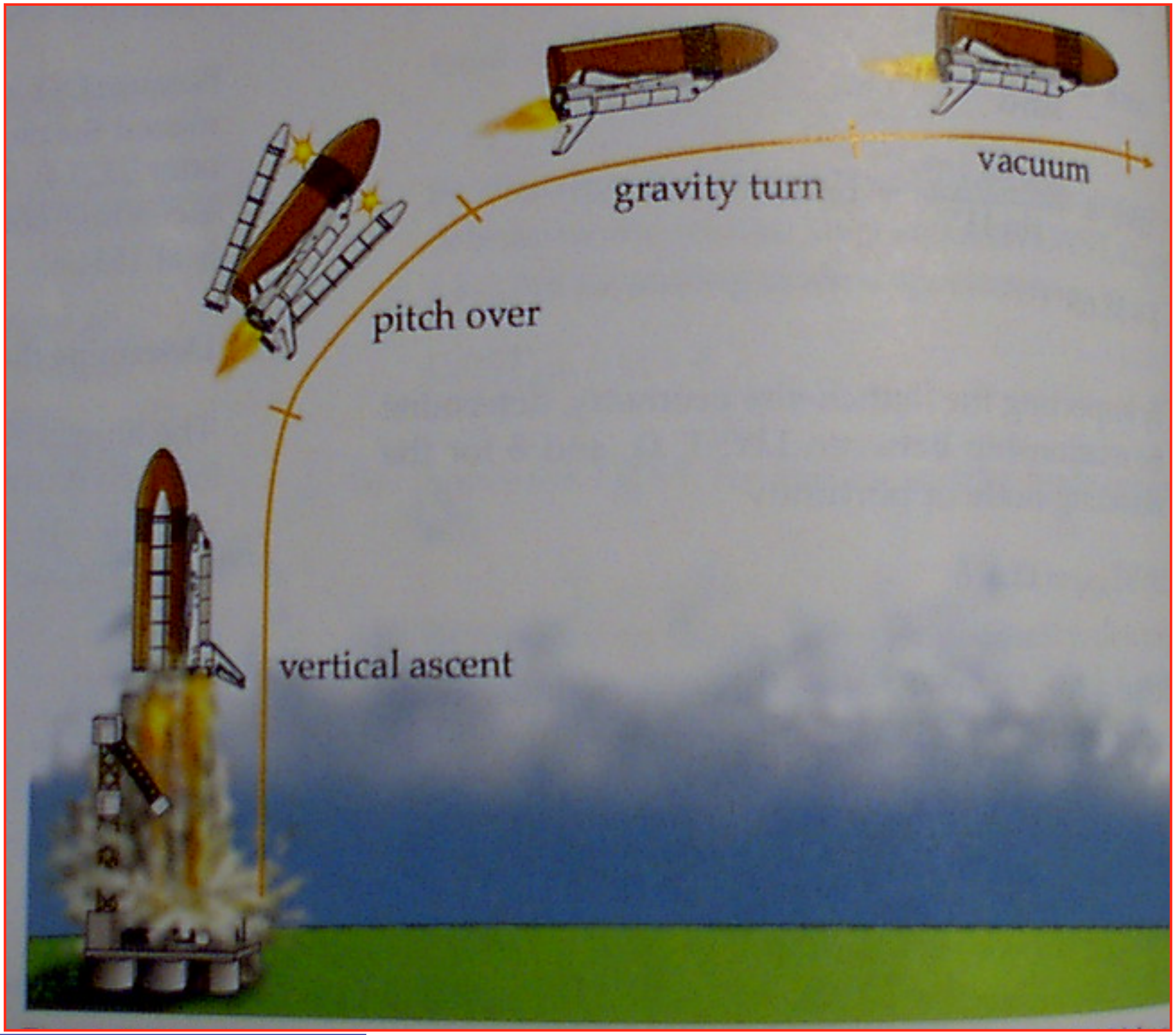
Section 3.1 Midcourse Review



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The Rocket Equation

- *Consider a rocket burn of duration t_{burn}*

Initial Mass

$$V_{final} = V_0 + g_0 I_{sp} \ln\left[\frac{M_0}{M_{final}}\right]$$

Initial Velocity

Final Mass

Final Velocity

Propellant Budgeting Equation

- Solving for P_{mf}

$$\left(P_{mf}\right)_{burn} = e^{\left[\frac{\Delta V_{burn}}{g_0 I_{sp}}\right]} - 1$$

- Mass of Fuel and oxidizer required for a burn to give a specified ΔV

$$M_{fuel + oxidizer} = \left[M_{dry} + M_{payload}\right]\left[e^{\left[\frac{\Delta V_{burn}}{g_0 I_{sp}}\right]} - 1\right]$$

Specific Impulse

• *Specific Impulse is a scalable characterization of a rocket's Ability to deliver a certain (specific) impulse for a given weight of propellant*

$$I_{sp} = \frac{\mathrm{I}_{mpulse}}{g_0 M_{propellant}} = \frac{\int_0^t F_{thrust}\, dt}{g_0 \int_0^t \dot{m}_{propellant}\, dt}$$

Mean specific impulse

Specific Impulse *(cont'd)*

$$I_{sp} = \frac{1}{g_0} \frac{F_{thrust}}{\dot{m}_{propellant}} \rightarrow \dot{m}_e \equiv \dot{m}_{propellant} \rightarrow$$

$$I_{sp} = \frac{1}{g_0} \left[V_e + \frac{p_e A_e - p_\infty A_e}{\dot{m}_e} \right] \equiv \frac{C_e}{g_0}$$

"Units ~ seconds"

- *Effective Exhaust Velocity*

Available Delta V

$$\Delta V_{t_{burn}} = g_0 \cdot I_{sp} \left[\ln\left(1 + P_{mf}\right)\right] -$$

$$\text{"}combustion\ \Delta V\text{"}$$

$$\int_0^{t_{burn}} g(t)\cdot \sin\theta dt - \sqrt{\int_0^{t_{burn}} \frac{\rho V^3}{\beta} dt}$$

"*gravity loss*" "*drag loss*"

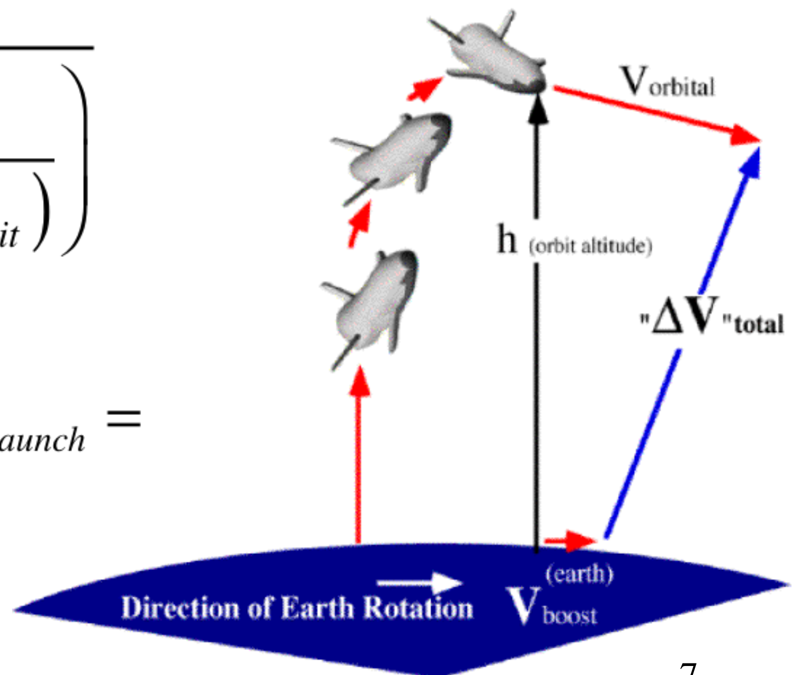
Required ΔV

- Root Sum Square of Required Kinetic Energy (Horizontal) + Potential Energy (Vertical)

$$\left(\Delta V_{required}\right)_{total} = \sqrt{\left(V_{orbital} - V_{\substack{"boost" \\ earth}}\right)^2 + \Delta V_{gravity}^{\;2}} =$$

$$\sqrt{\left(V_{orbital} - V_{\substack{"boost" \\ earth}}\right)^2 + \left(\frac{2\cdot\mu\cdot h_{orbit}}{R_\oplus\cdot\left(R_\oplus + h_{orbit}\right)}\right)}$$

$$V_{"boost"} = \left(R_\oplus + h_{launch}\right)\cdot\Omega_\oplus\cdot\cos\lambda\cdot\sin Az_{launch} =$$
$$\left(R_\oplus + h_{launch}\right)\cdot\Omega_\oplus\cdot\cos i$$

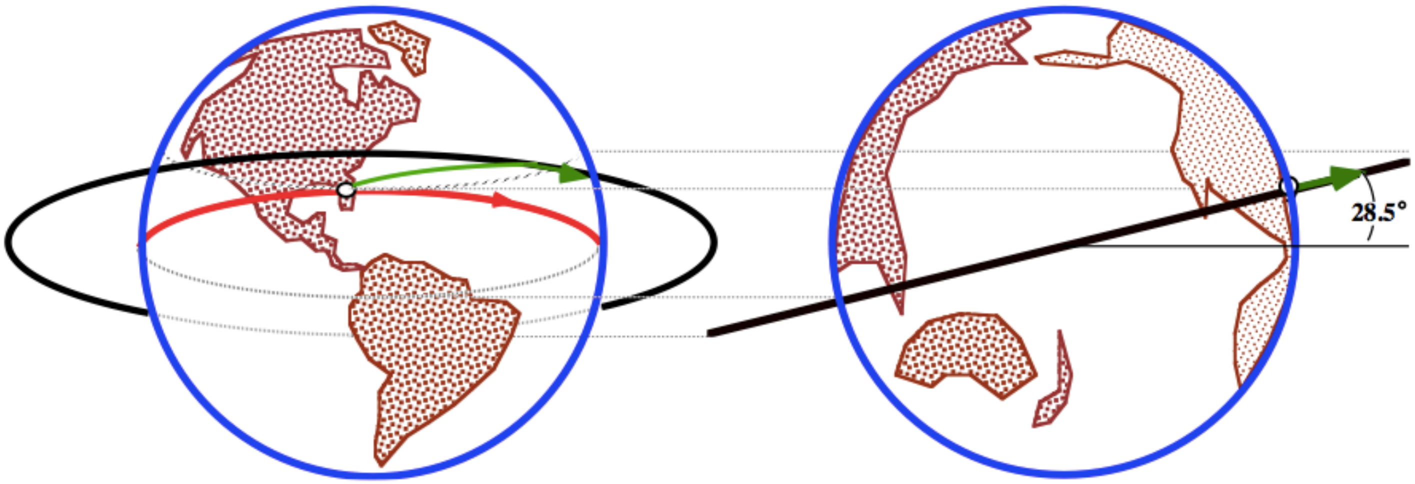


Required $\Delta V_{(2)}$

- Example Calculation, Shuttle to Due East LEO Orbit

Kennedy Space Center (KSC)
Due-East Launch
28.5° Inclination Orbit

110 km MECO



$$V_{"boost"} = \left(R_{\oplus} + h_{launch}\right) \cdot \Omega_{\oplus} \cdot \cos i_{orbit} =$$

$$6373.25\left(\frac{2\pi}{23\cdot 3600 + 56\cdot 60 + 4.1}\right)\cos\left(\frac{\pi}{180}28.5\right) = 0.408426 \ km/sec$$

Required $\Delta V_{(3)}$

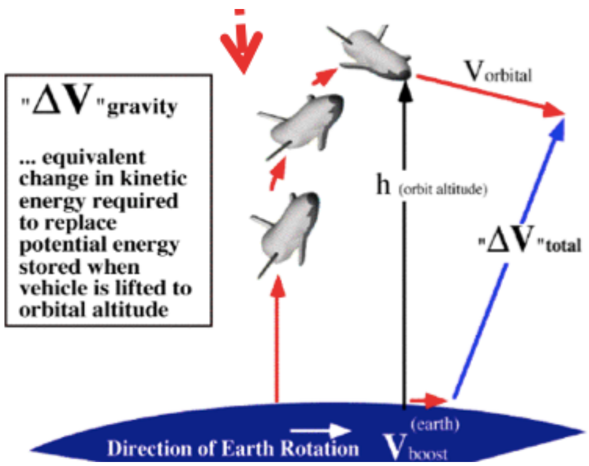
- Example Calculation, Shuttle to Due East LEO Orbit

- "Lift" Delta V… equivalent change in potential energy

$$\Delta V_{gravity} = \sqrt{2\frac{\mu \cdot h_{orbit}}{R_{earth} \cdot \left(R_{earth} + h_{orbit}\right)}} =$$

$$\left(\frac{2\,(3.9860044\cdot 10^{5})\,110}{6373.25\,(6373.25 + 110)}\right)^{0.5} = 1.45681$$

km/sec



110 km MECO

Required $\Delta V_{(3)}$

- Orbital Velocity …

$$V_{orbit} = \sqrt{\frac{\mu}{\left(R_{earth} + h_{orbit}\right)}} = \left(\frac{(3.9860044 \cdot 10^5)}{6373.25 + 110}\right)^{0.5} = 7.84102 \text{ km/sec}$$

$$\left(\Delta V_{total}\right)_{required} = \sqrt{\left(V_{orbit} - V_{"boost"}\right)^2 + \left(\Delta V_{gravity}\right)^2} =$$

$$\left(\left(7.84102 - 0.408426\right)^2 + 1.45732^2\right)^{0.5} = 7.57412 \; km/sec$$

110 km MECO

STS-114 Trajectory Example

"Return to Flight"
After Columbia Accident

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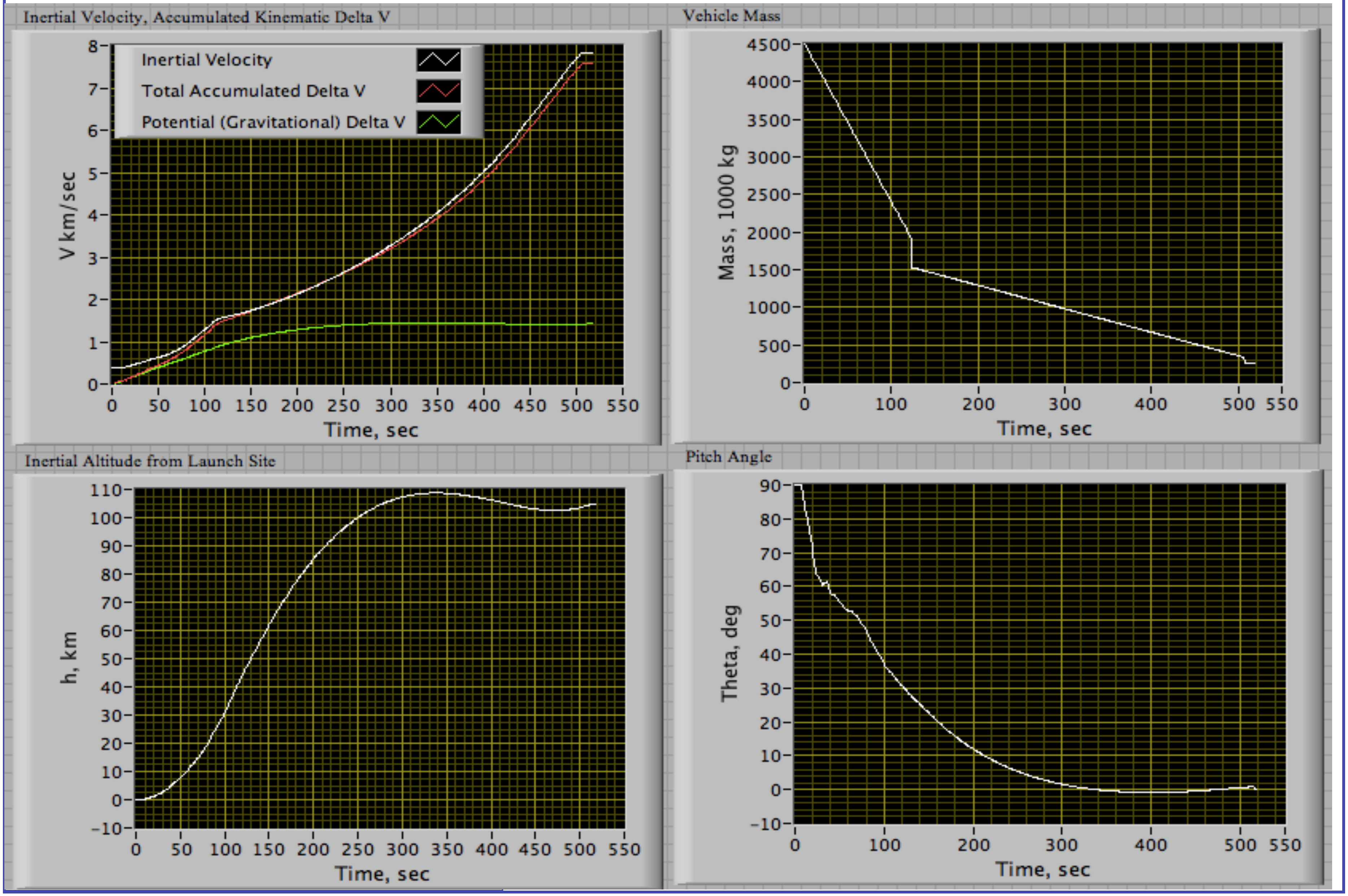
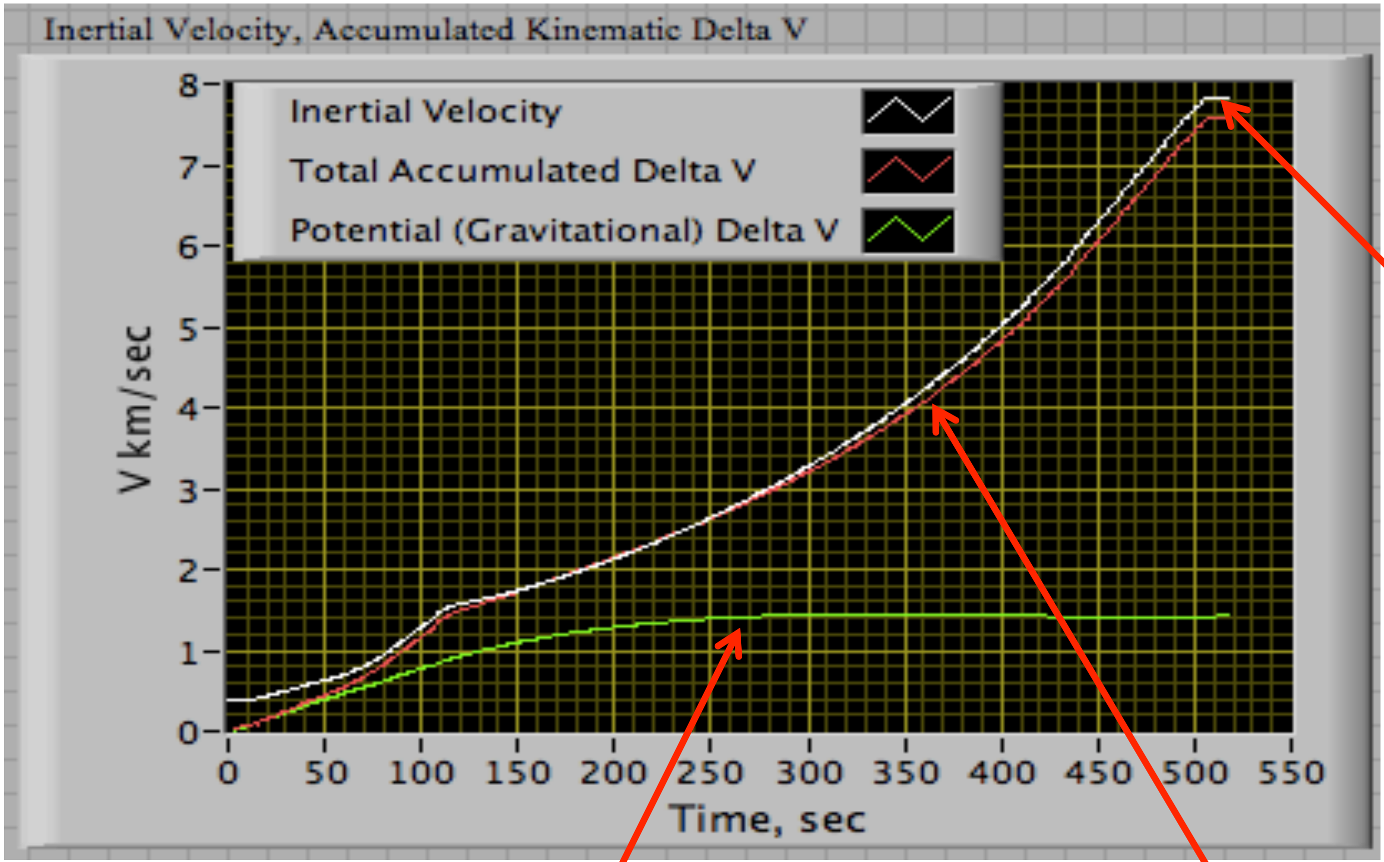
STS-114 Trajectory Example (2)



STS-114 Trajectory Example (3)

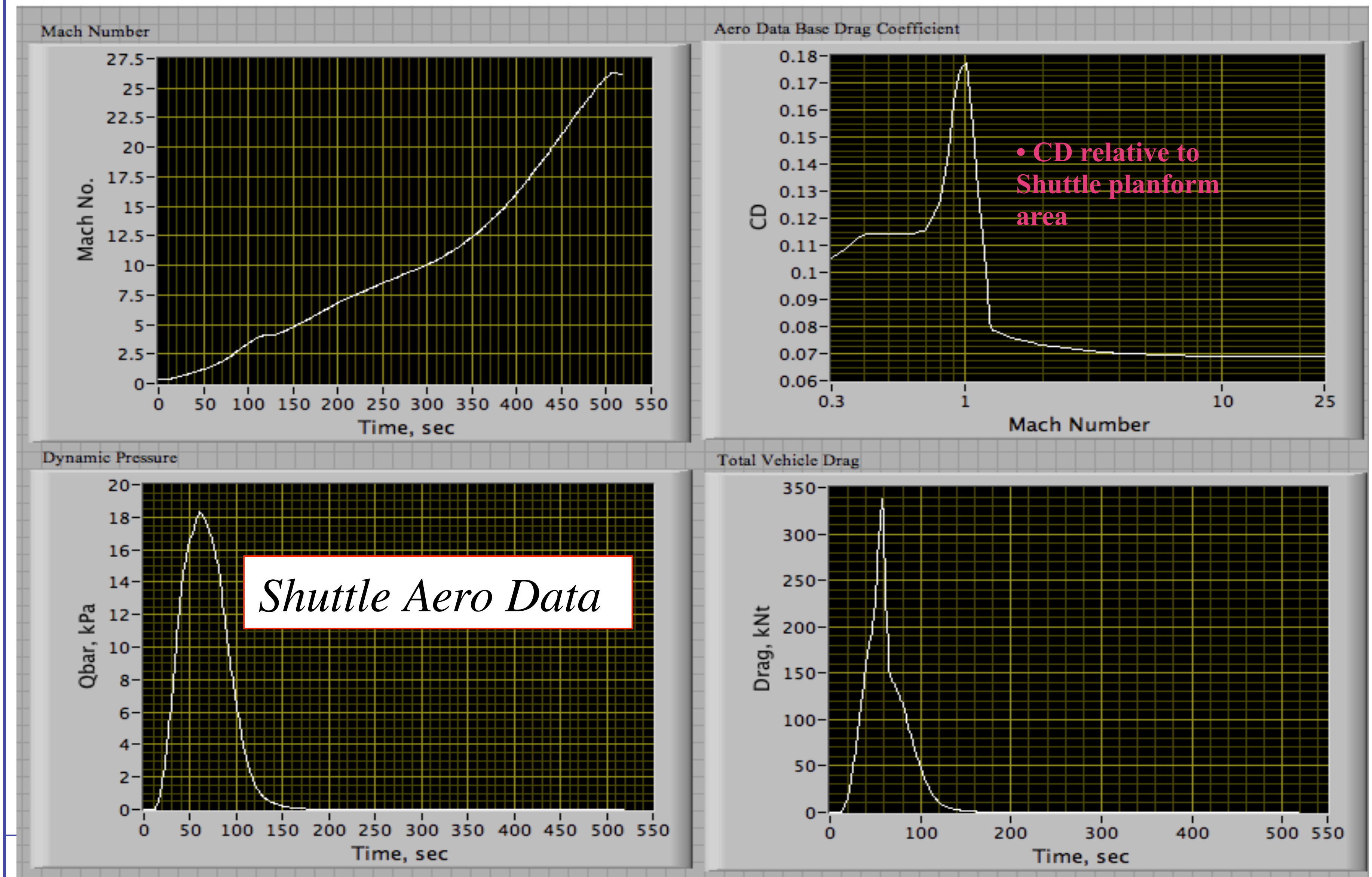


$$V_{orbit} = \sqrt{\frac{\mu}{R_{\oplus} + h_{orbit}}}$$

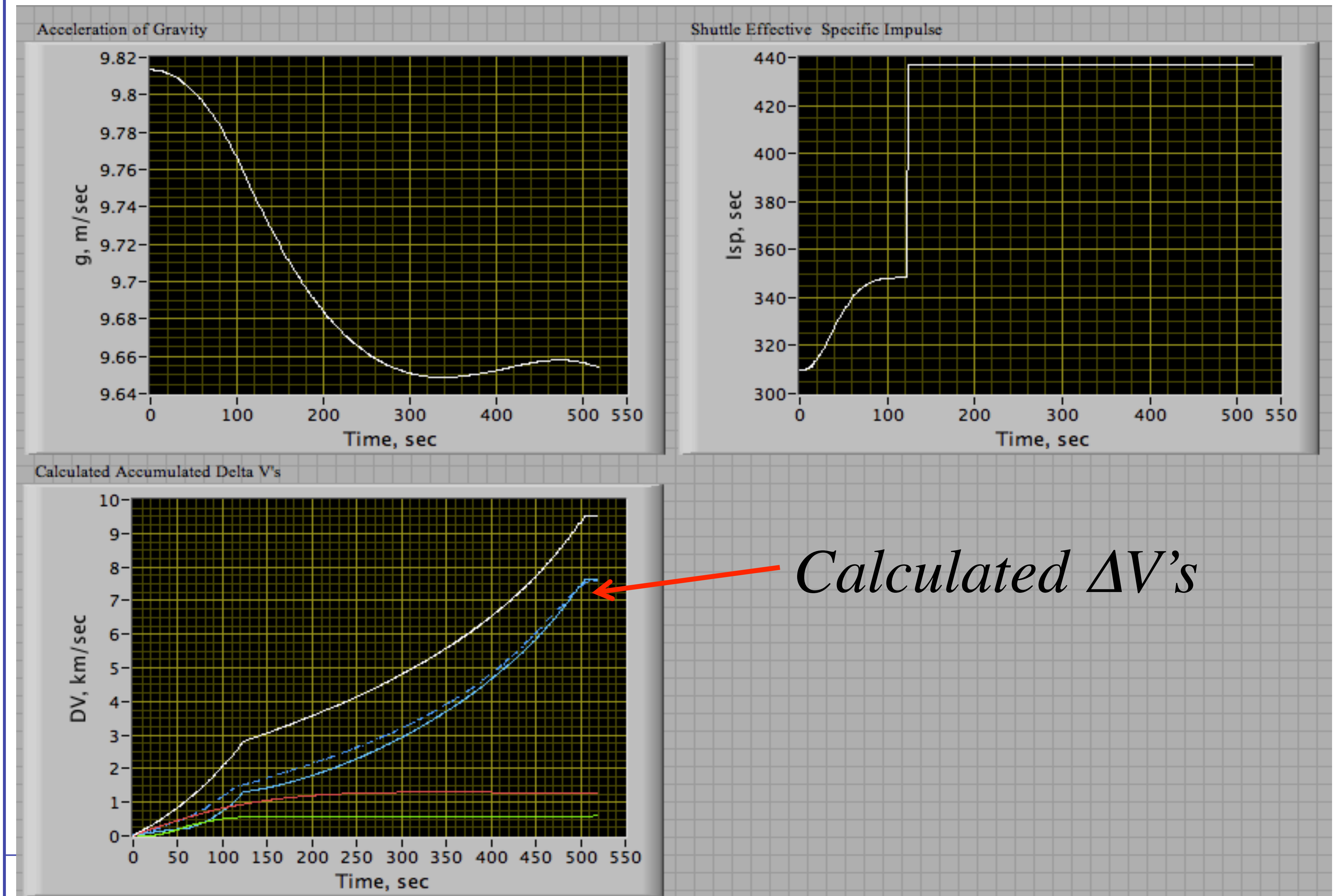
$$\Delta V_{\substack{grav \\ P.E.}} = \sqrt{2\frac{\mu \cdot h}{R_{\oplus} \cdot \left(R_{\oplus} + h\right)}}$$

$$\Delta V_{accumulated} = \sqrt{\left(V - V_{"boost"}\right)^2 + \Delta V_{\substack{grav \\ P.E.}}^{\ \ 2}}$$

STS-114 Trajectory Example (4)



STS-114 Trajectory Example (5)



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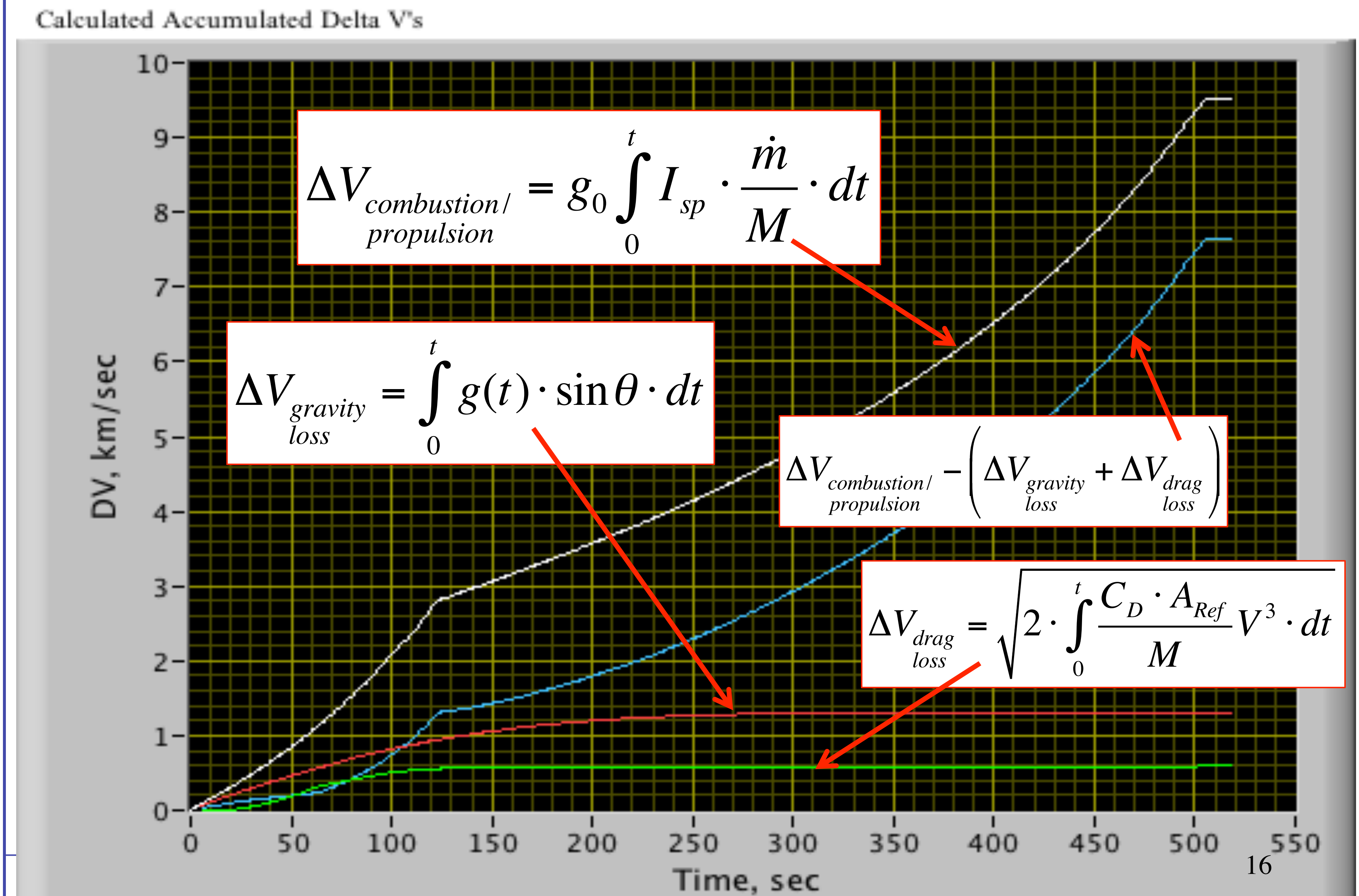
STS-114 Trajectory Example (6)



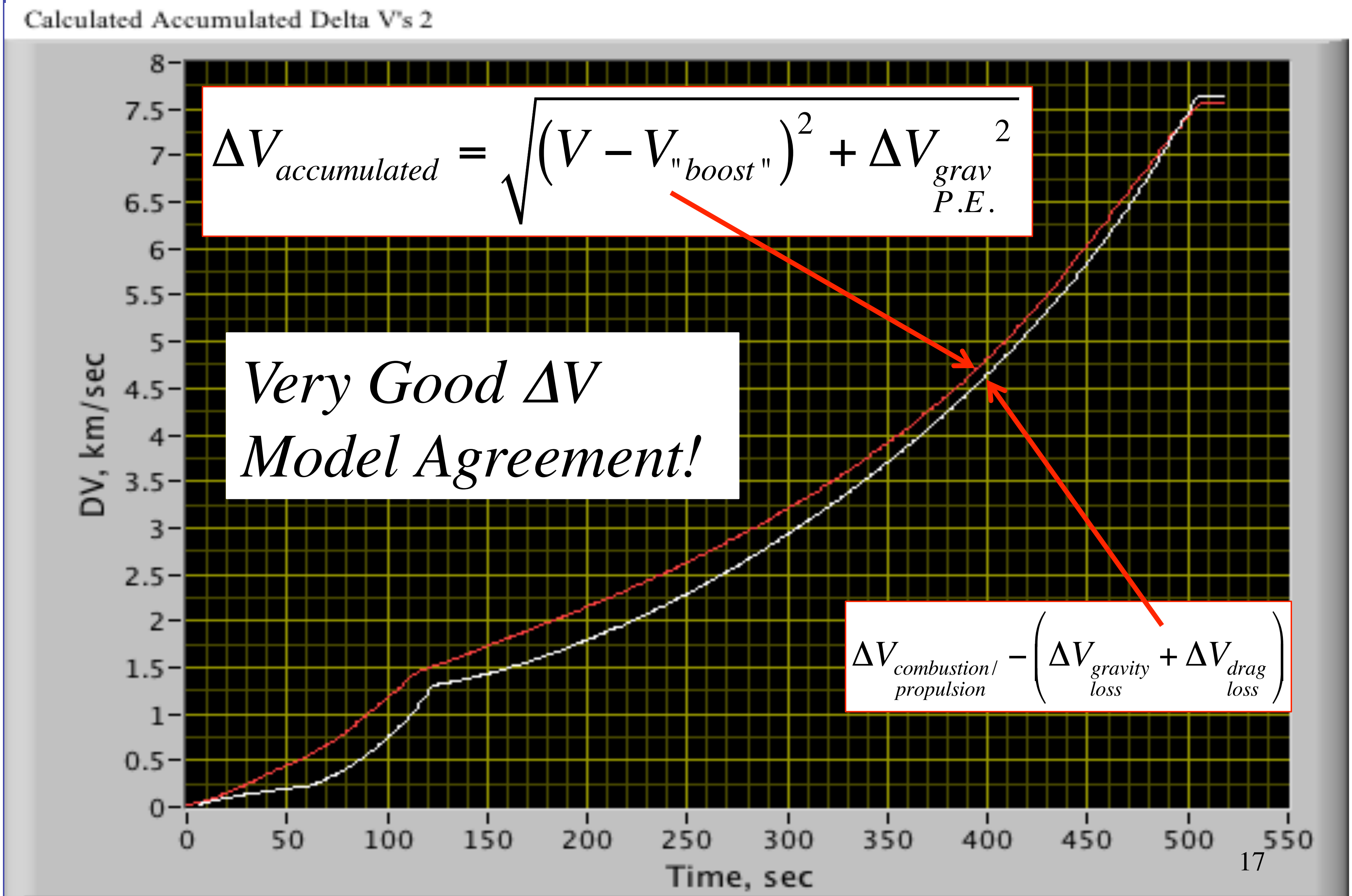
$$\Delta V_{\substack{combustion/\\propulsion}} = g_0 \int_0^t I_{sp} \cdot \frac{\dot{m}}{M} \cdot dt$$

$$\Delta V_{\substack{gravity\\loss}} = \int_0^t g(t) \cdot \sin\theta \cdot dt$$

$$\Delta V_{\substack{combustion/\\propulsion}} - \left(\Delta V_{\substack{gravity\\loss}} + \Delta V_{\substack{drag\\loss}}\right)$$

$$\Delta V_{\substack{drag\\loss}} = \sqrt{2 \cdot \int_0^t \frac{C_D \cdot A_{Ref}}{M} V^3 \cdot dt}$$

STS-114 Trajectory Example (7)



"Orbitology" Summary (1)

- **Kepler's First Law:** *In a two body universe, orbit of a satellite is a conic section with the Earth centered at one of the focii*

Circle: $r = a$

Ellipse: $r = \frac{a\,[1 - e^2]}{[1 + e\ \cos(v)]}$

Parabola: $r = \frac{2\,p}{[1 + \cos(v)]}$

Hyperbola: $r = \frac{a\,[e_{hyp}^2 - 1]}{[1 + e_{hyp}\cos(v)]}$

The Conic Sections:

r – radius vector

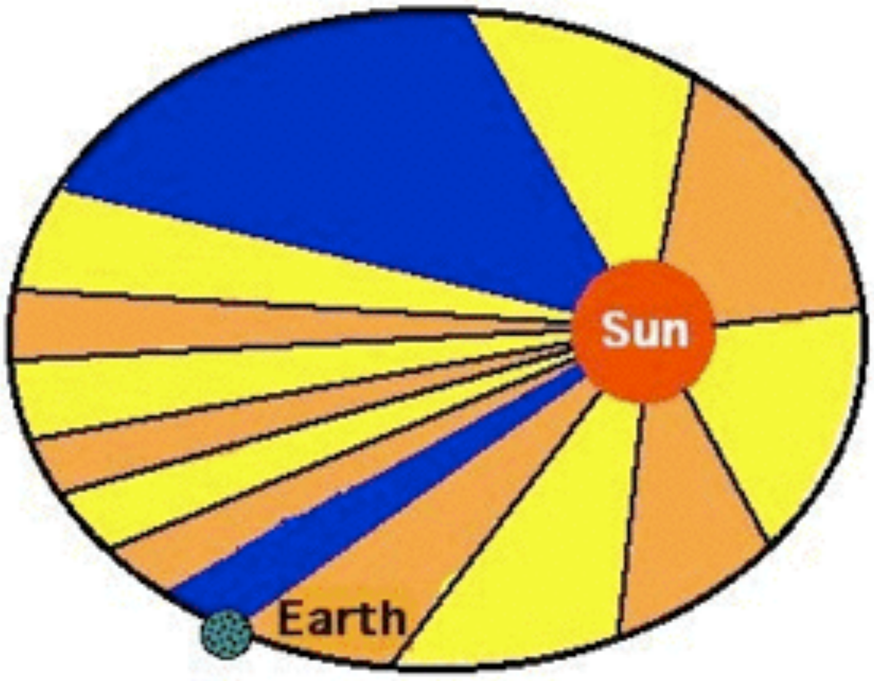
v – true anomaly

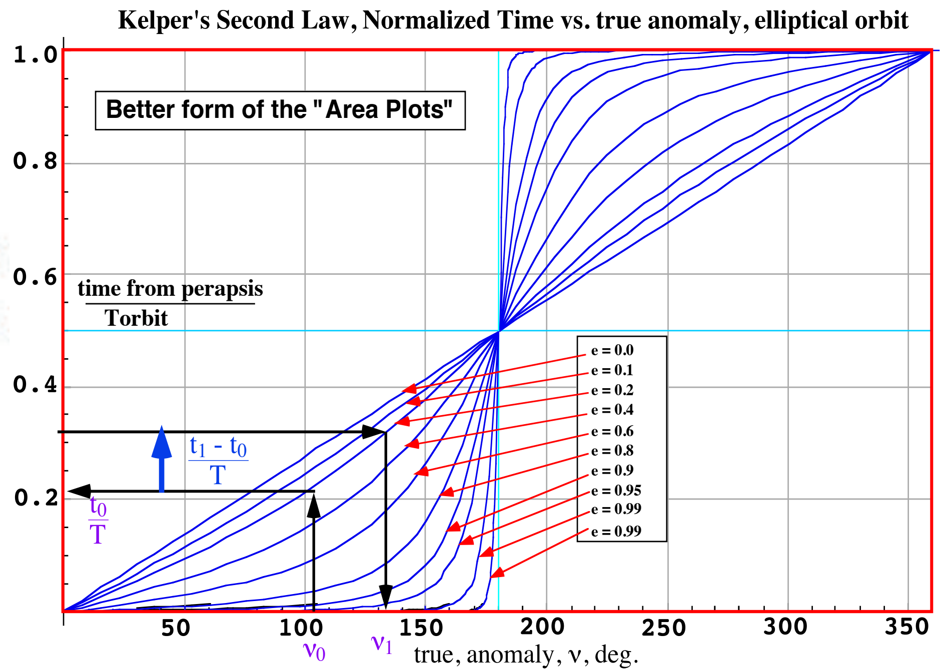
e – orbit eccentricity

a – orbit semi-major axis

"Orbitology" Review (2)

- **Kepler's Second Law:** *In a two body universe, radius vector from the Earth to the satellite sweeps out equal areas in equal times*





"Orbitology" Review (3)

Alternate Statement of of Kepler's Second law:

$$\frac{\bar{L}}{m} \equiv \bar{l} = \omega\, r^2\, \bar{i}_k \Rightarrow \omega\, r^2 = l \text{ (specific angular momentum)}$$

"The angular momentum of an orbiting object is constant"

- *Velocity Vector for an Elliptical Orbit*

$$\bar{V} = r(\nu)\,\omega\left[\frac{[e\sin(\nu)]}{[1 + e\cos(\nu)]}\bar{i}_r + \bar{i}_\nu\right]$$

$$\omega = \frac{\sqrt{\mu\, a\left[1 - e^2\right]}}{r^2}$$

$$\mu = G \cdot M_\oplus$$

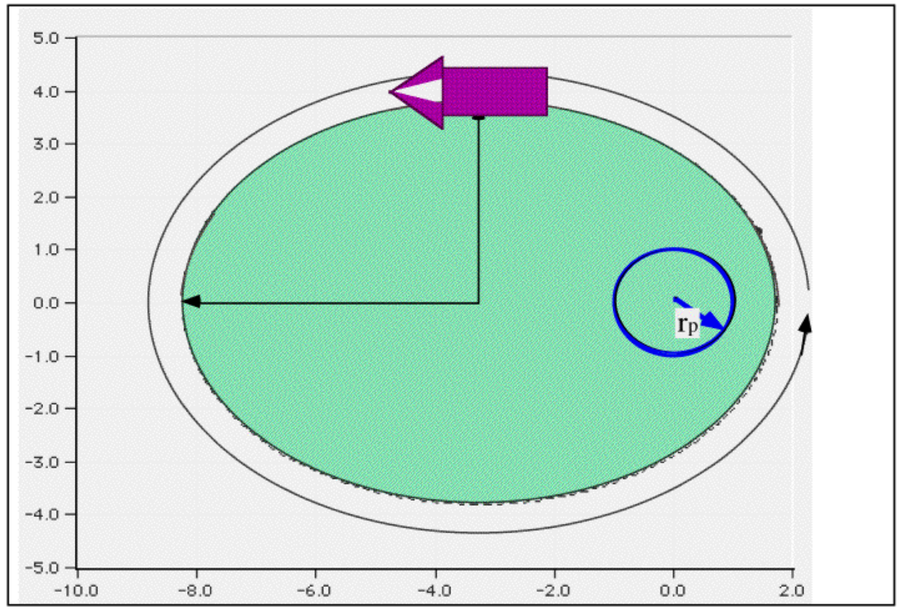
"Orbitology" Review (4)

Kepler's Third law:

Total Area of an Elliptical Orbit

Orbital Period

$$T = \frac{2\pi a^{3/2}}{\sqrt{\mu}}$$



$\mu = G M \Rightarrow$ planetary gravitational parameter

- $\mu_{earth} =$ 3.9860044 km^3/sec^2

$$\mu_{moon} = 4.903 \times 10^3 \frac{m^3}{sec^2}$$

$$\mu_{sun} = 1.327 \times 10^{20} \frac{m^3}{sec^2}$$

$$\mu_{Mars} = 4.269 \times 10^4 \frac{m^3}{sec^2}$$

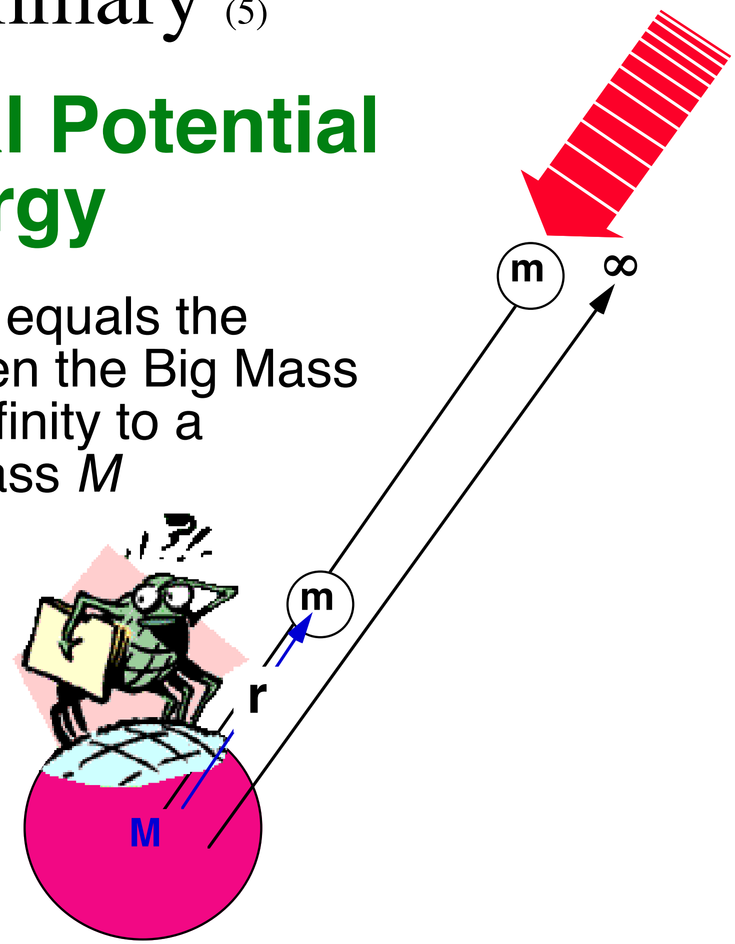
Orbitology Summary (5)

Gravitational Potential Energy

•**G***ravitational potential energy* equals the amount of energy released when the Big Mass M pulls the small mass *m* at infinity to a location *r* in the vicinity of a mass *M*

• **Energy of position**

$$P_{E_{grav}} \equiv E_{released} = \int_{\infty}^{\mathbf{r}} \mathbf{F} \cdot dr =$$



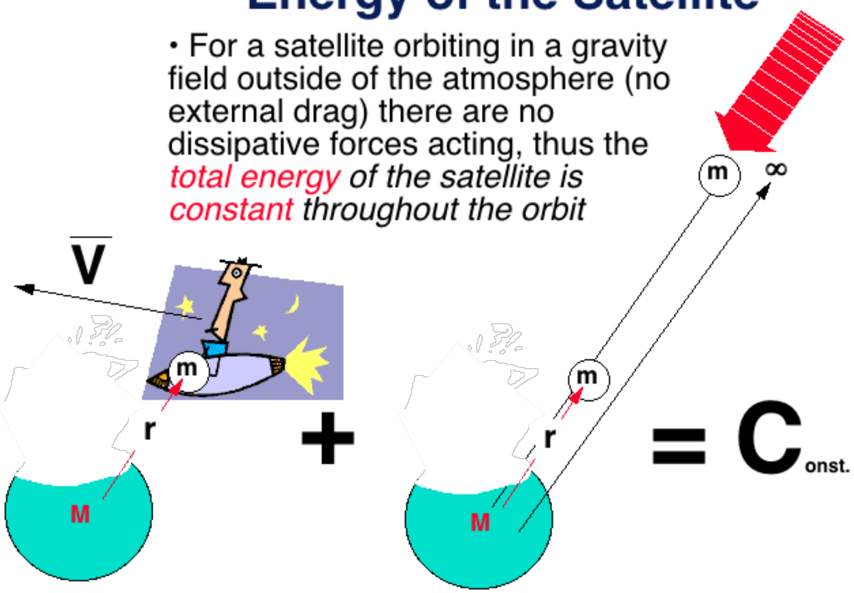
$$\int_{\infty}^{\mathbf{r}} \frac{G\,M\,m}{r^2}\, dr = -\,G\,M\,m\left[\frac{1}{\mathbf{r}} - \frac{1}{\infty}\right] = \boxed{-\frac{G\,M\,m}{\mathbf{r}}}$$

$$G\,M = \mu$$

Orbitology Summary (6)

Total (Mechanical) Energy of the Satellite

- For a satellite orbiting in a gravity field outside of the atmosphere (no external drag) there are no dissipative forces acting, thus the *total energy of the satellite is constant throughout the orbit*



Vis-Viva Equation

Elliptical Orbit

kinetic energy	potential energy	total energy
$\frac{V^2}{2}$	$-\frac{\mu}{r}$	$= -\frac{\mu}{2a}$

$$\frac{V^2}{2} - \frac{\mu}{r} = -\frac{\mu}{2a}$$

Orbitology Summary (7)

Vis-Viva Equation for All the Conic-Sections

Circle: $r = a \Rightarrow$ $$V = \sqrt{\mu\left[\frac{2}{a} - \frac{1}{a}\right]} = \sqrt{\frac{\mu}{a}}$$

Ellipse: $r = \frac{a\left[1 - e^2\right]}{\left[1 + e \cos(\nu)\right]} \Rightarrow$ $$V = \sqrt{\mu\left[\frac{2}{r} - \frac{1}{a}\right]}$$

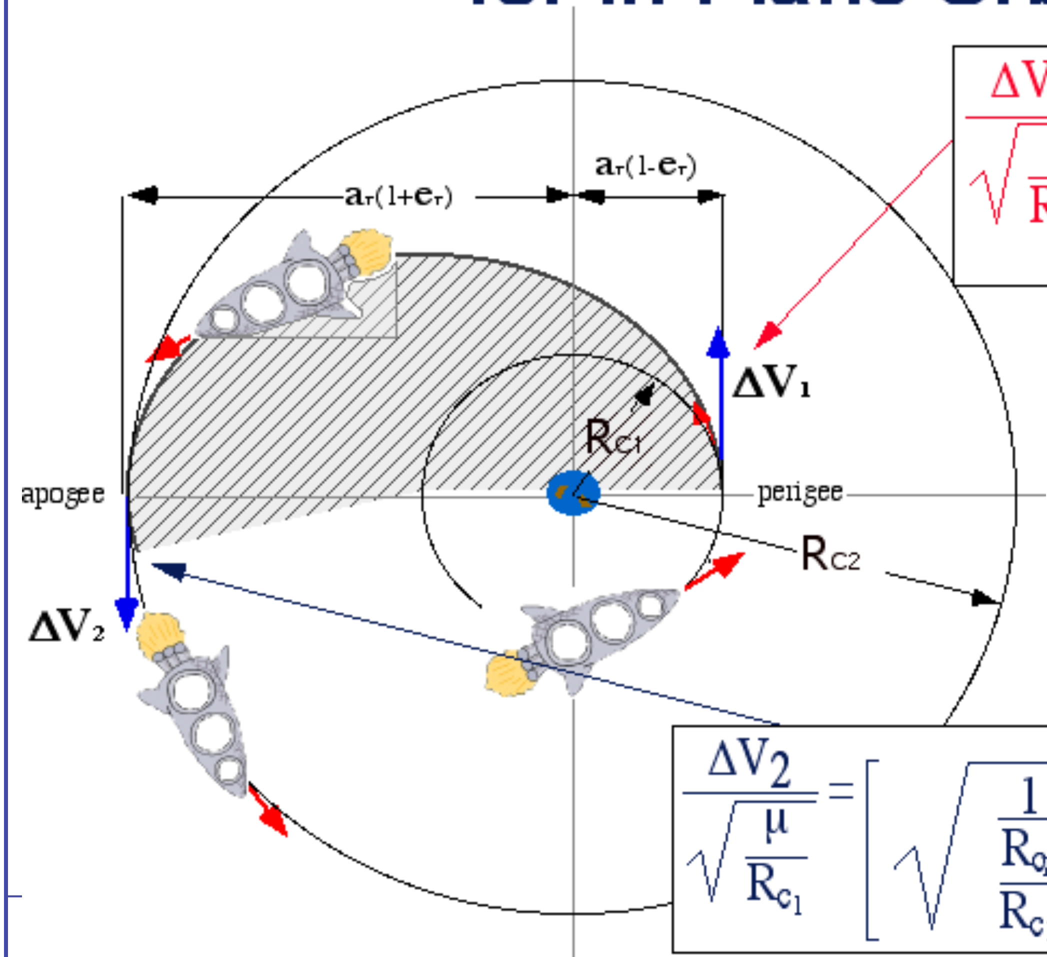
Parabola: $r = \frac{2p}{\left[1 + \cos(\nu)\right]} \Rightarrow$ $$V = \sqrt{\mu\left[\frac{2}{r} - \frac{1}{\infty}\right]} = \sqrt{\frac{2\mu}{r}}$$

Hyperbola: $r = \frac{a\left[e_{hyp}^2 - 1\right]}{\left[1 + e_{hyp} \cos(\nu)\right]} \Rightarrow$ $$V = \sqrt{\mu\left[\frac{2}{r} + \frac{1}{a}\right]}$$

Orbitology Summary (8)

- Hohmann Transfer

Total Delta-Vee Required for in-Plane Orbit Transfer



$$\frac{\Delta V_1}{\sqrt{\frac{\mu}{R_{c_1}}}} = \sqrt{2\left[1 - \frac{1}{\left[\frac{R_{c_2}}{R_{c_1}} + 1\right]}\right]} - 1$$

$$a_T = \frac{[R_{c_2} + R_{c_1}]}{2}$$

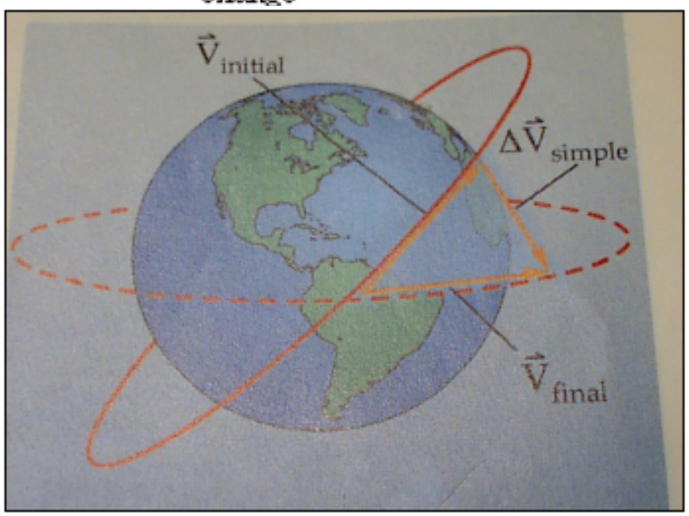
$$e_T = \frac{R_{c_2} - R_{c_1}}{[R_{c_2} + R_{c_1}]}$$

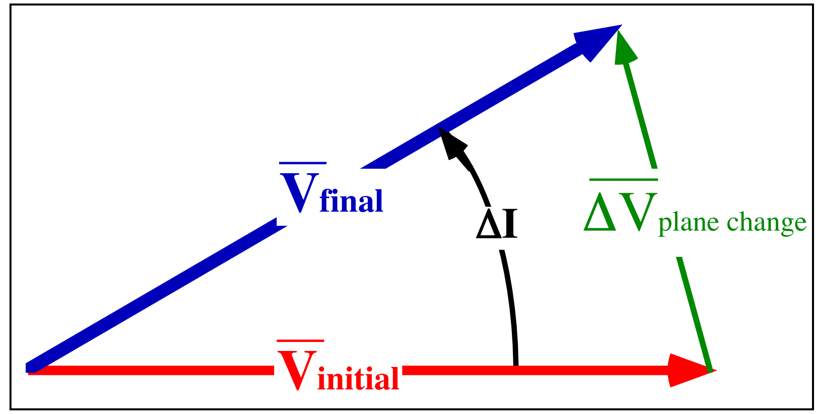
$$\frac{\Delta V_2}{\sqrt{\frac{\mu}{R_{c_1}}}} = \left[\sqrt{\frac{1}{\frac{R_{c_2}}{R_{c_1}}}} - \sqrt{2\left[\frac{1}{\left[\frac{R_{c_2}}{R_{c_1}}\right]} - \frac{1}{\left[\frac{R_{c_2}}{R_{c_1}} + 1\right]}\right]}\right]$$

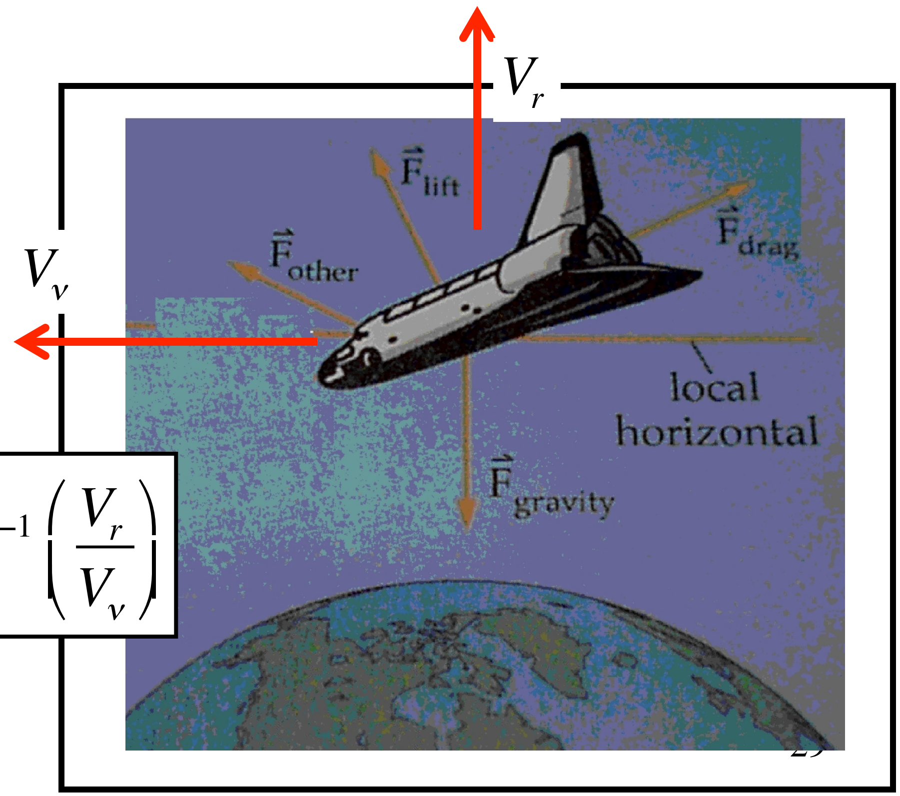
Orbitology Summary (9)

- **Simple Plane Change**

$$\left|\Delta V\right|_{\text{simple plane change}} = 2\sin\left(\frac{\Delta i}{2}\right) V_\nu$$







$$V_r = \|V\| \cdot \sin(\gamma)$$

$$V_\nu = \|V\| \cdot \cos(\gamma)$$

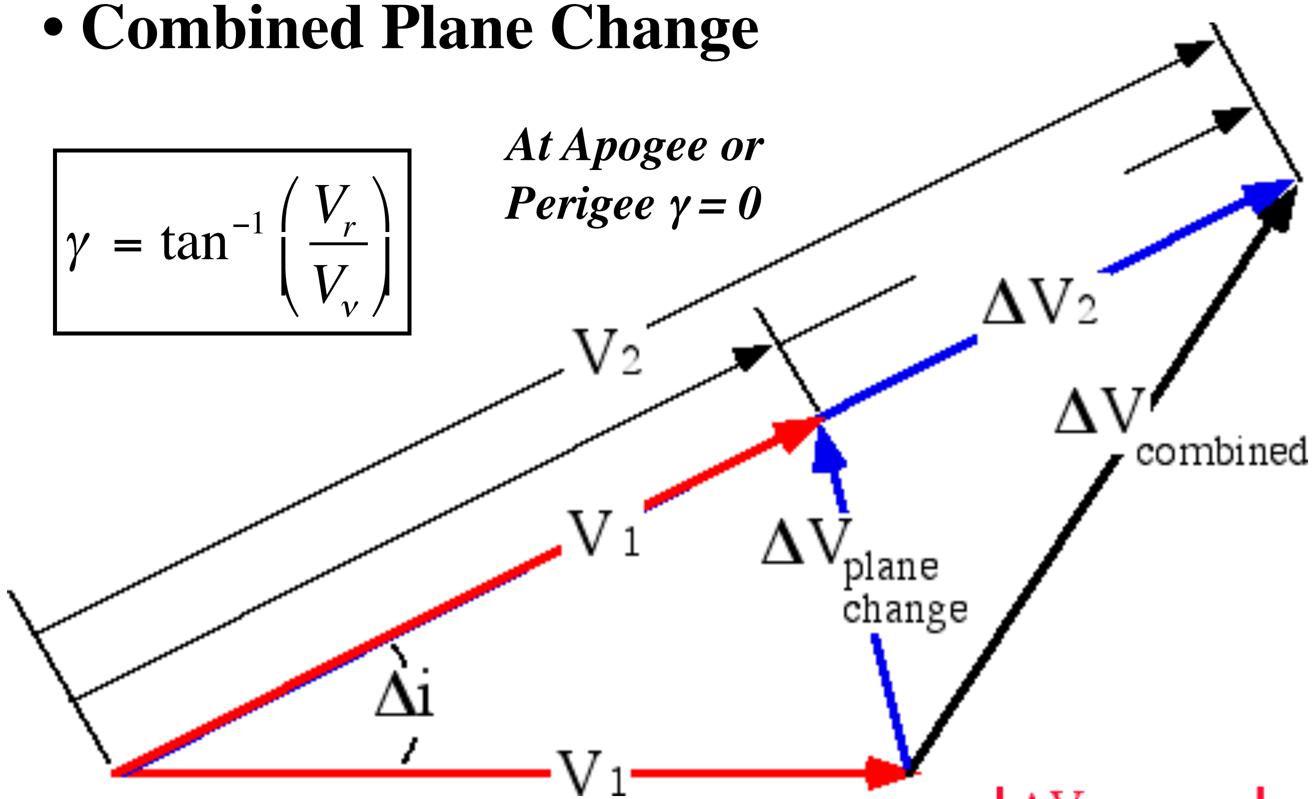
$$\gamma = \tan^{-1}\left(\frac{V_r}{V_\nu}\right)$$

Orbitology Summary (10)

- **Combined Plane Change**

$$\gamma = \tan^{-1}\left(\frac{V_r}{V_v}\right)$$

At Apogee or Perigee $\gamma = 0$



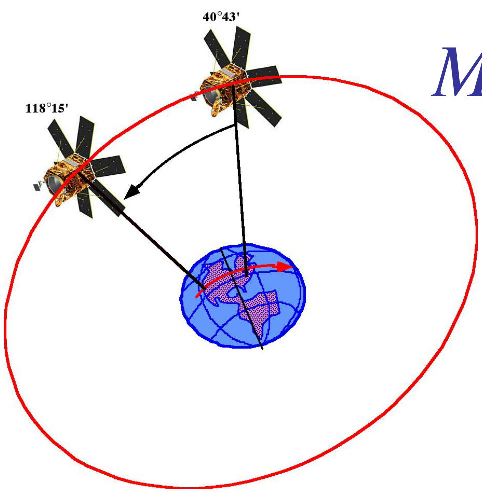
$$|\Delta V_{combined}| < |\Delta V_{\text{simple plane change}}| + |\Delta V_2|$$

$$\Delta V_{combined} = \sqrt{V_2^2 + V_1^2 - 2\,|V_2 \cos(\gamma_2)|\,|V_1 \cos(\gamma_1)| \cos(\Delta i)}$$

Homework 4

A Novel Application of the Rocket-Equation

Calculating the Fuel Budget for an Orbital Phasing Maneuver of a GeoStationary Satellite



TT&C Satellite

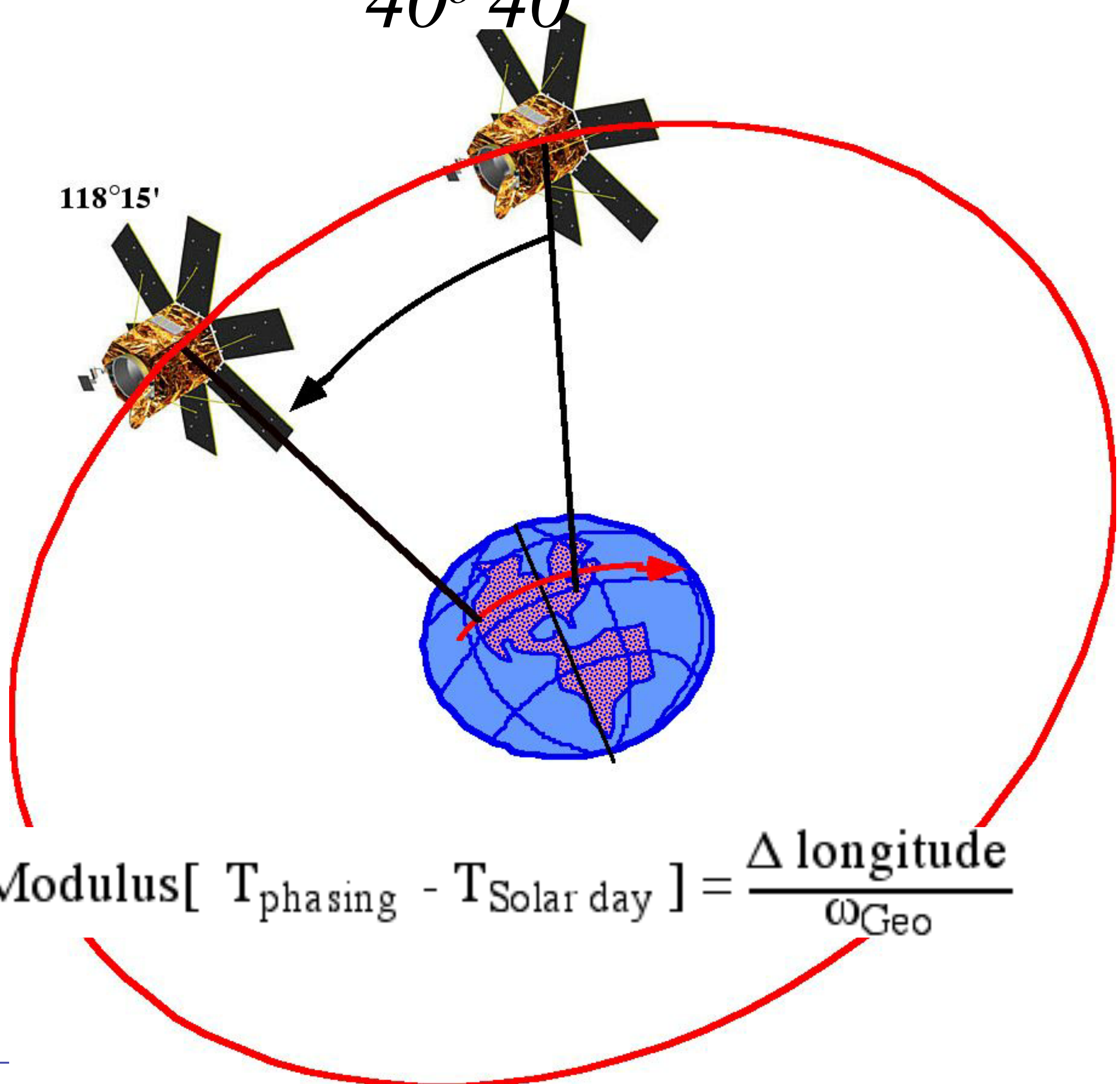
- *TT&C satellite used to monitor pacific coast battle has failed*

- *NACSOC has decided to transfer the functions of a spare Atlantic battle group satellite to the pacific until a replacement can be launched*

…design an Orbital phasing Maneuver that Allows Transfer of a GEO Synchronous Communication Satellite from 40.40' west Longitude To 118.15' west longitude

Phasing Maneuver

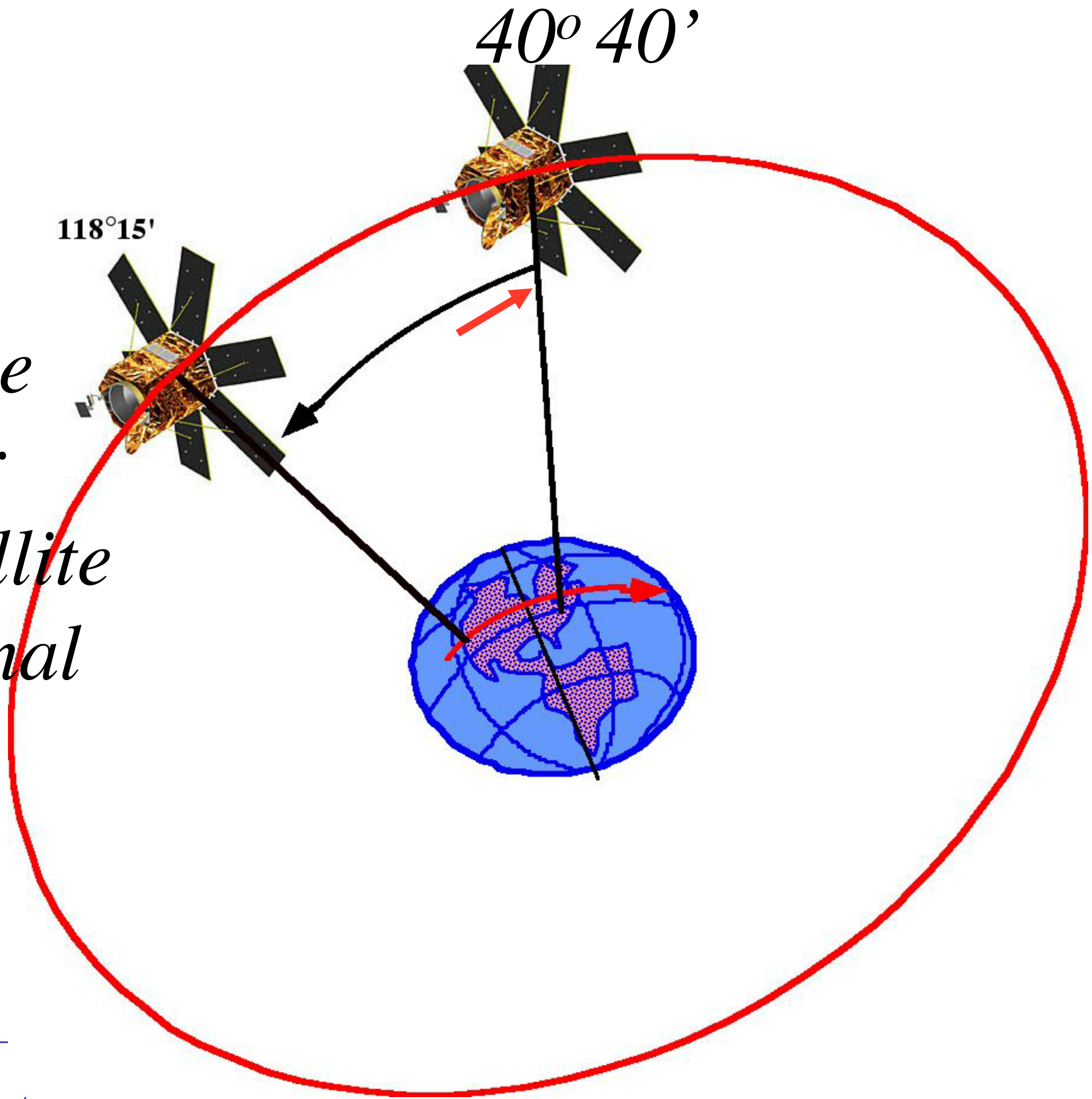
40^o 40'



$$\text{Modulus}[\ T_{phasing} - T_{Solar\ day}\ ] = \frac{\Delta\ \text{longitude}}{\omega_{Geo}}$$

Phasing Maneuver (part 2)

- *Design a Reverse Orbital Maneuver that Puts the Satellite Back to the Original Longitude after Mission has been accomplished*



What To Compute

- *Compute*

  … Phasing Orbit Parameters

  … Phasing Orbit Period

  … Required Delta V_1, Delta V_2

- *Assume R_{min} > 32,000 km*

  (to stay above Van Allen belts)

- *Note: It may take Multiple orbits of Phasing Orbit to accomplish this task*

What To Compute (cont'd)

- *Compute*

  …Burn time for Transfer Orbit Insertion

  …Burn Time for Final Orbit Insertion

  …Required Fuel Budget for Delta V_1, Delta V_2

Parameters of the Problem

Solar Day: 23 hrs, 56 min, 4.1 seconds

Gravitational Parameter: $\mu = 3.9860044 \times 10^5 \ \frac{km^3}{sec^2}$

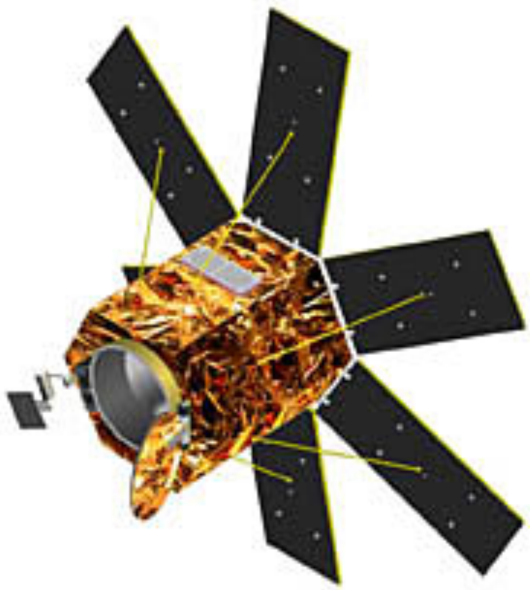
Original Longitude : 40 deg, 40 min West
Destination Longitude: 118 deg, 15 min West

Parameters of the Problem (cont'd)

Specific Impulse

Fuel	Oxidzer	Isp (s)
Liquid propellants		
Hydrogen (LH2)	Oxygen (LOX)	450
Kerosene (RP-4)	Oxygen (LOX)	260
Monomethyl hydrazine	Nitrogen Tetraoxide	310
Solid propellants		
Powered Al	Ammonium Perchlorate	270

Parameters of the Problem (Concluded)



- $F_{thruster} = 0.500\ kNt$
- *Spacecraft mass*

  1000 kg "Dry"