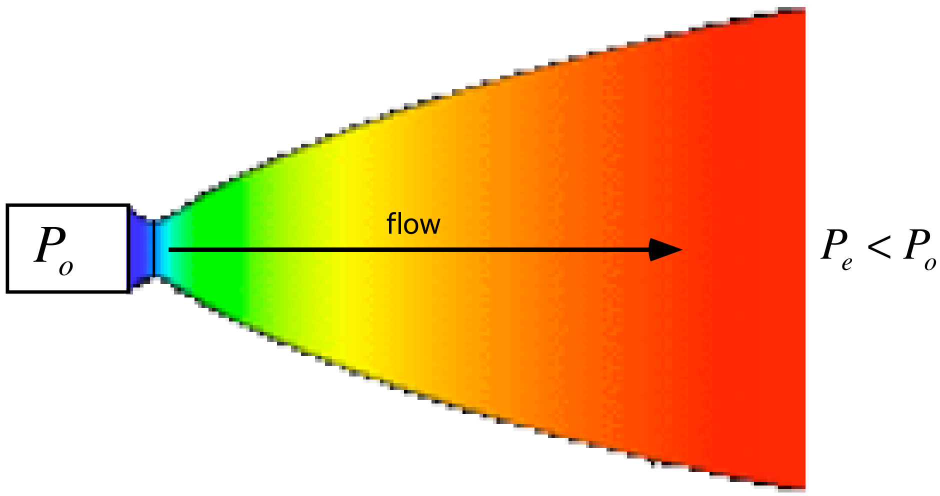
UtahState UNIVERSITY

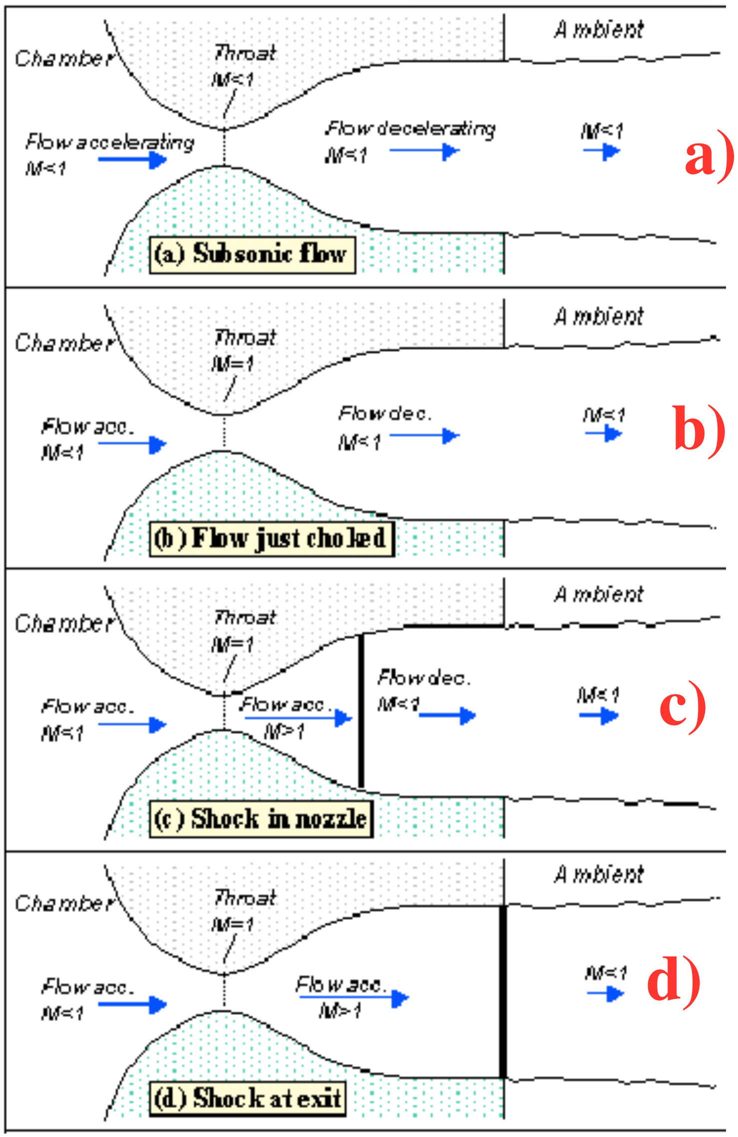
Mechanical & Aerospace Engineering

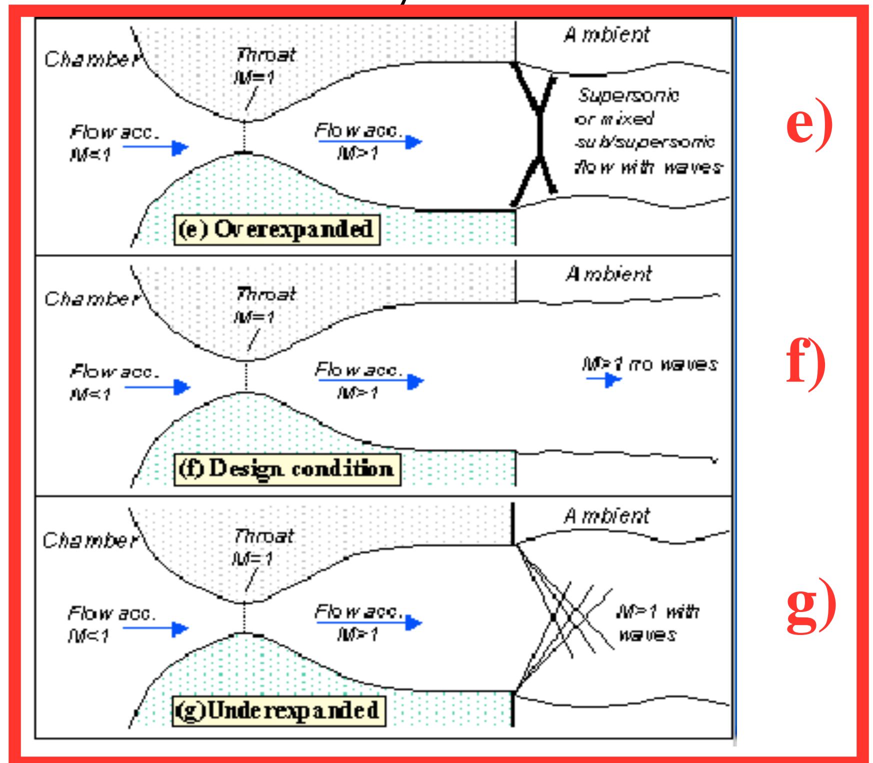
Section 5: Lecture 3
The Optimum Rocket Nozzle

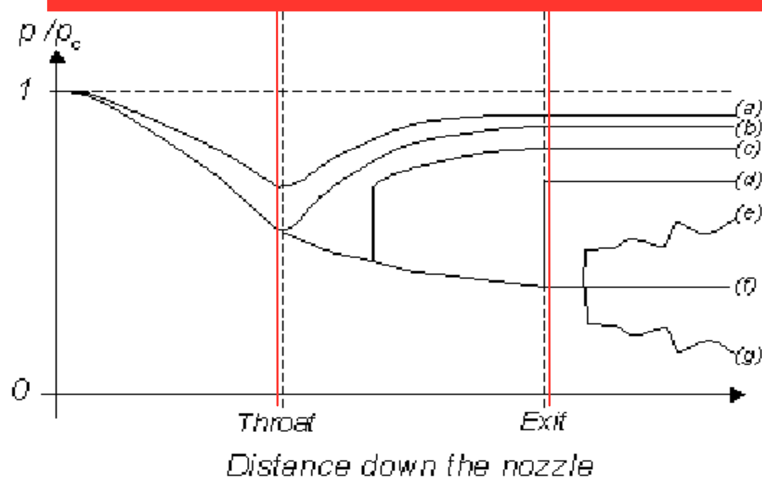


Not in Sutton and Biblarz

Nozzle Flow Summary







Rocket Thrust Equation

$$Thrust = \dot{m} V_{exit} + A_{exit}(p_{exit} - p_{\infty})$$

- Non dimensionalize as Thrust Coefficient

$$C_F = \frac{Thrust}{P_0 A_{throat}} = \frac{\dot{m} V_{exit}}{P_0 A_{throat}} + \frac{A_{exit}}{A_{throat}} \frac{(p_{exit} - p_{\infty})}{P_0}$$

- For a choked throat

$$\frac{\dot{m}}{A^* P_0} = \frac{1}{\sqrt{T_0}} \sqrt{\frac{\gamma}{R_g}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}} \longrightarrow C_F = \frac{Thrust}{P_0 A^*} = \frac{V_{exit}}{\sqrt{T_0}} \sqrt{\frac{\gamma}{R_g}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}} + \frac{A_e}{A^*} \frac{(p_{exit} - p_{\infty})}{P_0}$$

Rocket Thrust Equation (cont'd)

$$C_F = \frac{Thrust}{P_0 A^*} = \frac{V_{exit}}{\sqrt{T_0}} \sqrt{\frac{\gamma}{R_g}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}} + \frac{A_e}{A^*}\frac{(p_{exit} - p_\infty)}{P_0}$$

- For isentropic flow

$$V_{exit} = \sqrt{2c_p\left[T_{0_{exit}} - T_{exit}\right]} = \sqrt{2c_p T_{0_{exit}}}\left[1 - \frac{T_{exit}}{T_{0_{exit}}}\right]^{1/2}$$

- Also for isentropic flow

$$\frac{p_2}{p_1} = \left[\frac{T_2}{T_1}\right]^{\frac{\gamma}{\gamma-1}} \longrightarrow \frac{T_{exit}}{T_{0_{exit}}} = \left(\frac{p_{exit}}{P_{0_{exit}}}\right)^{\frac{\gamma-1}{\gamma}}$$

Rocket Thrust Equation (cont'd)

- Subbing into velocity equation

$$V_{exit} = \sqrt{2c_p\left[T_{0_{exit}} - T_{exit}\right]} = \sqrt{2c_pT_{0_{exit}}}\left[1-\left(\frac{p_{exit}}{P_{0_{exit}}}\right)^{\frac{\gamma-1}{\gamma}}\right]^{1/2}$$

- Subbing into the thrust coefficient equation

$$C_F = \frac{Thrust}{p_0A^*} = \frac{\sqrt{2c_pT_{0_{exit}}}\left[1-\left(\frac{p_{exit}}{P_{0_{exit}}}\right)^{\frac{\gamma-1}{\gamma}}\right]^{1/2}}{\sqrt{T_0}}\sqrt{\frac{\gamma}{R_g}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}} + \frac{A_{exit}}{A^*}\frac{(p_{exit}-p_\infty)}{P_0} =$$

$$\left[1-\left(\frac{p_{exit}}{P_{0_{exit}}}\right)^{\frac{\gamma-1}{\gamma}}\right]^{1/2}\sqrt{\frac{2c_p\gamma}{R_g}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}} + \frac{A_{exit}}{A^*}\frac{(p_{exit}-p_\infty)}{P_0}$$

Rocket Thrust Equation (cont'd)

- Simplifying

$$\frac{2c_p\gamma}{R_g} = \frac{2c_p\gamma}{c_p - c_v} = \frac{2\gamma}{1-\frac{1}{\gamma}} = \frac{2\gamma^2}{\gamma - 1}$$

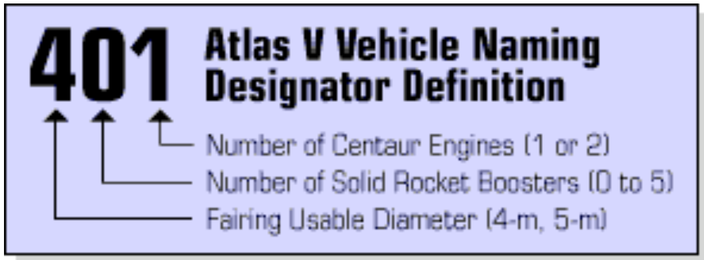
- Finally, for an isentropic nozzle $P_{0_{exit}} = P_0$

$$C_F = \frac{Thrust}{P_0 A^*} = \gamma\sqrt{\frac{2}{\gamma-1}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}\left[1-\left(\frac{p_{exit}}{P_0}\right)^{\frac{\gamma-1}{\gamma}}\right]^{1/2}} + \frac{A_{exit}}{A^*}\frac{(p_{exit}-p_\infty)}{P_0}$$

as derived last time

- **Non-dimensionalized thrust is a function of Nozzle pressure ratio and back pressure only**

Example: Atlas V 401 First Stage



- $Thrust_{vac}$ = 4152 kn
- $Thrust_{sl}$ = 3827 kn
- $I_{sp_{vac}}$ = 337.8 sec
- A_e/A_* = 36.87
- P_0 = 24.25 Mpa
- Lox/RP-1 Propellants
- Mixture ratio = 2.172:1
- Chamber pressure = 25.74 MPa

- Calculate γ

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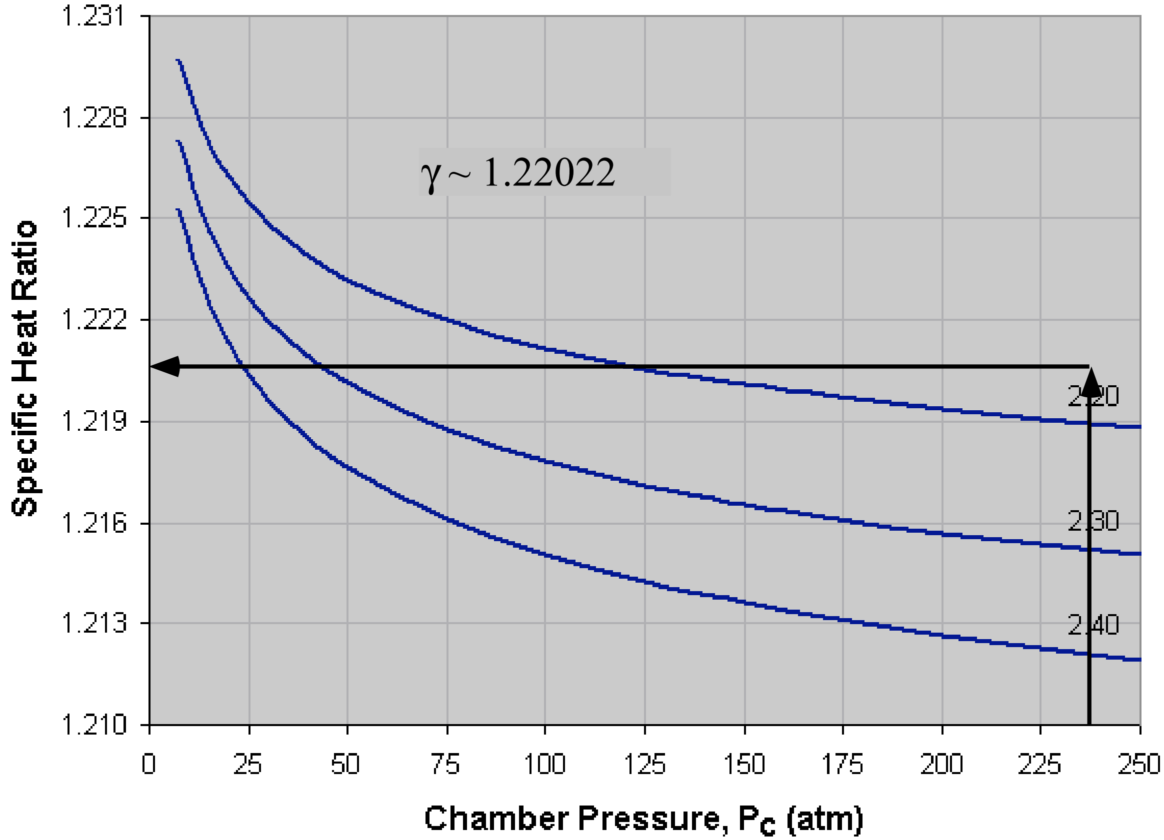
Example: Atlas V 401 First Stage



- $\gamma \sim 1.220122$

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Example: Atlas V 401 First Stage (cont'd)

- From Lecture 5.2

p_∞ Sea level -- 101.325 kpa

$$F_{vac} - F_{sl} = \left[\dot{m}_e V_e + \left(p_e A_e\right)\right] - \left[\dot{m}_e V_e + \left(p_e A_e - p_{sl} A_e\right)\right] = p_{sl} A_e$$

$$A_e = \frac{F_{vac} - F_{sl}}{p_{sl}} = \frac{4152000_{\frac{kg-m}{\sec^2}} - 3827000_{\frac{kg-m}{\sec^2}}}{101325_{\frac{kg-m}{\sec^2}/m^2}} = 3.2705_{m^2}$$

$$A^* = A_{exit} \Big/ \frac{A_{exit}}{A^*} = \frac{3.2705}{36.87} = 0.0870_{m^2}$$

Compute Isentropic Exit Pressure

- User Iterative Solve to Compute Exit Mach Number

$$\frac{A_{exit}}{A^*} = 36.87 = \left[\frac{1}{M_{exit}} \left[\left(\frac{2}{\gamma+1} \right) \left(1 + \frac{(\gamma-1)}{2} M_{exit}^{\ 2} \right) \right]^{\frac{\gamma+1}{2(\gamma-1)}} \right] \Rightarrow$$

$$M_{exit} = 4.2954$$

- Compute Exit Pressure

$$p_{exit} = \frac{P_{0_{exit}}}{\left(1 + \frac{\gamma-1}{2} M_{exit}^{\ 2}\right)^{\left(\frac{\gamma}{\gamma-1}\right)}} = \frac{24.25 \cdot 1000}{\left(1 + \frac{1.220122-1}{2} 4.2954^2\right)^{\left(\frac{1.220122}{1.220122-1}\right)}} = 51.95 \text{ kPa}$$

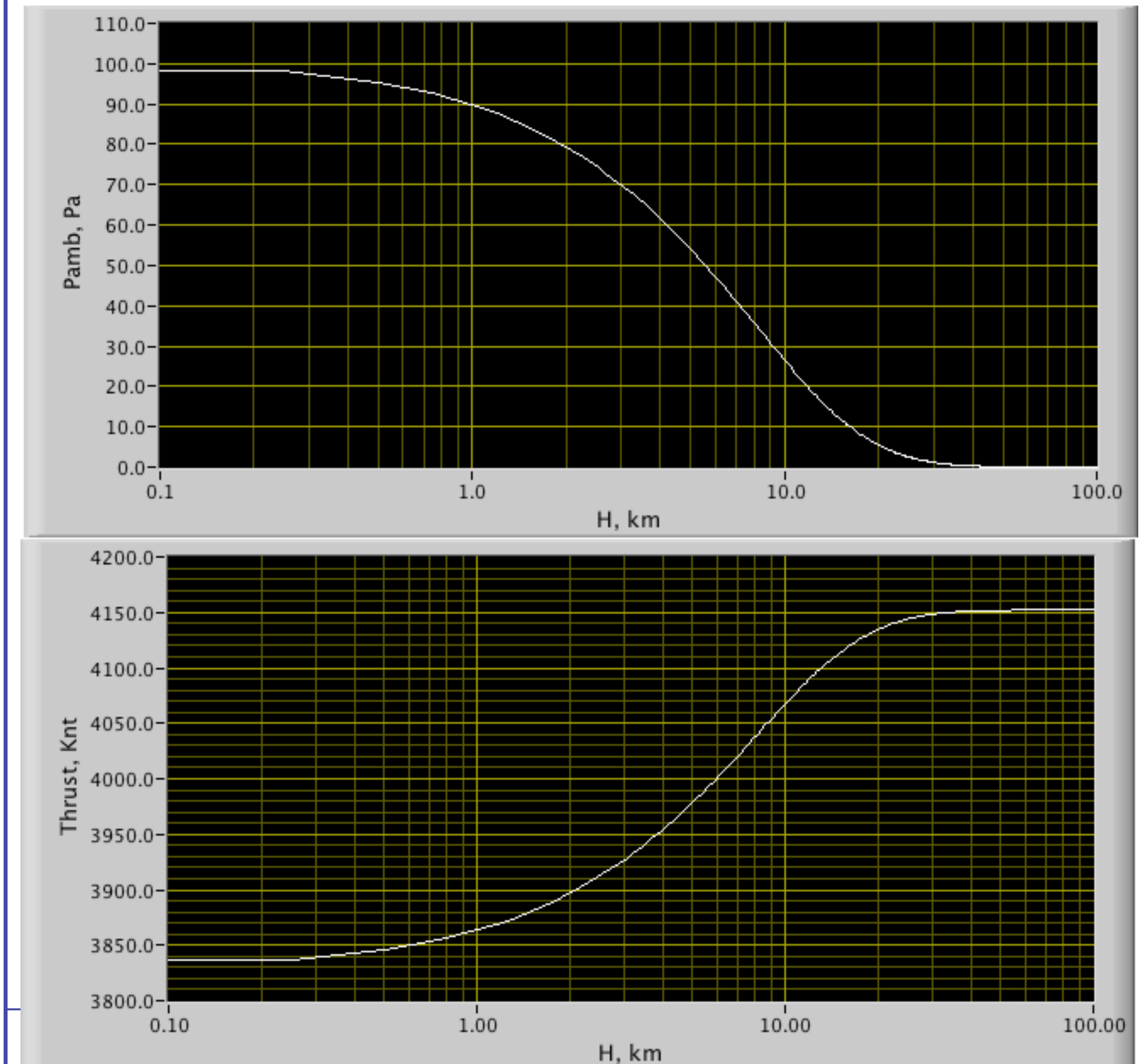
Look at Thrust as function of Altitude (p_∞)

- All the pieces we need now

$$Thrust = \gamma P_0 A^* \sqrt{\frac{2}{\gamma-1}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}\left[1-\left(\frac{p_{exit}}{P_0}\right)^{\frac{\gamma-1}{\gamma}}\right]^{1/2}} + A_{exit}(p_{exit} - p_\infty)$$

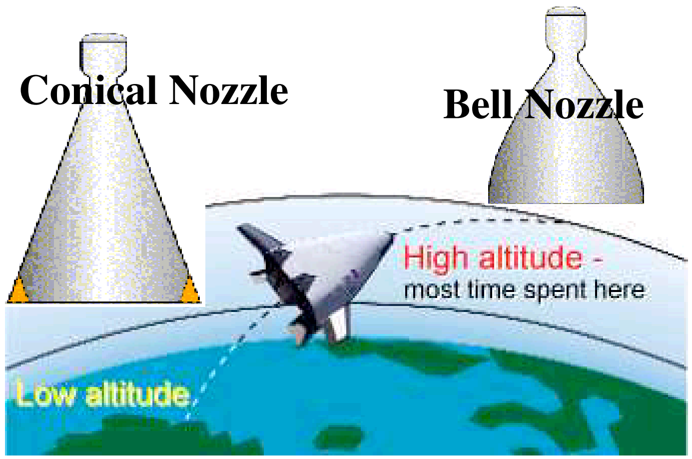
$$\begin{bmatrix} \gamma = 1.220122 \\ P_0 = 24.25_{Mpa} \\ p_{exit} = 51.95_{kPa} \\ A^* = 0.087_{m^2} \\ A_{exit} = 3.2705_{m^2} \end{bmatrix}$$

Look at Thrust as function of Altitude (p_∞) (cont'd)

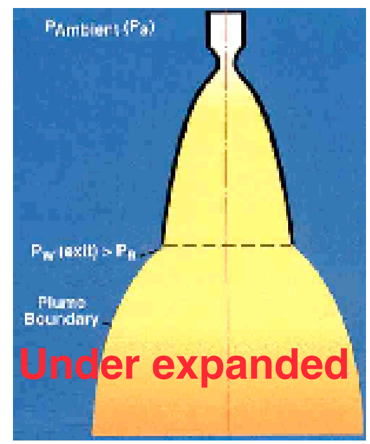


- Thrust increases With the logarithmic of altitude

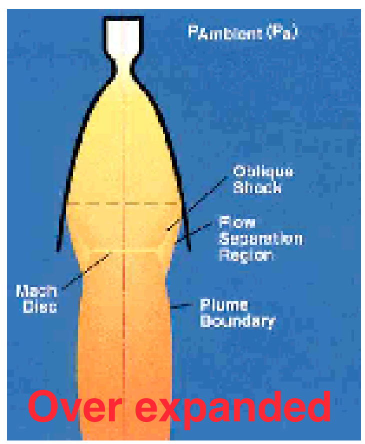
Exit Pressure has a dramatic effect on Nozzle performance



Vacuum (Space)



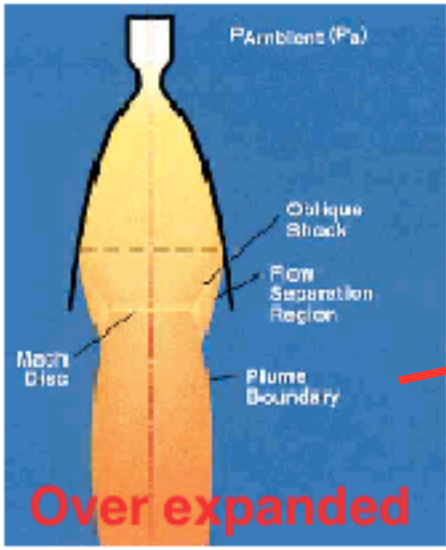
Lift off

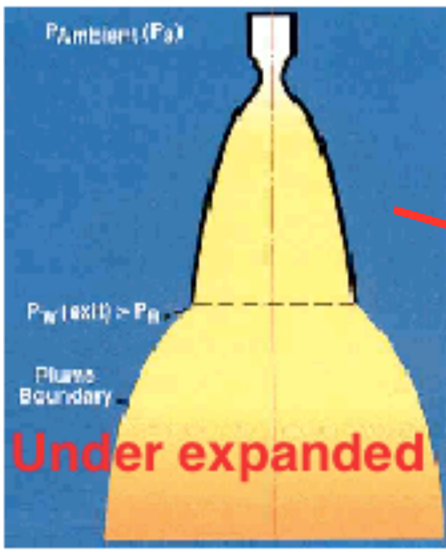


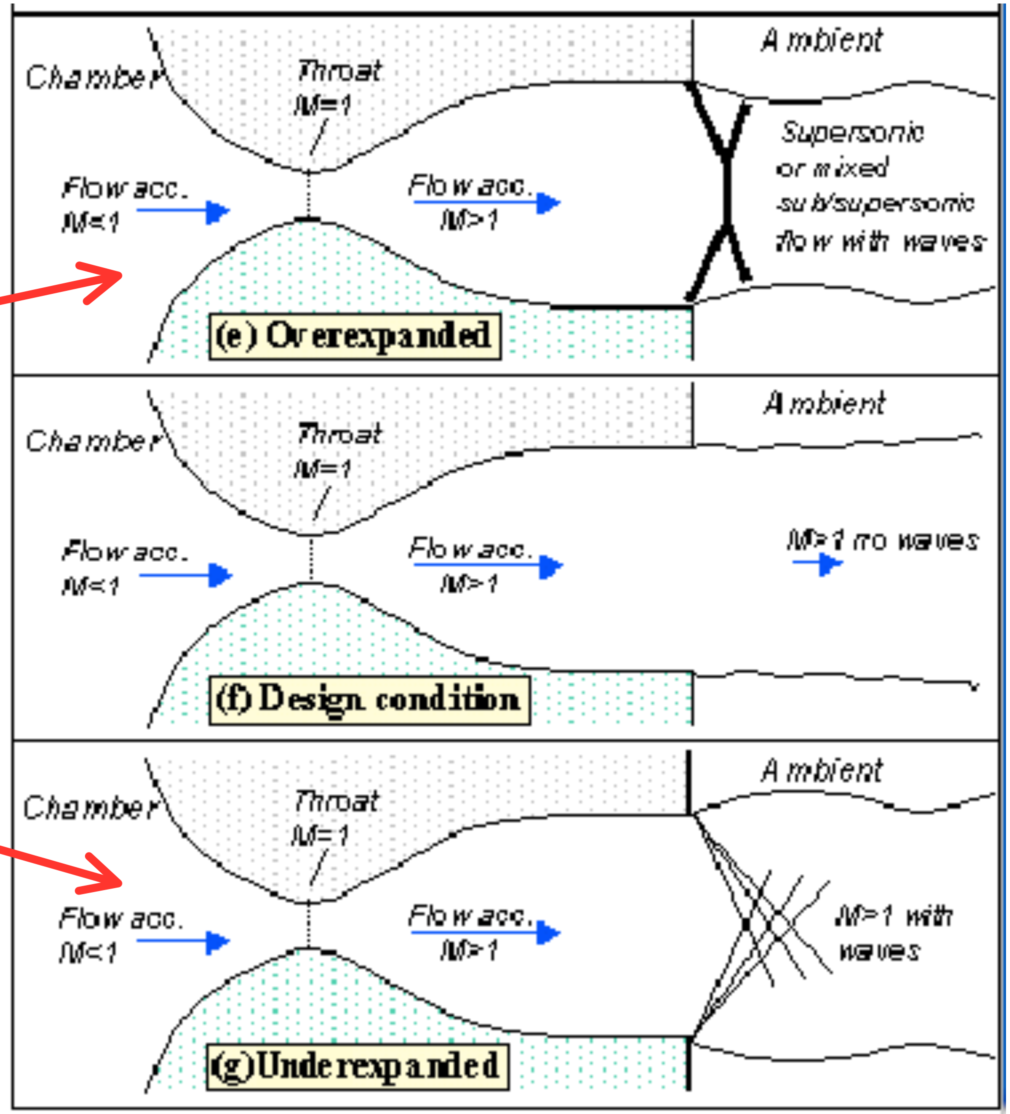
Large area ratio nozzles at sea level cause flow separation, performance losses, high nozzle structural loads

Bell constrains flow limiting performance

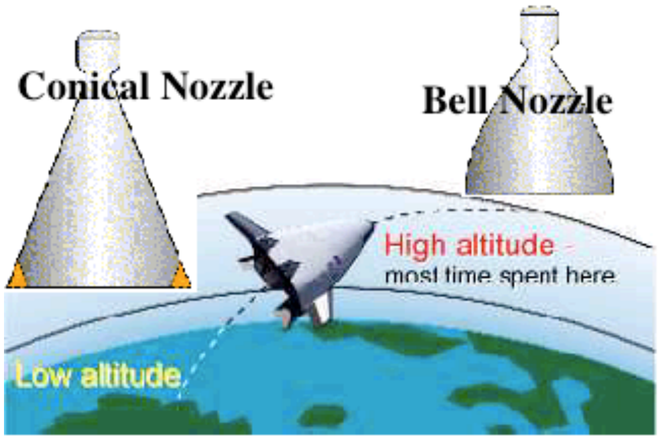
Next: The Optimum Nozzle (1)





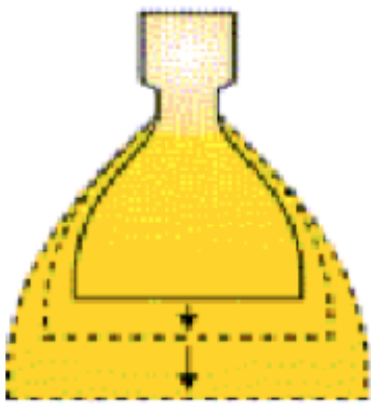


Next: The Optimum Nozzle (2)



$$Thrust = \dot{m} V_{exit} + A_{exit}(p_{exit} - p_\infty)$$

$$for\ given\ \dot{m} \rightarrow \boxed{\begin{array}{c} V_{exit} \propto \dfrac{A_{exit}}{A^*} \\ \dfrac{1}{P_{exit}} \propto \dfrac{A_{exit}}{A^*} \end{array}}$$



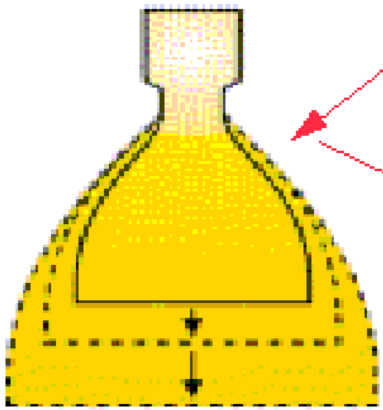
$$\rightarrow both\ \{V_{exit}, P_{exit}\}\ contribute\ to\ thrust$$

$$\rightarrow what\ \frac{A_{exit}}{A^*}\ is\ "optimal"?$$

The "Optimum Nozzle"

- Expanding nozzle increases V_{exit}, but decreases P_{exit} -- there is trade-off here
- It can be shown using variational calculus on the relationships from the previous pages that the Optimum nozzle performance occurs when

$$\frac{A_{exit}}{A_t} \Rightarrow p_{exit} = p_a$$



"telescoping nozzle"

Unfeasible because of the large weight penalty and complexity of deployment mechanisms, also requires that nozzle expand to very large area ratios

Lets Do the Calculus

- Prove that Maximum performance occurs when

$\frac{A_{exit}}{A^*}$ Is adjusted to give $p_{exit} = p_\infty$

Optimal Nozzle

- Show $\dfrac{A_{exit}}{A^*}$ is a function of $\dfrac{P_0}{p_{exit}}$

$$\frac{A_{exit}}{A^*} = \left[\frac{1}{M_{exit}} \left[\left(\frac{2}{\gamma+1} \right) \left(1 + \frac{(\gamma-1)}{2} M_{exit}^{\ 2} \right) \right]^{\frac{\gamma+1}{2(\gamma-1)}} \right] =$$

$$\frac{1}{M_{exit}} \sqrt{ \left[\left[\left(\frac{2}{\gamma+1} \right) \left(1 + \frac{(\gamma-1)}{2} M_{exit}^{\ 2} \right) \right]^{\frac{\gamma+1}{(\gamma-1)}} \right] }$$

Optimal Nozzle (cont'd)

$$M_{exit} = \sqrt{\left(\frac{2}{\gamma-1}\right)\left[\left(\frac{P_0}{p_{exit}}\right)^{\frac{(\gamma-1)}{\gamma}} - 1\right]} \Rightarrow$$

- Substitute in

$$\frac{A_{exit}}{A^*} = \sqrt{\frac{\left[\left(\frac{2}{\gamma+1}\right)\left(1+\frac{(\gamma-1)}{2}\left(\frac{2}{\gamma-1}\right)\left[\left(\frac{P_0}{p_{exit}}\right)^{\frac{(\gamma-1)}{\gamma}} - 1\right]\right)\right]^{\frac{\gamma+1}{(\gamma-1)}}}{\left(\frac{2}{\gamma-1}\right)\left[\left(\frac{P_0}{p_{exit}}\right)^{\frac{(\gamma-1)}{\gamma}} - 1\right]}} = \sqrt{\frac{\left[\left(\frac{2}{\gamma+1}\right)\left(\left(\frac{P_0}{p_{exit}}\right)^{\frac{(\gamma-1)}{\gamma}}\right)\right]^{\frac{\gamma+1}{(\gamma-1)}}}{\left(\frac{2}{\gamma-1}\right)\left[\left(\frac{P_0}{p_{exit}}\right)^{\frac{(\gamma-1)}{\gamma}} - 1\right]}} =$$

$$\sqrt{\frac{\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}}{\left(\frac{2}{\gamma-1}\right)}}\sqrt{\frac{\left(\left(\frac{P_0}{p_{exit}}\right)^{\frac{(\gamma-1)}{\gamma}}\right)^{\frac{\gamma+1}{(\gamma-1)}}}{\left[\left(\frac{P_0}{p_{exit}}\right)^{\frac{(\gamma-1)}{\gamma}} - 1\right]}} = \sqrt{\frac{\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}}{\left(\frac{2}{\gamma-1}\right)}}\sqrt{\frac{\left(\frac{P_0}{p_{exit}}\right)^{\frac{\gamma+1}{\gamma}}}{\left[\left(\frac{P_0}{p_{exit}}\right)^{\frac{(\gamma-1)}{\gamma}} - 1\right]}}$$

Optimal Nozzle (cont'd)

$$C_F = \frac{Thrust}{P_0 A^*} = \gamma \sqrt{\frac{2}{\gamma-1}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}} \left[1-\left(\frac{p_{exit}}{P_0}\right)^{\frac{\gamma-1}{\gamma}}\right]^{1/2} + \frac{A_{exit}}{A^*}\frac{(p_{exit}-p_\infty)}{P_0}$$

$$\frac{A_{exit}}{A^*} = \sqrt{\frac{\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}}{\left(\frac{2}{\gamma-1}\right)}} \sqrt{\frac{\left(\frac{P_0}{p_{exit}}\right)^{\frac{\gamma+1}{\gamma}}}{\left[\left(\frac{P_0}{p_{exit}}\right)^{\frac{(\gamma-1)}{\gamma}} - 1\right]}}$$

Optimal Nozzle (cont'd)

- Subbing into normalized thrust equation

$$\frac{Thrust}{P_0 A^*} = \gamma\sqrt{\frac{2}{\gamma-1}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}\left[1-\left(\frac{p_{exit}}{P_0}\right)^{\frac{\gamma-1}{\gamma}}\right]}^{1/2} + \sqrt{\frac{\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}}{\left(\frac{2}{\gamma-1}\right)}}\sqrt{\frac{\left(\frac{P_0}{p_{exit}}\right)^{\frac{\gamma+1}{\gamma}}}{\left[\left(\frac{P_0}{p_{exit}}\right)^{\frac{(\gamma-1)}{\gamma}}-1\right]}}\frac{(p_{exit}-p_\infty)}{P_0} =$$

$$\gamma\sqrt{\frac{2}{\gamma-1}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}}\left\{\left[1-\left(\frac{p_{exit}}{P_0}\right)^{\frac{\gamma-1}{\gamma}}\right]^{1/2} + \frac{1}{\gamma\sqrt{\frac{2}{\gamma-1}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}}}\sqrt{\frac{\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}}{\left(\frac{2}{\gamma-1}\right)}}\sqrt{\frac{\left(\frac{P_0}{p_{exit}}\right)^{\frac{\gamma+1}{\gamma}}}{\left[\left(\frac{P_0}{p_{exit}}\right)^{\frac{(\gamma-1)}{\gamma}}-1\right]}}\frac{(p_{exit}-p_\infty)}{P_0}\right\} =$$

$$\gamma\sqrt{\frac{2}{\gamma-1}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}}\left\{\left[1-\left(\frac{p_{exit}}{P_0}\right)^{\frac{\gamma-1}{\gamma}}\right]^{1/2} + \frac{\gamma-1}{2\gamma}\sqrt{\frac{\left(\frac{p_{exit}}{P_0}\right)^{\frac{\gamma+1}{-\gamma}}}{\left[\left(\frac{p_{exit}}{P_0}\right)^{\frac{(\gamma-1)}{-\gamma}}-1\right]}}\left[\frac{p_{exit}}{P_0}-\frac{p_\infty}{P_0}\right]\right\}$$

Optimal Nozzle (cont'd)

- Necessary condition for Maxim (Optimal) Thrust

$$\frac{\partial\left(\frac{Thrust}{P_0 A^*}\right)}{\partial p_{exit}} =$$

$$\frac{\partial}{\partial p_{exit}}\left[\gamma\sqrt{\frac{2}{\gamma-1}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}}\left\{\left[1-\left(\frac{p_{exit}}{P_0}\right)^{\frac{\gamma-1}{\gamma}}\right]^{1/2}+\frac{\gamma-1}{2\gamma}\sqrt{\frac{\left(\frac{p_{exit}}{P_0}\right)^{\frac{\gamma+1}{-\gamma}}}{\left[\left(\frac{p_{exit}}{P_0}\right)^{\frac{(\gamma-1)}{-\gamma}}-1\right]}}\left[\frac{p_{exit}}{P_0}-\frac{p_\infty}{P_0}\right]\right\}\right] =$$

$$\gamma\sqrt{\frac{2}{\gamma-1}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}}\frac{\partial}{\partial p_{exit}}\left[\left\{\left[1-\left(\frac{p_{exit}}{P_0}\right)^{\frac{\gamma-1}{\gamma}}\right]^{1/2}+\frac{\gamma-1}{2\gamma}\sqrt{\frac{\left(\frac{p_{exit}}{P_0}\right)^{\frac{\gamma+1}{-\gamma}}}{\left[\left(\frac{p_{exit}}{P_0}\right)^{\frac{(\gamma-1)}{-\gamma}}-1\right]}}\left[\frac{p_{exit}}{P_0}-\frac{p_\infty}{P_0}\right]\right\}\right] = 0$$

Optimal Nozzle (cont'd)

- Evaluating the derivative

$$\frac{\partial}{\partial p_{exit}}\left[\left\{\left[1-\left(\frac{p_{exit}}{P_0}\right)^{\frac{\gamma-1}{\gamma}}\right]^{1/2}+\frac{\gamma-1}{2\gamma}\sqrt{\frac{\left(\frac{p_{exit}}{P_0}\right)^{\frac{\gamma+1}{-\gamma}}}{\left[\left(\frac{p_{exit}}{P_0}\right)^{\frac{(\gamma-1)}{-\gamma}}-1\right]}}\left[\frac{p_{exit}}{P_0}-\frac{p_\infty}{P_0}\right]\right\}\right]=$$

$$\frac{(-1+\gamma)\left(\frac{P_{exit}}{P_0}-\frac{P_\infty}{P_0}\right)\left(-\frac{(1+\gamma)\left(\frac{P_{exit}}{P_0}\right)^{-1-\frac{1+\gamma}{\gamma}}}{\gamma\left(-1+\left(\frac{P_{exit}}{P_0}\right)^{-\frac{-1+\gamma}{\gamma}}\right)P_0}+\frac{(-1+\gamma)\left(\frac{P_{exit}}{P_0}\right)^{-1-\frac{-1+\gamma}{\gamma}-\frac{1+\gamma}{\gamma}}}{\gamma\left(-1+\left(\frac{P_{exit}}{P_0}\right)^{-\frac{-1+\gamma}{\gamma}}\right)^2P_0}\right)}{4\gamma\sqrt{\frac{\left(\frac{P_{exit}}{P_0}\right)^{-\frac{1+\gamma}{\gamma}}}{-1+\left(\frac{P_{exit}}{P_0}\right)^{-\frac{-1+\gamma}{\gamma}}}}}+$$

$$\frac{(-1+\gamma)\sqrt{\frac{\left(\frac{P_{exit}}{P_0}\right)^{-\frac{1+\gamma}{\gamma}}}{-1+\left(\frac{P_{exit}}{P_0}\right)^{-\frac{-1+\gamma}{\gamma}}}}}{2\gamma P_0}-\frac{(-1+\gamma)\left(\frac{P_{exit}}{P_0}\right)^{-1+\frac{-1+\gamma}{\gamma}}}{2\gamma\sqrt{1-\left(\frac{P_{exit}}{P_0}\right)^{\frac{-1+\gamma}{\gamma}}}\,P_0}$$

Let's try to get rid of This term

Optimal Nozzle (cont'd)

- Look at the term

$$\frac{(-1+\gamma)\sqrt{\frac{\left(\frac{P_{exit}}{P_0}\right)^{-\frac{1+\gamma}{\gamma}}}{-1+\left(\frac{P_{exit}}{P_0}\right)^{-\frac{-1+\gamma}{\gamma}}}}}{2\gamma P_0} - \frac{(-1+\gamma)\left(\frac{P_{exit}}{P_0}\right)^{-1+\frac{-1+\gamma}{\gamma}}}{2\gamma\sqrt{1-\left(\frac{P_{exit}}{P_0}\right)^{\frac{-1+\gamma}{\gamma}}}\,P_0} =$$

$$\left(\frac{\gamma-1}{2\gamma P_0}\right)\left\{\sqrt{\frac{\left(\frac{p_{exit}}{P_0}\right)^{\frac{\gamma+1}{-\gamma}}}{\left[\left(\frac{p_{exit}}{P_0}\right)^{\frac{(\gamma-1)}{-\gamma}}-1\right]}} - \left(\frac{p_{exit}}{P_0}\right)^{\frac{1}{-\gamma}}\sqrt{\frac{1}{\left[1-\left(\frac{p_{exit}}{P_0}\right)^{\frac{(\gamma-1)}{\gamma}}\right]}}\right\}$$

Optimal Nozzle (cont'd)

- Look at the term

Bring Inside

$$\left(\frac{\gamma-1}{2\gamma P_0}\right)\left\{\sqrt{\frac{\left(\frac{p_{exit}}{P_0}\right)^{\frac{\gamma+1}{-\gamma}}}{\left[\left(\frac{p_{exit}}{P_0}\right)^{\frac{(\gamma-1)}{-\gamma}}-1\right]}}-\left(\frac{p_{exit}}{P_0}\right)^{\frac{1}{-\gamma}}\sqrt{\frac{1}{\left[1-\left(\frac{p_{exit}}{P_0}\right)^{\frac{(\gamma-1)}{\gamma}}\right]}}\right\}=$$

$$\left(\frac{\gamma-1}{2\gamma P_0}\right)\left\{\sqrt{\frac{\left(\frac{p_{exit}}{P_0}\right)^{\frac{\gamma+1}{-\gamma}}}{\left[\left(\frac{p_{exit}}{P_0}\right)^{\frac{(\gamma-1)}{-\gamma}}-1\right]}}-\sqrt{\frac{\left(\frac{p_{exit}}{P_0}\right)^{\frac{1}{-\gamma}}\left(\frac{p_{exit}}{P_0}\right)^{\frac{1}{-\gamma}}}{\left[1-\left(\frac{p_{exit}}{P_0}\right)^{\frac{(\gamma-1)}{\gamma}}\right]}}\right\}$$

Optimal Nozzle (cont'd)

- Look at the term

Factor Out

$$\left(\frac{\gamma-1}{2\gamma P_0}\right)\left\{\sqrt{\frac{\left(\frac{p_{exit}}{P_0}\right)^{\frac{\gamma+1}{-\gamma}}}{\left[\left(\frac{p_{exit}}{P_0}\right)^{\frac{(\gamma-1)}{-\gamma}}-1\right]}}-\sqrt{\frac{\left(\frac{p_{exit}}{P_0}\right)^{\frac{1}{-\gamma}}\left(\frac{p_{exit}}{P_0}\right)^{\frac{1}{-\gamma}}}{\left[1-\left(\frac{p_{exit}}{P_0}\right)^{\frac{(\gamma-1)}{\gamma}}\right]}}\right\}=$$

$$\left(\frac{p_{exit}}{P_0}\right)^{\frac{(\gamma-1)}{-\gamma}}$$

$$\left(\frac{\gamma-1}{2\gamma P_0}\right)\left\{\sqrt{\frac{\left(\frac{p_{exit}}{P_0}\right)^{\frac{\gamma+1}{-\gamma}}}{\left[\left(\frac{p_{exit}}{P_0}\right)^{\frac{(\gamma-1)}{-\gamma}}-1\right]}}-\sqrt{\frac{\left(\frac{p_{exit}}{P_0}\right)^{\frac{1}{-\gamma}}\left(\frac{p_{exit}}{P_0}\right)^{\frac{1}{-\gamma}}\left(\frac{p_{exit}}{P_0}\right)^{\frac{(\gamma-1)}{-\gamma}}}{\left[\left(\frac{p_{exit}}{P_0}\right)^{\frac{(\gamma-1)}{-\gamma}}-1\right]}}\right\}$$

Optimal Nozzle (cont'd)

- Look at the term

$$\left(\frac{\gamma-1}{2\gamma P_0}\right)\left\{\sqrt{\frac{\left(\frac{p_{exit}}{P_0}\right)^{\frac{\gamma+1}{-\gamma}}}{\left[\left(\frac{p_{exit}}{P_0}\right)^{\frac{(\gamma-1)}{-\gamma}}-1\right]}}-\sqrt{\frac{\left(\frac{p_{exit}}{P_0}\right)^{\frac{1}{-\gamma}}\left(\frac{p_{exit}}{P_0}\right)^{\frac{1}{-\gamma}}\left(\frac{p_{exit}}{P_0}\right)^{\frac{(\gamma-1)}{-\gamma}}}{\left[\left(\frac{p_{exit}}{P_0}\right)^{\frac{(\gamma-1)}{-\gamma}}-1\right]}}\right\}=$$

Collect Exponents

$$\left(\frac{\gamma-1}{2\gamma P_0}\right)\left\{\sqrt{\frac{\left(\frac{p_{exit}}{P_0}\right)^{\frac{\gamma+1}{-\gamma}}}{\left[\left(\frac{p_{exit}}{P_0}\right)^{\frac{(\gamma-1)}{-\gamma}}-1\right]}}-\sqrt{\frac{\left(\frac{p_{exit}}{P_0}\right)^{\frac{\gamma+1}{-\gamma}}}{\left[\left(\frac{p_{exit}}{P_0}\right)^{\frac{(\gamma-1)}{-\gamma}}-1\right]}}\right\}=0$$

Good!

Optimal Nozzle (cont'd)

- and the derivative reduces to

$$\frac{\partial}{\partial p_{exit}}\left[\left\{\left[1-\left(\frac{p_{exit}}{P_0}\right)^{\frac{\gamma-1}{\gamma}}\right]^{1/2}+\frac{\gamma-1}{2\gamma}\sqrt{\frac{\left(\frac{p_{exit}}{P_0}\right)^{\frac{\gamma+1}{-\gamma}}}{\left[\left(\frac{p_{exit}}{P_0}\right)^{\frac{(\gamma-1)}{-\gamma}}-1\right]}}\left[\frac{p_{exit}}{P_0}-\frac{p_\infty}{P_0}\right]\right\}\right]=$$

$$\frac{(-1+\gamma)\left(\frac{p_{exit}}{P_0}-\frac{p_\infty}{P_0}\right)\left(-\frac{(1+\gamma)\left(\frac{p_{exit}}{P_0}\right)^{-1-\frac{1+\gamma}{\gamma}}}{\gamma\left(-1+\left(\frac{p_{exit}}{P_0}\right)^{-\frac{-1+\gamma}{\gamma}}\right)P_0}+\frac{(-1+\gamma)\left(\frac{p_{exit}}{P_0}\right)^{-1-\frac{-1+\gamma}{\gamma}-\frac{1+\gamma}{\gamma}}}{\gamma\left(-1+\left(\frac{p_{exit}}{P_0}\right)^{-\frac{-1+\gamma}{\gamma}}\right)^2P_0}\right)}{4\gamma\sqrt{\frac{\left(\frac{p_{exit}}{P_0}\right)^{-\frac{1+\gamma}{\gamma}}}{-1+\left(\frac{p_{exit}}{P_0}\right)^{-\frac{-1+\gamma}{\gamma}}}}}$$

Optimal Nozzle (concluded)

- Find Condition where

$$\frac{(-1+\gamma)\left(\frac{P_{exit}}{P_0}-\frac{P_\infty}{P_0}\right)\left(-\frac{(1+\gamma)\left(\frac{P_{exit}}{P_0}\right)^{-1-\frac{1+\gamma}{\gamma}}}{\gamma\left(-1+\left(\frac{P_{exit}}{P_0}\right)^{-\frac{-1+\gamma}{\gamma}}\right)P_0}+\frac{(-1+\gamma)\left(\frac{P_{exit}}{P_0}\right)^{-1-\frac{-1+\gamma}{\gamma}-\frac{1+\gamma}{\gamma}}}{\gamma\left(-1+\left(\frac{P_{exit}}{P_0}\right)^{-\frac{-1+\gamma}{\gamma}}\right)^2 P_0}\right)}{4\gamma\sqrt{\frac{\left(\frac{P_{exit}}{P_0}\right)^{-\frac{1+\gamma}{\gamma}}}{-1+\left(\frac{P_{exit}}{P_0}\right)^{-\frac{-1+\gamma}{\gamma}}}}}=0$$

$$\left[\frac{p_{exit}}{P_0}-\frac{p_\infty}{P_0}\right]=0 \Rightarrow \boxed{p_{exit}=p_\infty}$$

- Condition for Optimality *(maximum Isp)*

Optimal Thrust Equation

$$Thrust_{opt} = \gamma P_0 A^* \sqrt{\frac{2}{\gamma - 1}\left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma+1}{(\gamma-1)}}\left[1 - \left(\frac{p_{exit}}{P_0}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

$$\frac{A_{exit}}{A^*} = \sqrt{\frac{\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}}{\left(\frac{2}{\gamma-1}\right)}}\sqrt{\frac{\left(\frac{P_0}{p_\infty}\right)^{\frac{\gamma+1}{\gamma}}}{\left[\left(\frac{P_0}{p_\infty}\right)^{\frac{(\gamma-1)}{\gamma}} - 1\right]}} \Rightarrow forces ... p_{exit} = p_\infty$$

Rocket Nozzle Design Point

$$Thrust_{opt} = \gamma P_0 A^* \sqrt{\frac{2}{\gamma - 1}\left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma+1}{(\gamma-1)}}\left[1 - \left(\frac{p_{exit}}{P_0}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

$$\frac{A_{exit}}{A^*} = \sqrt{\frac{\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}}{\left(\frac{2}{\gamma-1}\right)}}\sqrt{\frac{\left(\frac{P_0}{p_\infty}\right)^{\frac{\gamma+1}{\gamma}}}{\left[\left(\frac{P_0}{p_\infty}\right)^{\frac{(\gamma-1)}{\gamma}} - 1\right]}} \Rightarrow forces ... p_{exit} = p_\infty$$

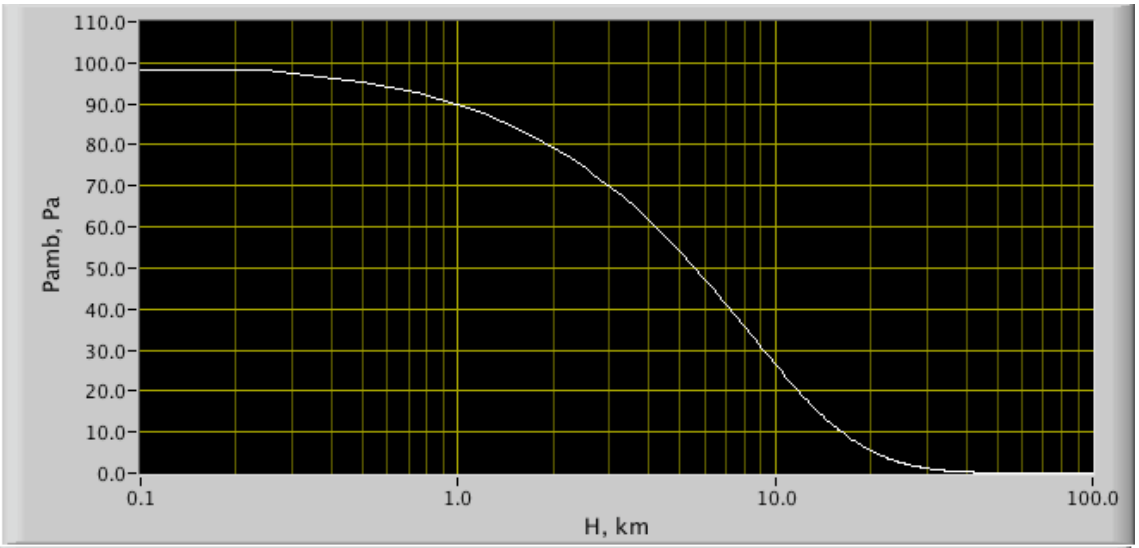
Atlas V, Revisited

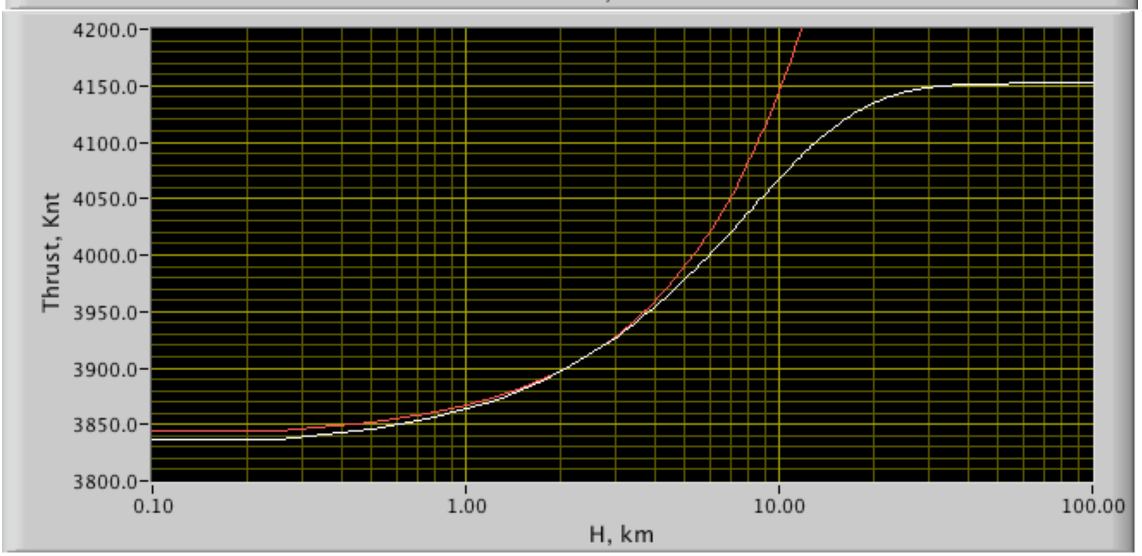
- Re-do the Atlas V plots for Optimal Nozzle

i.e. Let

$$\frac{A_{exit}}{A^*} = \sqrt{\frac{\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}}{\left(\frac{2}{\gamma-1}\right)}}\sqrt{\frac{\left(\frac{P_0}{p_\infty}\right)^{\frac{\gamma+1}{\gamma}}}{\left[\left(\frac{P_0}{p_\infty}\right)^{\frac{(\gamma-1)}{\gamma}}-1\right]}} \Rightarrow forces...p_{exit} = p_\infty$$

Atlas V, Revisited (cont'd)





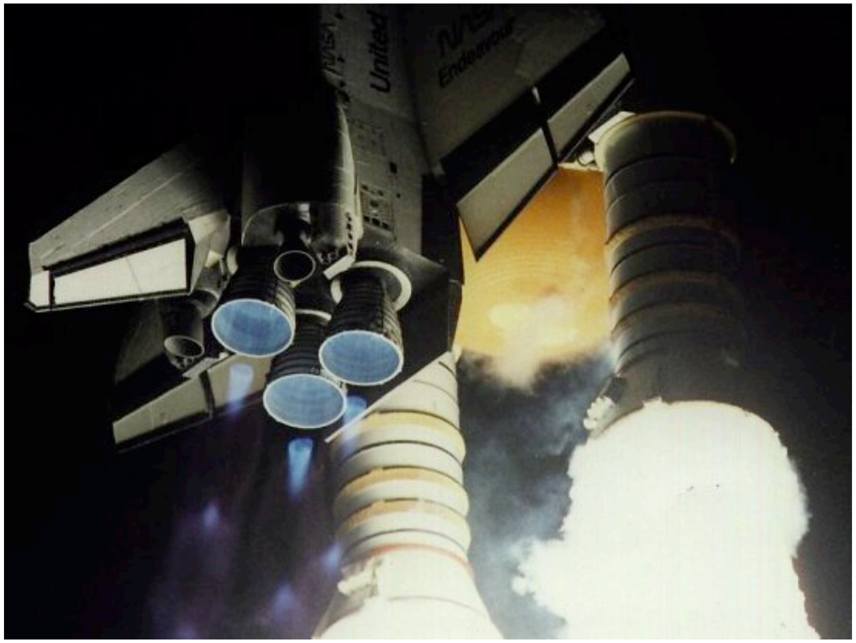
- ATLAS V
First stage is
Optimized for
Maximum
performance
At~ 3k altitude

7000 ft.

How About Space Shuttle SSME

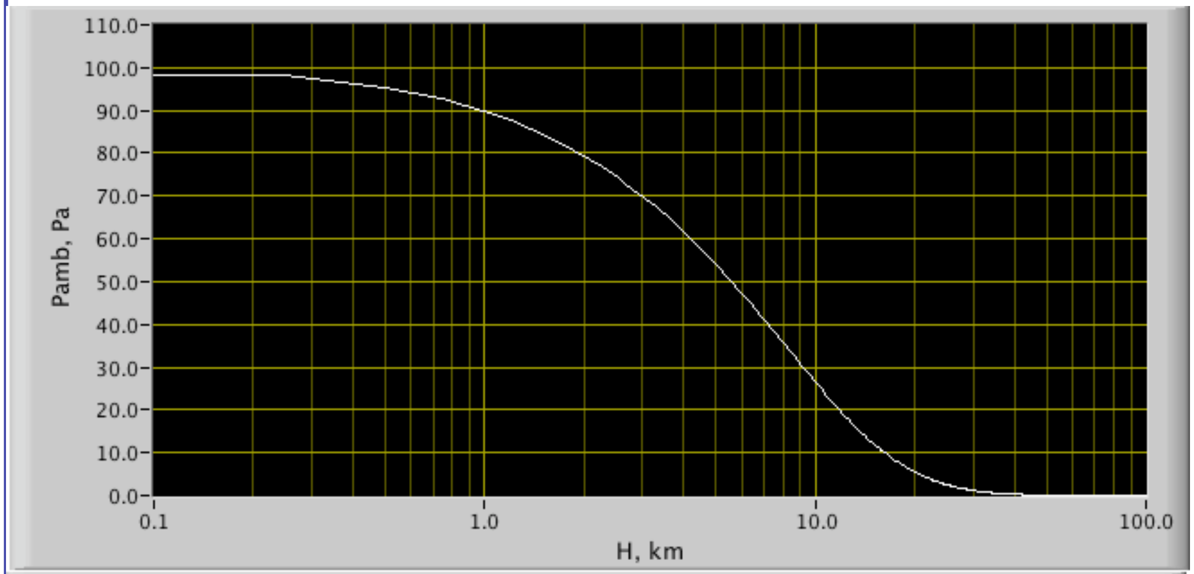
Per Engine (3)

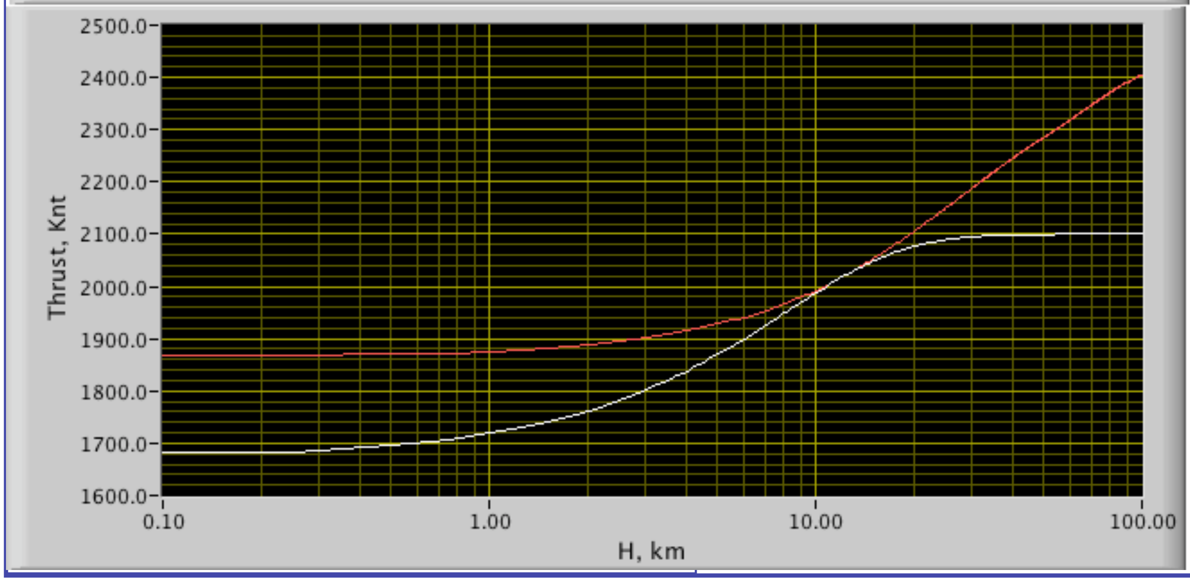
- $Thrust_{vac}$ = 2100.00 kn
- $Thrust_{sl}$ = 1670.00 kn
- $I_{sp_{vac}}$ = 452.55sec
- A_e/A_* = 77.52
- Lox/LH2 Propellants



- γ=1.196

How About Space Shuttle SSME (cont'd)





- SSME is Optimized for Maximum performance At~ 12.5k Altitude

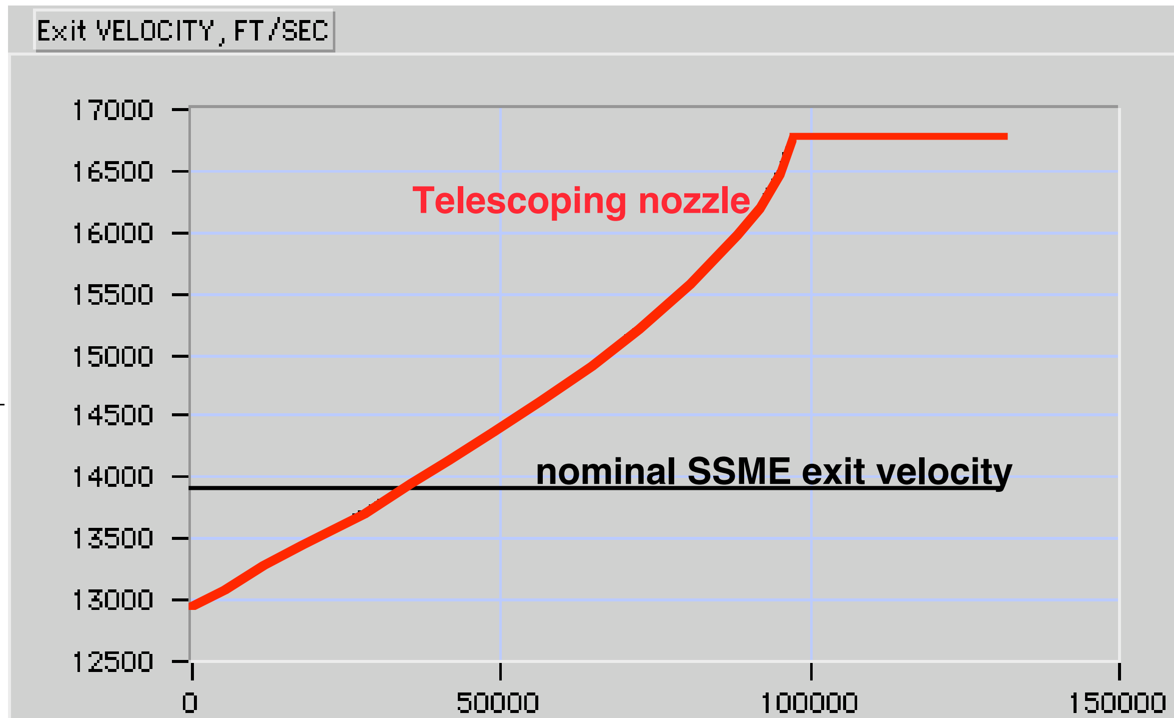
~ 40,000 ft

How About Space Shuttle SSME (cont'd)

"Optimum Nozzle" (cont'd)

- **Exit Velocity**

V_{exit}, $\frac{ft}{sec}$

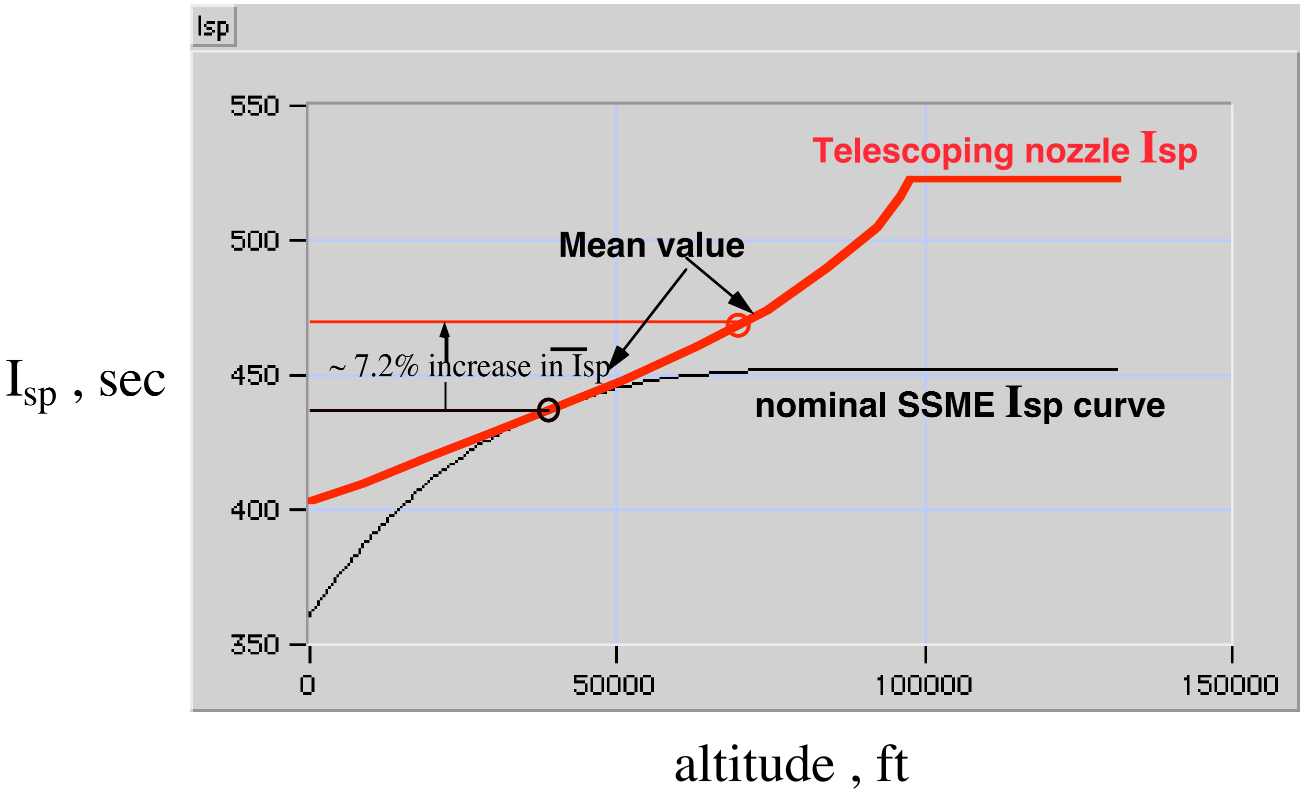


altitude , ft

How About Space Shuttle SSME (cont'd)

"Optimum Nozzle" (concluded)

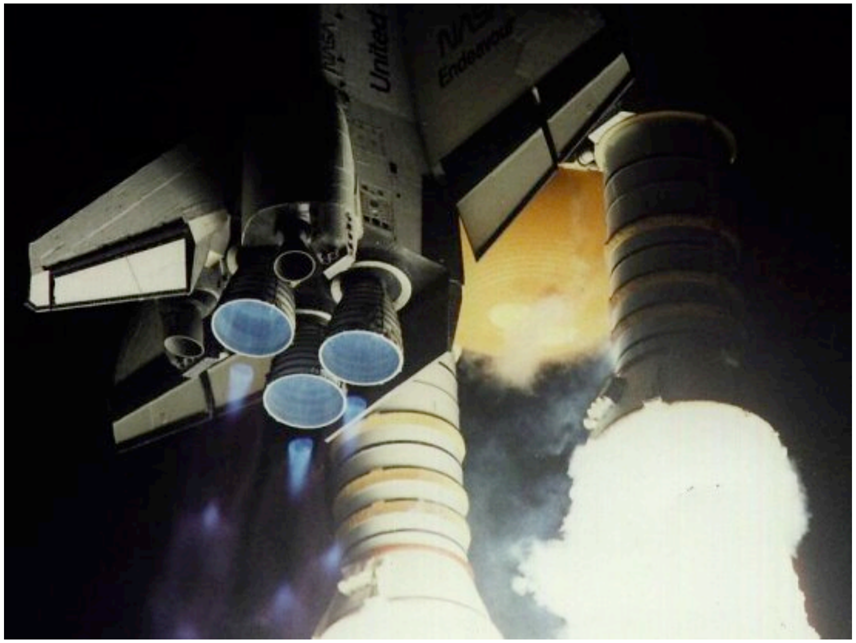
- **Isp**



How About Space Shuttle SRB

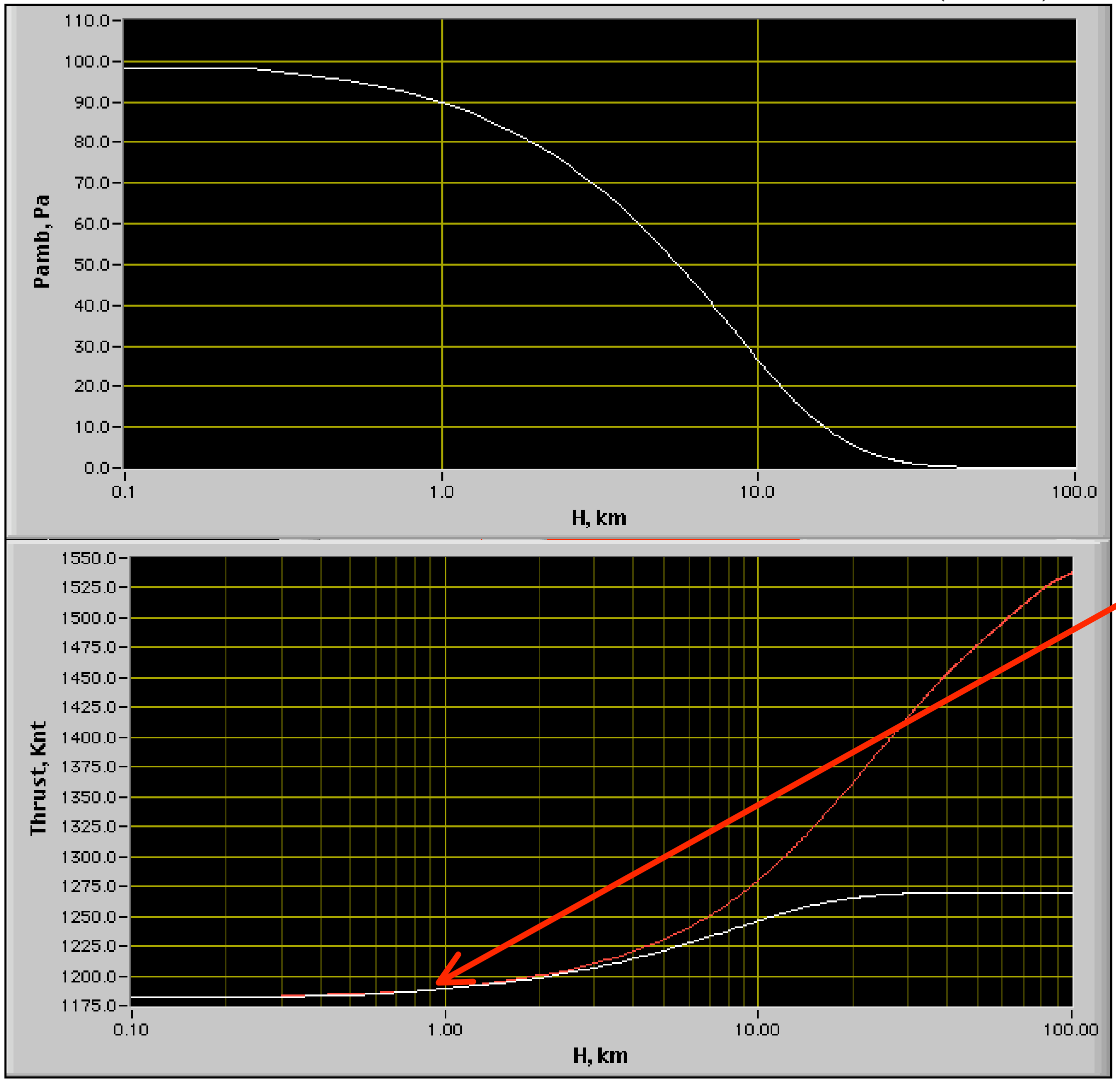
Per Motor (2)

- $Thrust_{vac}$ = 1270.00 kn
- $Thrust_{sl}$ = 1179.00 kn
- $I_{sp_{vac}}$ = 267.30 sec
- A_e/A_* = 7.50
- P_0 = 6.33 Mpa
- PABM (Solid) Propellant



- γ=1.262480

How About Space Shuttle SRB (cont'd)



- SRB is Optimized for Maximum performance At <1k altitude

  3280 ft.

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Solve for Design Altitude of Given Nozzle

$$\frac{A_{exit}}{A^*} = \sqrt{\frac{\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}}{\left(\frac{2}{\gamma-1}\right)}}\sqrt{\frac{\left(\frac{P_0}{p_\infty}\right)^{\frac{\gamma+1}{\gamma}}}{\left[\left(\frac{P_0}{p_\infty}\right)^{\frac{(\gamma-1)}{\gamma}}-1\right]}} \Rightarrow rewrite...as$$

$$\left(\frac{2}{\gamma-1}\right)\left[\left(\frac{P_0}{p_\infty}\right)^{\frac{(\gamma-1)}{\gamma}}-1\right]\left(\frac{A_{exit}}{A^*}\right)^2-\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}\left(\frac{P_0}{p_\infty}\right)^{\frac{\gamma+1}{\gamma}}=0$$

Solve for Design Altitude of Given Nozzle

(cont'd)

Factor out $\left(\dfrac{p_\infty}{P_0}\right)^{\frac{\gamma+1}{\gamma}}$

$$\left(\frac{2}{\gamma-1}\right)\left[\left(\frac{P_0}{p_\infty}\right)^{\frac{(\gamma-1)}{\gamma}}-1\right]\left(\frac{A_{exit}}{A^*}\right)^2-\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}\left(\frac{P_0}{p_\infty}\right)^{\frac{\gamma+1}{\gamma}}=0$$

$$\left(\frac{2}{\gamma-1}\right)\left(\frac{p_\infty}{P_0}\right)^{\frac{\gamma+1}{\gamma}}\left[\left(\frac{P_0}{p_\infty}\right)^{\frac{(\gamma-1)}{\gamma}}-1\right]\left(\frac{A_{exit}}{A^*}\right)^2-\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}=0$$

$$\left(\frac{2}{\gamma-1}\right)\left[\left(\frac{p_\infty}{P_0}\right)^{\frac{\gamma+1}{\gamma}}\left(\frac{p_\infty}{P_0}\right)^{-\frac{(\gamma-1)}{\gamma}}-\left(\frac{p_\infty}{P_0}\right)^{\frac{\gamma+1}{\gamma}}\right]\left(\frac{A_{exit}}{A^*}\right)^2-\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}=0$$

Solve for Design Altitude of Given Nozzle

(cont'd)

Simplify

$$\left(\frac{2}{\gamma-1}\right)\left[\left(\frac{p_\infty}{P_0}\right)^{\frac{2}{\gamma}}-\left(\frac{p_\infty}{P_0}\right)^{\frac{(\gamma+1)}{\gamma}}\right]\left(\frac{A_{exit}}{A^*}\right)^2-\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}=0$$

$$\left[\left(\frac{p_\infty}{P_0}\right)^{\frac{2}{\gamma}}-\left(\frac{p_\infty}{P_0}\right)^{\frac{(\gamma+1)}{\gamma}}\right]-\frac{\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}}{\left(\frac{2}{\gamma-1}\right)\left(\frac{A_{exit}}{A^*}\right)^2}=0$$

Solve for Design Altitude of Given Nozzle

(cont'd)

- Newton Again?

- No … there is an easier way

- User Iterative Solve to Compute Exit Mach Number

$$\frac{A_{exit}}{A^*} = 36.87 = \left[\frac{1}{M_{exit}} \left[\left(\frac{2}{\gamma + 1} \right) \left(1 + \frac{(\gamma - 1)}{2} M_{exit}{}^2 \right) \right]^{\frac{\gamma+1}{2(\gamma-1)}} \right] \Rightarrow$$

$$M_{exit} = 4.2954$$

Solve for Design Altitude of Given Nozzle

(cont'd)

- Compute Exit Pressure

$$p_{exit} = \frac{P_{0_{exit}}}{\left(1 + \frac{\gamma - 1}{2} M_{exit}^{\ 2}\right)^{\left(\frac{\gamma}{\gamma - 1}\right)}} = \frac{25.7 \cdot 1000}{\left(1 + \frac{1.220122 - 1}{2} 4.2954^2\right)^{\left(\frac{1.220122}{1.220122 - 1}\right)}} = 55.06 \text{ kPa}$$

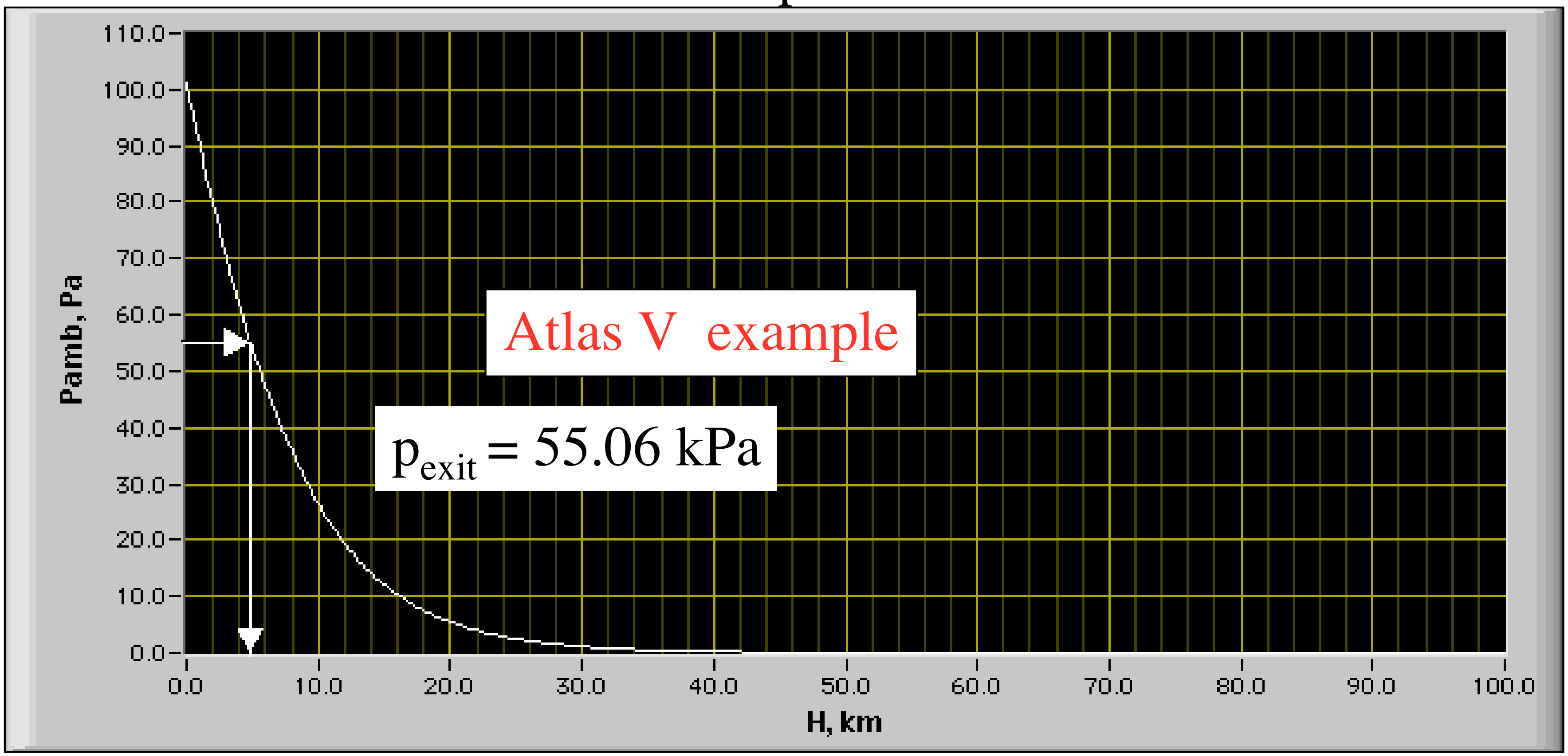
- Set

$$\boxed{p_{exit} = p_{\infty}}_{opt}$$

Solve for Design Altitude of Given Nozzle

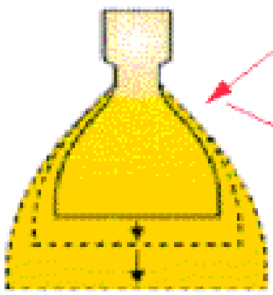
(cont'd)

- Table look up of US 1977 Standard Atmosphere or World GRAM 99 Atmosphere



Space Shuttle Optimum Nozzle?

$$\frac{A_{exit}}{A_t} \Rightarrow p_{exit} = p_a$$



"telescoping nozzle"

Unfeasible because of the large weight penalty and complexity of deployment mechanisms, also requires that nozzle expand to very large area ratios

What is A/A* Optimal for SSME at 80,000 ft altitude (24.384 km)?

At 80kft

$$p_\infty = 2.76144 \text{ kPa}$$

$$p_\infty = 2.76144 \text{ kPa} \rightarrow \frac{P_0}{p_\infty} = \frac{18.9 \times 10}{2.76144} = 6844.3$$

$$\rightarrow M_{exit} = \sqrt{\frac{2}{\gamma - 1}\left[\left(\frac{P_0}{p_\infty}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]} = \left(\frac{2}{1.196 - 1}\left(\left(\frac{18900}{2.76144}\right)^{\frac{1.196-1}{1.196}} - 1\right)\right)^{0.5} =$$

5.7592

Space Shuttle Optimum Nozzle? (cont'd)

$$p_\infty = 2.76144 \text{ kPa} \rightarrow \frac{P_0}{p_\infty} = \frac{18.9 \times 10}{2.76144} = 6844.3$$

$$\rightarrow M_{exit} = \sqrt{\frac{2}{\gamma - 1}\left[\left(\frac{P_0}{p_\infty}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]} = 5.7592$$

What is A/A^* Optimal for SSME at 80,000 ft altitude (24.384 km)?

At 80kft

$$p_\infty = 2.76144 \text{ kPa}$$

$$\frac{A}{A^*} = \frac{1}{M}\left[\left(\frac{2}{\gamma + 1}\right)\left(1 + \frac{(\gamma - 1)}{2} M^2\right)\right]^{\frac{\gamma+1}{2(\gamma-1)}} =$$

$$\frac{\left(\left(\frac{2}{1.196 + 1}\right)\left(1 + \frac{1.196 - 1}{2}\left(5.7592^2\right)\right)\right)^{\frac{1.196 + 1}{2\,(1.196 - 1)}}}{5.7592} = 340.98$$

(originally 77.52)

Space Shuttle Optimum Nozzle? (cont'd)

$$p_\infty = 2.76144 \text{ kPa} \rightarrow \frac{P_0}{p_\infty} = \frac{18.9 \times 10}{2.76144} = 6844.3$$

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(originally 77.52)

Space Shuttle Optimum Nozzle? (cont'd)

$$\frac{A}{A^*} = \frac{1}{M}\left[\left(\frac{2}{\gamma+1}\right)\left(1+\frac{(\gamma-1)}{2}M^2\right)\right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

$$= 340.98$$

What is A/A* Optimal for SSME at 80,000 ft altitude (24.384 km)?

At 80kft

$$p_\infty = 2.76144 \text{ kPa}$$

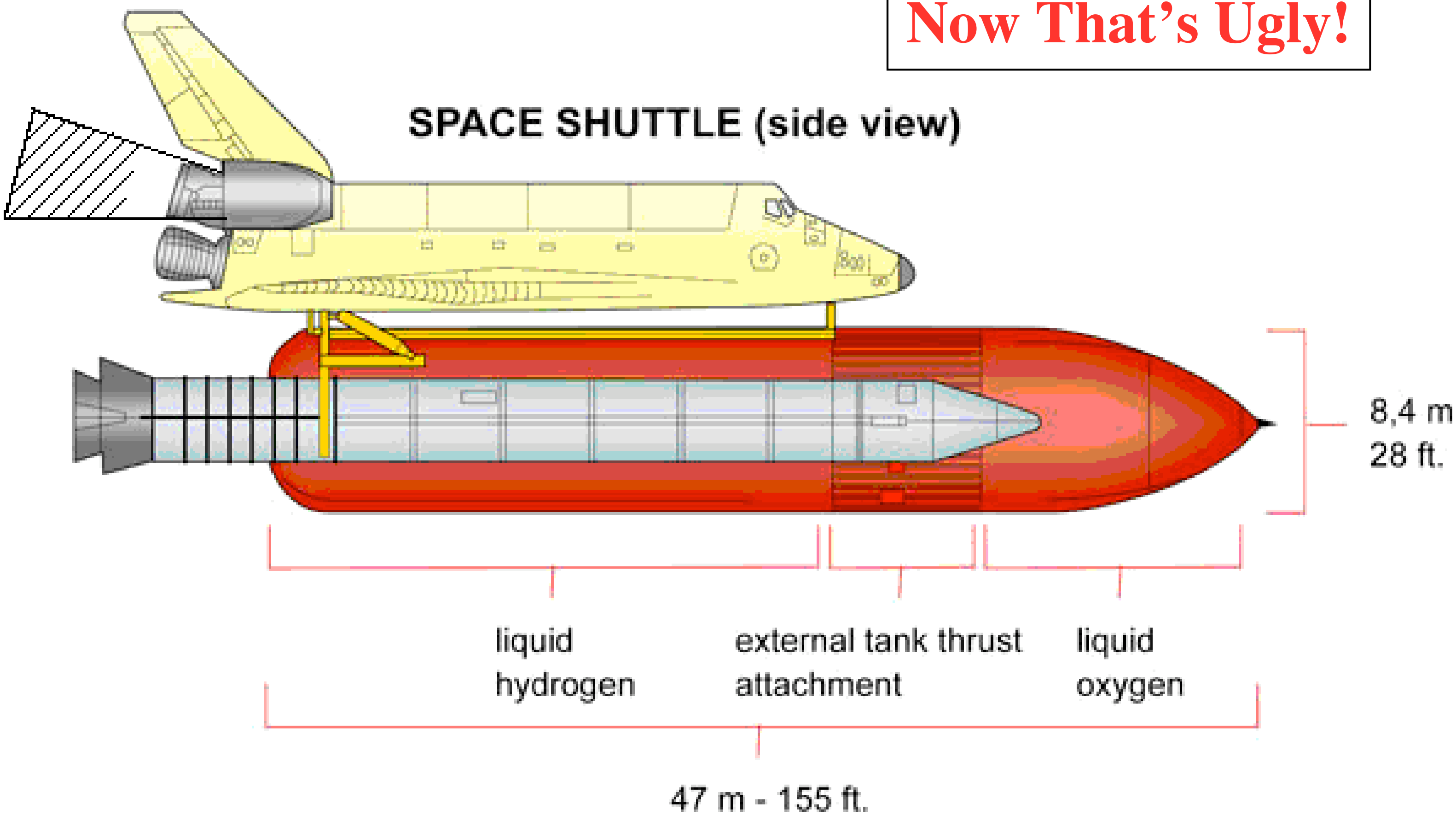
- Compute Throat Area

$$\left(\frac{26}{100}\right)^2 \frac{\pi}{4} = 0.05297 \text{ m}^2$$

→ Aexit=18.062 m^2---> 4.8 (15.7 ft) meters in diameter
As opposed to 2.286 meters for original shuttle

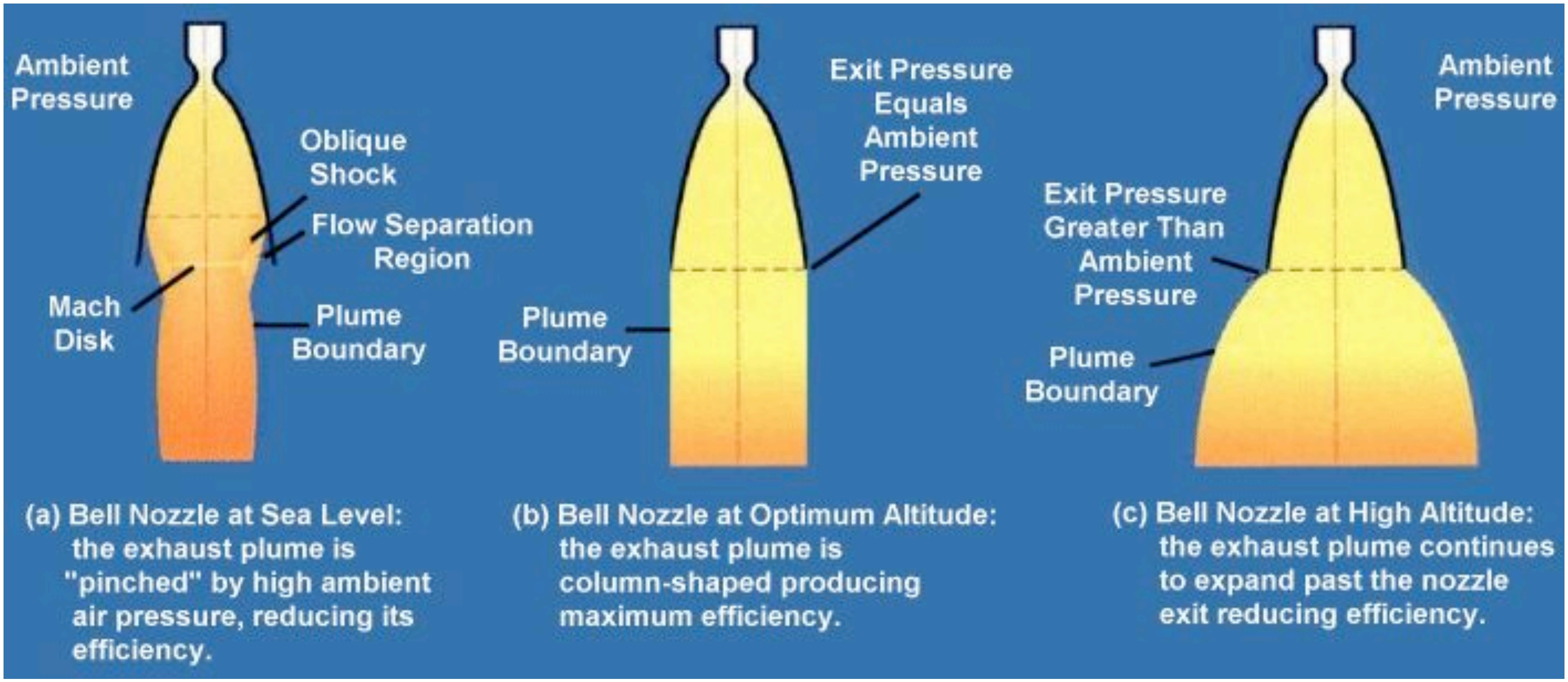
Space Shuttle Optimum Nozzle? (cont'd)

Now That's Ugly!



- So What are the Alternatives?

Optimal Nozzle Summary



Credit: Aerospace web

Optimal Nozzle Summary (cont'd)

- Thrust equation $Thrust = \dot{m} V_{exit} + A_{exit}(p_{exit} - p_{\infty})$

can be re-written as

$$Thrust = \gamma P_0 A^* \sqrt{\frac{2}{\gamma - 1}\left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma+1}{(\gamma-1)}}} \left[1 - \left(\frac{p_{exit}}{P_0}\right)^{\frac{\gamma-1}{\gamma}}\right]^{1/2} + A_{exit}(p_{exit} - p_{\infty})$$

and

$$\frac{A_{exit}}{A^*} = \sqrt{\frac{\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}}{\left(\frac{2}{\gamma-1}\right)}} \sqrt{\frac{\left(\frac{P_0}{p_{exit}}\right)^{\frac{\gamma+1}{\gamma}}}{\left[\left(\frac{P_0}{p_{exit}}\right)^{\frac{(\gamma-1)}{\gamma}} - 1\right]}}$$

Optimal Nozzle Summary (cont'd)

- Eliminating A_{exit} from the expression

$$Thrust = \gamma P_0 A^* \sqrt{\frac{2}{\gamma-1}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}} \left\{ \left[1-\left(\frac{p_{exit}}{P_0}\right)^{\frac{\gamma-1}{\gamma}}\right]^{1/2} + \frac{\gamma-1}{2\gamma}\sqrt{\frac{\left(\frac{p_{exit}}{P_0}\right)^{\frac{\gamma+1}{-\gamma}}}{\left[\left(\frac{p_{exit}}{P_0}\right)^{\frac{(\gamma-1)}{-\gamma}}-1\right]}}\left[\frac{p_{exit}}{P_0}-\frac{p_\infty}{P_0}\right] \right\}$$

P_0, γ, driven by combustion process, only p_e is effected by nozzle

- Optimal Nozzle given by

$$\frac{\partial\left(\frac{Thrust}{P_0 A^*}\right)}{\partial p_{exit}} = 0 \Rightarrow \boxed{p_{exit} = p_\infty}_{opt}$$

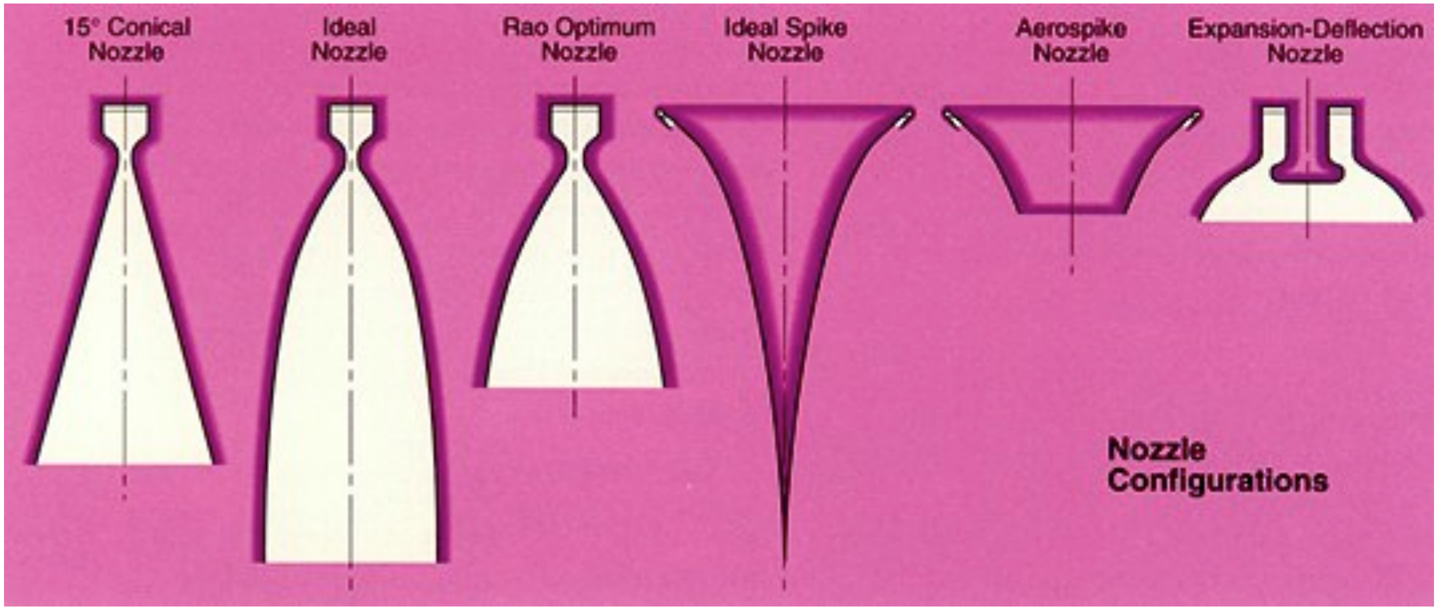
Optimal Nozzle Summary (cont'd)

- Optimal Thrust (or thrust at design condition)

$$Thrust_{opt} = \gamma P_0 A^* \sqrt{\frac{2}{\gamma - 1}\left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma+1}{(\gamma-1)}}\left[1 - \left(\frac{p_\infty}{P_0}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

$$\frac{A_{exit}}{A^*} = \sqrt{\frac{\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}}{\left(\frac{2}{\gamma-1}\right)}}\sqrt{\frac{\left(\frac{P_0}{p_\infty}\right)^{\frac{\gamma+1}{\gamma}}}{\left[\left(\frac{P_0}{p_\infty}\right)^{\frac{(\gamma-1)}{\gamma}} - 1\right]}} \Rightarrow forces ... p_{exit} = p_\infty$$

Optimal Nozzle Summary (concluded)

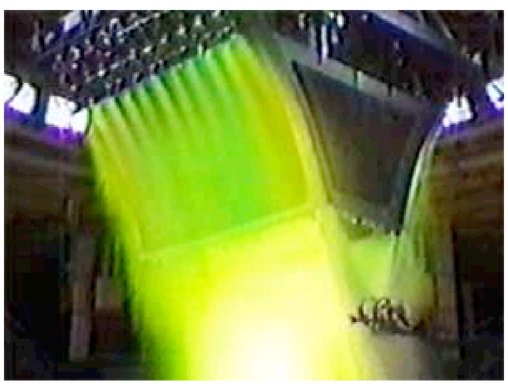


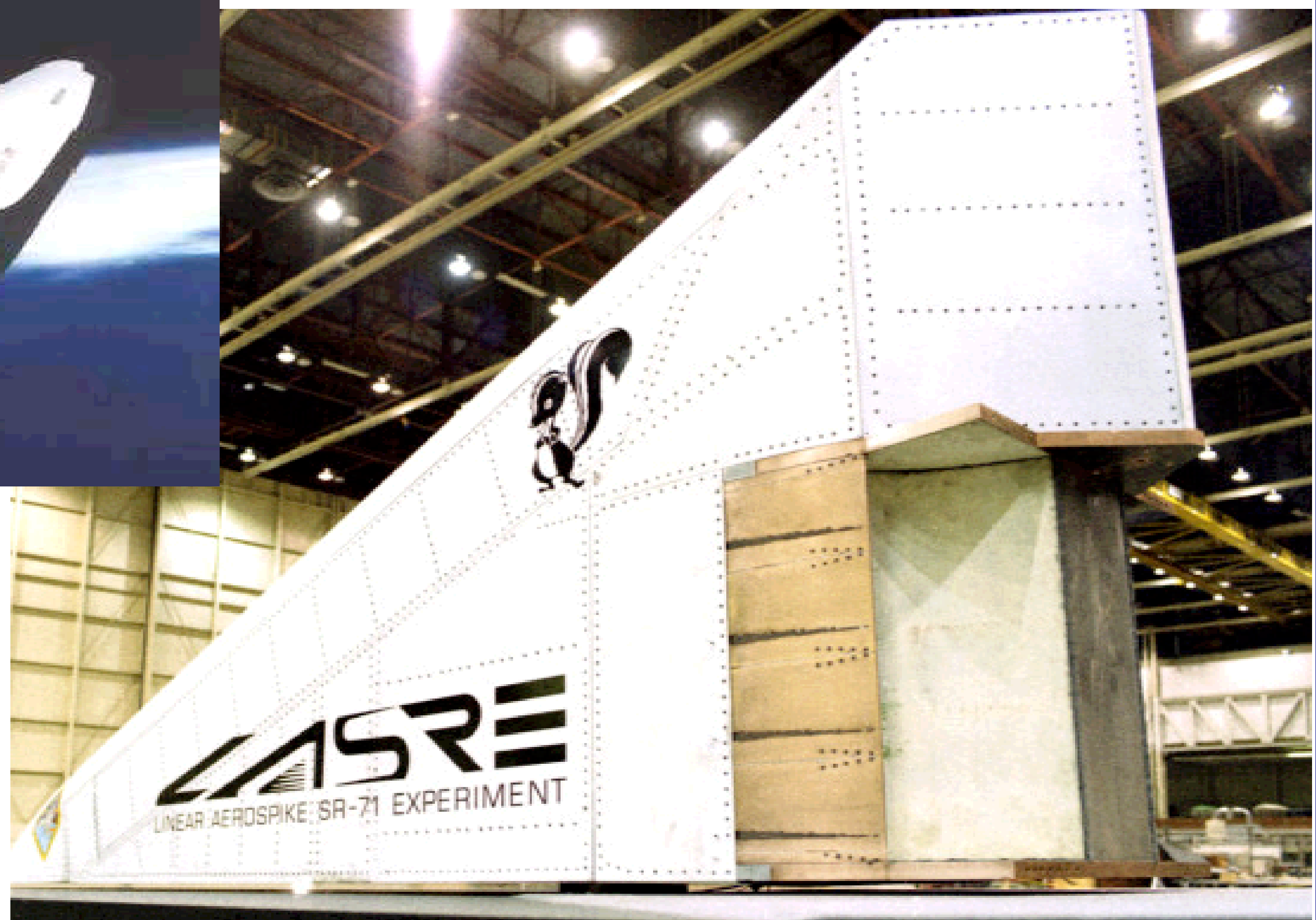
- *Optimum nozzle configuration for a particular mission depends upon system trades involving performance, thermal issues, weight, fabrication, vehicle integration and cost.*

http://www.k-makris.gr/RocketTechnology/ Nozzle_Design/nozzle_design.htm

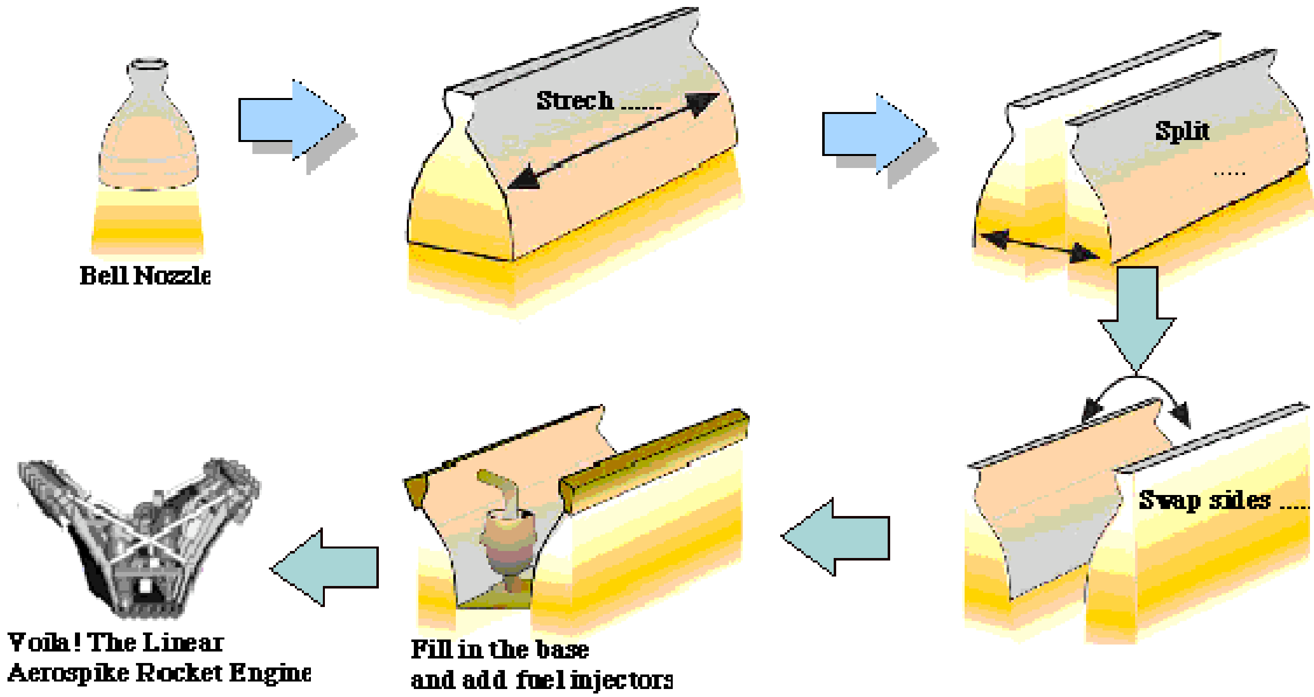
"The Linear Aerospike Rocket Engine"

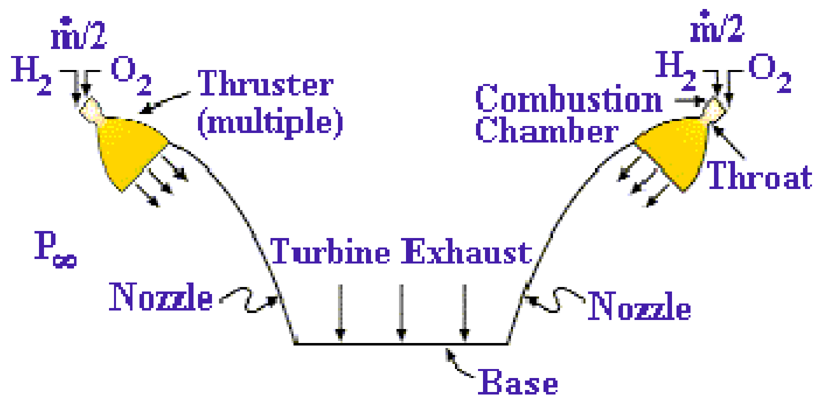
… Which leads us to the … real alternative





A New Nozzle Shape





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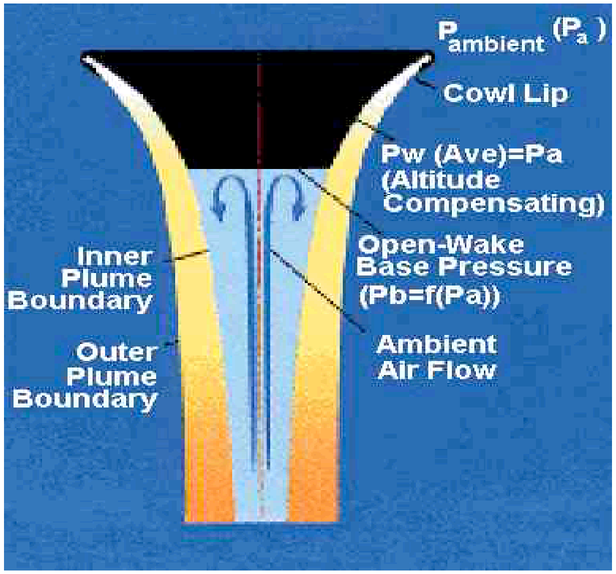
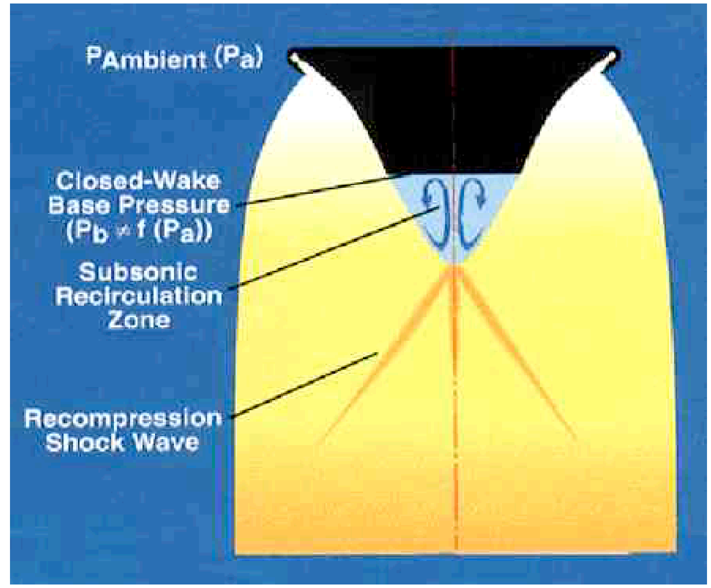
Linear Aerospike Rocket Engine

Nozzle has same effect as telescope nozzle

Lift off



Vacuum (Space)



$$F = F_{Thruster} + F_{Ramp} + F_{Base}$$

$$F_{Thruster} = \cos\theta\,(\dot{m}V_{exit} + A_{exit}(P_{exit} - P_{\infty}))$$

$$F_{Ramp} = \int^{A_{Ramp}} (P_{Ramp} - P_{\infty})\, dA$$

$$F_{Base} = A_{Base}\,(P_{Base} - P_{\infty})$$

- **Aerospike's flow unconstrained, allows best performance**

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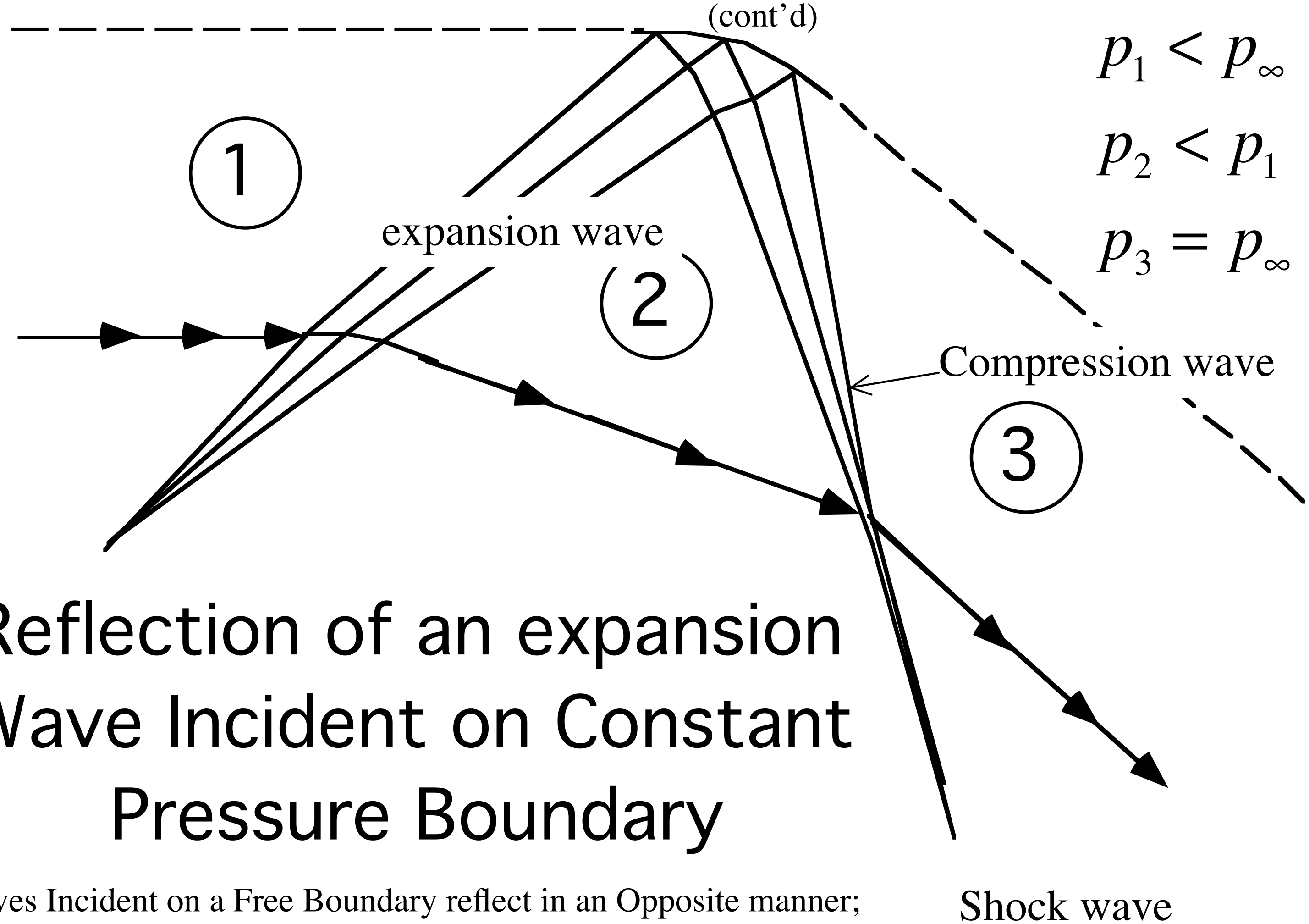
Wave reflections from a free boundary

(cont'd)



$p_1 < p_\infty$

$p_2 < p_1$

$p_3 = p_\infty > p_2$

Reflection of an expansion Wave Incident on Constant Pressure Boundary

Waves Incident on a Free Boundary reflect in an Opposite manner; *Compression wave reflects as expansion wave, expansion wave reflects as compression wave*

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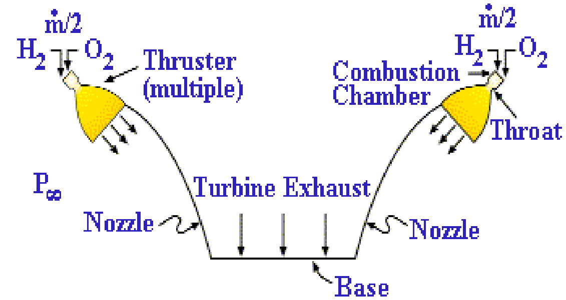
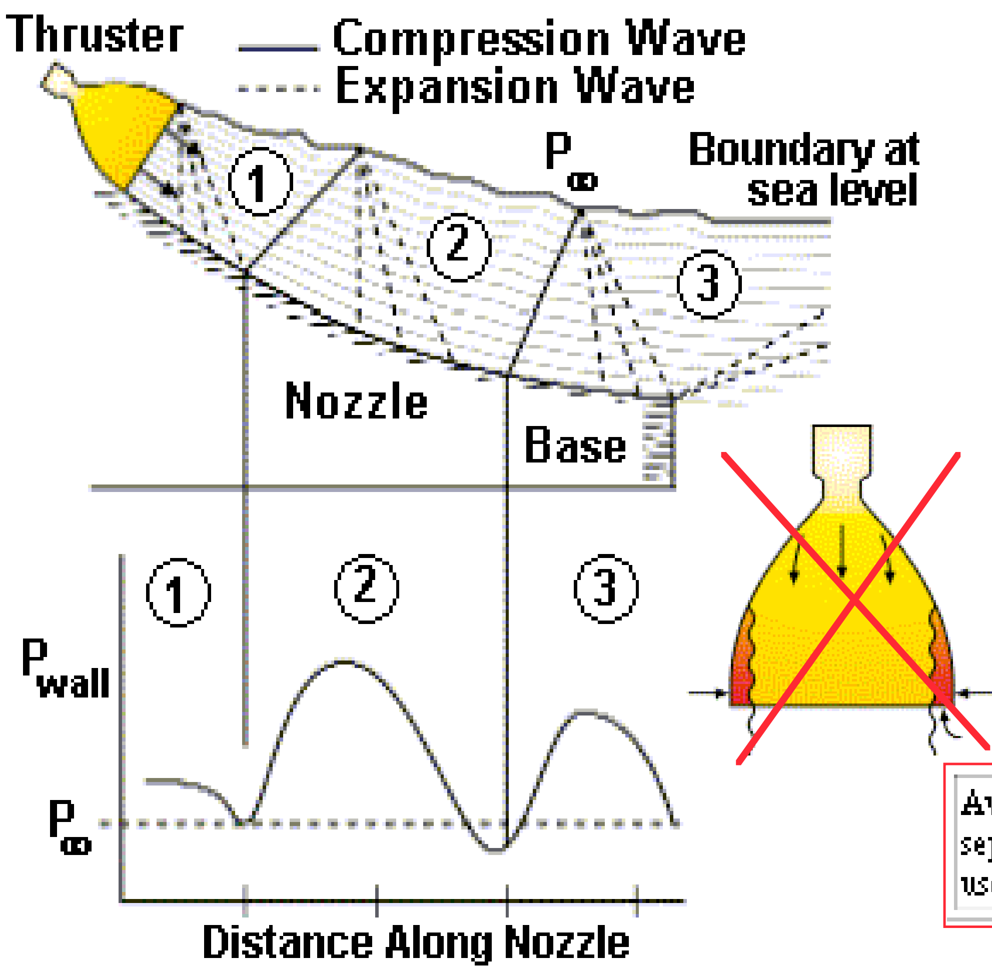
Linear Aerospike Rocket Engine

(cont'd)



Low Altitude Aerodynamics

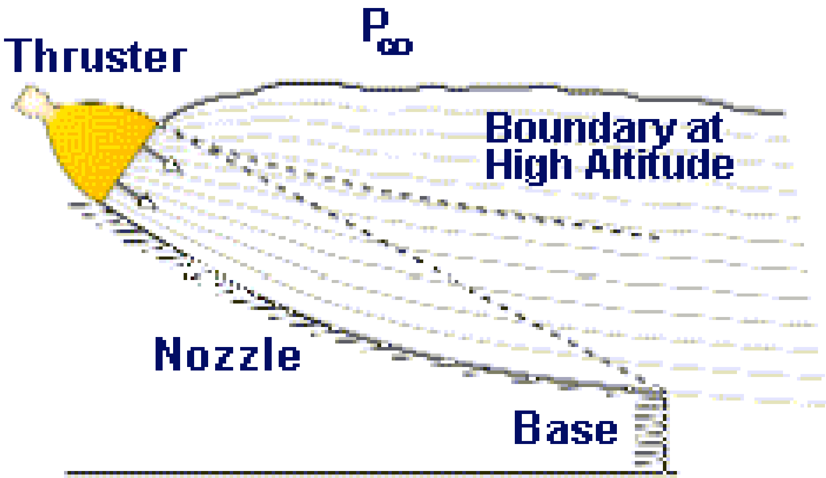


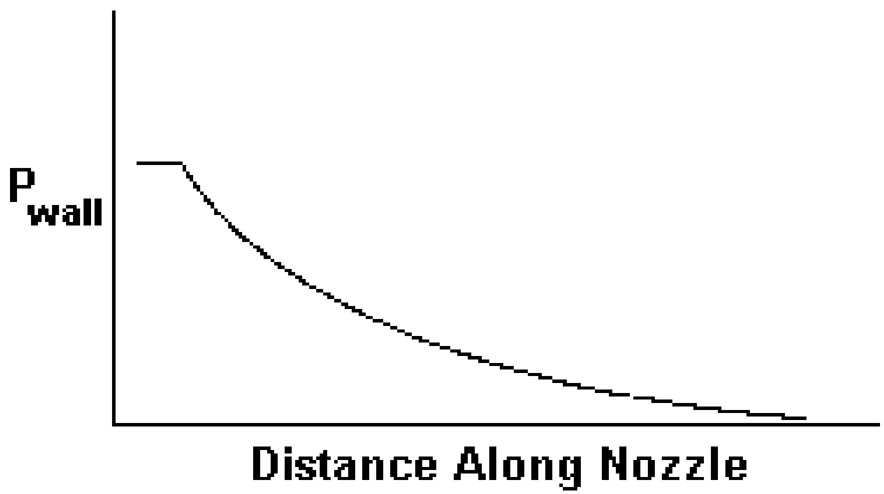
- Thruster flow discharges to ramp
- Expansion waves turn flow axially
- Ramp curves, turns flow axially (at low altitudes)
- Turning causes compression wave from (1) to (2) - nozzle pressure increases
- Compression wave reflects off boundary causing expansion waves
- Flow crosses expansion waves in (2) - nozzle pressure decreases
- Ramp continues to curve and turn flow
- Process repeats (2) to (3)

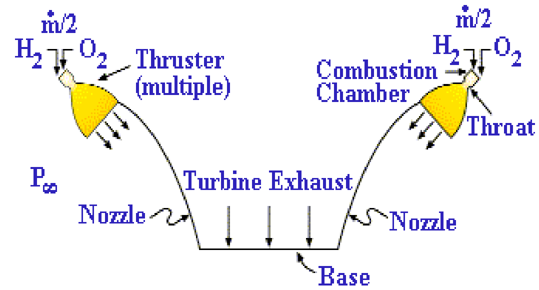
Average nozzle pressure > P, therefore no losses or separation, therefore large area ratio nozzle can be used, enabling SSTO

Linear Aerospike Rocket Engine
(cont'd)

High Altitude Aerodynamics



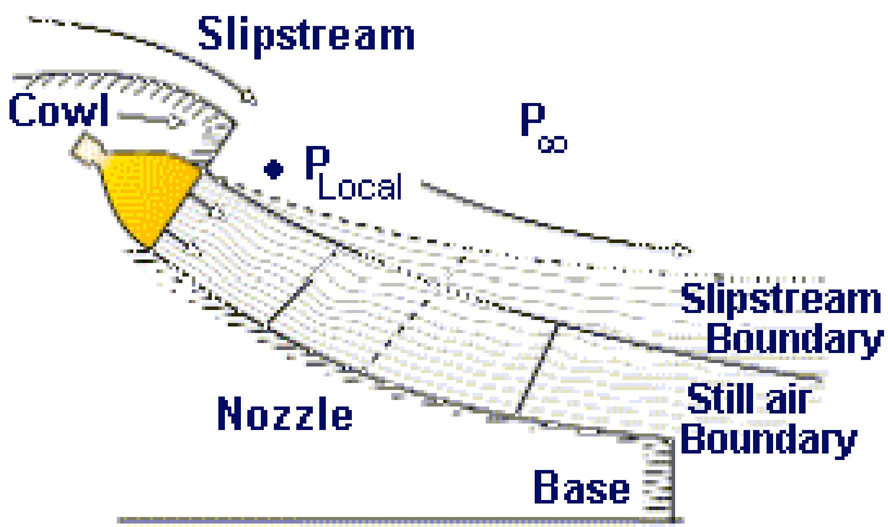


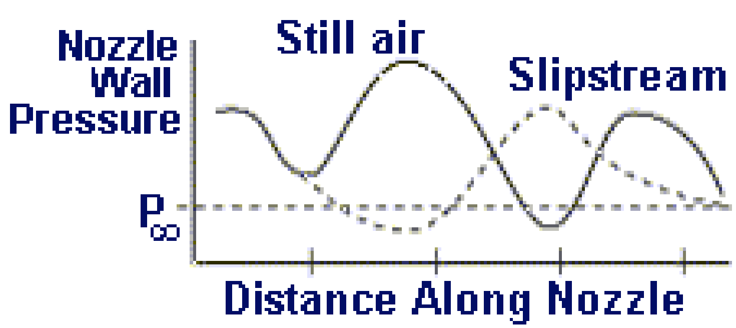


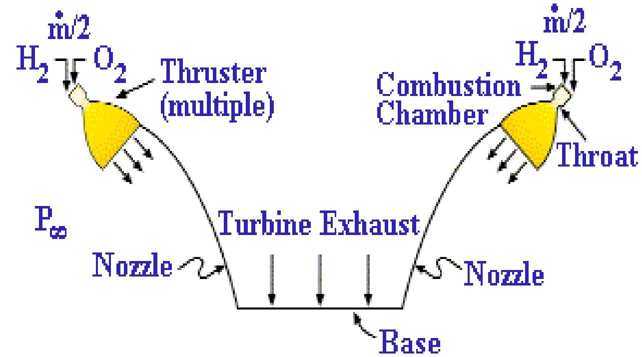
-
- Thruster flow discharges to ramp
- Expansion waves turn flow axially
- No compression waves exist - all flow turning done by expansion waves
- Nozzle behaves likes a bell

Linear Aerospike Rocket Engine
(concluded)

SlipStream effects





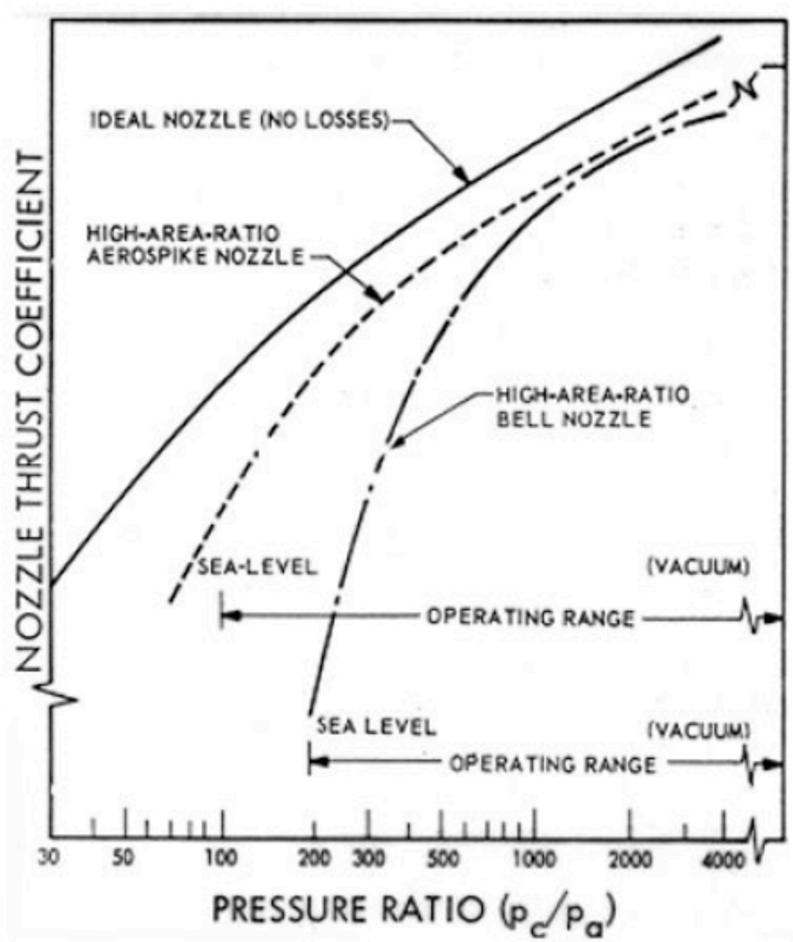


- Air streaming over cowl lowers local pressure - $P_{Local} < P_{infinity}$
- Exhaust plume expands beyond still air case
- Expansion and compression wave systems move aft from still air case

- Resulting recompression Delays Nozzle separation

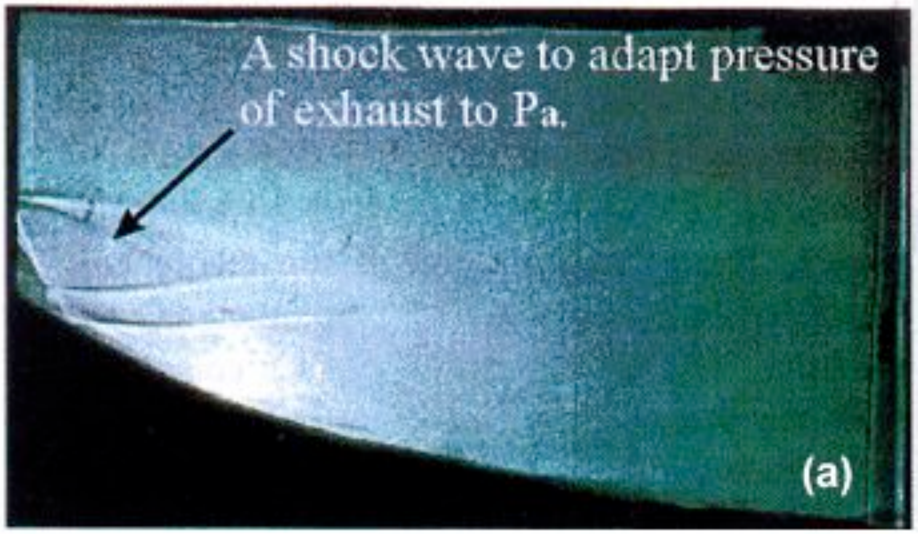
Bottom Line is that the Linear Aerospike engine realizes about 50% of thetheoretical Isp gains offeredby the Telescoping nozzle

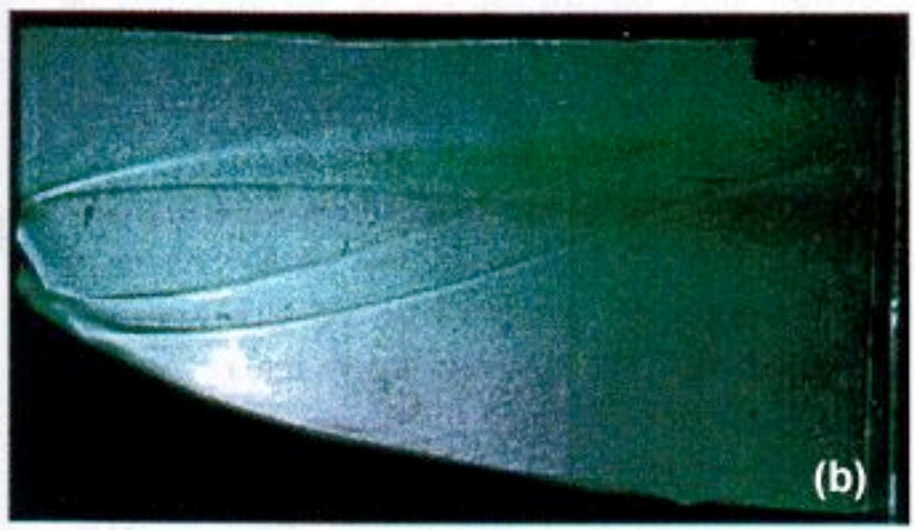
Performance Comparison



[from Huzel and Huang, 1967]

- Although less than Ideal
  The significant Isp recovery of Spike Nozzles offer significant advantage





- **Shadowgraph flow visualization of an ideal isentropic spike at**

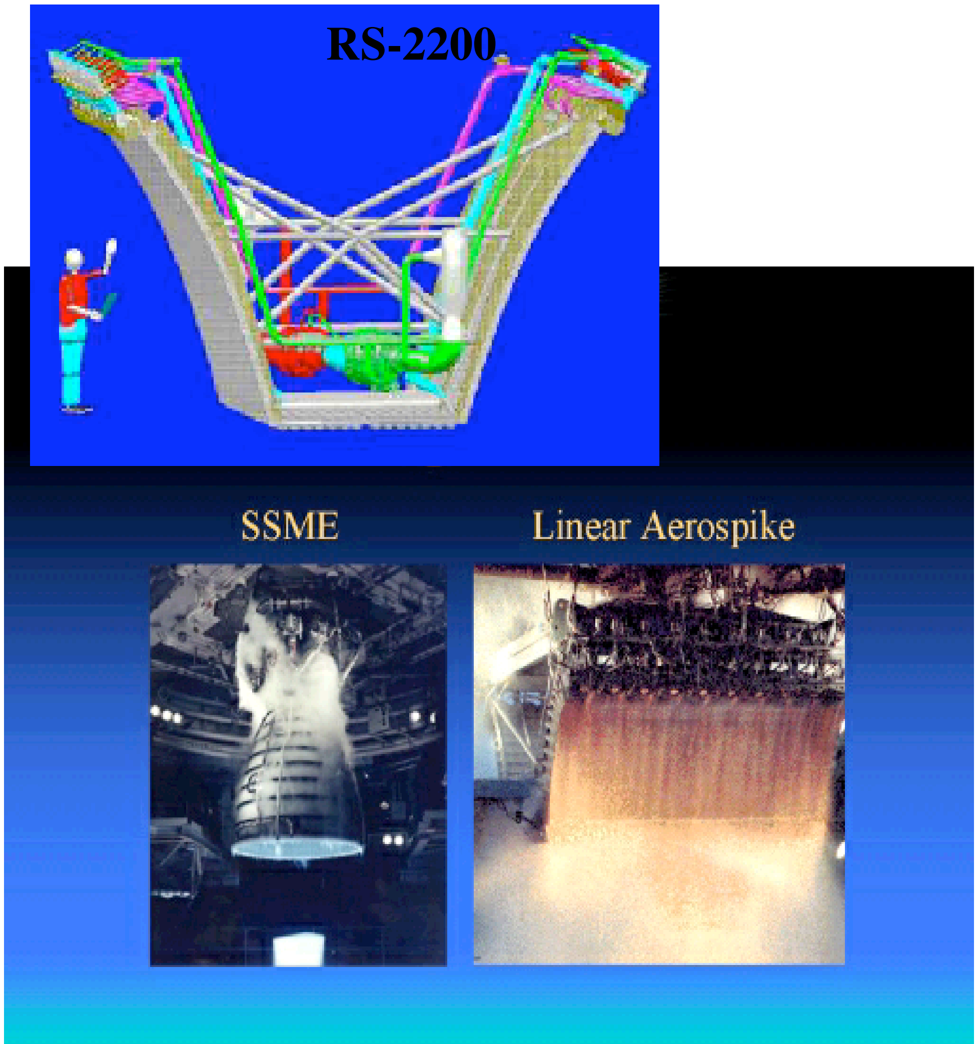
(a) low altitude and
(b) high altitude conditions

[from Tomita et al, 1998]

Credit: Aerospace web

Linear Aerospike Engine Comparison to SSME

Credit Rocketdyne



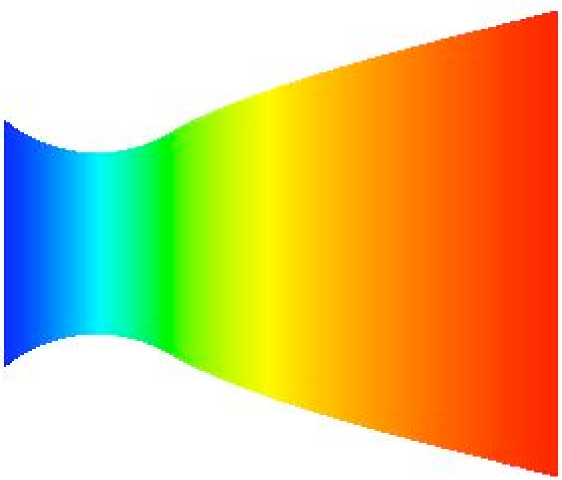
<u>RS-2200: (Venture-Star)</u>
Manufacturer: Boeing Rocketdyne
Weight: 8000 lbs.
Max Thrust: 520,000 lbf (Liftoff)
564,000 lbf (Space)
I_{sp}: 420 sec (Liftoff)
460 sec (Space)
Mean Isp: 453.3

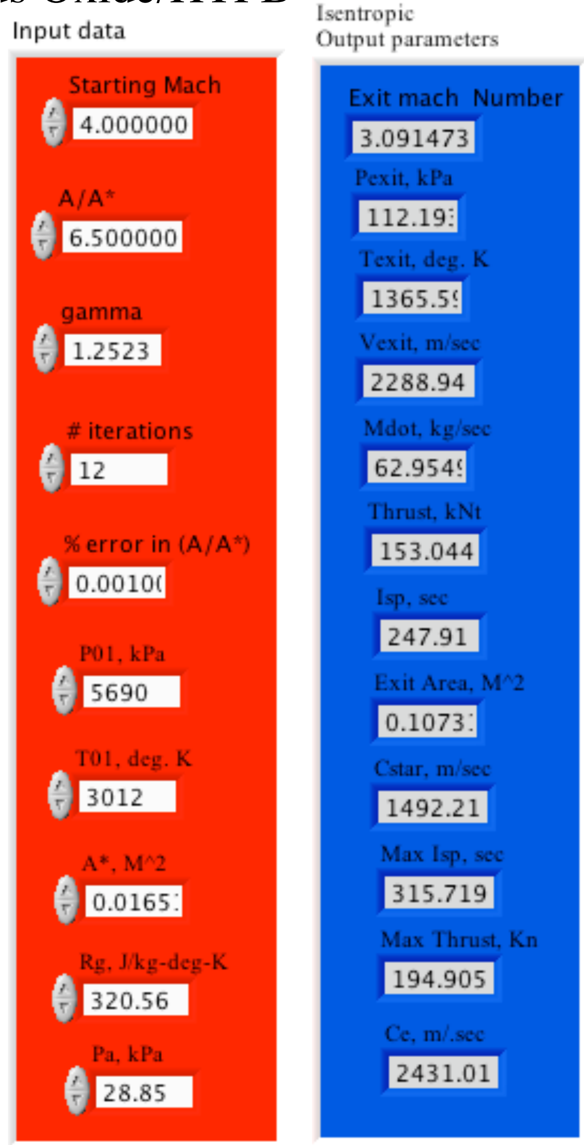
<u>SSME: (Shuttle (Block IIa)</u>
Manufacturer: Boeing Rocketdyne
Weight: 7,480 lbs.
Max Thrust: 418,660 lbf (Liftoff)
512,950 lbf (Space)
I_{sp}: 360 sec (Liftoff)
452.4 sec (Space)
Mean Isp: 437.0

3.7% better performance
~52% of the theoretical telecoping Nozzle I_{sp} gains

Computational Example

- Conventional Nozzle, Hybrid Motor, Nitrous Oxide/HTPB 5.7:1 mixture ratio
- Low expansion ratio nozzle: $A_{exit}/A^{*}=6.50$
- Operating @ 70,000 ft altitude
- I_{sp} = **247.91 sec**





Computational Example (cont'd)

UtahState UNIVERSITY

Mechanical & Aerospace Engineering

- Aerospike Nozzle, Hybrid Motor, Nitrous Oxide/HTPB
  5.7:1 mixture ratio, $A_{exit}/A^{*}=6.50$
- Operating @ 70,000 ft altitude --> I_{sp} = **274.35 sec**

Aerospike Nozzle Design by Method of Characteristics

Aerospike Nozzle Data

Thruster exit area ratio [Aei/At]	1.25	Chamber temperature	3012.435
Aerospike expansion ratio [Ae/At]	6.5	Chamber pressure	5.69E+6
Ratio of specific heats for exhaust	1.252	Aerospike thrust	1.53E+6
Gas constant of exhaust [Rgas]	320.56	Ramp base radius [Rbase]	0.
Aerospike pressure ratio [Pc/Pa]	197.2	CF - Thrust coefficient	1.635
Thruster internal circular radius [Ra]	0.3	CF - Vacuum thrust coefficient	1.662
Radius to lip of cowl [Re]	.5	Isp - Specific Impulse	274.35
Aerospike length from origin [Lnozzle]	1.75	Isp - Vacuum specific impulse	278.9

View Data

1

LOCATION

Plot Types

- (•) Contours
- ( ) F(x) verses x
- ( ) F(x) verses PR

Plot Contours

256

LEVELS

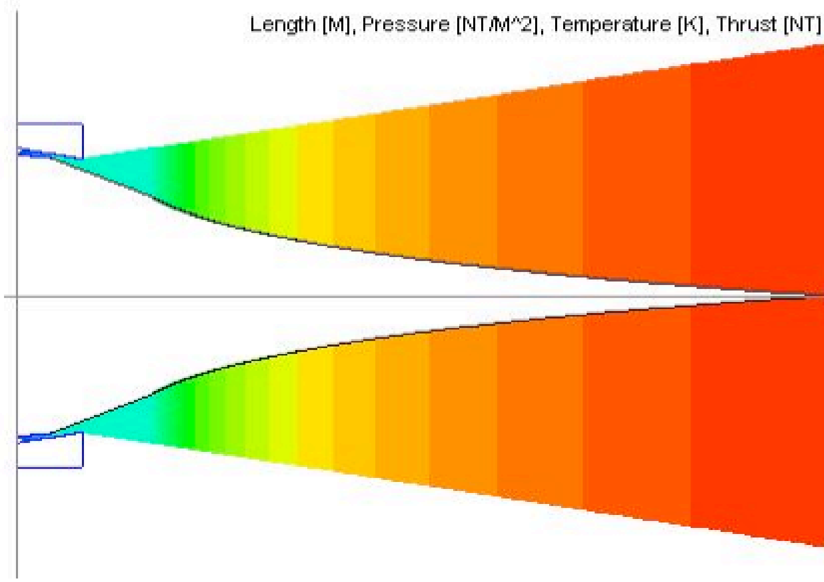
3.091

STOP

1.0

- (•) Mn
- ( ) P/Pc
- ( ) T/Tc
- ( ) R/Rc

Plot Progress

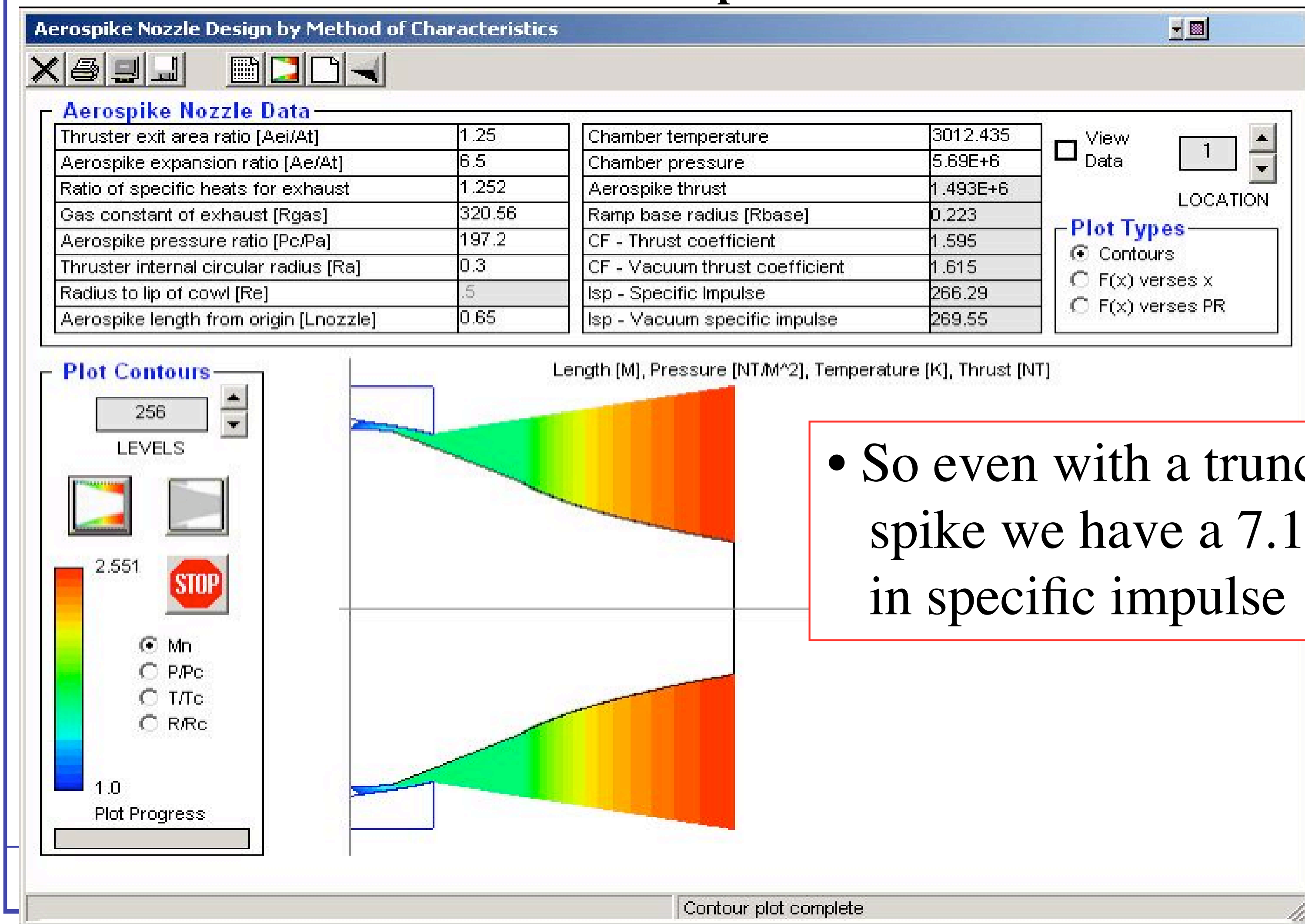


Contour plot complete

- 10.1% increase in specific impulse

Computational Example (cont'd)

- Truncated Aerospike Nozzle, Hybrid Motor, Nitrous Oxide/HTPB 5.7:1 mixture ratio, $A_{exit}/A^{*}=6.50$
- Operating @ 70,000 ft altitude --> I_{sp} **= 266.29 sec**



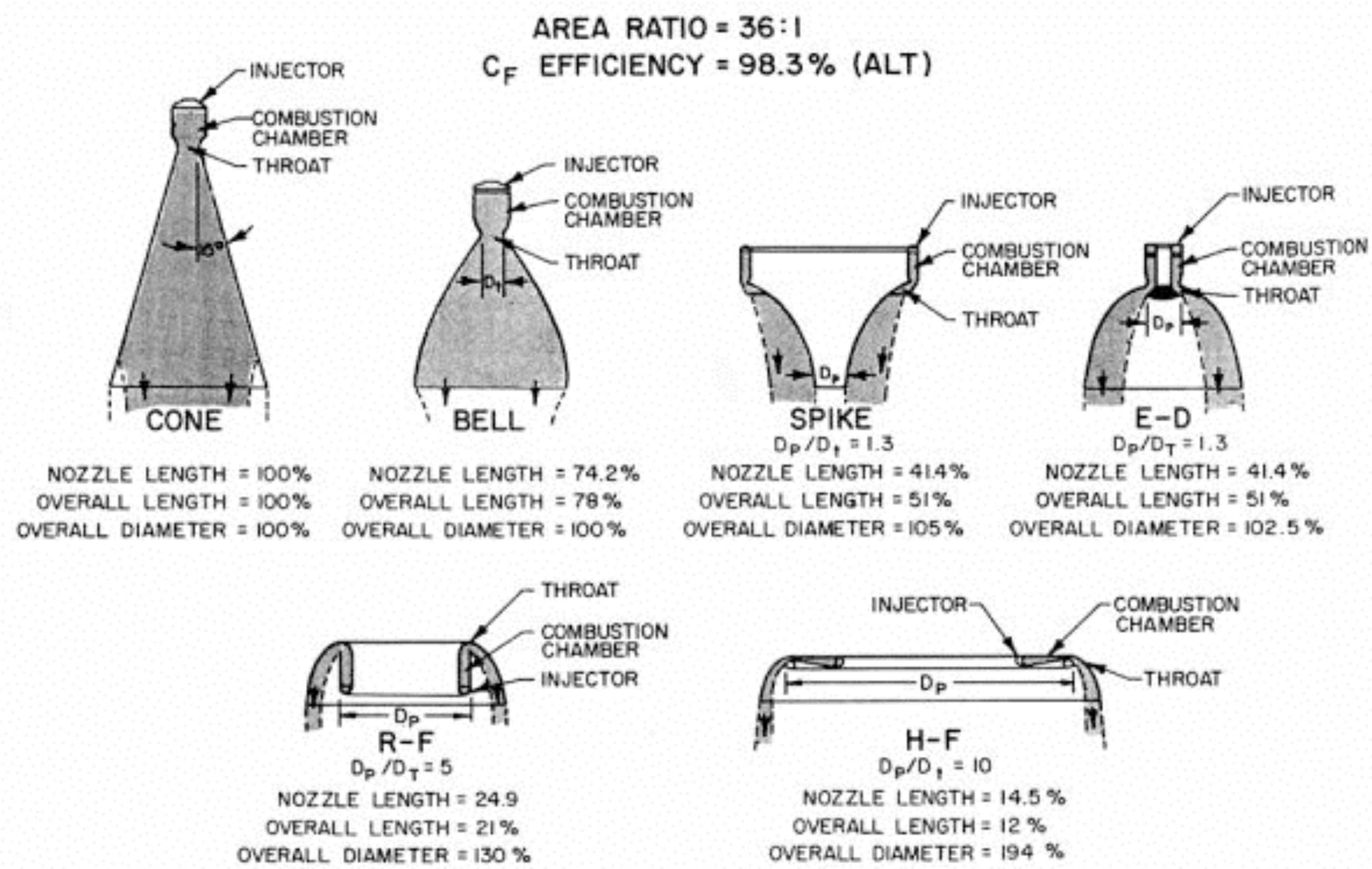
- So even with a truncated spike we have a 7.1% increase in specific impulse

Advantages of Aerospike High Expansion Ratio Experimental Nozzle

- Truncated aerospike nozzles can be as short as 25% the length of a conventional bell nozzle.
  - Provide savings in packing volume and weight for space vehicles.
- Aerospike nozzles allow higher expansion ratio than conventional nozzle for a given space vehicle base area.
  - Increase vacuum thrust and specific impulse.
- For missions to the Moon and Mars, advanced nozzles can increase the thrust and specific impulse by 5-6%, resulting in a 8-9% decrease in propellant mass.
- Lower total vehicle mass and provide extra margin for the mass inclusion of other critical vehicle systems.
- New nozzle technology also applicable to RCS, space tugs, etc…

Spike Nozzle … Other advantages

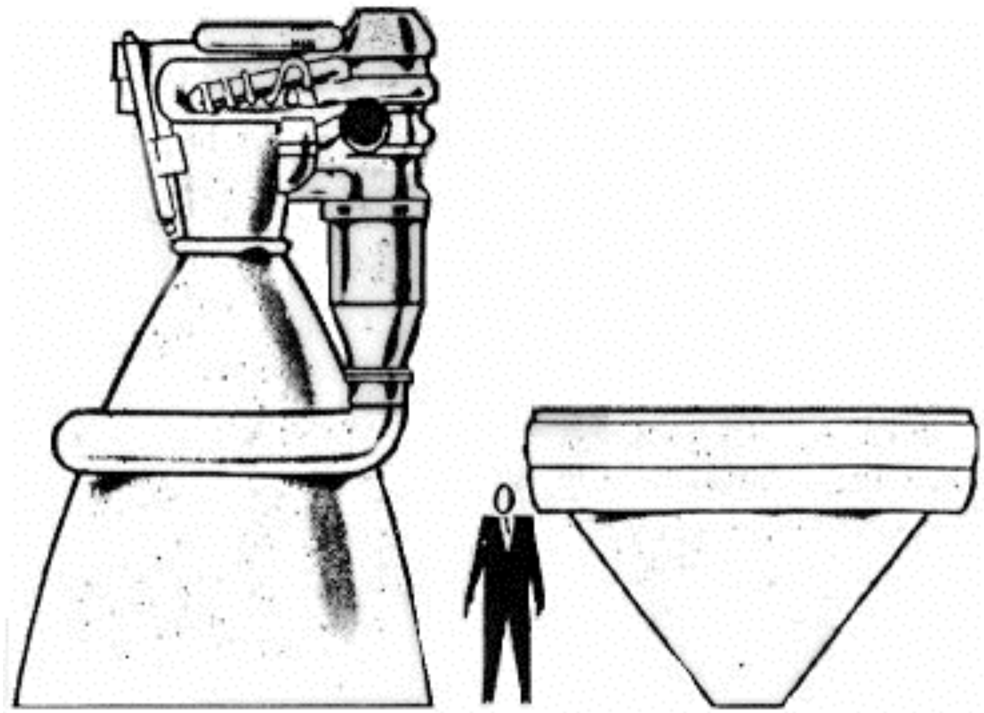
- Higher expansion ratio for smaller size



Credit: Aerospace web

Spike Nozzle … Other advantages (cont'd)

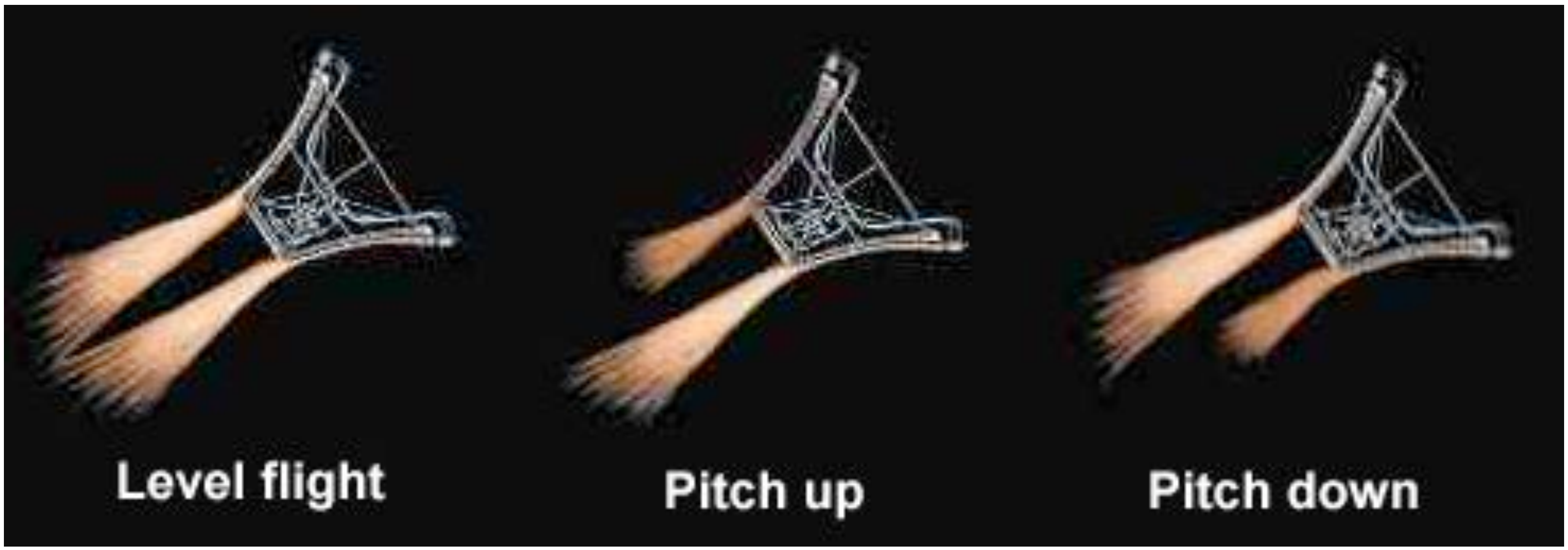
- Higher expansion ratio for smaller size II



Credit: Aerospace web

Spike Nozzle … Other advantages (cont'd)

- Thrust vectoring without Gimbals



Credit: Aerospace web

Spike Nozzle … Disadvantages)

•**Disadvantages:**

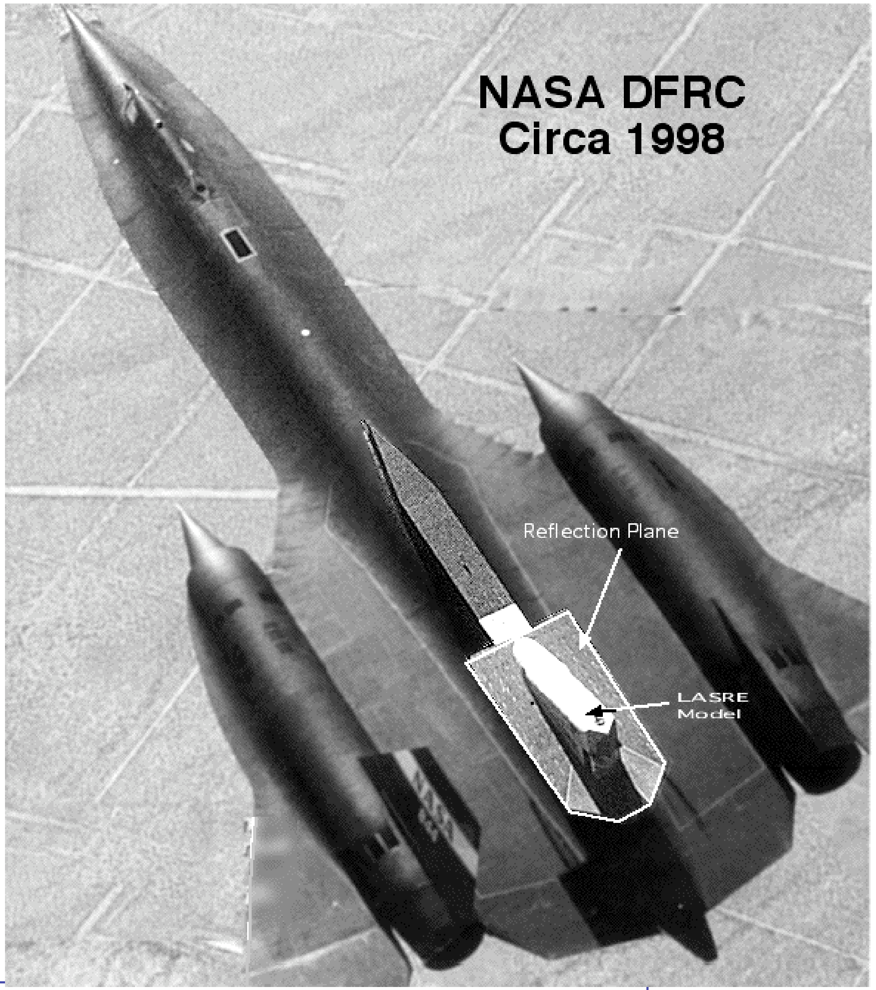
•**Cooling:** The central spike experiences far greater heat fluxes
•than does a bell nozzle. This problem can be addressed by truncating
•the spike to reduce the exposed area and by passing cold cryogenically-
•cooled fuel through the spike. The secondary flow also helps to cool the
•centerbody.

•**Manufacturing:** The aerospike is more complex and difficult
•to manufacture than the bell nozzle. As a result, it is more costly.

•**Flight experience:** No aerospike engine has ever flown in a rocket
•application. As a result, little flight design experience has been gained.

The LASRE Flight Experiment

Linear Aerospike SR-71 Experiment (LASRE)



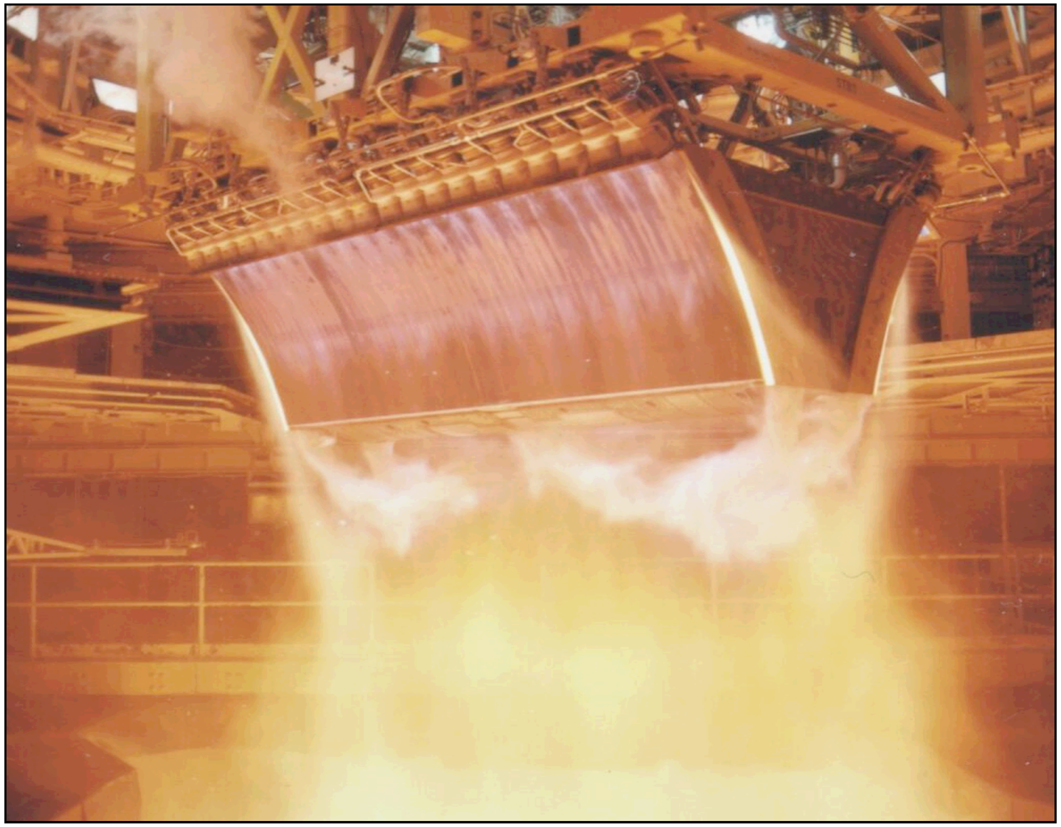
- Flight test of a 20% X-33 model with single linear-aerospike rocket engine
- Tests intended to demonstrate engine effectiveness and measure plume interactions
- Model was instrumented with 6-DOF load-cell balance and extensive surface pressure matrix
- Tests performed for flight conditions varying from Mach 0.6 to Mach 2.0
- Linear Aerospike Engine Never Successfully Fired In Flight

Full Scale Test of RS-2200 Rocket Engine

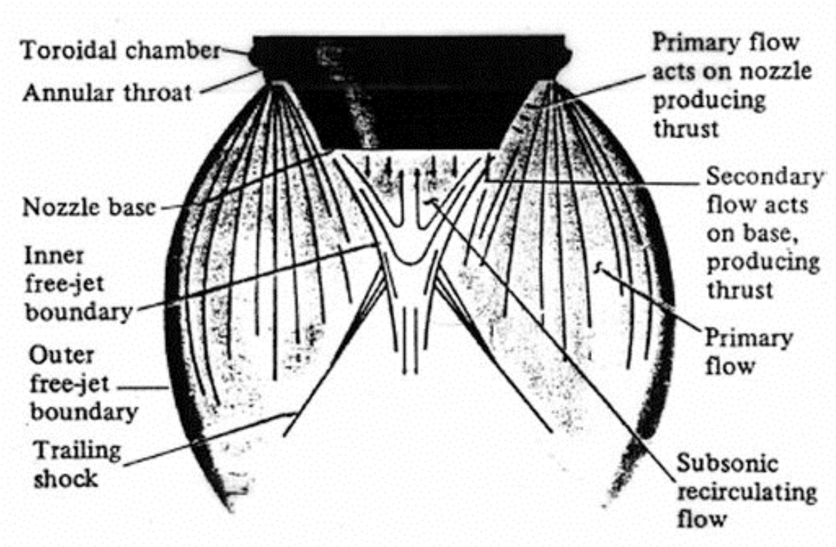
• July 12, 2001
NASA Stennis Space Center
Louisana

• Slip Stream Effects on Nozzle Plume Still never measured In-Flight

• Buuuut .... the Aerospike is Still a Viable Option in the toolbox for creating the 500 sec Isp engine



So Let's Go Test One!



Credit: Aerospace web