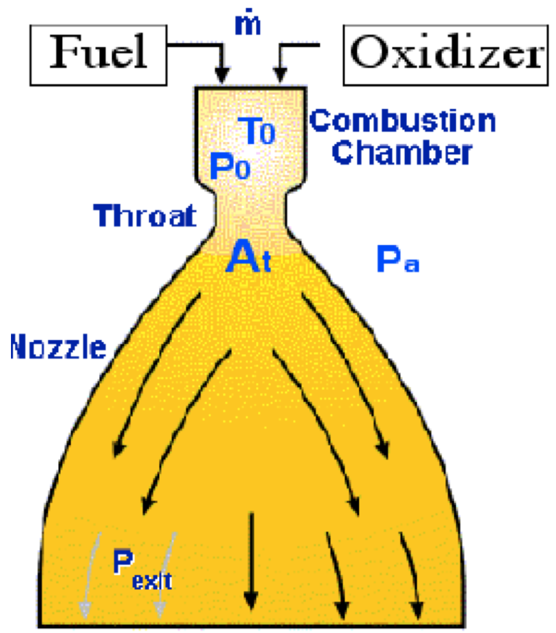
Utah State University

Mechanical & Aerospace Engineering

Section 5, Lecture *4*: *Non-Adiabatic* Adjustments to the 1-D DeLaval Rocket Flow Equations



Material Taken from Sutton and Biblarz, Sections 3.5-3.8

Factors Resulting in NON-IDEAL Rocket Performance

- Incomplete combustion (will defer this discussion to propellant analysis section) Section 7

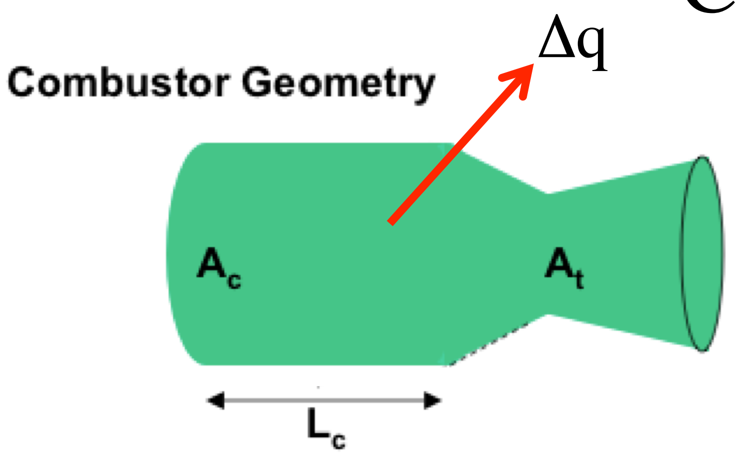
- *Heat Loss from combustor and nozzle*

- Flow Separation and Shockwaves in Nozzle
(will defer ***Section 5, MAE 5420)***

- *Nozzle Discharge Effects*

- *Transient Chamber Pressure*

Look at Effects of Heat Transfer in Combustor



- Look at Combustion (Stagnation) temperature

- From First Law of Thermodynamics

$$i)\ dh = dq + vdp = dq + \frac{dp}{\rho}$$

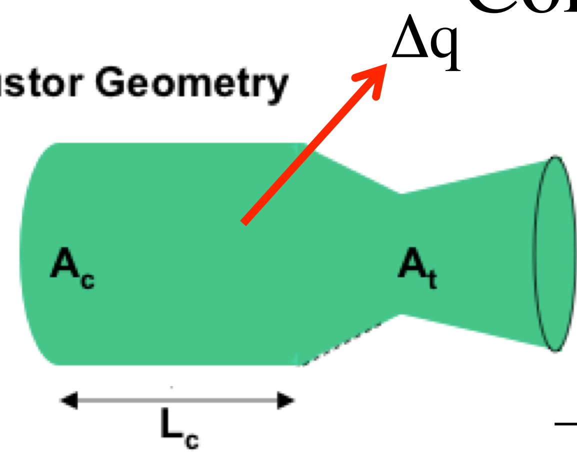
- Assuming approximately 1-D Steady Flow

$$ii)\ \rho V = const \rightarrow d\rho V = -\rho dV$$

$$iii)\ \mathrm{p}+\rho V^2 = const \rightarrow d\rho = -d\rho V^2 - 2\rho V dV$$

Look at Effects of Heat Transfer in Combustor (cont'd)

Combustor Geometry



- Combining i), ii), and iii)

$$dh = dq + \frac{-d\rho V^2 - 2\rho V dV}{\rho} = \frac{\rho V dV - 2\rho V dV}{\rho} = -VdV$$

$$\rightarrow \boxed{dq = dh + VdV}$$

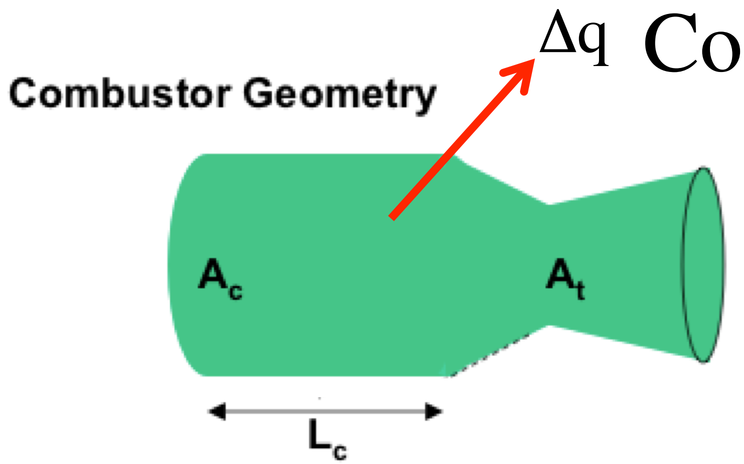
Assuming combustor velocities ahead of contraction are small

$$\Delta q_{combustor} = \iint_{combustor} dh = \iint_{combustor} c_p dT$$

For a short combustor ……

$$\Delta q_{combustor} = \frac{\dot{q}_{wall}}{\dot{m}} = c_p\left[T_{0_{ideal}} - T_{0_{actial}}\right] \rightarrow \boxed{T_{0_{actial}} = T_{0_{ideal}} - \frac{1}{c_p}\left(\frac{\dot{q}_{wall}}{\dot{m}}\right)}$$

Look at Effects of Heat Transfer in Combustor (cont'd)



- Look at effective η^*

- Assuming same combustion products

$$\eta^* = \frac{\overline{C^*}}{C^*} = \sqrt{\frac{T_{0_{actual}}}{T_{0_{ideal}}}} = \sqrt{\frac{T_{0_{ideal}} - \frac{1}{c_p}\left(\frac{\dot{q}_{wall}}{\dot{m}}\right)}{T_{0_{ideal}}}} = \sqrt{1 - \frac{1}{c_p T_{0_{ideal}}}\left(\frac{\dot{q}_{wall}}{\dot{m}}\right)}$$

Look at Effects of Heat Transfer in Δq Combustor (cont'd)

$$\rightarrow \overline{C^*} = \frac{\sqrt{\gamma R_u}}{\gamma \sqrt{\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}}} \sqrt{\frac{T_{o_{actual}}}{M_W}} =$$

- Look at effective C*

$$\frac{\sqrt{R_g}}{\sqrt{\gamma \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}}} \sqrt{T_{0_{ideal}} - \frac{1}{c_p}\left(\frac{\dot{q}_{wall}}{\dot{m}}\right)} = \sqrt{\frac{R_g T_{0_{ideal}} - \frac{\gamma-1}{\gamma}\left(\frac{\dot{q}_{wall}}{\dot{m}}\right)}{\gamma \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}}}$$

Look at Effects of Heat Transfer in Δq Combustor (cont'd)

- But recall for a constant combustor pressure, choking mass flow is a function of Stagnation temperature

$$\dot{m} = A^* \sqrt{\frac{\gamma}{R_g}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}} \frac{p_0}{\sqrt{T_0}}$$

- So we have an iterative problem

Look at Effects of Heat Transfer in Combustor (cont'd)

Iteration Sequence: Given: $\dot{q}_{wall}, T_{0_{ideal}}, \gamma, M_W, P_0, c_p$

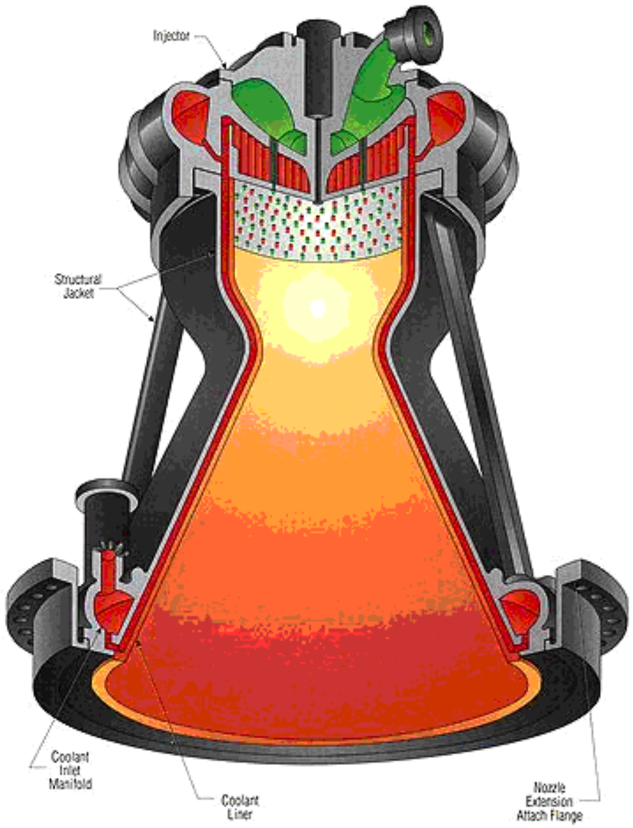
$$\dot{m} = A^* \sqrt{\frac{\gamma}{R_g}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}} \frac{p_0}{\sqrt{T_0}}$$

$$\overline{C}^* = \frac{\sqrt{\gamma R_u}}{\gamma \sqrt{\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}}} \sqrt{\frac{T_{o_{actual}}}{M_W}}$$

$$T_{0_{actial}} = T_{0_{ideal}} - \frac{1}{c_p}\left(\frac{\dot{q}_{wall}}{\dot{m}}\right)$$

SSME Example Revisited

- In order to Cool the Nozzle wall, the Shuttle Uses regenerative Cooling where propellant is circulated through channels in the Combustor walls *(and in the nozzle also … but we'll get to that Later)*



- To keep the burner within temperature limits regenerative heat fluxes as high as 100 BTU/in^2-sec .. Are required …

How Does this effect the engine Performance?

SSME Example Revisited (cont'd)

Calculate heat transfer rate

$$S_{area} \simeq 0.4884\ m^2 = 756.95\ in^2$$

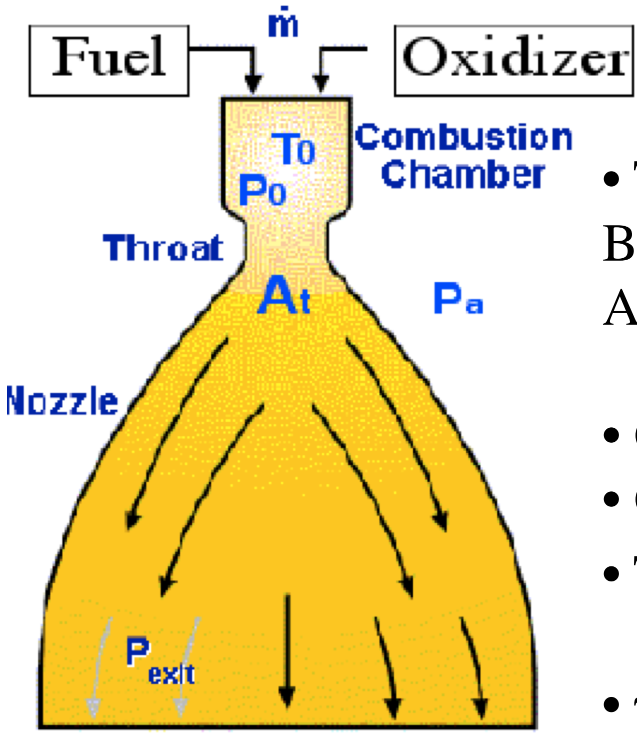
Heat Flux = 100 BTU/in^2-sec

$$\dot{q}_{wall} = 100\ \frac{BTU}{in^2\ \sec} \times 756.95_{in^2} \times 1054.35\ \frac{\text{joule}}{\text{BTU}} = 79809.2_{kWatt}$$

16.342 kW/cm^2

* Space Propulsion Analysis and Design [Ronald W. Humble, Gary N. Henry, and Wiley J. Larson - McGraw-Hill, 1995]

SSME Example Revisited (cont'd)



- The Space Shuttle Main Engines Burn LOX/LH2 for Propellants with A ratio of LOX:LH2 =6:1

- Combustor Pressure, P_0=18.94 Mpa,
- Combustor temperature, T_0 = 3615°K,
- Throat diameter, A^*=0.05785 m^2

- γ=1.196, MW=13.6 kg/kg-mol

$$\dot{q}_{wall} = 79809.2_{kWatt}$$

SSME Example Revisited (cont'd)

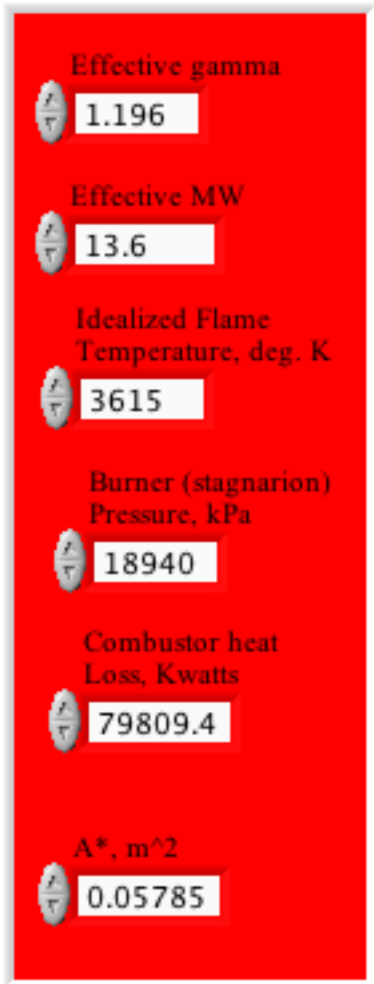
- Iterative Algorithm (use $T_{0_{actual}}$ as convergence criterion)

$$T_{0_{actial}} = T_{0_{ideal}} - \frac{1}{c_p}\left(\frac{\dot{q}_{wall}}{\dot{m}}\right)$$

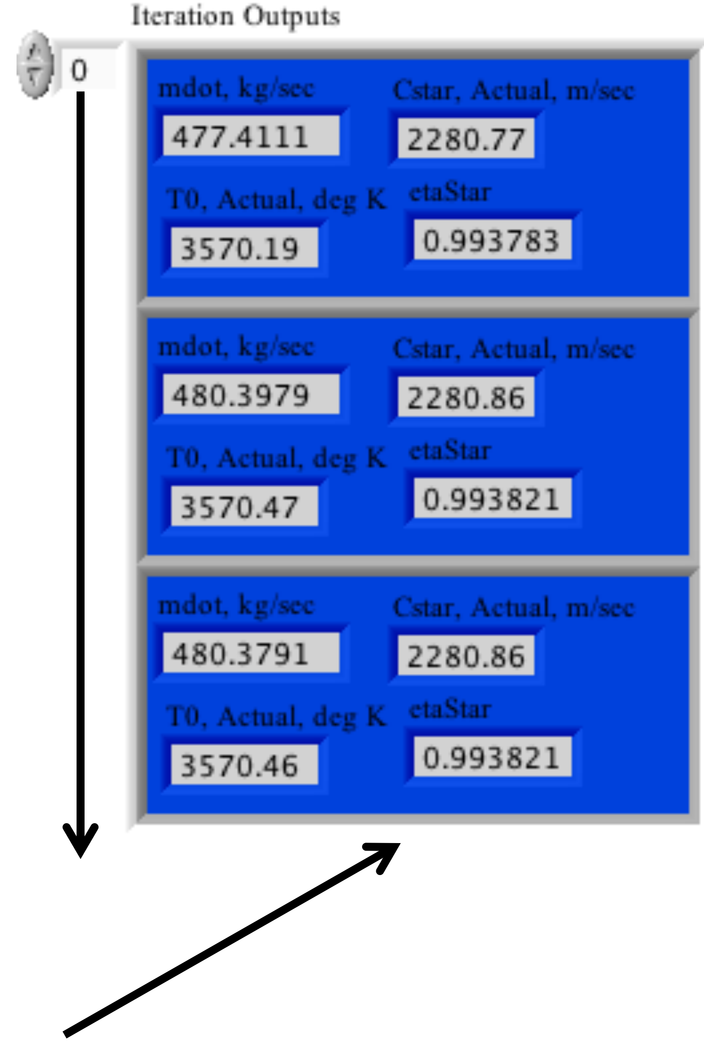
$$\dot{m} = A^* \sqrt{\frac{\gamma}{R_g}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}} \frac{p_0}{\sqrt{T_{0_{actial}}}}$$

Iterate ....

SSME Example Revisited (cont'd)



- So the regenerative Cooling Doesn't Hurt you a Lot



SSME Example Revisited (cont'd)

- Compare regenerative heat loss per unit mass to enthalpy per unit mass of combustion gases

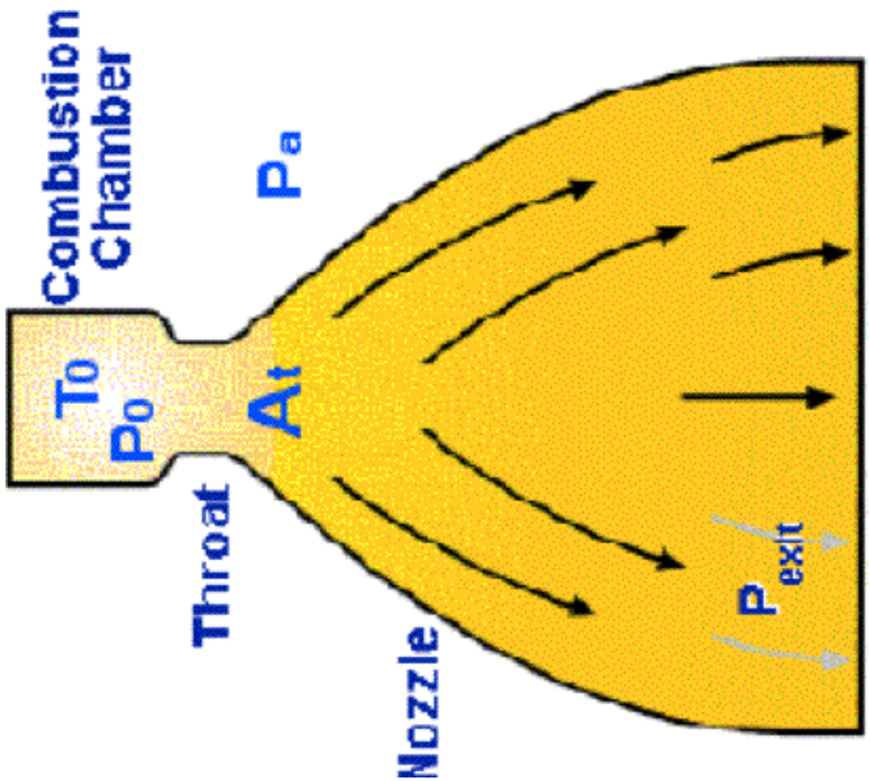
$$\left(\frac{\dot{q}_{wall}}{\dot{m}}\right) = \frac{79809.2_{kWatt\,kWatt}}{480.3791_{kg/\sec}} = 163.1_{kJ/kg}$$

$$h_{combustor} = c_p T_{0\,actual} = 3.73051_{kJ/kg^oK} \times 3568.34_{^oK} = 12027.13_{kJ/kg}$$

$$\rightarrow \frac{\left(\frac{\dot{q}_{wall}}{\dot{m}}\right)}{h_{combustor}} = \frac{163.1}{12027.13} = 1.356\%$$

- So it is only a small fraction of the energy that is used for combustor cooling

Nozzle Heat Loss Correction



- Consider Nozzle with $P_{0\,c}$, $T_{0\,c}$ leaving combustion chamber
- Assume nozzle is shock-free
- What happens at to exit plane conditions with heat loss in the nozzle?

- From earlier Analysis $T_{0_2} = T_{0_1} - \frac{1}{c_p}\left(\frac{\dot{q}_{nozzle}}{\dot{m}}\right)$

(1) … exit conditions without heat loss

(2) … exit conditions with heat loss

Nozzle Loss Correction (cont'd)

- Quasi 1-D momentum equation

$$p_1 + \rho_1 V_1^2 = p_2 + \rho_2 V_2^2 \downarrow$$

$$p_1 + \left(\frac{p_1}{R_g T_1}\right) V_1^2 = p_2 + \left(\frac{p_2}{R_g T_2}\right) V_2^2 \downarrow$$

$$p_1 \left[1 + \gamma M_1^2\right] = p_2 \left[1 + \gamma M_2^2\right] \downarrow$$

$$\frac{p_2}{p_1} = \frac{\left[1 + \gamma M_1^2\right]}{\left[1 + \gamma M_2^2\right]}$$

(1) … exit conditions without heat loss

(2) … exit conditions with heat loss

Nozzle Loss Correction (cont'd)

- Quasi 1-D continuity equation

$$\rho_1 V_1 = \rho_2 V_2 \rightarrow \frac{\rho_1}{\rho_2} = \frac{V_2}{V_1} \downarrow$$

Plug into Momentum eqn.

$$\frac{T_2}{T_1} = \frac{p_2}{p_1}\frac{\rho_1}{\rho_2} = \frac{p_2}{p_1}\frac{V_2}{V_1} = \frac{\left[1+\gamma M_1^{\ 2}\right]}{\left[1+\gamma M_2^{\ 2}\right]}\frac{V_2}{V_1}$$

$$\rightarrow \frac{T_2}{T_1} = \left\{\frac{\left[1+\gamma M_1^{\ 2}\right]}{\left[1+\gamma M_2^{\ 2}\right]}\frac{M_2}{M_1}\right\}^2$$

(1) … exit conditions without heat loss

(2) … exit conditions with heat loss

Nozzle Loss Correction (cont'd)

- But from definition for stagnation conditions

$$\frac{T_2}{T_1} = \frac{T_{0_1}\left[1+\frac{\gamma-1}{2}M_1^2\right]}{T_{0_2}\left[1+\frac{\gamma-1}{2}M_2^2\right]}$$

$$\left[\frac{M_2}{M_1}\frac{\left[1+\gamma M_1^2\right]}{\left[1+\gamma M_2^2\right]}\right]^2 = \frac{T_{02}}{T_{0_1}} \times \frac{\left[1+\frac{\gamma-1}{2}M_1^2\right]}{\left[1+\frac{\gamma-1}{2}M_2^2\right]}$$

- and from earlier

$$\frac{T_2}{T_1} = \left\{\frac{\left[1+\gamma M_1^2\right]}{\left[1+\gamma M_2^2\right]}\frac{M_2}{M_1}\right\}^2$$

$$\frac{M_2^2\left[1+\frac{\gamma-1}{2}M_2^2\right]}{\left[1+\gamma M_2^2\right]^2} = \left[\frac{T_{0_2}}{T_{0_1}}\right]\frac{M_1^2\left[1+\frac{\gamma-1}{2}M_1^2\right]}{\left[1+\gamma M_1^2\right]^2}$$

(1) … exit conditions without heat loss

(2) … exit conditions with heat loss

Nozzle Loss Correction (cont'd)

Solve for M_2

$$\frac{M_2^{\,2}\left[1+\frac{\gamma-1}{2}M_2^{\,2}\right]}{\left[1+\gamma M_2^{\,2}\right]^2} = \left[\frac{T_{0_2}}{T_{0_1}}\right]\frac{M_1^{\,2}\left[1+\frac{\gamma-1}{2}M_1^{\,2}\right]}{\left[1+\gamma M_1^{\,2}\right]^2}$$

- Let

$$F(M_1) = \left[\frac{T_{02}}{T_{0_1}}\right]\frac{M_1^{\,2}\left[1+\frac{\gamma-1}{2}M_1^{\,2}\right]}{\left[1+\gamma M_1^{\,2}\right]^2}$$

Nozzle Loss Correction (cont'd)

- Regroup in terms of M²

$$\frac{M_2^2\left[1+\frac{\gamma-1}{2}M_2^2\right]}{\left[1+\gamma M_2^2\right]^2}=\left[\frac{T_{02}}{T_{0_1}}\right]\frac{M_1^2\left[1+\frac{\gamma-1}{2}M_1^2\right]}{\left[1+\gamma M_1^2\right]^2}\downarrow$$

$$M_2^2\left[1+\frac{\gamma-1}{2}M_2^2\right]-F(M_1)\left[1+\gamma M_2^2\right]^2=0$$

$$F(M_1)=\left[\frac{T_{02}}{T_{0_1}}\right]\frac{M_1^2\left[1+\frac{\gamma-1}{2}M_1^2\right]}{\left[1+\gamma M_1^2\right]^2}$$

Nozzle Loss Correction (cont'd)

- Sub in and collect terms in powers of M_2

$$\frac{\gamma-1}{2}M_2^4+M_2^2-F(M_1)\left[1+\gamma M_2^2\right]^2=0$$

$$\frac{\gamma-1}{2}M_2^4+M_2^2-F(M_1)\left[1+2\gamma M_2^2+\gamma^2 M_2^4\right]=0$$

$$\left[\frac{\gamma-1}{2}-\gamma^2 F(M_1)\right]M_2^4+\left[1-F(M_1)2\gamma\right]M_2^2-F(M_1)=0$$

- **Quartic Equation, … but quadratic in M_2^2**

Nozzle Loss Correction (cont'd)

- How about stagnation pressure?

$$\frac{p_{0_2}}{p_{0_1}} = \frac{p_2}{p_1}\frac{\left[1+\frac{\gamma-1}{2}M_2^{\,2}\right]^{\frac{\gamma}{\gamma-1}}}{\left[1+\frac{\gamma-1}{2}M_1^{\,2}\right]} \rightarrow \frac{p_2}{p_1} = \frac{\left[1+\gamma M_1^{\,2}\right]}{\left[1+\gamma M_2^{\,2}\right]}$$

$$\frac{p_{0_2}}{p_{0_1}} = \frac{\left[1+\gamma M_1^{\,2}\right]}{\left[1+\gamma M_2^{\,2}\right]}\frac{\left[1+\frac{\gamma-1}{2}M_2^{\,2}\right]^{\frac{\gamma}{\gamma-1}}}{\left[1+\frac{\gamma-1}{2}M_1^{\,2}\right]}$$

Nozzle Heat Loss …..

- ***Calculation procedure:***
- Given T_{01}, P_{01}, γ, c_p, R_g, A_{exit}/A_*, $\left(\frac{\dot{q}_{nozzle}}{\dot{m}}\right)$
- 2) Compute adiabatic exit properties,
  $p_1$exit, T_1exit, M_1exit, V_1exit, Thrust, $\dot{m}$
- 3) Compute T_{0_2}/T_{0_1} $$T_{0_2} = T_{0_1} - \frac{1}{c_p}\left(\frac{\dot{q}_{nozzle}}{\dot{m}}\right)$$
- 4) Compute M_2exit from quartic

$$\left[\frac{\gamma-1}{2} - \gamma^2 F(M_1)\right] M_2^{\,4} + \left[1 - F(M_1)2\gamma\right] M_2^{\,2} - F(M_1) = 0$$

Nozzle Heat Loss …..(cont'd)

- ***Calculation procedure:*** *cont'd*

- 5) Compute T_2

$$\frac{T_2}{T_1} = \left\{ \frac{\left[1+\gamma M_1^{\,2}\right]}{\left[1+\gamma M_2^{\,2}\right]} \frac{M_2}{M_1} \right\}^2$$

- 6) Compute p_2

$$p_2 = p_1 \frac{\left[1+\gamma M_1^{\,2}\right]}{\left[1+\gamma M_2^{\,2}\right]}$$

- 7) Compute $P_{0\,2}$

$$\frac{p_{0_2}}{p_{0_1}} = \frac{\left[1+\gamma M_1^{\,2}\right]}{\left[1+\gamma M_2^{\,2}\right]} \frac{\left[1+\frac{\gamma-1}{2} M_2^{\,2}\right]^{\frac{\gamma}{\gamma-1}}}{\left[1+\frac{\gamma-1}{2} M_1^{\,2}\right]}$$

Nozzle Heat Loss …..(cont'd)

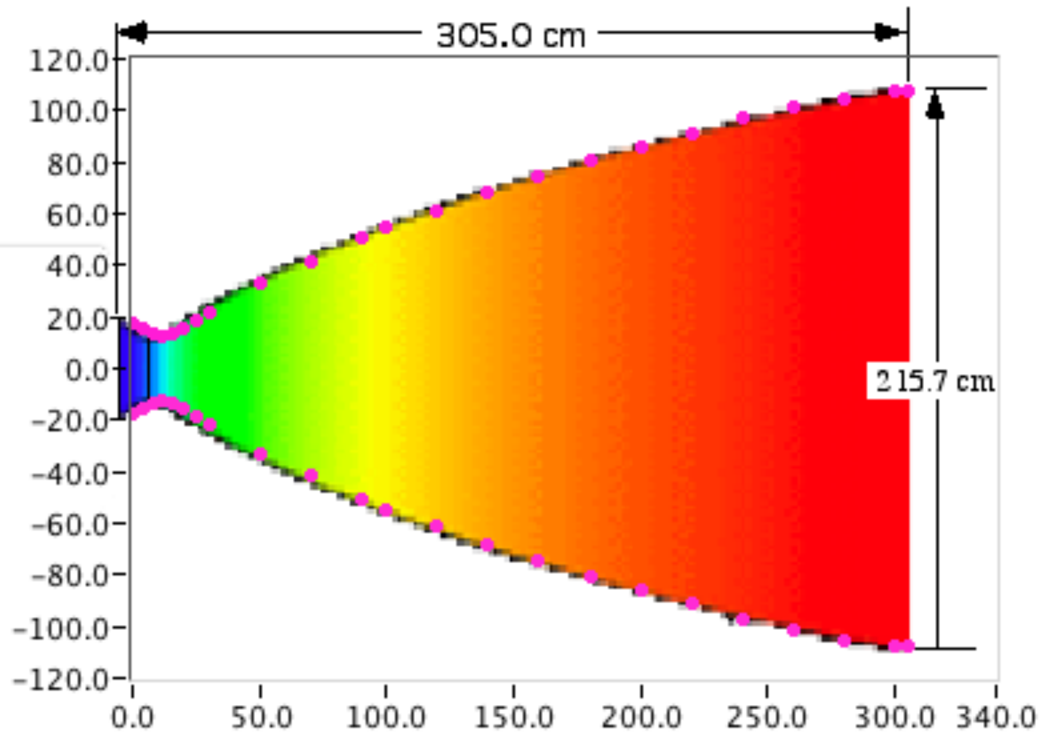
- ***Calculation procedure:*** ***cont'd***

- 8) Compute V_{2exit} $$V_2 exit = M_2 exit \times \sqrt{\gamma R_g T_2 exit}$$

- 9) Compute non-adiabatic exit properties,
  $p_2$exit, T_2exit, M_2exit, V_2exit, Thrust, Isp

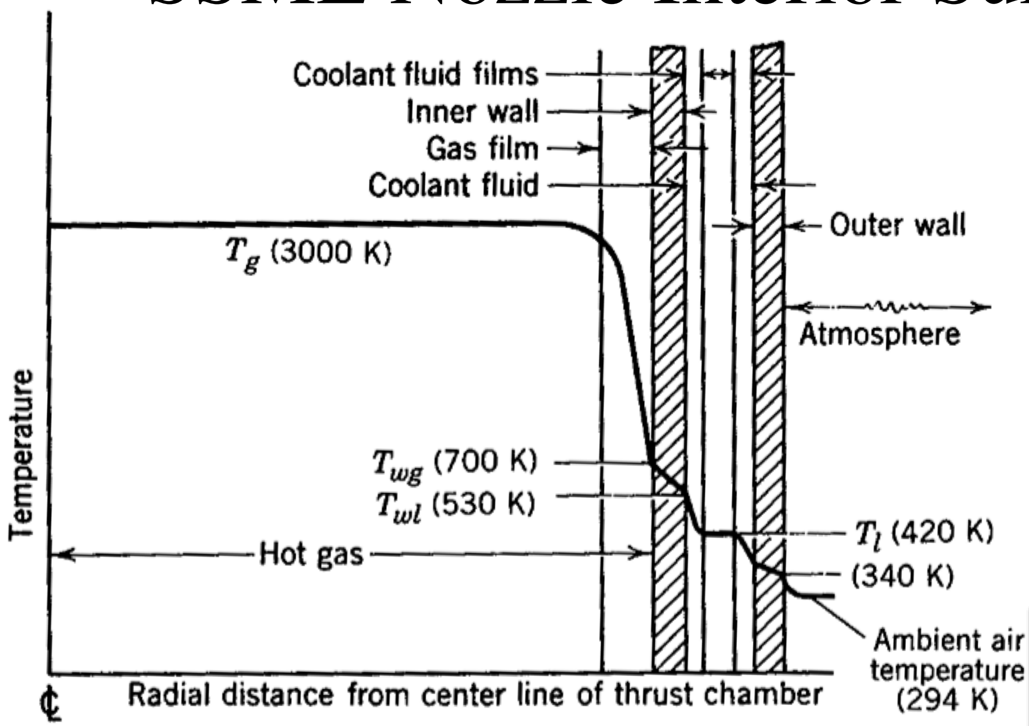
SSME Nozzle Interior Surface Area

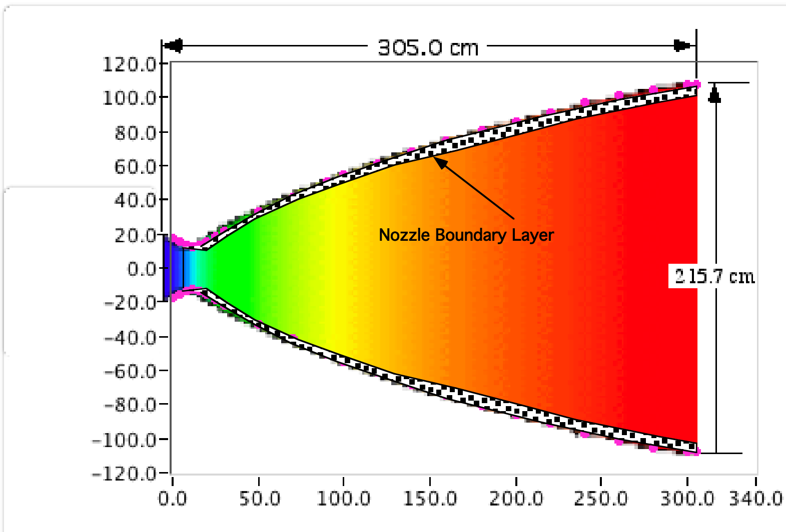
Nozzle
Surface Area, M^2
6.4947



$$\dot{q}_{wall} = 1.25_{\frac{BTU}{in^2\,\mathrm{sec}}} \times \frac{1054.35}{1000000}_{\frac{\mathrm{MJ}}{\mathrm{BTU}}} \times \left[\frac{1.0_{in}}{2.54_{cm}}\right]^2 \times 64947_{cm^2} = 13.26_{Mwatt}$$

SSME Nozzle Interior Surface Heat Transfer





SSME Nozzle with Heat Loss + Combustor Heat Loss

- $\dot{q}_{wall} = 13.26_{Mwatt}$
- Continuing from previous example

Combustor exit properties

mdot, kg/sec	480.3791
Cstar, Actual, m/sec	2280.86
T0, Actual, deg K	3570.46
etaStar	0.993821
Burner Exit Pressure, kPa	18940
qdot Mwatt/	79.8094

Nozzle Output Thermal Data

Cp, J/kg Deg-K 2	3730.51
T02, deg. K	3563.0
T02/T01	0.9979
qdot Mwatt/	13.2622
Q MJ/kg	0.027607

M2 Solutions

M21	0.12028
M22	22.060
M1	0.3468
M2	4.6969
FM	0.09301
M out	4.6969

Nozzle Exit conditions

P2, kPa	17.3126
T2, deg. K	1099.1
P02, kPa	19463.4
V2, m/sec	4210.68
Ce, m/.sec	4372.26
Thrust, kNt	2100.34
Isp, sec	445.876
M out	4.69703
Corrected Mdot, kg/sec	480.37

Compare to Isentropic Analysis (no Heat Loss in Combustor or Nozzle)

Isentropic Analysis:		***With Heat Loss Analysis:***	
T_0	= 3615 °K	T_0	= 3563 °K
P_0	= 18940 kPa	P_0	= 19463 kPa
dm/dt	= 477.47 kg/sec	dm/dt	= 480.37 kg/sec
M_{exit}	= 4.6771	M_{exit}	= 4.6970
V_{exit}	= 4288.61	V_{exit}	= 4210.68
p_{exit}	= 17.455 kPa	p_{exit}	= 17.313 kPa
T_{exit}	= 1149.9 °K	T_{exit}	= 1099.1 °K
Thrust	= 2125.69 kNt	Thrust	= 2100.3 kNt
I_{sp}	= 454.05 sec	I_{sp}	= 445.88 sec

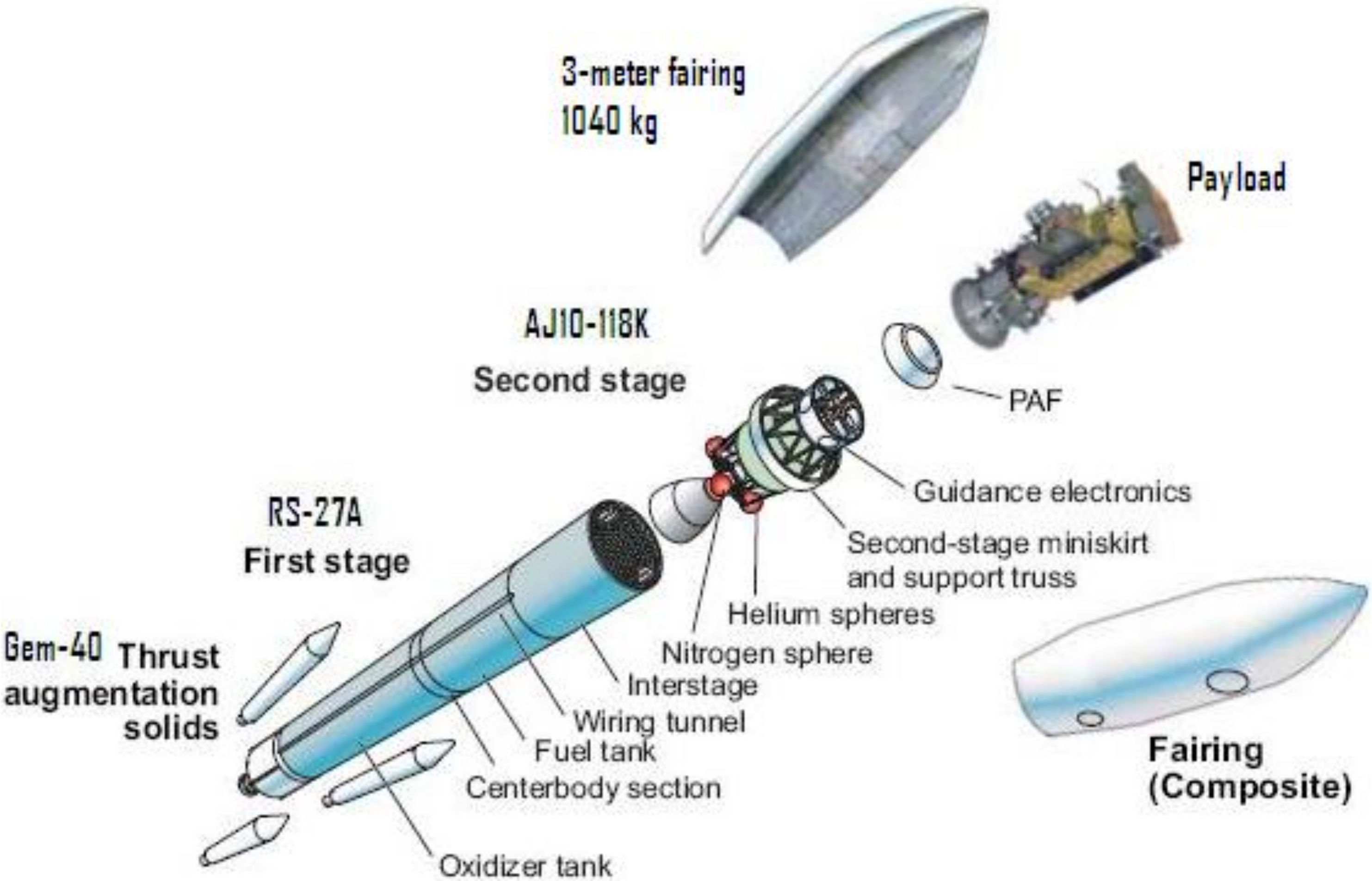
- **Heat Loss Costs you more Specific Impulse than Thrust**

$$\dot{q}_{combustor} = 79.809_{Mwatt}$$

$$\dot{q}_{nozzle} = 13.26_{Mwatt}$$

Homework 5

Delta II 7320 Launch Vehicle



A four digit designator has been used to distinguish between Delta configurations since the early 1970s. However, Boeing uses a new designation system for the Delta IV vehicles. The Delta III is currently available in only one configuration, and thus its four digit designator is rarely used. The Delta IV is available in two basic types, Medium and Heavy. These are designated DIV-M and DIV-H. The Medium can be enhanced ("Medium-Plus") with a larger fairing and strap-on boosters. These configurations are designated with a digit for the fairing diameter in meters, and digit for the number of strap-on boosters. Thus, the DIV-M+ (4,2) has a 4-m fairing and two strap-on boosters, while the DIV-M+ (5,4) has a 5-m fairing and four strap-on boosters.

Example

Delta 7925-10

First Digit

First-stage type, engine type, strap-on motor type

0: Long tank, MB-3 engine, Castor II

1: Extended long tank, MB-3 engine, Castor II

2: Extended long tank, RS-27 engine, Castor II

3: Extended long tank, RS-27 engine, Castor IV

4: Extended long tank, MB-3 engine, Castor IVA

5: Extended long tank, RS-27 engine, Castor IVA

6: Extra extended long tank, RS-27 engine, Castor IVA

7: Extra extended long tank, RS-27A engine, GEM-40

8: Delta III shortened first stage, RS-27A engine, GEM-46

Second Digit

Number of strap-on motors (3,4,6, or 9)

Third Digit

Second-stage engine

0: AJ-10-118 (Aerojet)

1: TR-201 (TRW)

2: AJ-10-118K (Aerojet)

3: RL10B-2 (Pratt & Whitney)

Fourth Digit

Third-stage motor

0: No third stage or unspecified

3: TE-364-3

4: TE-364-4

5: PAM-D derivative Star 48B

6: Star 37FM

Dash Number

Fairing type (optional)

None: Standard fairing (9.5 ft for Delta II)

-8: 8-ft fairing

-10: 10-ft composite fairing

-10L: 10-ft stretched composite fairing

Reference material

International Reference Guide to Space Launch Systems, 4th ed., Stephen J. Isakowitz, Joseph P. Hopkins, Jr., and Joshua B. Hopkins, American Institute of Aeronautics and Astronautics, Reston, VA, 2003. ISBN: 1-56347-591-X

Stage 1 Properties

- Boeing Delta II Rocket…Stage 1
  - Sea Level Thrust: 890kN
  - Vacuum Thrust: 1085.8 kN
  - **Nozzle Expansion Ratio: 15.2503:1**
  - **Conical Nozzle, 30.5 deg exit angle**
- Combustion Properties: (RS-27A Rocketdyne Engine)
  - Lox/Kerosene, Mixture Ratio: 2.24:1
  - **Chamber Pressure (P_0): 5161.463 kPa**
  - Combustion temperature (T_0): 3455 K
  - $\gamma = 1.2220$
  - $M_W = 21.28$ *kg/kg-mol*
  - Propellant Mass: 97.08 Metric Tons
  - Stage 1 Launch Mass: 101.8 Metric Tons

Gem 40 Augmentation Rocket Properties (ATK)

- 3 Boosters Total – Ground Lit
  - Sea Level Thrust: 442.95 kN
  - Vacuum Thrust: 499.20 kN
  - **Nozzle Expansion Ratio: 10.65:1**
  - **Conical Nozzle, 20 deg exit angle**
- Combustion Properties: (Gem 40)
  - Ap/Aluminum/HTPB
  - **Chamber Pressure (P_0): 5652.66 kPa**
  - Combustion temperature (T_0): 3600 K
  - γ = 1.2000
  - M_W = 28.15 *kg/kg-mol*
- Propellant Mass (Each): 11,765 kg
- Launch Mass: 13,080 kg

Stage II Properties

- Boeing Delta II Rocket…Stage 2 AJ10-118 Aerojet Engine

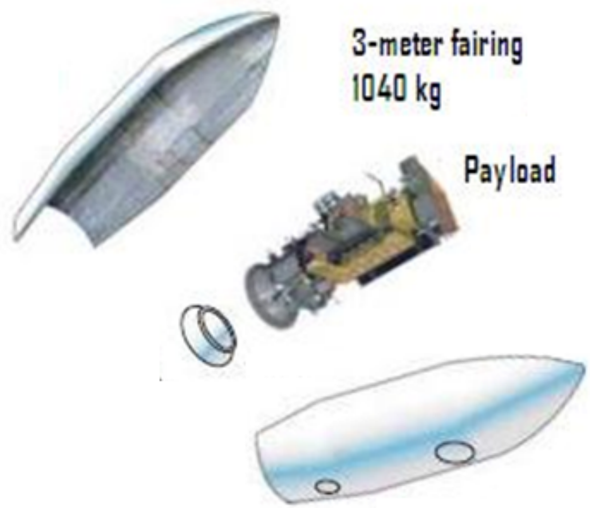
Propellants N_2O_4/Aerozine 50

- Vacuum I_{sp}: 319 seconds
- Vacuum Thrust: 43.657 kN
- Chamber Pressure: 5700 kPa
- Mixture Ratio: 1.8:1
- Nozzle Expansion Ratio: 65:1
- Bell nozzle, exit angle ~ 0 deg.

- Propellant Mass: 6004 kg
- Stage 2 Launch Mass: 6954 kg

Stage III Properties

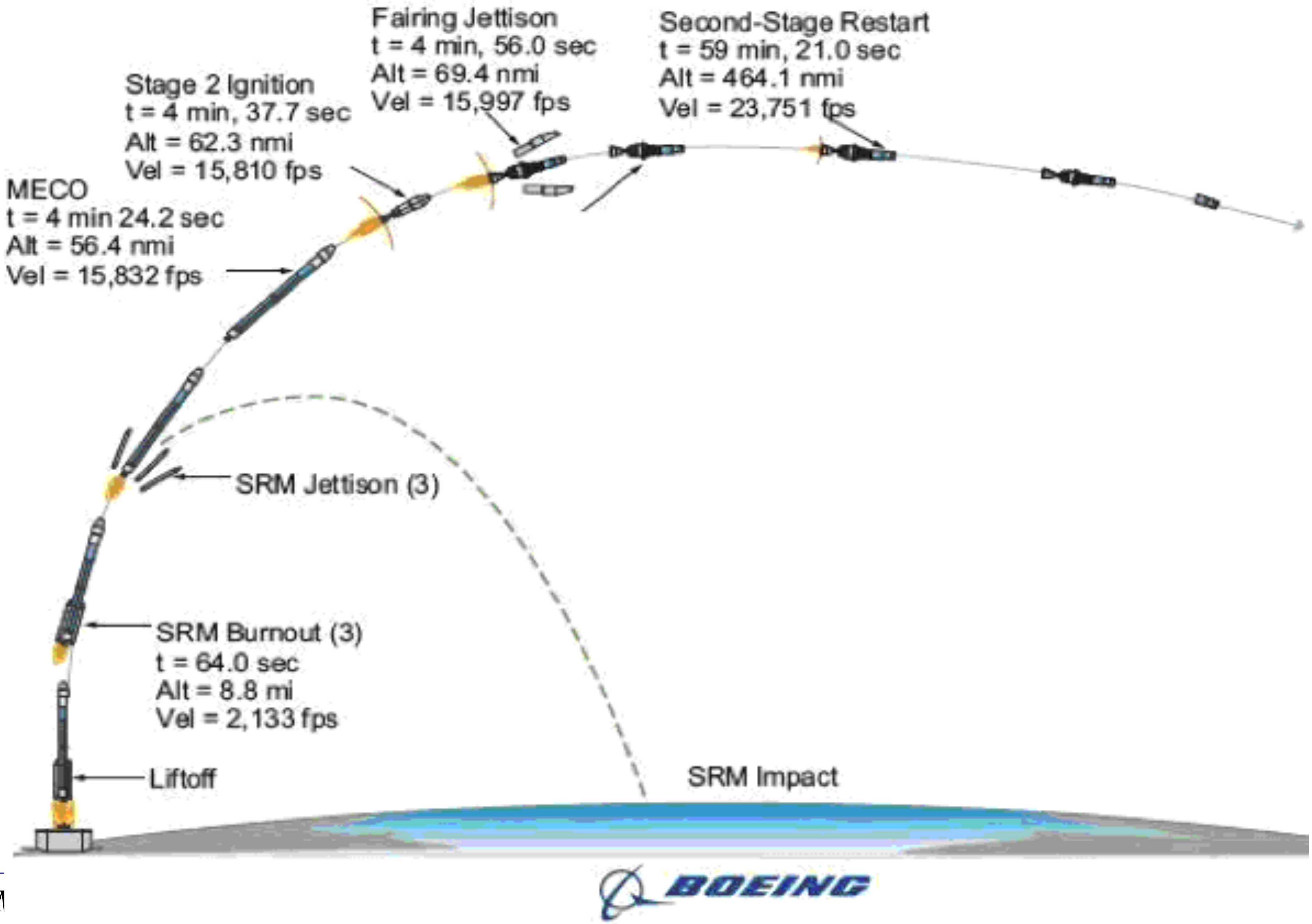
• Payload Inside of 3 (10 ft) meter shroud



• Payload Delivered to Orbit by stages 1-2 (no Kick motor burn)

• Shroud jettisoned prior to reaching orbit
3-meter Shroud weight ~ 1040 kg

Launch Profile



Problem Objectives (1)

• Estimate Total Payload mass that can be delivered to a 464.1 *nmi* (860.06 km) LEO orbit at inclination 28.7° … KSC Launch Due East

• Assume that all gravity losses occur while stage 1 (RS-27A) is burning and the vehicle flys ***"straight up"*** while Gem 40's are burning and then at ***30 deg pitch angle*** for remainder of RS-27A burn

$$\left[\left(\Delta V\right)_{\substack{gravity\\loss}}\right]_{stage} \approx \left[\frac{2}{3}\cdot g_{\left(h_{initial}\right)} + \frac{1}{3}\cdot g_{\left(h_{initial}\right)}\right]\cdot \sin\theta \cdot T_{burn}$$

$$..use...g_{(h)} = \frac{\mu}{R^2}..gravity\ \mathrm{model}$$

• Assume no gravity losses during stage 2 burn ..

Problem Objectives (2)

• Estimate Total Payload mass that can be delivered to a 464.1 *nmi* (860.06 km) LEO orbit at inclination 28.7° … KSC Launch Due East

• Assume 3% kinematic ΔελταV losses due to drag (includes interference from GEM 40 Boosters) During the stage 1 burns

$$\Delta V_{drag} = 0.03 \cdot g_0 \cdot I_{sp} \cdot \ln\left(\frac{M_{initial}}{M_{final}}\right)_{stage}$$

• Assume 1040 kg (2.9 meter) shroud + adapter weight

(not budgeted as part of payload) …. Jettisoned during stage 2 burn

• ***(be sure to account for conical nozzle exit thrust losses)***

Problem Objectives (3)

- 1) Calculate … total required delta V for the mission

… be sure to include

a) Required Orbital Velocity

b) Change in Potential Energy

c) Local Earth Rotational Velocity

2) Compare required delta V to available delta V … for each stage

… sure to account for

a) mass changes due to stage separation

b) gravity and drag losses during stage 1 burn

c) shroud jettison 4 min and 56 seconds into burn

You are going to have to iterate the payload weight until

"Available Delta V" = "Required Delta V"

Mission Requirements

- First establish Delta V requirements

Calculate

a) Final Orbital Velocity
b) "Boost Velocity" from earth along direction of launch
(use true local Earth radius at 28.7 deg latitude here)
c) Kinematic Delta V ($V_{orbital}$-V_{boost})
d) Gravitational Potential Delta V
e) Total Delta V

Calculate Stage 1 Booster properties first

- Need calculations of Mass flow, exit conditions to analyze altitude effects on performance

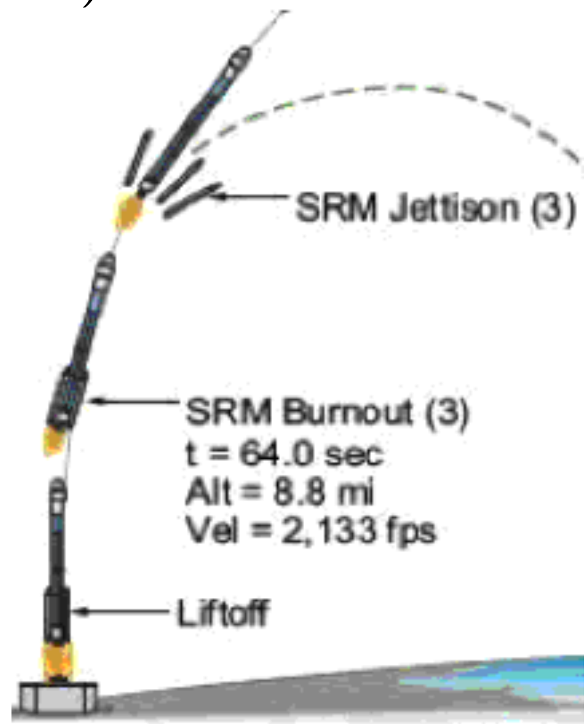
… Stage "1a"

• 3 x Gem 40 + Stage 1 (RS-27A)

-- Gem 40 Burnout Altitude ~ 8.8 *nmi* (16.31 km)

-- Calculate:

- ***i) Total Lift off Thrust***
- ***ii) Burn Time for Gem-40(s)***
- ***iii) Plot total Thrust profile during Burn "1a" vs Altitude***
- ***iv) Total propellant consumed during "stage 1a" burn***
- ***vi) Effective Specific Impulse (3 x Gem 40 + RS-27A over operating altitude range***
- ***vii) Stage masses at Gem40 burnout***



$$Use \rightarrow \left(I_{sp}\right)_{eff} = \frac{2}{3}\left[\left(I_{sp}\right)_{\substack{Rs-27A+\\ 3x\ Gem40}}\right]_{launch} + \frac{1}{3}\left[\left(I_{sp}\right)_{\substack{Rs-27A+\\ 3x\ Gem40}}\right]_{\substack{Gem40\\ Burnout}}$$

… Stage "1b"

Stage 1 (RS-27A) burning from Gem 40 Burnout Altitude ~ 8.8 *nmi* (16.31 km) to MECO altitude, 56.4 *nmi* (105.52 km)

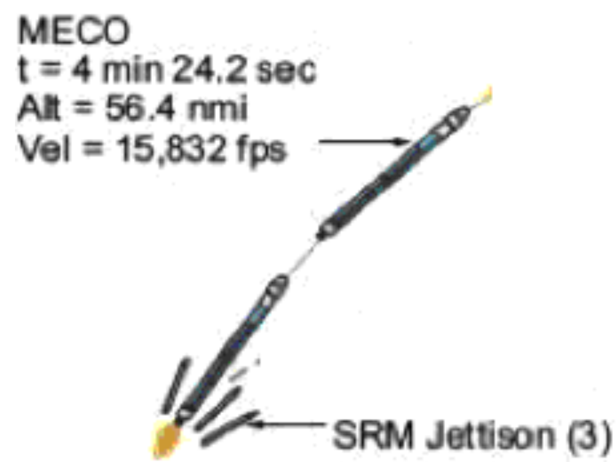
-- Calculate:

i) Burn Time from Gem-40(s) burnout to MECO

ii) Plot thrust profile during "1b" burn vs altitude

iii) Total propellant consumed during "stage 1b" burn

iv) Effective I_{sp} ***Over Altitude Range (16.31 km to 105.52 km)***

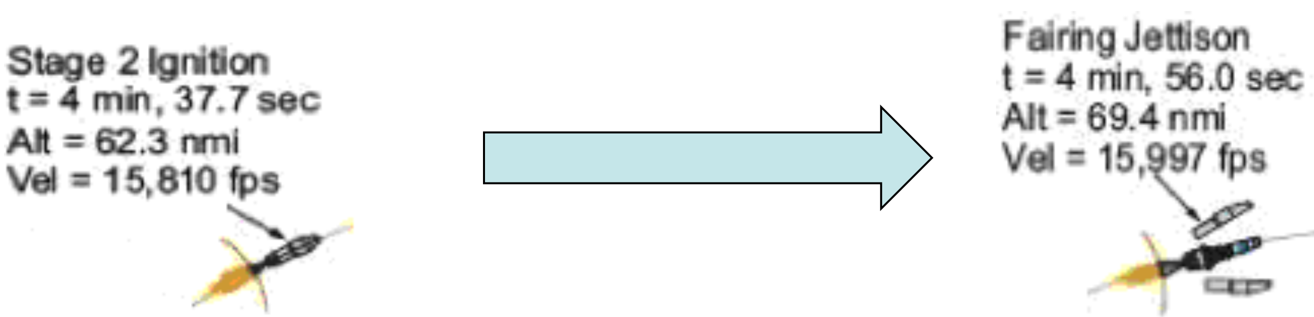


… Stage "2a"

Stage 2 (AJ10-118K Aerojet Engine) burning ignition (4 min 37.7 sec) to fairing jettison (4 min 56 sec) … Altitude ~ 62.3 *nmi* (115.45 km) to 69.4 *nmi* (128.61 km)

-- Calculate:

i) Stage "2a" massflow
ii) Stage "2a" burn time
iii) Total propellant consumed during "stage 2a" burn
iv) Initial Stage "2a" mass
v) Stage 2a "final mass" before shroud jettison
vi) Final Stage "2a" mass after shroud jettison



… Stage "2b"

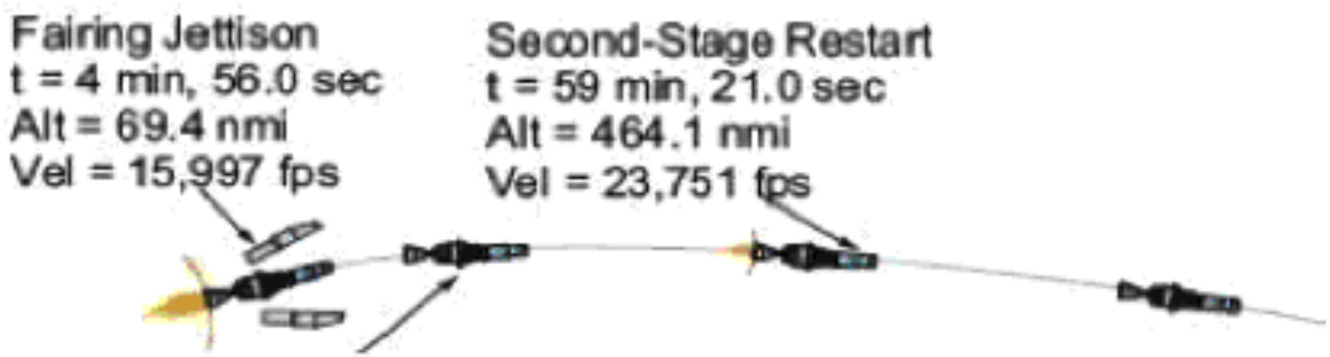
Stage 2 from Fairing Jettison to SECO … assume all propellant is consumed in stage

-- Calculate:

i) Total propellant consumed during "stage 2b" burn

ii) Initial and final masses (excluding payload)

Stage "2b"



… Stage Mass Fractions

For an assumed payload mass …. Calculate

i) Gross $\frac{M_{initial}}{M_{final}}$ for each "stage" … includes stuff each stage lifts

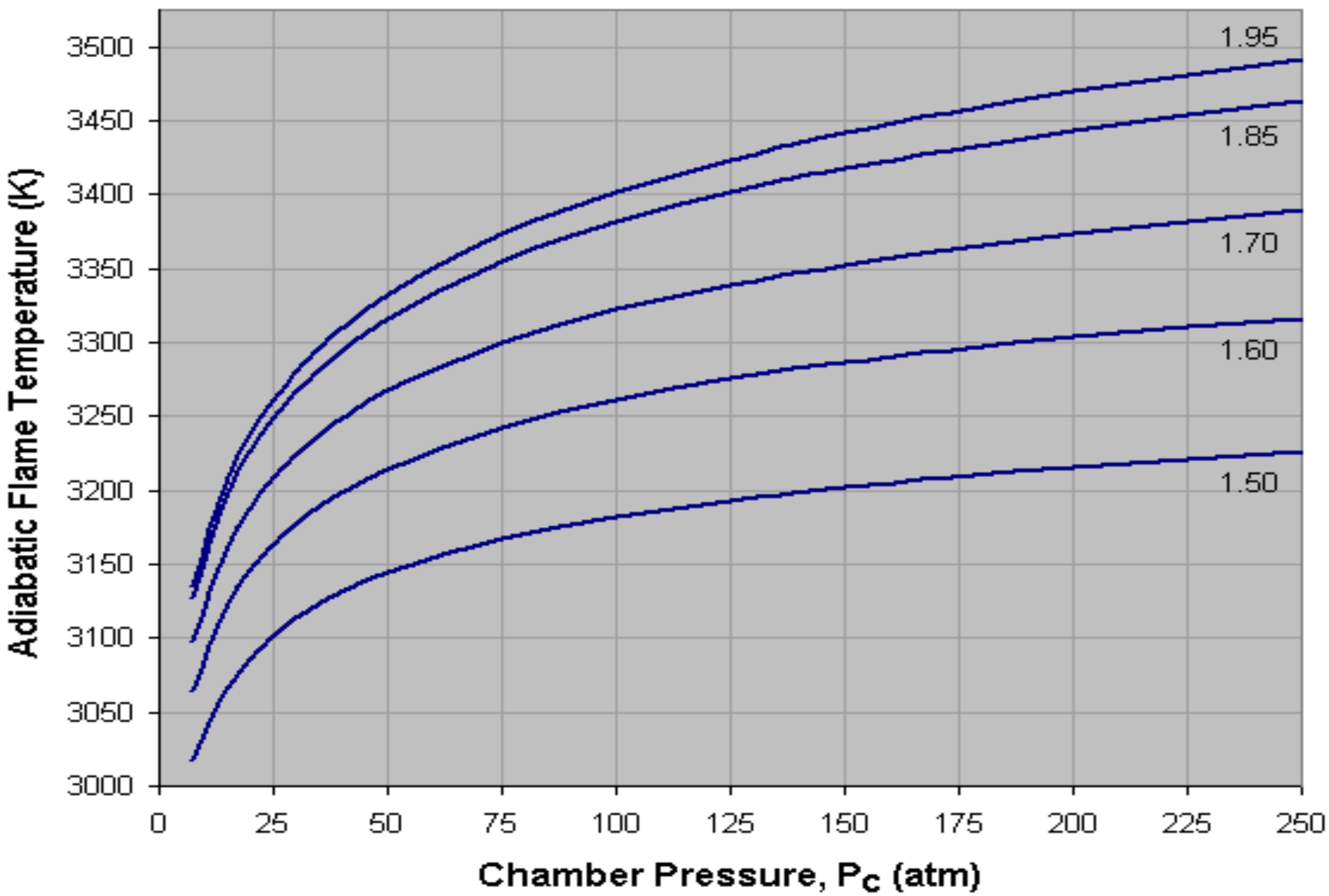
ii) ΔV For each "stage" (include gravity and drag losses .. Where appropriate ***(hint: work backwards from stage "2a")***

iii) Total available ΔV

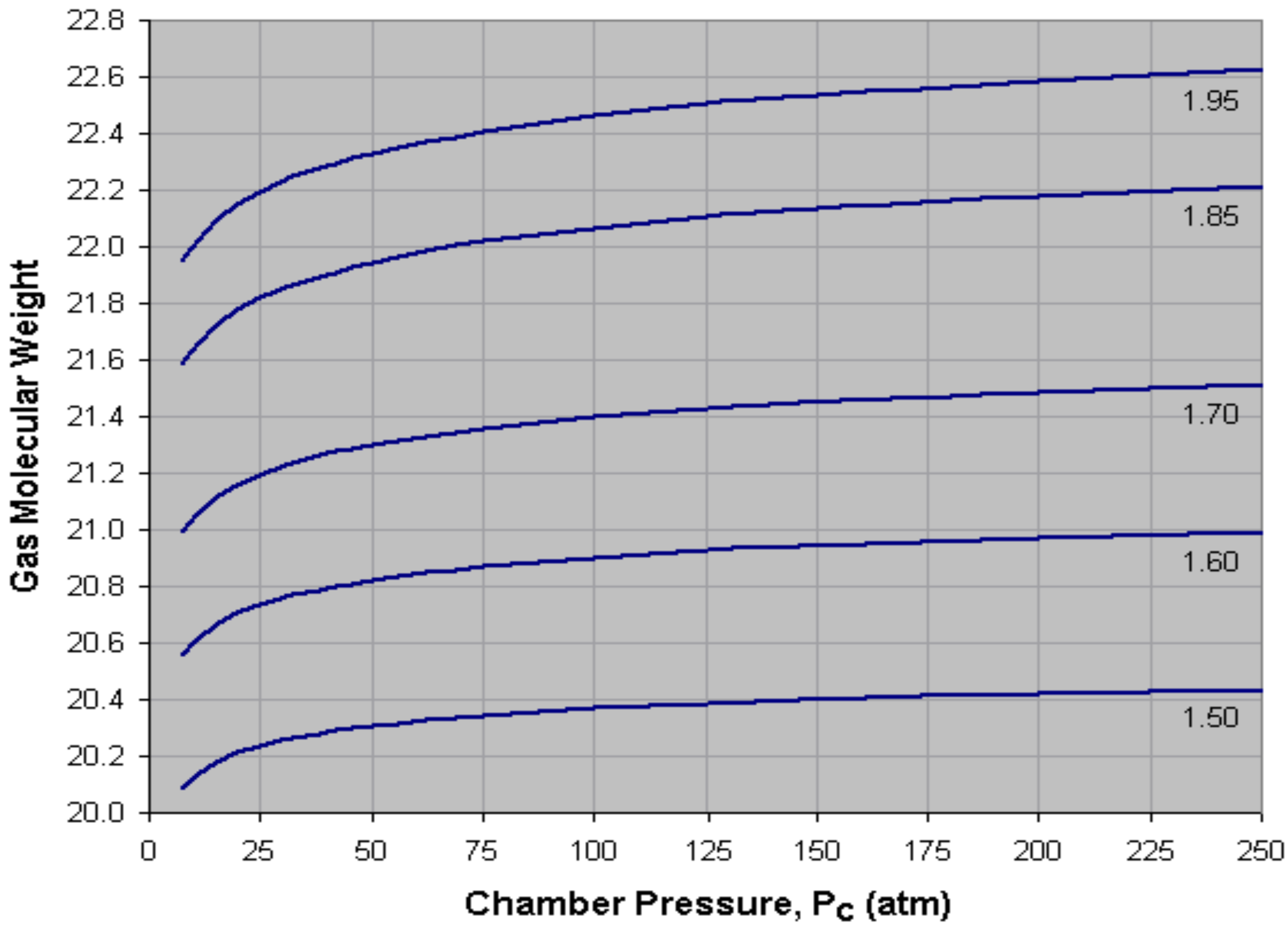
iv) Compare to $\Delta V_{available}$ *to* $\Delta V_{required}$

………… iterate until you get match

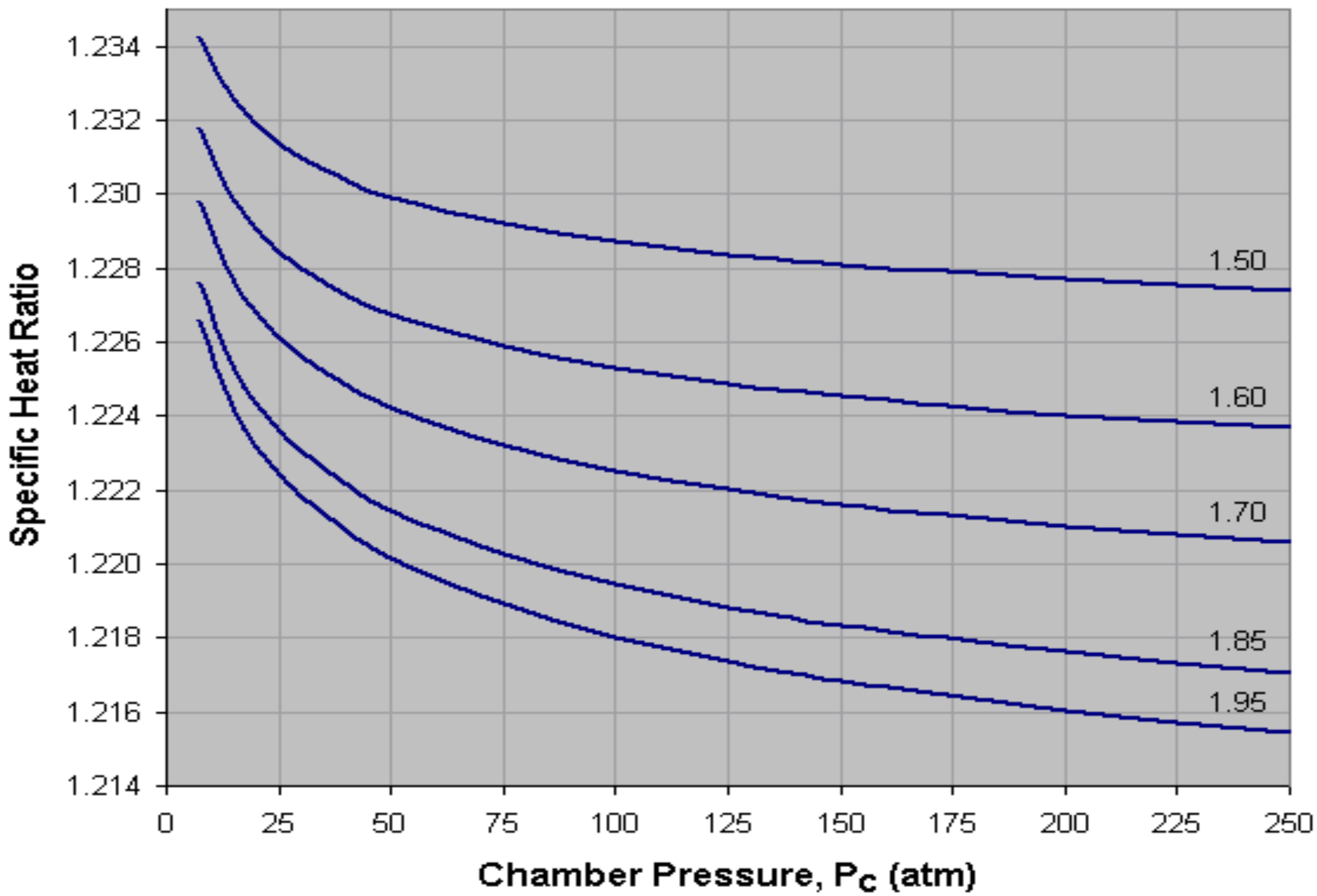
N_2O_4/Aerozine 50



N_2O_4/Aerozine 50



N_2O_4/Aerozine 50



N_2O_4/Aerozine 50