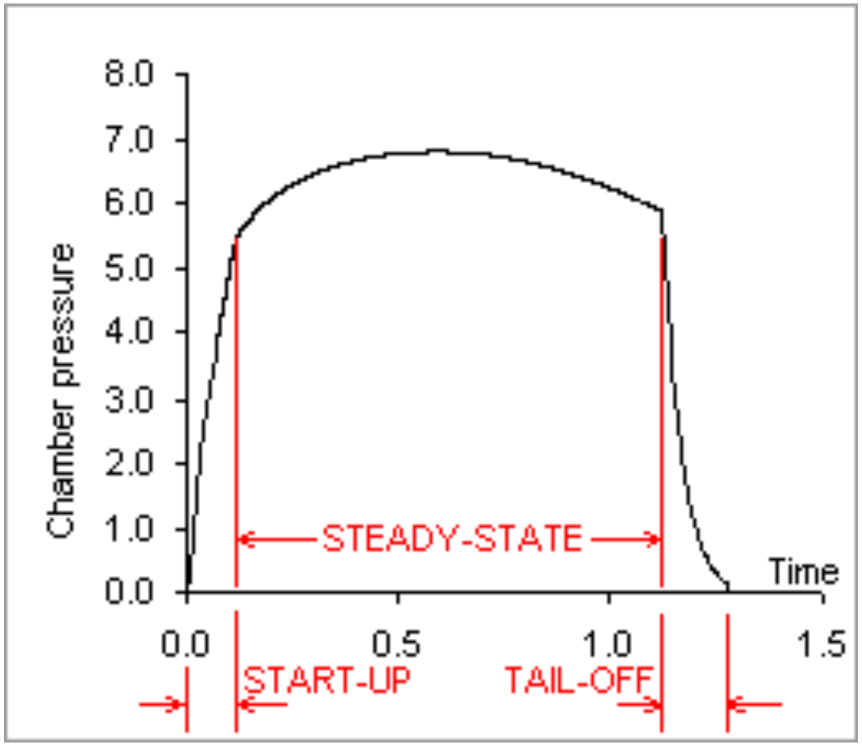
Modeling Transient Rocket Operation (Lecture 7.2: Solid Rockets)



- .. The primary goal of man is survival … food, shelter … basic necessities …

- ***A second aim of man is to build things that run very HOT and very LOUD and move really, really FAST ...***

- Sutton and Biblarz: Chapter 11
- Richard Nakka Web Page:

** http://members.aol.com/ricnakk/th_pres.html*

Transient Pressure Model

- Combustion Produces High temperature gaseous By-products
- Gases Escape Through Nozzle Throat
- Nozzle Throat Chokes (maximum mass flow)
- Since Gases cannot escape as fast as they are produced
  … Pressure builds up
- As Pressure Builds .. Choking mass flow grows
- Eventually Steady State Condition is reached

Choking Massflow per Unit Area

- maximum Massflow/area Occurs when When M=1
- Effect known as *Choking* in a Duct or Nozzle
- i.e. nozzle will Have a mach 1 throat

$$\left(\frac{\dot{m}}{A_c}\frac{\sqrt{T_0}}{p_0}\sqrt{R_g}\right)_{max} = \left(\frac{\dot{m}}{A^*}\frac{\sqrt{T_0}}{p_0}\sqrt{R_g}\right) =$$

$$\frac{\sqrt{\gamma}}{\left[1+\frac{(\gamma-1)}{2}\right]^{\frac{\gamma+1}{2(\gamma-1)}}} = \sqrt{\gamma\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}} \rightarrow$$

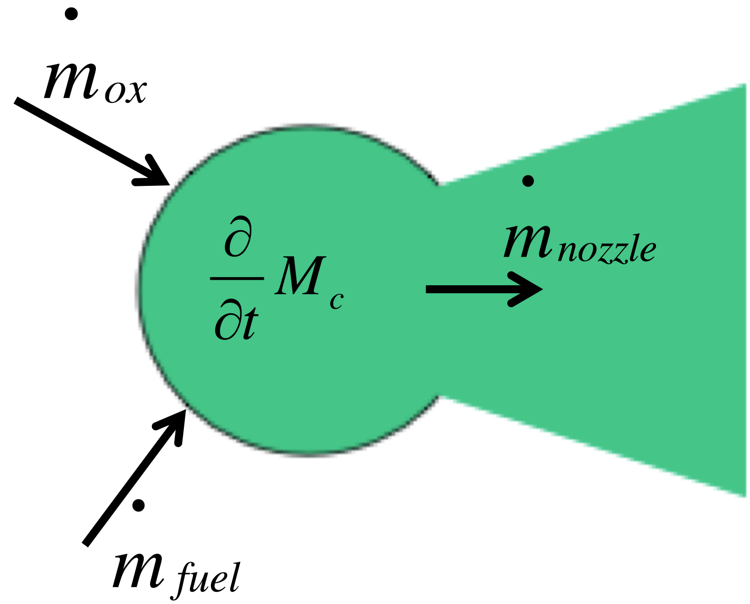
$$\frac{\dot{m}}{A^*} = \sqrt{\frac{\gamma}{R_g}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}}\frac{p_0}{\sqrt{T_0}}$$

Chamber Pressure Model

- Gaseous Mass Trapped in Chamber

$$\frac{\partial}{\partial t} M_c = \left[\dot{m}_{fuel} + \dot{m}_{ox} \right] - \dot{m}_{nozzle}$$

$$\frac{\partial}{\partial t} M_c = \frac{\partial}{\partial t}\left[r_c V_c \right] = \frac{\partial}{\partial t}\left[r_c \right] V_c + r_c \frac{\partial}{\partial t}\left[V_c \right]$$



- Assuming nozzle chokes immediately

$$\frac{\partial}{\partial t}\left[\rho_c \right] V_c + \rho_c \frac{\partial}{\partial t}\left[V_c \right] = \left[\dot{m}_{fuel} + \dot{m}_{ox} \right] - A^* \sqrt{\frac{\gamma}{R_g} \left(\frac{2}{\gamma + 1} \right)^{\frac{\gamma+1}{(\gamma-1)}}} \frac{P_0}{\sqrt{T_0}}$$

Chamber Pressure Model (cont'd)

- Using ideal gas law, *Assuming constant flame temperature*

$$\rho_c = \frac{P_0}{R_g T_0} \rightarrow \frac{\partial}{\partial t}\left[\rho_c\right] \approx \frac{1}{R_g T_0}\frac{\partial}{\partial t}\left[P_0\right]$$

- Subbing into mass flow equation

$$\frac{\partial P_0}{\partial t}\frac{V_c}{R_g T_0} + \frac{P_0}{R_g T_0}\frac{\partial V_c}{\partial t} = \left[\dot{m}_{fuel} + \dot{m}_{ox}\right] - \frac{R_g T_0}{V_c} A^* \sqrt{\frac{\gamma}{R_g}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}}\frac{P_0}{\sqrt{T_0}}$$

$$\frac{\partial P_0}{\partial t} + P_0 \frac{1}{V_c}\frac{\partial V_c}{\partial t} + \frac{R_g T_0}{V_c} A^* \sqrt{\frac{\gamma}{R_g}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}}\frac{P_0}{\sqrt{T_0}} = \frac{R_g T_0}{V_c}\left[\dot{m}_{fuel} + \dot{m}_{ox}\right]$$

$$\frac{\partial P_0}{\partial t} + P_0\left[\frac{1}{V_c}\frac{\partial V_c}{\partial t} + \frac{A^*}{V_c}\sqrt{\gamma R_g T\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}}\right] = \frac{R_g T_0}{V_c}\left[\dot{m}_{fuel} + \dot{m}_{ox}\right]$$

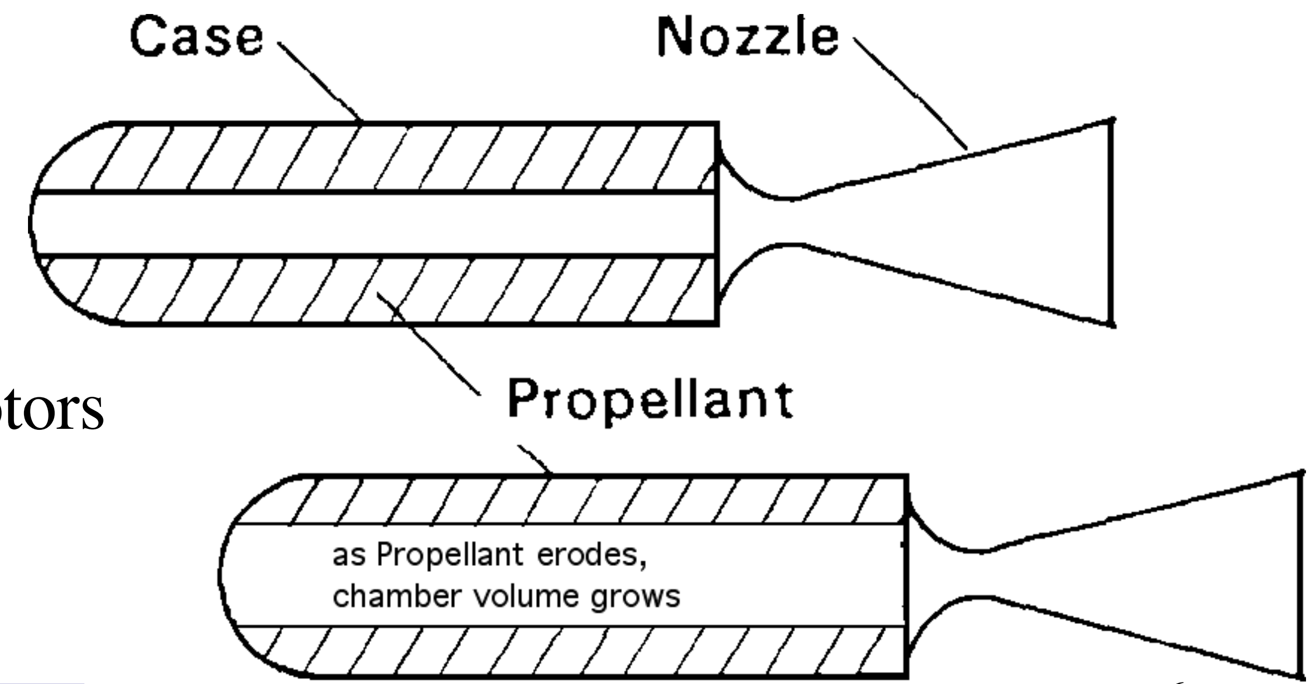
Transient Operation Model For Solid Rockets

- Revisit General Model

$$\frac{\partial P_0}{\partial t} + P_0\left[\frac{1}{V_c}\frac{\partial V_c}{\partial t} + \frac{A^*}{V_c}\sqrt{\gamma R_g T\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}}\right] = \frac{R_g T_0}{V_c}\left[\dot{m}_{propellant}\right]$$

- For Solid Rocket Motors

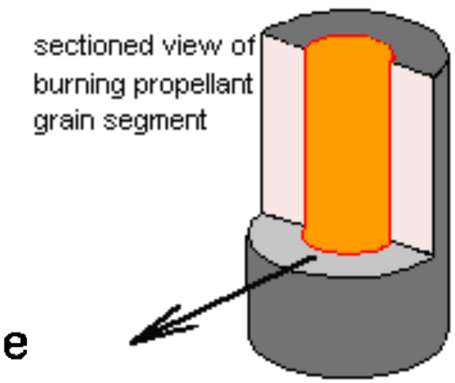
$$\frac{\partial V_c}{\partial t} \neq 0$$



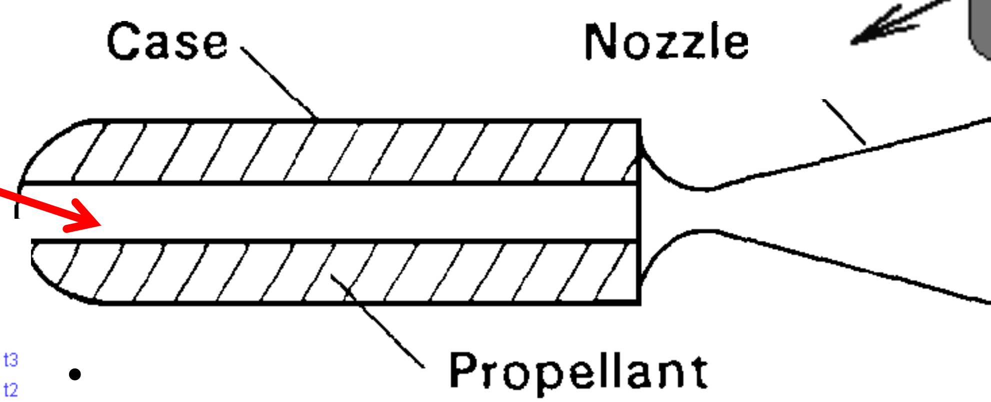
Solid Rocket Example

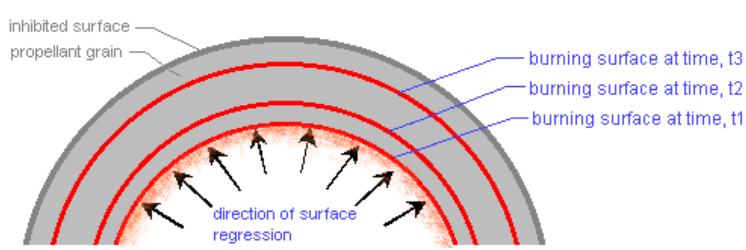
$$\frac{\partial P_0}{\partial t} + P_0\left[\frac{1}{V_c}\frac{\partial V_c}{\partial t} + \frac{A^*}{V_c}\sqrt{\gamma R_g T\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}}\right] = \frac{R_g T_0}{V_c}\left[\dot{m}_{propellant}\right]$$

$$\frac{\partial V_c}{\partial t} = A_{burn}\,\dot{r} \rightarrow \left[\begin{array}{l} A_{burn} = \text{Grain Surface Burn Area} \\ \dot{r} = \text{Grain Linear Regression Rate} \end{array}\right]$$

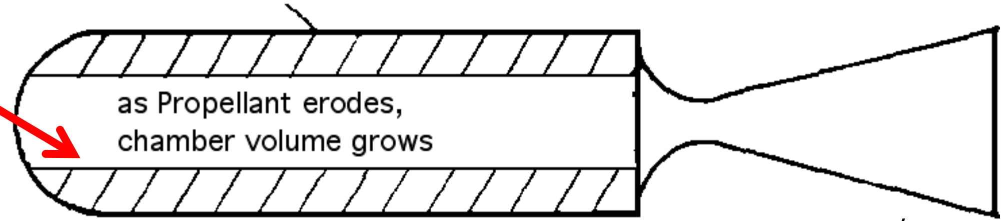


A_{burn}





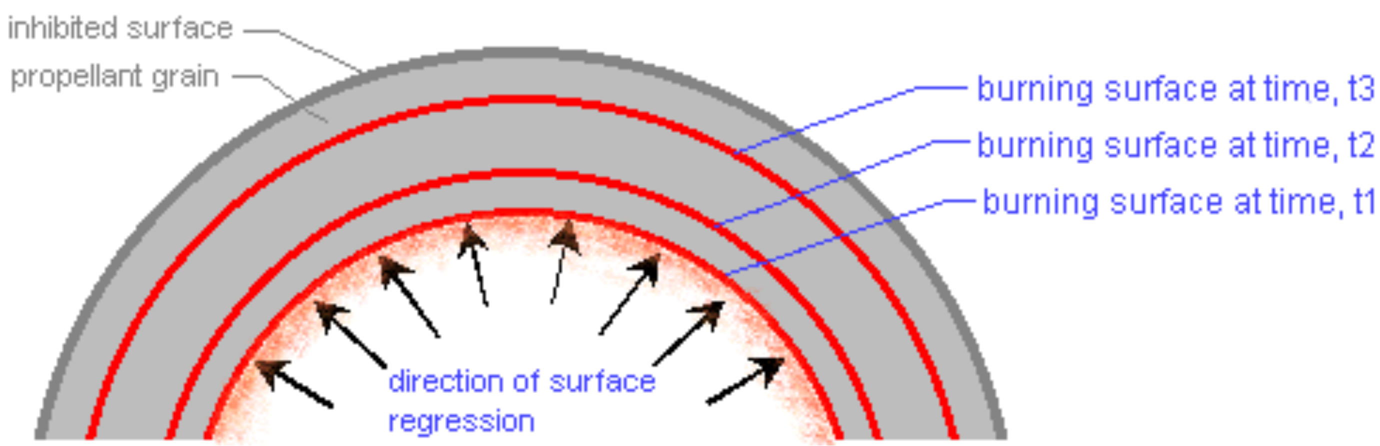
$\dot{r}$



Solid Rocket Example (cont'd)

$$\frac{\partial V_c}{\partial t} = A_{burn}\,\dot{r} \rightarrow \left[\begin{array}{l} A_{burn} = \text{Grain Surface Burn Area} \\ \dot{r} = \text{Grain Linear Regression Rate} \end{array} \right]$$

$$\left[\dot{m}_{propellant} \right] = \rho_p \cdot A_{burn} \cdot \dot{r}$$



EFFECT OF PRESSURE ON BURN RATE - Saint-Robert's Law

$$\dot{r} = ap^n$$

r = linear burning rate

a = an empirical constant moderately influenced by propellant grain temperature

n = burning rate pressure exponent

$$\dot{r} = aP_o^{\,n} \rightarrow \{a,n\} \rightarrow \text{empirically derived constants}$$

Solid Rocket Example (cont'd)

- Propellant burn rate may be expressed in terms of the chamber pressure by the Saint Robert's law …

$$\dot{r} = aP_o^{\,n} \rightarrow \{a,n\} \rightarrow \text{empirically derived constants}$$

$$\frac{\partial P_0}{\partial t} = \frac{A_{burn} a P_o^{\,n}}{V_c}\left[\rho_p R_g T_0 - P_0\right] - P_0\left[\frac{A^*}{V_c}\sqrt{\gamma R_g T_0\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}}\right]$$

- careful with units on a …

$$a \sim \frac{m}{\sec}\left(\frac{1}{kPa}\right)^n$$

Solid Rocket Example (cont'd)

$$\log\left[\dot{r}\right] = n\log\left[P\right] + y_0$$

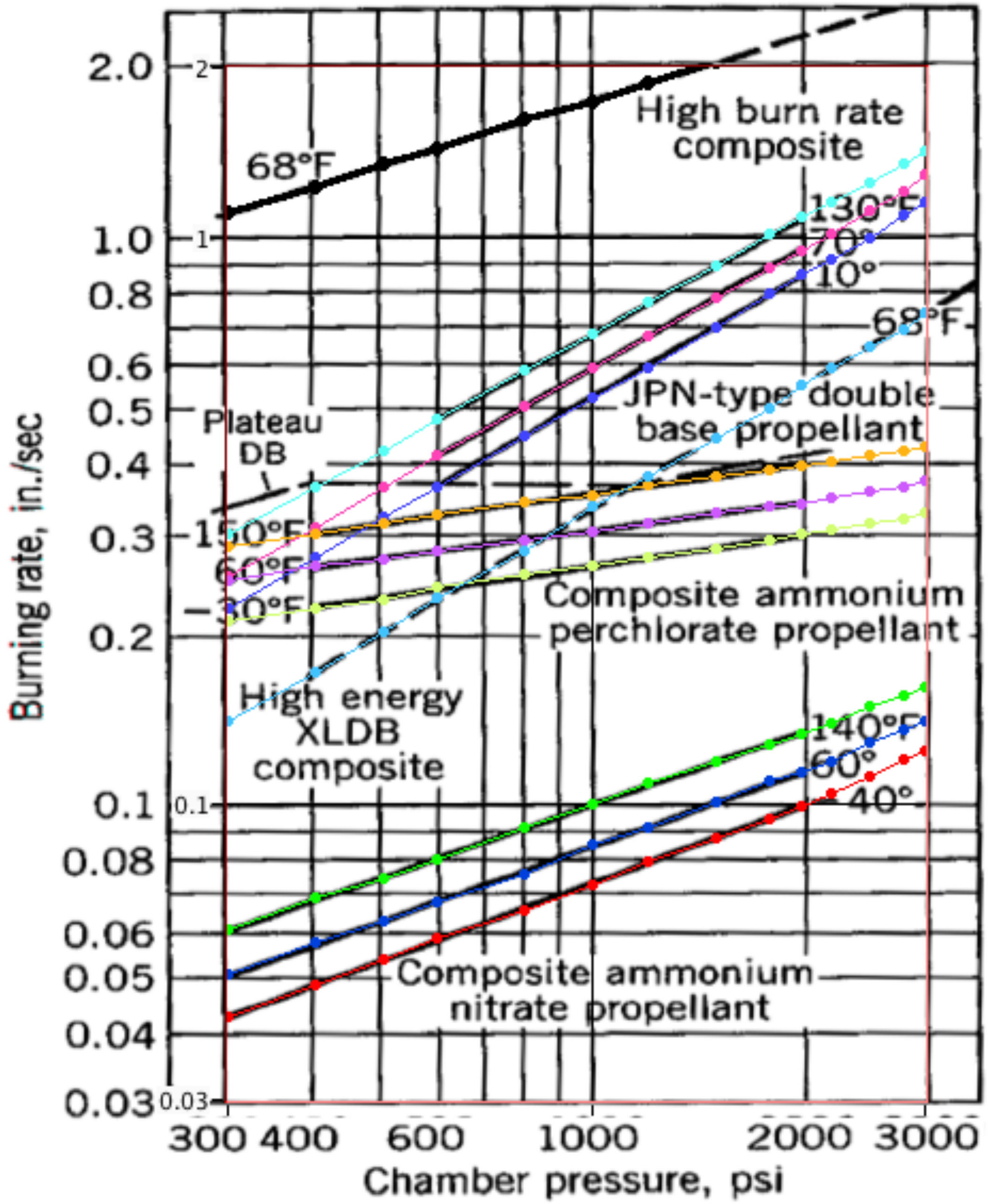
$$\dot{r} = e^{Log\left[P_o^{\,n}\right]+y_0} = e^{y_0} P_o^{\,n} \rightarrow e^{y_0} \equiv a$$

$\rightarrow$

$$\dot{r} = aP_o^{\,n}$$

- careful with units on a …

Sutton and Biblarz, Chapter 11



Solid Rocket Example (cont'd)

Solid Propellant Saint Robert's Curve Fit (P$_0$-psia, rdot- in/sec)

Propellant Name	*n*	*a (in/sec-psia^n)*
Composite Ammonium Nitrate, -40F	0.463474	0.002965
Composite Ammonium Nitrate, 60F	0.445084	0.003909
Composite Ammonium Nitrate, 140F	0.426803	0.005243
High Energy XLDB Composite	0.720473	0.002293
Composite Ammonium Perchlorate, -30F	0.187867	0.072001
Composite Ammonium Perchlorate, 60F	0.170286	0.094044
Composite Ammonium Perchlorate, 150F	0.172255	0.107348
JPN-type Double Base, 10F	0.712606	0.003818
JPN-type Double Base, 70F	0.701667	0.004624
JPN-type Double Base, 130F	0.678433	0.006260
High Burn Rate Composite @ 68F	0.380710	0.126409

$$\dot{r} = aP_o^{\,n}$$

- Input, Psia
- Output, in/sec

Solid Rocket Example (cont'd)

Propellant, Saint Robert's Curve Fit (P0-kPa, rdot- cm/sec)

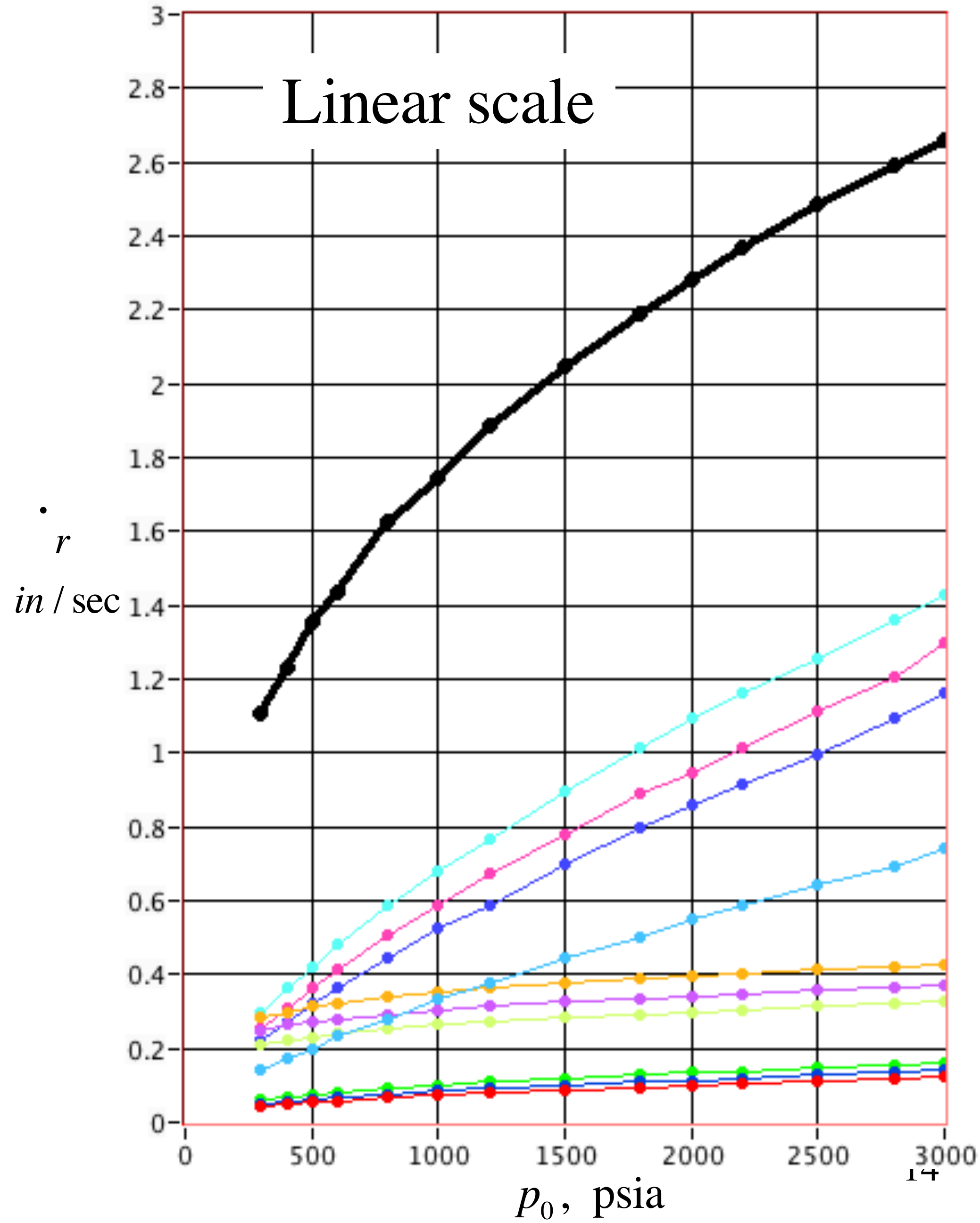
propellant name	*n*	*a (cm/sec-kPa^n)*
Composite Ammonium Nitrate, -40F	0.463474	0.003077
Composite Ammonium Nitrate, 60F	0.445084	0.004204
Composite Ammonium Nitrate, 140F	0.426803	0.005841
High Energy XLDB Composite	0.720473	0.001449
Composite Ammonium Perchlorate, -30F	0.187867	0.127245
Composite Ammonium Perchlorate, 60F	0.170286	0.171940
Composite Ammonium Perchlorate, 150F	0.172255	0.195519
JPN-type Double Base, 10F	0.712606	0.002450
JPN-type Double Base, 70F	0.701667	0.003030
JPN-type Double Base, 130F	0.678433	0.004291
High Burn Rate Composite @ 68F	0.380710	0.153949

$$\dot{r} = aP_o^{\,n}$$

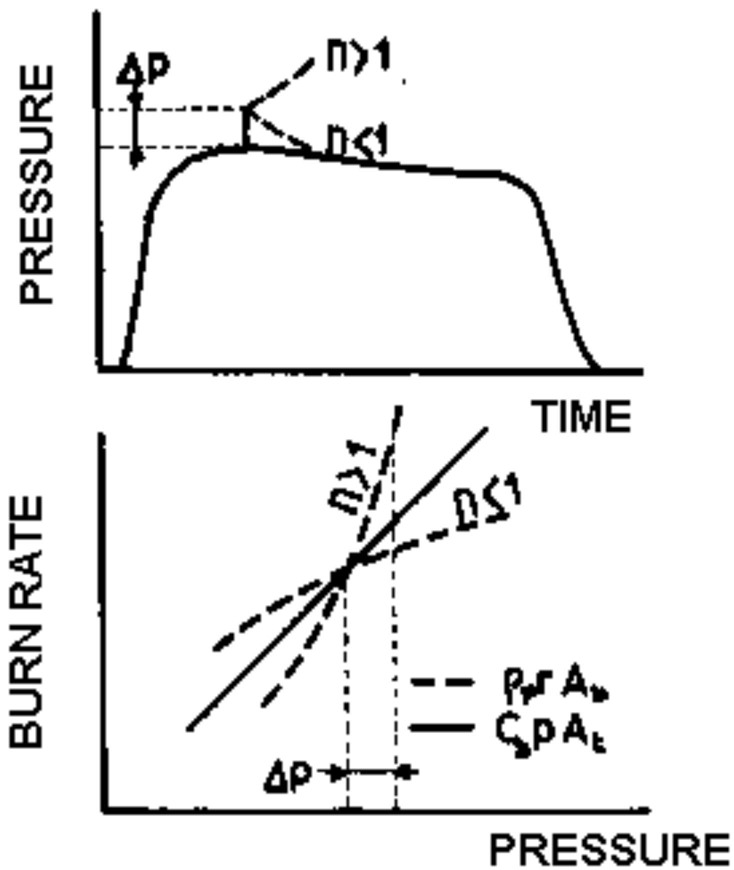
- Input, kPa
- Output, cm/sec

Solid Rocket Example (cont'd)

- *Propellant Burn rate is extremely sensitive to exponent, n*
- *Stable operation requires 0.001 < n < 0.990*
- *High values of n make for a propellant whose burn rate is sensitive to chamber pressure*



Exponent Effect on Burn Rate (Pressure Excursion)



Source: Barrere et al., Raketenantriebe, Fig 5.1 (1961)

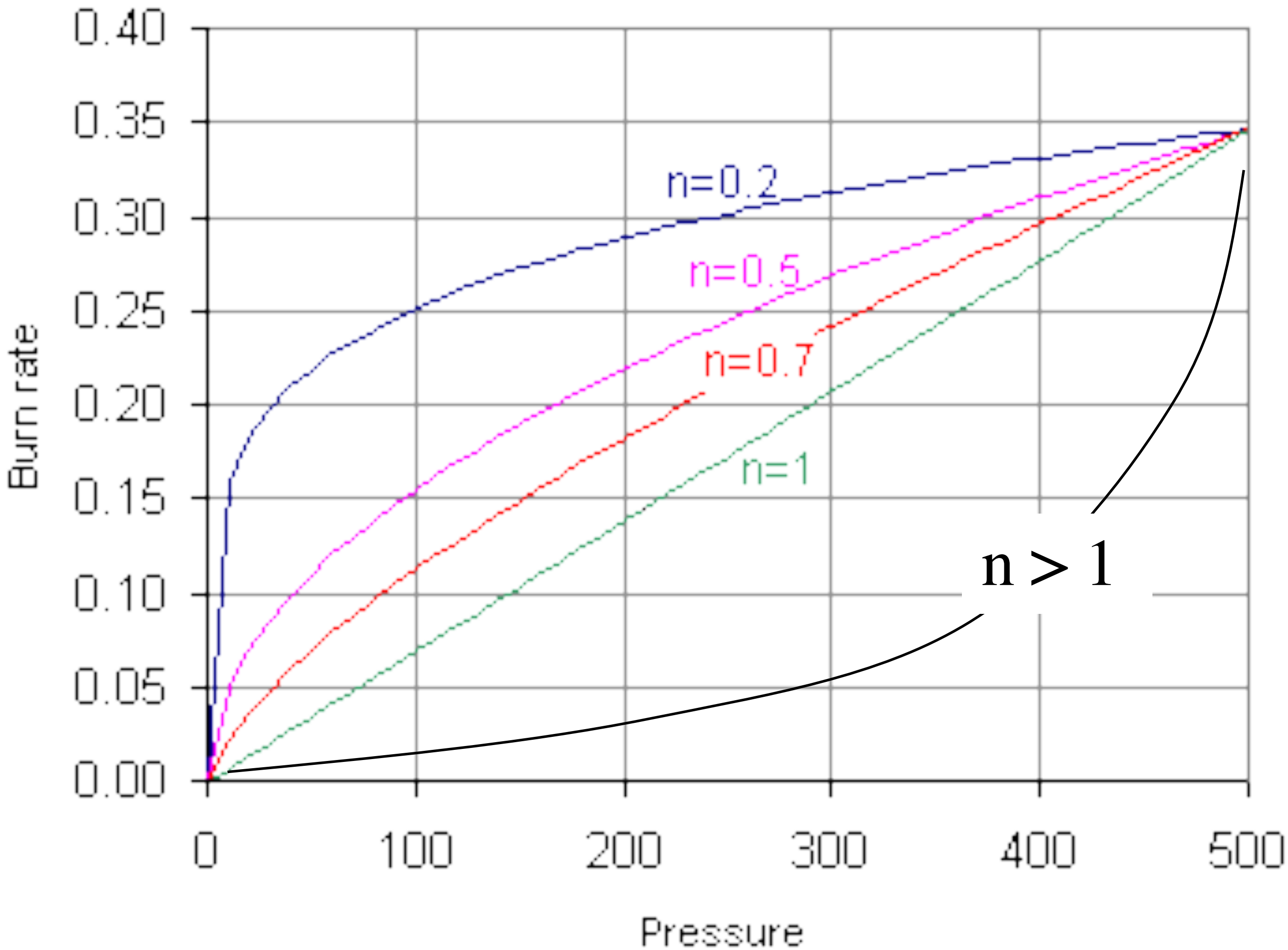
Effect of Burn Exponent

n>1 : Slight positive pressure excursion might lead to explosion of the chamber.

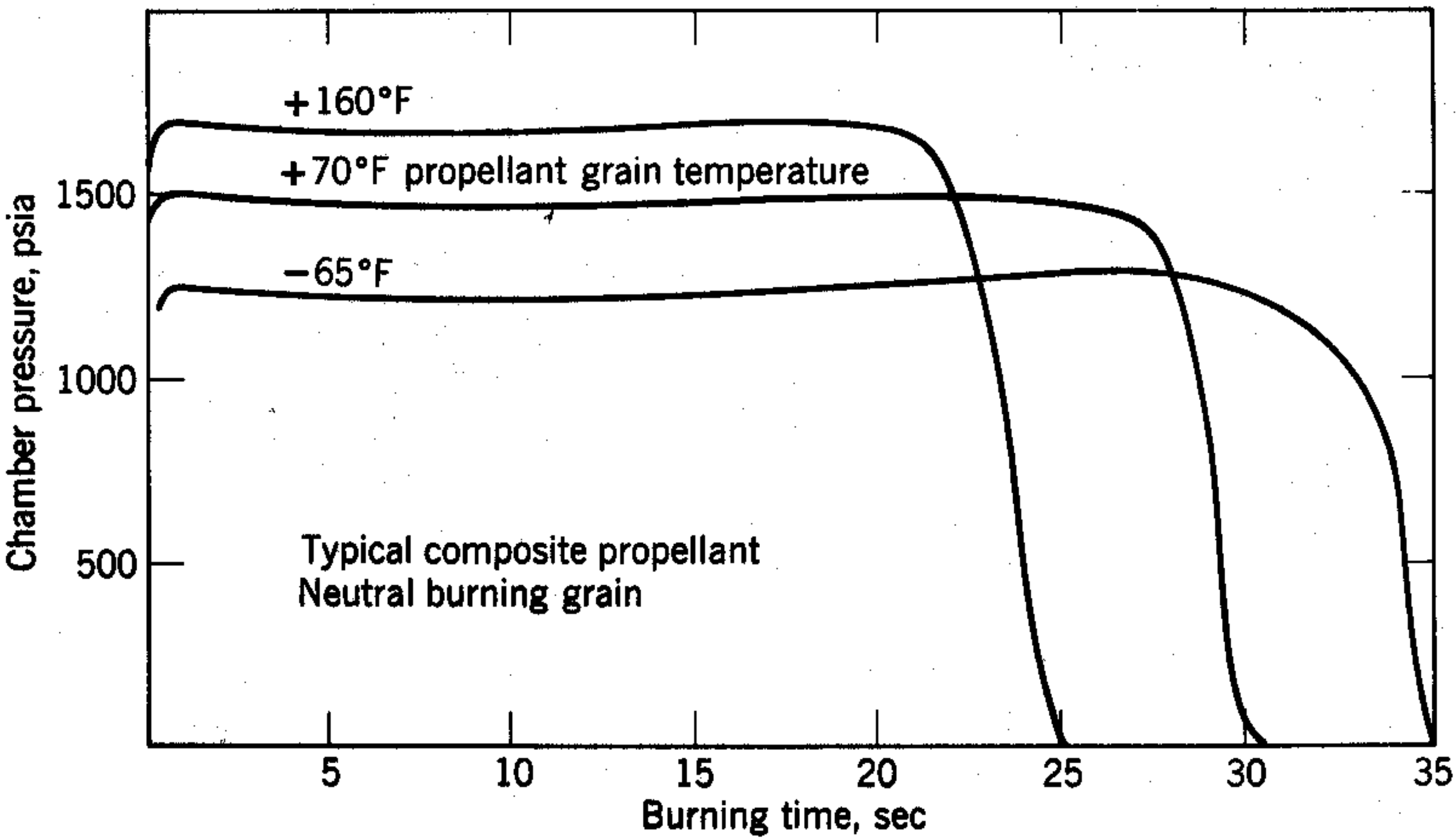
n ~ < 0.8: Maximum pressure exponent tolerated with typical solid rocket propellants.

n < 0: Slight negative pressure excursion might lead to continuing decay of chamber pressure and premature extinguishment of propellant.

Effect of Burn Exponent (2)



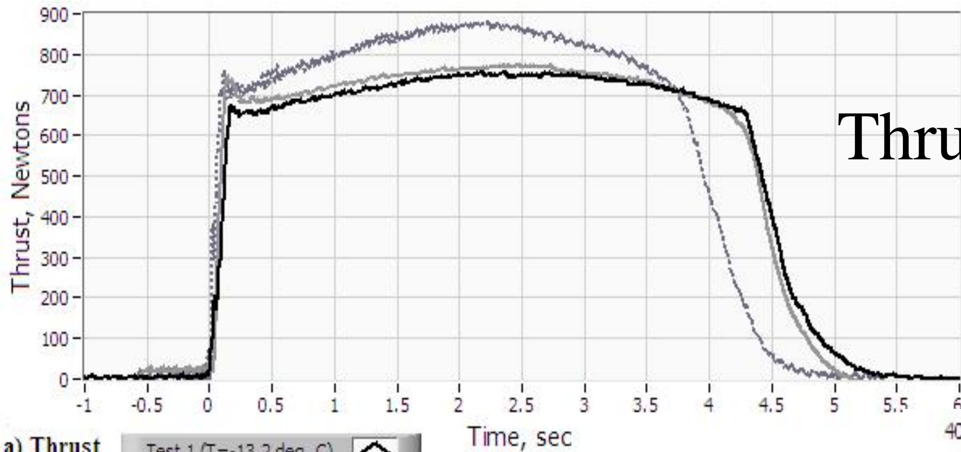
Grain Temperature Effect on Burn Rate



Sutton, & Biblarz p. 268 (1986)

Example from (2009) USLI Design Team Motor Tests (2)

Motor Burn Profiles

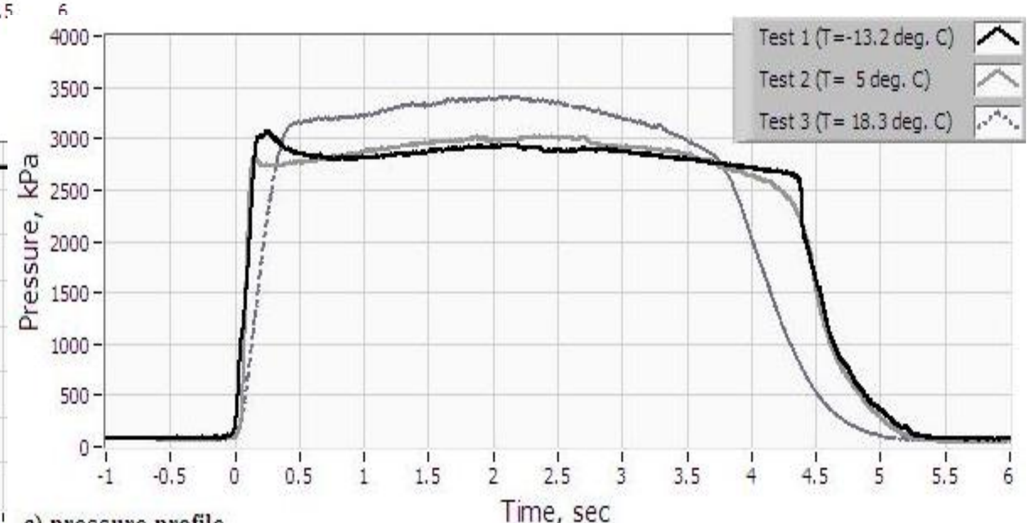


a) Thrust

Thrust

b) Impulse

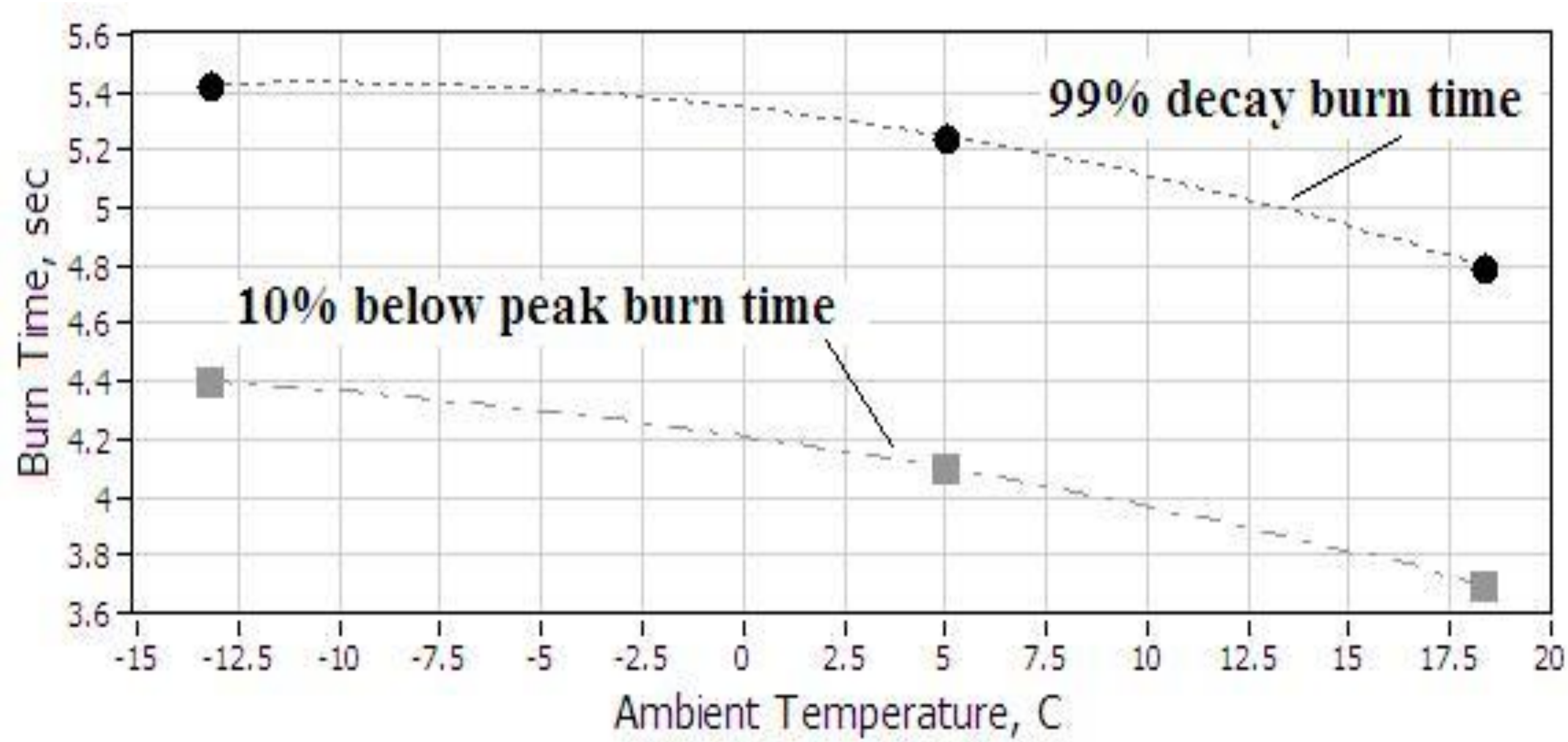
Impulse



c) pressure profile

Chamber pressure

Example from (2009) USLI Design Team Motor Tests (3)



Measures of Burn Time

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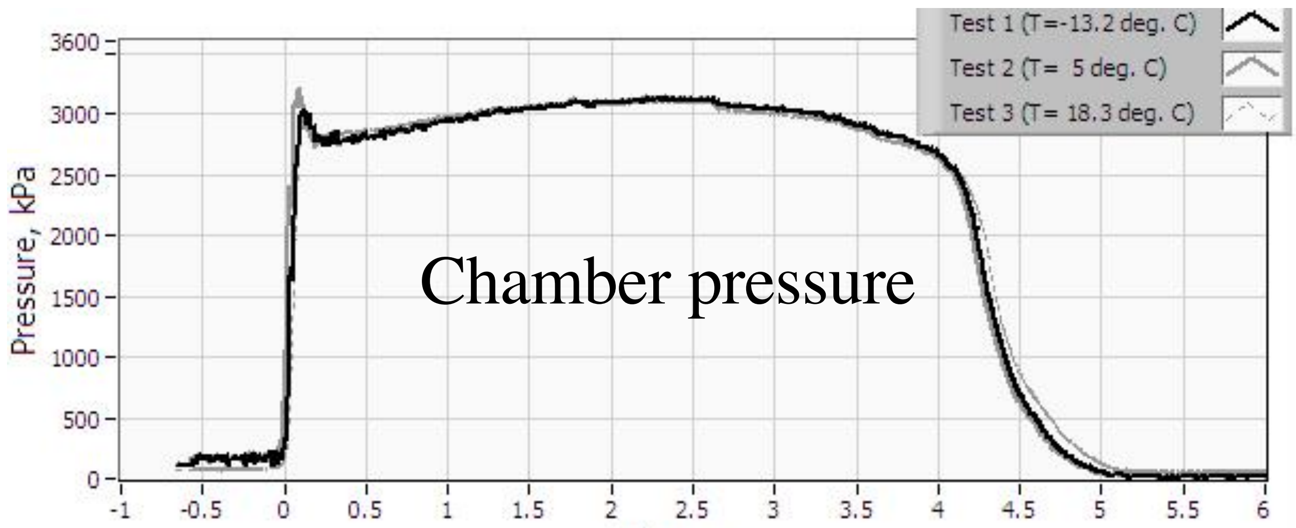
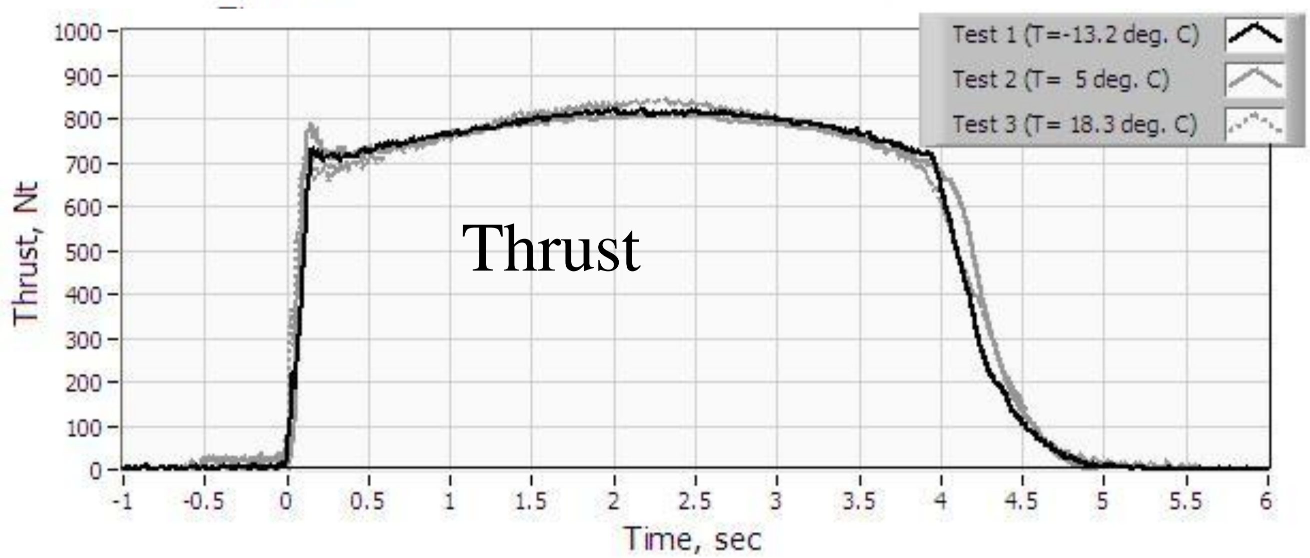
Mechanical & Aerospace Engineering

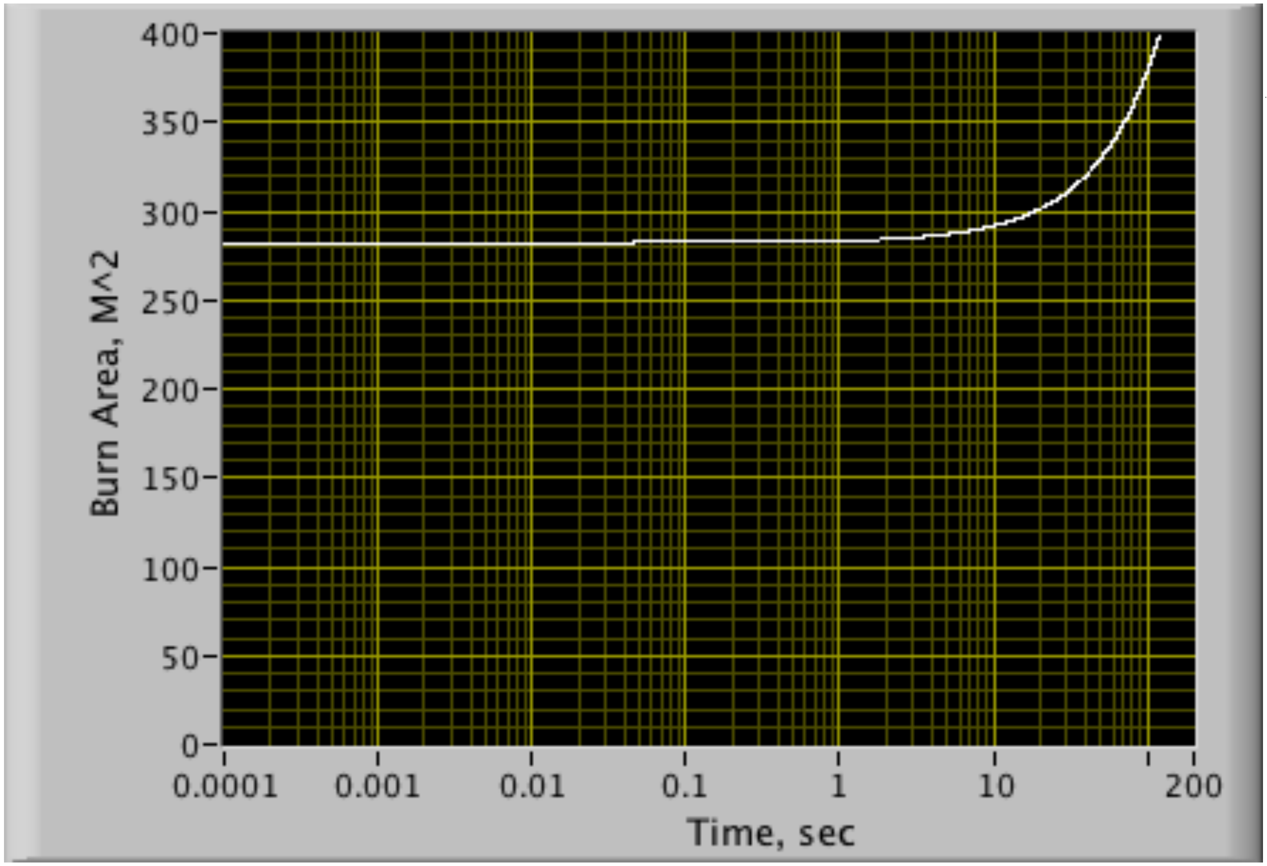
Example from (2009) USLI Design Team Motor Tests (4)

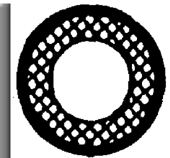
Temperature Adjusted Burn Profiles

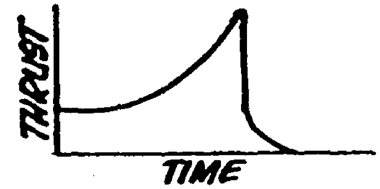




Burn Rate Revisited







- Show that for Uniform Cylindrical Propellant grain necessarily leads to Progressive burn

$$\frac{\partial P_0}{\partial t} = \frac{A_{burn} a P_o^{\ n}}{V_c}\left[\rho_p R_g T_0 - P_0\right] - P_0\left[\frac{A^*}{V_c}\sqrt{\gamma R_g T_0\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}}\right]$$

Burn Rate Revisited (2)

$$\frac{\partial P_0}{\partial t} = \frac{A_{burn} a P_o^{\,n}}{V_c}\left[\rho_p R_g T_0 - P_0\right] - P_0\left[\frac{A^*}{V_c}\sqrt{\gamma R_g T_0 \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}}\right] =$$

$$\rightarrow C^* = \left(\frac{P_0 A^*}{\dot{m}_{exit}}\right) = \frac{\sqrt{\gamma R_g T_0}}{\gamma\sqrt{\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}}}$$

$$\frac{\partial P_0}{\partial t} = R_g T_0\left(\frac{A_{burn} a P_o^{\,n}}{V_c}\left[\rho_p - \rho_0\right] - \left(\frac{P_0 A^*}{V_c C^*}\right)\right)$$

Burn Rate Revisited (3)

$$\frac{\partial P_0}{\partial t} = R_g T_0 \left(\frac{A_{burn} a P_o^{\ n}}{V_c} \left[\rho_p - \rho_0 \right] - \left(\frac{P_0 A^*}{V_c C^*} \right) \right)$$

$$\rho_p >> \rho_0 \rightarrow \frac{\partial P_0}{\partial t} \approx R_g T_0 \left(\frac{A_{burn} a P_o^{\ n}}{V_c} \rho_p - \left(\frac{P_0 A^*}{V_c C^*} \right) \right)$$

- Cylindrical Port … $\frac{A_{burn}}{V_{burn}} = \frac{2\pi \cdot r \cdot L}{\pi \cdot r^2 \cdot L} = \frac{2}{r}$

$$\frac{\partial P_0}{\partial t} \approx R_g T_0 \left(\frac{2 a P_o^{\ n}}{r} \rho_p - \left(\frac{P_0 A^*}{\pi \cdot r^2 \cdot L \cdot C^*} \right) \right)$$

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Burn Rate Revisited (4)

- Cylindrical Port … $\frac{A_{burn}}{V_{burn}} = \frac{2\pi \cdot r \cdot L}{\pi \cdot r^2 \cdot L} = \frac{2}{r}$

$$\frac{\partial P_0}{\partial t} \approx R_g T_0 \left(\frac{2aP_o^{\ n}}{r} \rho_p - \left(\frac{P_0 A^*}{\pi \cdot r^2 \cdot L \cdot C^*} \right) \right)$$

$$\longrightarrow \left(\frac{1}{\pi \cdot r^2} \right) \cdot \left(\frac{P_0 A^*}{L \cdot C^*} \right) > 0 \rightarrow \quad \rightarrow \frac{\partial P_0}{\partial t} > 2 \cdot \rho_p \cdot R_g \cdot T_0 \cdot \left(\frac{aP_o^{\ n}}{r} \right)$$

$$\longrightarrow \dot{r} = aP_o^{\ n} \rightarrow \frac{\partial P_0}{\partial t} > 2 \cdot \rho_p \cdot R_g \cdot T_0 \cdot \left(\frac{\dot{r}}{r} \right) = 2 \cdot \rho_p \cdot R_g \cdot T_0 \cdot \frac{\partial}{dt} \left(\ln(r) \right)$$

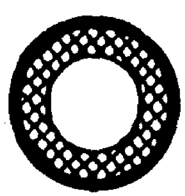
Burn Rate Revisited (5)

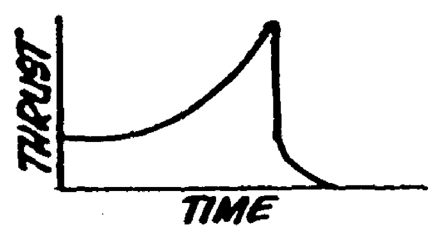
- Cylindrical Port … $\frac{A_{burn}}{V_{burn}} = \frac{2\pi \cdot r \cdot L}{\pi \cdot r^2 \cdot L} = \frac{2}{r}$

$$\dot{r} = aP_o^{\,n} \rightarrow \frac{\partial P_0}{\partial t} > 2 \cdot \rho_p \cdot R_g \cdot T_0 \cdot \left(\frac{\dot{r}}{r}\right) = 2 \cdot \rho_p \cdot R_g \cdot T_0 \cdot \frac{\partial}{dt}\left(\ln(r)\right)$$

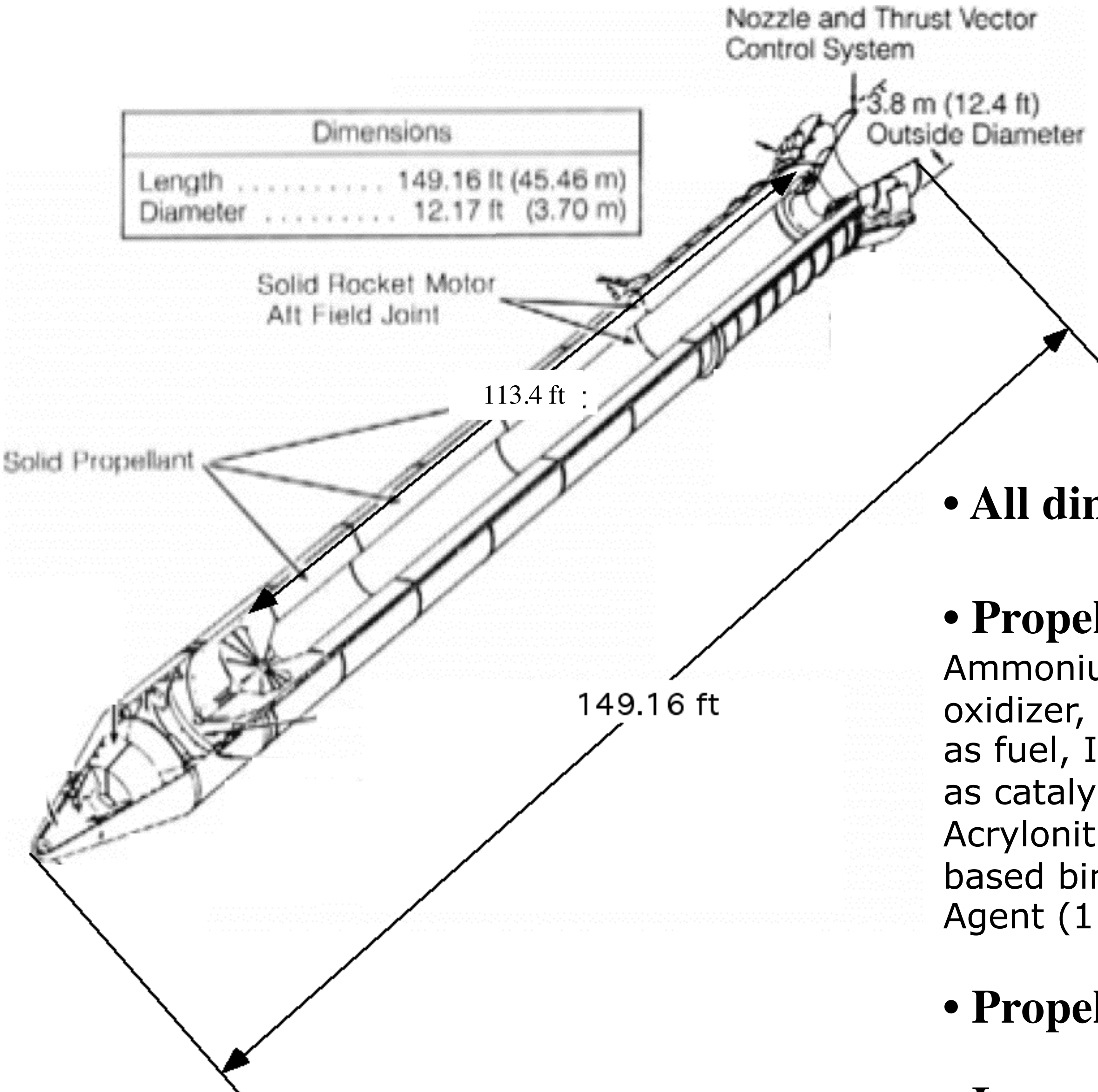
$$\frac{\partial P_0}{\partial t} > 2 \cdot \rho_p \cdot R_g \cdot T_0 \cdot \frac{\partial}{dt}\left(\ln(r)\right) \rightarrow \boxed{P_{0(t)} > P_{0(0)} + 2 \cdot \rho_p \cdot R_g \cdot T_0 \cdot \ln\left(\frac{r_{(t)}}{r_{(0)}}\right)}$$

For a Cylindrical Port Burn, Pressure Rises Logarithmically with time





Space Shuttle RSRM Numerical Example



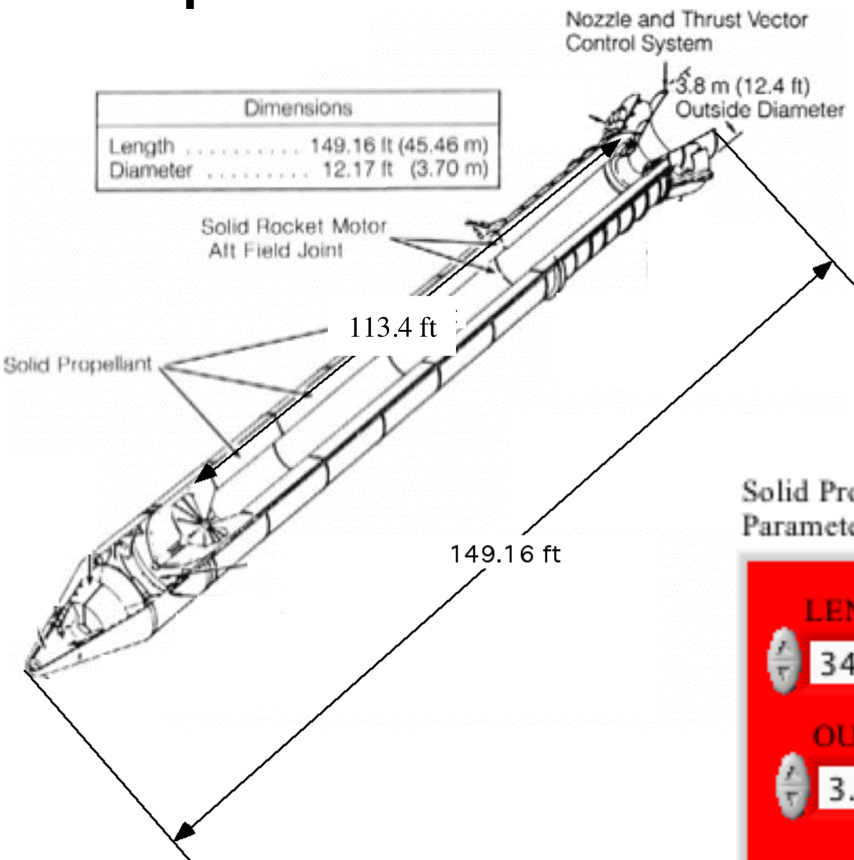
- **All dimensions are approximate**

- **Propellant:**
Ammonium Perchlorate (69.6%) as oxidizer, Aluminum Powder (16%) as fuel, Iron Oxidizer Powder (0.4%) as catalyst, Polybutadiene Acrylic Acid Acrylonitrile (12.04%) **(PBAN)** as rubber-based binder, Epoxy Curing Agent (1.96%)

- **Propellant Density .. 1760 kg/m^3**

- **Launch Propellant mass … 502,0 kg**

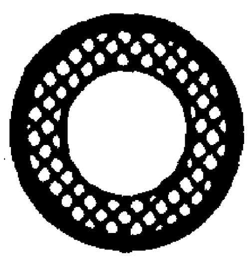
Space Shuttle RSRM Numerical Example (cont'd)



- D_{exit} = 3.8 m ---> A_{exit} = 11.341 m^2
- A_{exit}/A^* = 7.78
  --> A^*=1.458 m^2

- **Propellant Density .. 1760 kg/m^3**
- **Launch Propellant mass … 502,000 kg**

Internal Grain Pattern



Solid Propellant Grain Parameters

LENGTH, M
34.57

OUTSIDE DIAMETER, M
3.66

INSIDE DIAMETER, M
1.7

PROPELLANT DENSITY KG/M^3
1760

Solid Propellant Grain Parameters 2

Propellant thickness,M
1.96

PROPELLANT Volume M^3
285.24

PROPELLANT MASS, KG
502022

PROPELLANT MASS, Mton
502.022

Chamber Cross Section Area, M2
2.2698

Propellant Cross Section Area, M^2
8.25108

Chamber Volume, M^3
78.467

Propellant Burn area, M^2
184.628

Space Shuttle RSRM Numerical Example (cont'd)

Propellant, Saint Robert's Curve Fit

propellant name	*n*	*a (cm/sec-kPa^n)*
Composite Ammonium Perchlorate, 60F	0.172	0.192

$$\frac{\partial P_0}{\partial t} = \frac{A_{burn} a P_o^{\ n}}{V_c}\left[\rho_p R_g T_0 - P_0\right] - P_0\left[\frac{A^*}{V_c}\sqrt{\gamma R_g T_0\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}}\right]$$

$$\left[\begin{array}{c} A_{burn} = 2\pi R_{chamber} L_{prop} \\ V_c = \pi R_{chamber}^{\ \ 2} L_{prop} \end{array}\right] \rightarrow R_{chamber} = R_{i_{initial}} + \int_0^t \dot{r}\,dt = R_{i_{initial}} + \int_0^t a P_o^{\ n} dt$$

Space Shuttle RSRM Numerical Example (cont'd)

- Use Trapezoidal rule or Runge-Kutta to integrate

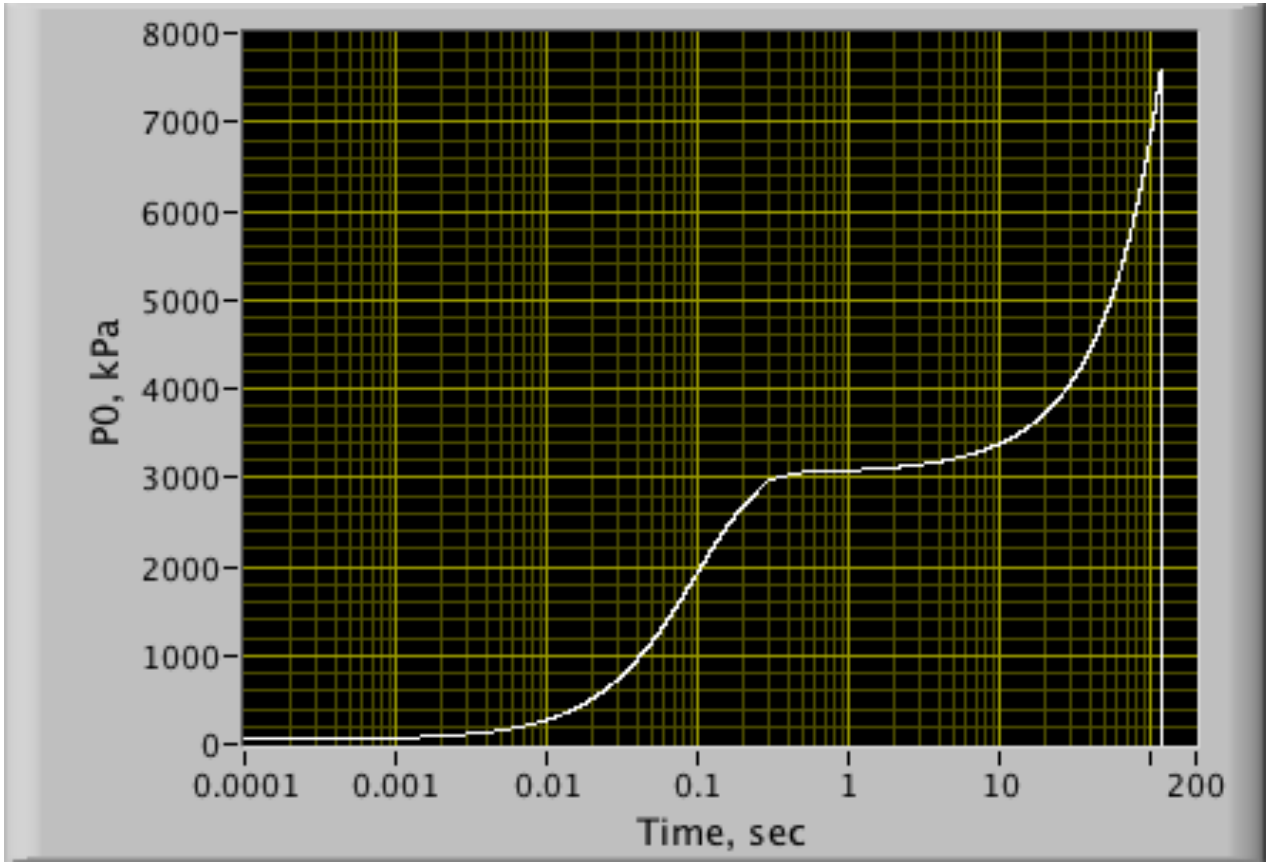
$$\frac{\partial P_0}{\partial t} = \frac{A_{burn} a P_o{}^n}{V_c}\left[\rho_p R_g T_0 - P_0\right] - P_0\left[\frac{A^*}{V_c}\sqrt{\gamma R_g T_0\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}}\right]$$

- Recursive propagation of chamber diameter

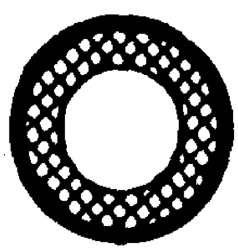
$$R_{burn_{k+1}} = R_{i_{initial}} + \int_0^{(k+1)\Delta t} \dot{r}dt = R_{i_{initial}} + \int_0^{(k)\Delta t} \dot{r}dt + \int_{(k)\Delta t}^{(k+1)\Delta t} \dot{r}dt \rightarrow$$

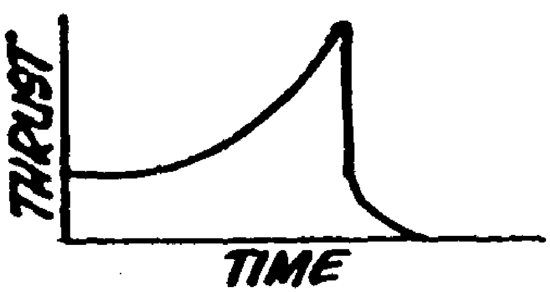
$$R_{burn_{k+1}} = R_{burn_k} + \int_{(k)\Delta t}^{(k+1)\Delta t} \dot{r}dt \approx R_{burn_k} + \dot{r}\,\Delta t = R_{burn_k} + aP_o{}^n{}_k\,\Delta t$$

RSRM Burn Time History



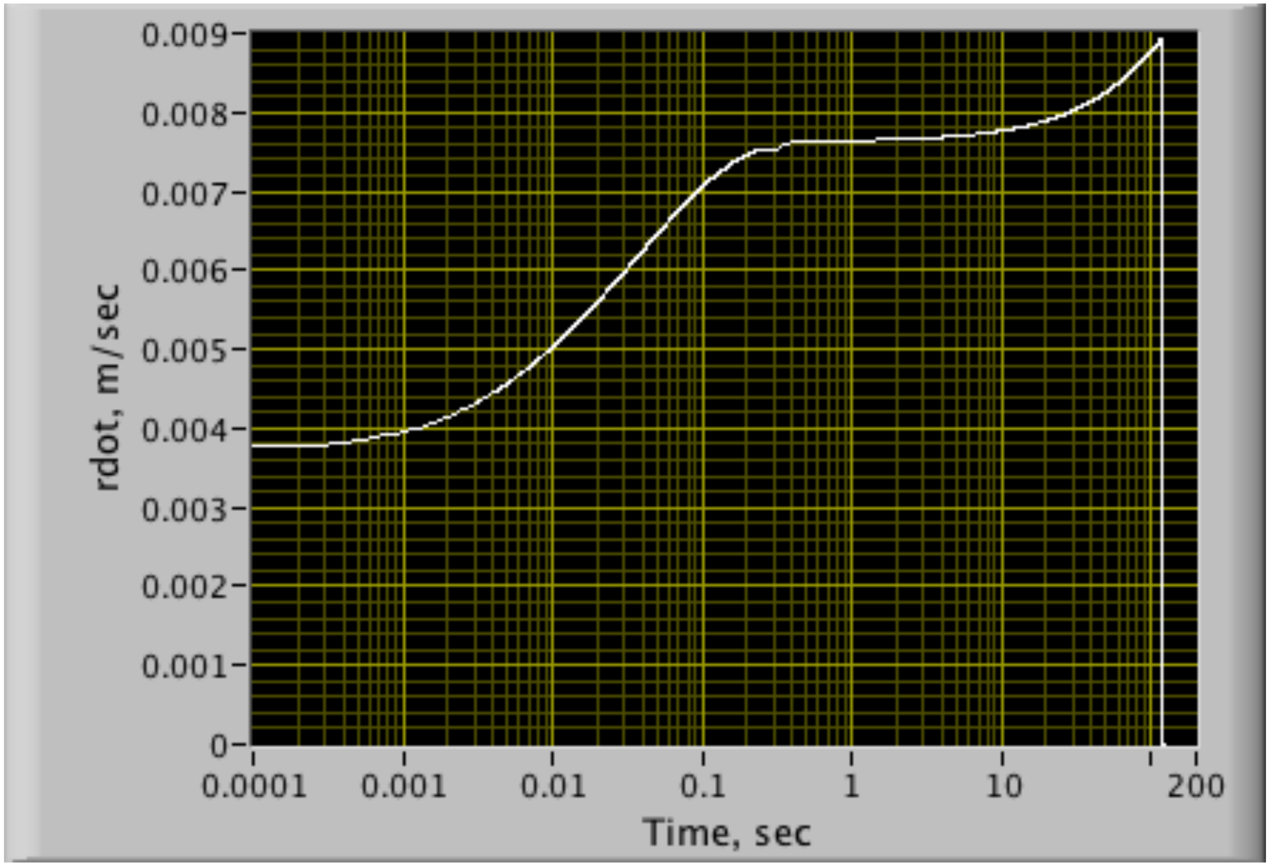
- "Progressive Burn Pattern"



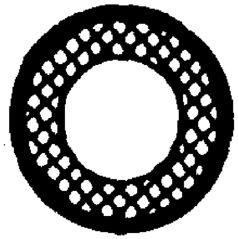


Burn Time: 121 seconds

RSRM Burn Time History (cont'd)

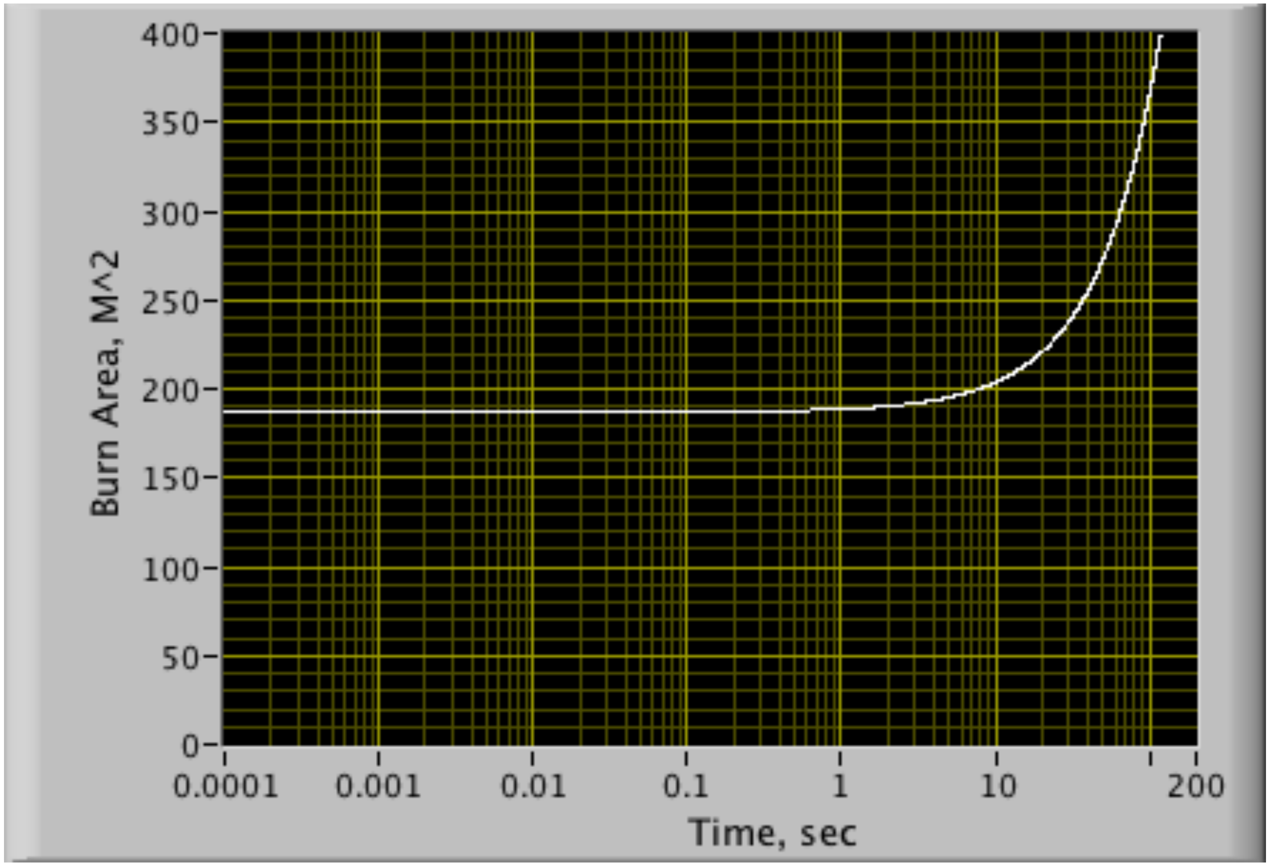


- "Progressive Burn Pattern"

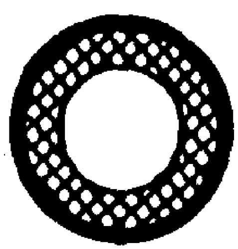


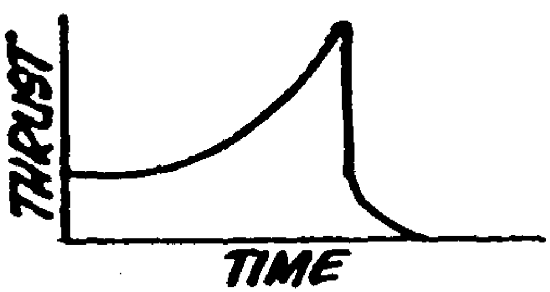
Burn Time: 121 seconds

RSRM Burn Time History (cont'd)



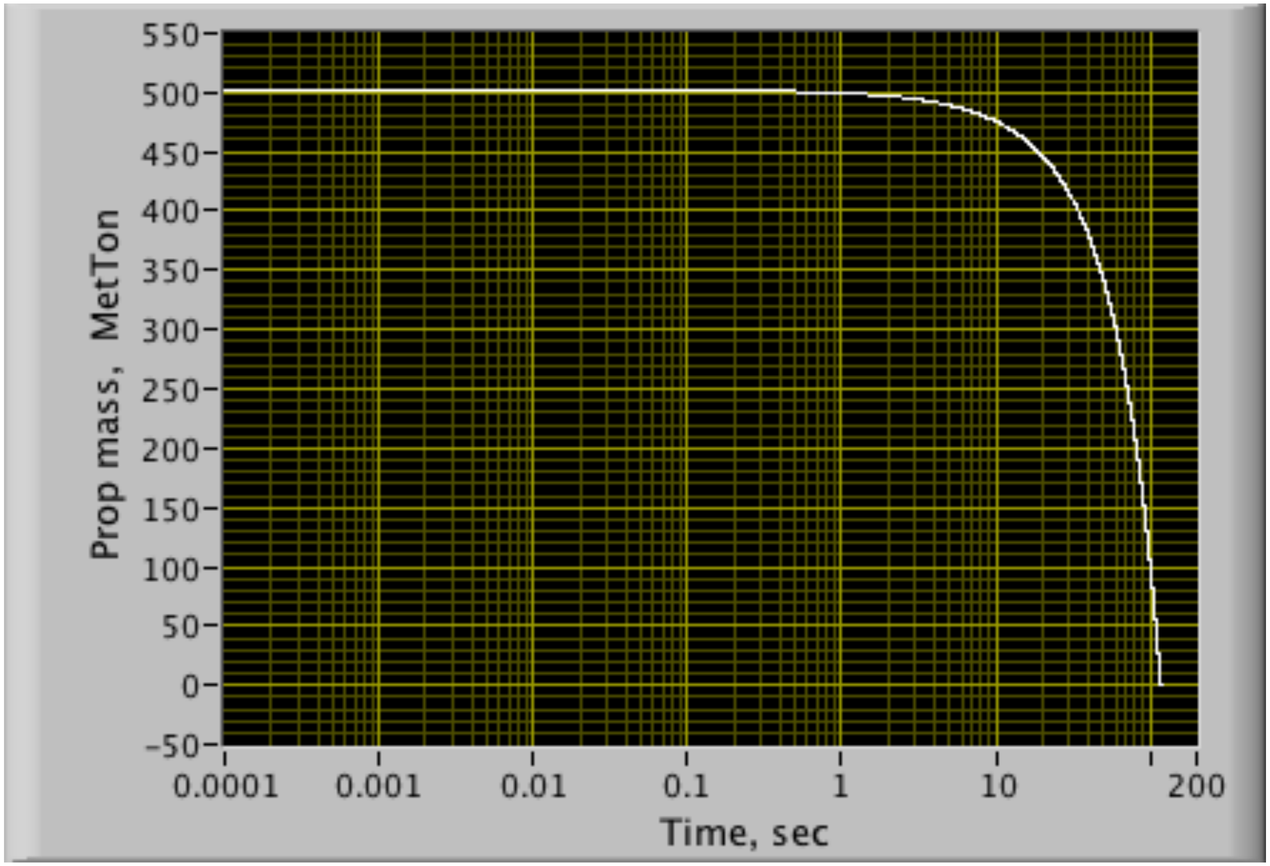
- "Progressive Burn Pattern"





Burn Time: 121 seconds

RSRM Burn Time History (cont'd)

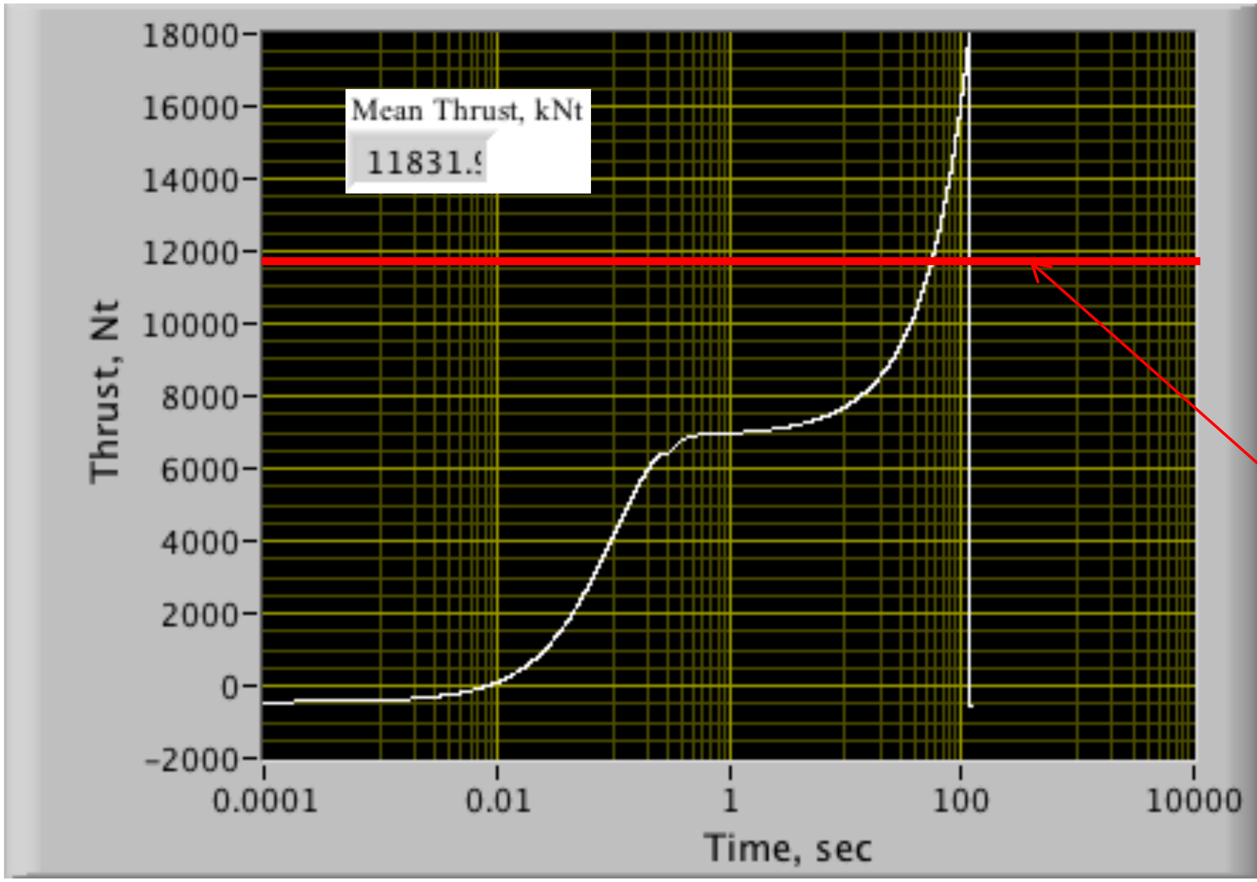


- "Progressive Burn Pattern"

Burn Time: 121 seconds

$$\frac{\partial P_0}{\partial t} = \frac{A_{burn} a P_o^{\,n}}{V_c}\left[\rho_p R_g T_0 - P_0\right] - P_0\left[\frac{A^*}{V_c}\sqrt{\gamma R_g T_0\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}}\right]$$

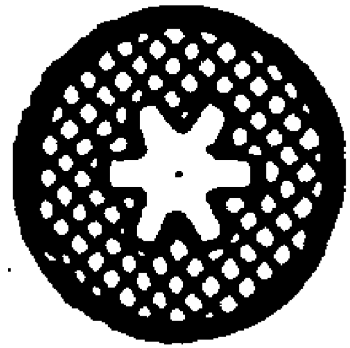
RSRM Burn Time History (cont'd)

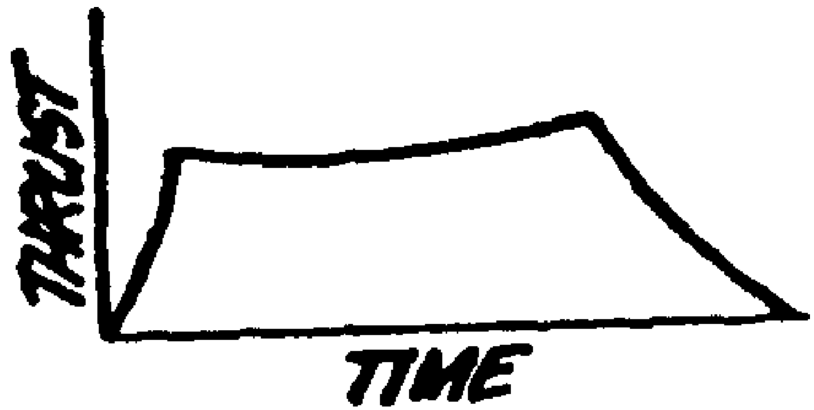


- "Progressive Burn Pattern"
- Quoted "Nominal Thrust" of SRB (11,780 kNt)

So our model is "pretty good"

Burn Area Revisited (1)

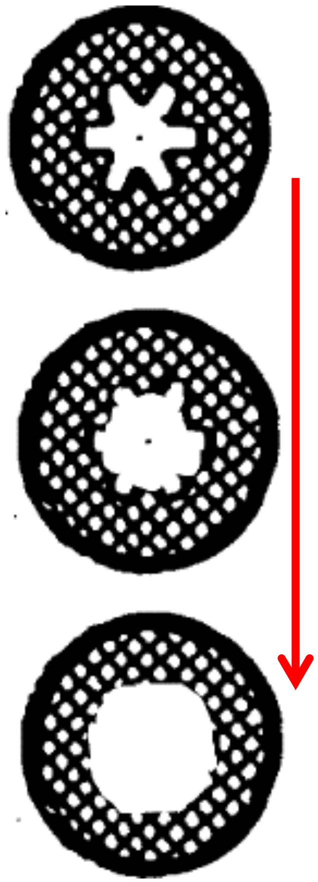




- What happens to the burn area with this pattern?

$$\frac{\partial P_0}{\partial t} = \frac{A_{burn} a P_o^{\,n}}{V_c}\left[\rho_p R_g T_0 - P_0\right] - P_0\left[\frac{A^*}{V_c}\sqrt{\gamma R_g T_0 \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}}\right]$$

Burn Area Revisited (2)



"tips burn first"

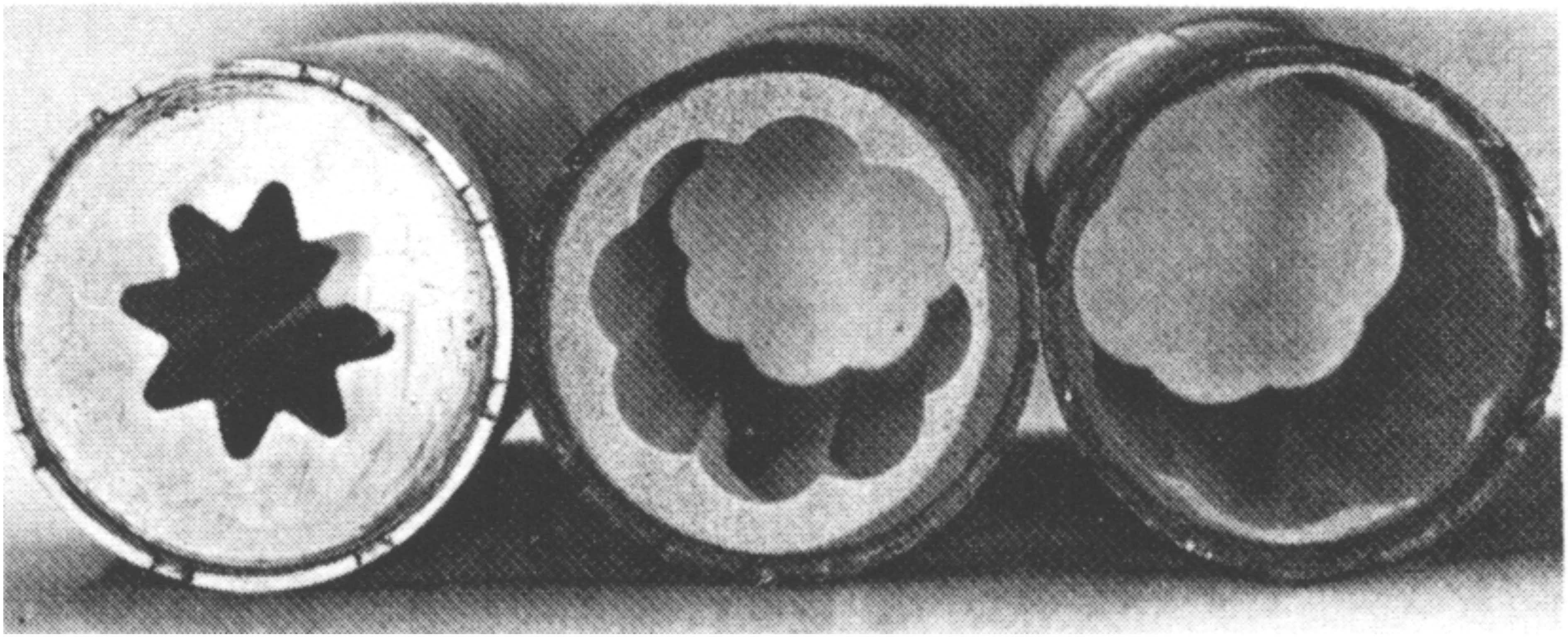
time

- Burn Area stays relatively constant
- Burn Volume Goes Down
- Ratio of Burn Area to Chamber Volume goes ….. *Down! Fast!*
- *Result is a more shaped burn profile*

$$\frac{\partial P_0}{\partial t} = \frac{A_{burn} a P_o^{\,n}}{V_c}\left[\rho_p R_g T_0 - P_0\right] - P_0\left[\frac{A^*}{V_c}\sqrt{\gamma R_g T_0\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}}\right]$$

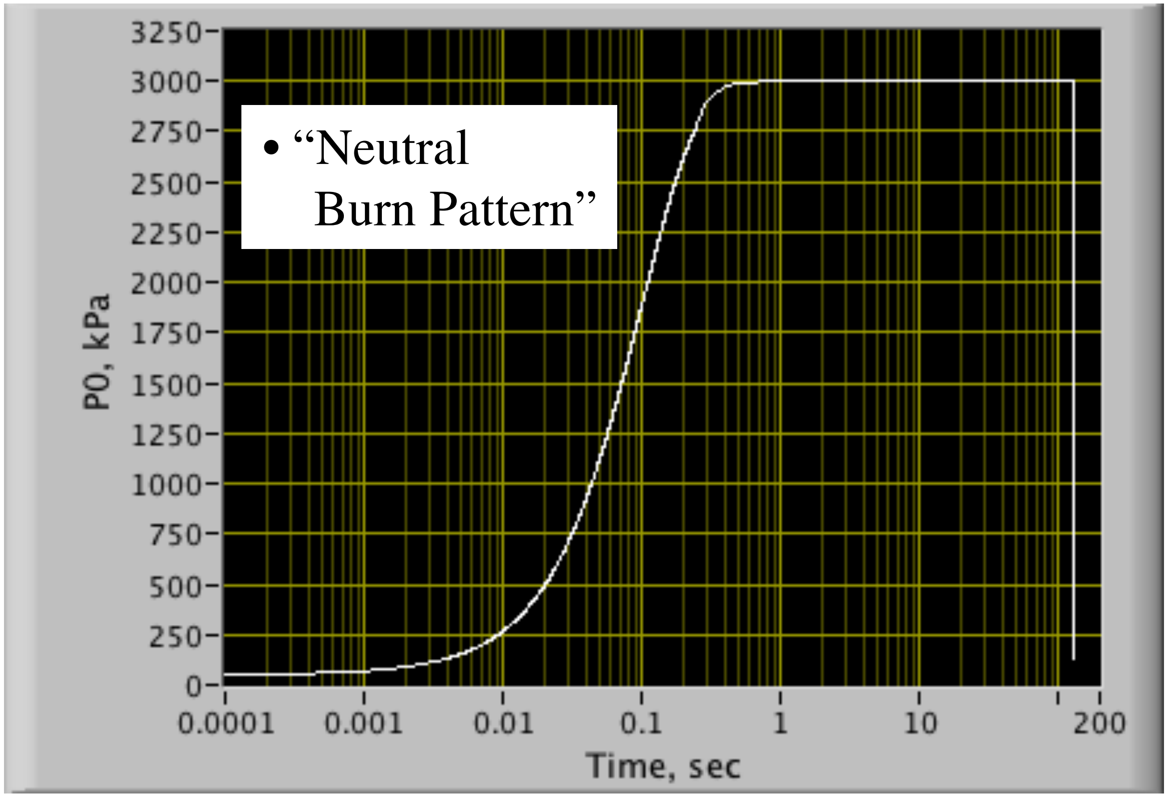
SHAPE OF PROPELLANT GRAINS QUENCHED AT DIFFERENT TIMES

Life History of Solid Motor Shown

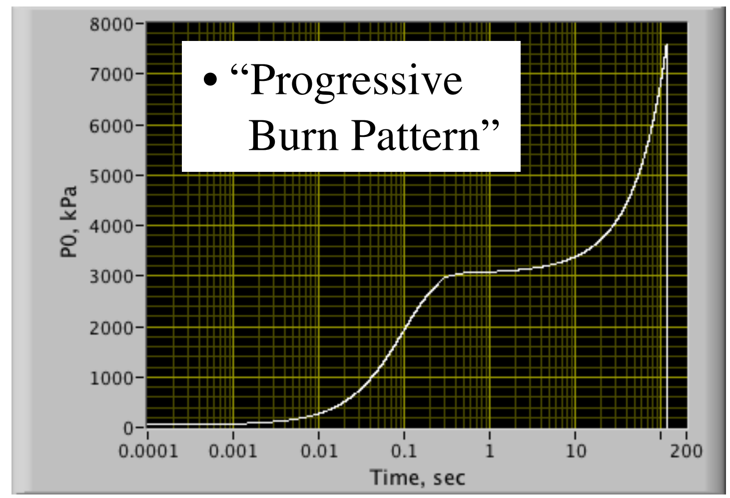


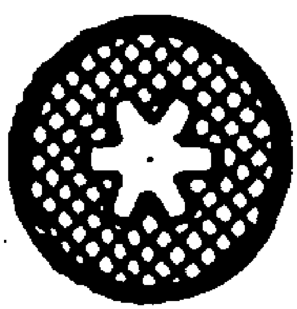
Start condition **Quenched at 1.5 s** **Quenched at 2.5 s**

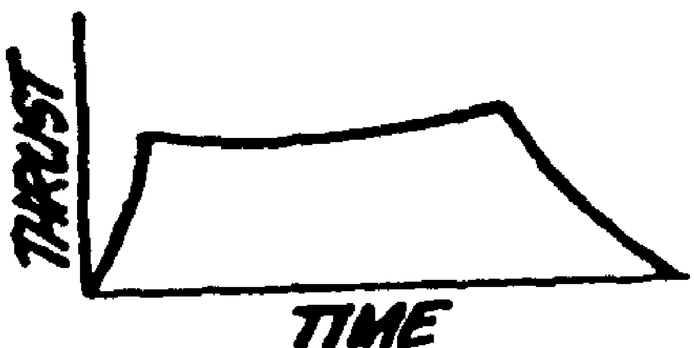
Modified RSRM Burn Time History



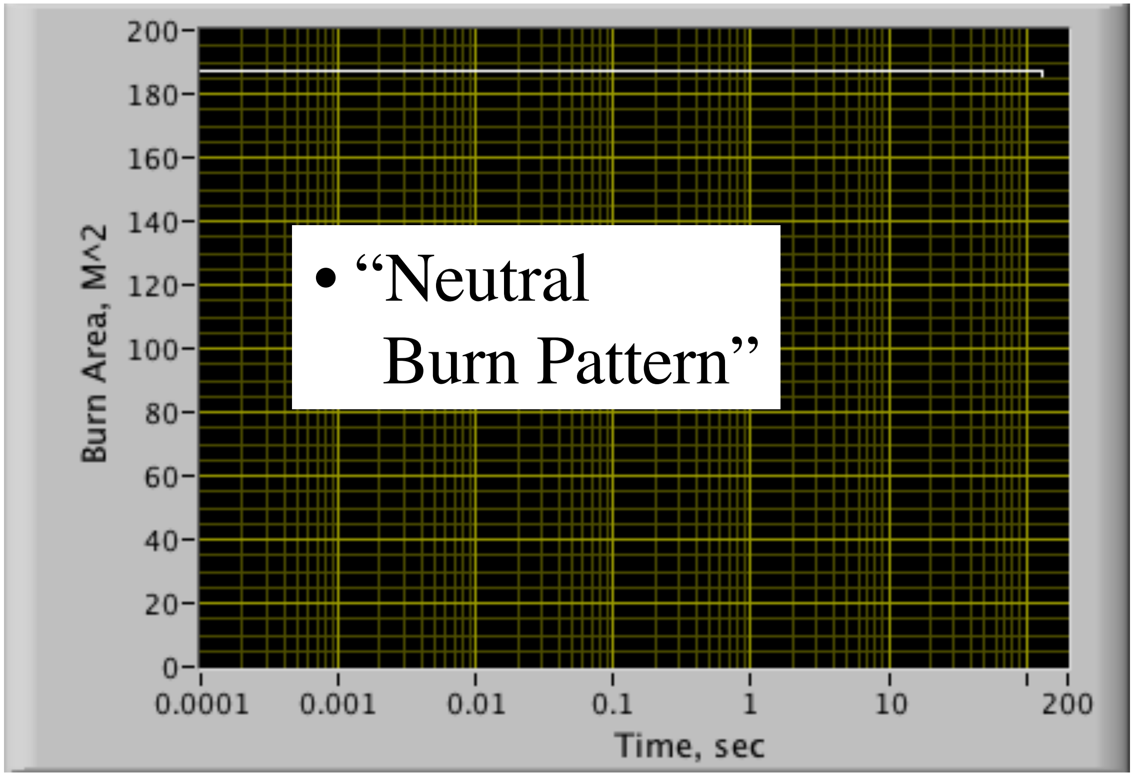
Burn Time: 129 seconds



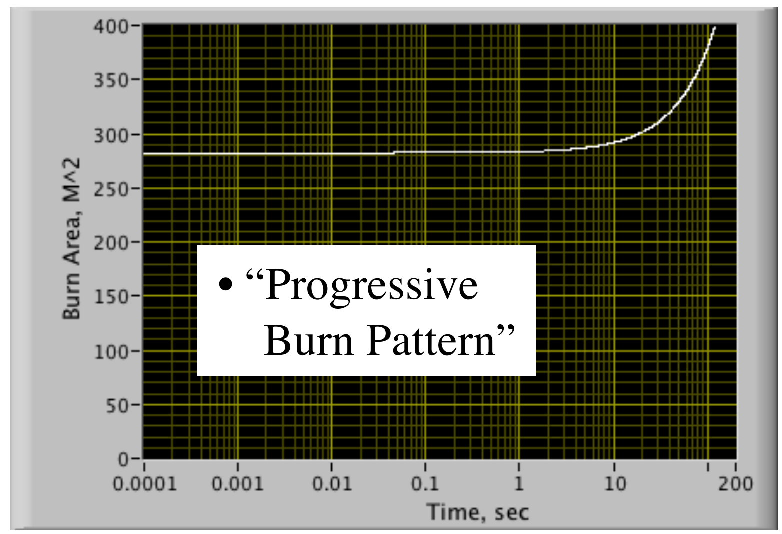


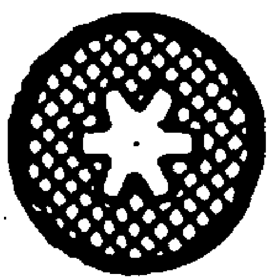


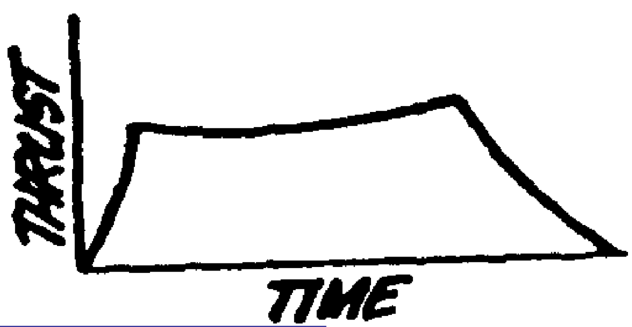
Modified RSRM Burn Time History (cont'd)



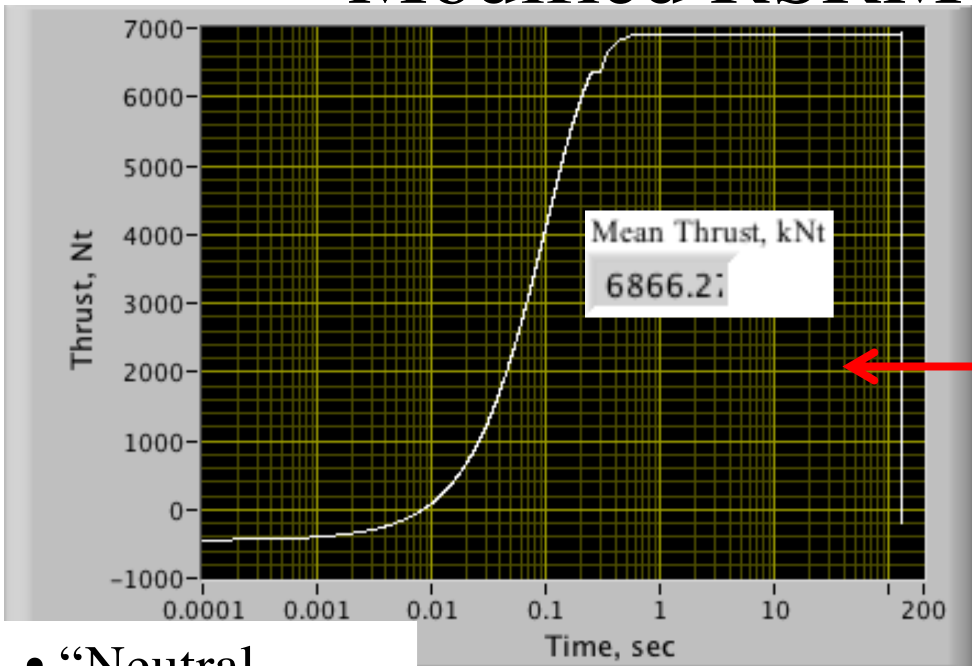
Burn Time: 129 seconds







Modified RSRM Burn Time History (cont'd)



$$\int_0^{t_{burn}} F_{thrust_{SL}} \, dt =$$

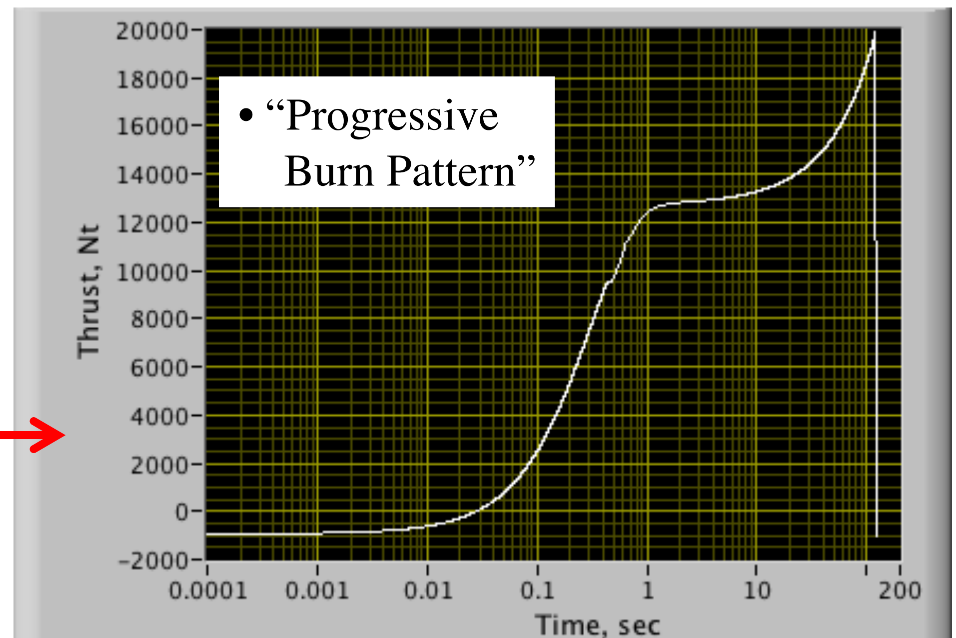
$$6.86627 \cdot 129 = 887.75_{MNt-\text{sec}}$$

- Significantly Less delivered impulse

- "Neutral Burn Pattern"

$$\int_0^{t_{burn}} F_{thrust_{SL}} \, dt =$$

$$11.832 \cdot 121 = 1431.7_{MNt-\text{sec}}$$

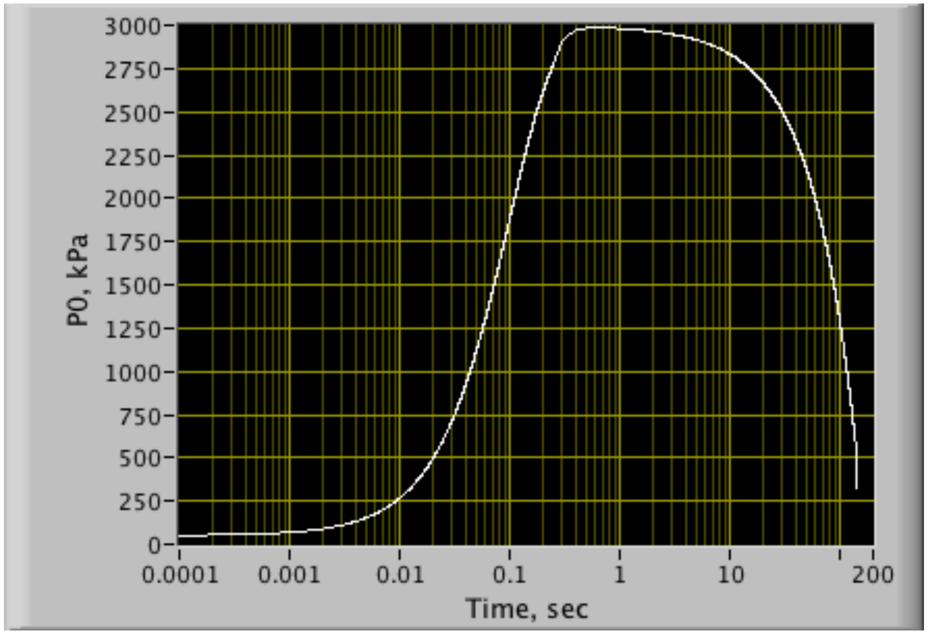


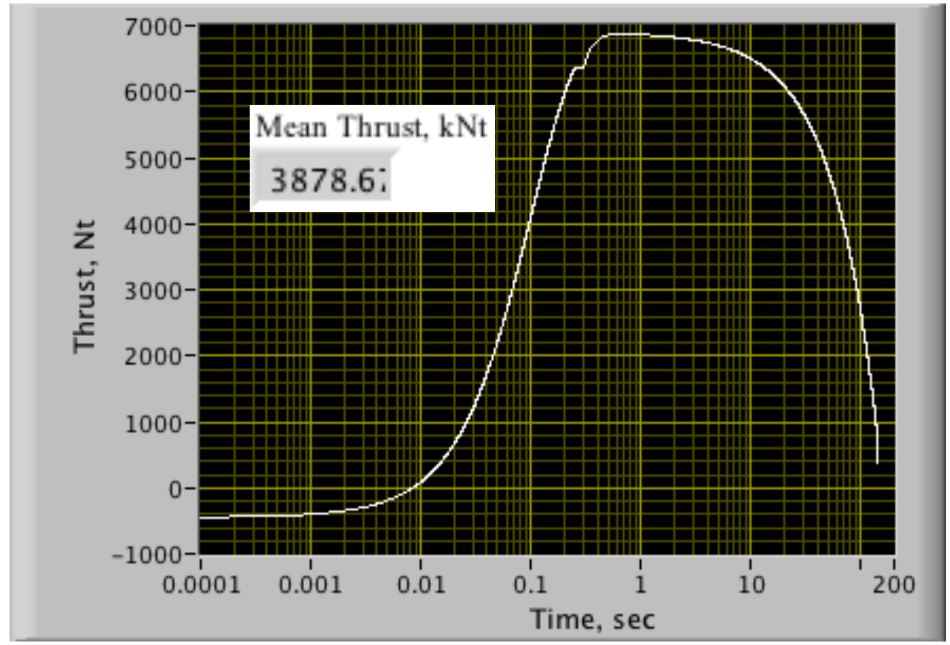
Finally Look at grain pattern

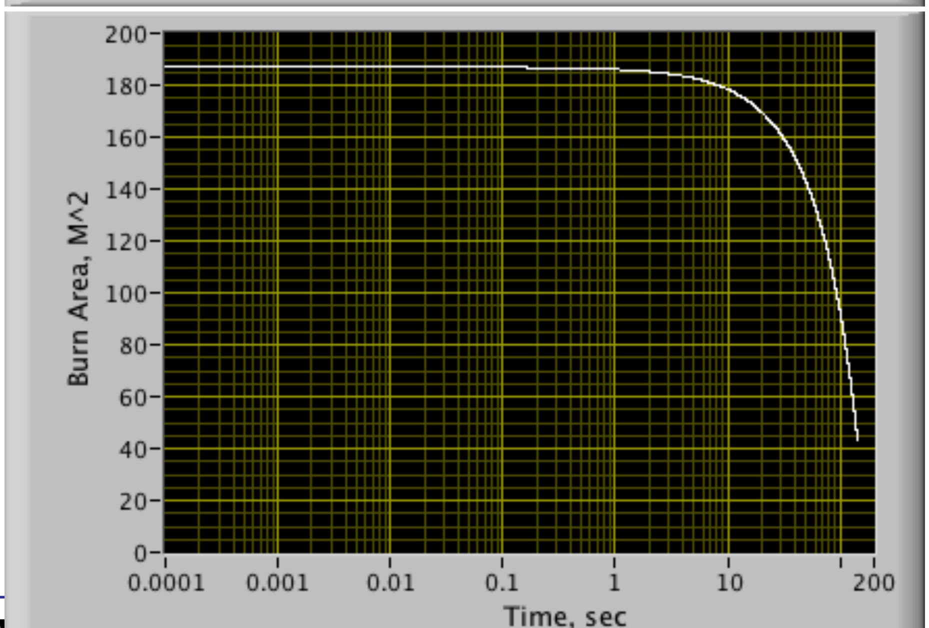
- "*Regressive Grain pattern*" … Burn surface area actually shrinks As propellant is burned

"Dendrite Grain"

Modified RSRM for Dendrite Grain Pattern





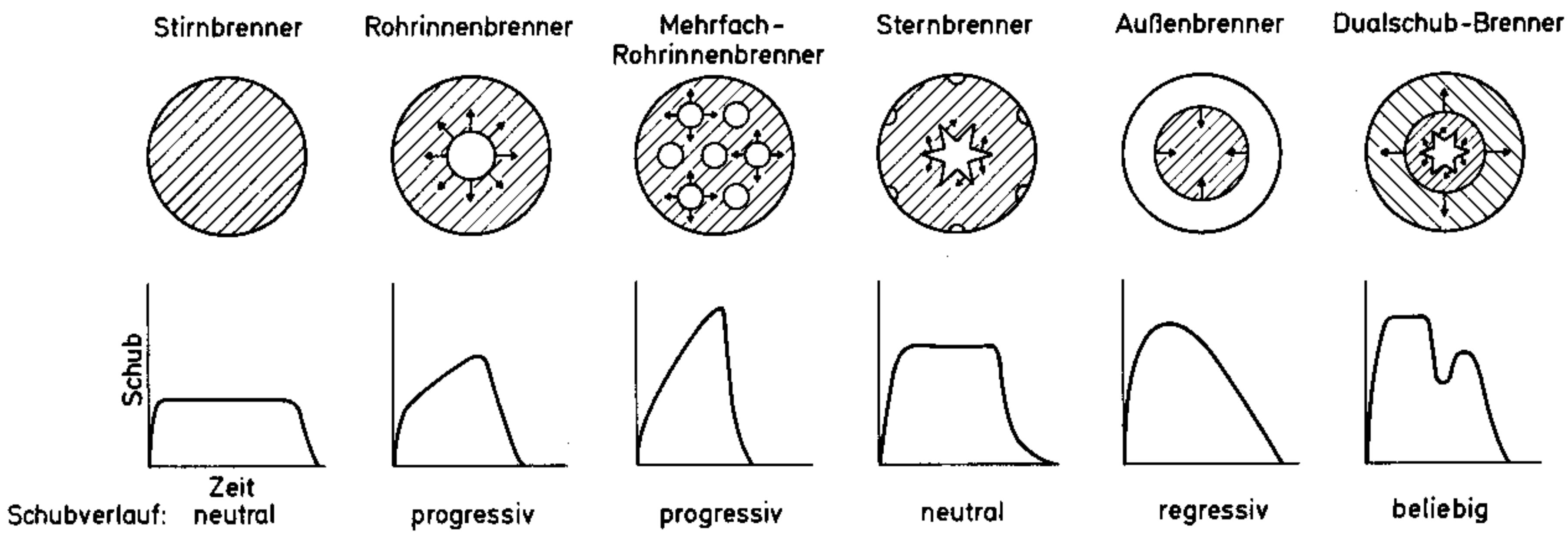


$$\int_0^{t_{burn}} F_{thrust_{SL}}\, dt =$$

$$3.87867 \cdot 142 = 550.77_{MNt-\mathrm{sec}}$$

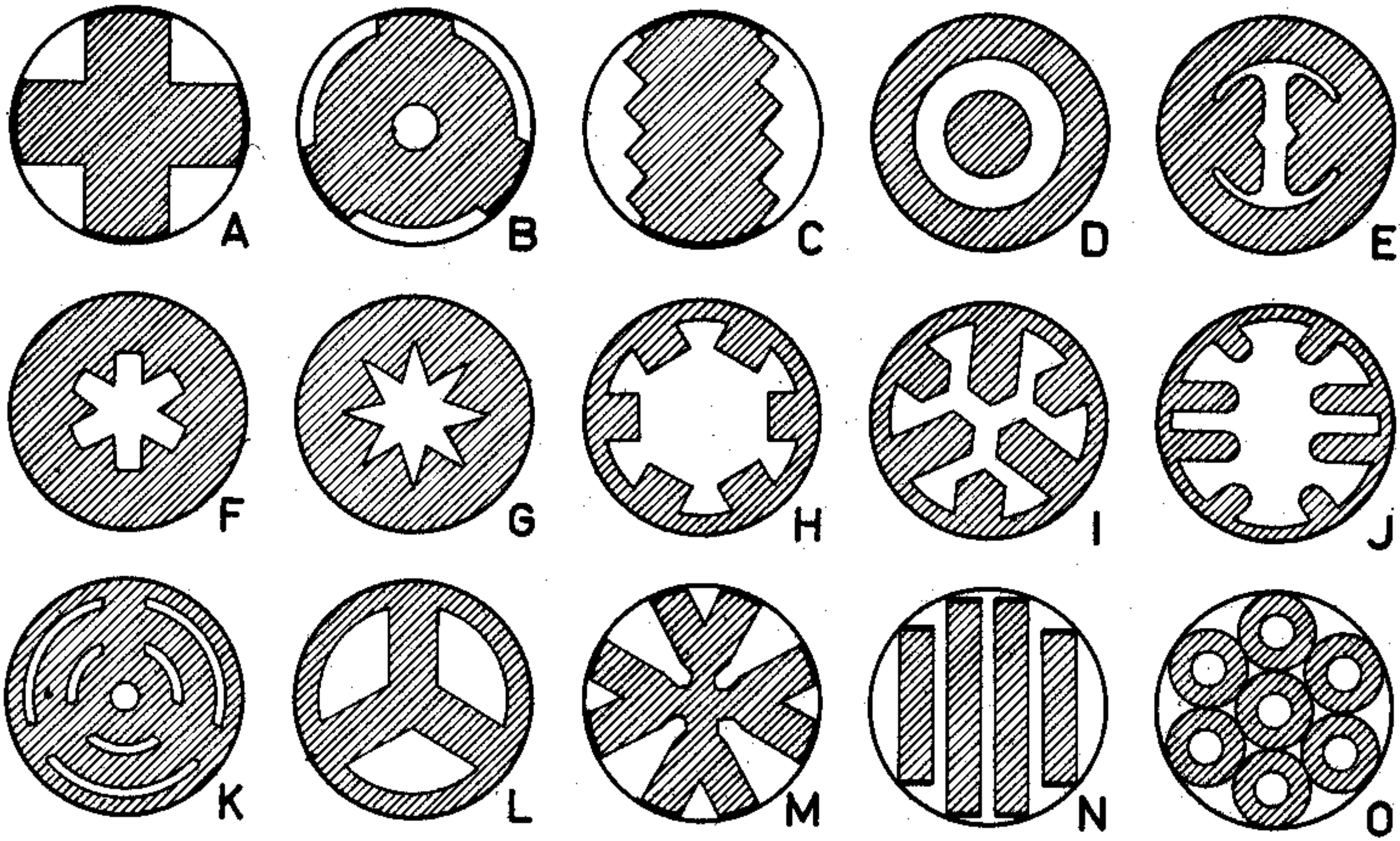
Burn Time: 142 seconds

SOLID ROCKET MOTOR GRAIN DESIGNS



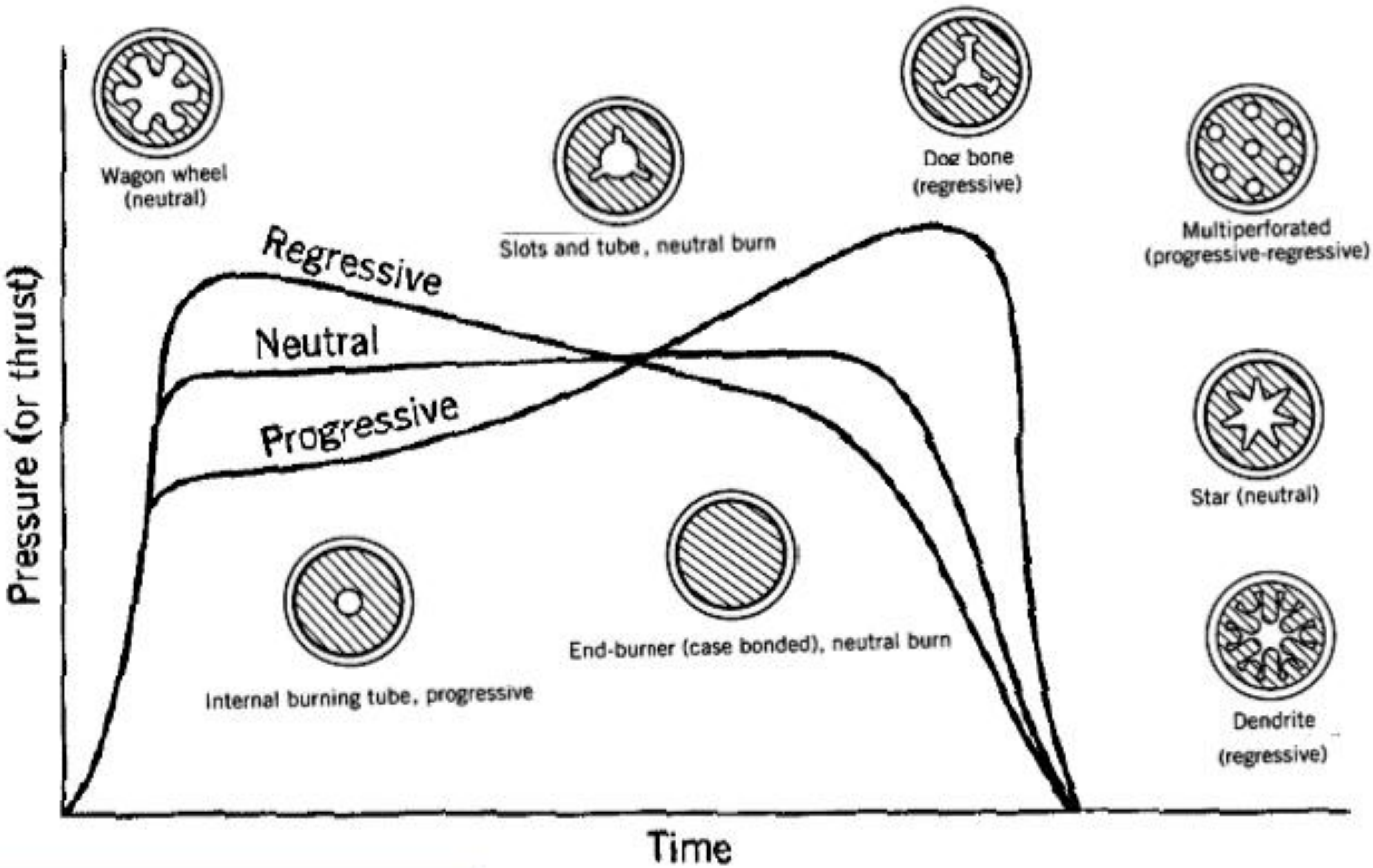
Source: Köhler, Feststoffraketenantriebe, Vol. 1, p. 122 (1972)

EXAMPLES OF SOLID ROCKET MOTOR UNUSUAL GRAIN DESIGNS



Source: Barrere et al., Raketenantriebe, p. 321, Fig 6.1 (1961)

Solid Rocket Burn Summary



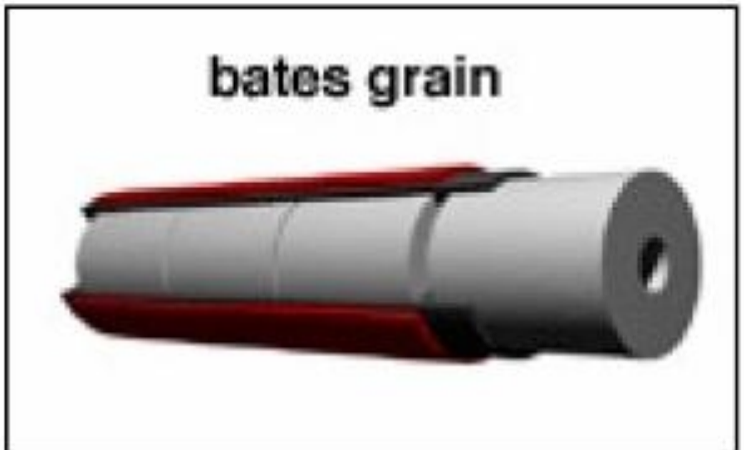
SOLID ROCKET MOTOR
GRAIN DESIGN PROGRAMS

- **Grain Design Program (GDP-Light)**
- **http://home.vianetworks.nl/users/aed/gdp/gdp.htm**

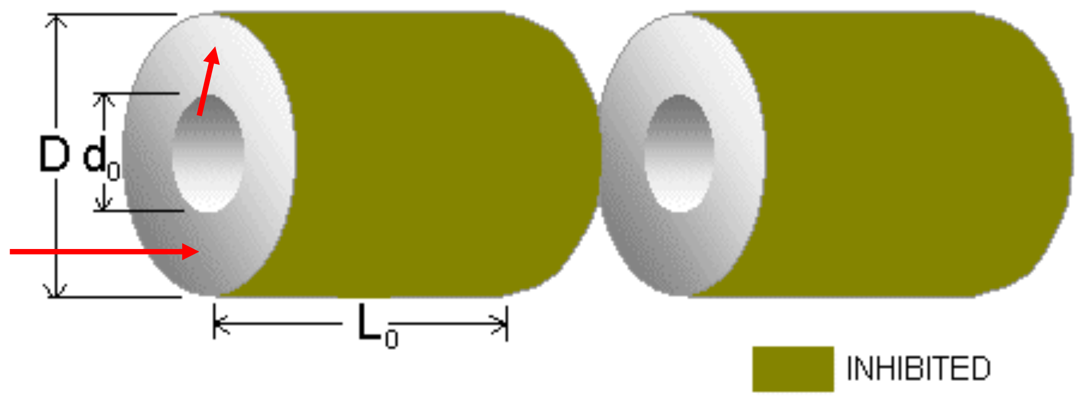
- **Useful Code to test new and unusual grain shapes to achieve certain thrust profiles or minimize slivers and residual burning.**

The "Bates Grain" Geometry

Simple Modification to Cylindrical Port to Give More Even Burn Pattern

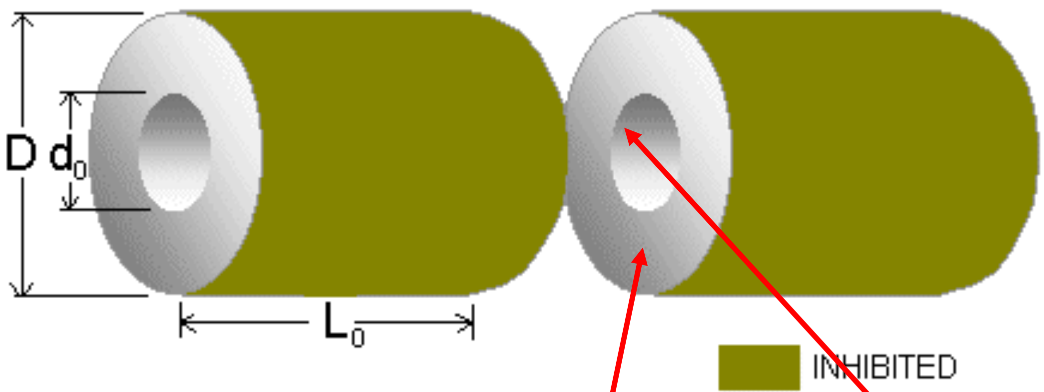


the bates grain uses multi-ple grain segments



Grain segments burn from "inside" and along the "ends"

The "Bates Grain" Geometry (2)



Look at Burn evolution of

$$\frac{A_{burn}}{V_{burn}}$$

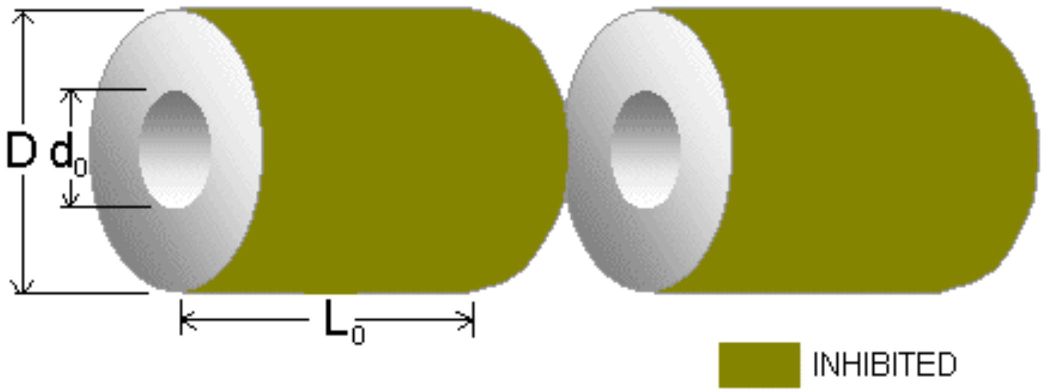
$$s_{regression} = \int_t \dot{r} \cdot dt$$

$$\rightarrow \quad A_{burn} = 2\pi \cdot \left(\frac{D_0^{\,2} - d^2}{4} \right) + L \cdot \pi \cdot d$$

end of segment   interior of grain

For each grain segment

The "Bates Grain" Geometry (3)



Look at Burn evolution of $\frac{A_{burn}}{V_{burn}}$

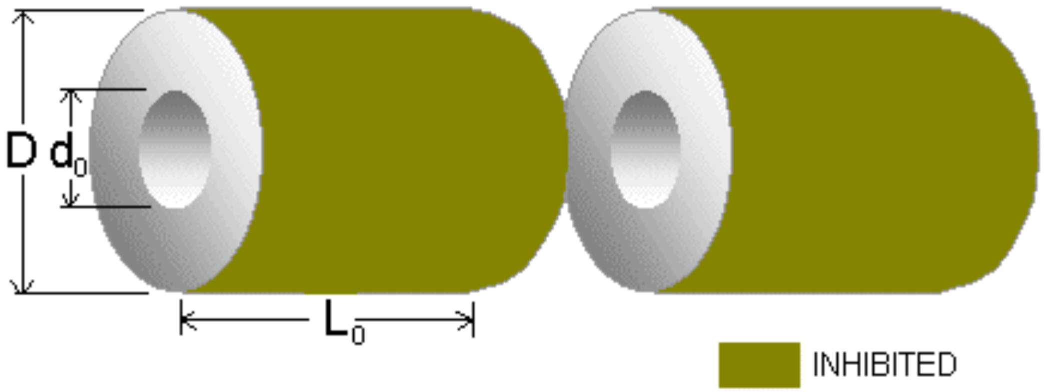
$$\left. \begin{array}{l} \text{regressing interior} \\ \text{surface diameter} + \\ \text{ends of segment} \end{array} \right| \rightarrow \left. \begin{array}{l} d = d_0 + 2 \cdot s \\ L = L_0 - 2 \cdot s \end{array} \right|$$

For each grain segment

$$A_{burn} = 2\pi \cdot \left(\frac{D_0{}^2 - d^2}{4} \right) + L \cdot \pi \cdot d =$$

$$\frac{\pi}{2} \cdot \left(D_0{}^2 - \left(d_0 + 2 \cdot s \right)^2 \right) + \pi \cdot \left(L_0 - 2 \cdot s \right) \cdot \left(d_0 + 2 \cdot s \right)$$

The "Bates Grain" Geometry (4)



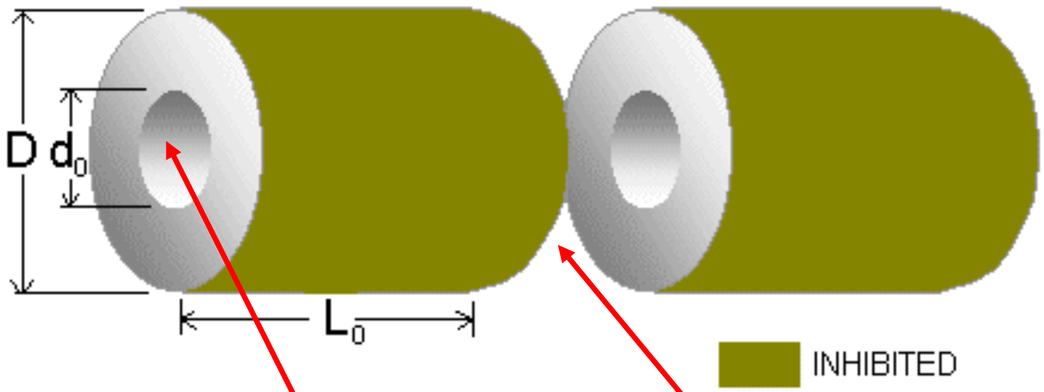
Look at Burn evolution of $\frac{A_{burn}}{V_{burn}}$

regressing interior surface diameter + ends of segment $\rightarrow$ $d = d_0 + 2 \cdot s$, $L = L_0 - 2 \cdot s$

For N grain segments

$$\left(A_{burn}\right)_{total} = N \cdot \pi \cdot \left[\frac{\left(D_0^{\,2} - \left(d_0 + 2 \cdot s\right)^2\right)}{2} + \left(L_0 - 2 \cdot s\right) \cdot \left(d_0 + 2 \cdot s\right) \right]$$

The "Bates Grain" Geometry (5)



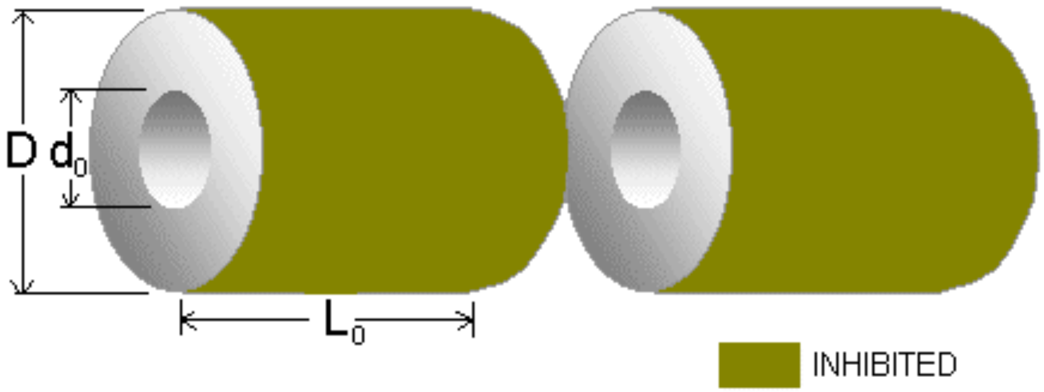
Look at Burn evolution of $\frac{A_{burn}}{V_{burn}}$

Look at total chamber burn volume (empty space) at any time

$$(V_{ol})_{total} = N \cdot \left[\pi \cdot \frac{d^2}{4} \cdot L \quad + \quad \pi \frac{D_0^{\ 2}}{4} \cdot (2 \cdot s) \right]$$

Port Volume …. "burn end" volume

The "Bates Grain" Geometry (6)



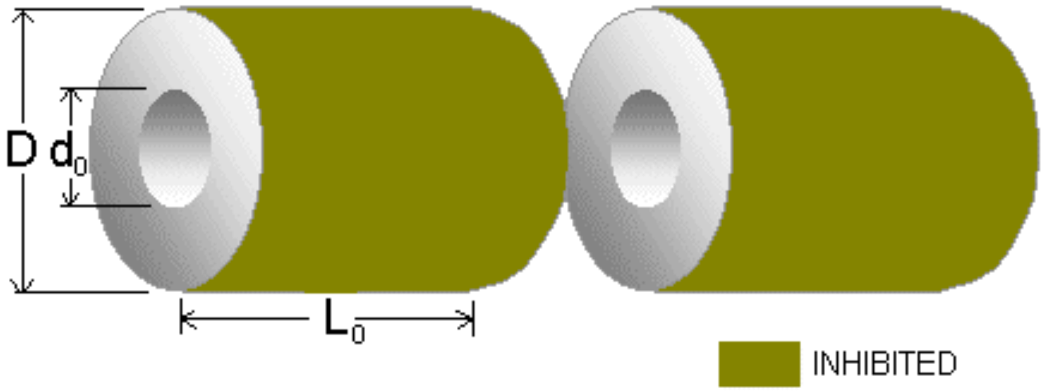
Look at Burn evolution of $\frac{A_{burn}}{V_{burn}}$

Allowing for regression from original geometry

$$\left(V_{ol}\right)_{total} = N \cdot \left[\pi \cdot \frac{d^2}{4} \cdot L + \pi \frac{D_0{}^2}{4} \cdot (2 \cdot s) \right] =$$

$$\frac{N \cdot \pi}{4} \left[\left(d_0 + 2 \cdot s\right)^2 \cdot \left(L_0 - 2 \cdot s\right) + D_0{}^2 \cdot (2 \cdot s) \right]$$

The "Bates Grain" Geometry (8)



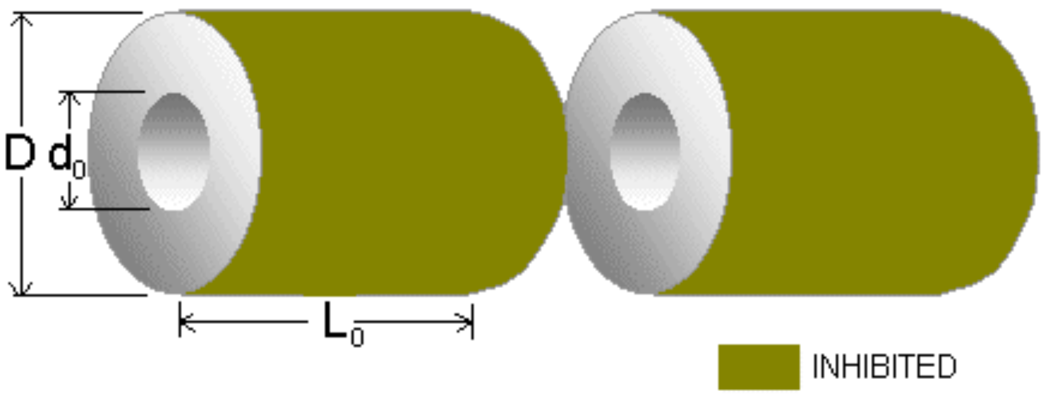
Look at Burn evolution of $\frac{A_{burn}}{V_{burn}}$

$$\left(A_{burn}\right)_{total} = N \cdot \pi \cdot \left[\frac{\left(D_0^{\,2} - \left(d_0 + 2 \cdot s\right)^2\right)}{2} + \left(L_0 - 2 \cdot s\right) \cdot \left(d_0 + 2 \cdot s\right) \right]$$

$$\left(V_{ol}\right)_{total} = \frac{N \cdot \pi}{4} \left[\left(d_0 + 2 \cdot s\right)^2 \cdot \left(L_0 - 2 \cdot s\right) \; + \; D_0^{\,2} \cdot \left(2 \cdot s\right) \right]$$

$$\frac{A_{burn}}{V_{burn}} = \frac{N \cdot \pi \cdot \left[\frac{\left(D_0^{\,2} - \left(d_0 + 2 \cdot s\right)^2\right)}{2} + \left(L_0 - 2 \cdot s\right) \cdot \left(d_0 + 2 \cdot s\right) \right]}{\frac{N \cdot \pi}{4} \left[\left(d_0 + 2 \cdot s\right)^2 \cdot \left(L_0 - 2 \cdot s\right) \; + \; D_0^{\,2} \cdot \left(2 \cdot s\right) \right]} = \frac{4 \left[\frac{\left(1 - \left(\frac{d_0 + 2 \cdot s}{D_0}\right)^2\right)}{2} + \left(1 - 2 \cdot \frac{s}{L_0}\right) \cdot \left(\frac{d_0 + 2 \cdot s}{D_0}\right) \right]}{\left[\left(\frac{d_0 + 2 \cdot s}{D_0}\right)^2 \cdot \left(1 - 2 \cdot \frac{s}{L_0}\right) \; + \; \left(2 \cdot \frac{s}{L_0}\right) \right]}$$

The "Bates Grain" Geometry (9)



Summary of Algorithm

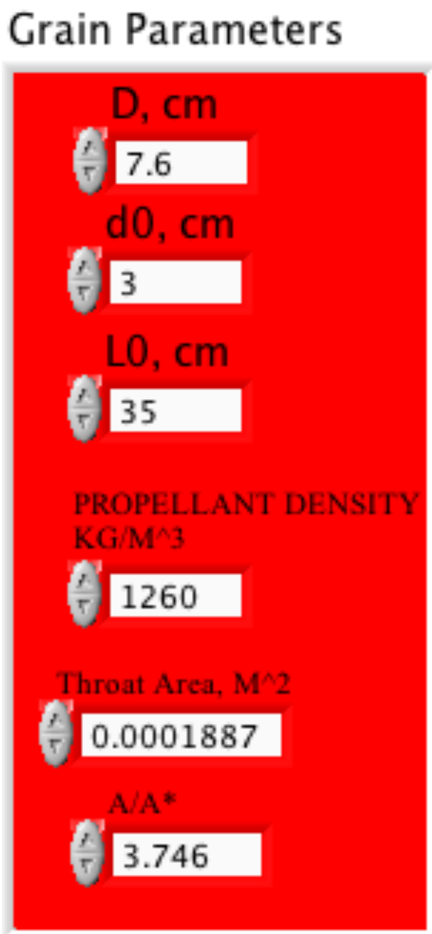
$$\dot{r} = a \cdot P_o^{\,n}$$

$$s_{regression} = \int_t \dot{r} \cdot dt \quad \rightarrow$$

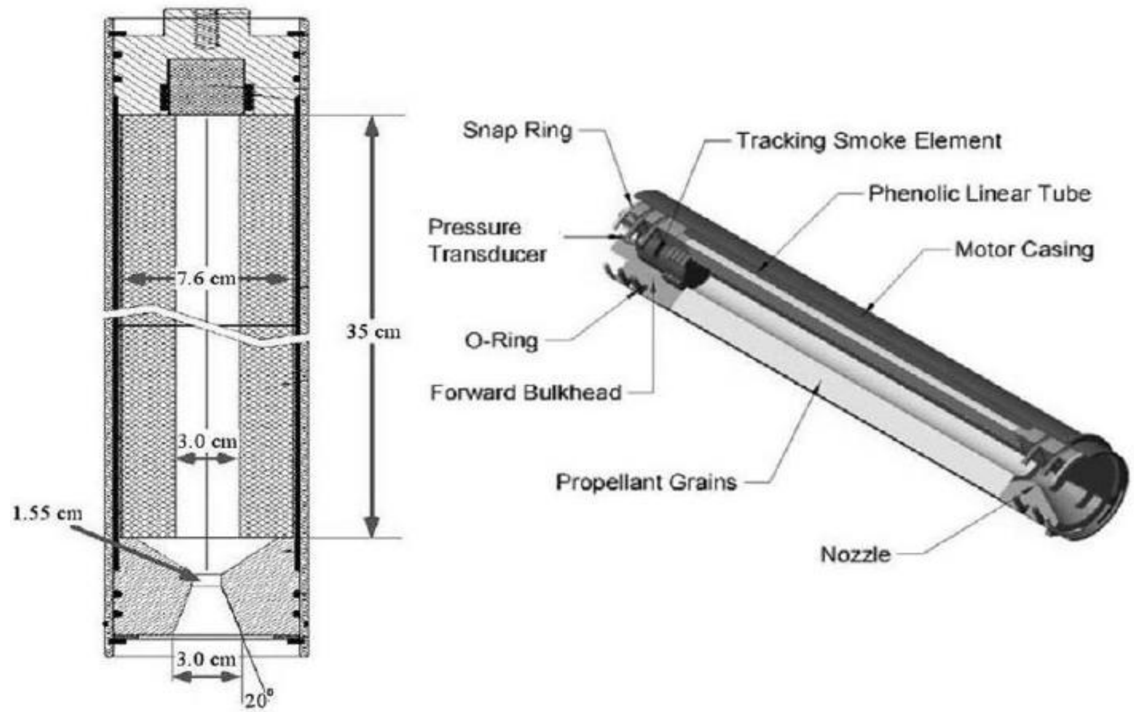
$$\left(A_{burn}\right)_{total} = N \cdot \pi \cdot \left[\frac{\left(D_0^{\,2} - \left(d_0 + 2 \cdot s\right)^2\right)}{2} + \left(L_0 - 2 \cdot s\right) \cdot \left(d_0 + 2 \cdot s\right) \right]$$

$$\left(V_{ol}\right)_{total} = \frac{N \cdot \pi}{4} \left[\left(d_0 + 2 \cdot s\right)^2 \cdot \left(L_0 - 2 \cdot s\right) \; + \; D_0^{\,2} \cdot \left(2 \cdot s\right) \right]$$

Comparative Burn Example



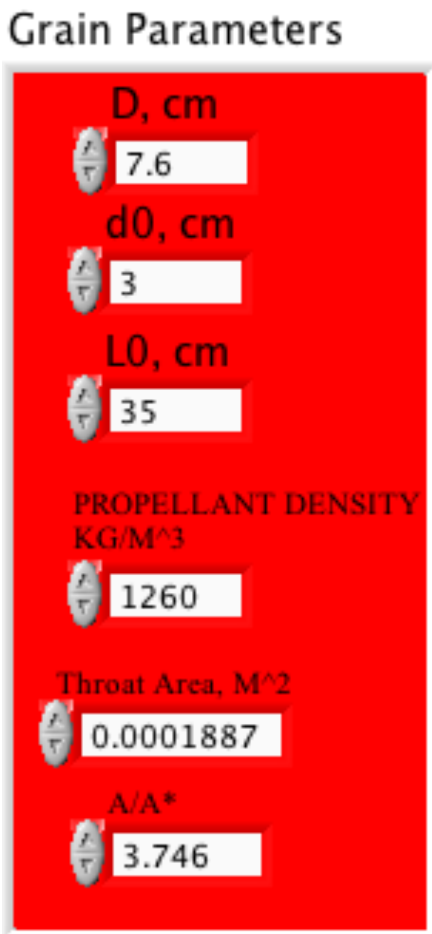
Fuel grain shape effect on a small Solid propelled motor (AMW L777)

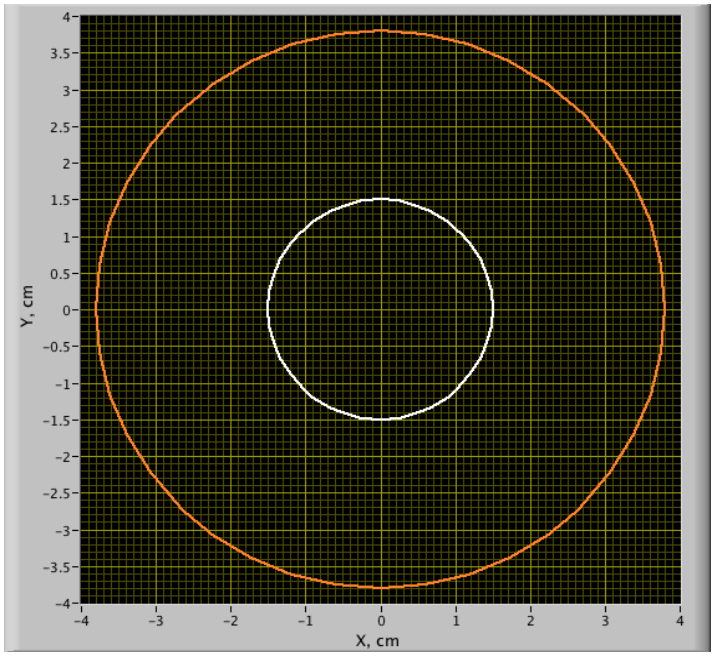


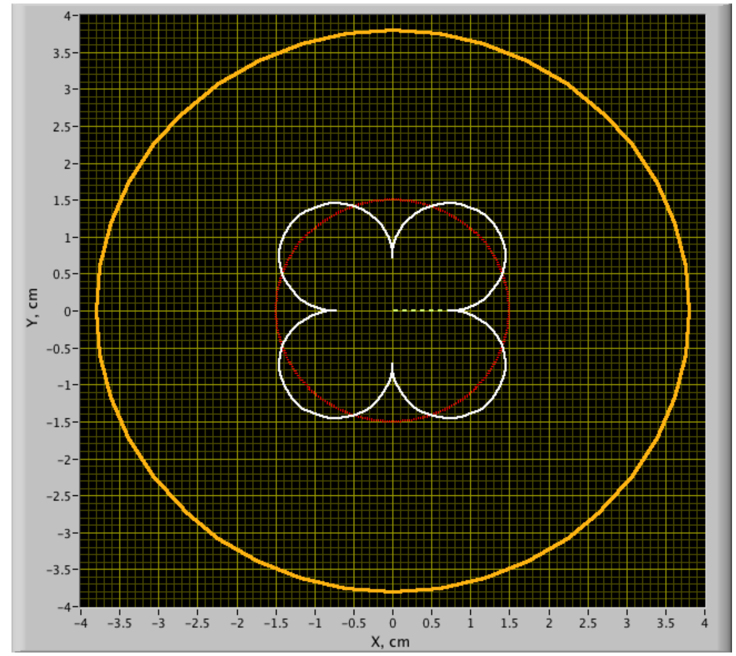
1.68885 kg of propellant

First compare 3-grain bates configuration against Hypothetical 6-grain configuration

Comparative Burn Example (2)

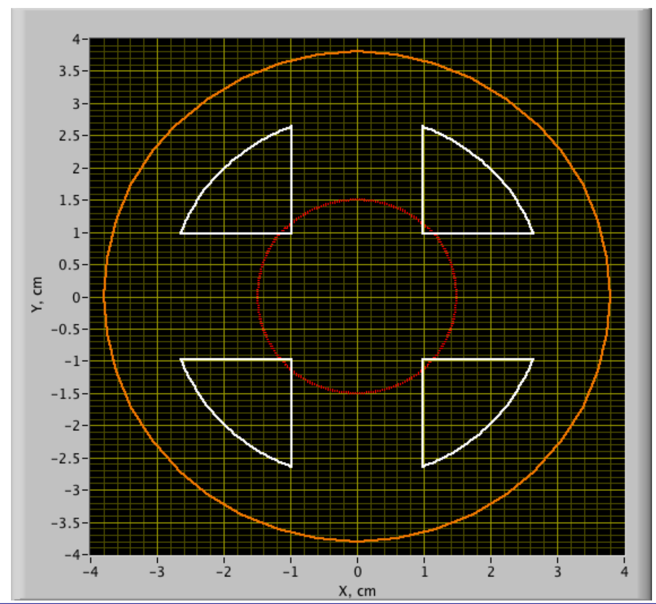






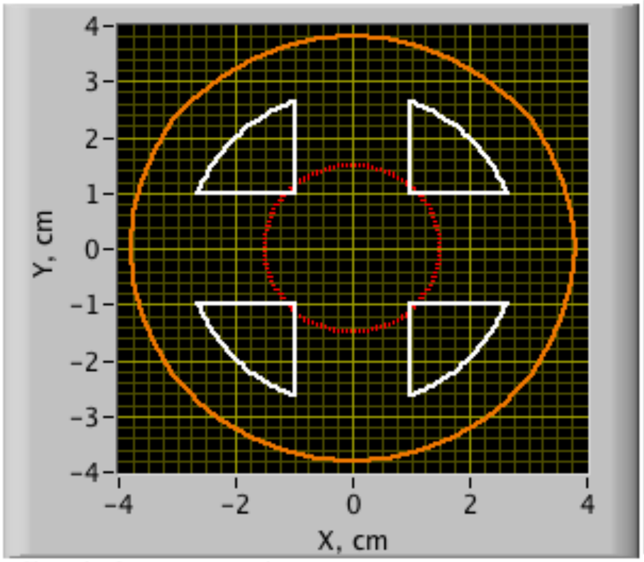
1.68885 kg
of propellant

Then compare more intricate port patterns

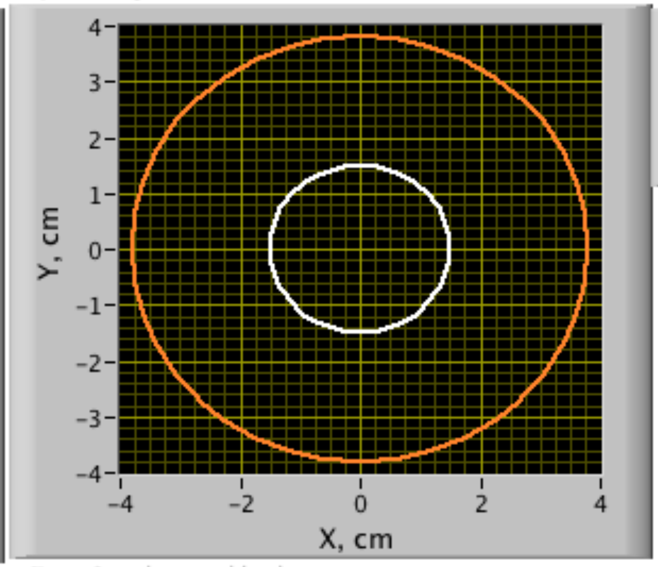


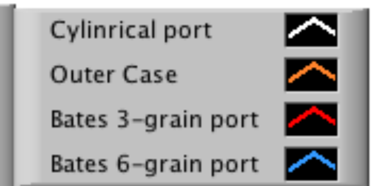
Comparative Burn Example (3)

SNC 4-port cross section



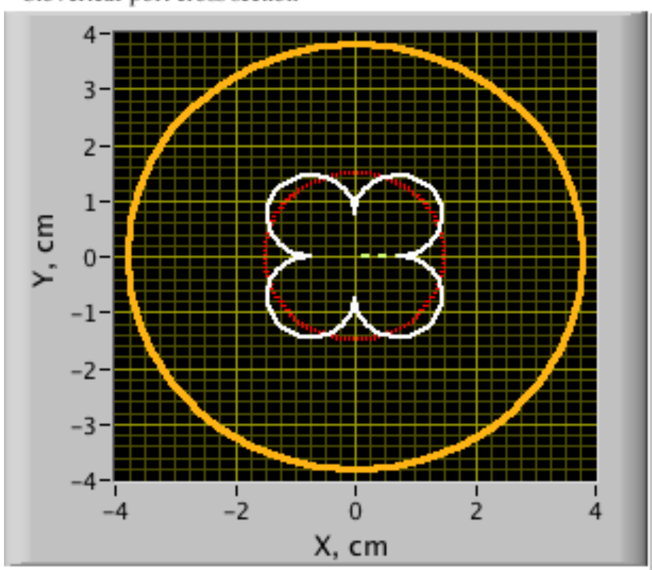
Cyindrical port cross section



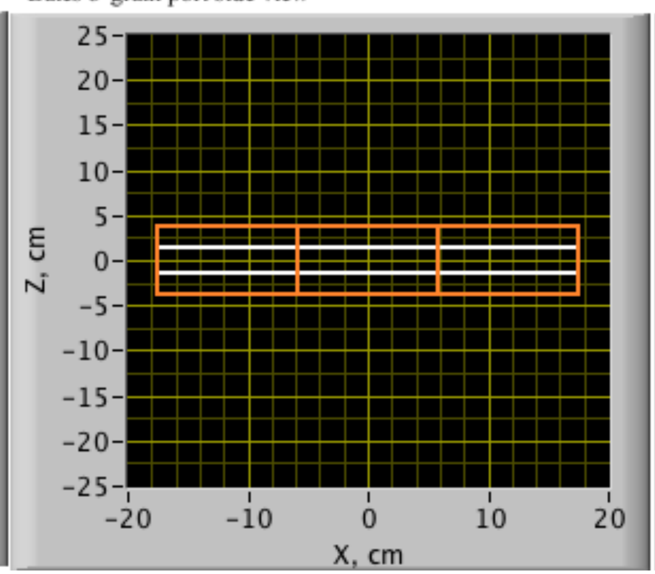


- Initial Grain Patterns
- Propellant Mass: 1.68885 kg

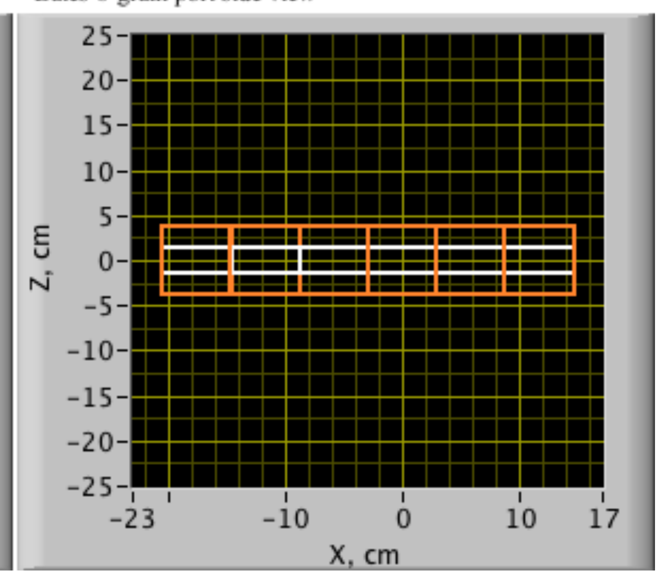
Cloverleaf port cross section



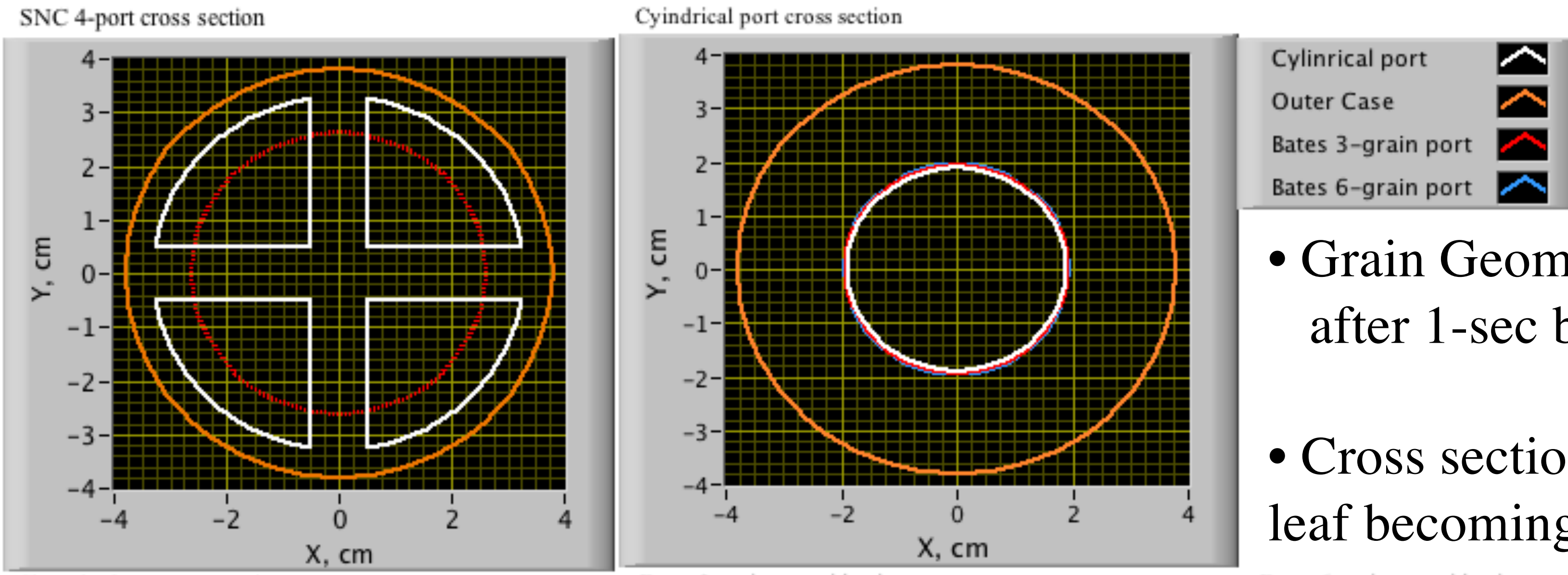
Bates 3-grain port side view



Bates 6-grain port side view

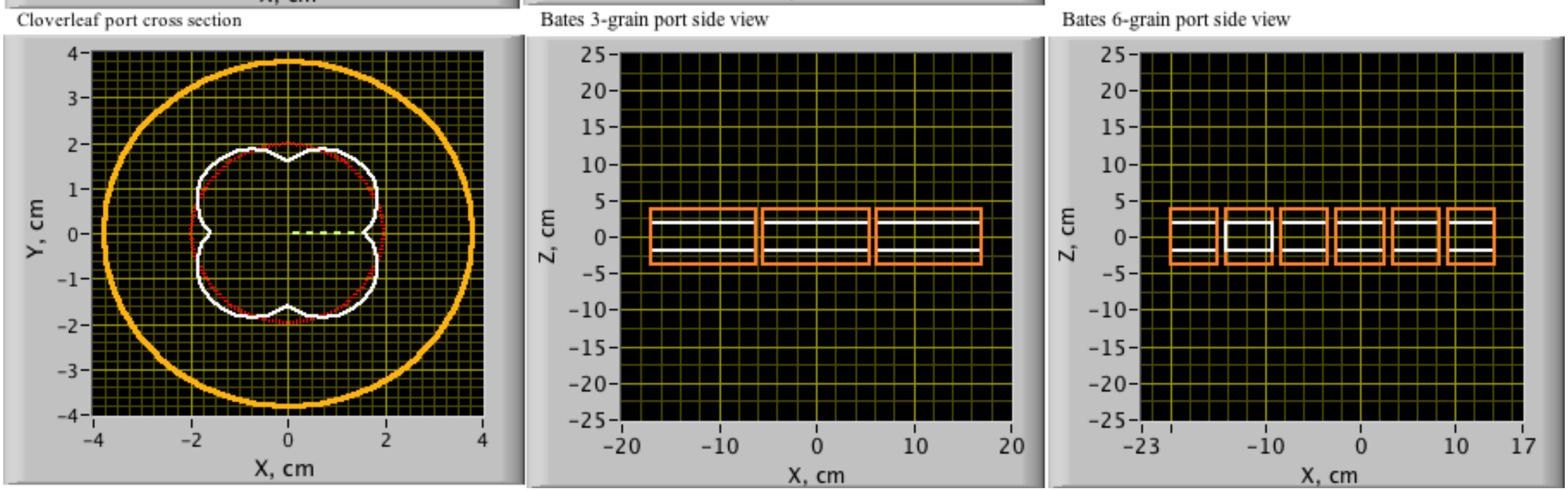


Comparative Burn Example (4)



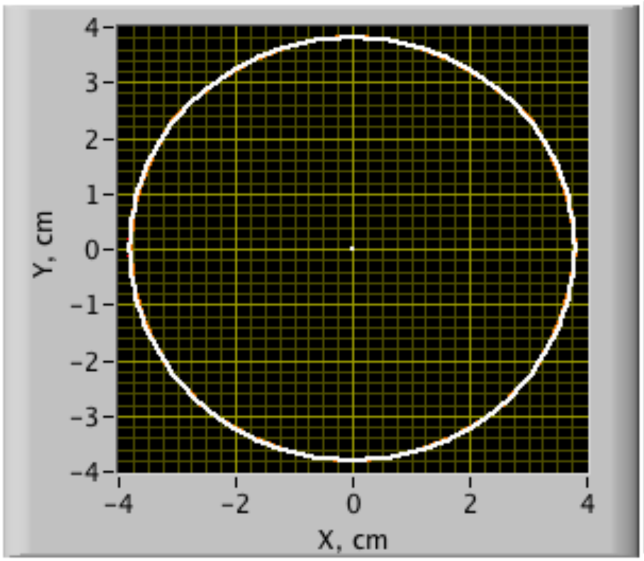
- Grain Geometry after 1-sec burn

- Cross section of clover leaf becoming circular

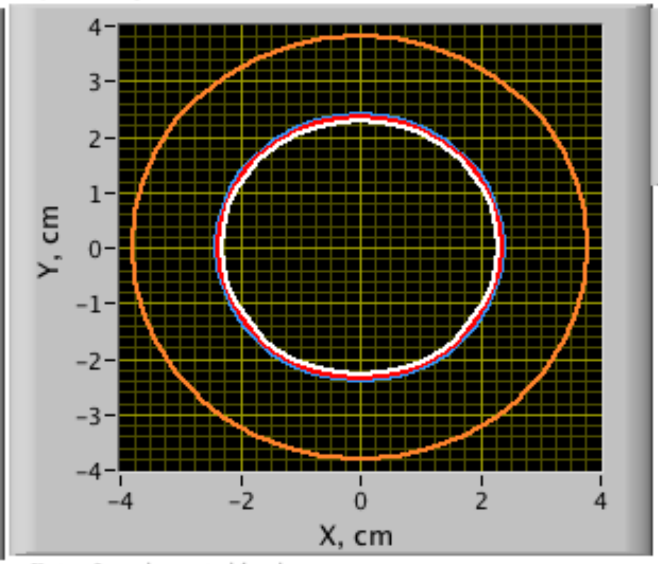


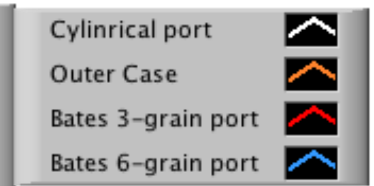
Comparative Burn Example (5)

SNC 4-port cross section



Cyindrical port cross section

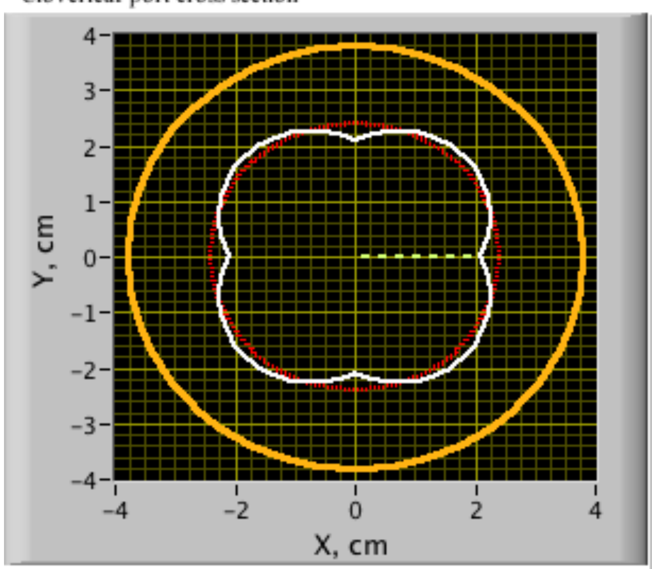




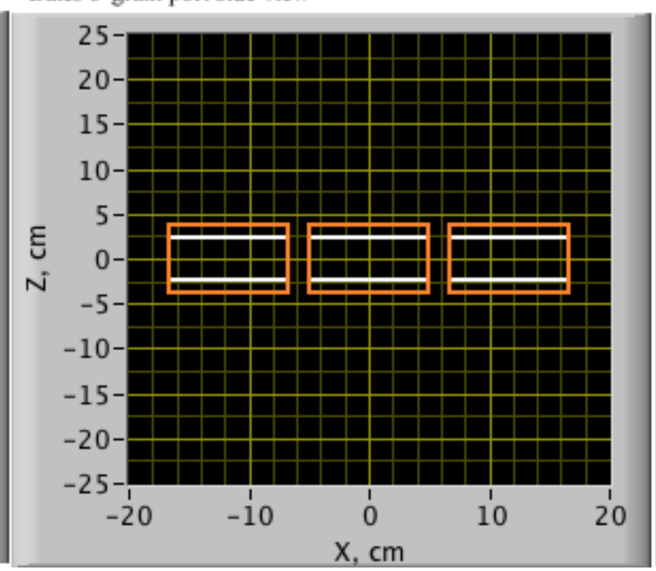
- Grain Geometry after 2-sec burn

- 4-Port grain completely burned

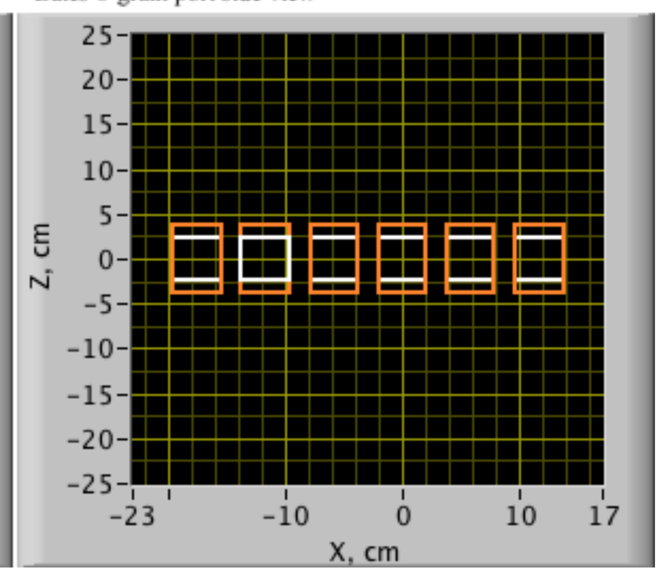
Cloverleaf port cross section



Bates 3-grain port side view

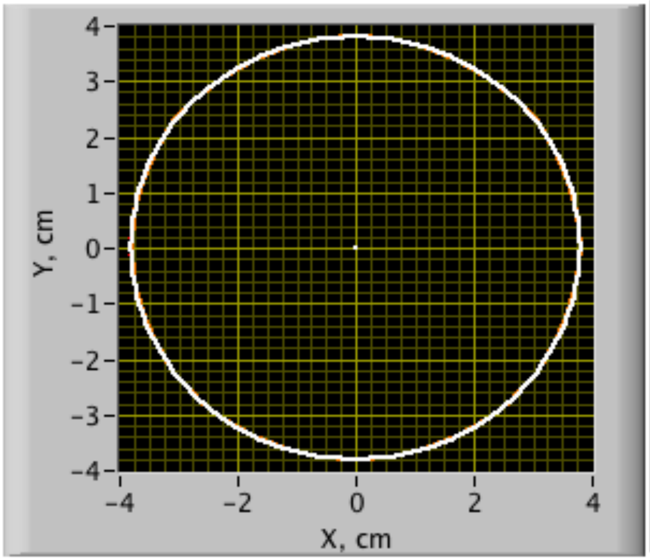


Bates 6-grain port side view

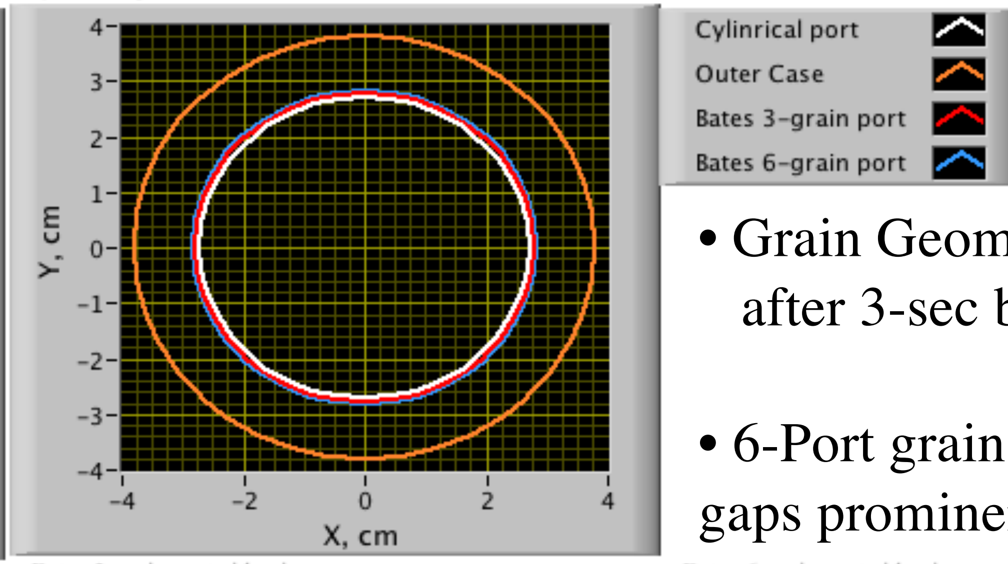


Comparative Burn Example (6)

SNC 4-port cross section



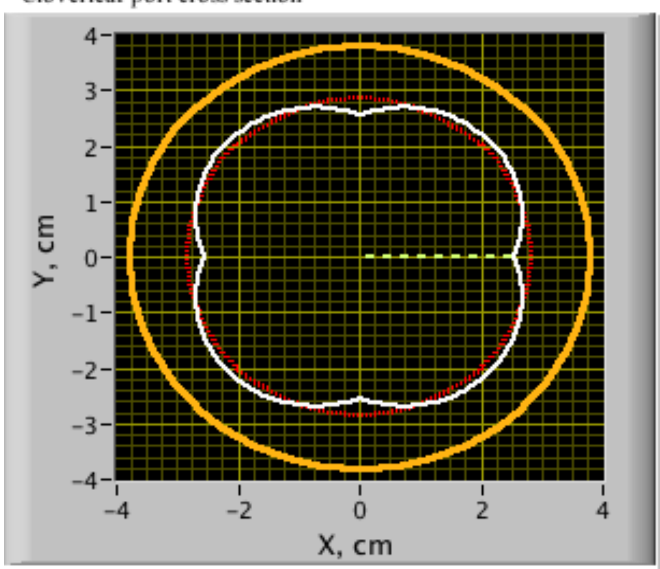
Cyindrical port cross section



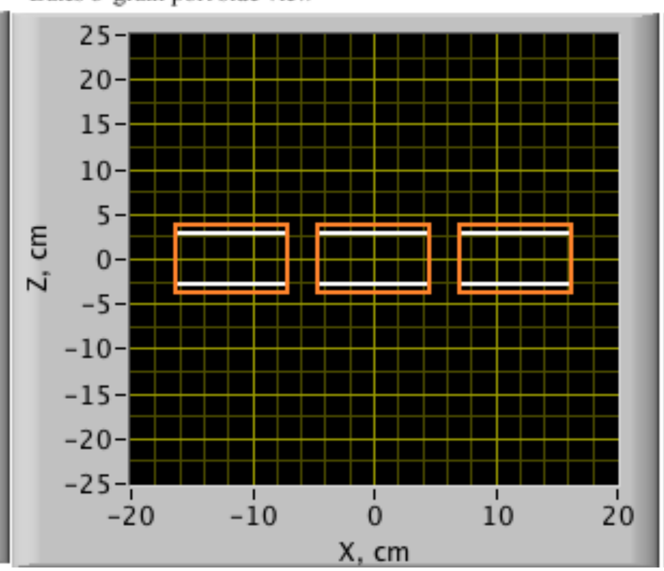
• Grain Geometry after 3-sec burn

• 6-Port grain longitudinal gaps prominent

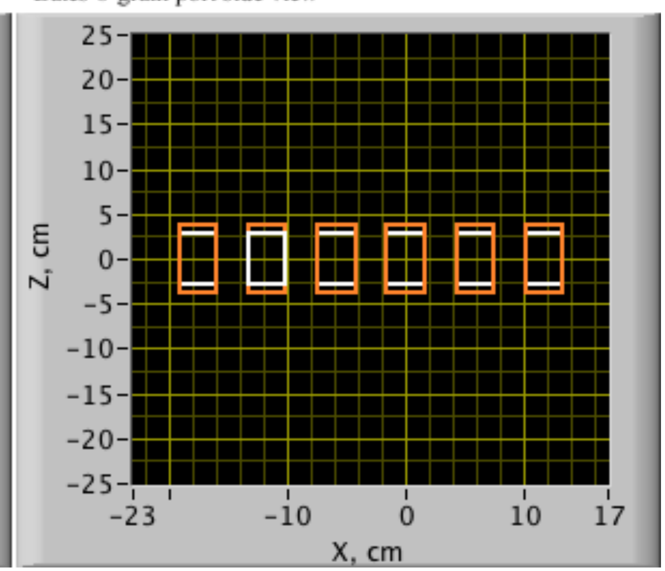
Cloverleaf port cross section



Bates 3-grain port side view

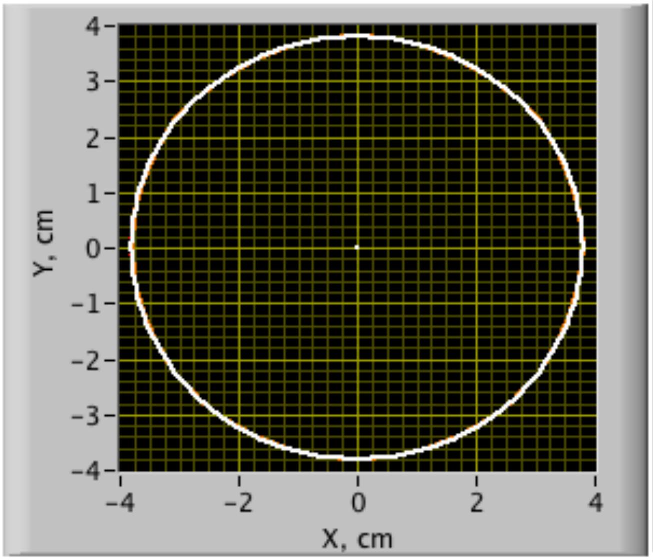


Bates 6-grain port side view

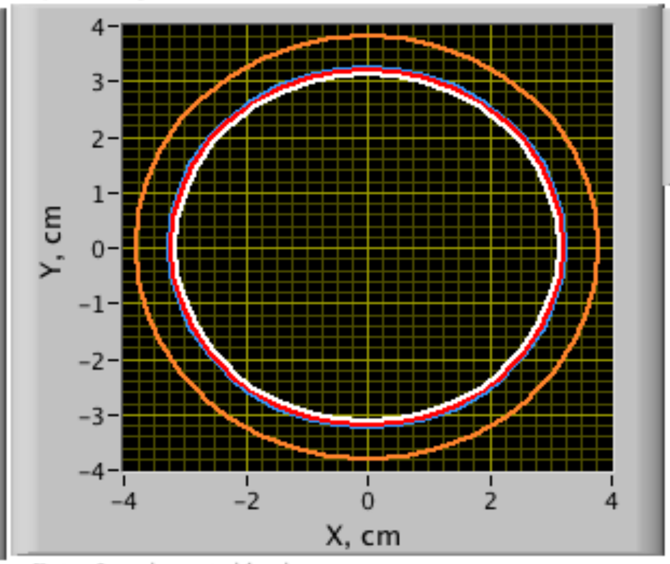


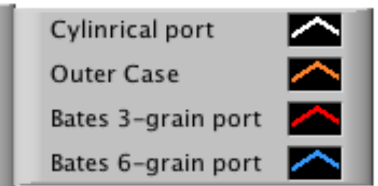
Comparative Burn Example (7)

SNC 4-port cross section



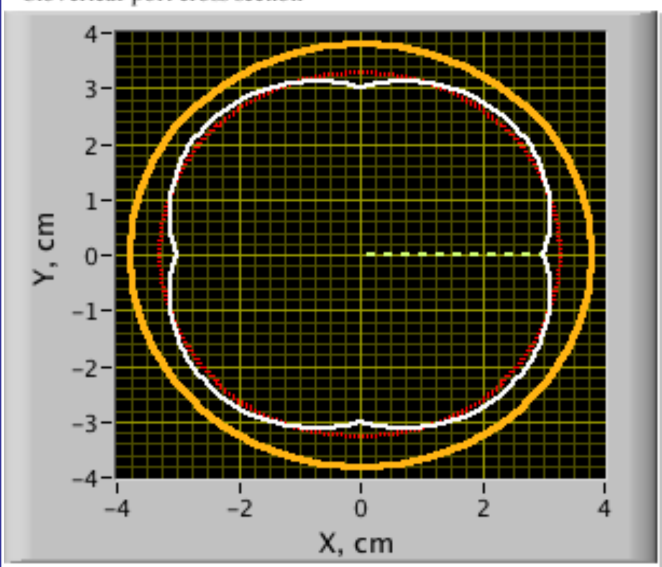
Cyindrical port cross section



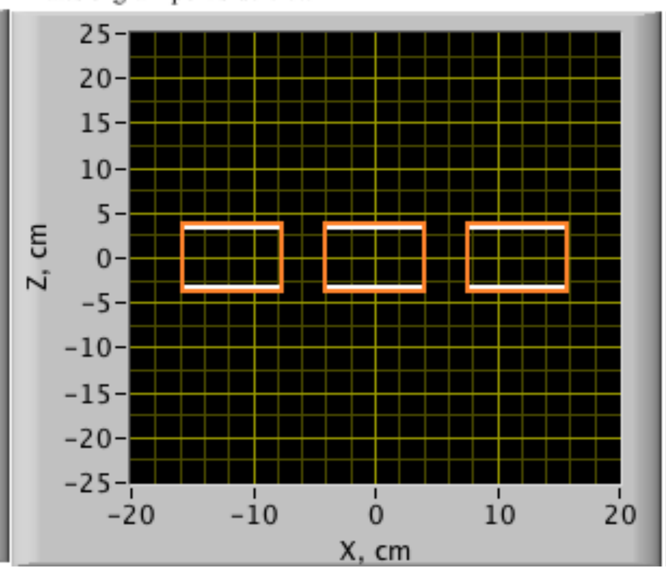


- Grain Geometry after 4-sec burn
- Cylindrical cross sections burning equally

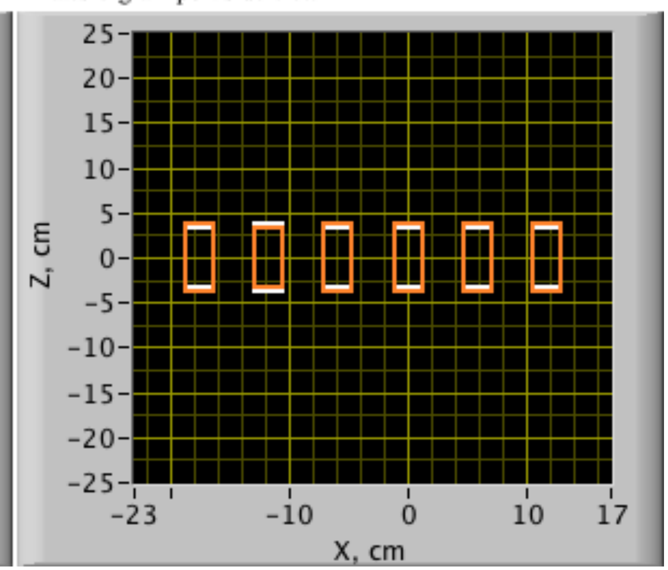
Cloverleaf port cross section



Bates 3-grain port side view

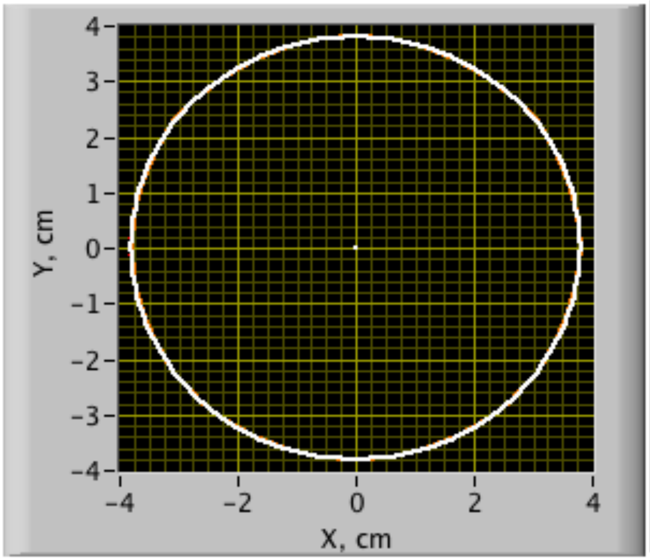


Bates 6-grain port side view

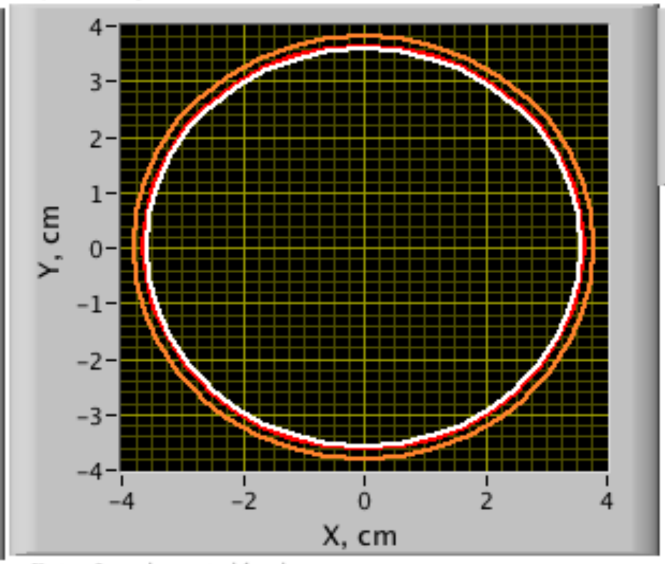


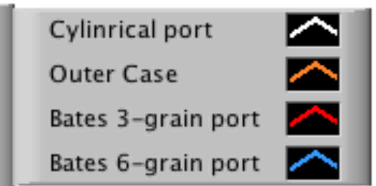
Comparative Burn Example (8)

SNC 4-port cross section



Cyindrical port cross section

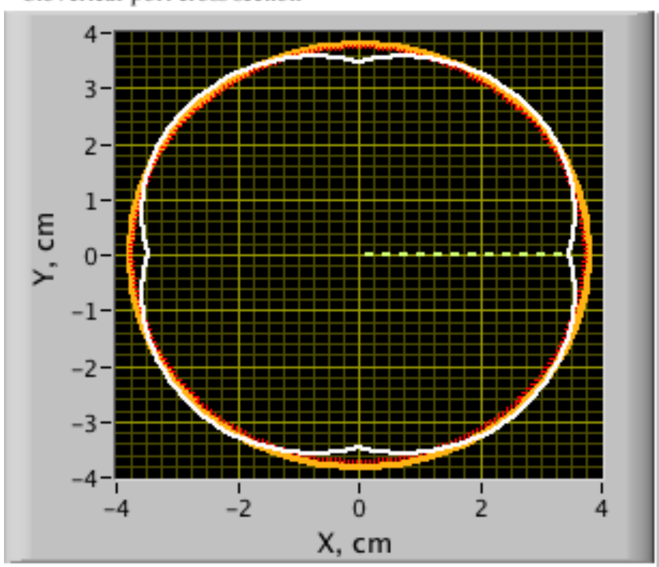




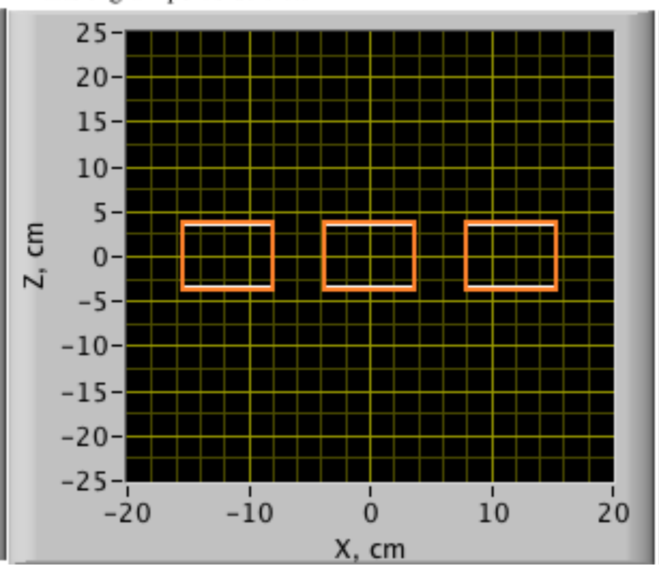
- Grain Geometry after 5-sec burn

- Clover Leaf grain essentially consumed

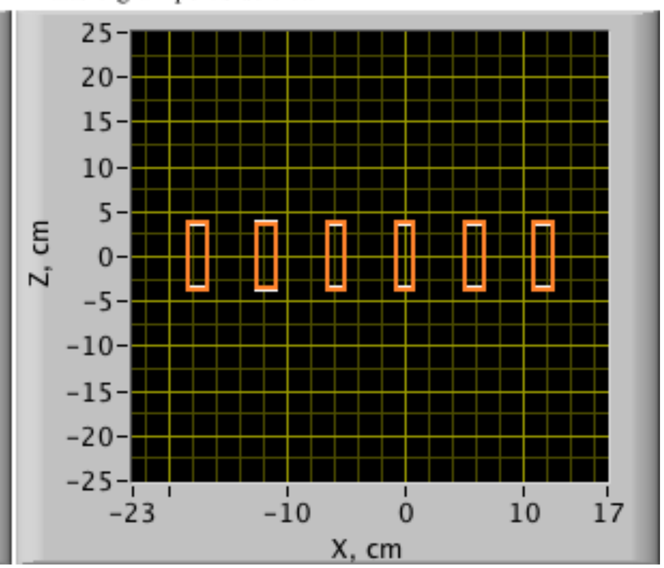
Cloverleaf port cross section



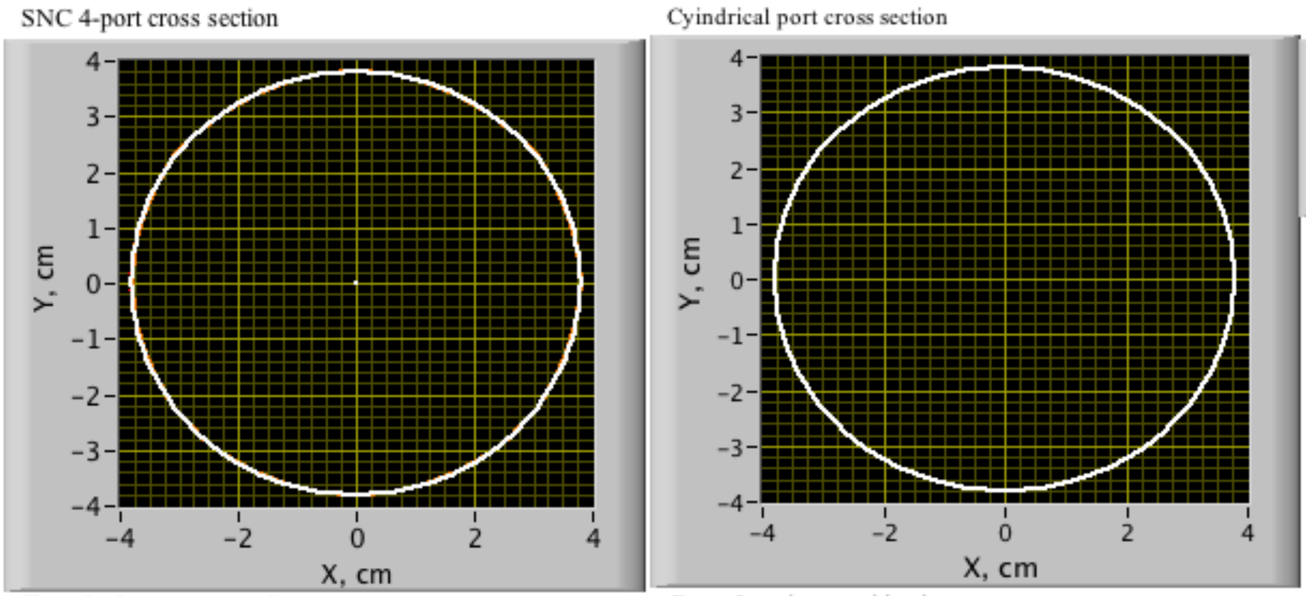
Bates 3-grain port side view

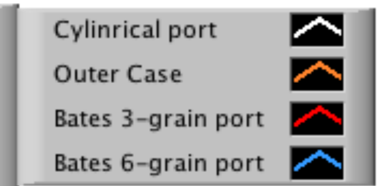


Bates 6-grain port side view

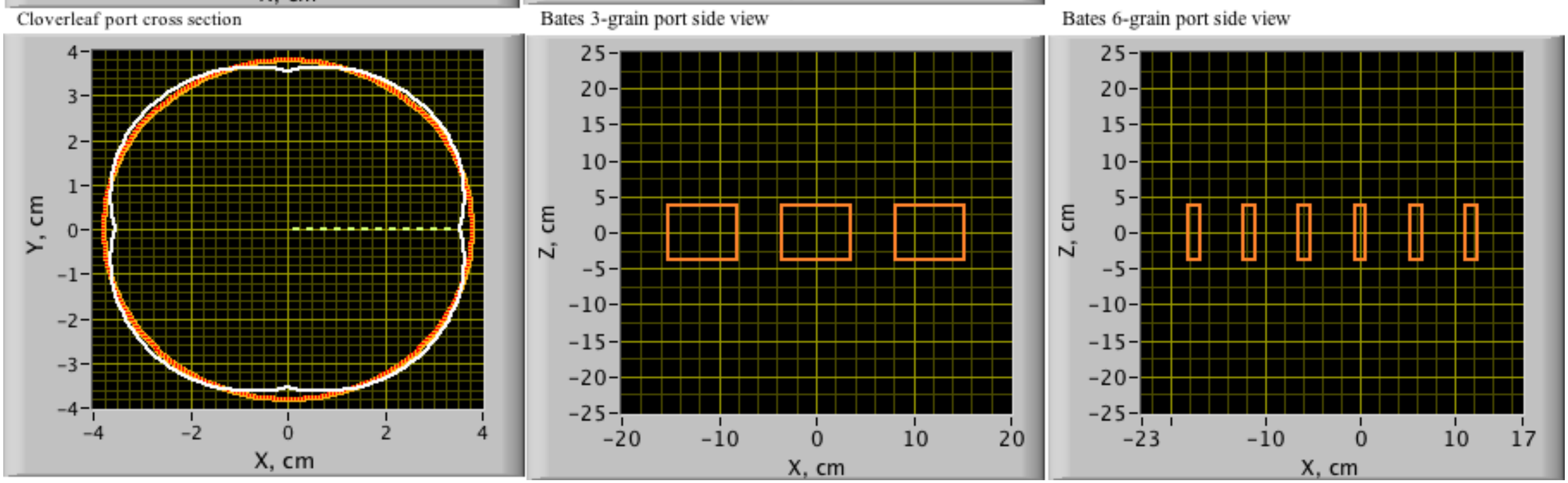


Comparative Burn Example (9)

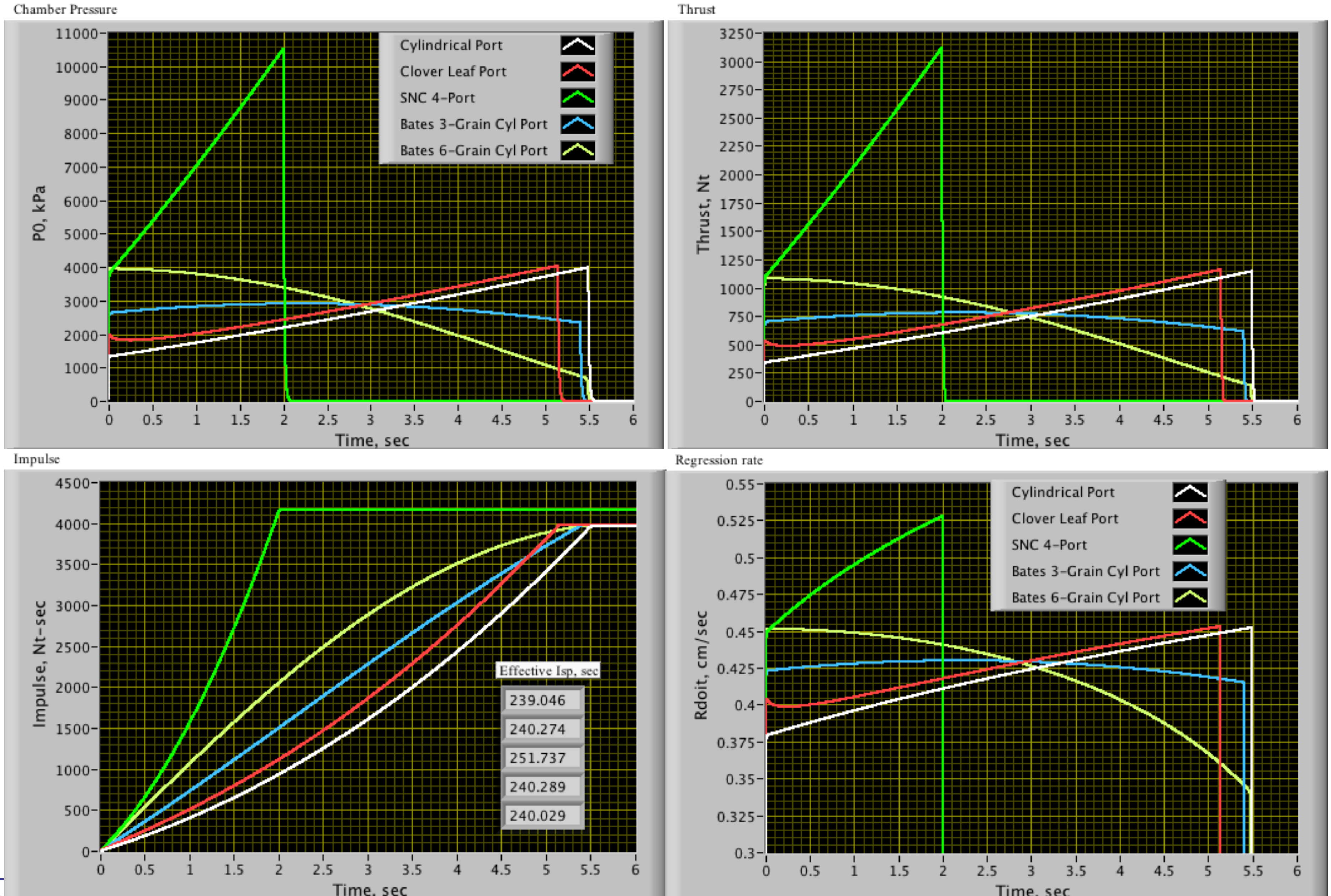




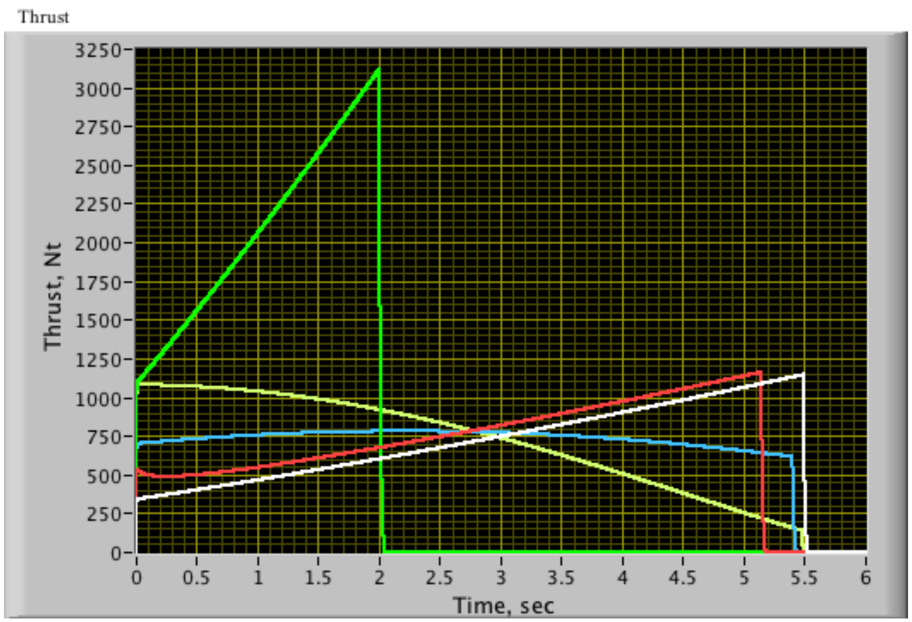
- Grain Geometry after 6-sec burn
- All Cylindrical port propellant consumed

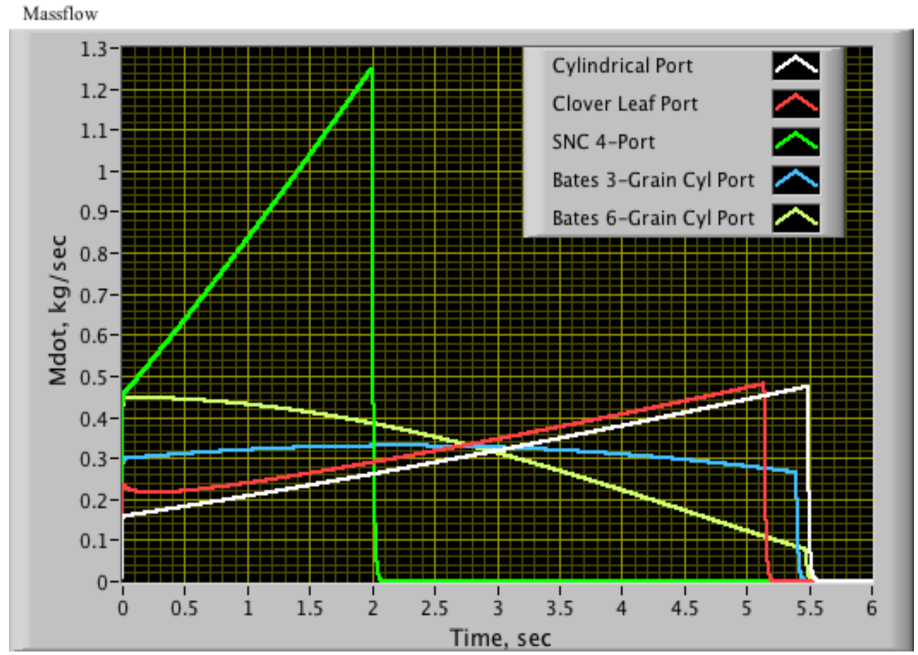


Comparative Burn Example (10)

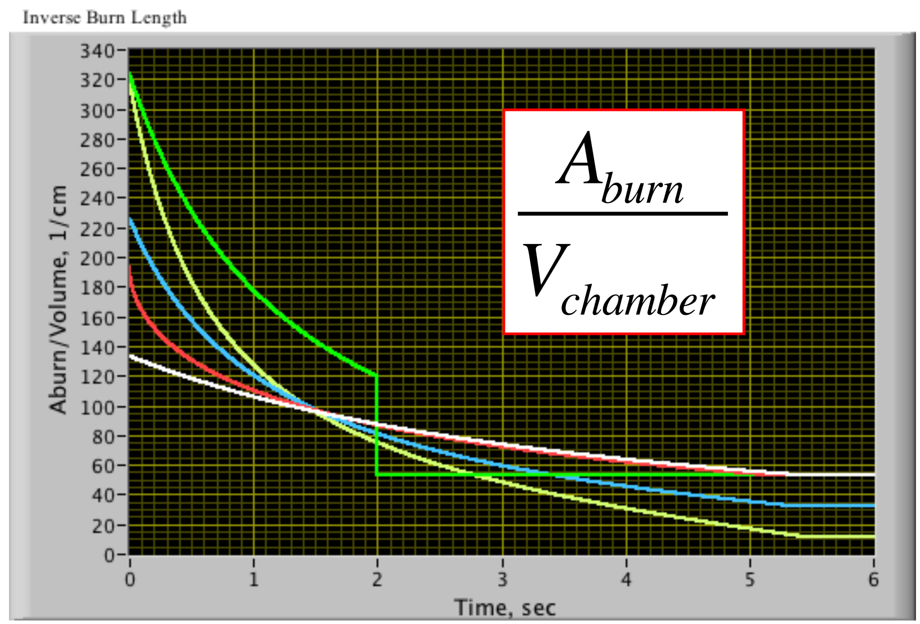


Comparative Burn Example (11)





Effective Isp, sec	Peak Thrust, Nt
239.046	1144.35
240.274	1159.84
251.737	3113.44
240.289	780.581
240.029	1080.45



Effects of Erosive Burning and Propellant Grain Fracture on Solid Propellants

NASA SP—8039

NASA
SPACE VEHICLE
DESIGN CRITERIA
(CHEMICAL PROPULSION)

SOLID ROCKET MOTOR
PERFORMANCE ANALYSIS
AND PREDICTION

NASA

CASE FILE
COPY

MAY 1971

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

Erosive Burning

1) **When high velocity or high mass flow hot gas from upstream combustion passes over** a downstream burning surface in a solid rocket motor local, chaotic increase in propellant burning rate results; phenomenon referred to as erosive burning.

2) **Two types of erosive burning;** velocity-based erosive burning and mass flux-based erosive burning. AP/composite propellants are more sensitive to the effect of the hot gas velocity flowing past burning propellant surface, some propellants (hybrids in particular) are more sensitive to the effect of the mass flux of the hot gas over the burning surface

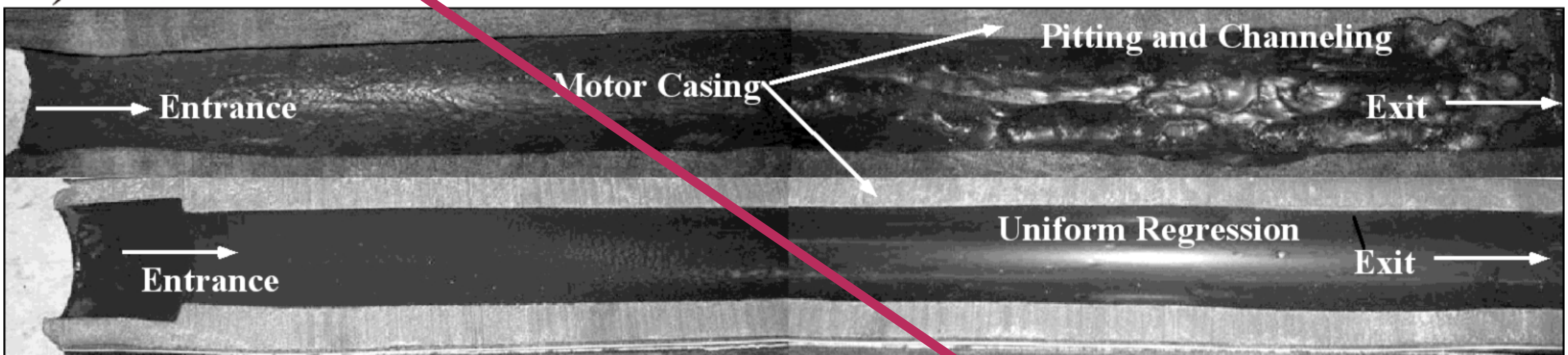
3) **Distinct thresholds for core combustion gas velocity and core mass flux** for the onset of velocity-based erosive burning and mass flux-based erosive burning.

4) **Erosive Burning** is nearly accompanied by a random burn rate element, making for a high variability on trust and total impulse levels for a given class of motors

Erosive Burning (2)

… St. Roberts Law strictly only "works" for non-erosive grain burns … Erosive burns are complex, typically chaotic, and hard to predict / analyze

a) Test 1

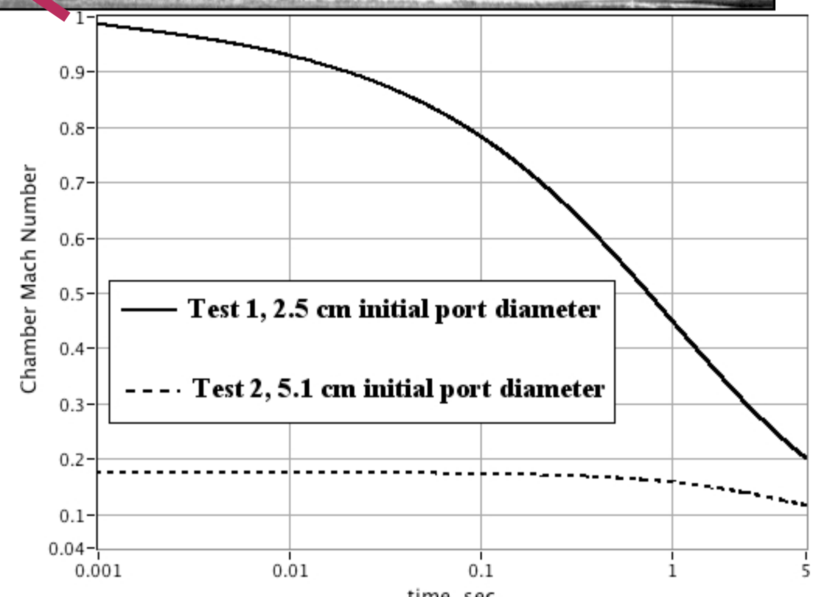


b) Test 2

Erosion Properties Highly Dependent on Chamber Flow Velocity

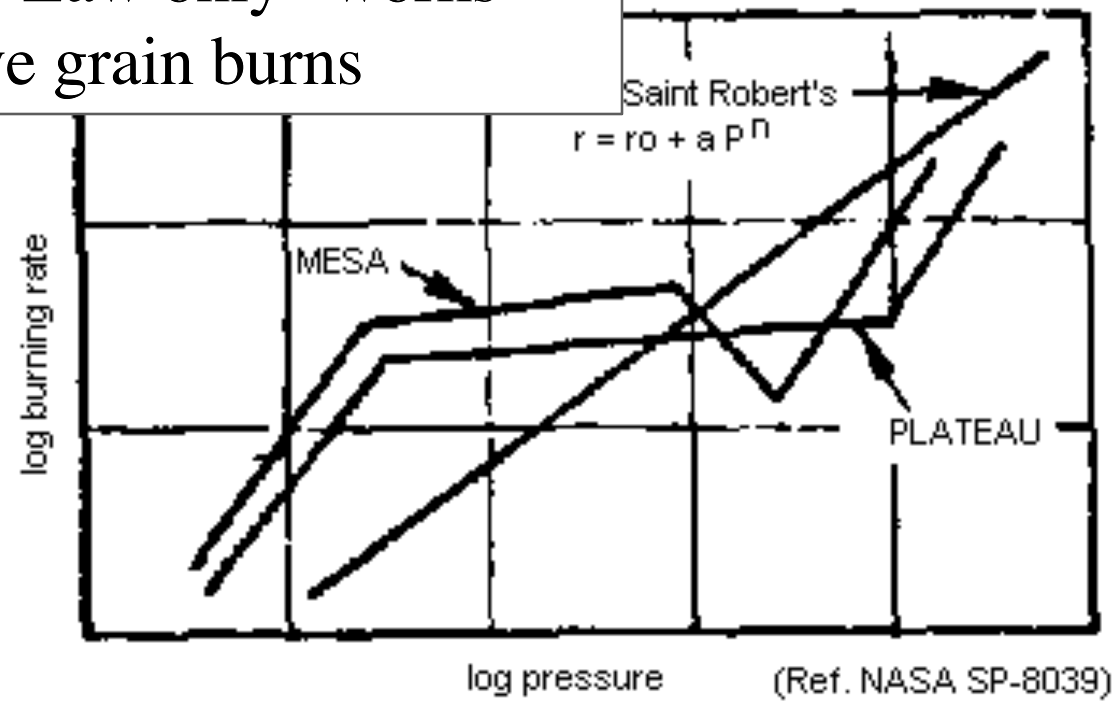
Eilers/Whitmore, JPC 2007,

AIAA-2007-5349



Erosive Burning (3)

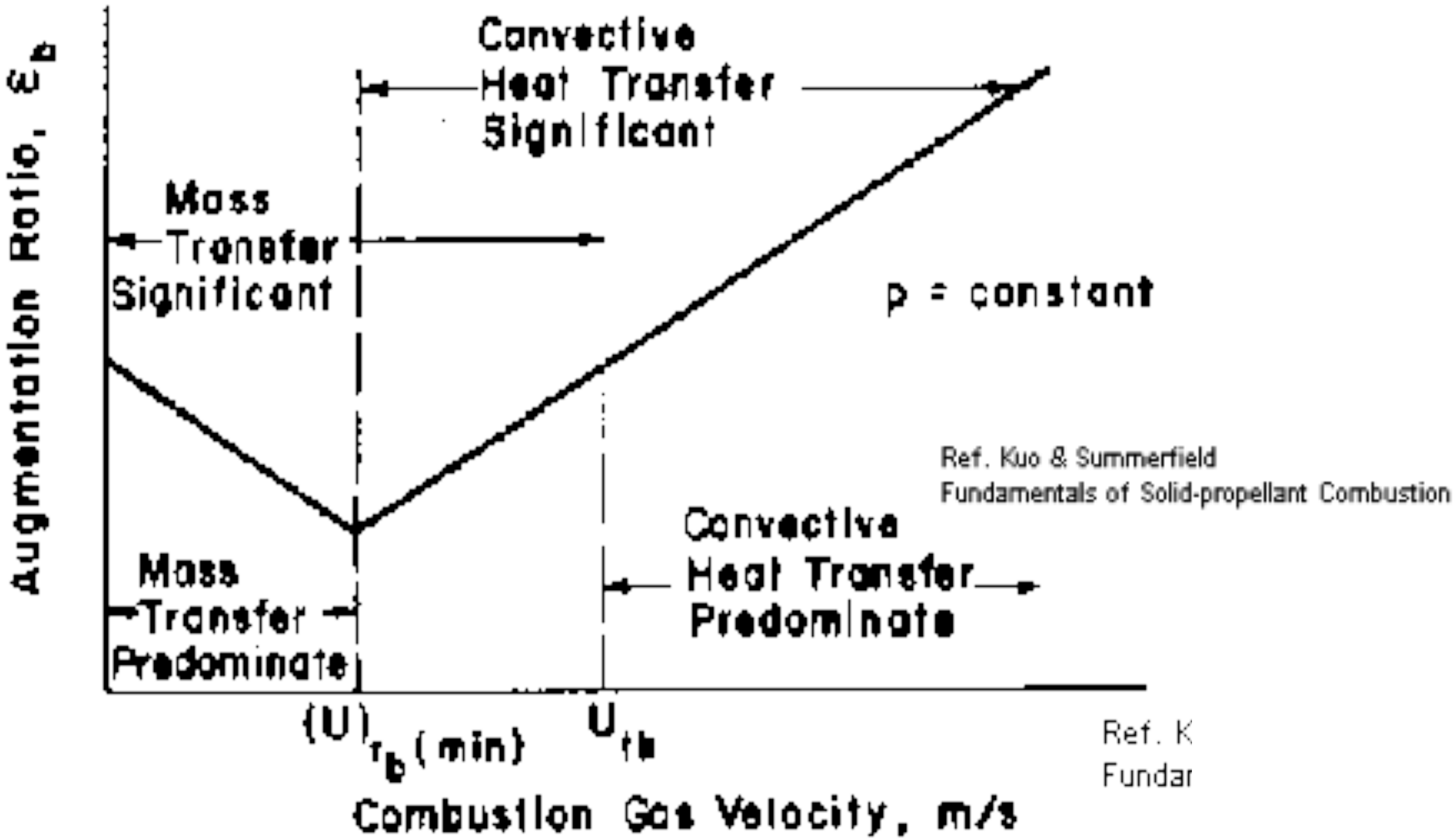
… St. Roberts Law only “works” for non-erosive grain burns



“Plateau effects” are result different surface regression rates --- pressure dependent

-- condensed phase combustion products also "pool" and retard heat transfer to the surface at elevated pressures.

Erosive Burning (4)

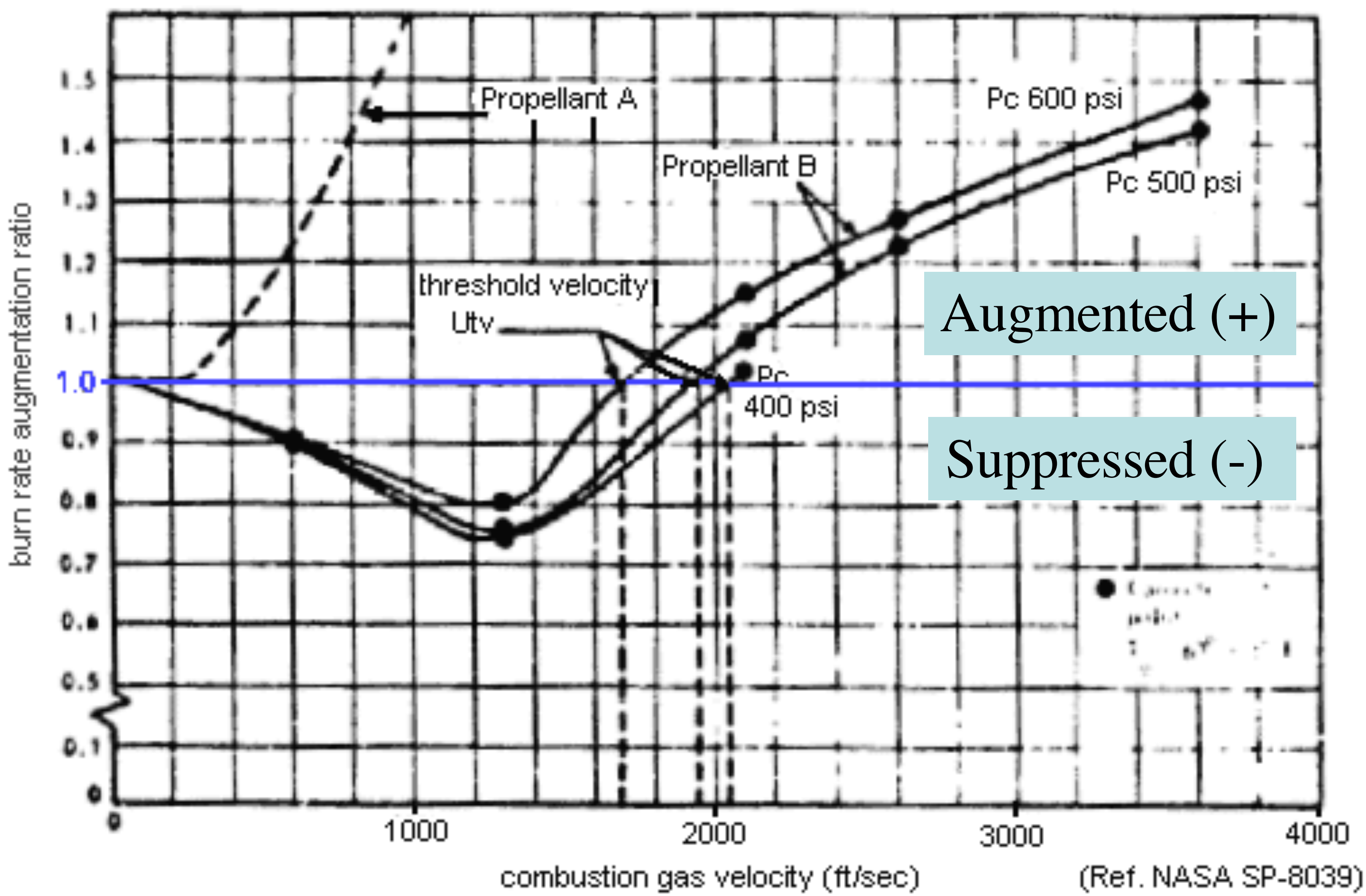


Heat Transfer Mechanisms as a Function of Combustion Gas Velocity

Erosive Burning (5)

- Most propellants have certain levels of combustion gas velocity (Mach number) that leads to an increased burning rate.
- ”Augmentation" of burn rate is referred to as *erosive burning*, chaotic and difficult to predict
- Physical mechanism -- increased convective heat transfer to the propellant surface – resulting from flow turbulence
- For many propellants, a *threshold* Mach number occurs.
- Below this flow level, no augmentation occurs, or a *decrease* in burn rate is experienced (negative erosive burning). *(Slag buildup)*
- *Both Augmented (+) and Suppressed (-) Erosive Burning result in Chaotic Behaviors*

Erosive Burning (6)



A Simple Erosive Burning Model

.. Mean Regression Rate Augmentation/Suppression

$$\dot{r}_{erosive} = \left(\dot{r}\right)_{\substack{Saint\\Roberts}} \cdot \left(\frac{1 + k \cdot \dfrac{M_{port}}{M_{crit}}}{1+k} \right) = aP_0^n \cdot \left(\frac{1 + k \cdot \dfrac{M_{port}}{M_{crit}}}{1+k} \right)$$

- *k ... empirical scale factor*
- M_{port} *... Port Mach number based on* A_{port}/A^*
- M_{crit} *... critical or thresh hold Mach number*

➢ Port Mach Number Above M_{crit} .. --> Burn rate is augmented

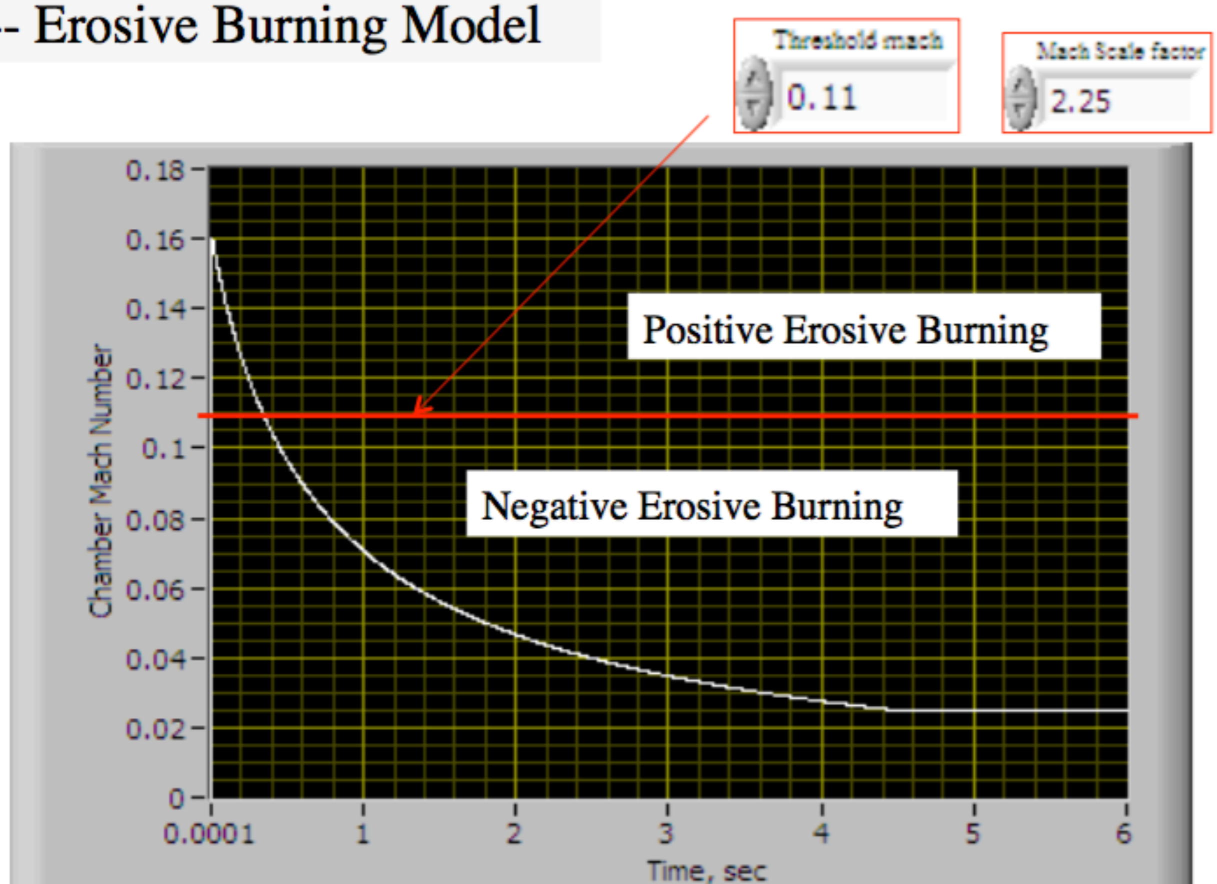
➢ Port Mach Number Above M_{crit} .. --> Burn rate is suppressed

A Simple Erosive Burning Model (2)

$$\dot{r}_{erosive} = \left(\dot{r}\right)_{\substack{Saint\\Roberts}} \cdot \left(\frac{1 + k \cdot \frac{M_{port}}{M_{crit}}}{1+k} \right) = aP_0^n \cdot \left(\frac{1 + k \cdot \frac{M_{port}}{M_{crit}}}{1+k} \right)$$

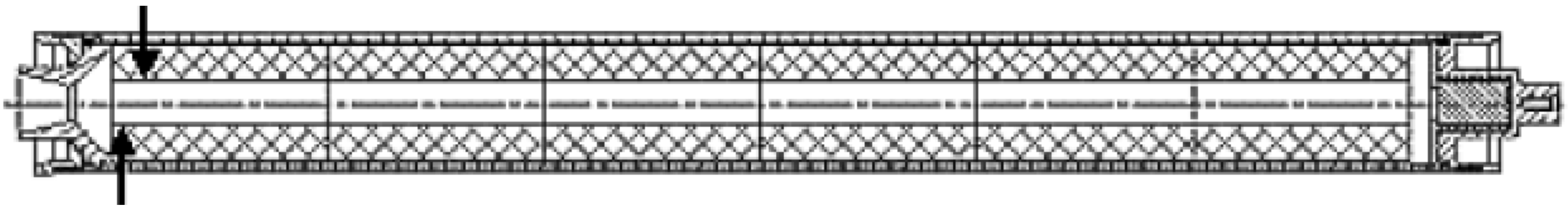
- *k … empirical scale factor*
- *M_{port} … Port Mach number based on A_{port}/A^**
- *M_{crit} … critical or thresh hold Mach number*

-- Erosive Burning Model



Preventing Erosive Burning

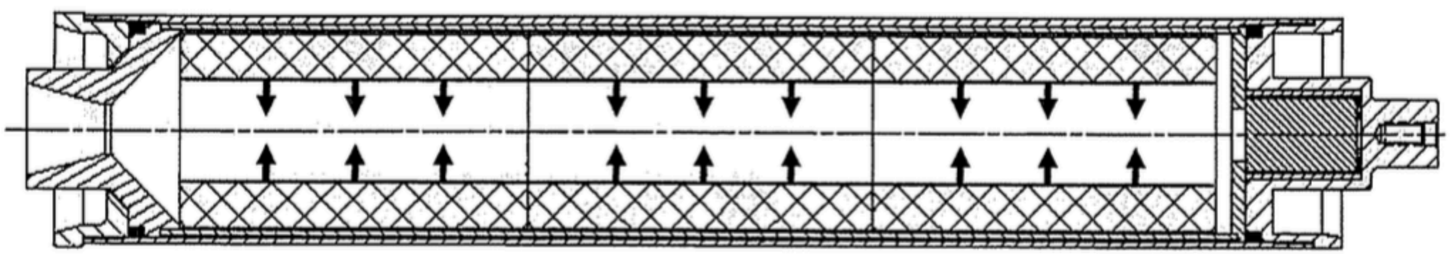
(From C. E. Rogers, "Erosive Burning Design Cruteria for High Power and Experimental/Amateur Solid Rocket Motors, High Power Rocketry, Vol. 36, No. 1, Jan. 2005)



..Effects of erosive burning can be minimized by designing the motor with a sufficiently large *port-to-throat area* ratio (A_{port}/A^*).
.. to keep combustor gas velocities below threshold Vaue

Preventing Erosive Burning (2)

For Any Motor Length (Short or Long), Reducing Core Diameter Increases Propellant Loading, More Propellant Loaded within Fixed Motor Volume. Maximizes Volumetric Loading (Total Impulse Installed within a Fixed Volume). Increased Rocket Flight Performance from Higher Total Impulse Installed within Volume or Length Available for Motor.



How Much Can Motor Core Diameter Be Reduced?

For Velocity-Based Erosive Burning:

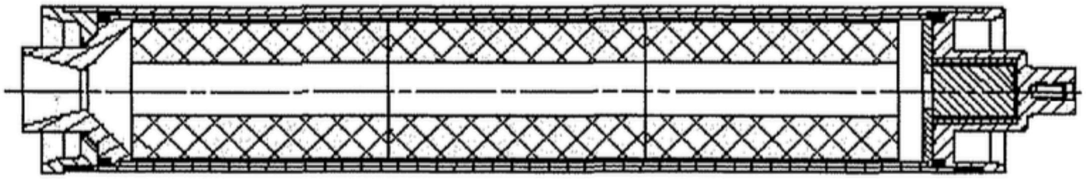
1) Reduced Core Diameter Reduces Port Area (A_p, the Core Cross-Sectional Area). Port Area Begins to Approach Fixed Throat Area (A_{th}). Port-to-Throat Area Ratio (A_p/A_{th}) Decreases, Core Mach Number Increases.
2) Increased Core Mach Number, Increased Velocity-Based Erosive Burning.

For Mass Flux-Based Erosive Burning:

1) Reduced Core Diameter Reduces Propellant Surface Area, Reducing Core Mass Flow Rate, but Port Area (Core Cross-Sectional Area) is Reduced at a Greater Rate. Result is an Increase in Core Mass Flux.
2) Increased Core Mass Flux, Increased Mass Flux-Based Erosive Burning.

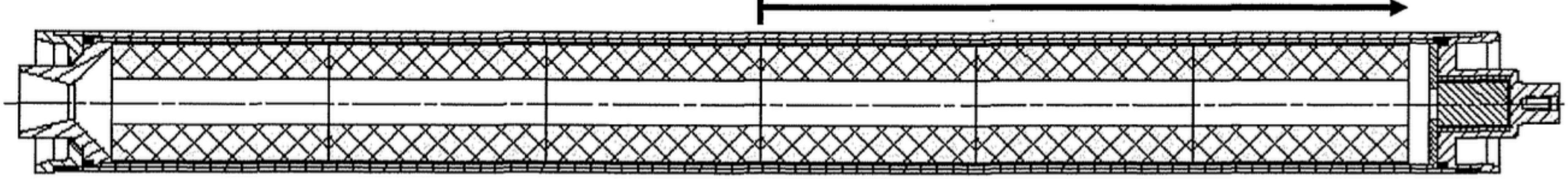
High Length-to-Diameter (L/D) Motors Increase Rocket Flight Performance.

Maximizes Total Impulse within a Given Frontal Area,
Minimizes Aerodynamic Drag in Minimum Diameter Rockets.



Keeping the Core Diameter the Same, Motor Propellant Grain Length is Increased.

How Much Can Motor Length Be Increased For a Given Motor Diameter?



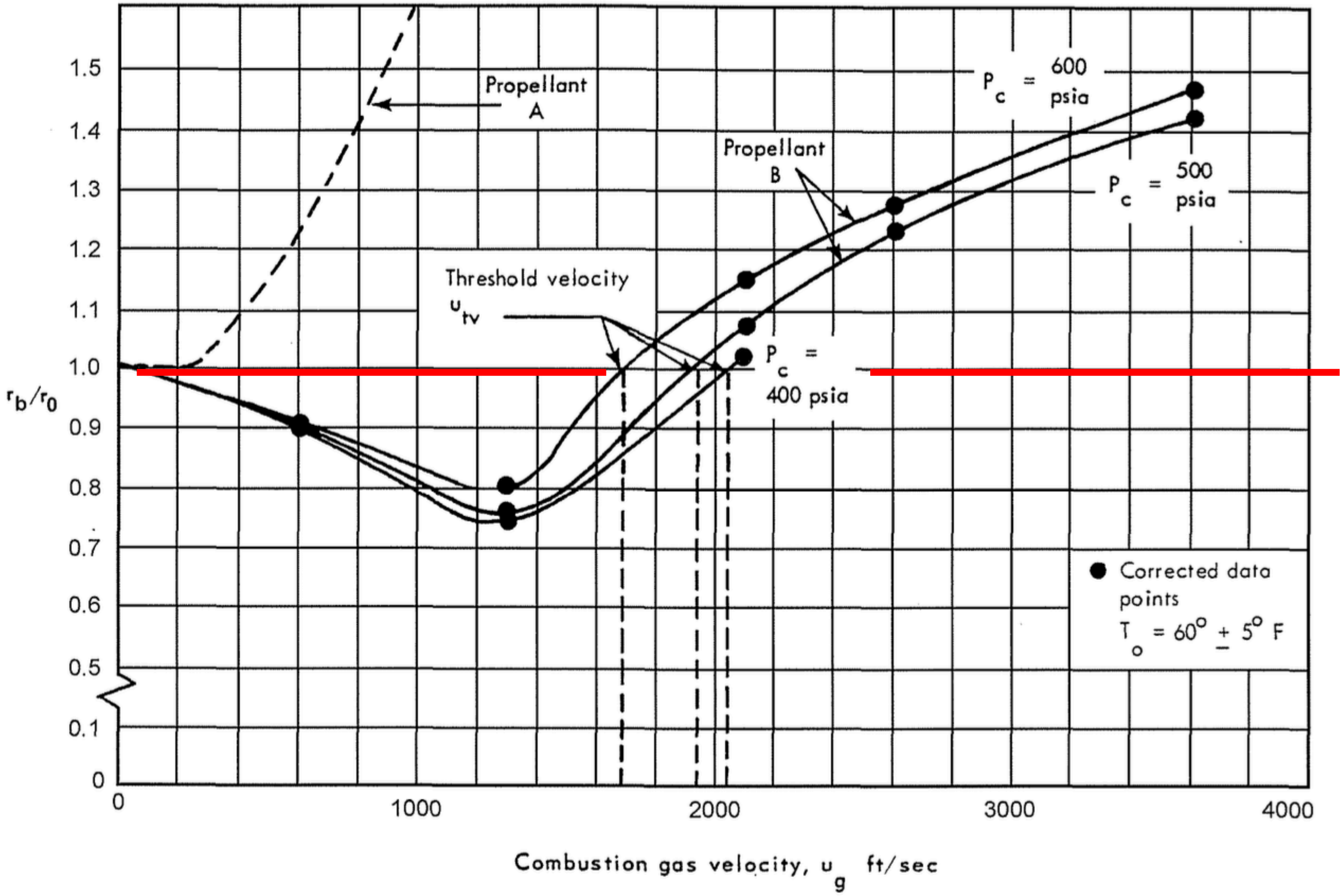
For Velocity-Based Erosive Burning:

1) Increased Propellant Grain Length Increases Propellant Surface Area.
2) For Same K_n, Increased Propellant Surface Area Requires Increase in Throat Area (A_{th}).
3) Increased Throat Area Approaches Port Area (A_p, the Core Cross-Sectional Area). Port-to-Throat Area Ratio (A_p/A_{th}) Decreases, Core Mach Number Increases, Increased Velocity-Based Erosive Burning.

For Mass Flux-Based Erosive Burning:

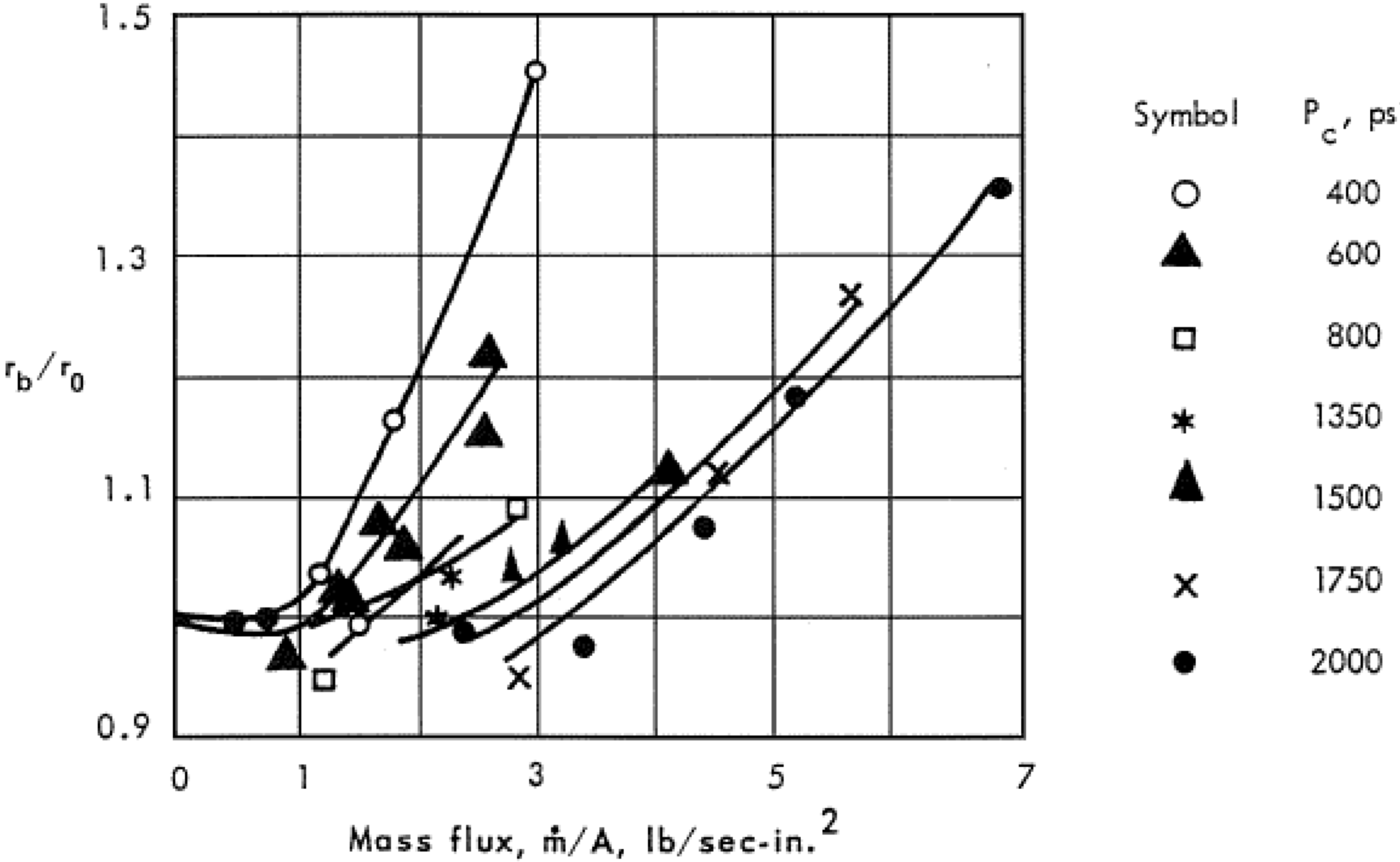
1) Increased Propellant Grain Length Increases Propellant Surface Area.
2) Increased Propellant Surface Area Increases Mass Flow Rate Down Core.
3) With Same Core Diameter, Port Area (Core Cross-Sectional Area) Remains the Same. Increased Core Mass Flow Rate through Same Core Cross-Sectional Area Results in Increased Core Mass Flux, Increased Mass Flux-Based Erosive Burning.

Preventing Erosive Burning (4)



Typical Effect of Combustion Gas Velocity on Burning Rate Augmentation - ***Velocity- Based Erosive Burning*****.**

Preventing Erosive Burning (5)



Typical Effect of Mass Flux on Burning Rate Augmentation - *Mass Flux-Based Erosive Burning*.

Rules of Thumb for Preventing Erosive Burning

- Non-Erosive (Safe Zone)

Mach Number

Core Mach Number < 0.50

For $\gamma = 1.2$; $A_{port}/A^* > 1.36$

Mass Flux

$P_0 = 400\text{-}600$ psia; Core Mass Flux < 1.0 lb/sec-in^2

$P_0 = 800$ psia; Core Mass Flux < 1.75 lb/sec-in^2

$P_0 = 1400$ psia; Core Mass Flux < 2.0 lb/sec-in^2

Rules of Thumb … (2)

- **Maximum Recommended Allowable Parameter at Erosive Burn Threshold: (Tickling Tail of the Dragon)**

Mach Number

Core Mach Number < 0.70

For $\gamma = 1.2$; $A_{port}/A^{*} > 1.10$

Mass Flux

$P_0 = 400\text{-}600$ *psia; Core Mass Flux < 2.0 lb/sec-in*^{2}

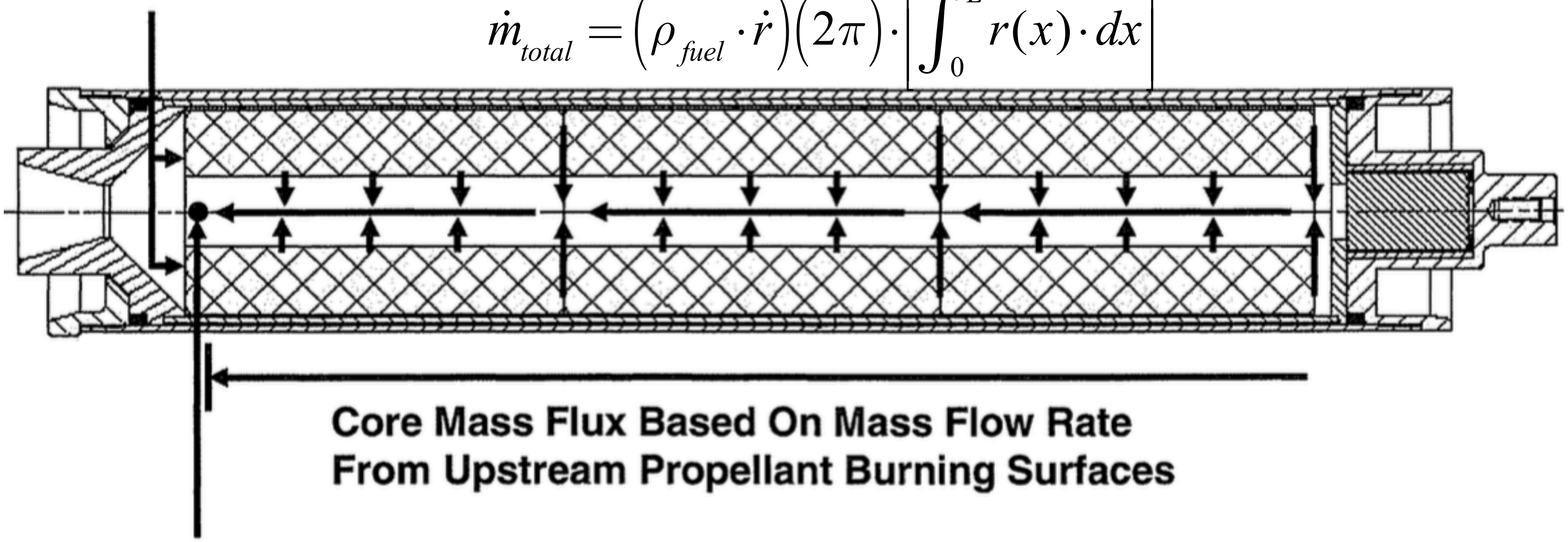
$P_0 = 800$ *psia; Core Mass Flux < 2.5 lb/sec-in*^{2}

$P_0 = 1400$ *psia; Core Mass Flux < 3.0 lb/sec-in*^{2}

Core Mass Flux limits for Max Recommended Erosivity should not be exceeded unless erosive Burning Characterization Tests are performed for propellant.

Fuel Port Core Geometry Design

$$\dot{m}_{total} = \left(\rho_{fuel} \cdot \dot{r}\right)\left(2\pi\right) \cdot \left[\int_0^L r(x) \cdot dx\right]$$



Core Mass Flux
at Aft End of Core.

Commonly Shortened to:

"Core Mass Flux"

For Erosive Burning Design Criteria
Core Mass Flux Based On Core Mass Flow Rate
Based On Non-Erosive Propellant Burn Rate

Most Important Core Mass Flux Condition is
the Highest Core Mass Flux, Which Occurs
at Aft End of Core at Motor Ignition.

Propellant Burning Surfaces Contributing to Core Mass Flux at Aft End of Core. … ***Core Mass Flux Based on Non-Erosive Burn Rate.***

Combined Core Mach Number/Core Mass Flux Erosive Burning Design Criteria for Motors With Constant Port Diameter

Step 1:

Initial Core Diameter Sized Based On Core Mach Number

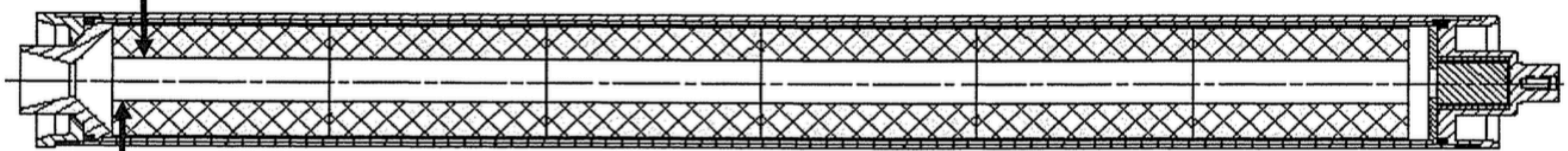
Non-Erosive; $M_a = 0.50$
$\gamma = 1.2$; $A_p/A_{th} = 1.36$

Max Erosive; $M_a = 0.70$
$\gamma = 1.2$; $A_p/A_{th} = 1.10$

Core Mach Number and Core Mass Flux Conditions are at Motor Ignition

Core Mass Flux Values Based on Non-Erosive Propellant Burn Rate

Maintain Constant Core Diameter From Head End to Aft End of Core



Step 2:

Check Core Mass Flux at Aft End of Core

If Core Mass Flux at Aft End of Core Higher Than Design Point Core Mass Flux, Increase Core Diameter to Reduce Core Mass Flux to Design Point Value.

Design Point Core Mass Flux (Recommended Values)

Non-Erosive;	p_c = 400-600 psia	Core Mass Flux ≤ 1.0 lb/sec-in^2
	p_c = 800 psia	Core Mass Flux ≤ 1.75 lb/sec-in^2
	p_c = 1400 psia	Core Mass Flux ≤ 2.0 lb/sec-in^2
Max Erosive;	p_c = 400 psia	Core Mass Flux = 2.0 lb/sec-in^2
	p_c = 600 psia	Core Mass Flux = 2.5 lb/sec-in^2
	p_c ≥ 800 psia	Core Mass Flux = 3.0 lb/sec-in^2

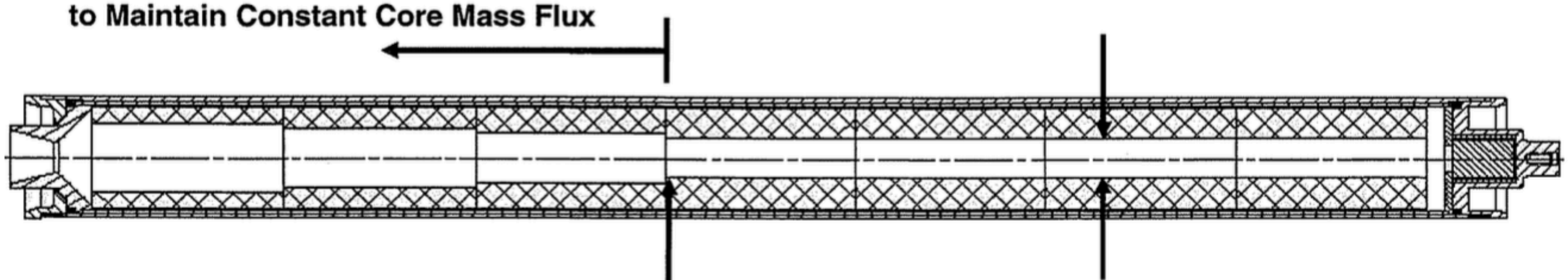
Constant Mass Flux Initial Port Design

Core Mach Number and Core Mass Flux Design Point Conditions are at Motor Ignition

Core Mass Flux Values Based on <u>Non-Erosive</u> Propellant Burn Rate

Provides Maximum Motor Length, Minimum Motor Core Diameter, Maximum Propellant Loading for a Given Level (Design Point) of Erosive Burning

Core Diameter Increased Past This Point to Maintain Constant Core Mass Flux



Initial Core Diameter Based On Design Point Core Mach Number

Non-Erosive; $M_a = 0.50$
$\gamma = 1.2$; $A_p/A_{th} = 1.36$

Max Erosive; $M_a = 0.70$
$\gamma = 1.2$; $A_p/A_{th} = 1.10$

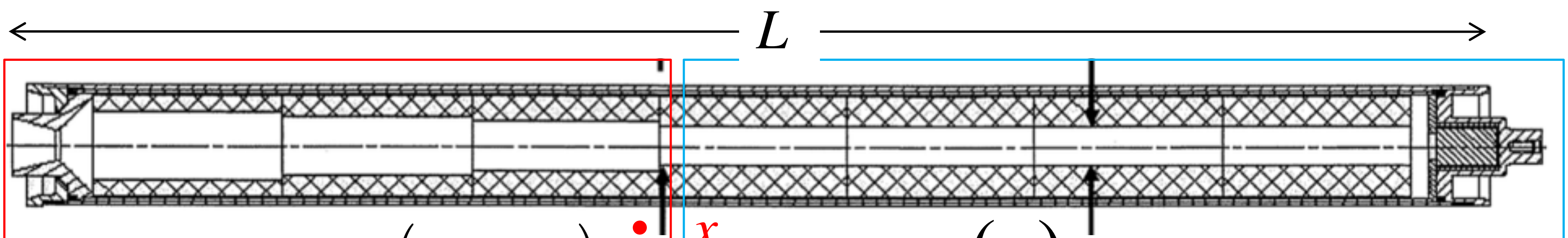
Design Point Core Mass Flux Achieved

Design Point Core Mass Flux (Recommended Values)

Non-Erosive;	p_c = 400-600 psia	Core Mass Flux ≤ 1.0 lb/sec-in^2
	p_c = 800 psia	Core Mass Flux ≤ 1.75 lb/sec-in^2
	p_c = 1400 psia	Core Mass Flux ≤ 2.0 lb/sec-in^2
Max Erosive;	p_c = 400 psia	Core Mass Flux = 2.0 lb/sec-in^2
	p_c = 600 psia	Core Mass Flux = 2.5 lb/sec-in^2
	p_c ≥ 800 psia	Core Mass Flux = 3.0 lb/sec-in^2

Constant Mass Flux Initial Port Design (2)

- Fix initial port radius until Flux > G_{max} at some critical location x_{cr}
- *Then grow port diameter to give constant massflux*



$$r(x) = r_0 + K \cdot (x - x_{cr})$$

$$r(x) = r_o$$

- *Up to* $x_{cr} \rightarrow r_{(x)} = r_0$

Augmented Burn Area

$$\rightarrow A_{burn} = 2\pi \cdot \left\{ \int_{x_{cr}}^{L} \left[r_0 + K \cdot (x - x_{cr}) \right] \cdot dx \quad + r_0 \cdot x_{cr} \right\} = 2\pi \cdot \left\{ r_0 \cdot (L - x_{cr}) + \int_{x_{cr}}^{L} \left[K \cdot (x - x_{cr}) \right] \cdot dx \quad + r_0 \cdot x_{cr} \right\} =$$

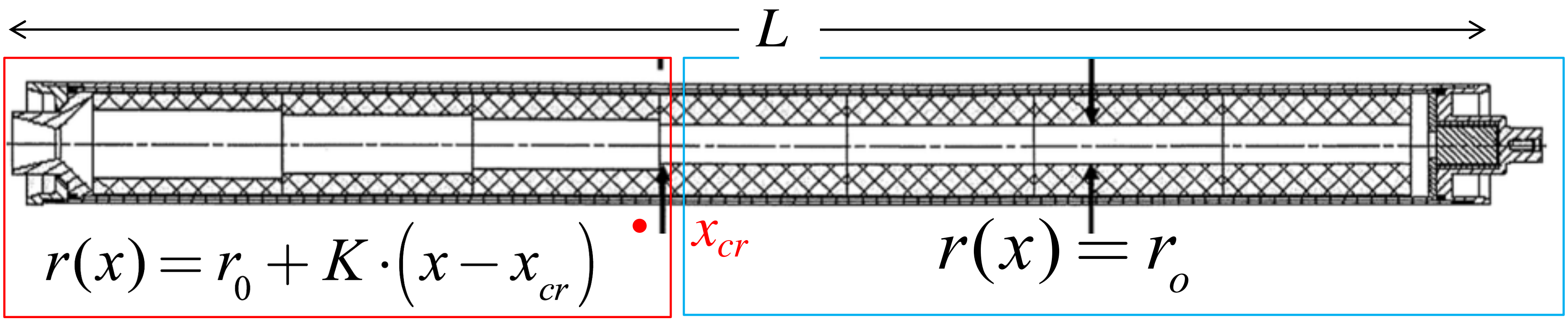
$$2\pi \cdot \left\{ r_0 \cdot L + \frac{1}{2} K (x - x_{cr})^2 \Big|_{x_{cr}}^{L} \right\} = 2\pi \cdot \left\{ r_0 \cdot L + \frac{1}{2} K (L - x_{cr})^2 \right\}$$

Augmented Massflow @ Exit

$$\rightarrow \dot{m}_{exit} = \rho_{fuel} \cdot \dot{r} \cdot A_{burn} = 2\pi \cdot \rho_{fuel} \cdot \dot{r} \cdot \left\{ r_0 \cdot L + \frac{1}{2} K (L - x_{cr})^2 \right\}$$

Constant Mass Flux Initial Port Design (3)

- Use Allowable G_{max} to find location of x_{cr}



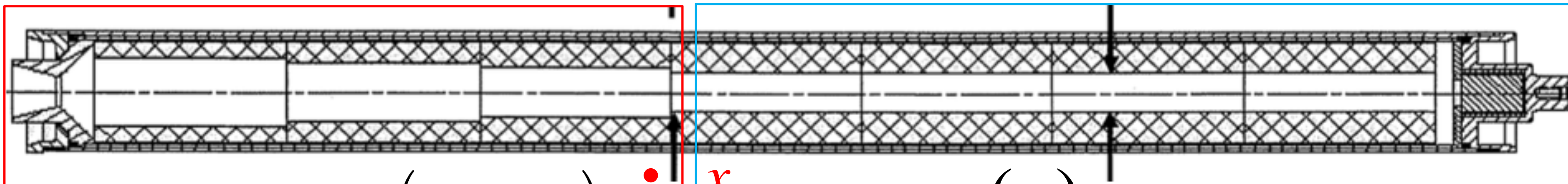
- At $x_{cr} \rightarrow \frac{\dot{m}_{x_{cr}}}{A_c} = G_{max}$ *for optimal propellant loading*

$$\rightarrow G_{max} = \frac{\rho_{fuel} \cdot \dot{r} \cdot A_{burn}}{\pi \cdot r_0^2} = \frac{2\pi \cdot \rho_{fuel} \cdot \dot{r} \cdot r_0 \cdot x_{cr}}{\pi \cdot r_0^2} = \frac{2 \cdot \rho_{fuel} \cdot \dot{r} \cdot x_{cr}}{r_0}$$

- *Solve for* x_{cr}

$$\rightarrow x_{cr} = \frac{G_{max} \cdot (r_0 / \dot{r})}{2 \cdot \rho_{fuel}}$$

Constant Mass Flux Initial Port Design (4)



$$r(x) = r_0 + K \cdot (x - x_{cr})$$

$$r(x) = r_o$$

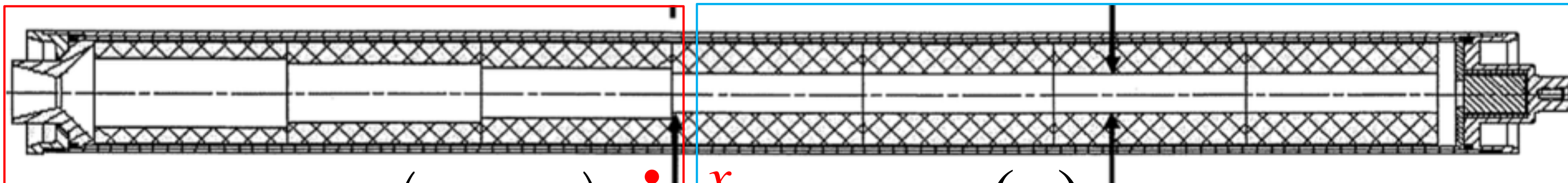
- Apply Max Flux Design Rule @ L

$$(G_{\max})_L \geq \frac{\dot{m}_L}{A_{c_x}} = \frac{2\pi \cdot (\rho_{fuel} \cdot \dot{r}) \cdot \left[r_0 \cdot L + \frac{1}{2} K \cdot (L - x_{cr})^2 \right]}{\pi \cdot r_L^2} = \frac{2\pi \cdot (\rho_{fuel} \cdot \dot{r}) \cdot \left[r_0 \cdot L + \frac{1}{2} K \cdot (L - x_{cr})^2 \right]}{\pi \cdot \left[r_0 + K \cdot (L - x_{cr}) \right]^2}$$

- *Solve for Threshold Value of K @ L*

$$K^2 + \left\{ \left(\frac{2r_0}{L - x_{cr}} \right) - \left(\frac{\rho_{fuel} \cdot \dot{r}}{G_{\max}} \right) \right\} \cdot K + \left(\frac{r_0}{L - x_{cr}} \right) \left\{ \left(\frac{r_0}{L - x_{cr}} \right) - 2 \left(\frac{\rho_{fuel} \cdot \dot{r}}{G_{\max}} \right) \cdot \left(\frac{L}{L - x_{cr}} \right) \right\} = 0$$

Constant Mass Flux Initial Port Design (5)



$$r(x) = r_0 + K\cdot\left(x - x_{cr}\right)$$

x_{cr}

$$r(x) = r_o$$

- Solve Quadratic Equation

$$K = \left\{\left(\frac{\rho_{fuel}\cdot\dot{r}}{2\cdot G_{\max}}\right) - \left(\frac{r_0}{L - x_{cr}}\right)\right\} \pm \sqrt{\left\{\left(\frac{\rho_{fuel}\cdot\dot{r}}{2\cdot G_{\max}}\right) - \left(\frac{r_0}{L - x_{cr}}\right)\right\}^2 + \left(\frac{r_0}{L - x_{cr}}\right)\left\{\left(\frac{\rho_{fuel}\cdot\dot{r}}{2\cdot G_{\max}}\right)\cdot\left(\frac{4\cdot L}{L - x_{cr}}\right) - \left(\frac{r_0}{L - x_{cr}}\right)\right\}}$$

Keep Root with Positive Value

.. K determines our radius growth slope at x_{cr}

Constant Massflux Design Example (6)

- Mean Combustion Pressure (Quasi Steady) of Augmented Grain

$$\frac{\partial P_0}{\partial t} = \frac{A_{burn} a P_o^{\,n}}{V_c}\left[\rho_p R_g T_0 - P_0\right] - P_0\left[\frac{A^*}{V_c}\sqrt{\gamma R_g T_0\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}}\right]$$

$$Quasi-Steady: \frac{A_{burn} a P_o^{\,n}}{V_c}\left[\rho_p R_g T_0 - P_0\right] = P_0\left[\frac{A^*}{V_c}\sqrt{\gamma R_g T_0\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}}\right]$$

$$\dot{r} = a P_o^{\,n} \rightarrow P_{0_{ss}}\left[\frac{A^*}{V_c}\sqrt{\gamma R_g T_0\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}} + \frac{A_{burn}\dot{r}}{V_c}\right] = \frac{A_{burn}\dot{r}}{V_c}\left[\rho_p R_g T_0\right]$$

$$\rightarrow Solve\ for\ P_0 \rightarrow \boxed{P_{0_{ss}} = \frac{\rho_p R_g T_0}{\left[\frac{A^*}{A_{burn}\dot{r}}\sqrt{\gamma R_g T_0\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}} + 1\right]}}$$

Constant Massflux Design Example (7)

• Mean Combustion Pressure (Quasi Steady), Account for Increased Burn Area due to expanded Port Radius

$$\rightarrow A_{burn} = 2\pi \cdot \left\{ r_0 \cdot L + \frac{1}{2} K \left(L - x_{cr} \right)^2 \right\}$$

$$P_{0_{ss}} = \frac{\rho_p R_g T_0}{\left[\frac{A^*}{A_{burn} \dot{r}} \sqrt{\gamma R_g T_0 \left(\frac{2}{\gamma+1} \right)^{\frac{\gamma+1}{(\gamma-1)}}} + 1 \right]} = \frac{\rho_p R_g T_0}{\left[\frac{\frac{A^*}{\dot{r}} \sqrt{\gamma R_g T_0 \left(\frac{2}{\gamma+1} \right)^{\frac{\gamma+1}{(\gamma-1)}}}}{2\pi \cdot \left\{ r_0 \cdot L + \frac{1}{2} K \left(L - x_{cr} \right)^2 \right\}} + 1 \right]}$$

Constant Massflux Design Example (8)

Pick $\left(G_{\max}\right)_L$ *for given pressure level*

$\rightarrow$ i.e. $400\text{-}600_{psia}\left(2{,}758\text{-}4{,}137_{kPa}\right) \rightarrow G\max \approx 1.00_{\ lbm/sec\text{-}in^2}\ (70.33_{g/sec\text{-}cm^2})$

$\rightarrow$ *Assume Starting* $\left(P_0\right)_{(j)}, \left(\dot{r}\right)_j = a \cdot \left(P_0^n\right)_{(j)}$

Calculate $\left(x_{cr}\right)_{(j)}$

$$\rightarrow \left(x_{cr}\right)_{(j)} = \frac{G_{max} \cdot \left(r_0 / \left(\dot{r}\right)_j\right)}{2 \cdot \rho_{fuel}}$$

- Algorithm, Iterate to convergence

Calculate $\left(K\right)_{(j)}$

$$\left(K\right)_{(j)} = \left\{\left(\frac{\rho_{fuel} \cdot \dot{r}}{2 \cdot G_{\max}}\right) - \left(\frac{r_0}{L - \left(x_{cr}\right)_{(j)}}\right)\right\} \pm \sqrt{\left[\left(\frac{\rho_{fuel} \cdot \left(\dot{r}\right)_j}{2 \cdot G_{\max}}\right) - \left(\frac{r_0}{L - \left(x_{cr}\right)_{(j)}}\right)\right]^2 + \left(\frac{r_0}{L - \left(x_{cr}\right)_{(j)}}\right)\left[\left(\frac{\rho_{fuel} \cdot \left(\dot{r}\right)_j}{2 \cdot G_{\max}}\right) \cdot \left(\frac{4 \cdot L}{L - \left(x_{cr}\right)_{(j)}}\right) - \left(\frac{r_0 \cdot L}{L - \left(x_{cr}\right)_{(j)}}\right)\right]}$$

Re– *calculate* $P_0, \dot{r}$

$$\rightarrow \left(P_0^n\right)_{(j+1)} = \frac{\rho_p R_g T_0}{\left[\dfrac{\dfrac{A^*}{\left(\dot{r}\right)_j}\sqrt{\gamma R_g T_0 \left(\dfrac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}}}{2\pi \cdot \left(r_0 \cdot L + \left\{r_0 + \left(K\right)_{(j)} \cdot \left(L - \left(x_{cr}\right)_{(j+1)}\right)^2\right\}\right)} + 1\right]}$$

$$\left(\dot{r}\right)_{j+1} = a \cdot \left(P_0^n\right)_{(j+1)}$$

Constant Massflux Design Example (9)

- Initial Erosion Compensated Port Geometry

Port Radius

$$x > x_{cr}$$

$$\rightarrow r(x) = r_0 + K \cdot (x - x_{cr})$$

else

$$\rightarrow r(x) = r_0$$

Burn Area

$$\rightarrow A_{burn} = 2\pi \cdot \left\{ r_0 \cdot L + \frac{1}{2} K \left(L - x_{cr} \right)^2 \right\}$$

Port Volume

$$V_{port} = \pi \cdot \left(x_{cr} \cdot r_0^2 + \int_{x_{cr}}^{L} r^2 \, dx \right) =$$

$$\pi \cdot \left(x_{cr} \cdot r_0^2 + \frac{(L - x_{cr})}{3} \cdot \left\{ \left(r_0 + K \cdot (L - x_{cr}) \right)^2 + r_0 \cdot \left(r_0 + K \cdot (L - x_{cr}) \right)^2 + r_0^2 \right\} \right)$$

Example Calculation

Motor Parameters

Geometry

- Upstream Port Diameter, d0, cm: 1
- Port Length, cm: 28
- Throat Diameter, cm: 0.85
- Grain Diameter, cm: 3

Properties of Propellant Combustion Products

- Effective gamma: 1.2
- Effective MW: 25
- Propellant Density KG/M^3: 1260
- Idealized Flame Temperature, deg. K: 2900

Saint Roberts Burn Parameters

- Burn Multiplier, a cm/sec-kPa^n: 0.25
- Burn Exponent, n: 0.175

- Start pressure, kPa: 3400
- Max Number of Iterations`: 20
- Port mass Flux Erosive Burning Threshold g/sec-cm^2 2: 70.33
- Convergence error: 0.00010
- Relaxation Factor: 0.7500(

Iteration Array

rdot, cm/sec	1.07398	1.07402	1.07404
mdot, g/sec	155.299	155.334	155.35
Kscale	0.0192409	0.0192416	0.019242
Mean Radius, cm	0.577377	0.577386	0.57739
Aburn, cm^2	115.191	115.193	115.195
xcrit2, cm	12.9932	12.9927	12.9924
P02.kPa	4145.58	4145.85	4145.97
Port Volume, cm^3	30.1076	30.1086	30.1091
EXIT Port Diameter, cm	1.57749	1.57753	1.57755
Aexit, cm^2	1.95444	1.95455	1.95459
Mdot, xcr, g/sec	55.2371	55.2371	55.2371
G, xcr, g/sec 2	70.33	70.33	70.33

# of iterations: 10

Example Calculation (2)

Propellant Mass, MassFlow

Uncompensated Initial Mass, g	221.671
Compensated Initial Mass, g	211.442
% mass loss	-4.72328
Uncompensated Propellant Massflow, g/sec	155.36
Compensated Propellant Massflow, g/sec	155.35
% massflow Loss	-0.00644265

Iteration Array (index: 8)

rdot, cm/sec	1.07398	1.07402	1.07404
mdot, g/sec	155.299	155.334	155.35
Kscale	0.0192409	0.0192416	0.019242
Mean Radius, cm	0.577377	0.577386	0.57739
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Aexit, cm^2	1.95444	1.95455	1.95459
Mdot, xcr, g/sec	55.2371	55.2371	55.2371
G, xcr, g/sec 2	70.33	70.33	70.33

Final Iteration

rdot, cm/sec	1.07404
mdot, g/sec	155.35
Kscale	0.019242
Mean Radius, cm	0.57739
Aburn, cm^2	115.195
xcrit2, cm	12.9924
P02.kPa	4145.97
Port Volume, cm^3	90.1009
EXIT Port Diameter, cm	1.57755
Aexit, cm^2	1.95459
Mdot, xcr, g/sec	55.2371
G, xcr, g/sec 2	70.33

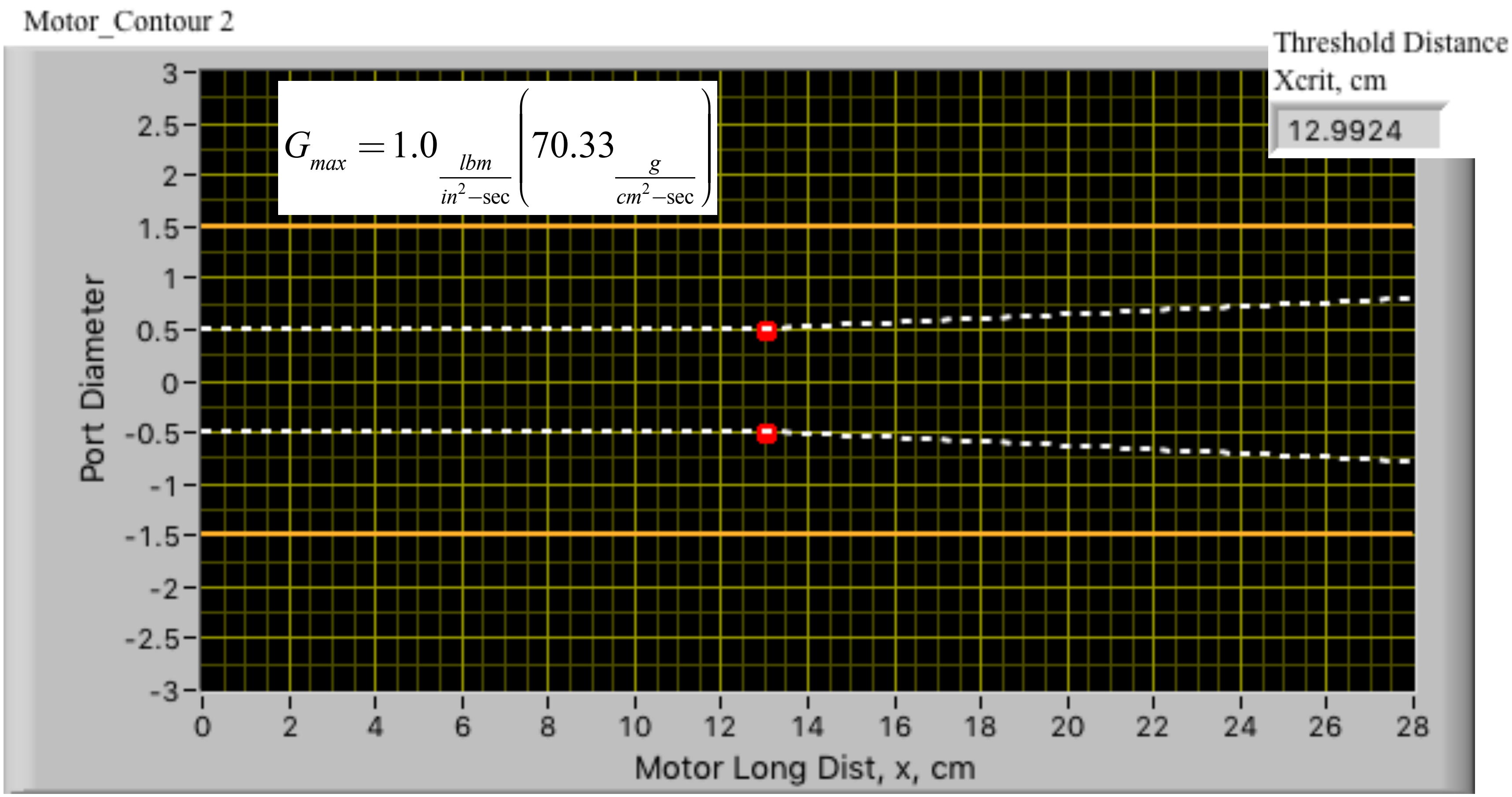
Threshold Distance Xcrit, cm

12.9924

Example Calculation (3)

- Adjusted Port Diameter →

$$K=\left\{\left(\frac{\rho_{fuel}\cdot\dot{r}}{2\cdot G_{\max}}\right)-\left(\frac{r_0}{L-x_{cr}}\right)\right\}\pm\sqrt{\left\{\left(\frac{\rho_{fuel}\cdot\dot{r}}{2\cdot G_{\max}}\right)-\left(\frac{r_0}{L-x_{cr}}\right)\right\}^2+\left(\frac{r_0}{L-x_{cr}}\right)\left\{\left(\frac{\rho_{fuel}\cdot\dot{r}}{2\cdot G_{\max}}\right)\cdot\left(\frac{4\cdot L}{L-x_{cr}}\right)-\left(\frac{r_0}{L-x_{cr}}\right)\right\}}$$



$$G_{max}=1.0\,\frac{lbm}{in^2-\sec}\left(70.33\,\frac{g}{cm^2-\sec}\right)$$

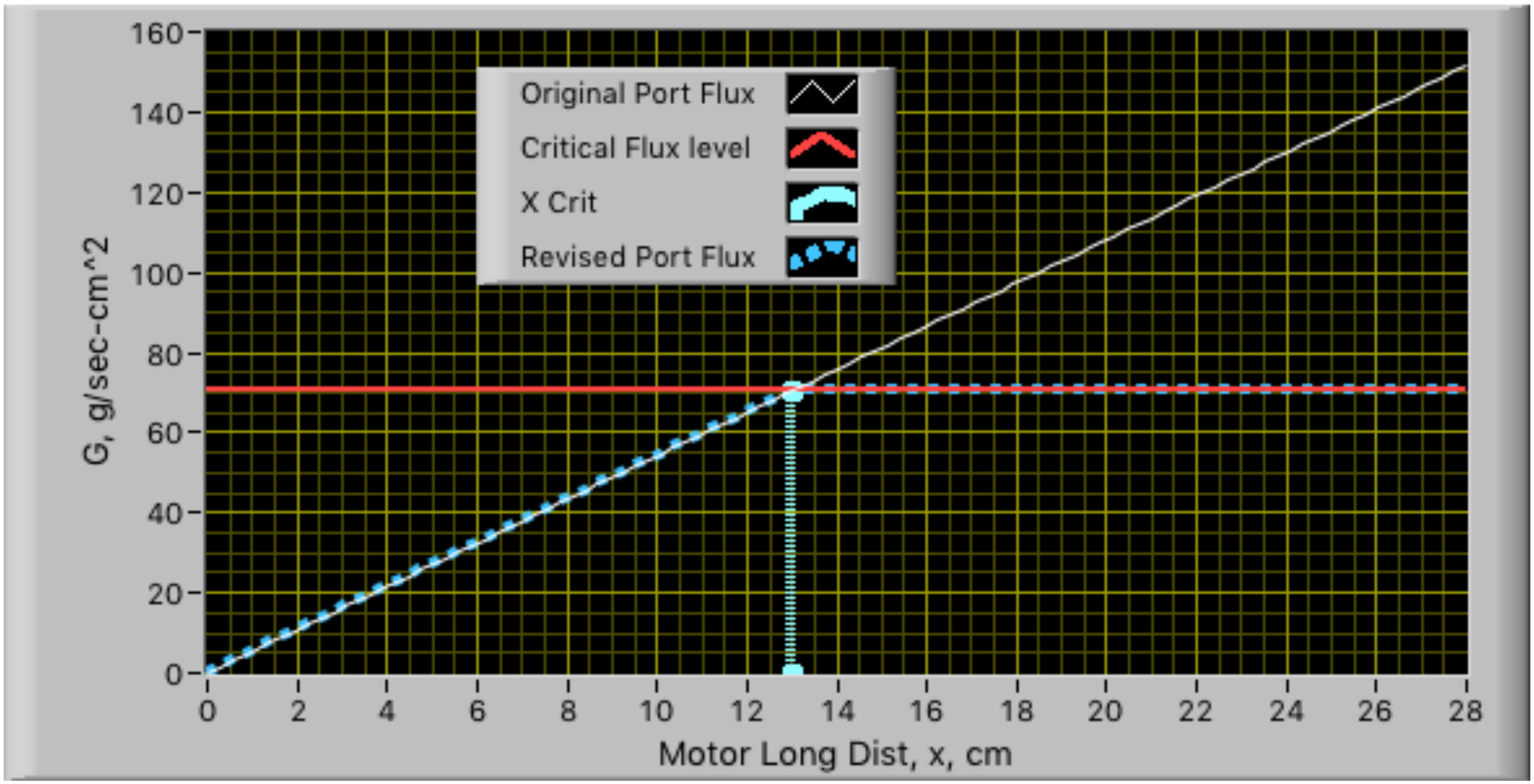
Constant Massflux Design Example (4)

- Adjusted Port Diameter →

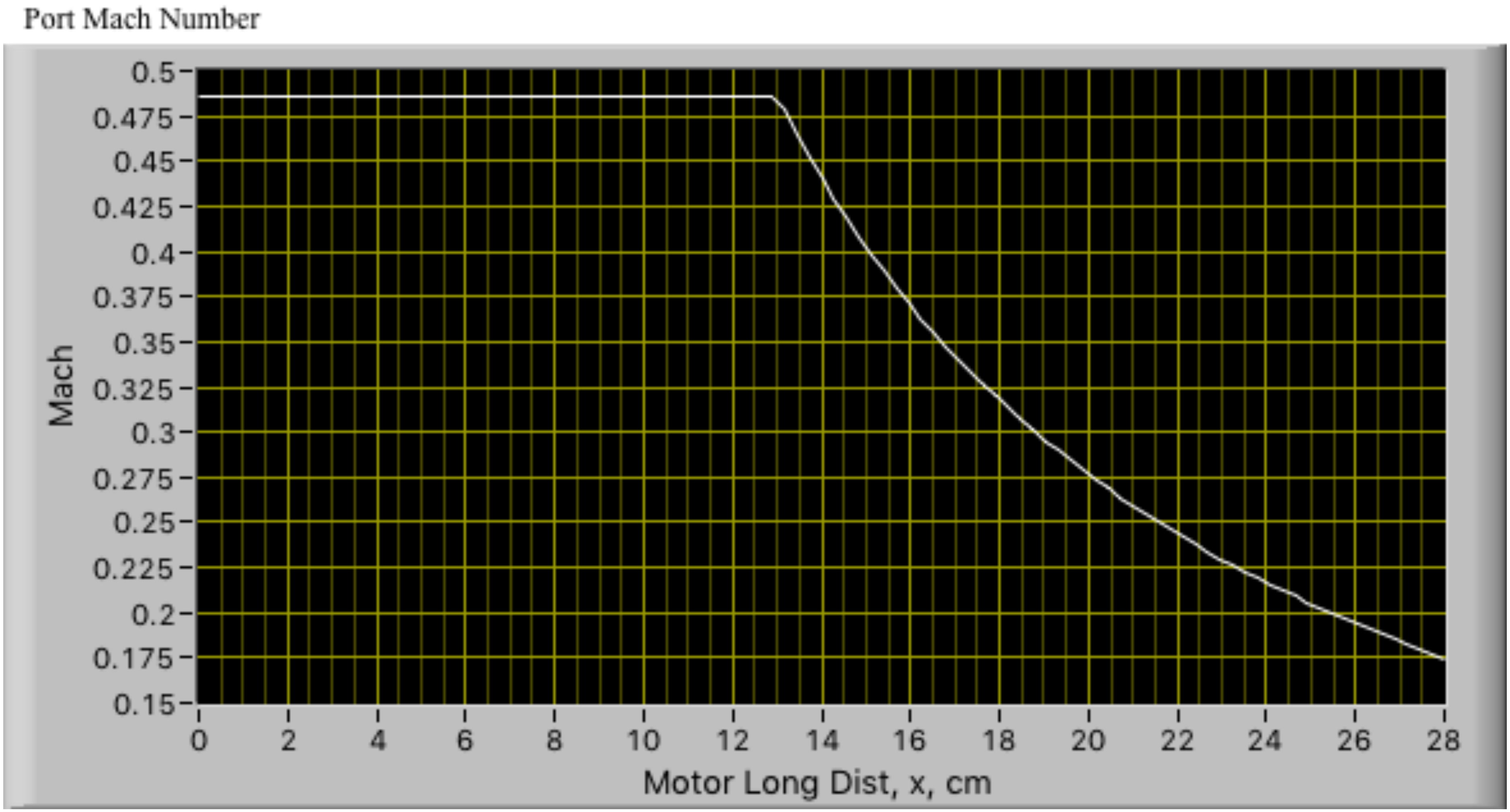
$$G_{max} = 1.0 \frac{lbm}{in^2 - sec} \left(70.33 \frac{g}{cm^2 - sec} \right)$$

Threshold Distance
Xcrit, cm
12.9924

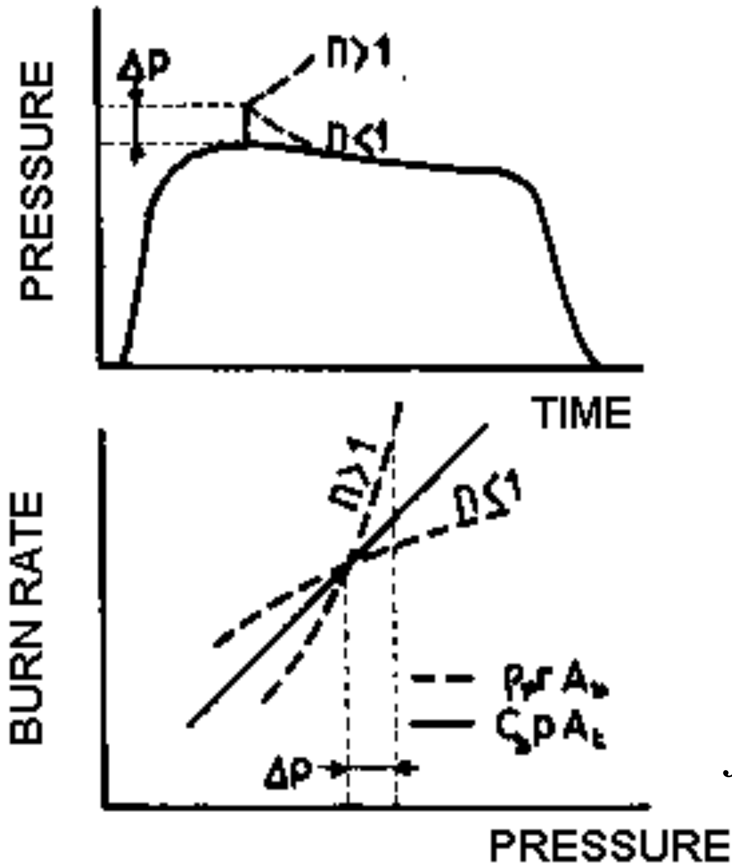
Massflux vs Length



Example Calculation (5)



Exponent Effect on Burn Rate (Pressure Excursion)



Source: Barrere et al., Raketenantriebe, Fig 5.1 (1961)

- *High values of burn exponent (**n**) make for a propellant whose burn rate is sensitive to chamber pressure*
- *Solid propellant motors with high burn rate profiles specially susceptible to fuel grain cracks and fractures*
- *Erosive burning can precipitate grain fracture*

What happens when a solid propellant grain factures?

- Solid propellant grain fracture events produce detrimental effects on motor performance, and can sometimes be catastrophic

- As crack propagates in the grain or along the grain/case interface, it creates additional burning surfaces

- Augmented burn area produces an excess of hot gas

- Excess mass flow strongly affects the chamber pressure rise and can (depending on the burn exponent) couple with the regression rate to produce a runaway burn and catastrophic failure.

What happens when a solid propellant grain factures? (2)

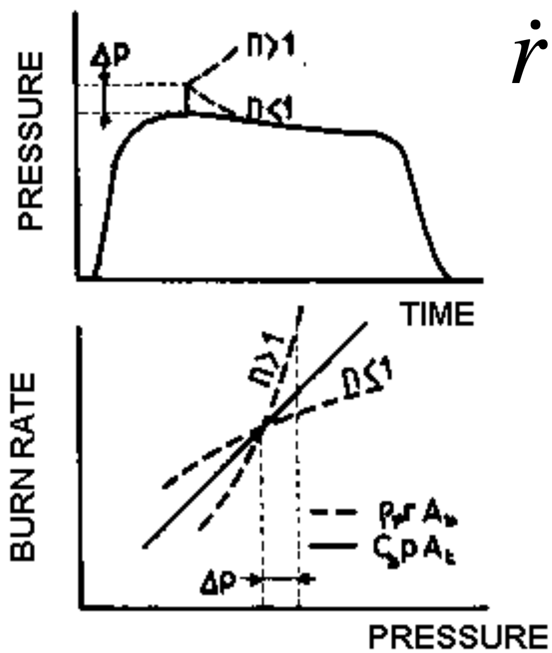
- A classical example of a catastrophic solid rocket failure resulting from propagation of a crack along the grain/case interface is the Titan IV accident August, 1998
- Aerodynamic effects associated with the grain shape near a slot and the interaction between core and cross flows resulted in a dramatic increase in the head end pressure of the motor.
- The crack extended to the propellant case bond and propagated along the interface between the fuel and case.
- This sequence of events eventually led to the choking of the core flow and resulted in the rocket exploding.

http://www.youtube.com/watch?v=ZFeZkrRE9wI

What happens when a solid propellant grain factures? (3)

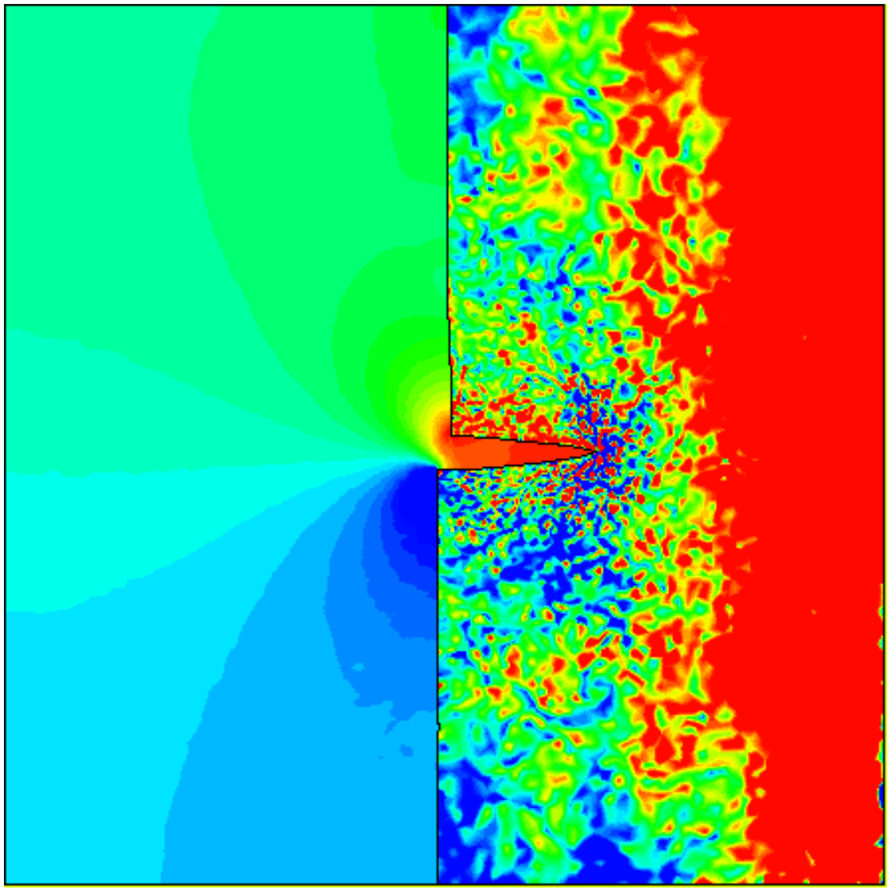
$$\frac{\partial P_0}{\partial t} = \frac{A_{burn} a P_o^{\;n}}{V_c}\left[\rho_p R_g T_0 - P_0\right] - P_0\left[\frac{A^*}{V_c}\sqrt{\gamma R_g T_0\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}}\right]$$

$$\dot{r} = a \cdot P_0^{\;n} \rightarrow \dot{m}_{propellant} = \rho_p \cdot A_{burn} \cdot \dot{r}$$



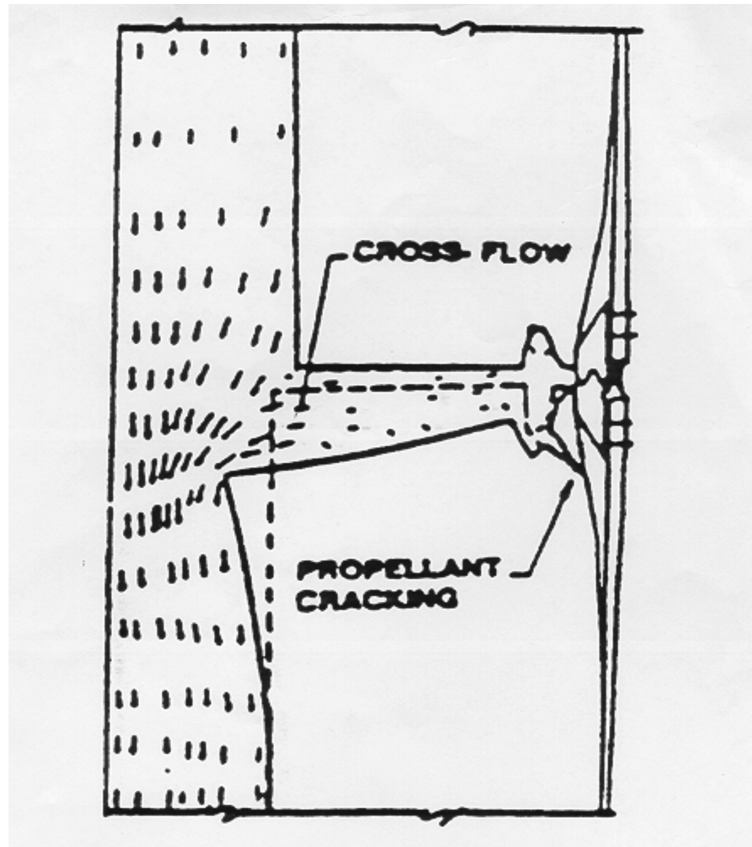
- Saint Robert's Pressure/Burn Rate Coupling
- Propellant Surface deformation
- Wall Burn Through

Extremely Complex Fluid/Structural Interaction Problem



Center for Simulation of Advanced Rockets

University of Illinois at Urbana-Champaign



State of the Art Modeling

Project 2

Build Unsteady Model of "Pike" .. Use Integrator of your choice

Calculate:

Chamber pressure profile
Regression rate profile
Massflow rate (compare to choking massflow)
Mass depletion vs time
plot Thrust profile
plot Total Impulse profile
Effective Mean Specific Impulse

Allow:

St. Robert's Parameter Input
Variable Step Size
Variable Thermodynamic Properties ***(as inputs to the problem)***
Erosive burn model for cylindrical port (Not Bates grain)

Project 2 (2)

Part 1, cylindrical port

Fuel Grain Geometry

$L_0 = 35$ cm

$D_0 = 7.6$ cm

$D_0 = 3$ cm

$\rho_{propellant} = 1260$ kg/M^3

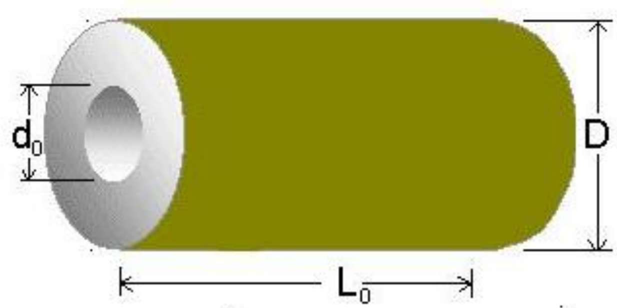
Nozzle Geometry

$A^* = 1.887$ cm^2

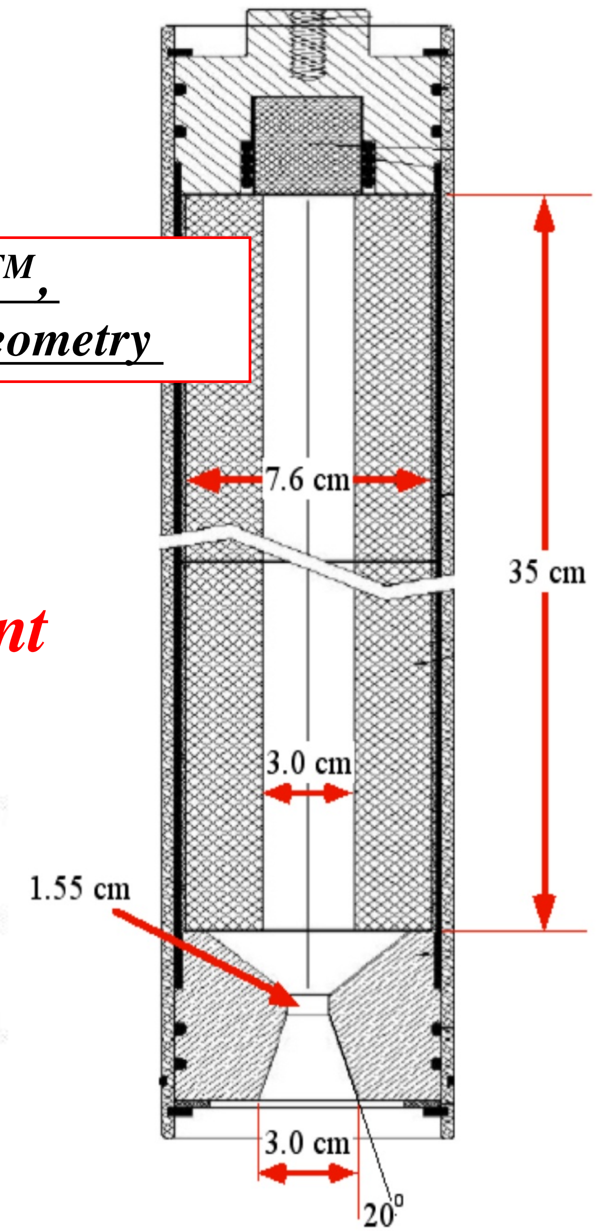
$A_{exit}/A^* = 4.0$

$\theta_{exit} = 20$ deg.

Single propellant segment



Animal Works™, L700 Motor Geometry



Assume ends are burn inhibited

Project 2 (3)

Combustion Gas Properties

$\gamma = 1.18$

$M_W = 23$ kg/kg-mol

$T_0 = 2900$ K

Burn Parameters

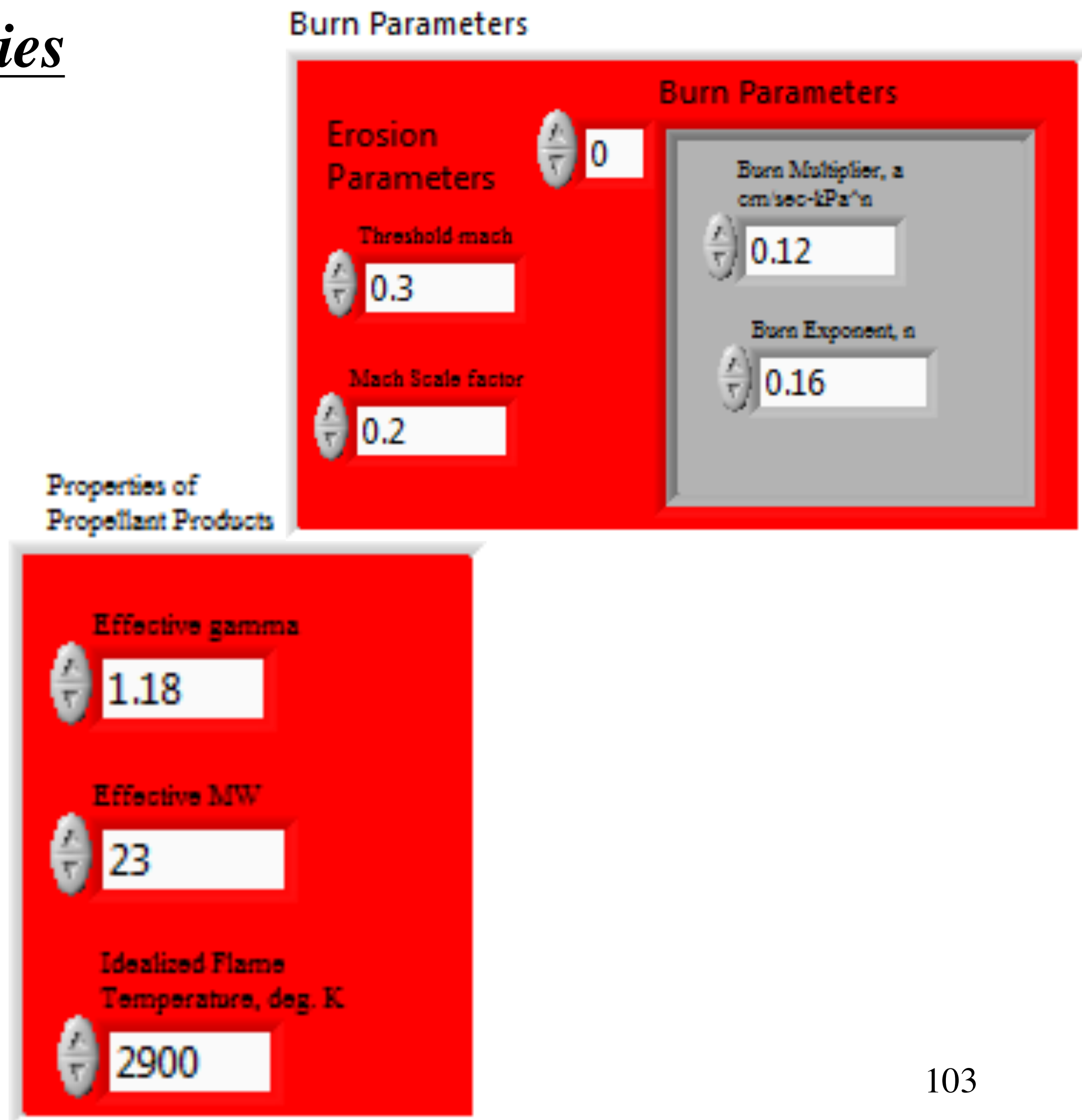
$a = 0.12$ cm/(sec-kPa ^{n})

$n = 0.16$

$M^{crit} = 0.3$

$k = 0.2$

(cylindrical port only)



Project 2 (4)

Part 1, cylindrical port

Fuel Grain Geometry

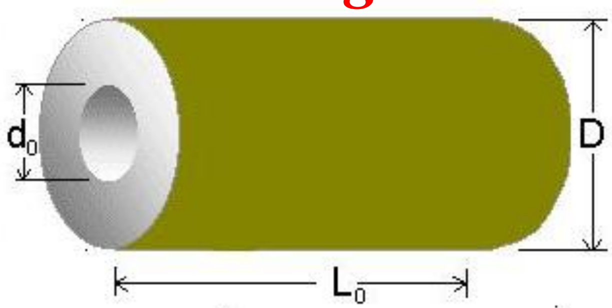
$L_0 = 35$ cm
$D_0 = 7.6$ cm
$D_0 = 3$ cm
$\rho_{propellant} = 1260$ kg/M^3

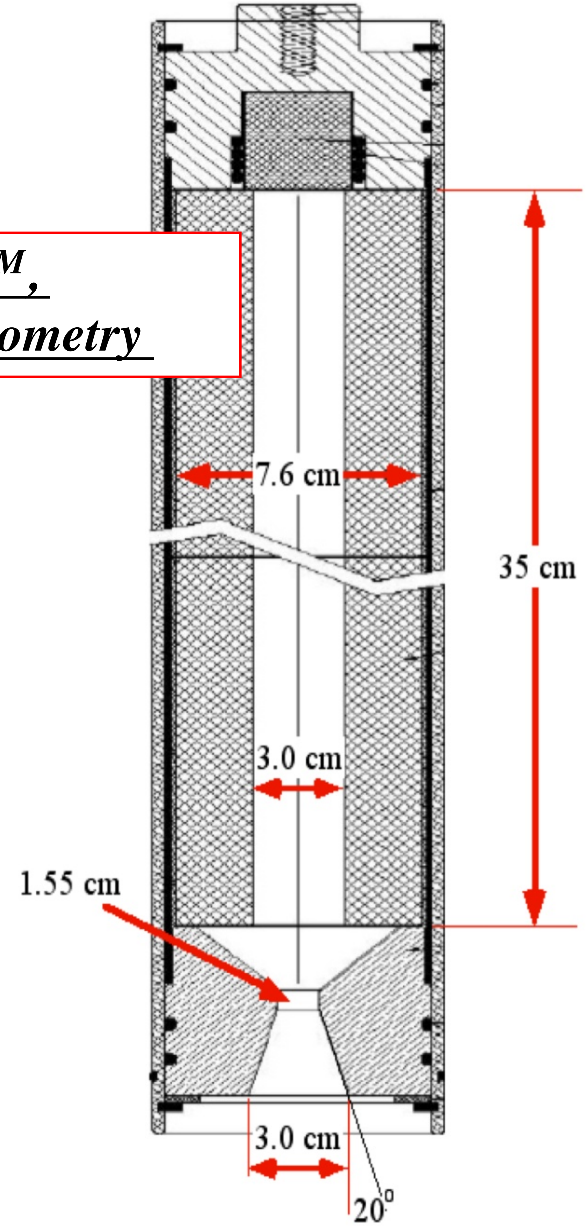
Animal Works™, L700 Motor Geometry

Repeat results
Using bates grain
With 3 segments

Nozzle Geometry

$A^* = 1.887$ cm ^{2}
$A_{exit}/A^* = 4.0$
$\theta_{exit} = 20$ deg.





Assume ends are not! burn inhibited

Project 2 (5)

Combustion Gas Properties

$\gamma = 1.18$

$M_W = 23$ kg/kg-mol

$T_0 = 2900$ K

Burn Parameters

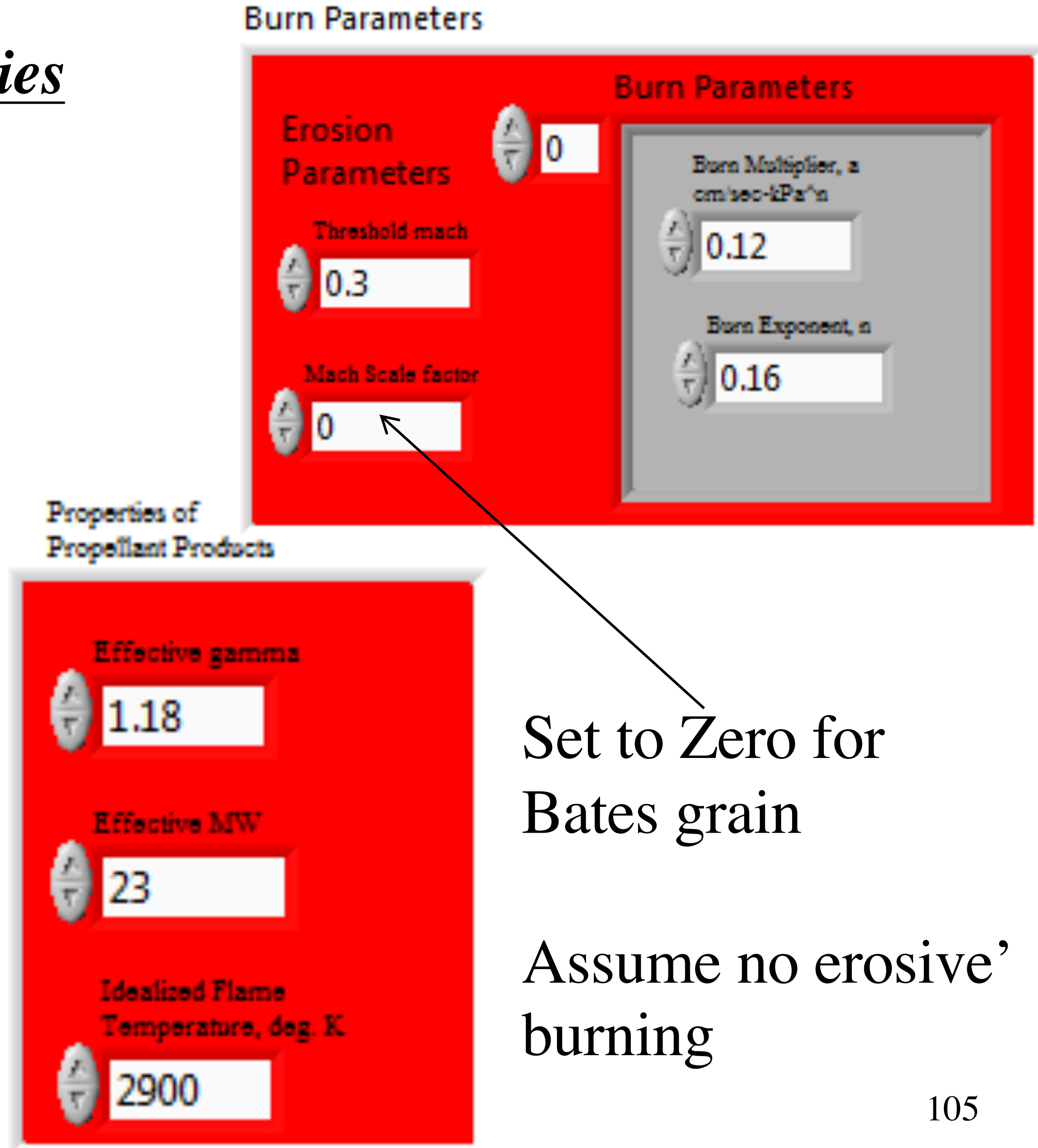
$a = 0.12$ cm/(sec-kPa ^{n})

$n=0.16$

$M^{crit} = 0.3$

$k = 0.2$

(cylindrical port only)



Set to Zero for Bates grain

Assume no erosive' burning

Project 2 (6)

Examine sensitivity of calculations to burn rate parameters, Critical Mach number (for erosion) … cylindrical port Only, Assume bates grain does not burn erosivley

What is the effect of Flame temperature (T_0)

Plot Regression rate versus Chamber pressure

Prepare report stating your results and conclusions

State Equation Formulation of Problem

$$\begin{bmatrix} \dot{P}_0 \\ \dot{r} \end{bmatrix} = \begin{bmatrix} \left(\frac{A_{burn} \cdot \dot{r}}{V_c}\right) \cdot \left(\rho_{propellant} \cdot R_g \cdot T_0 - P_0\right) - \left(\frac{A^*}{V_c}\right) \cdot P_0 \cdot \sqrt{\gamma \cdot R_g \cdot T_0 \cdot \left(\frac{2}{\gamma+1}\right)^{\left(\frac{\gamma+1}{\gamma-1}\right)}} \\ a \cdot P_0^n \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ r \end{bmatrix}_{t=0} = \begin{bmatrix} P_{ambient} \\ \frac{d_0}{2} \end{bmatrix} \rightarrow \boxed{s(t) = \int_0^t \dot{r} \cdot dt \approx r(t) - \frac{d_0}{2}}$$

$$\begin{bmatrix} k \equiv \textit{Erosion Constant}_{\text{(GRAIN DEPENDENT)}} \\ M_{crit} \equiv \textit{Critical Port Mach Number} \end{bmatrix}$$

$\rightarrow$ *State Equations for Erosive Burning:*

$$\begin{bmatrix} \dot{P}_0 \\ \dot{r} \end{bmatrix} = \begin{bmatrix} \left(\frac{A_{burn} \cdot \dot{r}}{V_c}\right) \cdot \left(\rho_{propellant} \cdot R_g \cdot T_0 - P_0\right) - \left(\frac{A^*}{V_c}\right) \cdot P_0 \cdot \sqrt{\gamma \cdot R_g \cdot T_0 \cdot \left(\frac{2}{\gamma+1}\right)^{\left(\frac{\gamma+1}{\gamma-1}\right)}} \\ \left(1 + k \cdot \frac{M_{port}}{M_{crit}}\right) \cdot a \cdot P_0^n / (1+k) \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ r \end{bmatrix}_{t=0} = \begin{bmatrix} P_{ambient} \\ \frac{d_0}{2} \end{bmatrix} \rightarrow \boxed{s(t) = \int_0^t \dot{r} \cdot dt \approx r(t) - \frac{d_0}{2}}$$

State Equation Formulation of Problem (2)

$\rightarrow$ *Cylindrical Port :*

$$A_{burn} = 2 \cdot \pi \cdot r \cdot L_{port}$$
$$V_c = \pi \cdot r^2 \cdot L_{port}$$
$$\rightarrow \begin{bmatrix} r \equiv Port\ Radius \\ L_{port} \equiv Port\ Length \end{bmatrix}$$

$\rightarrow$ *Bates Grain :*

$$A_{burn} = N \cdot \pi \cdot \left\{ \left[\frac{D_0^2 - (d_0 + 2 \cdot s)^2}{2} \right] + (L_0 - 2 \cdot s) \cdot (d_0 + 2 \cdot s) \right\}$$

$$V_c = \frac{N \cdot \pi}{4} \cdot \left\{ (d_0 + 2 \cdot s)^2 \cdot (L_0 - 2 \cdot s) + D_0^2 \cdot 2s \right\}$$

Do NOT! Use Erosive Burning for Bates Grain