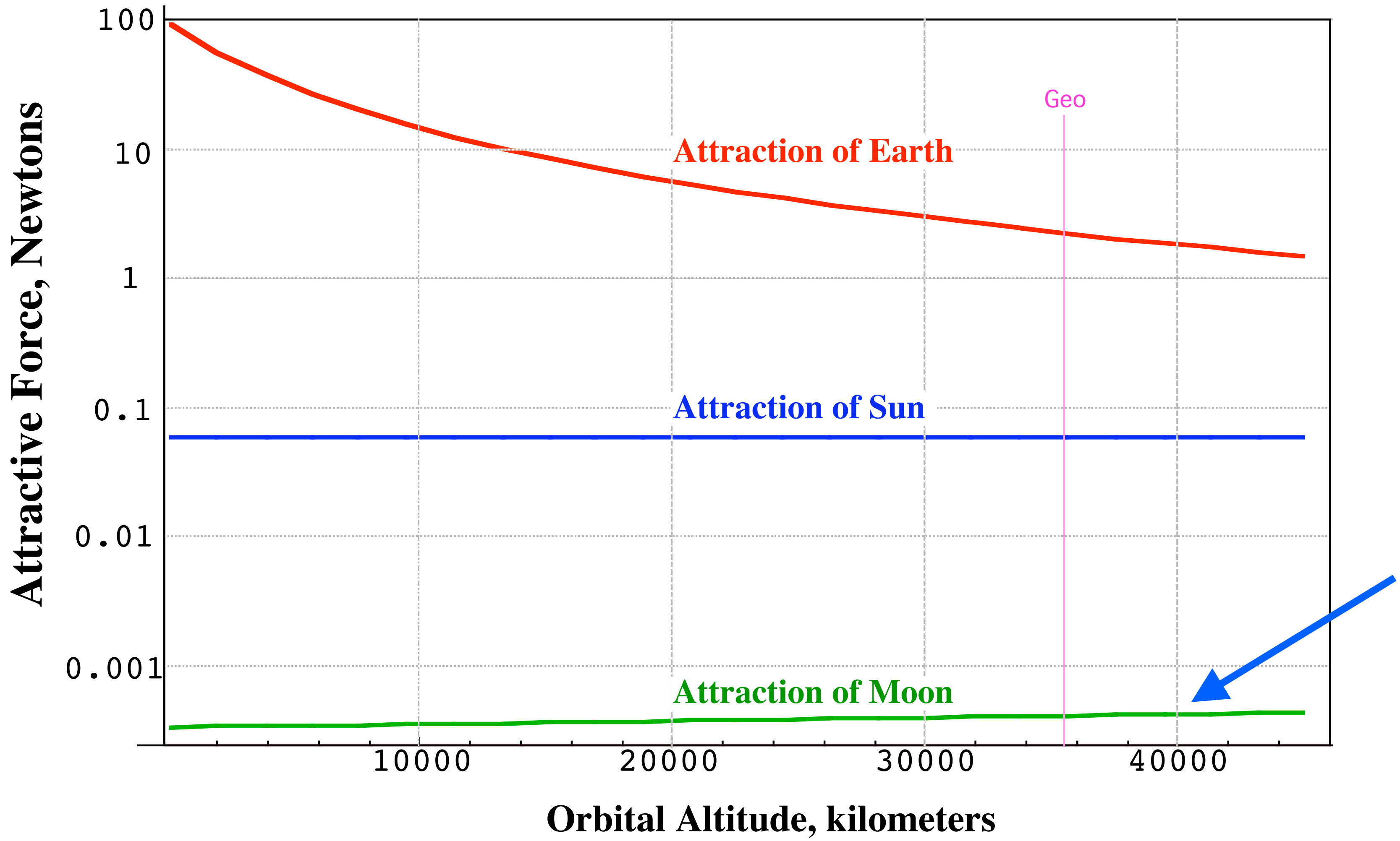


The *"n-Body"* Problem

- **If effects of Sun, Moon, earth, and Satellite mass are simultaneously modeled, gravitational interactions are extremely complex**

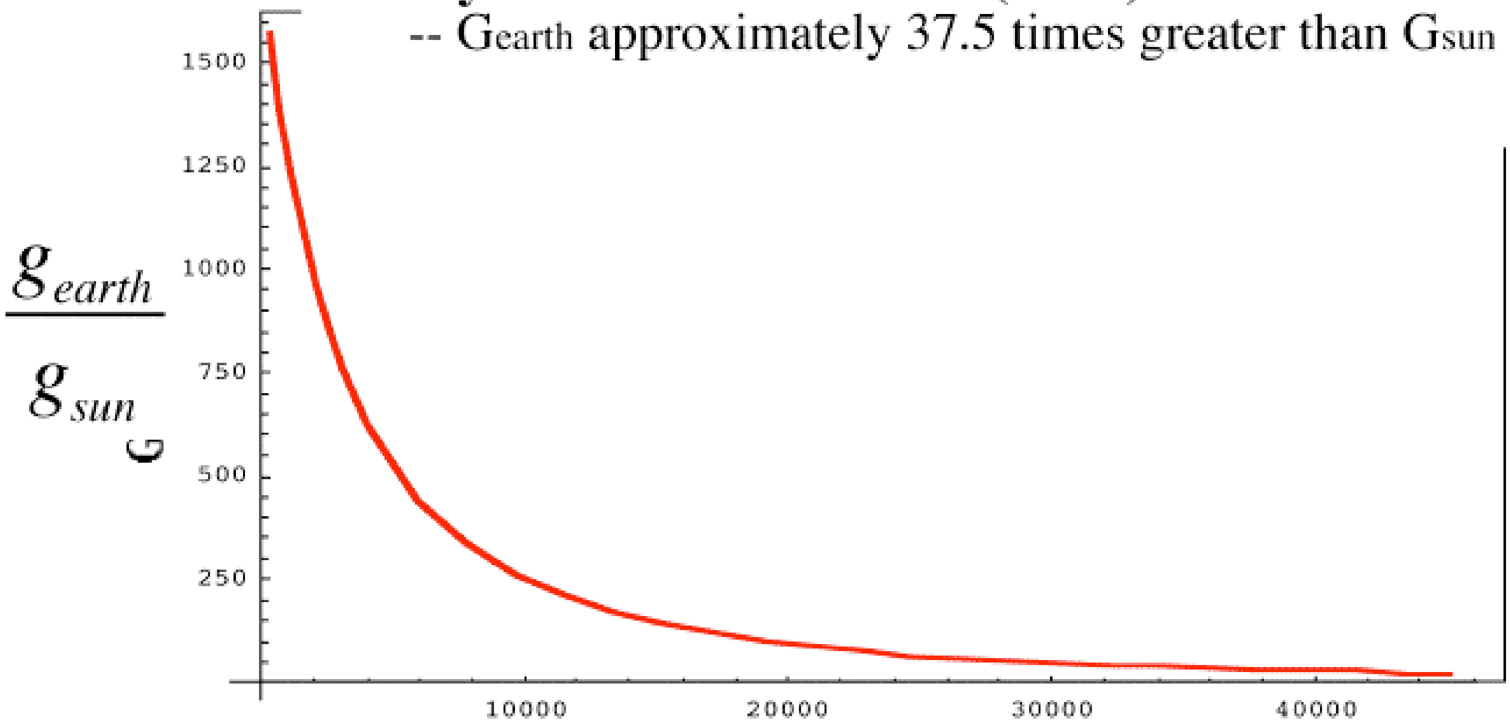


Gravitational Attraction on a 10,000 kg Spacecraft



Gravitational Attraction of Earth Relative to Sun

- **Low Earth Orbit (LEO)**
  -- G_{earth} approximately 2000 times greater than G_{sun}
- **Geosychronous Earth Orbit (GEO)**
  -- G_{earth} approximately 37.5 times greater than G_{sun}



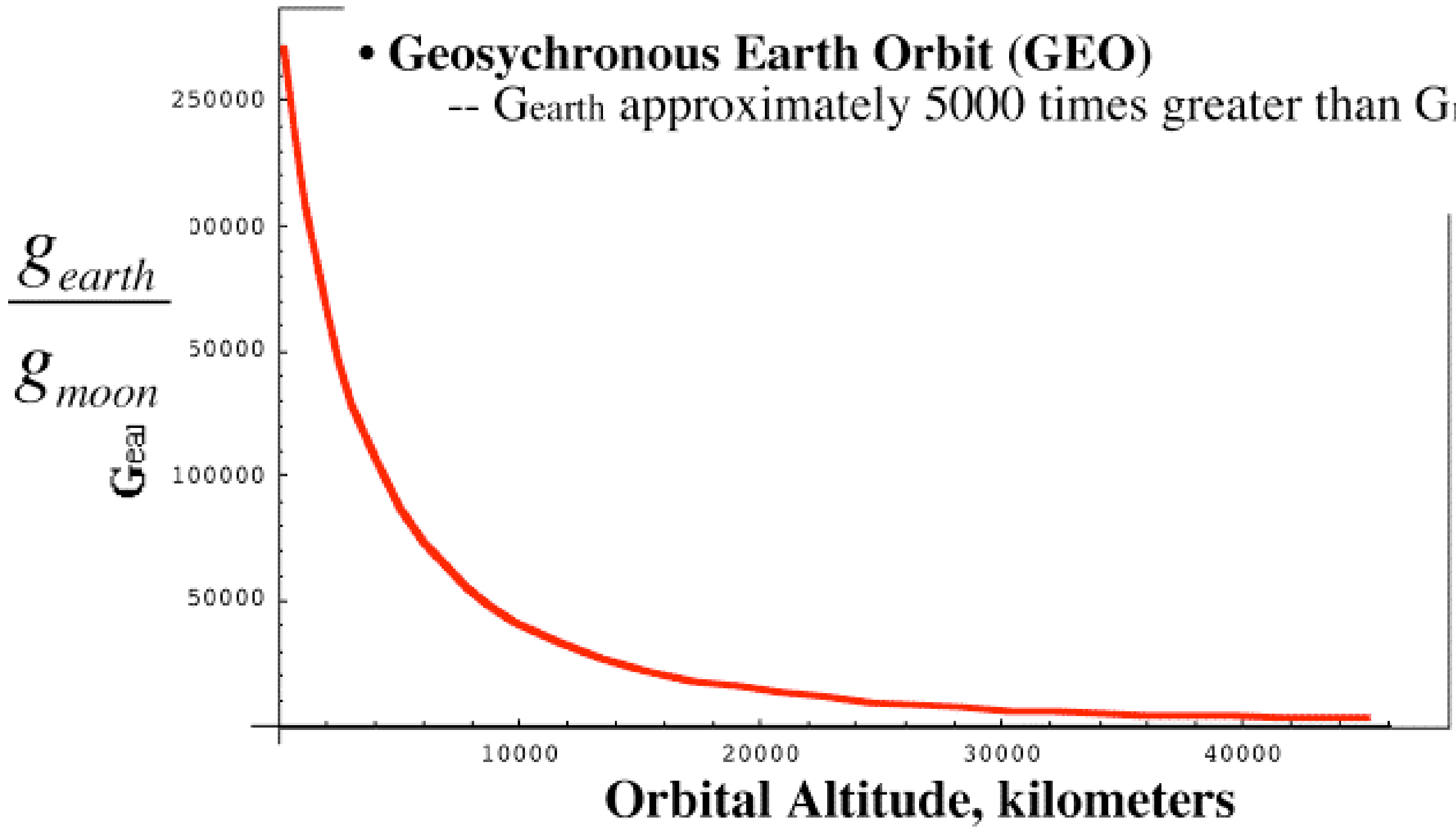
Orbital Altitude, kilometers

UtahState UNIVERSITY

Mechanical & Aerospace Engineering

Gravitational Attraction of Earth Relative to Moon

- **Low Earth Orbit (LEO)**
  -- G_{earth} approximately 27,000 times greater than G_{moon}
- **Geosychronous Earth Orbit (GEO)**
  -- G_{earth} approximately 5000 times greater than G_{moon}



The Two-Body Problem

- **For earth orbit, since the Earth's gravitational attraction is so much stronger than the Sun and Moon**

- **Can approximate most orbital dynamics by considering only the effects of the Earth on the satellite**
  (Clearly the effect of the satellite on the earth is negligible)

- **The-so called *restricted two-body universe***

- **Gravitational attractions of sun and moon are considered as *perturbations* to the two-body problem**

- **In the *two-body universe* ... if the effect of drag ignored the motions of the satellite are exactly described by *Kepler's Laws***

Kepler's laws:

Kepler

- Root of orbital mechanics traced back to laws of planetary motion for posed by Johannes Kepler, Imperial Mathematician to the Holy Roman Emperor, (1609 and 1619)

- *Kepler's laws* are a reasonable approximation of the dynamics of a small body orbiting around a much larger body in a 2-body universe

- Interesting to note that Kepler derived his laws of planetary motion by *observation only*.

- He did not have calculus available to assist him. *That* had to wait almost 100 years for *Sir Isaac Newton!*

Kepler's laws: (concluded)

Kepler

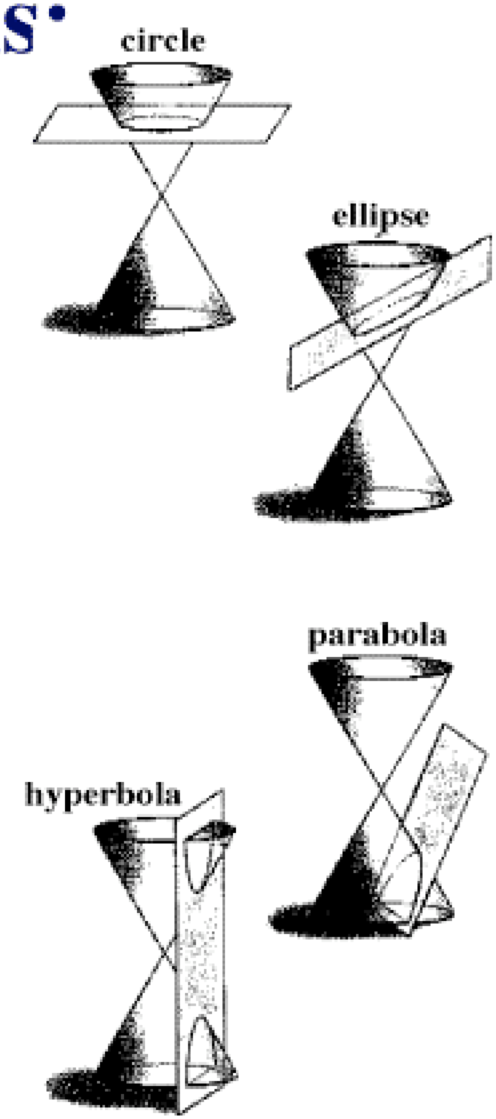
- **Kepler's First Law:** *In a two body universe, orbit of a satellite is a conic section with the Earth centered at one of the focii*

- **Kepler's Second Law:** *In a two body universe, radius vector from the Earth to the satellite sweeps out equal areas in equal times*

- **Kepler's Third Law:** *In a two body universe, square of the period of any object revolving about the Earth is in the same ratio as the cube of its mean distance*

- ***Later*** *We'll Derive These Laws from First Principles Using Newton's Laws*

Kepler's First Law:
Conic Sections



- 4 Possible orbital paths:

  Circle:

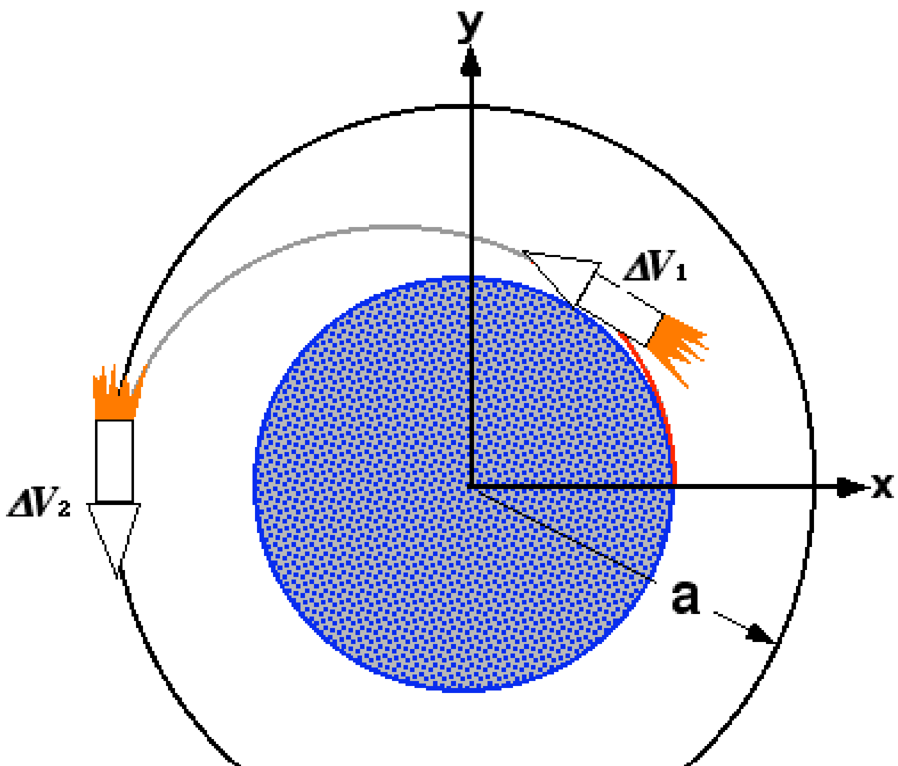
  Ellipse:

  Parabola:

  Hyperbola:

Circular Orbits:

Constant radius, a

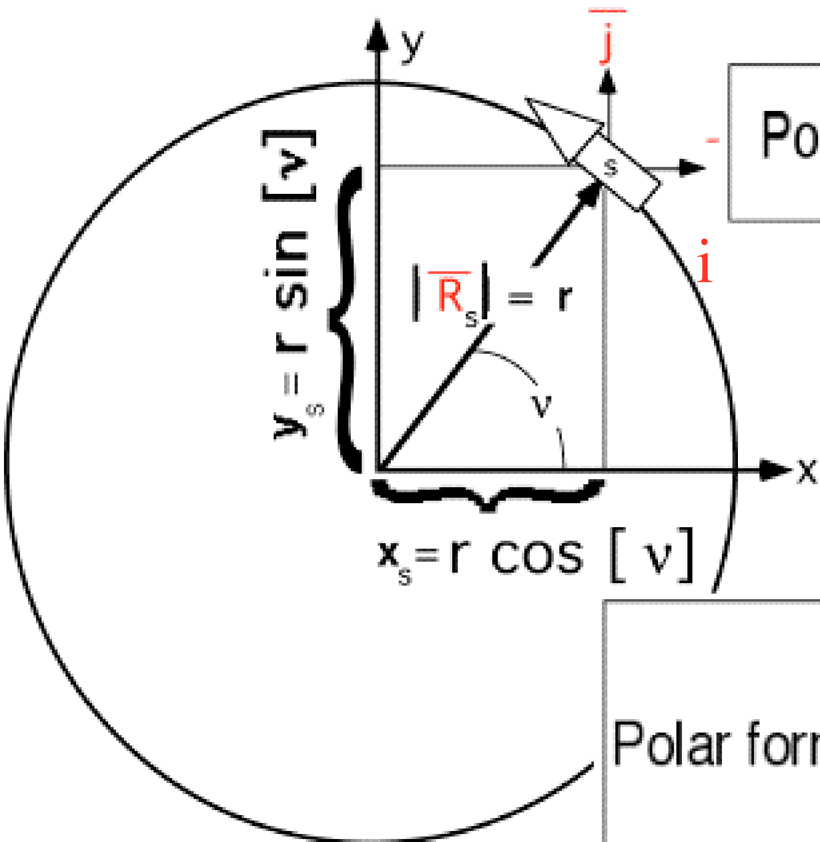


$$\left[\frac{x - x_c}{a}\right]^2 + \left[\frac{y - y_c}{a}\right]^2 = 1$$

for earth orbit

$x_c, y_c = \{0,0\}$

Circular Orbits: (cont;d)



Position vector: $\overline{R}_s = x_s \bar{i} + y_s \bar{j} \equiv \begin{bmatrix} x_s \\ y_s \end{bmatrix}$

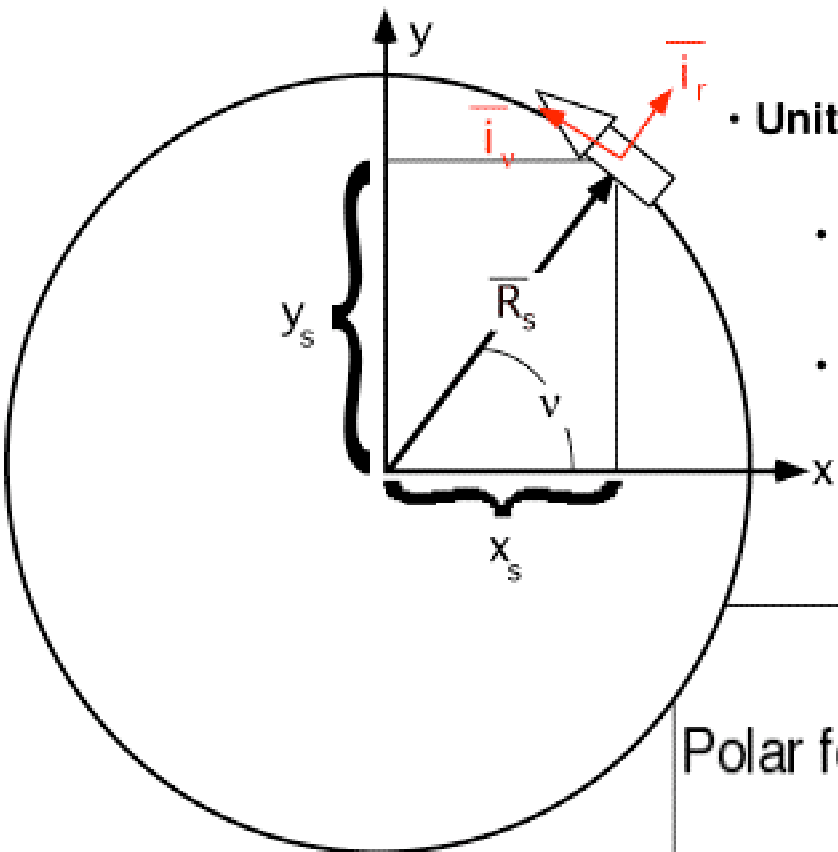
- Unit vectors {i,j} always stay pointed in same direction

Polar form: $\overline{R}_s = a\, \bar{i}_r = \sqrt{x_s^2 + y_s^2}\, \bar{i}_r \equiv \begin{bmatrix} a \\ 0 \end{bmatrix}_{polar}$

$$\nu = \tan^{-1}\left[\frac{y_s}{x_s}\right]$$

Circular Orbits

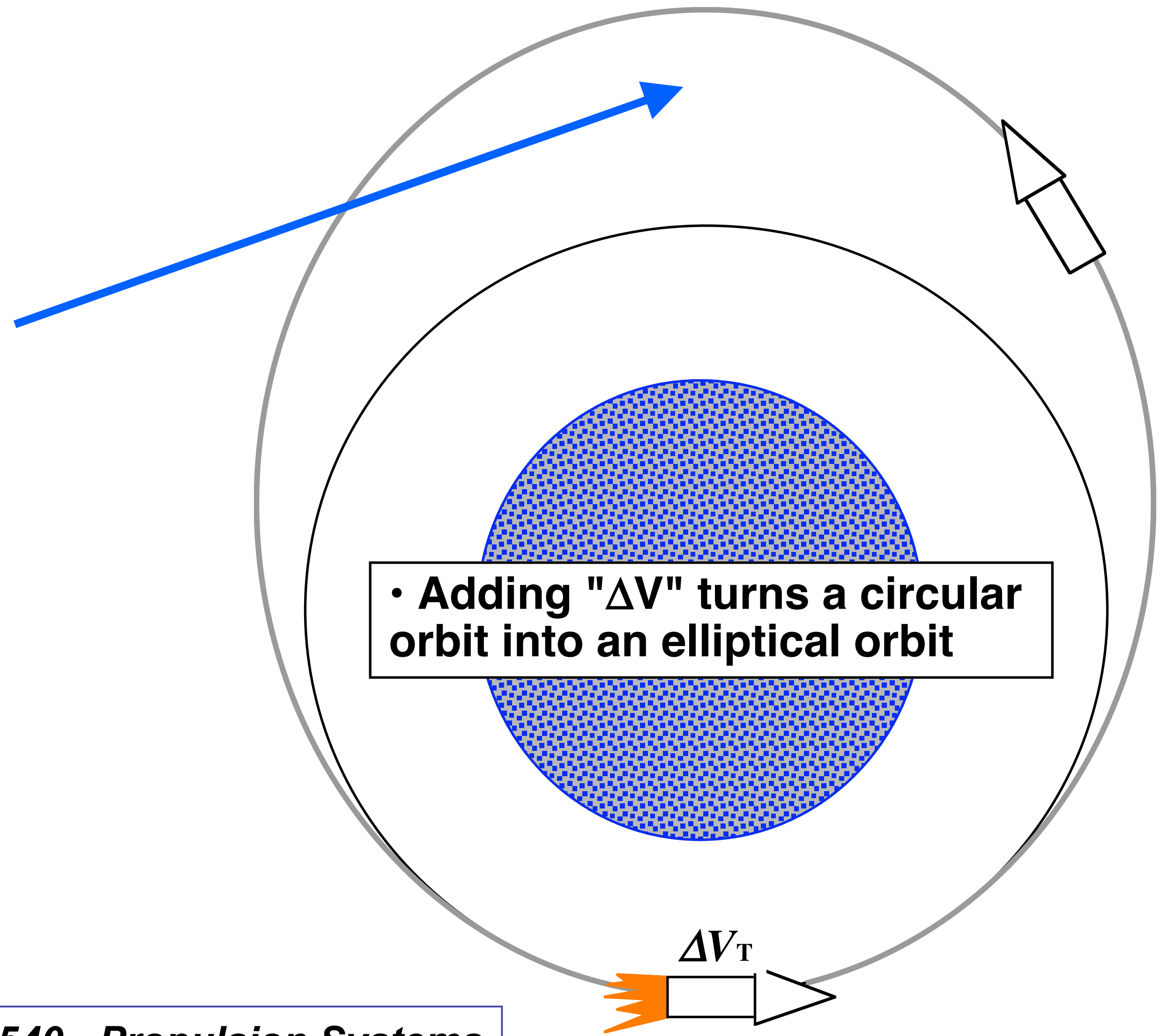
Polar (rotating) Reference Frame



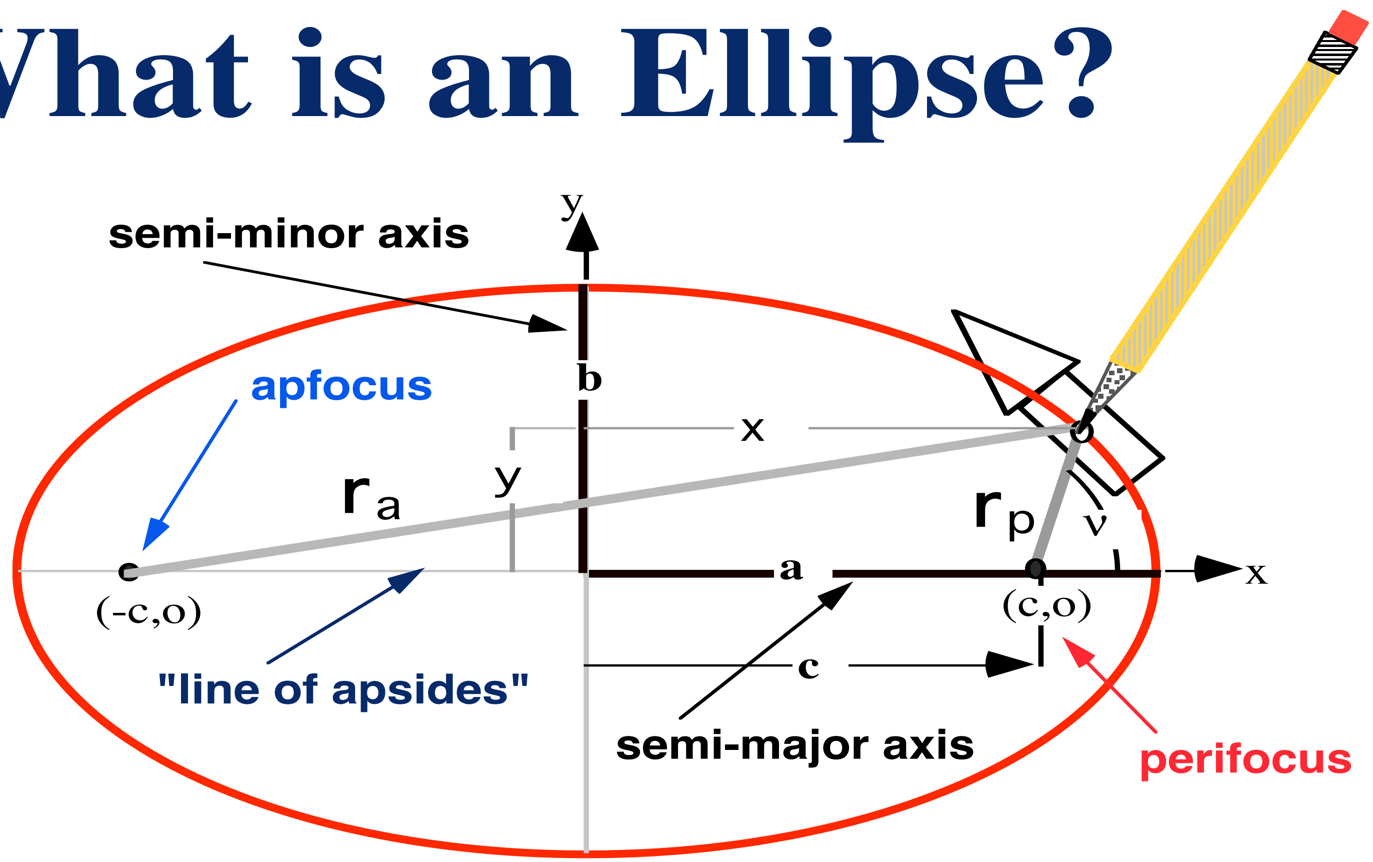
- Unit vectors $\{i_r, i_\nu\}$ are fixed to the spacecraft
  - i_r points along radial direction
  - i_ν is perpendicular to i_r

Polar form: $\bar{R}_S = r\ \bar{i}_r = \sqrt{x_s^2 + y_s^2}\ \bar{i}_r$, $\nu = \tan^{-1}\left[\frac{y_s}{x_s}\right]$ $\equiv \begin{bmatrix} r \\ 0 \end{bmatrix}_{polar}$

Elliptical Orbits:



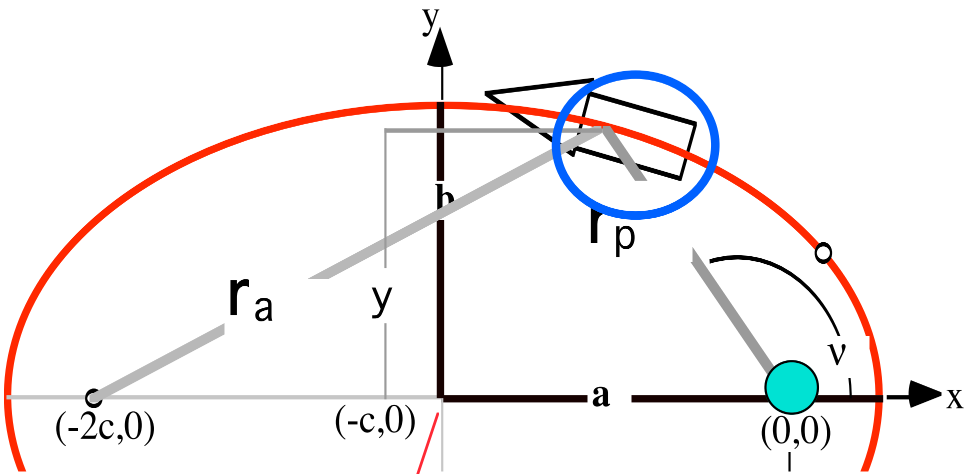
What is an Ellipse?



Geometry of an Ellipse

$$|r_a| + |r_p| = 2\,a$$

Ellipse Equation: Earth
Centered at origin

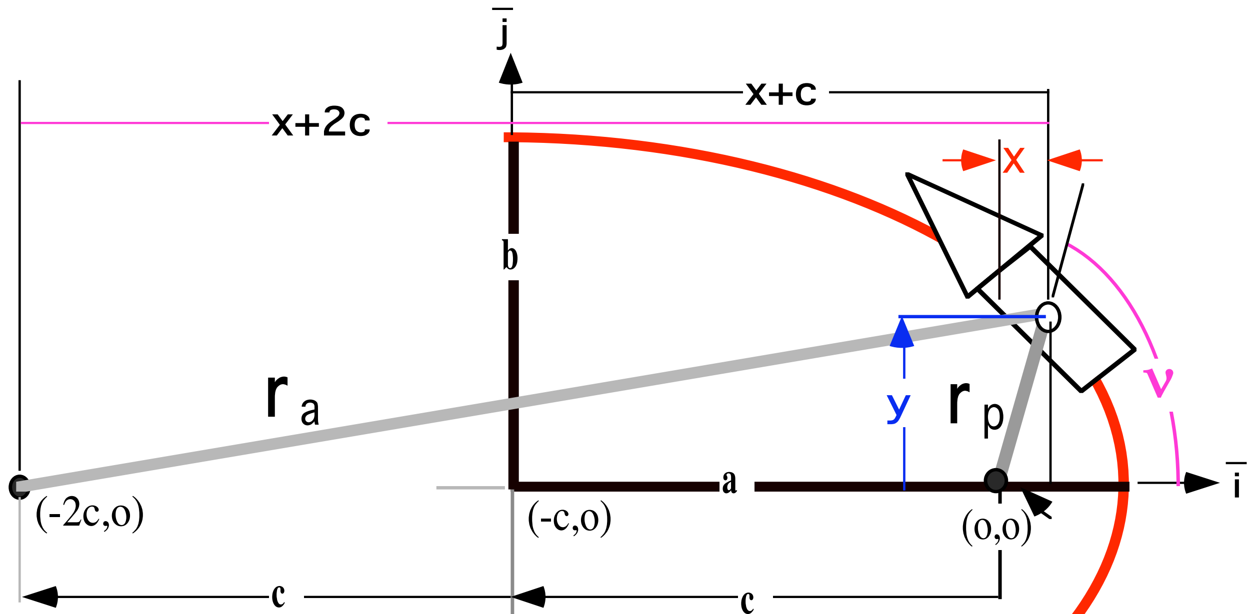


Earth (perifocus) centered at origin:

$$\frac{[x + c]^2}{a^2} + \frac{[y]^2}{b^2} = 1$$

Polar-Form of the Ellipse Equation

$$|r_a| + |r_p| = 2\,a \quad \Rightarrow \quad \sqrt{(x + 2c)^2 + y^2} + \sqrt{(x\,)^2 + y^2} = 2a$$



Geometry of an Ellipse
(Earth Centered at perifocus)

Polar-Form of the Ellipse Equation (cont'd)

$$\left|r_a\right| + \left|r_p\right| = 2\,a \quad \Rightarrow \sqrt{(x+2c)^2 + y^2} + \sqrt{(x\,)^2 + y^2} = 2a$$

$$\Downarrow$$

$$\sqrt{(x+2c)^2 + y^2} = 2a - \sqrt{(x\,)^2 + y^2}$$

$$\Downarrow$$

$$x^2 + 4c\,x + 4\,c^2 + y^2 = 4\,a^2 - 4\,a\sqrt{(x\,)^2 + y^2} + (x\,)^2 + y^2$$

$$\Downarrow$$

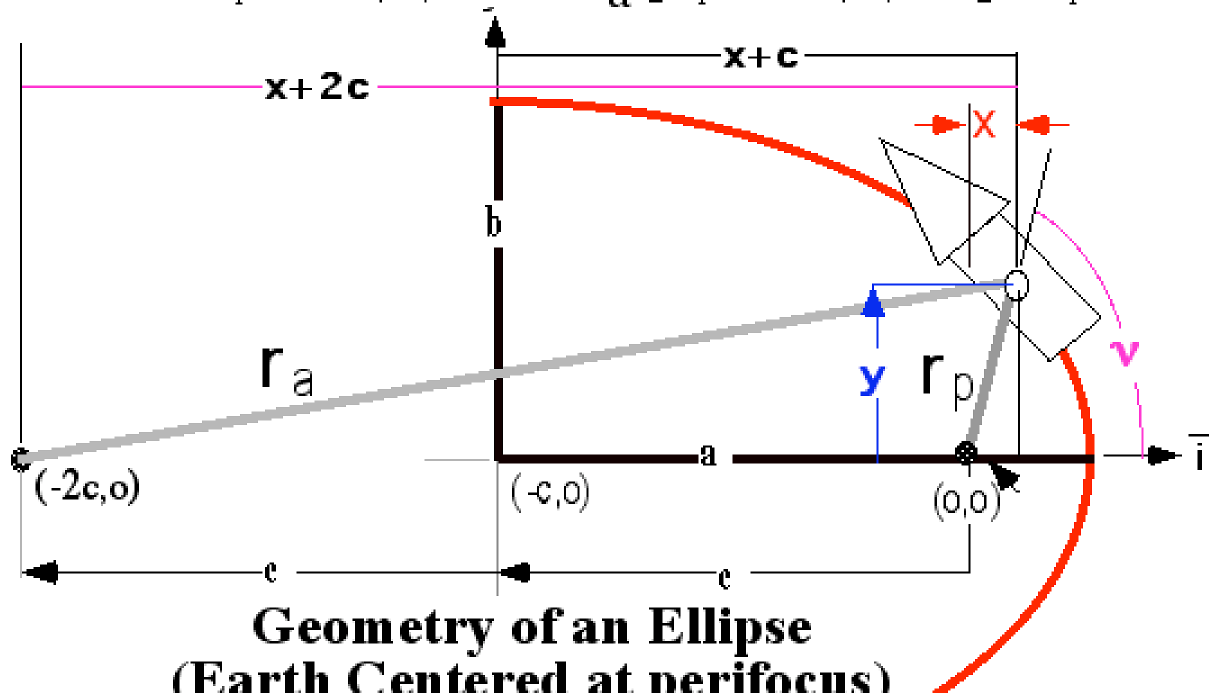
$$c\left[\,x + c\right] = a^2 - \;a\sqrt{(x\,)^2 + y^2} \Rightarrow a - \frac{c}{a}\left[\,x + c\right] = \sqrt{(x\,)^2 + y^2}$$

Polar-Form of Ellipse Equation (cont'd)

$$a - \frac{c}{a}\left[x + c\right] = \sqrt{(x)^2 + y^2} = r_p$$

$$\Downarrow$$

$$x = r_p \cos(\nu) \Rightarrow a - \frac{c}{a}\left[r_p \cos(\nu) + c\right] = r_p$$



Geometry of an Ellipse
(Earth Centered at perifocus)

Polar-Form of Ellipse Equation (cont'd)

- Substituting and simplifyiing

$$r_p(x) = a - \frac{c}{a}\left[r_p(x)\cos(\nu) + c\right] \Rightarrow r_p(x)\left[1 + \frac{c}{a}\cos(\nu)\right] = a - \frac{c^2}{a}$$

$$\Downarrow$$

$$r_p(x) = \frac{a - \dfrac{a^2 - b^2}{a}}{\left[1 + \dfrac{\sqrt{a^2 - b^2}}{a}\cos(\nu)\right]} = \frac{a - \dfrac{a^2 - b^2}{a}}{\left[1 + \dfrac{\sqrt{a^2 - b^2}}{a}\cos(\nu)\right]} =$$

$$\Downarrow$$

$$r_p(x) = a\,\frac{b^2}{a^2}\,\frac{1}{\left[1 + \sqrt{1 - \dfrac{b^2}{a^2}}\cos(\nu)\right]}$$

Polar-Form of the Ellipse Equation

(concluded)

- **Defining the elliptical *eccentricity* as**

$$e \equiv \sqrt{1 - \frac{b^2}{a^2}}$$

- **The *polar form* of the *ellipse equation* reduces to**

$$\boxed{r = \frac{a\left[1 - e^2\right]}{\left[1 + e\cos(\nu)\right]}}$$

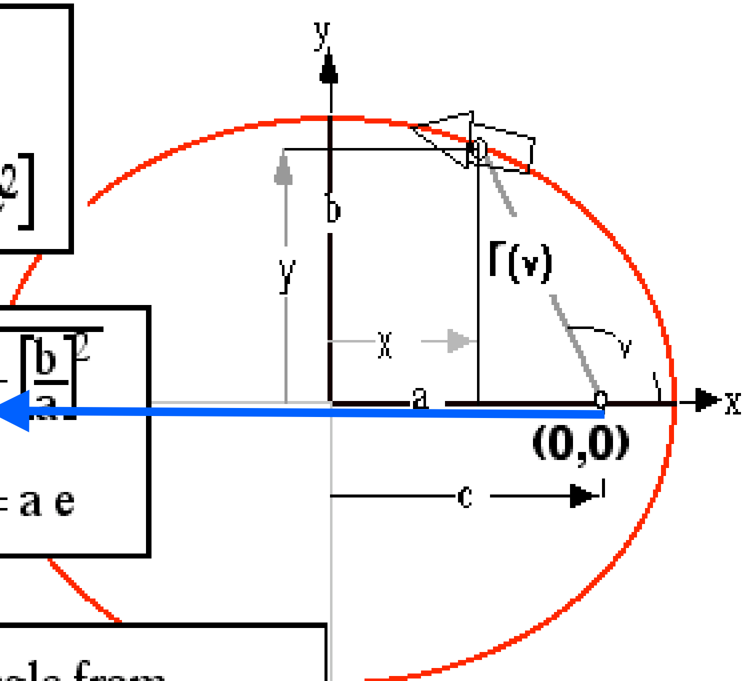
Parameters of the Elliptical Orbit

a: semi-major axis:

b: semi-minor axis: $b^2 = a^2\left[1 - e^2\right]$

e: orbital eccentricity $\Rightarrow e = \sqrt{1 - \left[\frac{b}{a}\right]^2}$

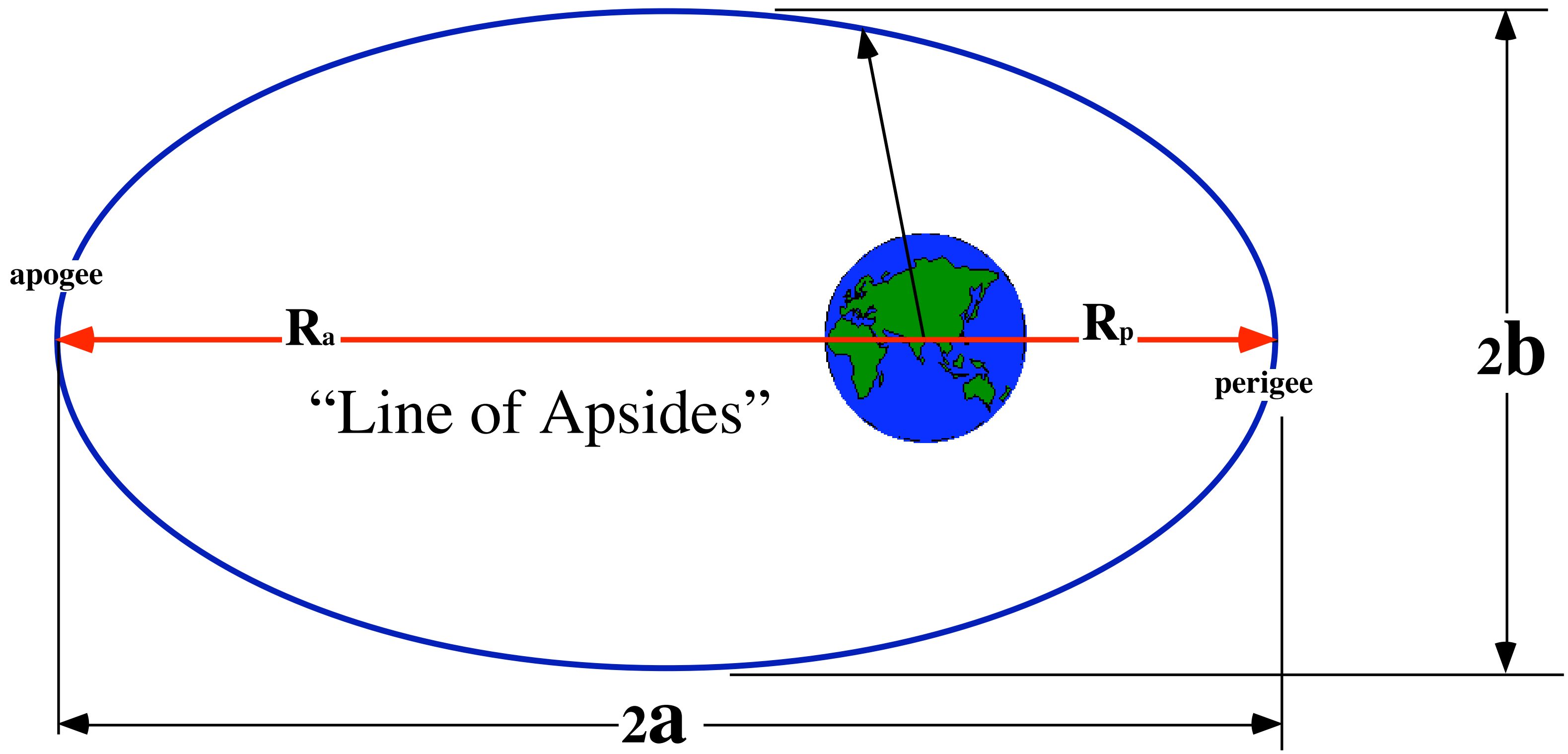
c: perifocus $\Rightarrow c = a\sqrt{1 - \left[\frac{b}{a}\right]^2} = a\,e$



ν: true anomaly ⇒ Angle from perapsis to satellite

(ν): orbital radius $\Rightarrow r(\nu) = \dfrac{a\left[1 - e^2\right]}{\left[1 + e\cos(\nu)\right]}$

Fundamental Elliptical Orbit Definitions



a -- Semi-major axis

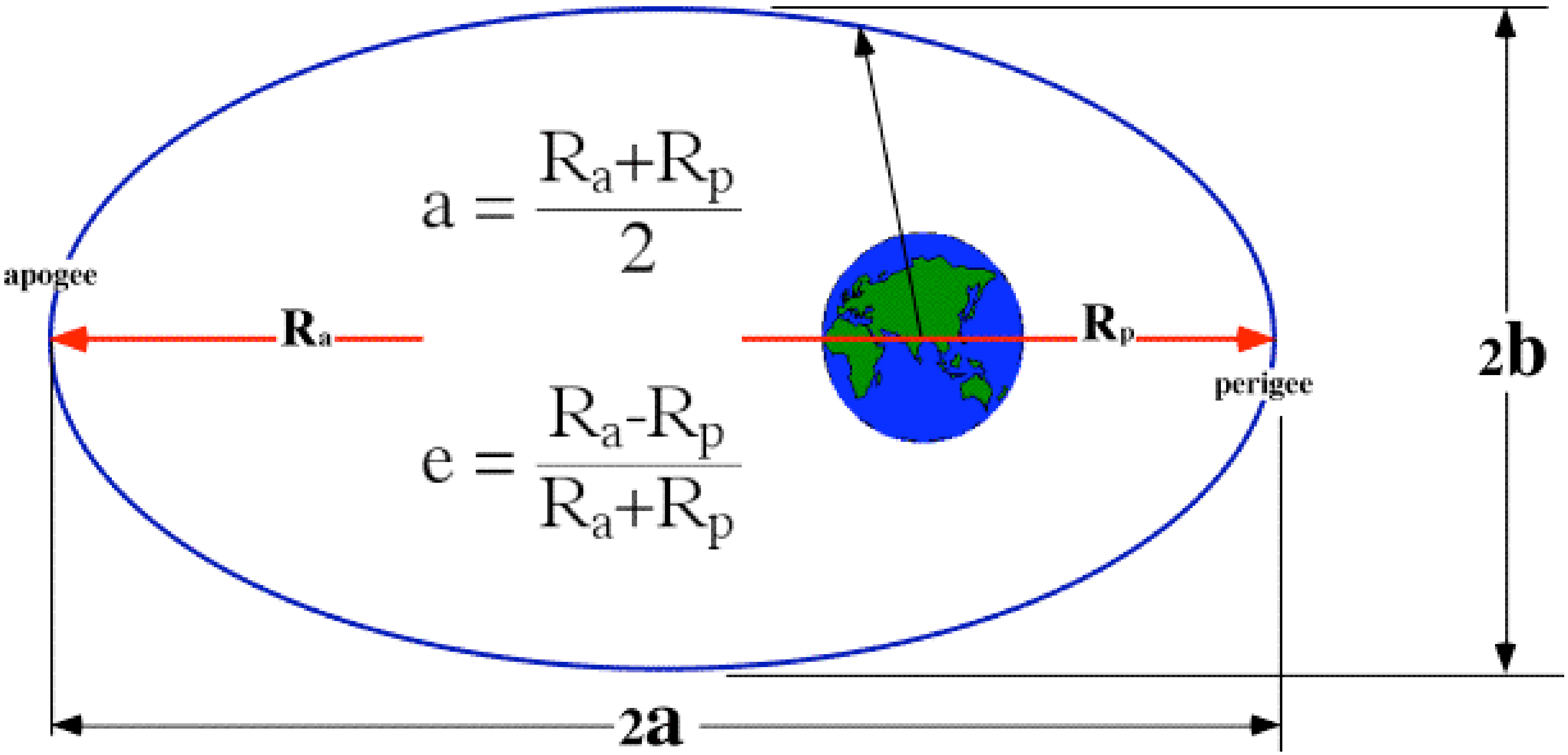
b -- Semi-minor axis

e -- Orbital eccentricity

$$e = \sqrt{1 - \left[\frac{b}{a}\right]^2}$$

Fundamental Elliptical Orbit Definitions

cont'd

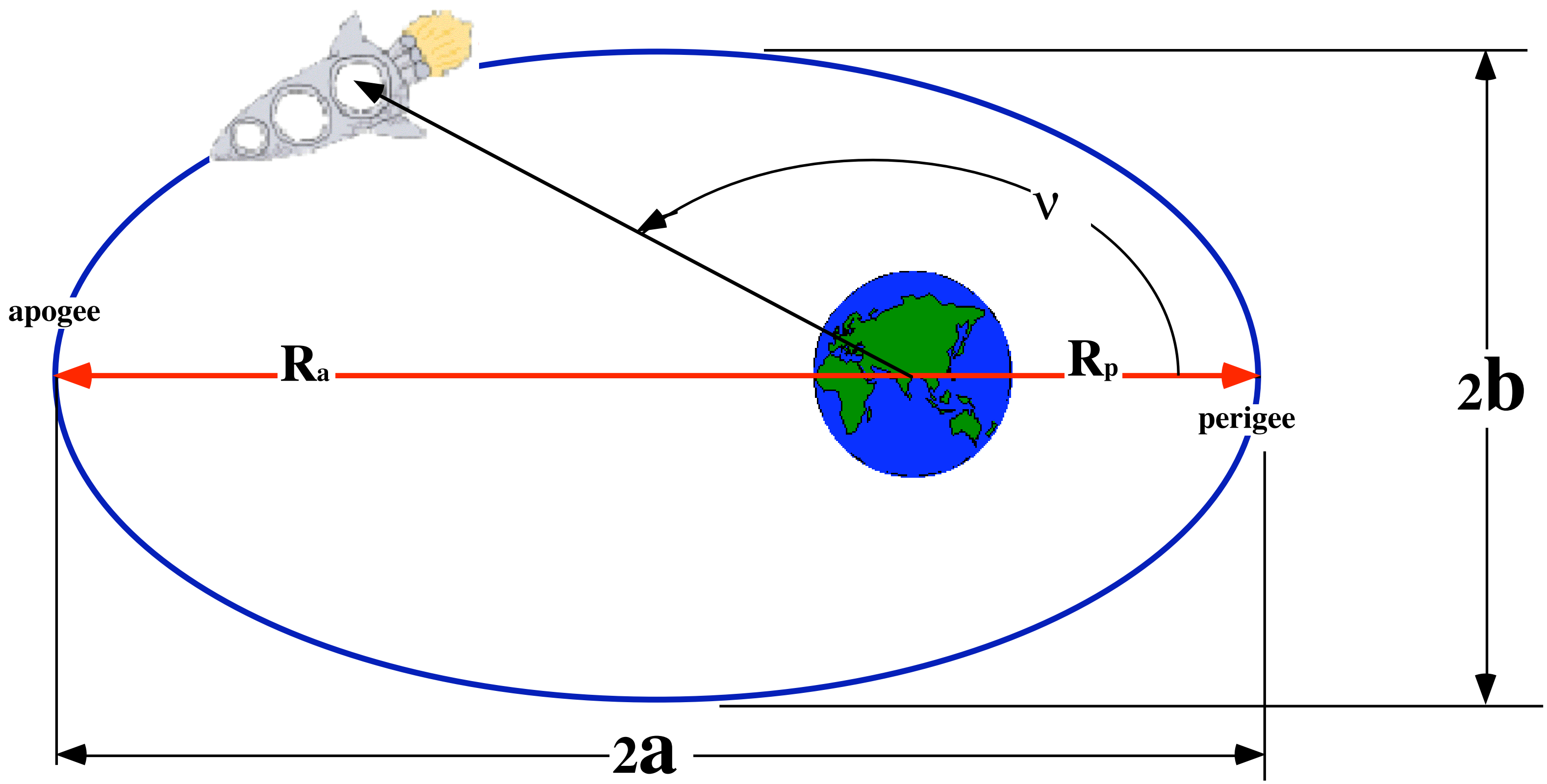


Orbit Apogee and Perigee *(closest and farthest approaches)*

… semi major axis and eccentricity related to apogee and perigee radius

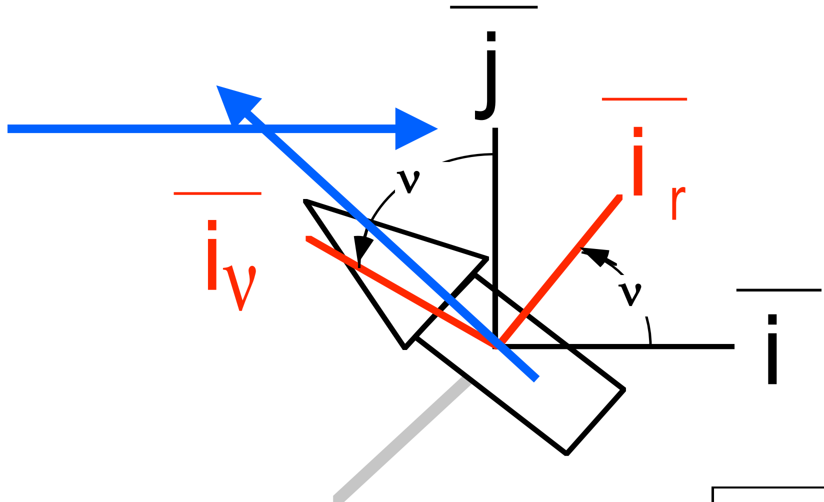
Fundamental Elliptical Orbit Definitions

cont'd



Location within an Orbit: "true anomaly" ν

Coordinate Transformations:



{i, j} fixed in space

Transform ⇒ polar ↑ inertial

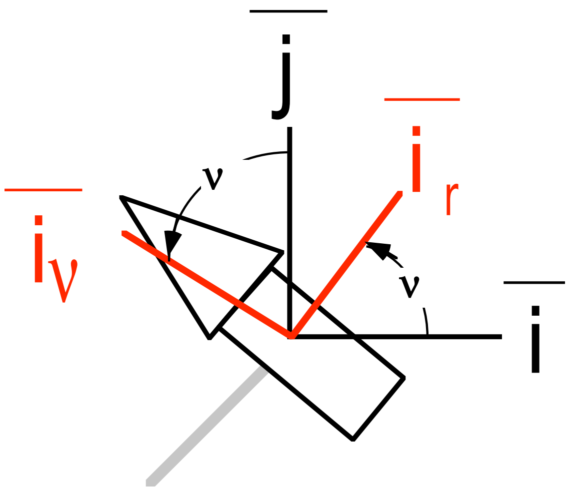
$$\bar{i} = \bar{i}_r \cos[\nu] - \bar{i}_\nu \sin[\nu]$$
$$\bar{j} = \bar{i}_r \sin[\nu] + \bar{i}_\nu \cos[\nu]$$

Transform ⇒ inertial ↑ polar

$$\bar{i}_r = \bar{i} \cos[\nu] + \bar{j} \sin[\nu]$$
$$\bar{i}_\nu = -\bar{i} \sin[\nu] + \bar{j} \cos[\nu]$$

Coordinate Transformations: (cont;d)

- **A Matrix "trick" for coordinate transform in 2-D**



Transform ⇒ polar ↑ inertial

$$\bar{i} = \bar{i}_r \cos[\nu] - \bar{i}_\nu \sin[\nu]$$
$$\bar{j} = \bar{i}_r \sin[\nu] + \bar{i}_\nu \cos[\nu]$$

Transform ⇒ inertial ↑ polar

$$\bar{i}_r = \bar{i} \cos[\nu] + \bar{j} \sin[\nu]$$
$$\bar{i}_\nu = -\bar{i} \sin[\nu] + \bar{j} \cos[\nu]$$

$$\begin{bmatrix} \bar{i}_r \\ \bar{i}_\nu \end{bmatrix} = \begin{bmatrix} \cos[\nu] & \sin[\nu] \\ -\sin[\nu] & \cos[\nu] \end{bmatrix} \begin{bmatrix} \bar{i} \\ \bar{j} \end{bmatrix}$$

$$\Downarrow$$

$$\begin{bmatrix} \bar{i} \\ \bar{j} \end{bmatrix} = \begin{bmatrix} \cos[\nu] & \sin[\nu] \\ -\sin[\nu] & \cos[\nu] \end{bmatrix}^T \begin{bmatrix} \bar{i}_r \\ \bar{i}_\nu \end{bmatrix} = \begin{bmatrix} \cos[\nu] & -\sin[\nu] \\ \sin[\nu] & \cos[\nu] \end{bmatrix} \begin{bmatrix} \bar{i}_r \\ \bar{i}_\nu \end{bmatrix}$$

Orbital Velocity

Velocity is the derivative of position ….

$$\vec{r} = \frac{a\left(1-e^2\right)}{1+e\cdot\cos(\nu)}\cdot\vec{i}_r \rightarrow \boxed{\begin{array}{l}\{a,e\} \Rightarrow constant \\ parameters\ of\ the\ orbit\end{array}}$$

$$\vec{V} = \frac{d}{dt}(\vec{r}) = \frac{d}{dt}\left(\frac{a\left(1-e^2\right)}{1+e\cdot\cos(\nu)}\right)\cdot\vec{i}_r + \frac{a\left(1-e^2\right)}{1+e\cdot\cos(\nu)}\cdot\frac{d}{dt}\left(\vec{i}_r\right)$$

Orbital Velocity (2)

$$Differentiate \quad r$$

$$\frac{d}{dt}\left(\frac{a\left(1-e^2\right)}{1+e\cdot\cos(v)}\right)=-\left(\frac{a\left(1-e^2\right)}{\left[1+e\cdot\cos(v)\right]^2}\right)\cdot\left(-e\cdot\sin(v)\cdot(\dot{v})\right)=$$

$$\left(\frac{a\left(1-e^2\right)}{\left[1+e\cdot\cos(v)\right]}\right)\cdot\left(\frac{e\cdot\sin(v)}{\left[1+e\cdot\cos(v)\right]}\right)\cdot(\dot{v})=\left(\frac{e\cdot\sin(v)}{\left[1+e\cdot\cos(v)\right]}\right)\cdot(r\cdot\dot{v})$$

$$Differentiate \quad \vec{i}$$

$$\frac{d}{dt}\left(\vec{i}_r\right)=\frac{d}{dt}\left(\ \vec{i}\cdot\cos(v)+j\cdot\sin(v)\ \right)=\left(\ \text{-}\vec{i}\cdot\sin(v)+j\cdot\cos(v)\ \right)\cdot\dot{v}=\dot{v}\cdot\vec{i}_v$$

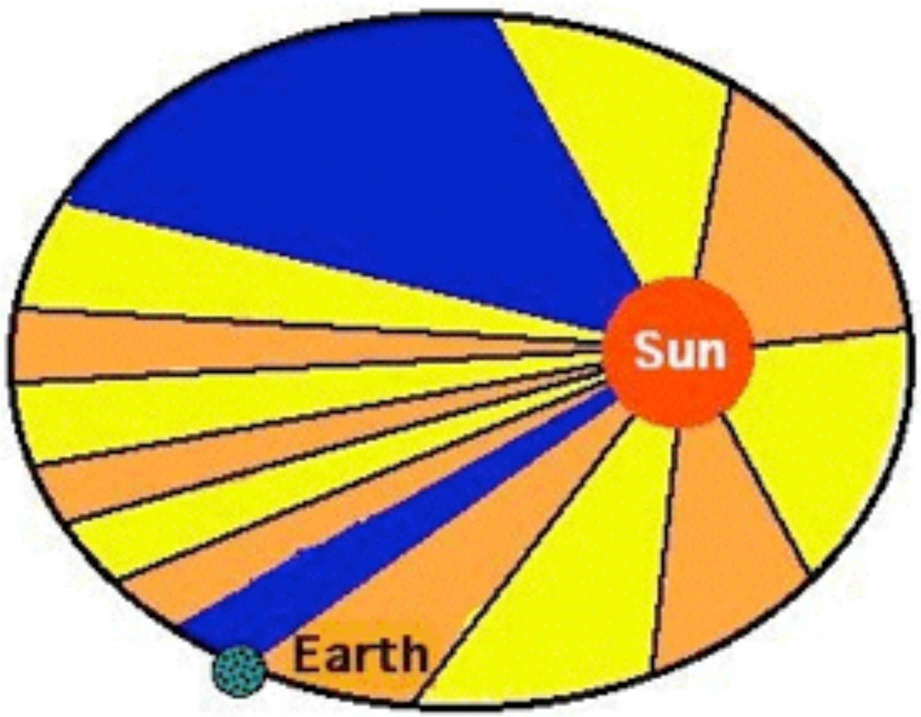
Orbital Velocity (3)

$$\vec{V} = \left(\frac{e \cdot \sin(\nu)}{\left[1 + e \cdot \cos(\nu)\right]} \cdot (r \cdot \dot{\nu}) \right) \cdot \vec{i}_r + \left(\frac{a(1 - e^2)}{1 + e \cdot \cos(\nu)} \right) \cdot \dot{\nu} \cdot \vec{i}_\nu =$$

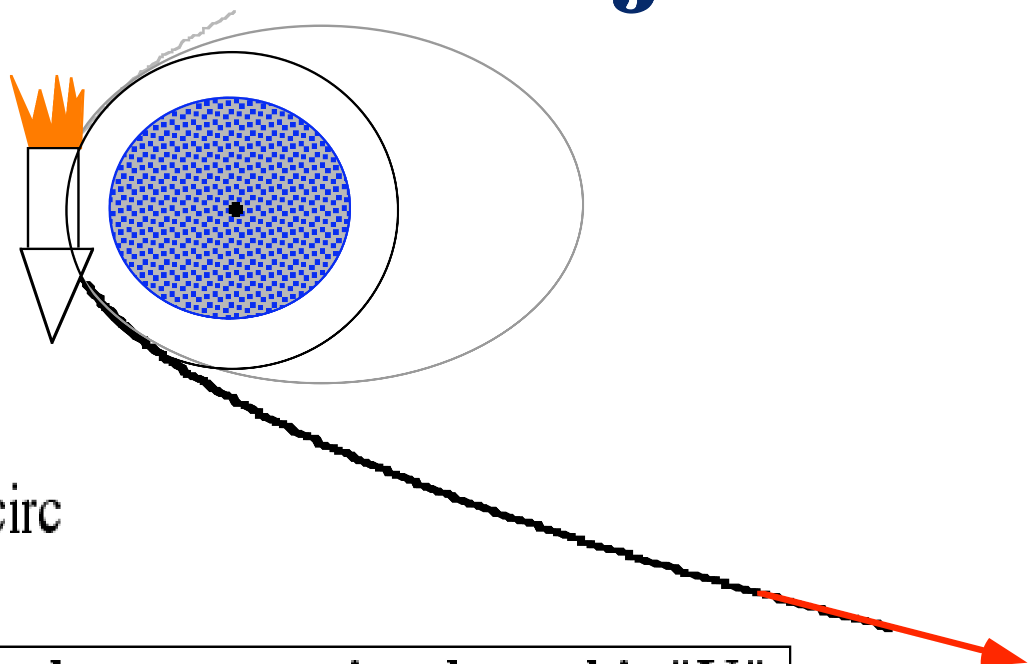
$$(r \cdot \dot{\nu}) \left[\left(\frac{e \cdot \sin(\nu)}{\left[1 + e \cdot \cos(\nu)\right]} \right) \cdot \vec{i}_r + \vec{i}_\nu \right]$$

Need Kepler's Second Law to get

$$\dot{\nu} = \omega$$


Sun
Earth

Parabolic Trajectories:



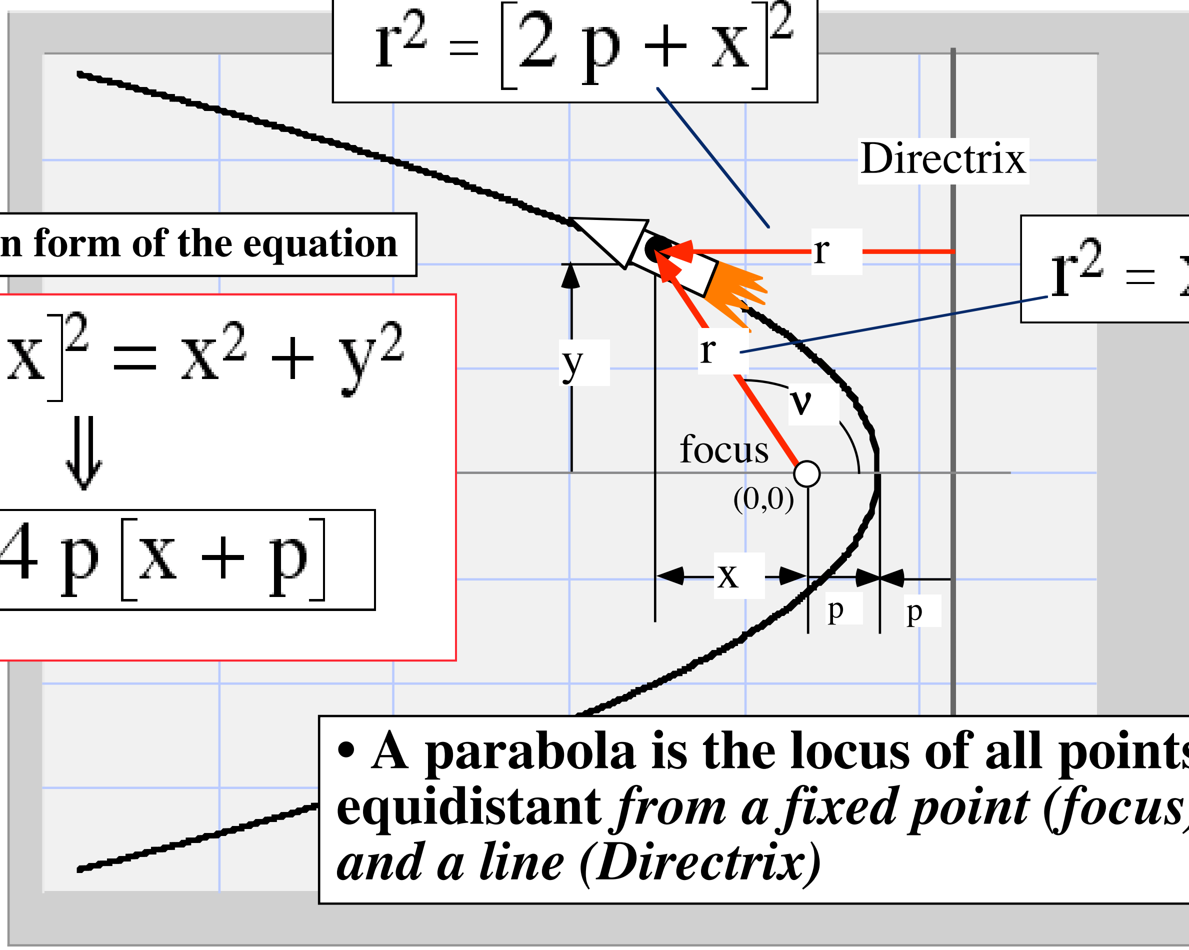
$$\Delta V_{esc} = \left[\sqrt{2} - 1\right] V_{circ}$$

• If we increase the current circular orbit "V" by a factor of $\sqrt{2}$; then the velocity becomes too great for the planet to contain the orbit

• Satellite *escapes* the planet on a parabolic trajectory

What is a Parabola?

$$r^2 = \left[2p + x\right]^2$$



$$r^2 = x^2 + y^2$$

- **Cartesian form of the equation**

$$\left[2p + x\right]^2 = x^2 + y^2$$

$$\Downarrow$$

$$y^2 = 4p\left[x + p\right]$$

- **A parabola is the locus of all points equidistant *from a fixed point (focus) and a line (Directrix)***

Parabola Equation

Polar Form

$$y = \mathbf{r} \sin(\nu)$$

$$x = -\mathbf{r} \cos(\nu)$$

$$\Rightarrow$$

$$\mathbf{y}^2 = 4p\left[p + x\right]$$

$$\Downarrow$$

$$\left[\mathbf{r} \sin(\nu)\right]^2 = 4p\left[p - \mathbf{r}\cos(\nu)\right]$$

$$\Downarrow$$

Use quadratic formula to solve

$$\left[\sin^2(\nu)\right]\mathbf{r}^2 + \left[4p\cos(\nu)\right]\mathbf{r} - 4p^2 = 0$$

Parabola Equation

Polar Form (cont'd)

- Solve for r using quadratic formula

$$a\,\mathbf{r}^2 + b\,\mathbf{r} + c = 0 \Rightarrow \mathbf{r} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\mathbf{r} = \frac{-4p\cos(\nu) \pm \sqrt{16p^2\cos^2(\nu) + 16p^2\sin^2(\nu)}}{2\sin^2(\nu)}$$

- Use some trig identities to simplify

$$\cos^2(\nu) + \sin^2(\nu) = 1$$

$$\sin^2(\nu) = 1 - \cos^2(\nu) = [1 - \cos(\nu)][1 + \cos(\nu)]$$

$$\boxed{\mathbf{r} = \frac{-2p[\cos(\nu) \pm 1]}{[1 - \cos(\nu)][1 + \cos(\nu)]}}$$

Parabola Equation

Polar Form (concluded)

- r must be ≥ 0, therefore pick - sign

$$\mathbf{r} = \frac{-2\,p\left[\cos(\nu) \pm 1\right]}{\left[1 - \cos(\nu)\right]\left[1 + \cos(\nu)\right]} = \frac{-2\,p\left[\cos(\nu) - 1\right]}{\left[1 - \cos(\nu)\right]\left[1 + \cos(\nu)\right]}$$

$$= \boxed{\frac{2\,p}{\left[1 + \cos(\nu)\right]}}$$

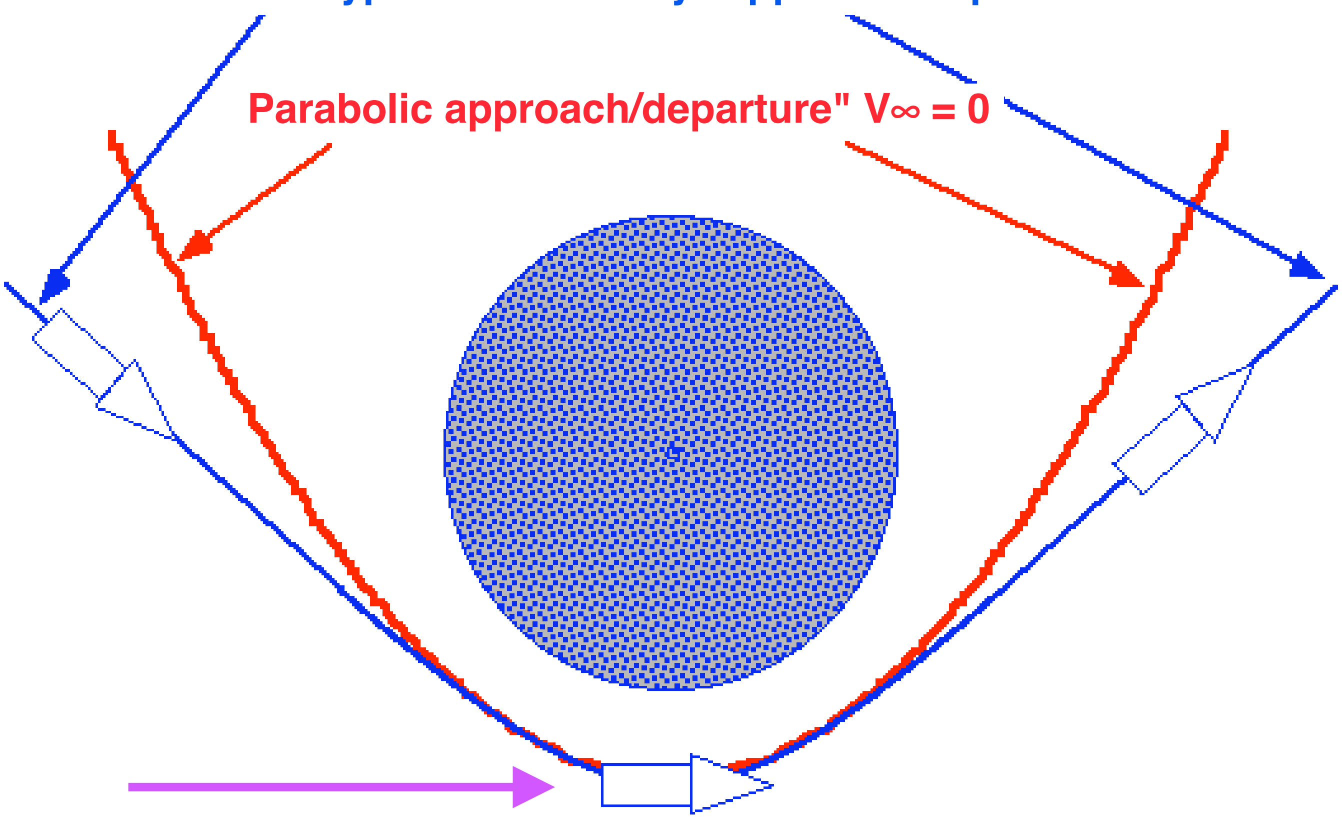
$$\{r_{max} @ \nu = \pi\},\ \frac{2\,p}{\left[1 + \cos(\pi)\right]} = \frac{2\,p}{\left[1 + (-1)\right]} = \infty$$

$$\{r_{min} @ \nu = 0\},\ \frac{2\,p}{\left[1 + \cos(0)\right]} = \frac{2\,p}{\left[1 + (1)\right]} = p$$

$p = r_{min}$

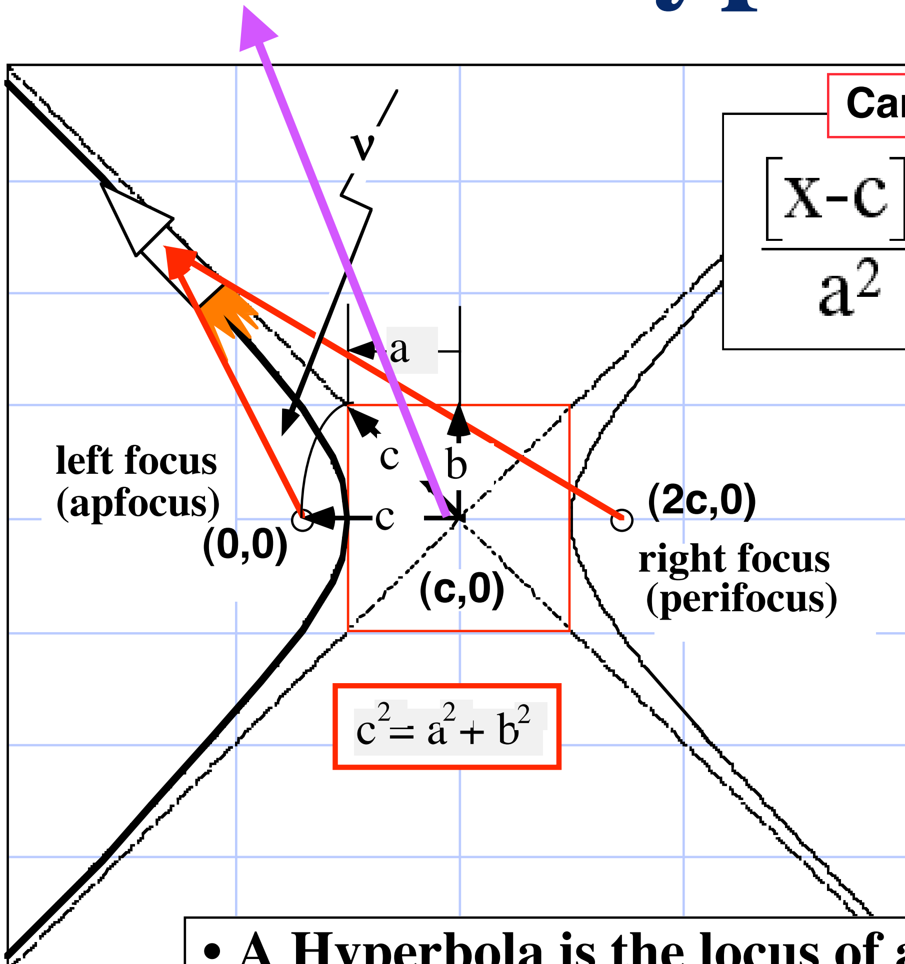
Perigee!

Hyperbolic Trajectories:



• If "V ∞" > 0 , then probe will approach and depart along a hyperbolic trajectory

What is a Hyperbola?



Cartesian Equation

$$\frac{[x-c]^2}{a^2} - \frac{y^2}{b^2} = 1$$

- **A Hyperbola is the locus of all points whose *difference* from two fixed points is constant**

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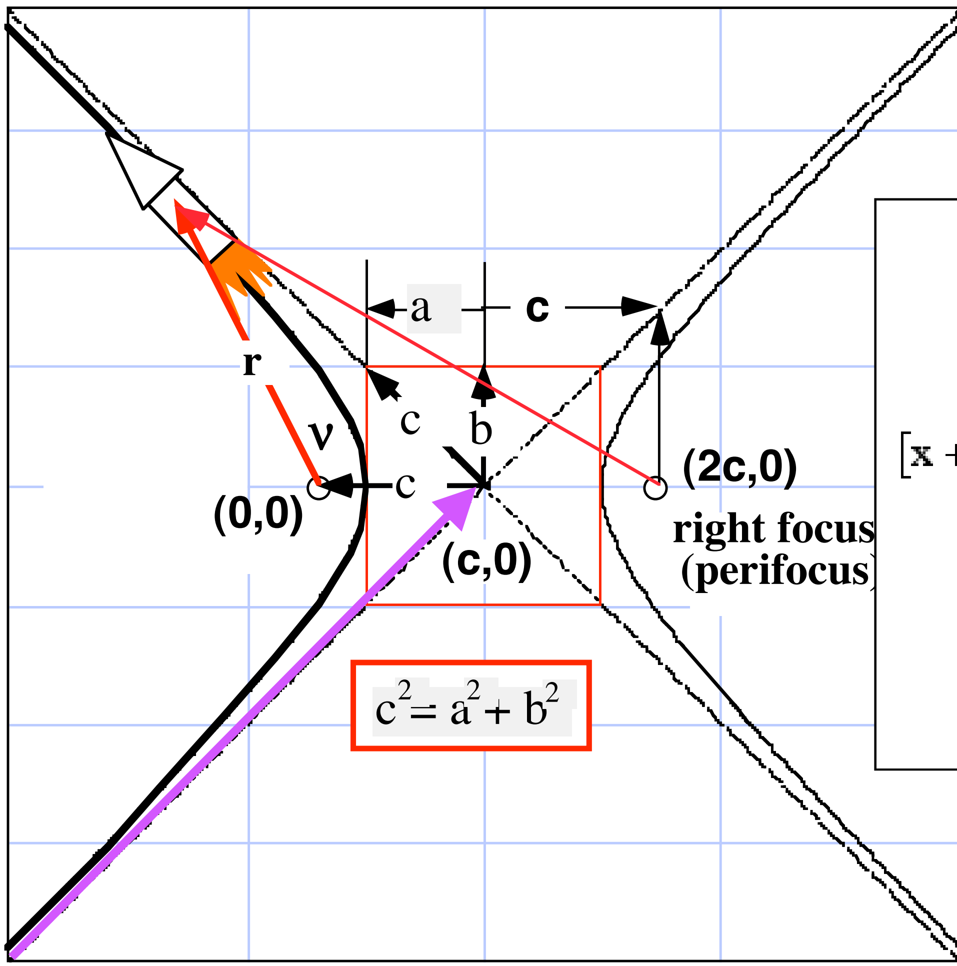
Mechanical & Aerospace Engineering

Hyperbolic Equation:

polar form



$$\sqrt{[x+2c]^2 + y^2} - \sqrt{x^2 + y^2} = 2a$$

$$\Downarrow$$

$$[x + 2c]^2 + y^2 = \left[2a + \sqrt{x^2 + y^2}\right]^2$$

$$\Downarrow$$

$$[x + 2c]^2 + y^2 = 4a^2 + 4a\sqrt{x^2 + y^2} + x^2 + y^2$$

$$\Downarrow$$

$$xc + c^2 = a^2 + a\sqrt{x^2 + y^2}$$

$$\Downarrow$$

$$\boxed{\frac{c}{a}[x+c] - a = \sqrt{x^2 + y^2}}$$

Hyperbolic Equation:

polar form (cont'd)

- From earlier analysis

$$\boxed{\frac{c}{a}\left[\,x+c\,\right]-a=\sqrt{x^2+y^2}=r}$$

$$\boxed{x = -\,r\cos(\nu)}$$

$$\frac{c}{a}\left[\,c-r\cos(\nu)\,\right]-a=r$$

$$\Downarrow$$

$$r\left[\,1+\frac{c}{a}\cos(\nu)\,\right]=\frac{c^2-a^2}{a}$$

$$\Downarrow$$

$$\frac{c^2-a^2}{a}=\frac{\left[a^2+b^2\right]-a^2}{a}=\frac{b^2}{a}$$

$$\Downarrow$$

$$\boxed{r=\frac{b^2}{a}\,\frac{1}{\left[\,1+\frac{c}{a}\cos(\nu)\,\right]}}$$

Hyperbolic Equation:

polar form (concluded)

- **Defining the "*hyperbolic eccentricity*"**

$$e_{hyp} = \sqrt{1 + \frac{b^2}{a^2}} \Rightarrow \frac{b^2}{a} = a\left[\ e_{hyp}^2 - 1\right]$$

$$e^2{}_{hyp} = \frac{a^2 + b^2}{b^2} = \frac{c^2}{b^2}$$

$$\boxed{\begin{array}{c} \Downarrow \\ r = \dfrac{a\left[\ e_{hyp}^2 - 1\right]}{\left[1 + e_{hyp}\cos(\nu)\right]} \end{array}}$$

Hyperbolic Asymptotes

- **What is the behavior of a Hyperbola at "infinity"**
i.e. along away from earth

$$\lim_{x,\, y \to \infty} \left[\frac{[x-c]^2}{a^2} - \frac{y^2}{b^2} = 1 \right] \Rightarrow \lim_{x,\, y \to \infty} \left[[x-c]^2 - \frac{a^2}{b^2} y^2 - a^2 = 0 \right]$$

$$\Downarrow$$

asymptotes

$$\Downarrow$$

$$[x-c]^2 = \frac{a^2}{b^2} y^2 \Rightarrow \boxed{y = \pm \frac{b}{a} [x-c]}$$

Hyperbolic Behavior at ∞

asymptotes

$$\Downarrow$$

$$\boxed{y = \pm \frac{b}{a}[x-c]} \Rightarrow r \sin(\nu) = \pm \frac{b}{a}[-r\cos(\nu) - c]$$

$$\lim_{r \to \infty} \left(\sin(\nu) + \left[\pm \frac{b}{a}\cos(\nu)\right] = -\pm \frac{b}{a}\frac{c}{r} \right) = \sin(\nu_\infty) + \left[\pm \frac{b}{a}\cos(\nu_\infty)\right] = 0$$

$$\Downarrow$$

$$\tan(\nu_\infty) = \mp \frac{b}{a} = \mp \sqrt{e_{hyp}^2 - 1} \Rightarrow$$

departure: $\nu_\infty = \pi - \tan^{-1}\left[\sqrt{e_{hyp}^2 - 1}\right]$

arrival: $\boxed{\nu_\infty = \pi + \tan^{-1}\left[\sqrt{e_{hyp}^2 - 1}\right]}$

Kepler's First Law

(Summary)

Circle: $\boxed{r = a}$

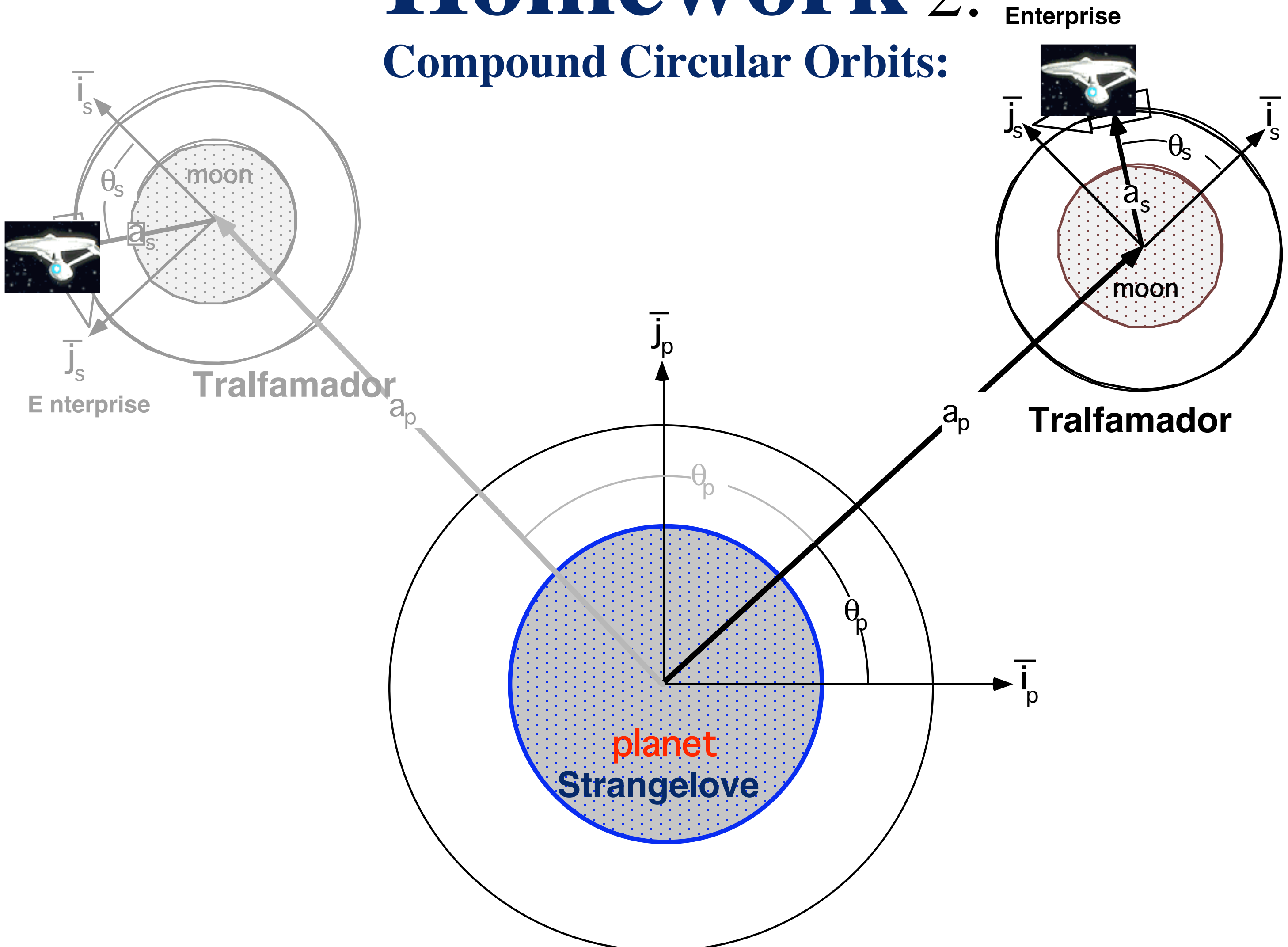
Ellipse: $\boxed{r = \dfrac{a\left[1 - e^2\right]}{\left[1 + e\ \cos(\nu)\right]}}$

Parabola: $\boxed{r = \dfrac{2\,p}{\left[1 + \cos(\nu)\right]}}$

Hyperbola: $\boxed{r = \dfrac{a\left[e_{hyp}^2 - 1\right]}{\left[1 + e_{hyp}\cos(\nu)\right]}}$

Homework ~~2~~ 3:

Compound Circular Orbits:



Homework: Compound Orbits

(cont'd)

- Starship *Enterprise* orbits alien moon *Tralfamador* in a circular orbit of radius a_s

- Moon orbits alien planet *Strangelove* in circular orbit with radius a_p

- Alien GPS system orbiting moon gives position relative to *Tralfamadorian* -fixed coordinate system.

- Due to gravitational damping *Tralfamador*, always keeps the same face directed towards *Strangelove*

Homework: Compound Orbits

(cont'd)

- **Compute the position vector of the *Enterprise relative to Strangelove* ... in the *Strangeloveian* -fixed coordinate system --** $\bar{R}_{sp}$

- **Solution should have a_s, a_p, θ_s, θ_p as parameters**

Hint 1 : $\bar{i}_s = (\bar{i}_r)_{planet}$

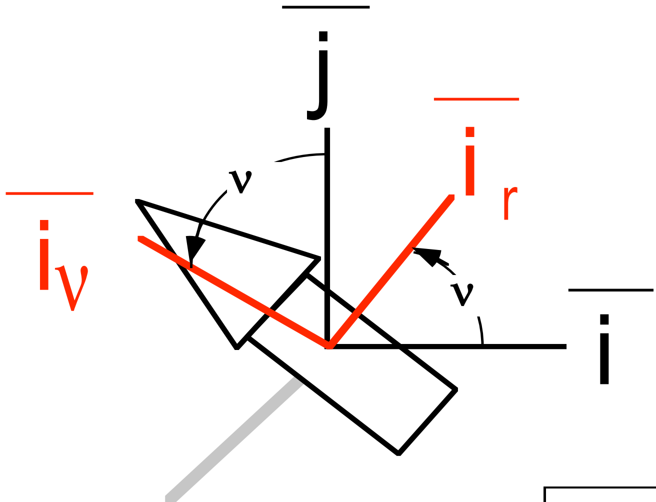
$\bar{j}_s = (\bar{i}_\theta)_{planet}$

Hint 2 : $\cos[a+b] = \cos[a]\cos[b] - \sin[a]\sin[b]$

$\sin[a+b] = \sin[a]\cos[b] + \cos[a]\sin[b]$

Hint 3 : $\bar{R}_{sp} = \bar{R}_s + \bar{R}_p$

Coordinate Transformations:



{i, j} fixed in space

Transform ⇒ polar ↑ inertial

$$\boxed{\begin{aligned} \bar{i} &= \bar{i}_r \cos[\nu] - \bar{i}_\nu \sin[\nu] \\ \bar{j} &= \bar{i}_r \sin[\nu] + \bar{i}_\nu \cos[\nu] \end{aligned}}$$

Transform ⇒ inertial ↑ polar

$$\boxed{\begin{aligned} \bar{i}_r &= \bar{i} \cos[\nu] + \bar{j} \sin[\nu] \\ \bar{i}_\nu &= -\bar{i} \sin[\nu] + \bar{j} \cos[\nu] \end{aligned}}$$

Homework: Circular Orbits

(concluded)

- Compute the velocity vector of the *Enterprise relative to Strangelove* ... in the *Strangeloveian* -fixed coordinate system.

Hint 4 :

$$\overline{V}_{sp} = \frac{d}{dt}\left[\overline{R}_{sp}\right] = \frac{d}{dt}\left[\overline{R}_s + \overline{R}_p\right]$$

$$\omega_s \equiv \frac{d}{dt}\left[\theta_s\right] \qquad \omega_p \equiv \frac{d}{dt}\left[\theta_p\right]$$