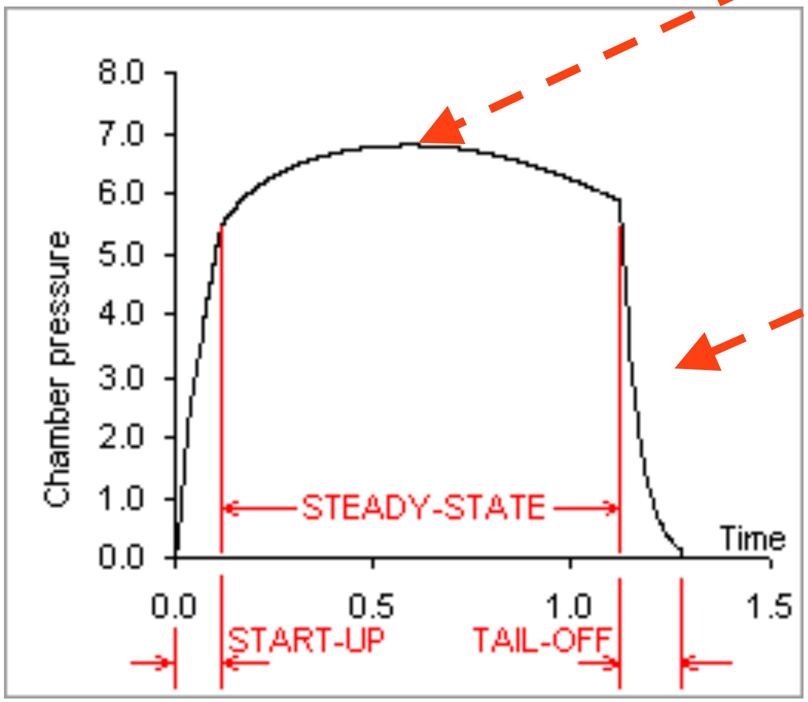
Modeling Transient Rocket Operation (Lecture 6.1: Liquid Rockets)



- .. The primary goal of man is survival … food, shelter
  … basic necessities …

- ***A second aim of man is to build things that run very HOT and very LOUD and move really, really FAST ...***

Material Taken from

1. Sutton and Biblarz: Section 6.1, Section 8.1, Chapter 11, Chapter 15, Appendix 4
2. Humble and Henry, " Space Propulsion Analysis and Design"
3. Richard Nakka Web Page: http://members.aol.com/ricnakk/th_pres.html

Transient Pressure Model

- Combustion Produces High temperature gaseous By-products

- Gases Escape Through Nozzle Throat

- Nozzle Throat Chokes (maximum mass flow)

- Since Gases cannot escape as fast as they are produced
  … Pressure builds up

- As Pressure Builds .. Choking mass flow grows

- Eventually Steady State Condition is reached

Choking Massflow per Unit Area

- maximum Massflow/area Occurs when When M=1
- Effect known as *Choking* in a Duct or Nozzle
- i.e. nozzle will Have a mach 1 throat

$$\left(\frac{\dot{m}}{A_c}\frac{\sqrt{T_0}}{p_0}\sqrt{R_g}\right)_{\max} = \left(\frac{\dot{m}}{A^*}\frac{\sqrt{T_0}}{p_0}\sqrt{R_g}\right) =$$

$$\frac{\sqrt{\gamma}}{\left[1+\frac{(\gamma-1)}{2}\right]^{\frac{\gamma+1}{2(\gamma-1)}}} = \sqrt{\gamma\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}} \rightarrow$$

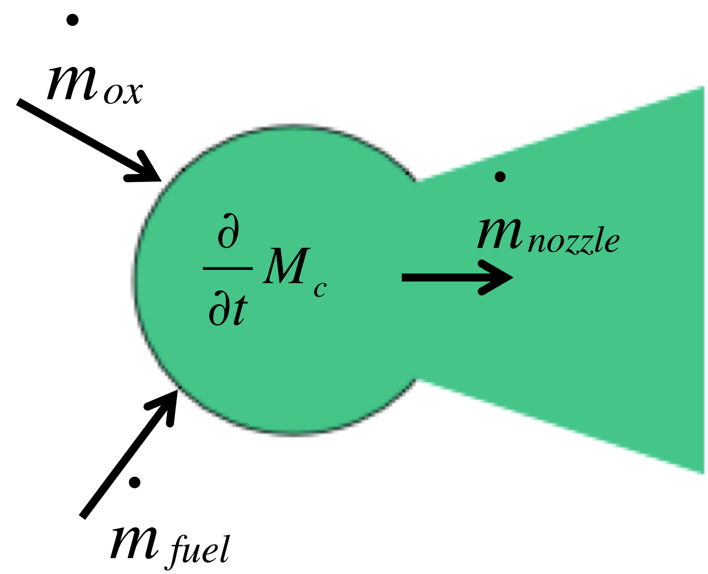
$$\boxed{\frac{\dot{m}}{A^*} = \sqrt{\frac{\gamma}{R_g}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}}\frac{p_0}{\sqrt{T_0}}}$$

Chamber Pressure Model

- Gaseous Mass Trapped in Chamber

$$\frac{\partial}{\partial t} M_c = \left[\dot{m}_{fuel} + \dot{m}_{ox} \right] - \dot{m}_{nozzle}$$

$$\frac{\partial}{\partial t} M_c = \frac{\partial}{\partial t}\left[\rho_c V_c\right] = \frac{\partial}{\partial t}\left[\rho_c\right] V_c + \rho_c \frac{\partial}{\partial t}\left[V_c\right]$$



- Assuming nozzle chokes immediately

$$\frac{\partial}{\partial t}\left[\rho_c\right] V_c + \rho_c \frac{\partial}{\partial t}\left[V_c\right] = \left[\dot{m}_{fuel} + \dot{m}_{ox} \right] - A^* \sqrt{\frac{\gamma}{R_g} \left(\frac{2}{\gamma + 1} \right)^{\frac{\gamma + 1}{(\gamma - 1)}}} \frac{P_0}{\sqrt{T_0}}$$

Chamber Pressure Model (cont'd)

- Using ideal gas law, *Assuming constant flame temperature*

$$\rho_c = \frac{P_0}{R_g T_0} \rightarrow \frac{\partial}{\partial t}\left[\rho_c\right] \approx \frac{1}{R_g T_0}\frac{\partial}{\partial t}\left[P_0\right]$$

- Subbing into mass flow equation

$$\frac{\partial P_0}{\partial t}\frac{V_c}{R_g T_0} + \frac{P_0}{R_g T_0}\frac{\partial V_c}{\partial t} = \left[\dot{m}_{fuel} + \dot{m}_{ox}\right] - \frac{R_g T_0}{V_c} A^* \sqrt{\frac{\gamma}{R_g}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}}\frac{P_0}{\sqrt{T_0}}$$

$$\frac{\partial P_0}{\partial t} + P_0 \frac{1}{V_c}\frac{\partial V_c}{\partial t} + \frac{R_g T_0}{V_c} A^* \sqrt{\frac{\gamma}{R_g}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}}\frac{P_0}{\sqrt{T_0}} = \frac{R_g T_0}{V_c}\left[\dot{m}_{fuel} + \dot{m}_{ox}\right]$$

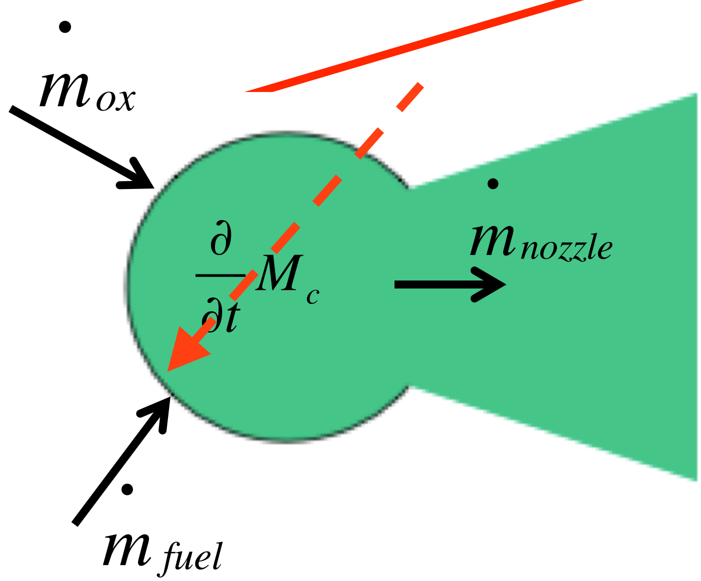
$$\frac{\partial P_0}{\partial t} + P_0\left[\frac{1}{V_c}\frac{\partial V_c}{\partial t} + \frac{A^*}{V_c}\sqrt{\gamma R_g T_0 \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}}\right] = \frac{R_g T_0}{V_c}\left[\dot{m}_{fuel} + \dot{m}_{ox}\right]$$

Chamber Pressure Model (cont'd)

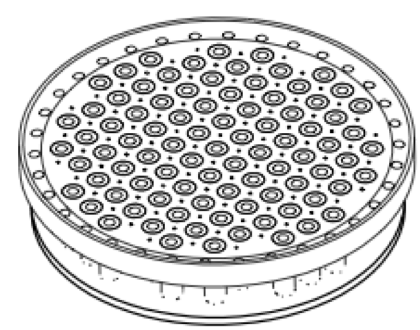
- Chamber pressure equation

$$\frac{\partial P_0}{\partial t} + P_0\left[\frac{1}{V_c}\frac{\partial V_c}{\partial t} + \frac{A^*}{V_c}\sqrt{\gamma R_g T\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}}\right] = \frac{R_g T_0}{V_c}\left[\dot{m}_{fuel} + \dot{m}_{ox}\right]$$

- How do we estimate mass flows?



Injector design



Incompressible Injector Equation

- Many Liquid Propellants are Essentially Incompressible Fluids

- Incompressible Fuel Examples:

  Kerosene (RP-1, RP-4), Ethanol, Methanol, UDMH (Unsymmetrical Dimthyl Hydrazine), MMH (Mono Methyl Hydrazine), Ammonia, Hydrazine, ~Liquid Hydrogen, etc.

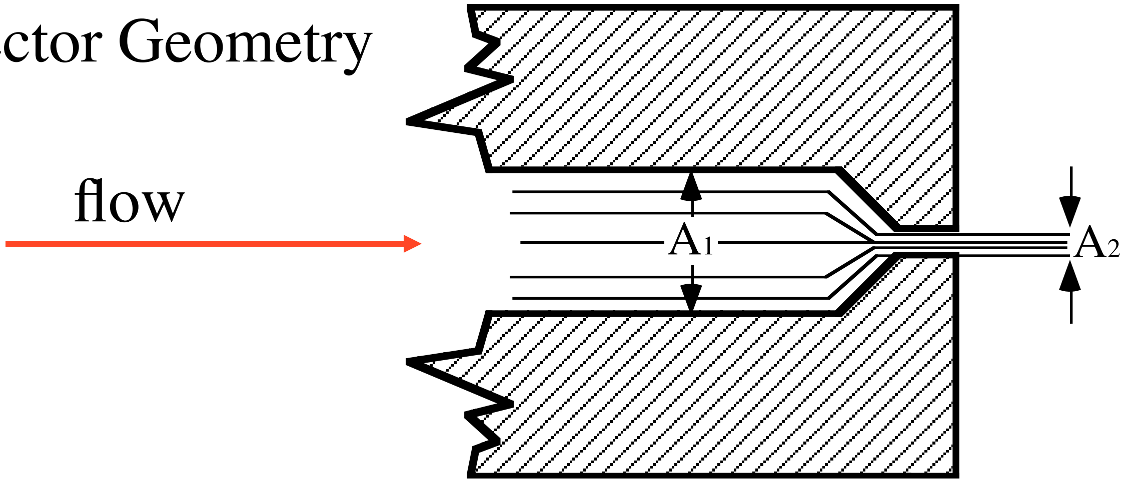
- Incompressible Oxidizer Examples:

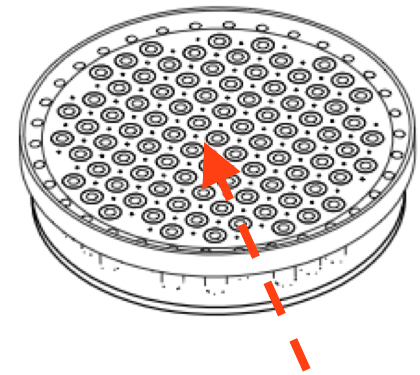
  Hydrogen Peroxide, Liquid Flourine, Nitrogen Tetraoxide, Nitric Acid, ~Liquid Oxygen, etc.

- Incompressible Assumption Allows Simplified Form of Injector Equations

Incompressible Injector Eq. (2)

- Injector Geometry





- Assume Liquid Propellants are incompressible ($\rho = const$)

- Momentum $p_1 + \frac{1}{2}\rho V_1^2 = p_2 + \frac{1}{2}\rho V_2^2$

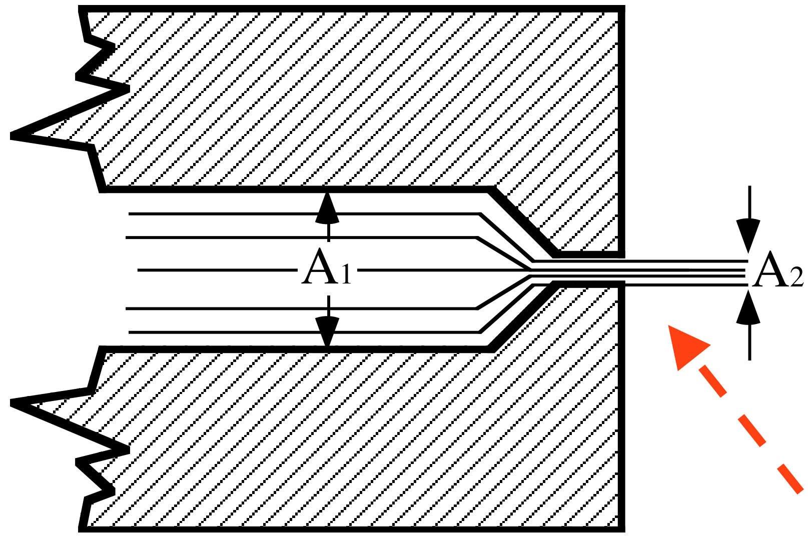
- Continuity $\rho A_1 V_1 = \rho A_2 V_2$

$$\rightarrow \quad p_1 - p_2 = \frac{1}{2}\rho V_2^2 \left[1 - \left(\frac{A_2}{A_1}\right)^2\right]$$

Incompressible Injector Eq. (3)

- Solve for V_2

$$V_2 = \frac{1}{\left[1 - \left(\frac{A_2}{A_1}\right)^2\right]^{\frac{1}{2}}} \sqrt{2\left(\frac{p_1 - p_2}{\rho}\right)}$$



- Friction effects in orifice will cause

$$V_{2_{actual}} < V_{2_{ideal}} \rightarrow V_{2_{actual}} \equiv C_v V_{2_{ideal}} \rightarrow$$

$$V_{2_{actual}} \equiv \frac{C_v}{\left[1 - \left(\frac{A_2}{A_1}\right)^2\right]^{\frac{1}{2}}} \sqrt{2\left(\frac{p_1 - p_2}{\rho}\right)}$$

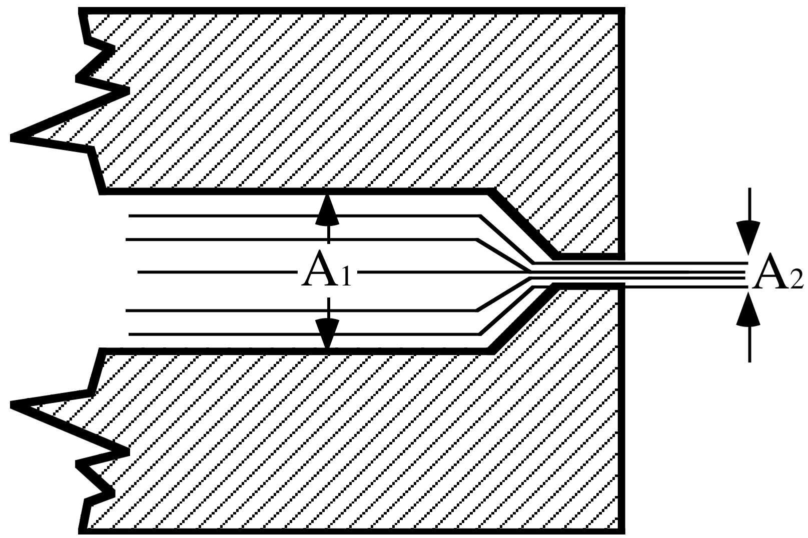
C_v --> "velocity coefficient"

- Define "Discharge Coefficient"

$$C_d \equiv \frac{C_v}{\left[1 - \left(\frac{A_2}{A_1}\right)^2\right]^{\frac{1}{2}}}$$

Incompressible Injector Eq. (4)

$$V_{2_{actual}} \equiv C_d \sqrt{2\left(\frac{p_1 - p_2}{\rho}\right)}$$

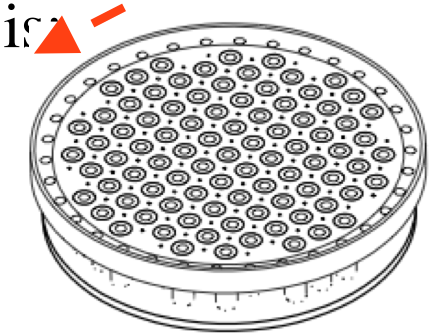


- Define Volumetric Flow as

$$Q_v = A_2 V_{2_{actual}} = A_2 C_d \sqrt{2\left(\frac{p_1 - p_2}{\rho}\right)}$$

- Finally Incompressible Massflow Equation is:

$$\boxed{\dot{m} = \rho Q_v = A_2 C_d \sqrt{2\rho\left(p_1 - p_2\right)}}$$



Compressible Injector Equation

- Some Common Propellants are in Gaseous Form
- Compressible Fuel/Oxidizer Examples:

Gaseous Hydrogen, Methane, Ethane, Gaseous oxygen (GOX

Accurate Injector Model requires Modeling of Flow Compressibility Effects

$$\frac{\dot{m}}{A} = \sqrt{\frac{\gamma}{R_g}} \frac{p_0}{\sqrt{T_0}} \frac{M}{\left[1 + \frac{(\gamma - 1)}{2} M^2\right]^{\frac{\gamma+1}{2(\gamma-1)}}}$$

General Massflow Equation for Compressible Flow

$$\frac{P_{out}}{P_{in}} = \frac{1}{\left[1 + \frac{\gamma - 1}{2} M^2\right]^{\frac{\gamma}{\gamma-1}}} \rightarrow M = \sqrt{\frac{2}{\gamma - 1}\left[\left(\frac{P_{in}}{P_{out}}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]}$$

General Pressure Ratio Equation for Compressible Flow

- Substitute and collect terms

Compressible Injector Equation [2]

$$\dot{m} = A\sqrt{\frac{\gamma}{R_g}}\cdot\frac{P_{in}}{T_0}\frac{\sqrt{\frac{2}{\gamma-1}\left[\left(\frac{P_{in}}{P_{out}}\right)^{\frac{\gamma-1}{\gamma}}-1\right]}}{\left(\frac{P_{in}}{P_{out}}\right)^{\frac{\gamma+1}{2\gamma}}} = AP_{in}\sqrt{\frac{\gamma}{R_gT_0}\frac{2}{\gamma-1}\left[\left(\frac{P_{in}}{P_{out}}\right)^{\frac{\gamma-1}{\gamma}}\left(\frac{P_{in}}{P_{out}}\right)^{-\frac{\gamma+1}{\gamma}}-\left(\frac{P_{in}}{P_{out}}\right)^{-\frac{\gamma+1}{\gamma}}\right]}$$

Simplify

$$\dot{m} = A\sqrt{\frac{P_{in}}{R_gT_0}\frac{2\gamma}{\gamma-1}P_{in}\left[\left(\frac{P_{in}}{P_{out}}\right)^{\frac{-2}{\gamma}}-\left(\frac{P_{out}}{P_{in}}\right)^{\frac{\gamma+1}{\gamma}}\right]} = A\sqrt{\frac{2\gamma}{\gamma-1}\rho_{in}P_{in}\left[\left(\frac{P_{out}}{P_{in}}\right)^{\frac{2}{\gamma}}-\left(\frac{P_{out}}{P_{in}}\right)^{\frac{\gamma+1}{\gamma}}\right]}$$

Allow for Non-isentropic pressure losses (C_d)

$$\dot{m} = C_dA\sqrt{\frac{2\gamma}{\gamma-1}\rho_{in}P_{in}\left[\left(\frac{P_{out}}{P_{in}}\right)^{\frac{2}{\gamma}}-\left(\frac{P_{out}}{P_{in}}\right)^{\frac{\gamma+1}{\gamma}}\right]}$$

1-D "Lossy" Mass Flow Equations, Collected

Unchoked Flow

$$\left(\frac{P_{in}}{P_{out}}\right)_{critical} < \left(\frac{\gamma+1}{2}\right)^{\frac{\gamma}{\gamma-1}} \Rightarrow \boxed{\begin{array}{c}1.8929\\ \gamma=1.4\end{array}}$$

$$\dot{m} = C_d A \sqrt{\frac{2\gamma}{\gamma-1}\rho_{in}P_{in}\left[\left(\frac{P_{out}}{P_{in}}\right)^{\frac{2}{\gamma}} - \left(\frac{P_{out}}{P_{in}}\right)^{\frac{\gamma+1}{\gamma}}\right]}$$

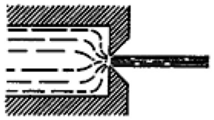
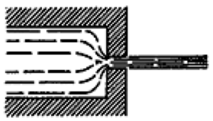
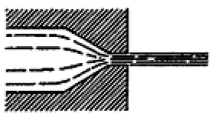
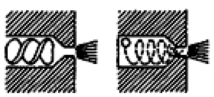
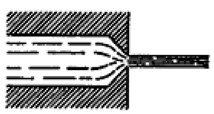
Choked Flow:

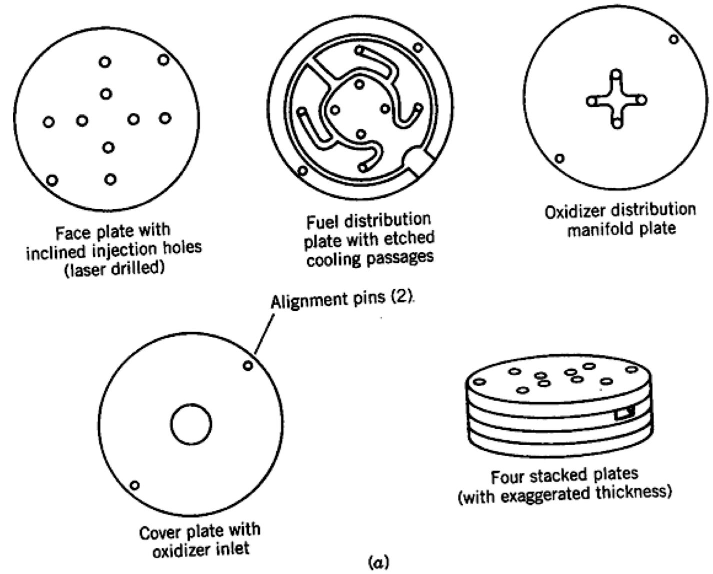
$$\left(\frac{P_{in}}{P_{out}}\right)_{critical} \geq \left(\frac{\gamma+1}{2}\right)^{\frac{\gamma}{\gamma-1}} \Rightarrow \boxed{\begin{array}{c}1.8929\\ \gamma=1.4\end{array}}$$

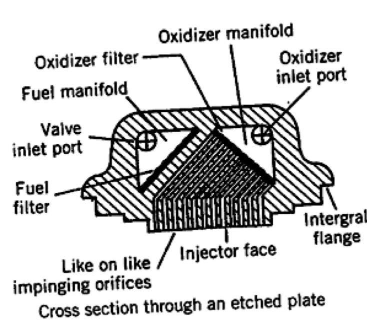
$$\dot{m} = C_d A_{in}^{*} \cdot \sqrt{\gamma P_{in} \cdot \rho_{in}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}}$$

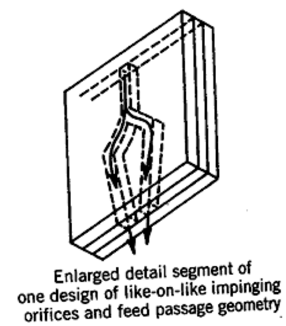
Injector Port Design Options

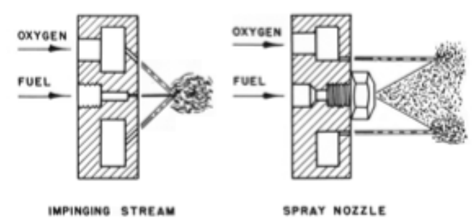
TABLE 8–2. Injector Discharge Coefficients

Orifice Type	Diagram	Diameter (mm)	Discharge Coefficient
Sharp-edged orifice		Above 2.5 Below 2.5	0.61 0.65 approx.
Short-tube with rounded entrance $L/D > 3.0$		1.00 1.57 1.00 (with $L/D \sim 1.0$)	0.88 0.90 0.70
Short tube with conical entrance		0.50 1.00 1.57 2.54 3.18	0.7 0.82 0.76 0.84–0.80 0.84–0.78
Short tube with spiral effect		1.0–6.4	0.2–0.55
Sharp-edged cone		1.00 1.57	0.70–0.69 0.72





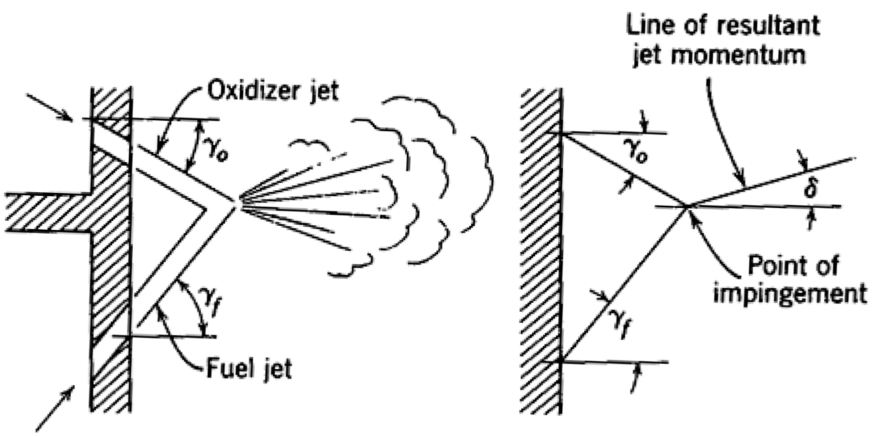




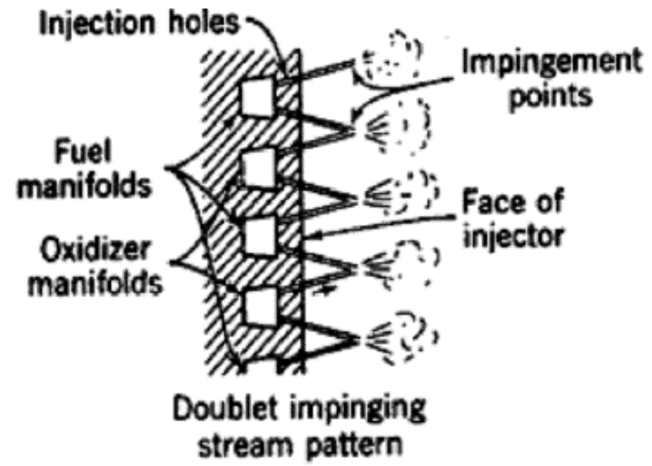
Injector Design (cont'd))

- **Mixing patterns**

$$\dot{m}_{fuel} + \dot{m}_{ox} = A_{fuel} C_{d_f} \sqrt{2\rho_f \left(p_f - P_0\right)} + A_{ox} C_{d_{ox}} \sqrt{2\rho_{ox} \left(p_{ox} - P_0\right)}$$



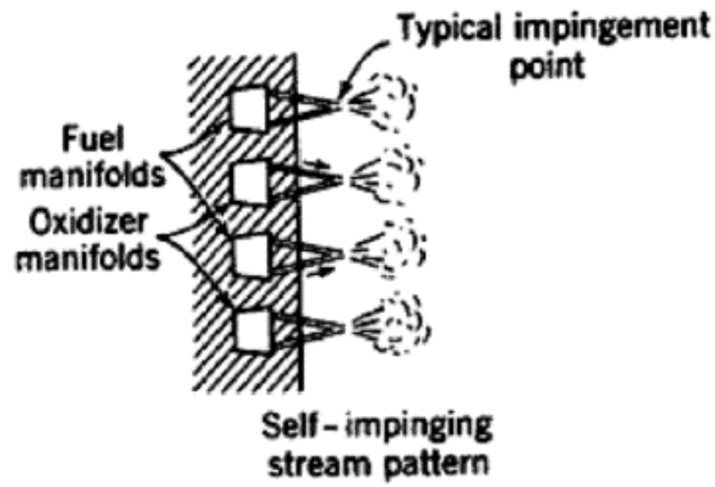
Angular relation of doublet impinging-stream injection pattern.



- Resultant momentum stream

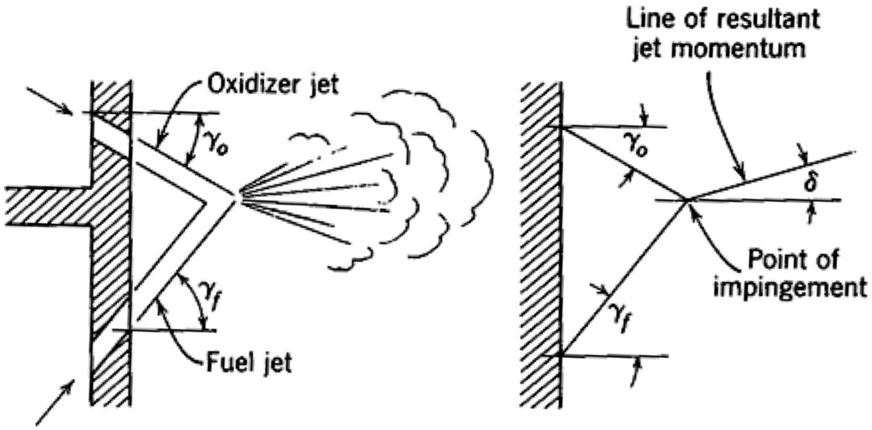
$$\left(\dot{m}_{ox} + \dot{m}_f\right) V_{ox+f} \cos\left(\delta\right) = \dot{m}_{ox} V_{ox} \cos\left(\gamma_{ox}\right) + \dot{m}_f V_f \cos\left(\gamma_f\right)$$

$$\left(\dot{m}_{ox} + \dot{m}_f\right) V_{ox+f} \sin\left(\delta\right) = \dot{m}_{ox} V_{ox} \sin\left(\gamma_{ox}\right) + \dot{m}_f V_f \sin\left(\gamma_f\right)$$



Injector Design (cont'd)

- **Mixing patterns**



Angular relation of doublet impinging-stream injection pattern.

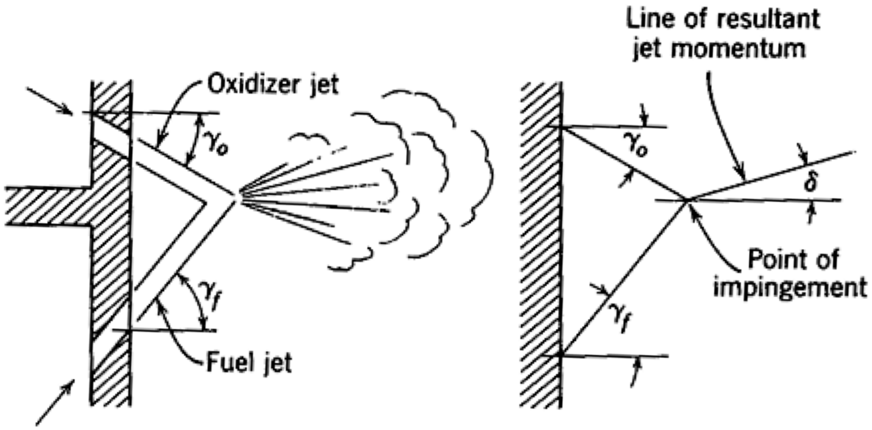
$$\dot{m}_{fuel} + \dot{m}_{ox} = A_{fuel} C_{d_f} \sqrt{2\rho_f \left(p_f - P_0\right)} + A_{ox} C_{d_{ox}} \sqrt{2\rho_{ox} \left(p_{ox} - P_0\right)}$$

- Desirable for … $\delta \sim 0$ *(resultant momentum is directed axially)*

$$\longrightarrow \quad \delta \sim 0 \quad \boxed{\dot{m}_{ox} V_{ox} \sin\left(\gamma_{ox}\right) = \dot{m}_f V_f \sin\left(\gamma_f\right)}$$

Injector Design (concluded)

- **Mixing patterns**



Angular relation of doublet impinging-stream injection pattern.

Condition for $\delta \sim 0$ $\qquad \dot{m}_{ox} V_{ox} \sin(\gamma_{ox}) = \dot{m}_f V_f \sin(\gamma_f)$

- But ....

$$V_{actual} \equiv C_d \sqrt{2\left(\frac{p_1 - p_2}{\rho}\right)}$$

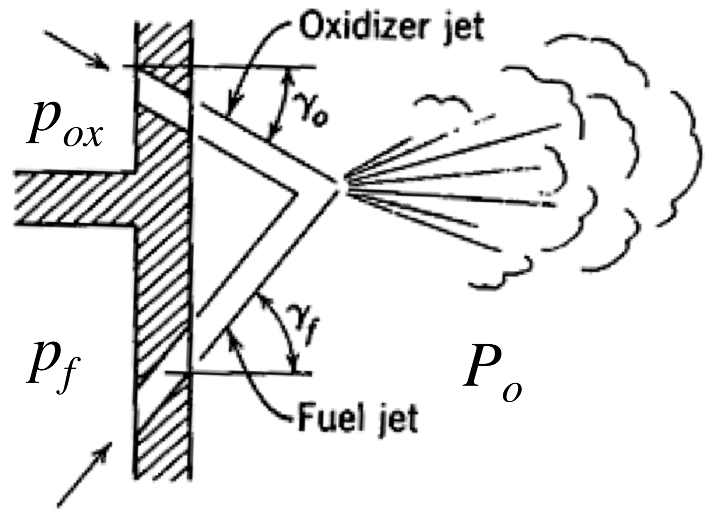
$$\dot{m} = A C_d \sqrt{2\rho(p_1 - p_2)}$$

Injector Design (concluded)

- **Subbing in**

$$A_{ox} C_{d_{ox}} \sqrt{2\rho_{ox}\left(p_{ox} - P_0\right)} \left[C_{d_{ox}} \sqrt{2\left(\frac{p_{ox} - P_0}{\rho_{ox}}\right)} \right] \sin\left(\gamma_{ox}\right) =$$

$$A_{fuel} C_{d_f} \sqrt{2\rho_f\left(p_f - P_0\right)} \left[C_{d_f} \sqrt{2\left(\frac{p_f - P_0}{\rho_f}\right)} \right] \sin\left(\gamma_f\right)$$



- **Collecting terms**

$$\rightarrow A_{ox} C_{d_{ox}}{}^2 \left(p_{ox} - P_0\right) \sin\left(\gamma_{ox}\right) = A_{fuel} C_{d_f}{}^2 \left(p_f - P_0\right) \sin\left(\gamma_f\right)$$

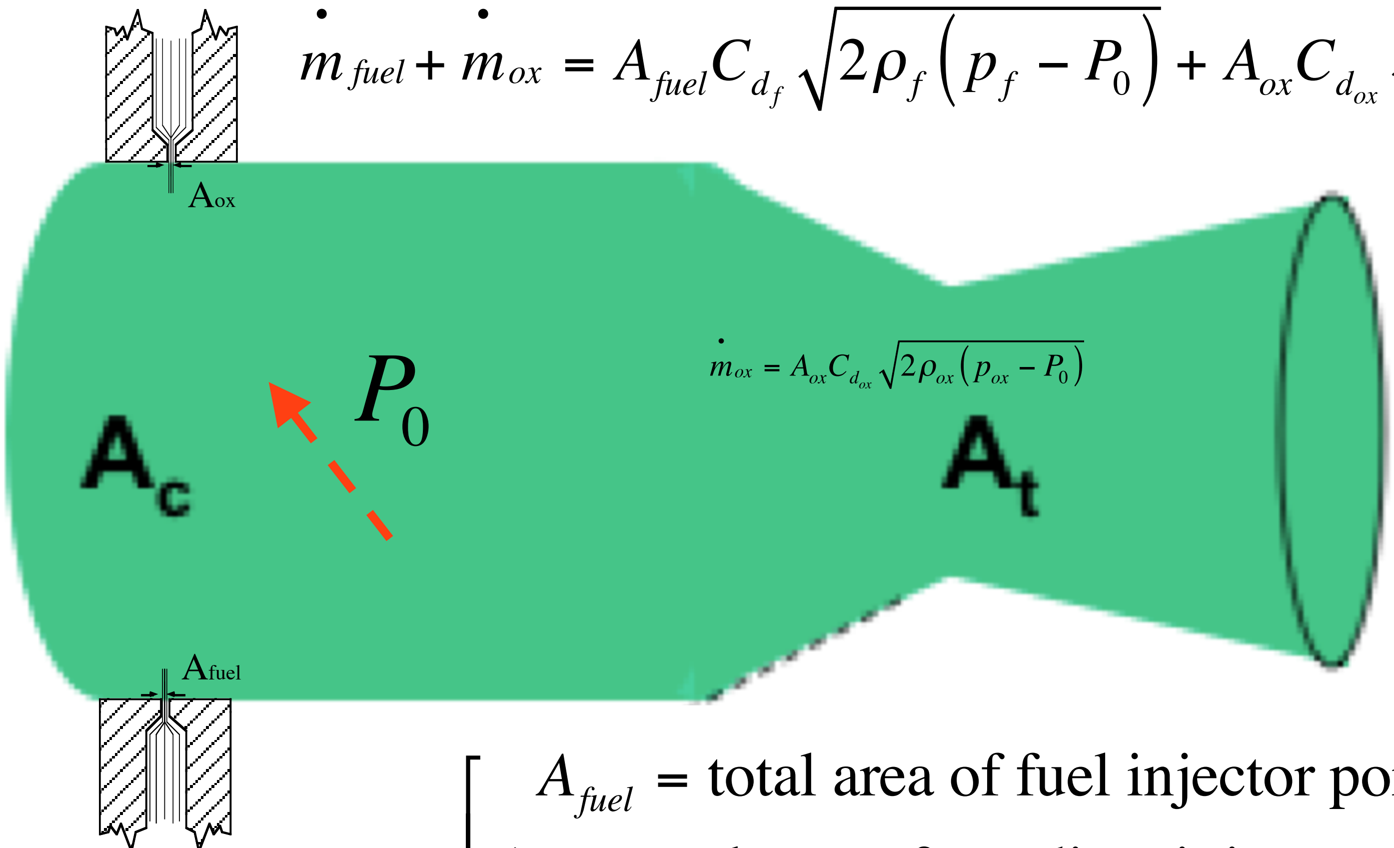
$$\boxed{\frac{\sin\left(\gamma_{ox}\right)}{\sin\left(\gamma_f\right)} = \frac{A_{fuel} C_{d_f}{}^2 \left(p_f - P_0\right)}{A_{ox} C_{d_{ox}}{}^2 \left(p_{ox} - P_0\right)}}$$

- Design criterion for injection angle

Return to Liquid Rocket Example (cont'd)

- Total Massflow for Incompressible propellants

$$\dot{m}_{fuel} + \dot{m}_{ox} = A_{fuel} C_{d_f} \sqrt{2\rho_f \left(p_f - P_0\right)} + A_{ox} C_{d_{ox}} \sqrt{2\rho_{ox} \left(p_{ox} - P_0\right)}$$



$$\begin{bmatrix} A_{fuel} = \text{total area of fuel injector ports} \\ A_{ox} = \text{total area of oxydizer injector ports} \end{bmatrix}$$

Liquid Rocket Example (cont'd)

- Substitute into ODE for combustor pressure

$$\frac{\partial P_0}{\partial t} + P_0 \left[\frac{A^*}{V_c} \sqrt{\gamma R_g T_0 \left(\frac{2}{\gamma+1} \right)^{\frac{\gamma+1}{(\gamma-1)}}} \right] = \frac{R_g T_0}{V_c} \left[A_{fuel} C_{d_f} \sqrt{2\rho_f \left(p_f - P_0 \right)} + A_{ox} C_{d_{ox}} \sqrt{2\rho_{ox} \left(p_{ox} - P_0 \right)} \right]$$

$$\frac{\partial P_0}{\partial t} + P_0 \left[\frac{A^*}{V_c} \sqrt{\gamma R_g T_0 \left(\frac{2}{\gamma+1} \right)^{\frac{\gamma+1}{(\gamma-1)}}} \right] =$$

$$\frac{R_g T_0}{V_c} \left[A_{fuel} C_{d_f} \sqrt{2\rho_f \left(p_f - P_0 \right)} + A_{ox} C_{d_{ox}} \sqrt{2\rho_{ox} \left(p_{ox} - P_0 \right)} \right]$$

Incompressible Fluids

Liquid Rocket Example (cont'd)

- Define ***O/F*** or "*mixture ratio*" of the propellants

$$\overset{O/F}{M_R} = \frac{\dot{m}_{ox}}{\dot{m}_{fuel}} = \frac{A_{ox} C_{d_{ox}} \sqrt{2\rho_{ox}\left(p_{ox} - P_0\right)}}{A_{fuel} C_{d_f} \sqrt{2\rho_f\left(p_f - P_0\right)}} \rightarrow \dot{m}_{ox} = M_R \dot{m}_{fuel}$$

- Liquid Rocket Model O.D.E.

$$\boxed{\frac{\partial P_0}{\partial t} = \frac{R_g T_0}{V_c}\left[1 + M_R\right] A_{fuel} C_{d_f} \sqrt{2\rho_f\left(p_f - P_0\right)} - P_0\left[\frac{A^*}{V_c}\sqrt{\gamma R_g T_0 \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}}\right]}$$

$$\frac{\partial P_0}{\partial t} = F\left[P_0, \left(\left\{R_g, T_0, \gamma\right\}, \left\{V_c, A^*\right\}, \left\{A_{fuel}, C_{d_f}, \rho_f, p_f\right\}, \left\{A_{ox}, C_{d_{ox}}, \rho_{ox}, p_{ox}\right\}\right)\right]$$

combustion products

chamber geometry

fuel injector geometry/pressure

oxydizer injector geometry/pressure

Liquid Rocket Example (cont'd)

- Steady State behavior $\frac{\partial P_0}{\partial t} = 0$

$$\frac{R_g T_0}{V_c}\left[1 + M_R\right] A_{fuel} C_{d_f} \sqrt{2\rho_f\left(p_f - P_0\right)} = P_0 \left[\frac{A^*}{V_c}\sqrt{\gamma R_g T_0 \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma+1}{(\gamma-1)}}}\right]$$

- Squaring both sides and Collecting terms

$$P_0^{\,2}\left[\left(\frac{A^*}{\left[1 + M_R\right] A_{fuel} C_{d_f}}\right)^2 \left(\frac{\gamma}{2\rho_f R_g T_0}\left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma+1}{(\gamma-1)}}\right)\right] + P_0 - p_f = 0$$

Liquid Rocket Example (cont'd)

- Solve for P_0

$$P_{0_{ss}} = \frac{-1 + \sqrt{1 + 4p_f \left(\frac{A^*}{[1+M_R]A_{fuel}C_{d_f}}\right)^2 \left(\frac{\gamma}{2\rho_f R_g T_0}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}\right)}}{2\left(\frac{A^*}{[1+M_R]A_{fuel}C_{d_f}}\right)^2 \left(\frac{\gamma}{2\rho_f R_g T_0}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}\right)}$$

$$M_R = \frac{\dot{m}_{ox}}{\dot{m}_{fuel}} = \frac{A_{ox}C_{d_{ox}}\sqrt{2\rho_{ox}\left(p_{ox} - P_0\right)}}{A_{fuel}C_{d_f}\sqrt{2\rho_f\left(p_f - P_0\right)}} \rightarrow \dot{m}_{ox} = M_R\,\dot{m}_{fuel}$$

Liquid Rocket Example (cont'd)

- "Tail-Off" Pressure $\left[\dot{m}_{fuel} + \dot{m}_{ox}\right] = 0$

$$\frac{\partial P_0}{\partial t} + P_0 \left[\frac{A^*}{V_c} \sqrt{\gamma R_g T_0 \left(\frac{2}{\gamma + 1} \right)^{\frac{\gamma+1}{(\gamma-1)}}} \right] = 0$$

$$P_0(t)_{\text{tail off}} = P_{0_{ss}} \left[e^{-\frac{A^*}{V_c} \sqrt{\gamma R_g T_0 \left(\frac{2}{\gamma+1} \right)^{\frac{\gamma+1}{(\gamma-1)}}} \left(t - t_{burnout}\right)} \right]$$

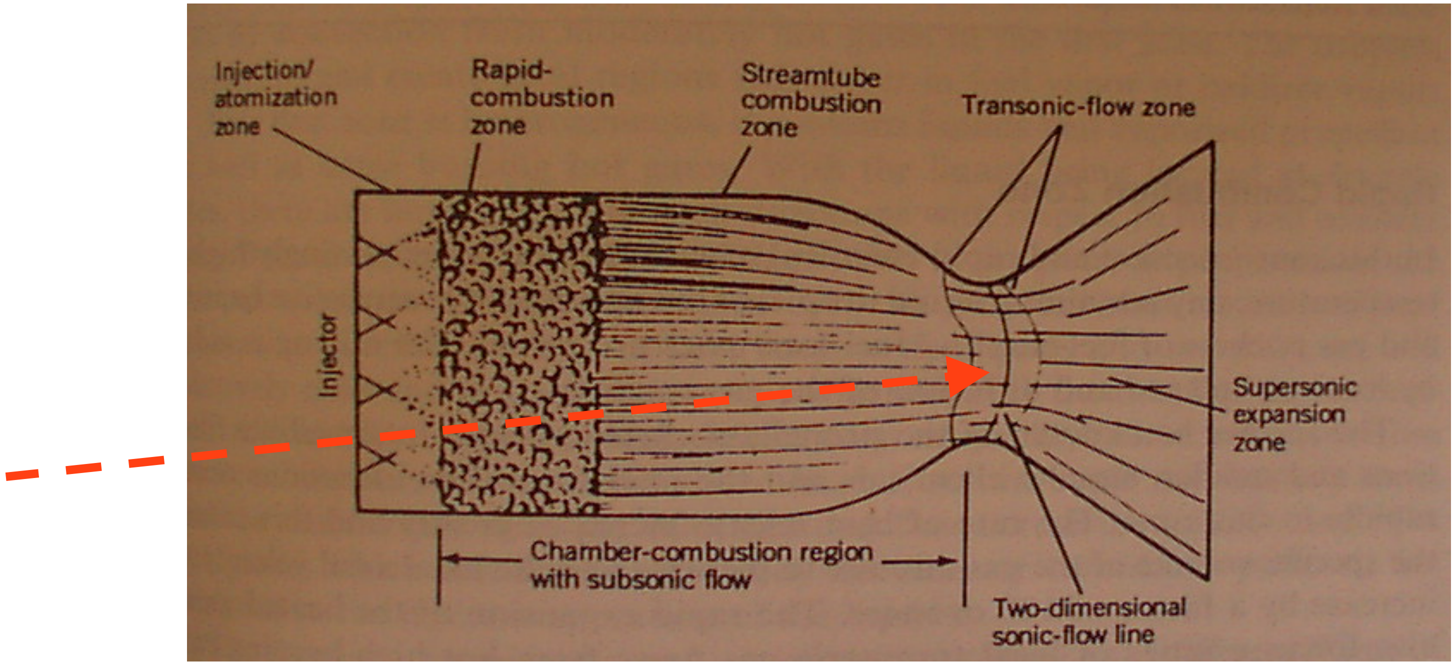
Liquid Rocket Example (cont'd)

- Combustor "Response Time"

$$P_0(t)_{\substack{tail\\off}} = P_{0_{ss}} \cdot e^{-\frac{A^*}{V_c}\cdot\left(\sqrt{(\gamma R_g T_0)\cdot\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}}\right)(t-t_{burnout})} = P_{0_{ss}} \cdot e^{-\left(\frac{t-t_{burnout}}{\tau_{combustor}}\right)}$$

$$\boxed{\tau_{combustor} = \frac{V_c}{A^*}\cdot\frac{1}{\sqrt{(\gamma R_g T_0)\cdot\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}}} \rightarrow \text{"combustor response time"}}$$

What happens in combustion chamber?



- The total combustion process, from injection of the reactants until completion of the chemical reactions and conversion of the products into hot gases, requires finite amounts of time and volume.

- How do we distribute the chamber volume length, diameter, convergence section to insure "good combustion?"

What happens in combustion chamber? (2)

- Combustion chamber serves as an envelope to retain the propellants for a sufficient period to ensure complete mixing and combustion.

- Combustion "residence time" … MUST MATCH propellant reaction rates ….

$$\tau_{combustor} = \frac{V_c}{A^*} \cdot \frac{1}{\sqrt{\left(\gamma R_g T_0\right) \cdot \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}}}$$

…. Match $V_c/A^* = L^*$ *(Lstar)* to propellant reaction rates to Get best combustion characteristics

What happens in combustion chamber? (3)

- Conventional method of establishing the L^* of a new thrust chamber design largely relies on past experience with similar propellants and engine size.

- Under a given set of operating conditions, value of the minimum required L^* evaluated by actual firings of thrust chambers.

- With throat area and minimum required L^* established, the appropriate chamber volume distribution

- L^* is significantly greater than the linear length between injector face and throat plane.

Characteristic Length parameter, L*

Characteristic length, L*

- Optimum L_c must be determined experimentally
- How to scale L_c from one engine size to another?
- Characteristic length, L^*
- Value of L^* tabulated for different propellants

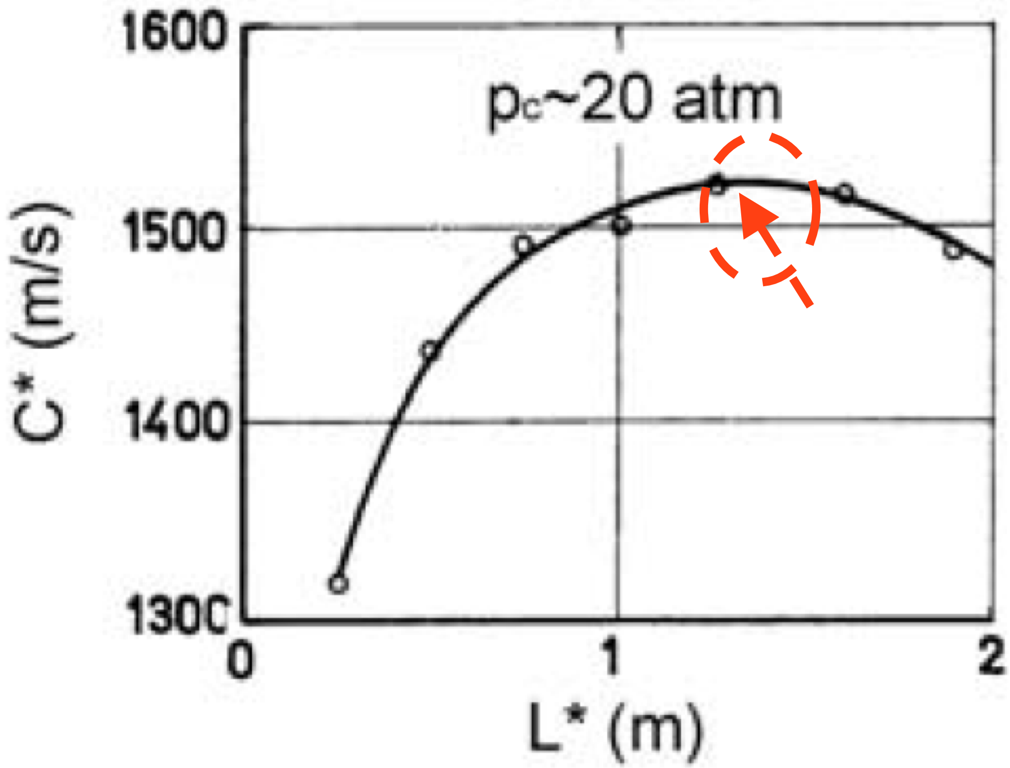
$$L^* = \frac{V_c}{A_t}$$

Propellant	L^* (m)
LOX-kerosene	1.5-2.5
LOX-ethanol	2.5-3
HNO_3-UDMH	1.5-2

Key parameter for combustion stability

L^* has big effect on performance

Characteristic length, L*



Hydrazine / HNO_3

Characteristic Length L^* for various propellants

Ranges of Combustor Characteristic Length

Propellants	Characteristic Length L*	
	Low(m)	High(m)
Liquid fluorine / hydrazine	0.61	0.71
Liquid fluorine / gaseous H_2	0.56	0.66
Liquid fluorine / liquid H_2	0.64	0.76
Nitric acid/hydrazine	0.76	0.89
N_2O_4 / hydrazine	0.60	0.89
Liquid O_2/ ammonia	0.76	1.02
Liquid O_2 / gaseous H_2	0.56	0.71
Liquid O_2 / liquid H_2	0.76	1.02
Liquid O_2 / RP-1	1.02	1.27
H_2O_2 / RP-1 (incl. catalyst)	1.52	1.78

- *Space Propulsion Analysis
- and Design [Ronald
- W. Humble,
- Gary N. Henry, and
- Wiley J. Larson -
- McGraw-Hill, 1995]

L^* is experimentally determined for given propellants and includes Effects of mixture ratio, massflow, etc. "High/Low" ranges

Characteristic Length L^* for various propellants (2)

Table 1: Chamber Characteristic Length, L*

Propellant Combination	L*, cm
Nitric acid/hydrazine-base fuel	76-89
Nitrogen tetroxide/hydrazine-base fuel	76-89
Hydrogen peroxide/RP-1 (including catalyst bed)	152-178
Liquid oxygen/RP-1	102-127
Liquid oxygen/ammonia	76-102
Liquid oxygen/liquid hydrogen (GH_2 injection)	56-71
Liquid oxygen/liquid hydrogen (LH_2 injection)	76-102
Liquid fluorine/liquid hydrogen (GH_2 injection)	56-66
Liquid fluorine/liquid hydrogen (LH_2 injection)	64-76
Liquid fluorine/hydrazine	61-71
Chlorine trifluoride/hydrazine-base fuel	51-89

•http://www.braeunig.us/space/propuls.htm

Combustion chamber scaling

• The *contraction ratio* is defined as the major cross-sectional area of the combuster divided by the throat area.

• Typically, large engines are constructed with a low contraction ratio and a comparatively long length

• Smaller chambers use a large contraction ratio with a shorter length, … still providing sufficient *L** for adequate vaporization and combustion dwell-time.

• As a good place to start, the process of sizing a new combustion Chamber examines the dimensions of previously successful designs in the same size class and plotting such data in a rational manner.

Combustion chamber scaling (2)

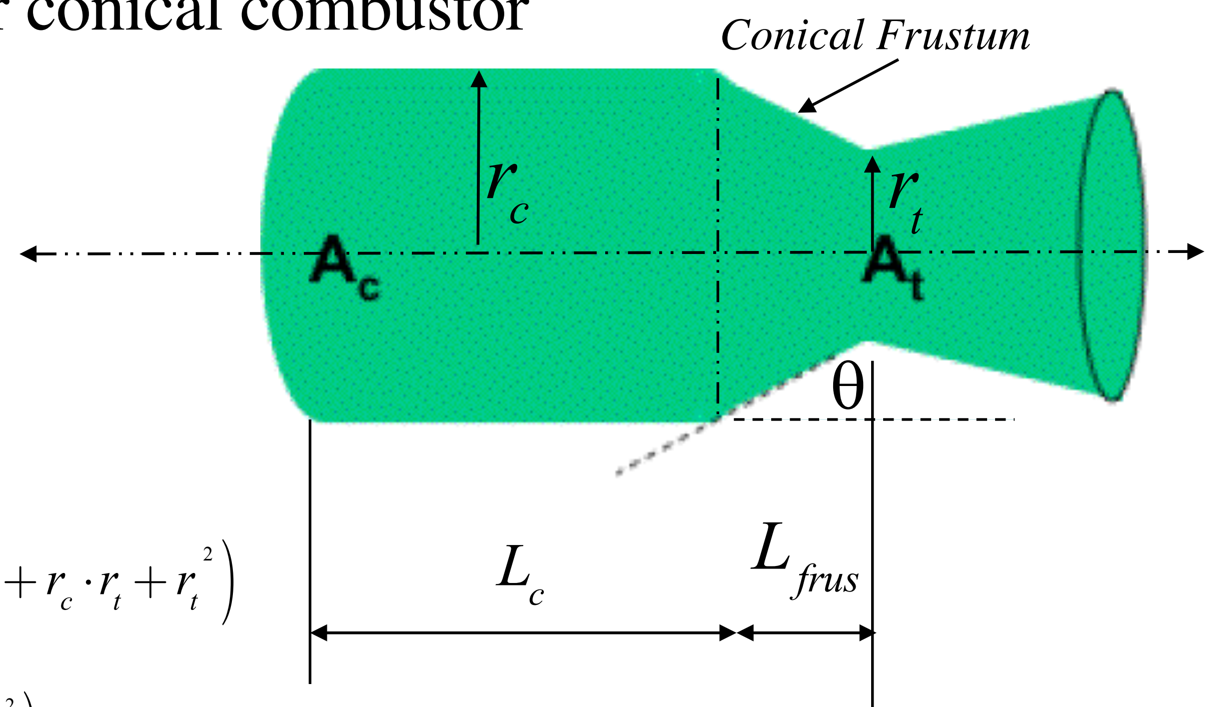
- The throat size of a new engine can be generated with a fair degree of confidence, so it makes sense to plot the data from historical sources in relation to throat diameter.

- Three geometrical shapes have been used in combustion chamber design – spherical, near-spherical, and cylindrical - with the cylindrical chamber being used most frequently in the United States.

- Compared to a cylindrical chamber of the same volume, a spherical or near-spherical chamber offers the advantage of less cooling surface and weight; however, the spherical chamber is more difficult to manufacture and has provided poorer performance in other respects.

Combustion chamber scaling (3)

For a cylindrical chamber with a conical contraction section
The volume can be computed directly using geometric
Relationships . … e.g for conical combustor



$$\varepsilon_c = \frac{A_c}{A_t} = \left(\frac{r_c}{r_t}\right)^2$$

$$L_{frus} = \frac{r_c - r_t}{\tan\theta}$$

$$V_c = V_{cyl} + V_{frus} = A_c \cdot L_c + \frac{\pi}{3} \cdot L_{frus} \cdot \left(r_c^2 + r_c \cdot r_t + r_t^2\right)$$

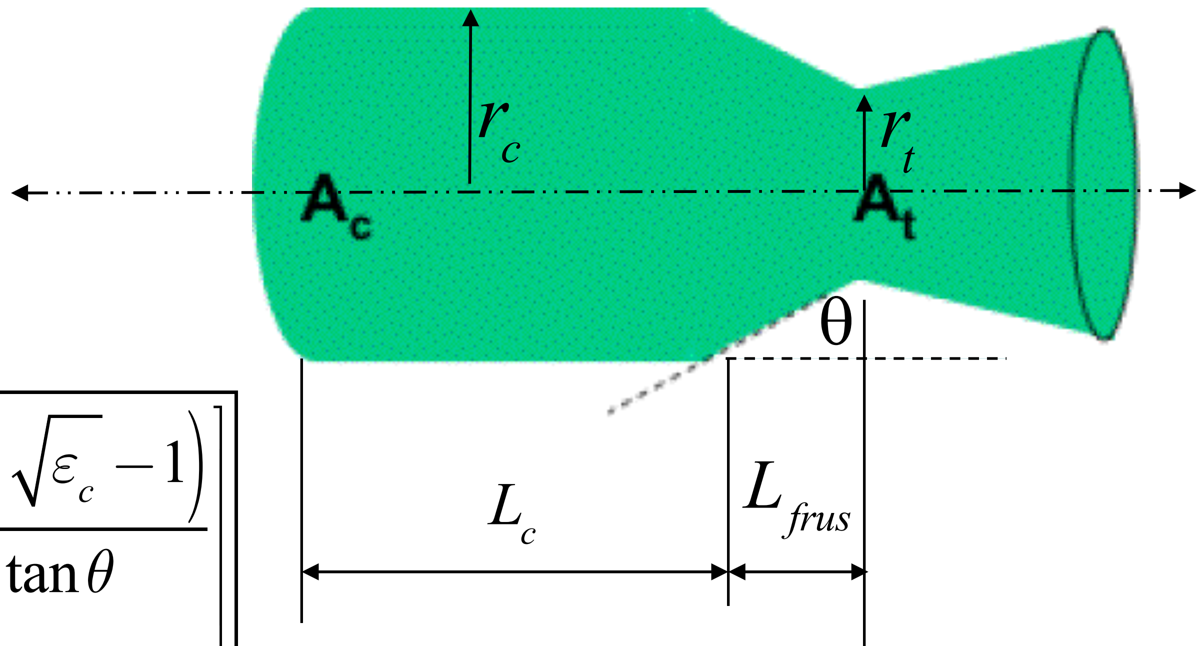
$$= A_t \cdot \varepsilon_c \cdot L_c + \frac{\pi}{3} \cdot \left(\frac{r_c - r_t}{\tan\theta}\right) \cdot \left(r_c^2 + r_c \cdot r_t + r_t^2\right) =$$

$$A_t \cdot \varepsilon_c \cdot L_c + \frac{1}{3}\frac{\pi \cdot r_t^2 \cdot r_t}{\tan\theta} \cdot \left(\left(\frac{r_c}{r_t}\right)^3 - 1\right) = A_t \cdot \left[\varepsilon_c \cdot L_c + \frac{1}{3} \cdot \sqrt{\frac{A_t}{\pi}} \cdot \frac{\left(\varepsilon_c \cdot \sqrt{\varepsilon_c} - 1\right)}{\tan\theta}\right]$$

Combustion chamber scaling (4)

For a cylindrical chamber with a conical contraction section
The volume can be computed directly using geometric relationships

$$\varepsilon_c = \frac{A_c}{A_t}$$



$$\boxed{V_c = A_t \cdot \left[\varepsilon_c \cdot L_c + \frac{1}{3} \cdot \sqrt{\frac{A_t}{\pi}} \cdot \frac{\left(\varepsilon_c \cdot \sqrt{\varepsilon_c} - 1\right)}{\tan\theta} \right]}$$

- What is the scaling relationship between the chamber length and the throat contraction ratio, ε_c ?

L^* … used to scale combustor dimensions

- Given throat geometry and propellants, calculate V_c, L_c, ε_c

- Select L^* mid range from previous table

- Use empirical formula for contraction ratio based on throat diameter

$$\varepsilon_c = \frac{A_c}{A_t} \approx \frac{8.0}{D_t^{3/5}} + 1.25$$

….. **D_t is the throat diameter in centimeters. valid for $1/5 < A_c/A_t < 13.5$ …. and 0.7cm $< D_t <$ 200cm**

"rule of thumb"

- The effective chamber length is derived from

$$\boxed{V_c = A_t \cdot \left[\varepsilon_c \cdot L_c + \frac{1}{3} \cdot \sqrt{\frac{A_t}{\pi}} \cdot \frac{\left(\varepsilon_c \cdot \sqrt{\varepsilon_c} - 1\right)}{\tan\theta}\right]}$$

SSME Combustor Example Revisited

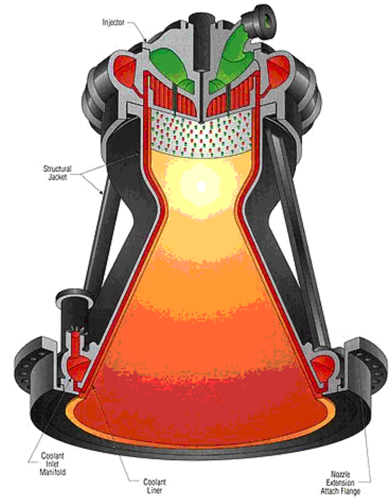
- *Rules of Thumb**

"Characteristic Length"

$$L^* = \frac{V_c}{A_t}$$

Combustor Cross sectional area

$$\varepsilon_c = \frac{A_c}{A_t} \approx \frac{8.0}{D_t^{3/5}} + 1.25 \quad D_t \sim \text{cm}$$



- SSME Throat Diameter, 27.1 cm
- Compute Throat Area

$$\frac{\left(\frac{27.1}{100}\right)^2 \pi}{4} = 0.05768 \ \text{m}^2$$

$$\boxed{A_c = \left(8\left(27.1^{-0.6}\right) + 1.25\right) 0.05768 = 0.1358 \ \text{m}^2}$$

* **Space Propulsion Analysis and Design [Ronald W. Humble, Gary N. Henry, and Wiley J. Larson - McGraw-Hill, 1995]**

SSME Combustor Example Revisited (cont'd)

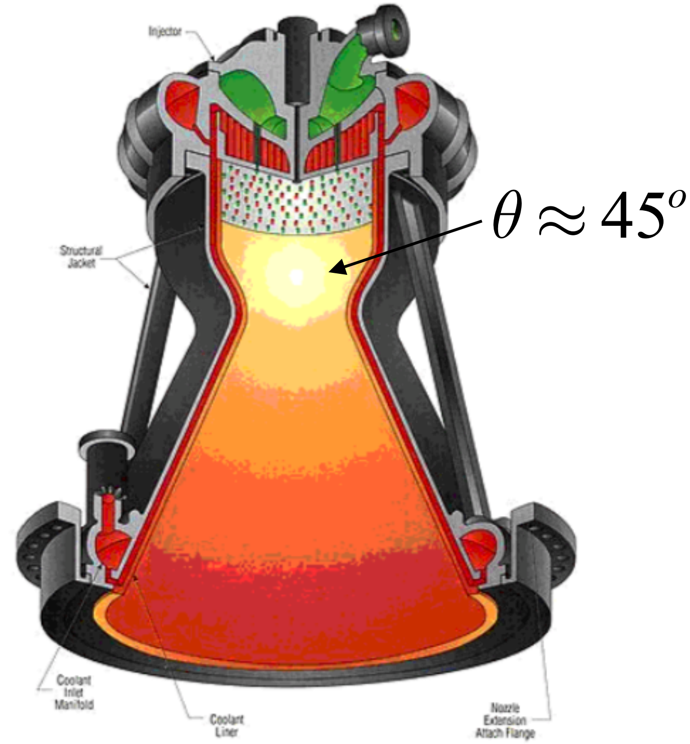
$$A_c = (8\,(27.1^{-0.6}) + 1.25)\,0.05768 = 0.1358\ \text{m}^2$$

- Shuttle Combustor Diameter,

$$D_c = \left(\frac{0.1358 \cdot 4}{\pi}\right)^{0.5} 100 = 41.58\ \text{cm}$$

- Contraction ratio

$$\varepsilon_c = \frac{A_c}{A_t} = \frac{0.1358_{m^2}}{0.05768_{m^2}} = 2.354$$

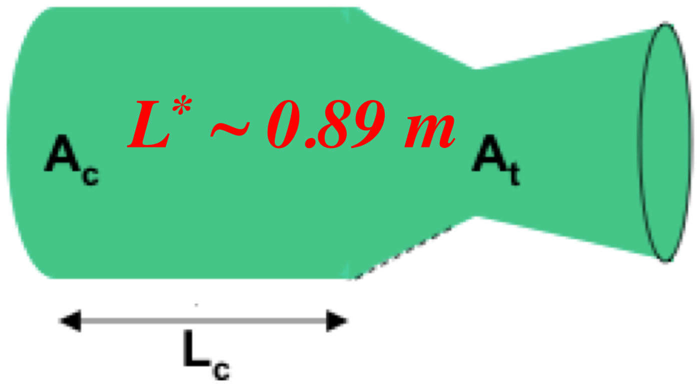


* **Space Propulsion Analysis and Design [Ronald W. Humble, Gary N. Henry, and Wiley J. Larson - McGraw-Hill, 1995]**

SSME Combustor Example Revisited (cont'd)

$$L^* = \frac{V_c}{A_t}$$

- Compute chamber volume based on "mid range" value for L^*



Ranges of Combustor Characteristic Length *

Propellants	Characteristic Length L*	
	Low(m)	High(m)
Liquid O_2/ ammonia	0.76	1.02
Liquid O_2 / gaseous H_2	0.56	0.71
Liquid O_2 / liquid H_2	0.76	1.02
Liquid O_2 / RP-1	1.02	1.27
H_2O_2 / RP-1 (incl. catalyst)	1.52	1.78

$$V_c \simeq \left(\left(\frac{0.76 + 1.02}{2}\right) 0.05768\right)$$

$$= 0.05134\ m^3$$

* Space Propulsion Analysis and Design [Ronald W. Humble, Gary N. Henry, and Wiley J. Larson - McGraw-Hill, 1995]

L^* used to scale combustor dimensions (cont'd)

$$\boxed{V_c = A_t \cdot \left[\varepsilon_c \cdot L_c + \frac{1}{3} \cdot \sqrt{\frac{A_t}{\pi}} \cdot \frac{\left(\varepsilon_c \cdot \sqrt{\varepsilon_c} - 1 \right)}{\tan\theta} \right]}$$

$L^* \sim 0.89_m$ $\quad \theta \approx 45^o$

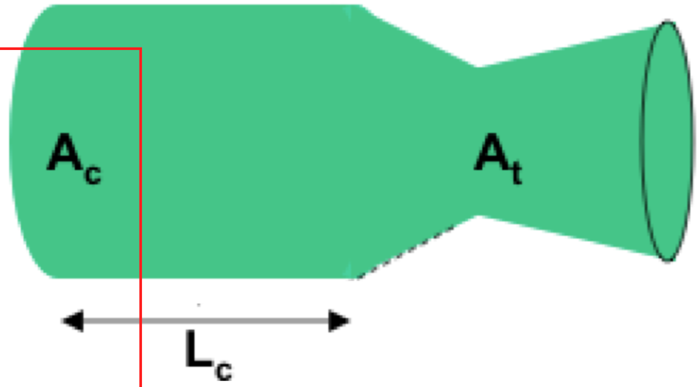
$\varepsilon_c = 2.345 \quad A_t = 576.8_{cm^2}$

Solve for L_c

$$L_c = \frac{1}{\varepsilon_c}\left[\frac{V_c}{A_t} - \frac{1}{3} \cdot \sqrt{\frac{A_t}{\pi}} \cdot \frac{\left(\varepsilon_c \cdot \sqrt{\varepsilon_c} - 1 \right)}{\tan\theta} \right] \approx \frac{1}{\varepsilon_c}\left[L^* - \frac{1}{3} \cdot \sqrt{\frac{A_t}{\pi}} \cdot \frac{\left(\varepsilon_c \cdot \sqrt{\varepsilon_c} - 1 \right)}{\tan\theta} \right]$$

$$\frac{1}{2.354}\left(0.89 \cdot 100 - \frac{1}{3}\left(\frac{576.8}{\pi}\right)^{0.5} \frac{\left(2.345\left(2.345^{0.5}\right) - 1\right)}{\tan\left(\frac{\pi}{180} 45\right)} \right) = 32.84\ cm$$

SSME Combustor Example Revisited (cont'd)



… Compare to "Effective" combustor length dimension..

$$V_c/A_c = L_{eff} = \frac{0.05134}{0.1358}100 = 37.81 \text{ cm}$$

What is burner surface area?

$$A_{surf} = 2 \cdot \pi \cdot r_c \cdot L_c + \pi \cdot (r_c + r_t) \cdot \sqrt{(r_c - r_t)^2 + L_{frus}^2} =$$

$$2\pi 20.79 \cdot 32.84 + \pi\,(20.79 + 13.55)\,\left((20.79 - 13.55)^2 + 7.24^2\right)^{0.5}$$

$= 5394.4\ cm^2$

Published number

0.54884 m²

Pretty good estimate

$$r_c = 20.79_{cm}$$

$$r_2 = 13.55_{cm}$$

$$L_{frus} = \frac{r_c - r_t}{\tan\theta} = \frac{20.79 - 13.55}{1} = 7.24_{cm}$$

$$L_c = 32.84_{cm}$$

$L_c + L_{frus} =$ *32.84 cm + 7.24 cm = 40.08 cm*

SSME Example

- Compute Nominal Steady State Operating Combustor Pressure

$$P_{0_{ss}} = \frac{-1+\sqrt{1+4p_f\left(\frac{A^*}{[1+M_R]A_{fuel}C_{d_f}}\right)^2\left(\frac{\gamma}{2\rho_f R_g T_0}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}\right)}}{2\left(\frac{A^*}{[1+M_R]A_{fuel}C_{d_f}}\right)^2\left(\frac{\gamma}{2\rho_f R_g T_0}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}\right)}$$

Properties of Propellant Products 2

- Effective gamma: 1.196
- Effective MW: 13.6
- Idealized Flame Temperature, deg. K: 3615

Combustor Output data

- Rg, J/kg Deg-K: 611.3539
- Cp, J/kg Deg-K 2: 3730.51
- Cstar, Ideal, m/sec: 2295.04

Chamber /Nozzle geometry

- Volume, M^3: 0.05119
- Throat Area, M^2: 0.05785
- A/A*: 77.5

- Ox, Mdot, kg/sec: 67.9644
- Fuel, Mdot, kg/sec: 409.952
- Mixture, ratio: 6.03186
- Mdot, total kg/sec: 477.916

Fuel Injector Properties

- Injector Area, m^2: 0.002877
- Discharge Coefficient: 0.81
- Fuel density, kg/M^3: 71
- Injector Pressure, kPa: 24950
- Total Fuel mass, kg: 35477.5

Oxydizer Injector Properties

- Injector Area, m^2: 0.0043308
- Discharge Coefficient: 0.81
- Oxydizer density, kg/M^3: 1140
- Injector Pressure, kPa: 24950
- Total Oxydizer mass, kg: 213995

SSME Example (cont'd)

- Compute Nominal Steady State Operating Combustor Pressure

$$P_{0_{ss}} = \frac{-1 + \sqrt{1 + 4 p_f \left(\frac{A^*}{[1 + M_R] A_{fuel} C_{d_f}} \right)^2 \left(\frac{\gamma}{2 \rho_f R_g T_0} \left(\frac{2}{\gamma + 1} \right)^{\frac{\gamma+1}{(\gamma-1)}} \right)}}{2 \left(\frac{A^*}{[1 + M_R] A_{fuel} C_{d_f}} \right)^2 \left(\frac{\gamma}{2 \rho_f R_g T_0} \left(\frac{2}{\gamma + 1} \right)^{\frac{\gamma+1}{(\gamma-1)}} \right)} =$$

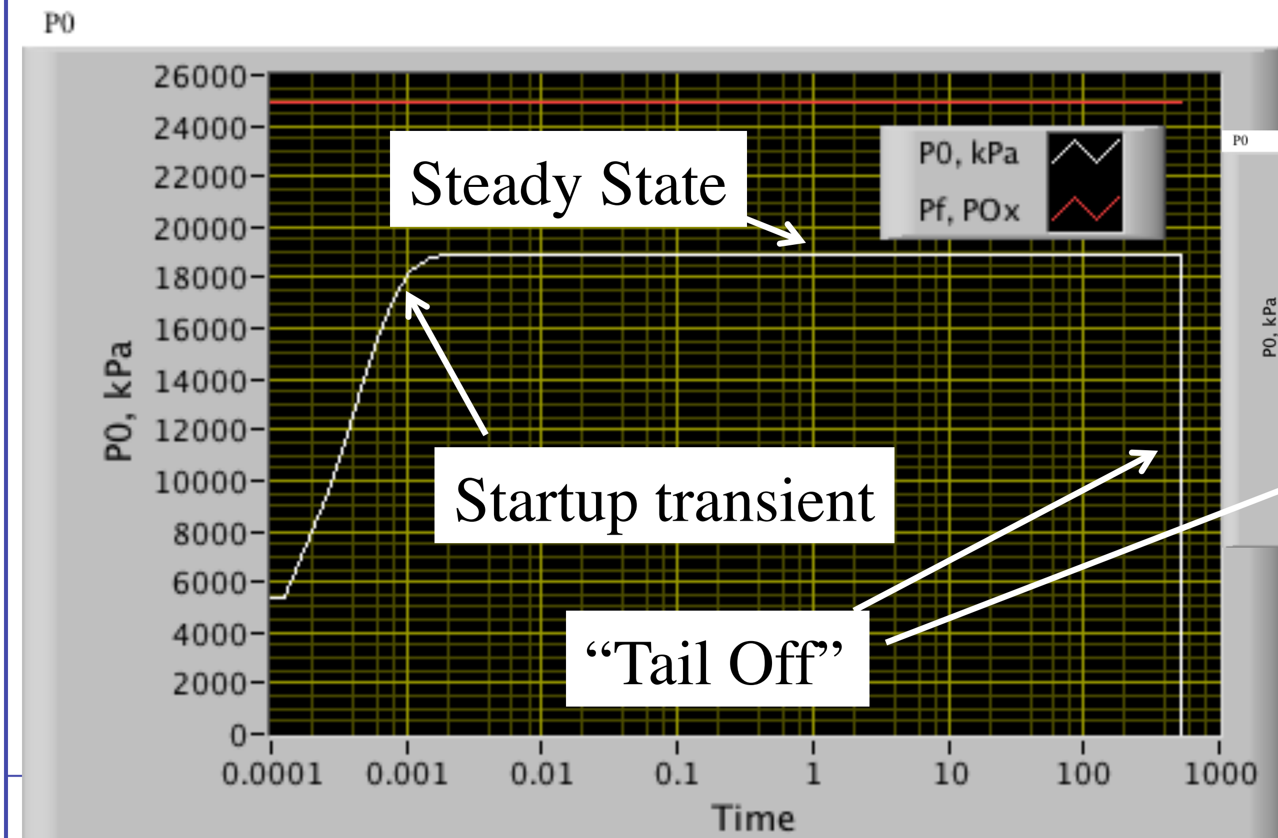
$$\frac{\left(1 + \left(4 \cdot 24950 \cdot 10^3 \left(\left(\frac{0.05785}{(1 + 6.03186)\,(0.002877 \cdot 0.81)} \right)^2 \left(\frac{1.196}{2 \cdot 71 \cdot 611.3539 \cdot 3615} \right) \left(\frac{2}{1.196 + 1} \right)^{\left(\frac{1.196+1}{1.196-1}\right)} \right)\right)\right)^{0.5} - 1}{2 \left(\left(\frac{0.05785}{(1 + 6.03186)\,(0.002877 \cdot 0.81)} \right)^2 \left(\frac{1.196}{2 \cdot 71 \cdot 611.3539 \cdot 3615} \right) \left(\frac{2}{1.196 + 1} \right)^{\left(\frac{1.196+1}{1.196-1}\right)} \right) 1000}$$

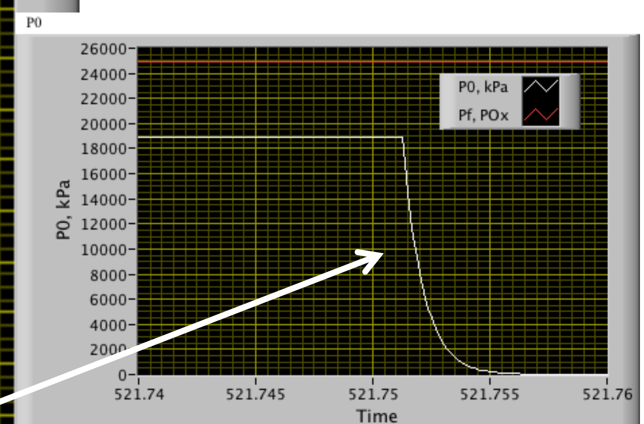
$= 18960.03$ kPa .... !

… and we know from earlier (5.2) this is "correct" answer

SSME: Look at Transient Response

$$\frac{\partial P_0}{\partial t} = \frac{R_g T_0}{V_c}\left[1 + M_R\right] A_{fuel} C_{d_f} \sqrt{2\rho_f \left(p_f - P_0\right)} - P_0 \left[\frac{A^*}{V_c}\sqrt{\gamma R_g T_0 \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma+1}{(\gamma-1)}}}\right]$$





- At these high Combustor Pressure Levels … response is "fast"

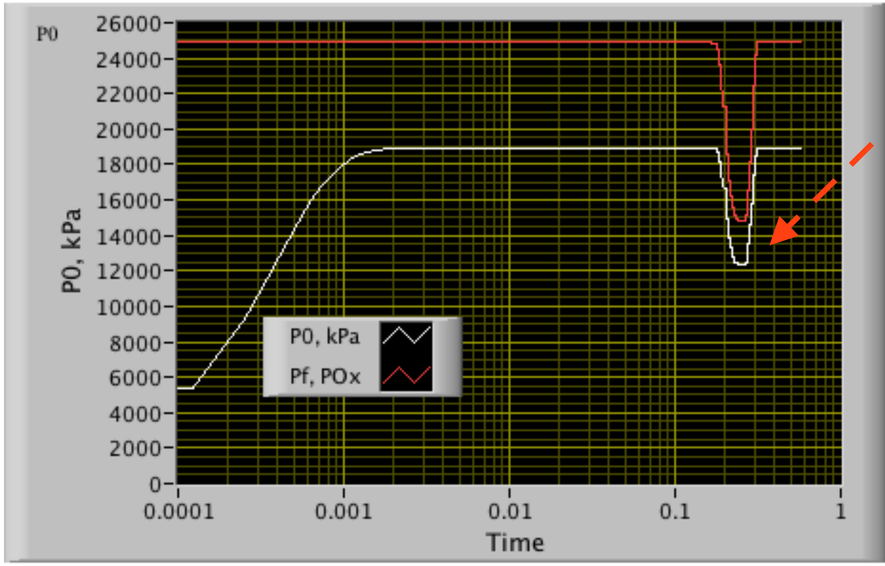
A Mechanism for Throttling

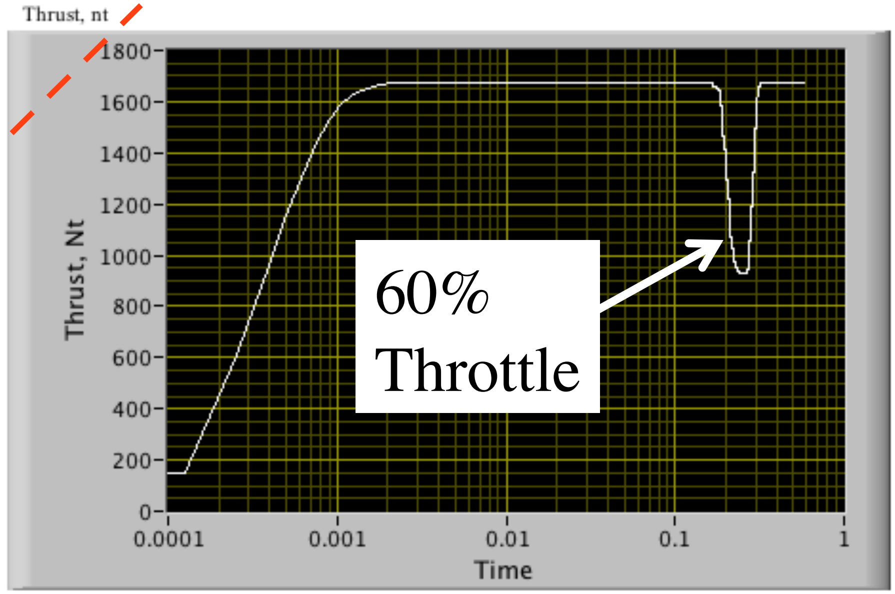
$$\frac{\partial P_0}{\partial t} = -P_0\left[\frac{A^*}{V_c}\sqrt{\gamma R_g T_0\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}}\right] +$$

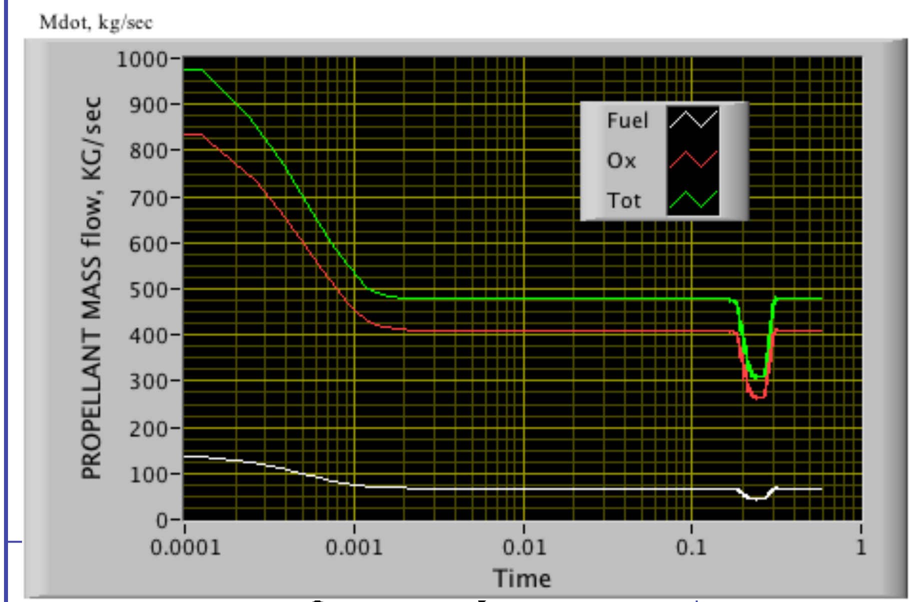
$$\frac{R_g T_0}{V_c}\left[A_{fuel}C_{d_f}\sqrt{2\rho_f\left(\boxed{\left\{\frac{T_h\%}{100}\times p_f\right\}}-P_0\right)} + A_{ox}C_{d_{ox}}\sqrt{2\rho_{ox}\left(\boxed{\left\{\frac{T_h\%}{100}\times p_{ox}\right\}}-P_0\right)}\right]$$

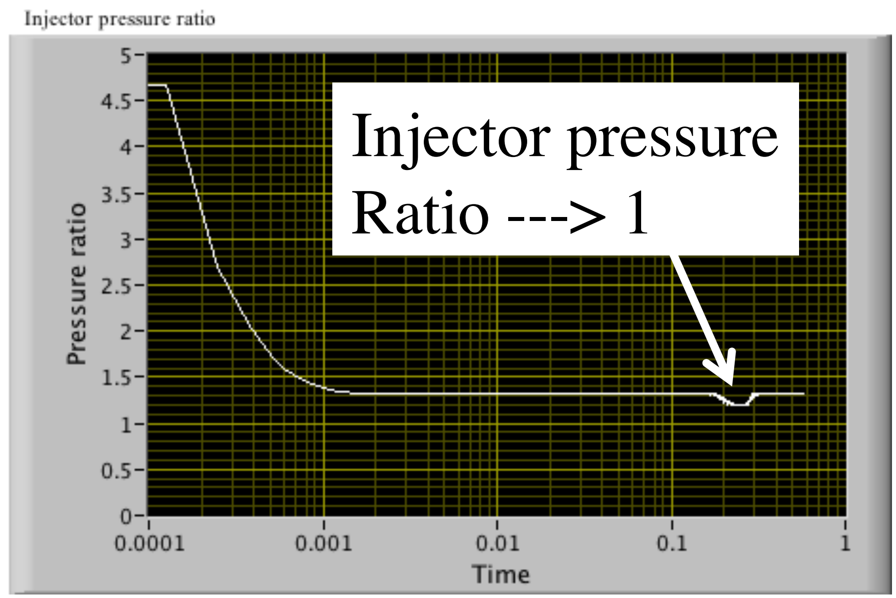
- By modulating the injector pressures, we can throttle the rocket engine

A Mechanism for Throttling (cont'd)





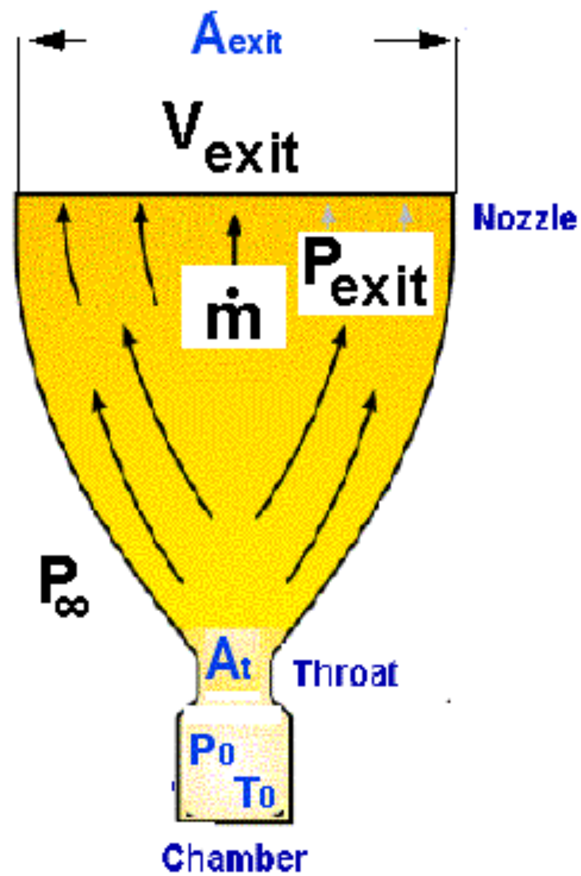




Deep-Throttle Rocket Engines

- **In theory rocket engine motor can be throttled back until the throat is no longer sonic by reducing propellant Flow rate *(injector pressure)***

- **Difficult problem in practice.**



$$Thrust = \dot{m} V_{exit} + A_{exit} (P_{exit} - P_\infty)$$

Deep-Throttle Rocket Engines

(cont'd)

- **Essential for pressure drop across injector > 25% of chamber pressure**

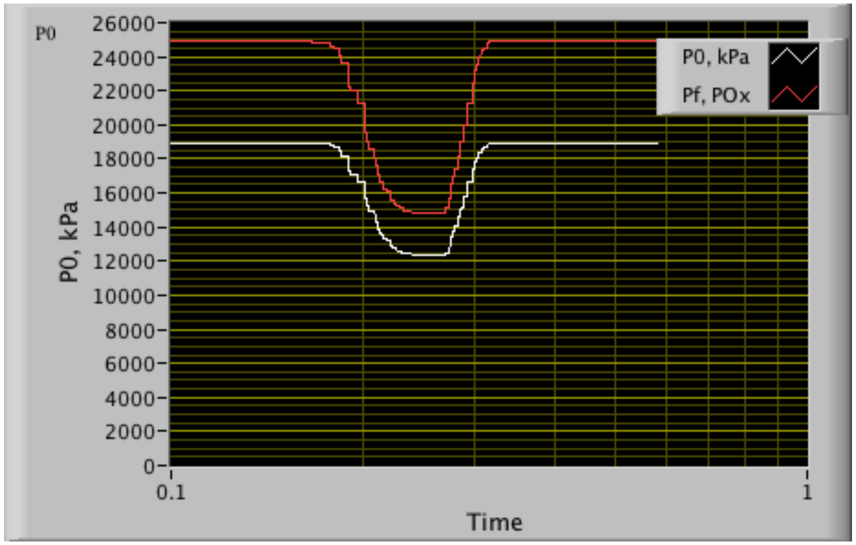
    -- Pressure ratio insures propellant flow rates are independent of fluctuations in chamber pressure.

- **Fixed geometry injectors**

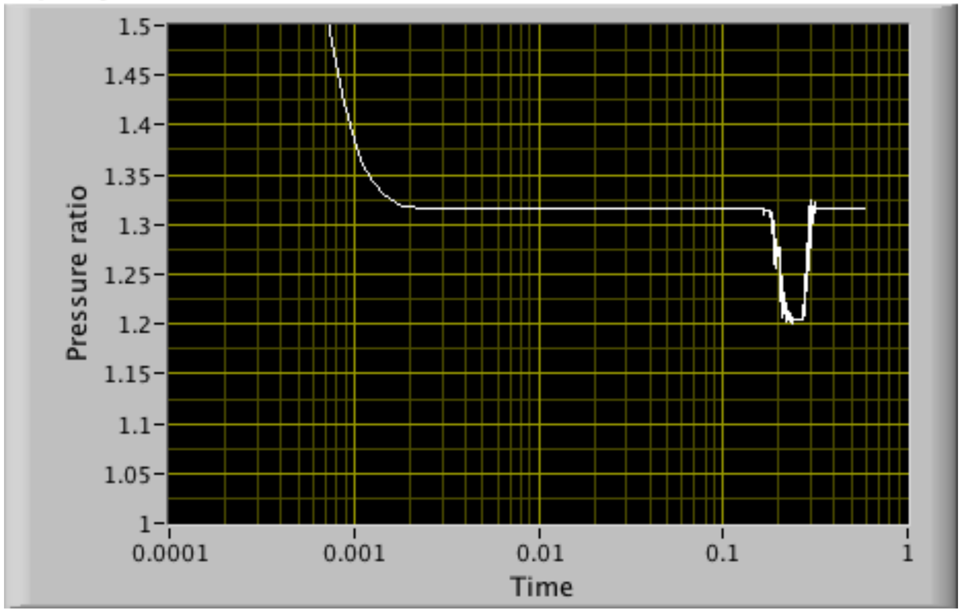
    Reduction of Propellant flow rates causes injector pressure to drop faster than the chamber pressure
    … until injector pressure becomes so low that coupling between chamber and propellant feed system occurs
    … causing combustor instability (a.k.a explosion or flameout)

Deep-Throttle Rocket Engines

(cont'd)



Injector pressure ratio



Deep-Throttle Rocket Engines

(cont'd)

- **Typically rocket motor with fixed injector geometry can be throttled down to 60-70% of nominal thrust without serious stability problems (SSME)**

- **Highest Operational "turndown ratio" Engine**

  Lunar Module descent engine, 10:1 turndown ratio
  Even with a variable-geometry injector there were stability Problems, Thrust levels between 100% and 65% were never used because mixture ratio was so hard to properly control.

- **Obviously this is a big Challenge**

  Current State-of-The-Art
  "Pintle-injectors"

Deep-Throttle Rocket Engines

(cont'd)

- **Northrop Grumman (TRW) TR-106**
  - Pintle-injection (similar to LEM descent engine)

• **Low Cost Pintle Engine (LCPE)**

The key element of the LCPE's design is its single element coaxial pintle injector, used to introduce propellants into the combustion chamber.

(LCPE)

• **Moveable pintle injector attributes include deep throttle capability, 10:1 turndown ratio**

TRW has tested more than 50 different pintle injector engines, using more than 25 different propellant combinations with complete combustion stability

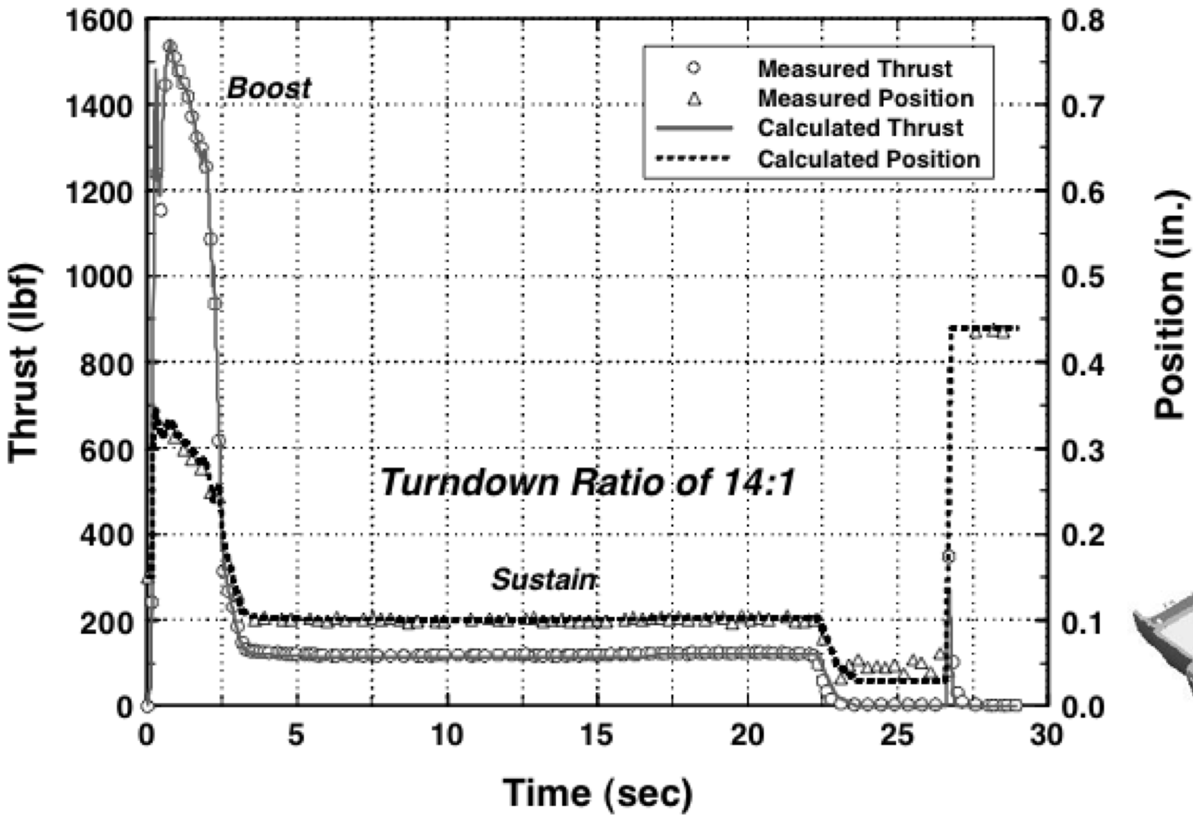
Range in size from the 100-pound thrust liquid apogee engine used on NASA's Chandra X-ray Observatory to the 10,0-pound thrust Delta and LMDE engines.

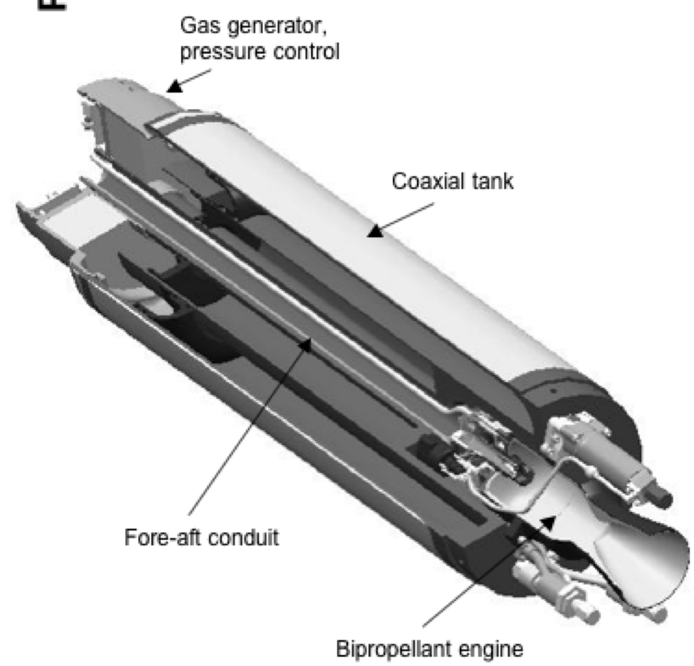
• **Small Scale Test Version?**

Deep-Throttle Rocket Engines

(cont'd)

• **Small Scale Test Version?**





Deep-Throttle Rocket Engines

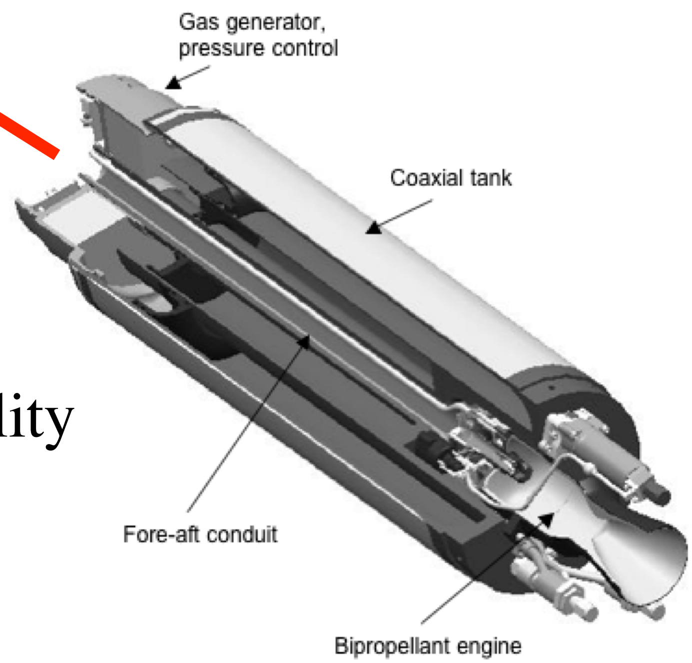
(cont'd)

- **Pintle Injectors**

$$\frac{\partial P_0}{\partial t} = \frac{R_g T_0}{V_c}\left[1 + M_R\right]\underline{A_{fuel} C_{d_f}}\sqrt{2\rho_f\left(p_f - P_0\right)} - P_0\left[\frac{A^*}{V_c}\sqrt{\gamma R_g T_0\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}}\right]$$

- By moving the pintle shaft
  Propellant mass flow is modulated
  by changing the effective injector area

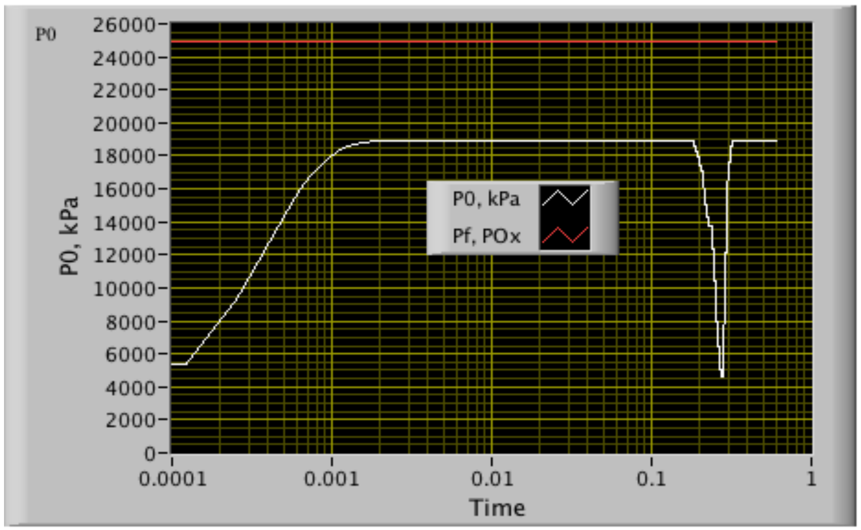
- Injection pressure remains relatively constant … reducing combustion instability problem



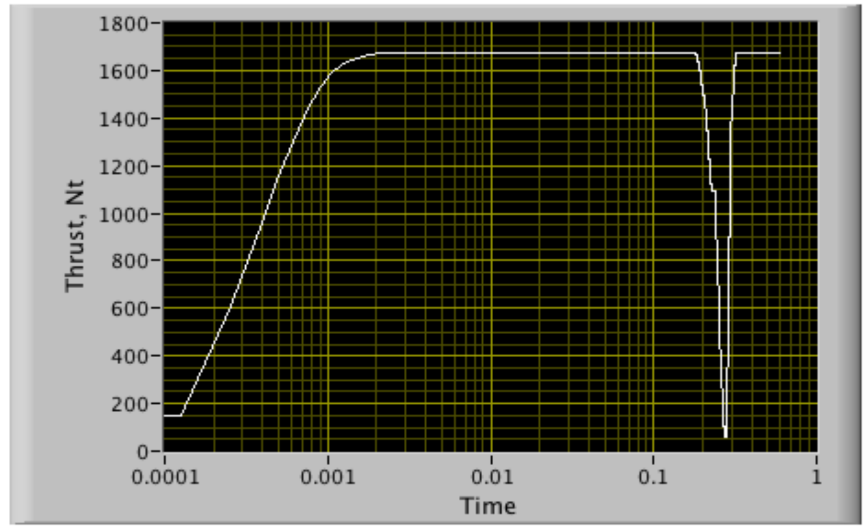
Deep-Throttle Rocket Engines

(cont'd)

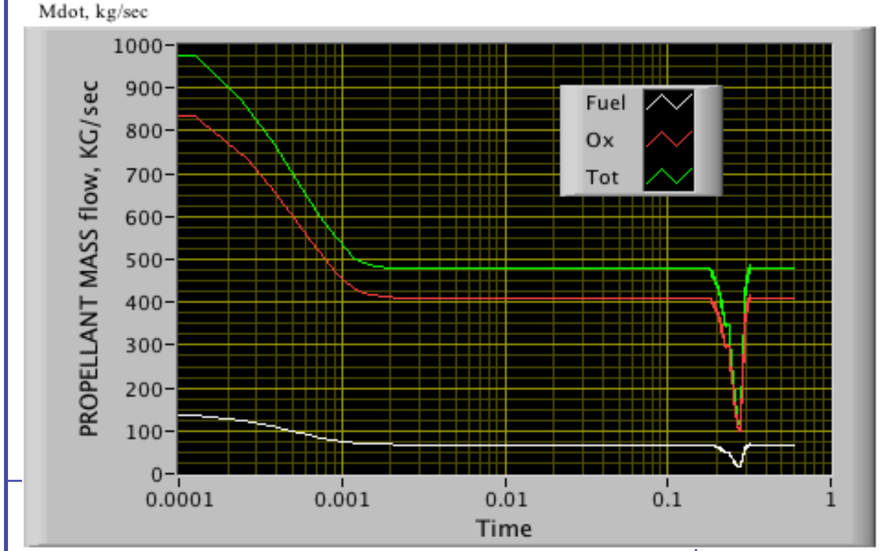
- **SSME Throttle Simulation using Pintle Injector (Area Modulation)**



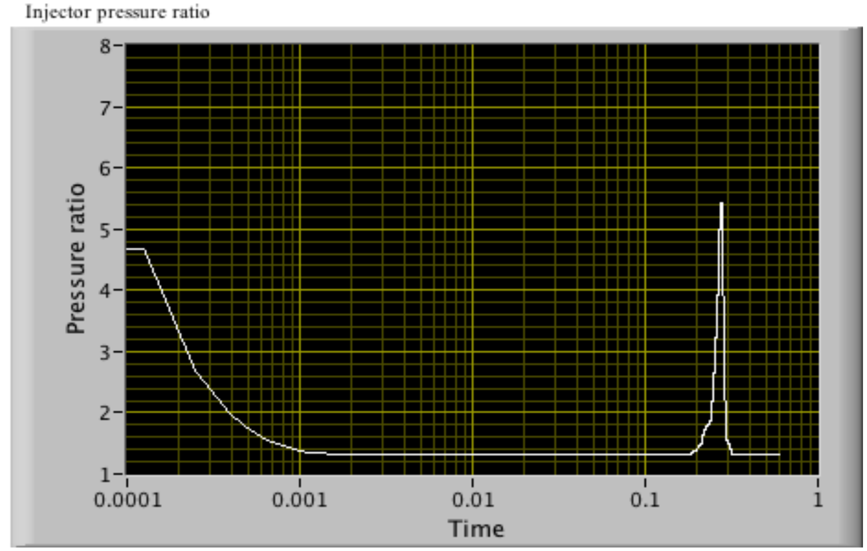
Thrust, nt



Mdot, kg/sec

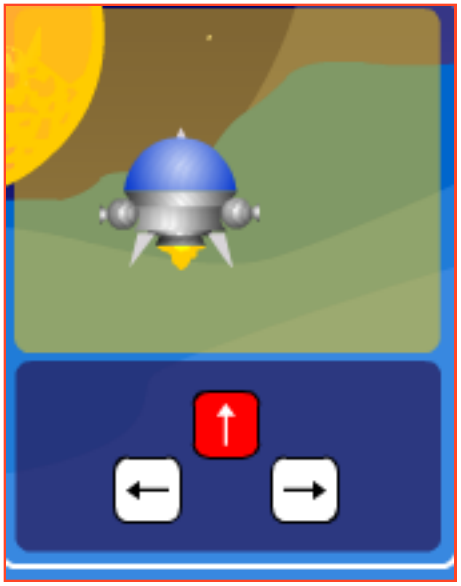


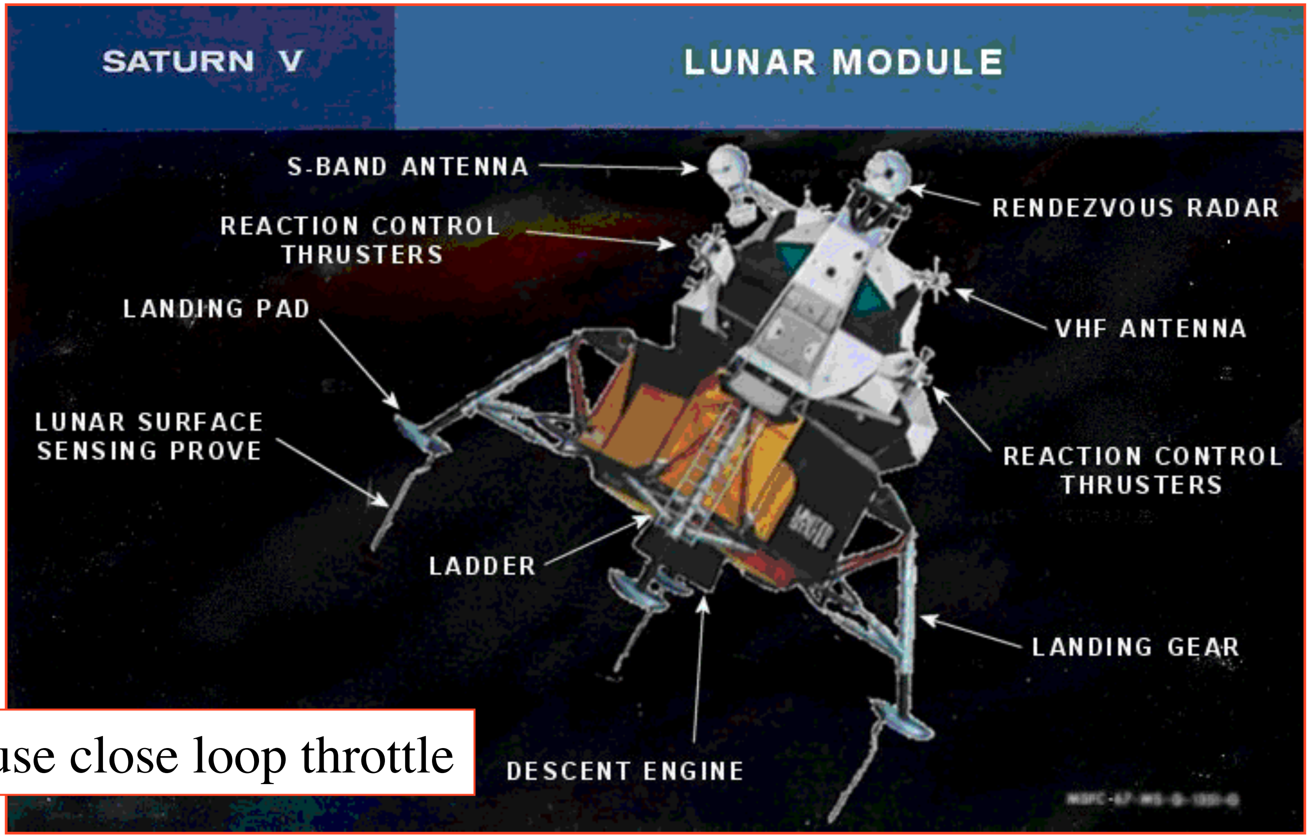
Injector pressure ratio



Applications of Deep Throttling (I)

- Lunar Lander … Throttling key element in ability to perform precision landing … Apollo --> Open loop throttle under control of mission commander





New LM will likely use close loop throttle

Homework 5, Assigned *Monday March 15*, Due Monday *Wednesday March 24* 8 pts Total

1) 1 points, Propellant Performance
2) 2 points, Combustor Cooling
3) 1 points, Combustor Geometry
4) 2 points, Combustor Pressure, Injector Design
5) 2 points, Optimal Performance

1) Propellant Performance (1 Point)

- A Bi-Propellant Rocket Engine Burns LOX/LH_2 … with an O/F ratio of 5.33333

… compute the Ideal characteristic velocity, C* and ideal specific impulse (infinite nozzle)

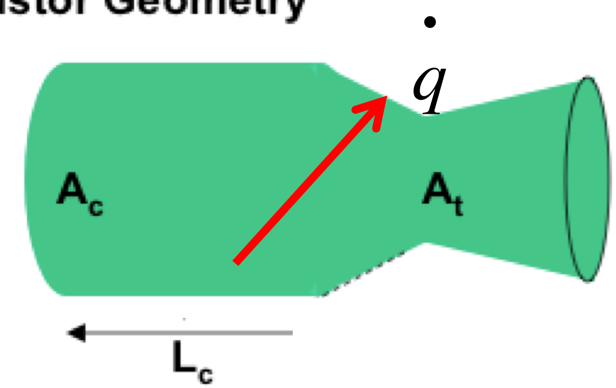
2) Non-Ideal Combustion Temperature (~~1~~ 2 point)

- For the same Rocket in (1) and (2) …Assume that there is regenerative cooling in the combustor …..

When operating in steady state mode
if the regenerative cooling system removes

700 kWatts / kg/sec of propellant mass flow
entering the combustor

Combustor Geometry



Combustor

Calculate the actual stagnation temperature at the throat of the combustor and the efficiency of the combustion-->

$$\eta^* = \frac{1}{C^*_{ideal}} \cdot \left(\frac{P_0 \cdot A^*}{\dot{m}} \right) = \frac{C^*_{actual}}{C^*_{ideal}}$$

Assume that heat transfer has no effect on combustion products and that Flow is “frozen” with species mix defined by O/F = 5.333

3) Combustor Geometry (1 point)

- For the same Rocket in (1) and (2) …

Ranges of Combustor Characteristic Length

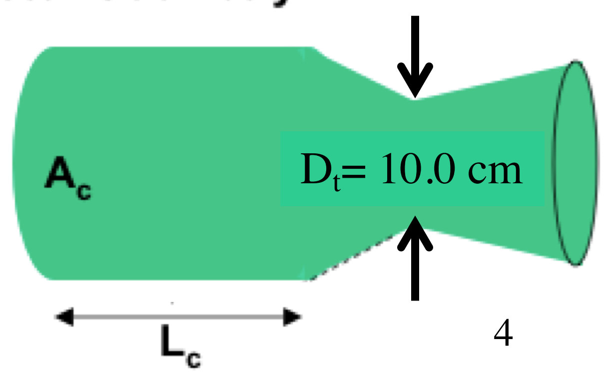
Propellants	Characteristic Length L^*	
	Low(m)	High(m)
Liquid fluorine / hydrazine	0.61	0.71
Liquid fluorine / gaseous H_2	0.56	0.66
Liquid fluorine / liquid H_2	0.64	0.76
Nitric acid/hydrazine	0.76	0.89
N_2O_4 / hydrazine	0.60	0.89
Liquid O_2/ ammonia	0.76	1.02
Liquid O_2 / gaseous H_2	0.56	0.71
Liquid O_2 / liquid H_2	0.76	1.02
Liquid O_2 / RP-1	1.02	1.27
H_2O_2 / RP-1 (incl. catalyst)	1.52	1.78

- Given the following Characteristic length Values,

Calculate a mid range value for The combustor Volume based on

$$L^*_{mean} = \frac{L^*_{high} + L^*_{low}}{2}$$

Combustor Geometry



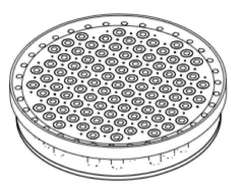
4) Steady Combustor Pressure (2 points)

- For the same Rocket in (1) and (2), and the combustor L^* geometry from (3) …Use the true stagnation temperature calculated in (2), and assume that the combustion products are frozen as in (1) … for the following injector properties .. ***Calculate the steady state combustor pressure (assume incompressible propellants)***
- ***What LOX Injector feed pressure is require to give the O/F ratio calculated in Part 1?***
- **LOX Injector**

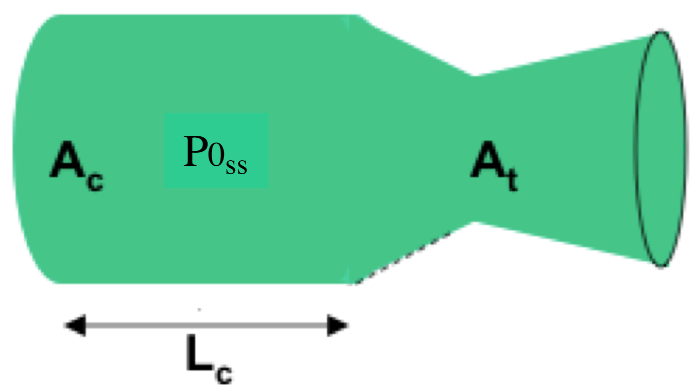
  Port diameter:0.1841 cm
  100 injector ports
  LOX density: 1.140 g/cm^3
  Cd (discharge coeff.): 0.75

- **LH_2 Injector**

  Port diameter:0.1596 cm
  100 injector ports
  LH_2 density: 71 g/cm^3
  Injector pressure 3700 kPa
  Cd (discharge coeff.): 0.75



Combustor



Base calculations on actual combustor temperature

5) Design Specific impulse (2 points)

- Given the results of problem (1), (2), (3) and (4)

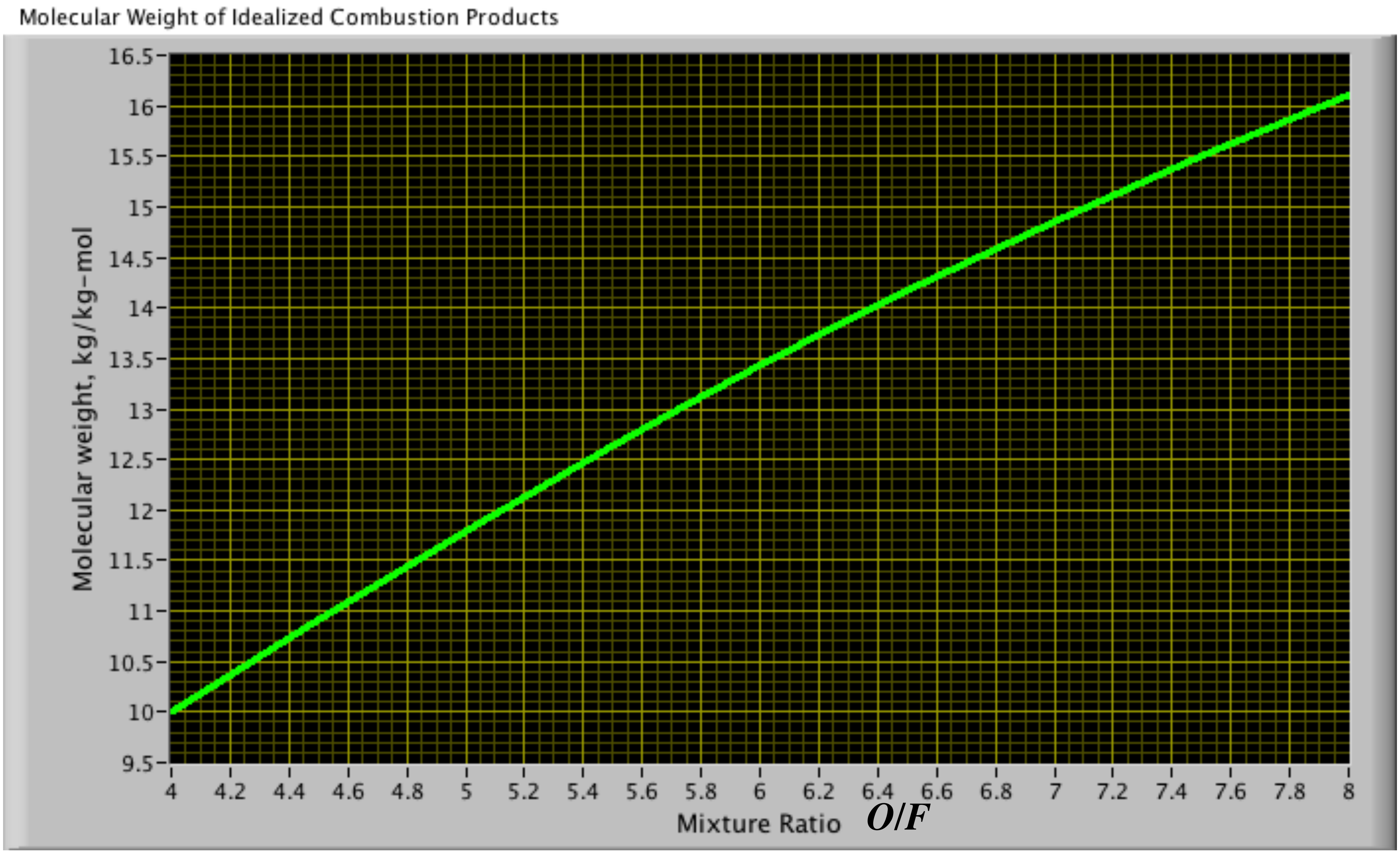
calculate the Optimal (design) Specific Impulse when the nozzle has A design altitude of 20 km (p_∞ = 5.4748 kPa)

…. Assume nozzle exit divergence angle is zero, *be sure to include effects of combustor heat loss*

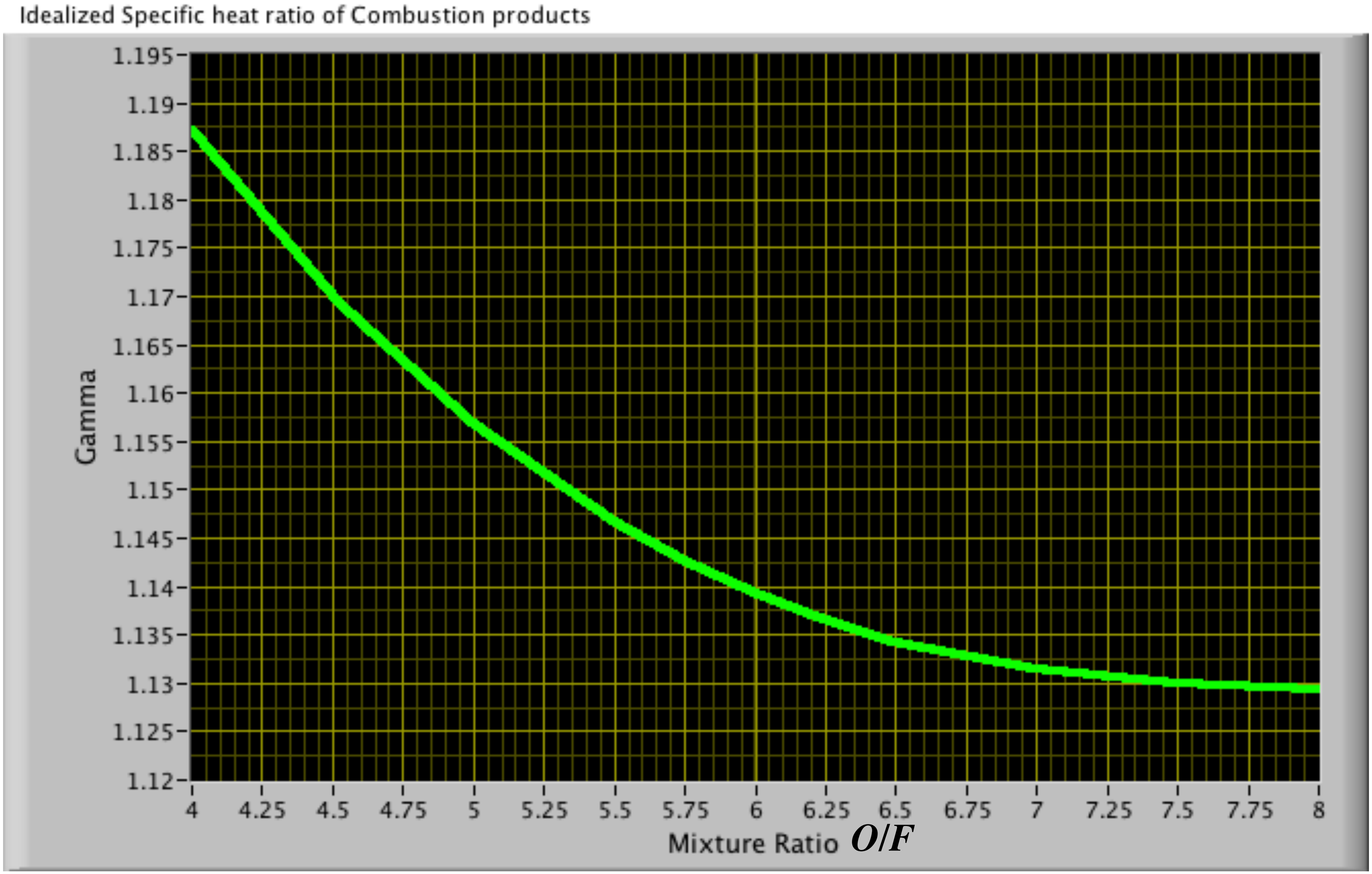
Calculate the expansion ratio of this nozzle

What Thrust and Isp Penalty would you pay if you used a conical nozzle of same expansion ratio; but with an exit angle of 20 degrees

Propellant Combustion Properties



Propellant Combustion Properties



Propellant Combustion Properties