UtahState UNIVERSITY

Mechanical & Aerospace Engineering

Final Project, Spring 2022

- **Assignment is due on or before the Final exam, Monday, May 2(11:59 AM MDT)**

NASA "Mighty Eagle" Autonomous Robotic Lunar Lander Test Platform

20 Points Total

NASA Marshall Space Flight Center and the Johns Hopkins Applied Physics Laboratory have been working together since 2005 to develop technologies and mission concepts for a new generation of small, versatile robotic landers to land on airless bodies, including the moon and asteroids, in our solar system.

As part of this larger effort, APL and the Marshall Space Flight Center worked with the Von Braun Center for Science and Innovation to construct a prototype *hydrogen-peroxide monopropellant-fueled* robotic lander that has been given the name *Mighty Eagle*.

Required Data for Report *(2 pts for proper formatting)*

1. **CEA Program Setup** ***(Step 1)*** ***(5 pts)***
   - Screen Shots of program setup
   - Plots of Chamber conditions, T_0, γ, M_w, c^* as a function of P0, %H2O2 Concentration
   - Plots of C_F, c^* and I_{sp} as a function of P0, %H2O2 Concentration
   - Discussion of results

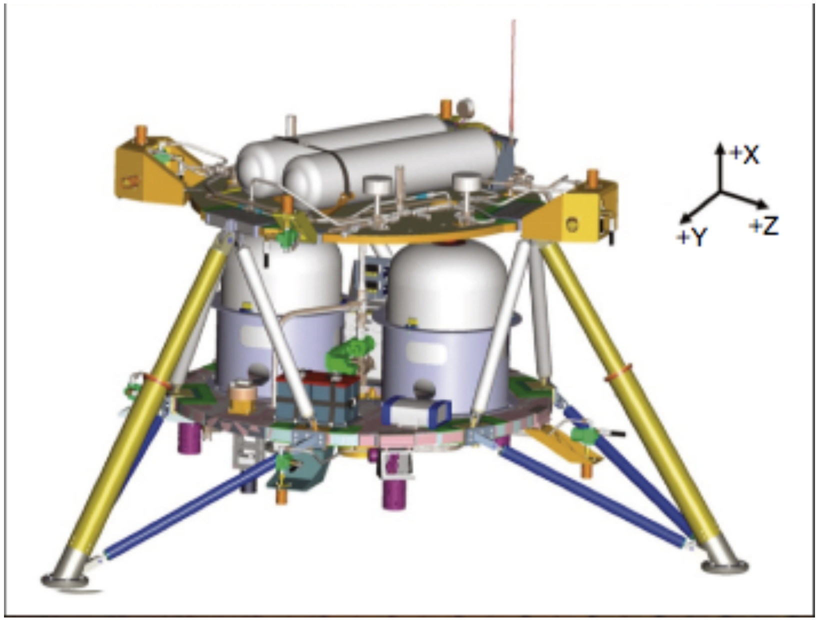
2. **Optimal Performance Analysis** ***(Step 2)*** ***(3 pts)***
   - 90% H_2O_2 concentration, Plots of Nozzle exit pressure versus chamber pressure.
   - Optimal Chamber pressure for EGC thruster.
   - Engine Thrust and I_{sp} Specific Impulse @Optimal Chamber pressure

3. **Throttle Area Schedule** ***(Step 3)*** ***(4 pts)***
   - Solution for Pintle $C_d A_{pintle}$ in terms of steady-state chamber pressure, $P_{0_{ss}}$
   - Plot of injector area as a function of chamber pressure, P_0
   - Plot of Massflow as a function of the Pintle Injector area
   - Plot of *massflow as a function of thrust* for EGC Engine
   - Pintle Area Settings at Minimum and Maximum Thrust levels for EGC Engine

4. **Hover Control Schedule** ***(Step 4)*** ***(4 pts)***
   - Plot of EGC thrust, total thrust, Isp for Hover Control as a function of time
   - Plot of EGC Pintle Area for Hover Control as a function of time
   - Plot of Vehicle Mass, consumed propellant as a function of the time

5. **Total Burn Time** ***(Step 5)*** ***(2 pts)***
   - Mean Isp calculation
   - Rocket equation, total burn time calculation
   - Comparison to burn time from part 4.

Background (1)



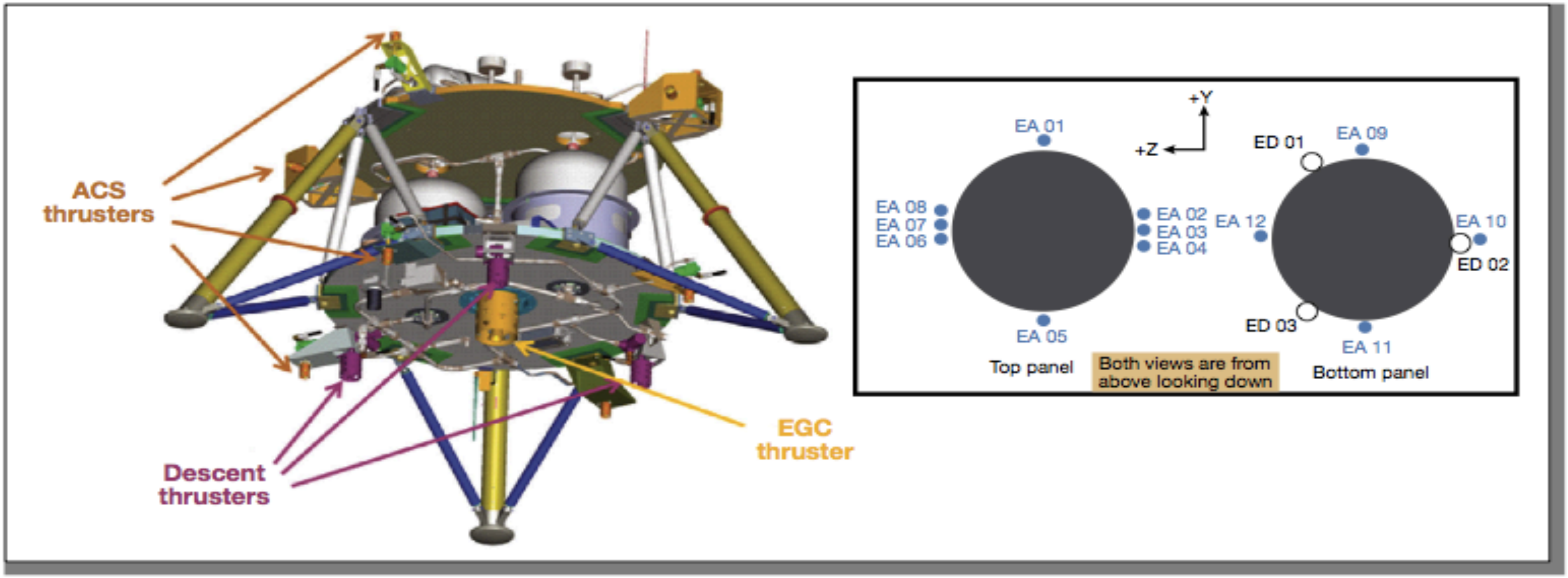
• Hydrogen peroxide was chosen for the prototype system because its decomposition by products, steam and oxygen, are both nontoxic, and it provides sufficient energy density to achieve the target flight times.

• A blowdown 90% pure hydrogen peroxide monopropellant propulsion system that is pressurized using regulated high-purity nitrogen provides actuation for both the attitude control system (ACS) and the descent control systems.

• A large throttleable monopropellant engine provides *Earth gravity cancellation* (EGC). The EGC engine nominally produces a thrust of five-sixths the weight of the lander throughout the flight to approximately simulate lunar gravity for the rest of the system by nulling the difference between Earth and lunar gravity.

• A fixed EGC engine was chosen over a gimbaled design to minimize system complexity, cost, and schedule constraints.

Background *(2)*



- The propulsion system was built by Dynetics in collaboration with MSFC and APL, feeds 16 mono-propellant hydrogen-peroxide thrusters: twelve 44.5 N (10 lbf) attitude control thrusters, three 250 N (60 lbf) descent engines, and the throttleable EGC engine with a thrust range from approximately 1000 N (225 lbf) to 3114 N (700 lbf).

- The 12 attitude thrusters are grouped into six coupled pairs to allow torque to be applied independently to each of the three rotation axes of the vehicle. The three fixed descent engines provide the vertical thrust to control the vehicle's altitude and descent rate.

Background (3)

Description	Size
Dry mass	207 kg
Descent thrusters	3 total
Descent thrust (each)	60 lbf
ACS thrusters	12 total
ACS thrust (each)	10 lbf
Max propellant mass	193 kg
Propellant	H_2O_2
Pressurisation	Nitrogen
Height	4 ft
Diameter	8 ft
Payload	3D camera

- Propulsion System
  - H_2O_2 Decomposition with Silver catalyst
  - EGC engine – 1000 N (225 lbf) to 3114 N (700 lbf).
  - 3 x Descent Thrusters – 270 N (60 lbf) Ea.
  - 12 x Attitude Control – 44.5 N (10 lbf) Ea.
  - 116 kg Max H_2O_2 Loading
- High purity nitrogen pressurant
  - 3000 psi tank pressure
  - Regulated down to 750 psi (5170 kPa) feed pressure
  - 7 kilograms (15 lb) of pressurant.
- Fully Fueled Mass 400 kg (882 lbf)

Background (4)

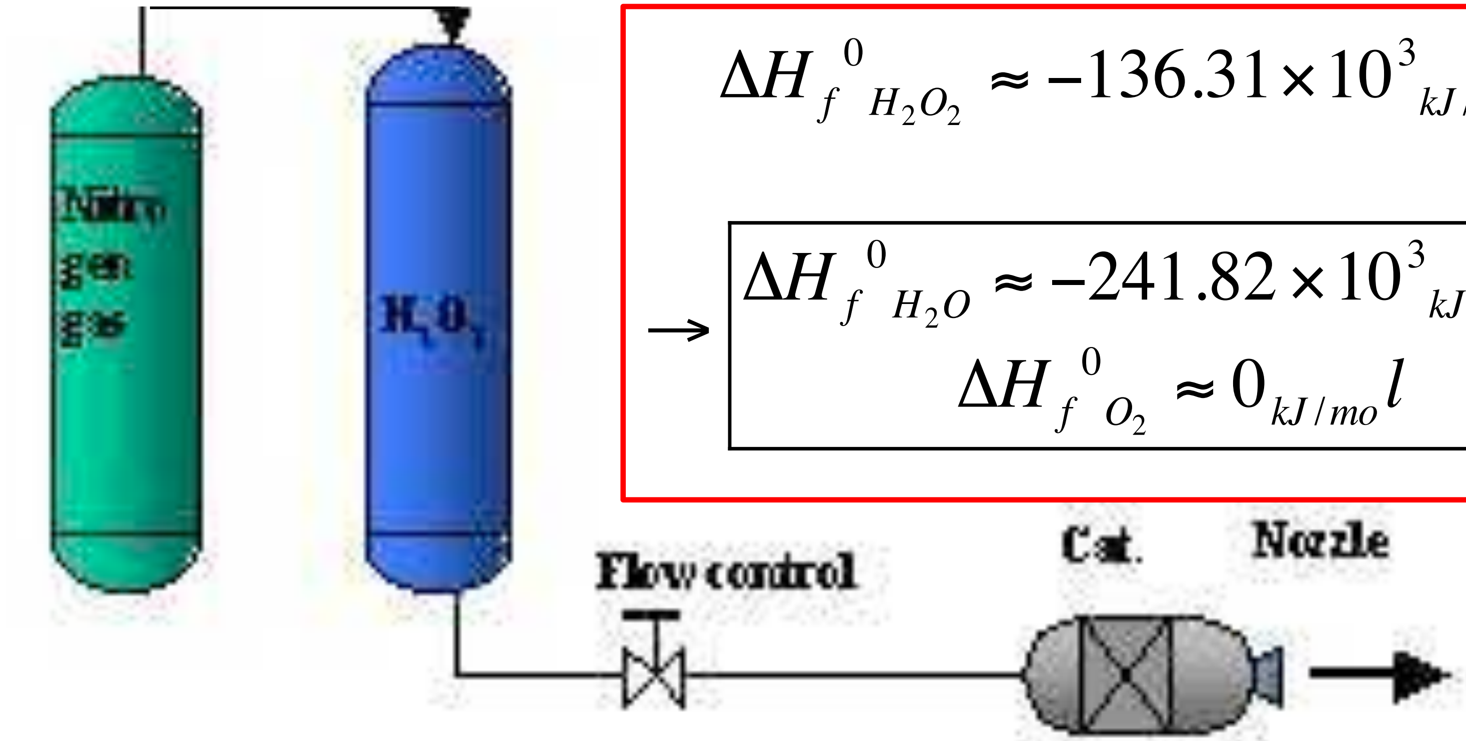
Mono-Propellant Hydrogen Peroxide Thruster

$$H_2O_2 \rightarrow H_2O + \frac{1}{2}O_2 + 105.51\times10^3{}_{kJ/kg-mol\ heat} \rightarrow -3100_{kJ/kg}$$

$$\Delta H_f^0{}_{H_2O_2} \approx -136.31\times10^3{}_{kJ/kg-mol}$$

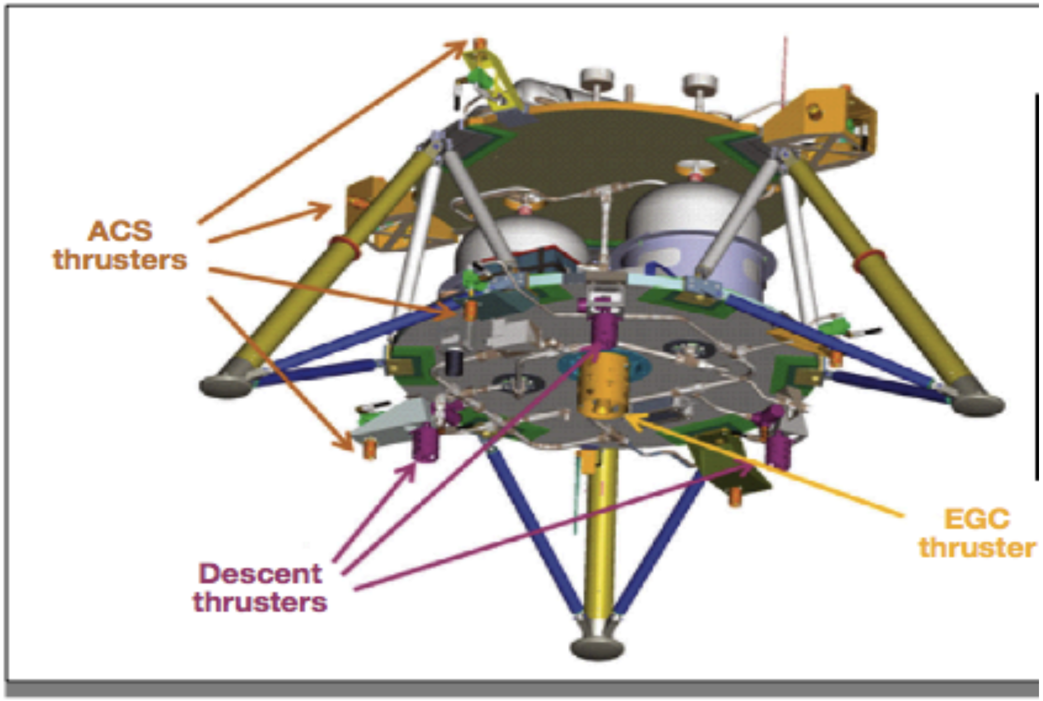
$$\rightarrow \Delta H_f^0{}_{H_2O} \approx -241.82\times10^3{}_{kJ/kg-mol}$$

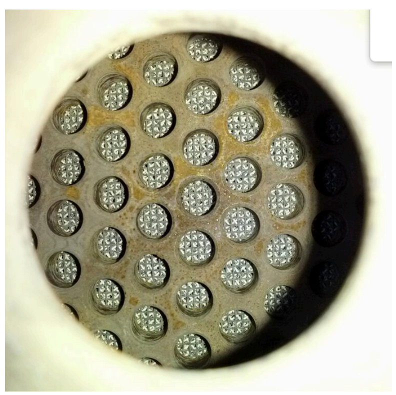
$$\Delta H_f^0{}_{O_2} \approx 0_{kJ/mol}$$



Monopropellant rocket with pressurized fuel tank

Background (5)





ECG Catbed Outlet

- EGC engine operates continuously and nominally offsets 5/6th of the vehicle weight
- EGC also Used to Hover the vehicle
- Variable Area Injector Pintile Valve Throttle Control on EGC System
- ECG Nozzle Expansion Ratio – 5.7:1
- Conical Nozzle with exit angle 15.4 ^{o}
- Throat Area $A^* = 4.5\ cm^2$
- Nominal Operating Altitude ~ 1200 ft.
- $P_\infty \sim 97$ kPa

CEA Program Setup *(Step 1)* ***(5 pts)***

- **Install CEA program and Update Java Runtime (JRE) on your computer**
- **Set up input file to run as "Rocket" Problem with a range of combustion pressures from 1000 kPa (145 psia) to 6000 kPa (870 psia)**
- **Set up problem to calculate performance of H_2O_2/H_2O monopropellant mixture for H_2O_2 mass concentrations varying from 70% to 95% (i.e. 70, 75, 80, 85, 90, 95%) .. Assume Liquid H_2O_2 and H2O at 298 K**
- **Run code in "equilibrium", with "infinite" combustor contraction ratio, Define a 5.7:1 expansion ratio, and allow for shifting equilibrium in CEA**
- **At Operating Altitude Ambient Pressure at MSFC test site is 97 kPa (14.07 psia)**

Final Project *(Step 1) (2)*

- **Estimate the Combustion temperature and associated properties using Gibbs free-energy and available-heat calculations**

- **H_2O_2 = "oxidizer", H_2O = "fuel", reactants at standard conditions, 298 K**

- **Define output file (.plt) with sufficient outputs to calculate C_F, c^* *and* I_{sp}**

- **Read .plt file**

  - **For Each (Chamber) Pressure Value, Plot T_0, γ, M_w, c^* for the combustion chamber conditions *as a function of %H2O2 Concentration …. What can you conclude about the effects of chamber pressure on Mono-propellant decomposition (combustion chamber properties)?***

  - **For Each (Chamber) Pressure Value, Plot C_F, c^* *and* I_{sp} *(exit conditions) as a function of %H2O2 Concentration … do these exit parameters vary with pressure?***

Optimal Performance Analysis *(Step 2)* ***(3 pts)***

- **For the 90% H_2O_2 concentration, From CEA .plt file, Plot the Nozzle exit pressure versus the prescribed chamber pressure.**
- **Interpolating the results from this plot, at what chamber pressure is the Mighty Eagle ECG Engine operating at Optimal (Design) Conditions.**
- **Insert this Value into the CEA pressure list, re-run code, and verify result give Pexit=97 kpa**
- **What is the Engine Thrust level and Specific Impulse when Operating at the Optimal (Design) Chamber pressure Setting?**

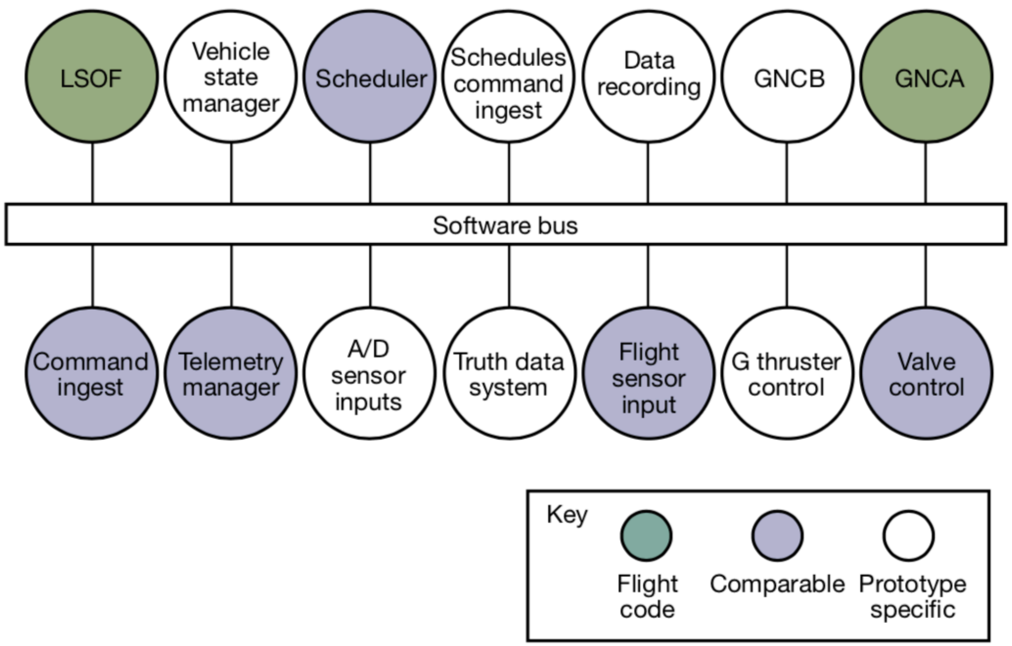
$$\frac{A_{exit}}{A^*} = 5.7 \qquad A^* = 4.5_{cm^2}$$

$$\left(p_\infty\right)_{operating} \approx 97_{kPa} \qquad \theta_{exit} = 15.4^o$$

Throttle Area Schedule *(Step 3)* *(4 pts)*

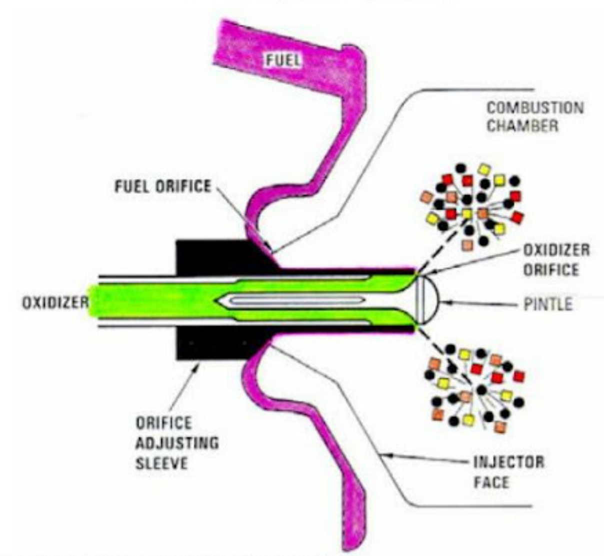
The large throttleable engine provides Earth gravity cancellation (EGC). The EGC engine nominally produces a thrust of five-sixths the weight of the lander throughout the flight to approximately simulate lunar gravity for the rest of the system by nulling the difference between Earth and lunar Gravity

- **Let's Design an Area Control Schedule for the Pintle Valve, Based on CEA Results**



Mighty Eagle: Modular software architecture.

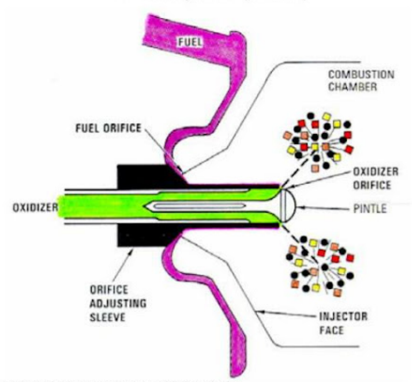
Pintle Injector (LMDE)



Dressler, G. A., "Summary of Deep Throttling Rocket Engines with Emphasis on Apollo LMDE," AIAA Paper 2006-5220

Throttle Area Schedule *(Step 3)* *(2)*

Pintle Injector (LMDE)



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For a Mono-propellant H_2O_2 Thruster, the Chamber ballistic equation can be modified as

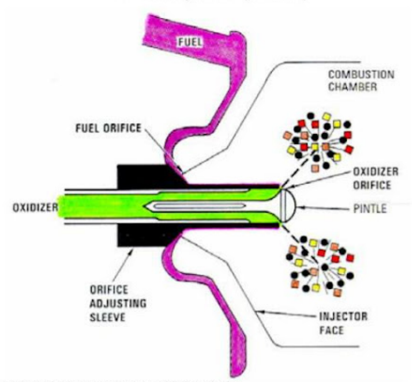
$$\frac{dP_0}{dt} + P_0 \cdot \left(\frac{A^*}{V_c} \cdot \sqrt{\gamma \cdot R_g \cdot T_0 \cdot \left(\frac{2}{\gamma+1}\right)^{\left(\frac{\gamma+1}{\gamma-1}\right)}} \right) = \frac{R_g \cdot T_0}{V_c} \cdot \left(C_d A_{pintle}\right) \cdot \sqrt{\rho_{H_2O_2} \cdot \left(p_{inj} - P_0\right)}$$

- **Given the upstream injection pressure, p_{inj}**
  - Solve for the Pintle Injector Discharge Area, $C_d A_{pintle}$ That gives the desired steady-state chamber pressure, $P_{0_{ss}}$

$$\left(C_d A_{pintle}\right) = f\left(\frac{A^*}{V_c}, \rho_{H_2O_2}, \left(\gamma, R_g, T_0\right), p_{inj}, P_{0ss} \right)$$

Throttle Area Schedule *(Step 3)* (3)

Pintle Injector (LMDE)



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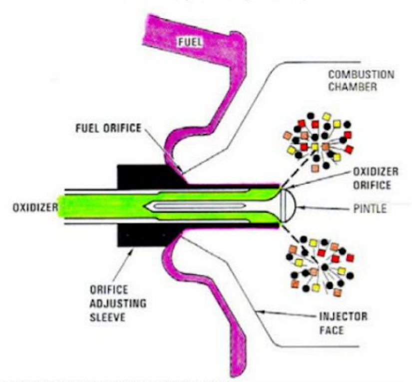
- **Using from the CEA analysis for 90% H_2O_2**
  - Plot the required injector area as a function of chamber pressure, $P_0 (P_{0ss})$
  - Plot the resulting Massflow as a function of the Pintle Injector area
    - *Assume*
    - p_{inj} = 750 psi (5170 kPa) feed pressure
    - $A^* = 4.5 \text{ cm}^2$
    - $C_d = 0.9$
    - *90% $\rho_{H2O2/H2O}$@298 K=1.387 g/cm^3*
    - Use CEA T_0 @ combustion chamber
    - Use γ, R_g @ Nozzle Throat (*) @ 90% peroxide concentration

Throttle Area Schedule *(Step 3)* (4)

- **Using the CEA analysis for 90% H_2O_2**



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- Plot the resulting *thrust level and specific impulse* as a function of P_0 and Pintle Injector area, A_{pintle}
- Plot *massflow as a function of thrust*
- *Assume* $p_\infty = 97$ kPa, $P_0 = P_{0\,ss}$
  - $\theta_{exit} = 15.4^o$
  - $A^* = 4.5\ cm^2$

USE CEA Output to get $(C_F)_{Opt}$

$$C_F = \frac{Thrust}{P_0 \cdot A^*} = \frac{\lambda \cdot \dot{m} \cdot V_{exit}}{P_0 \cdot A^*} + \frac{(p_{exit} - p_\infty) \cdot A_{exit}}{P_0 \cdot A^*} = \lambda \cdot (C_F)_{opt} + \frac{(p_{exit} - p_\infty) \cdot A_{exit}}{P_0 \cdot A^*} = \lambda \cdot (C_F)_{opt} + \left(\frac{p_{exit} - p_\infty}{P_0}\right) \cdot \left(\frac{A_{exit}}{A^*}\right)$$

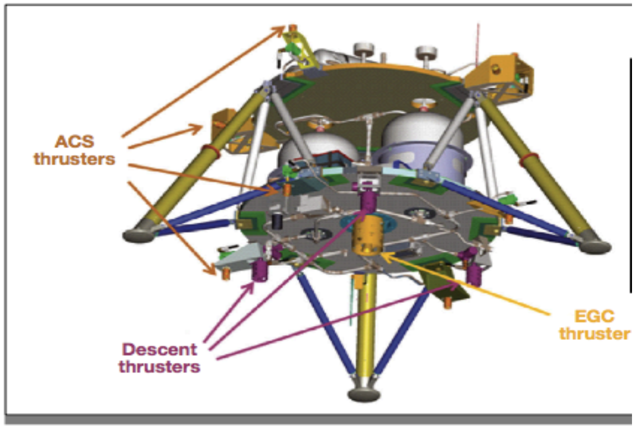
$$Thrust = C_F \cdot P_0 \cdot A^* \rightarrow I_{sp} = \frac{Thrust}{g_0 \cdot \dot{m}}$$

$$(C_d A_{pintle}) = f\left(\frac{A^*}{V_c}, \rho_{H_2O_2}, (\gamma, R_g, T_0), p_{inj}, P_{0ss}\right)$$

$$\lambda = \frac{1 + \cos\theta_{exit}}{2}$$

- What are the Pintle Area Settings at the Minimum and Maximum Thrust levels for the EGC Engine, 1000 N (225 lbf) to 3114 N (700 lbf).

Hover Control Schedule *(Step 4)* ***(4 pts)***



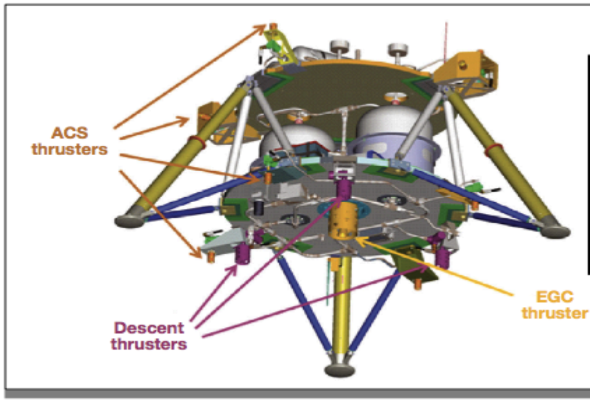
- Because the EGC engine operates continuously and offsets the vehicle weight, the total "Delta V" capacity of the EGC system is the primary driver limiting the vehicle flight time.

- **Vehicle mass as a Function of Time**
  - Ignore attitude control thrusters
  - Assume constant thrust level for the 3 descent thrusters
  - EGC engine has variable thrust and massflow
  - Descent thrusters operate at *optimal* I_{sp} level for H_2O_2

$$M_{(t)} = M_0 - \int_0^t \dot{m} \cdot dt = M_0 - \int_0^t \dot{m}_{EGC} \cdot dt - 3 \cdot \dot{m}_{descent} \cdot t$$

$$\dot{m}_{descent} \approx \frac{F_{descent}}{I_{sp_{opt}} \cdot g_0} \rightarrow F_{descent} = \mathbf{270\ N}$$

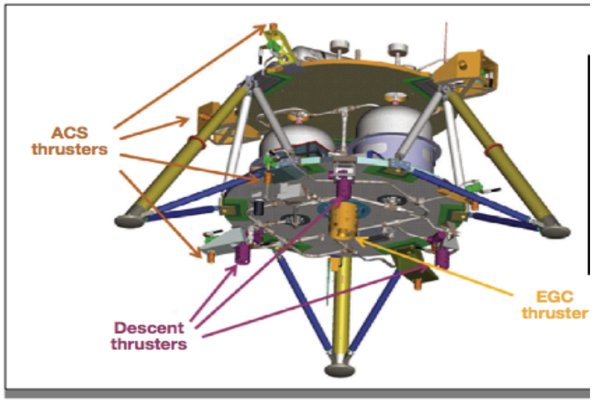
Hover Control Schedule *(Step 4)* *(2)*



- Using Data from massflow versus EGC Thrust Plot from Step 3
  - Plot the required EGC thrust level, and total thrust (including descent thrusters) required for Hover Control as a function of time
- Using Data from Thrust versus Pintle Area from Step 3
  - Plot the required EGC Pintle area required for Hover Control as a function of time

- Be sure to track the total vehicle mass and propellant load to calculate the total available burn time for the vehicle, $M_{dry} = 207\ kg$

Hover Control Schedule *(Step 4)* *(3)*



$$M_{(t)} = M_0 - \int_0^t \dot{m} \cdot dt = M_0 - \int_0^t \dot{m}_{EGC} \cdot dt - 3 \cdot \dot{m}_{descent} \cdot t$$

$$\dot{m}_{descent} \approx \frac{F_{descent}}{I_{sp_{opt}} \cdot g_0} \rightarrow F_{descent} = 270_N$$

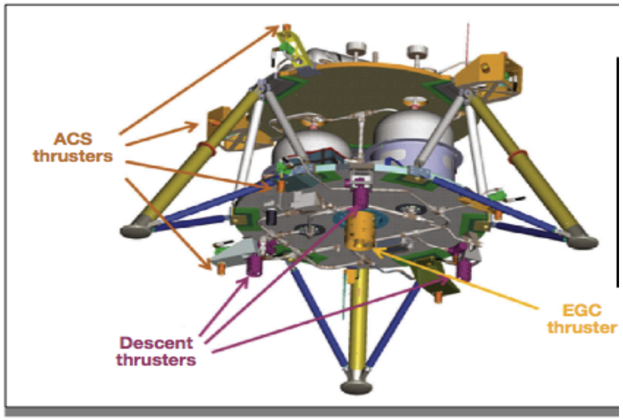
- **For Hover, vehicle acceleration level must equal 1-g**
- **Since descent Engine thrust is constant, EGC engine regulates the total thrust in hover,**

$$A_{ccl_{vert}} = 1g \rightarrow F_{EGC(t)} + 3 \cdot F_{descent} = g \cdot M_{(t)}$$

$$F_{EGC(t)} = g \cdot \left(M_0 - \int_0^t \dot{m}_{EGC} \cdot dt - 3 \cdot \frac{\dot{m}_{descent}}{1000} \cdot t \right) - 3 \cdot F_{descent}$$

Assume $g = g_0 = 9.8067\ m/sec^2$

Total Burn Time (Part 5) (2 pts)



- Check your integrated burn time against the rocket equation

Propulsive ΔV loss from acting against gravity….

$$(\Delta V)_{\substack{gravity\\loss}} = \int_0^{T_{burn}} g(t)\cdot \sin\theta \cdot dt$$

$$Hover \rightarrow \theta = 90^o \rightarrow$$

$$\Delta V_{hover} = g \cdot t_{burn} = g_0 \cdot I_{sp} \cdot \ln\left(\frac{M_0}{M_{(tburn)}}\right)$$

$$\rightarrow t_{burn} \approx \left(I_{sp}\right)_{avg} \cdot \ln\left(\frac{M_0}{M_{(tburn)}}\right)$$

$$\left(I_{sp}\right)_{avg} = \frac{\int_0^{t_{burn}} F(t)\cdot dt}{M_{prop}} \rightarrow$$

$$F(t) = F_{EGC}(t) + 3 \cdot F_{descent}$$

$$M_{prop} = 193_{kg}$$

$$M_0 = 400_{kg}$$

$$F_{descent} = 270_N$$

Bonus Question *(1 pt)*

- **For the EGC Nozzle, Assuming the throat radius of curvature follows the rule**

• $R_1 \sim 0.75 \cdot D_{throat}$ (typical)

- **And …**

$$\frac{A_{exit}}{A^*} = 5.7 \qquad A^* = 4.5_{cm^2}$$

$$\left(p_\infty\right)_{operating} \approx 97_{kPa} \qquad \theta_{exit} = 15.4^o$$

- **At the optimal (design) Operating Conditions, What is the nozzle length?**
- **How does this value compare to the minimum length Nozzle for the optimal (design) operating conditions *(apply 2/3rds safety rule)***
- ***Use γ^* from CEA analysis at optimal (design) chamber pressure, throat location***
- ***Show calculations***

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   - Plot of Vehicle Mass, consumed propellant as a function of the time

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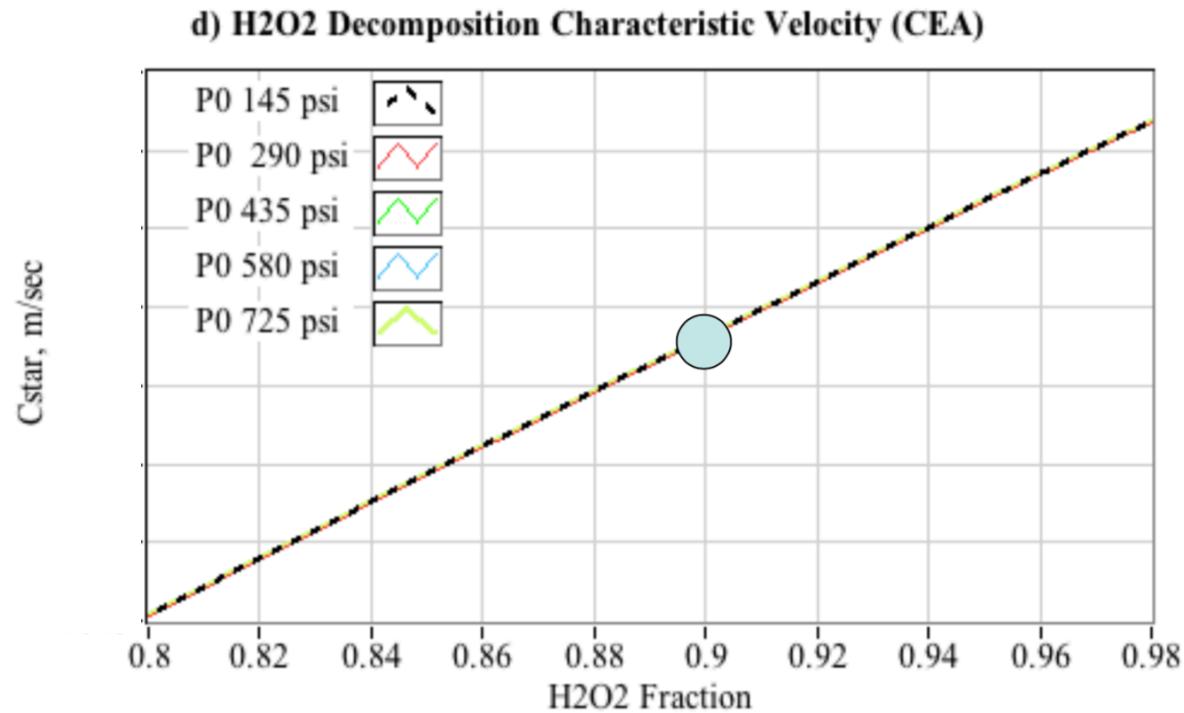
   - Mean Isp calculation
   - Rocket equation, total burn time calculation
   - Comparison to burn time from part 4.

Appendix, Hints and Suggestions



Hints and Suggestions (2)

- **Pick proper O/F Ratios to give correct peroxide concentrations**

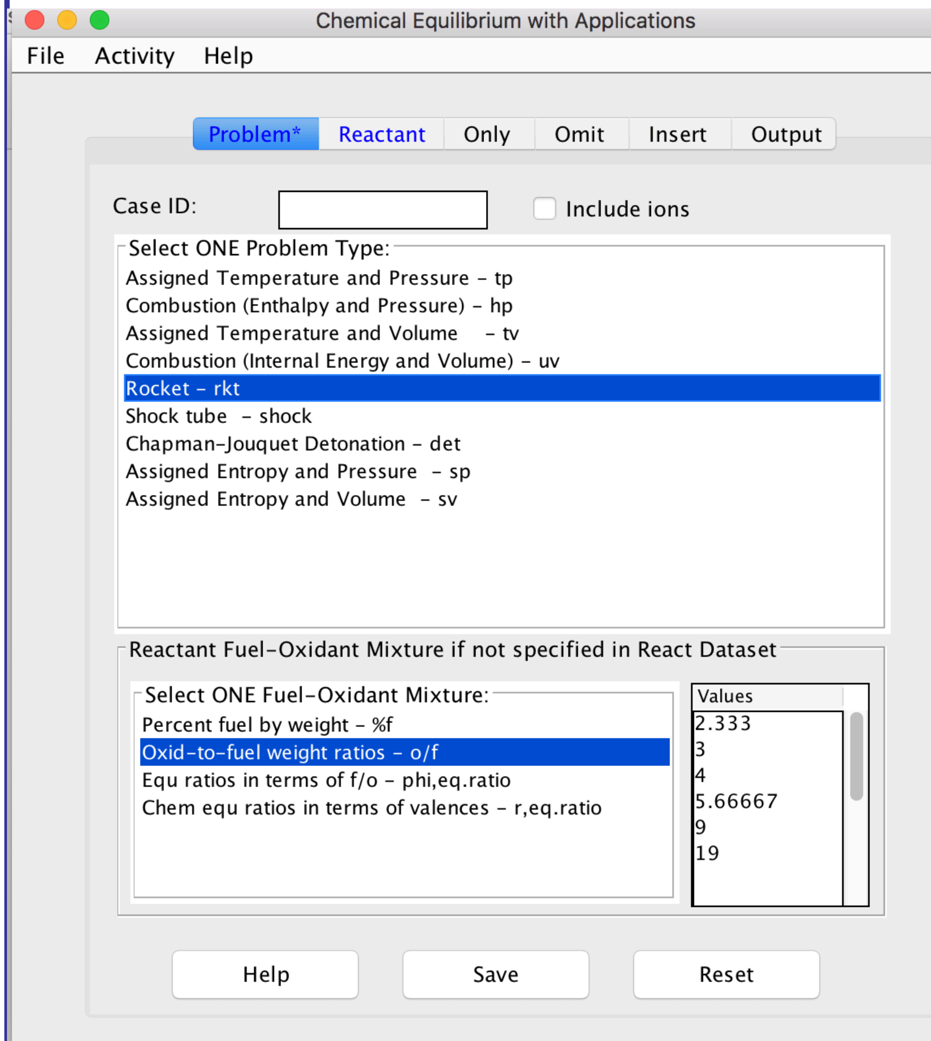
$$\frac{\%H_2O_2}{100} = \frac{M_{H_2O_2}}{M_{H_2O} + M_{H_2O_2}} = \frac{1}{\frac{M_{H_2O}}{M_{H_2O_2}} + 1} = \frac{1}{\frac{1}{O/F} + 1}$$

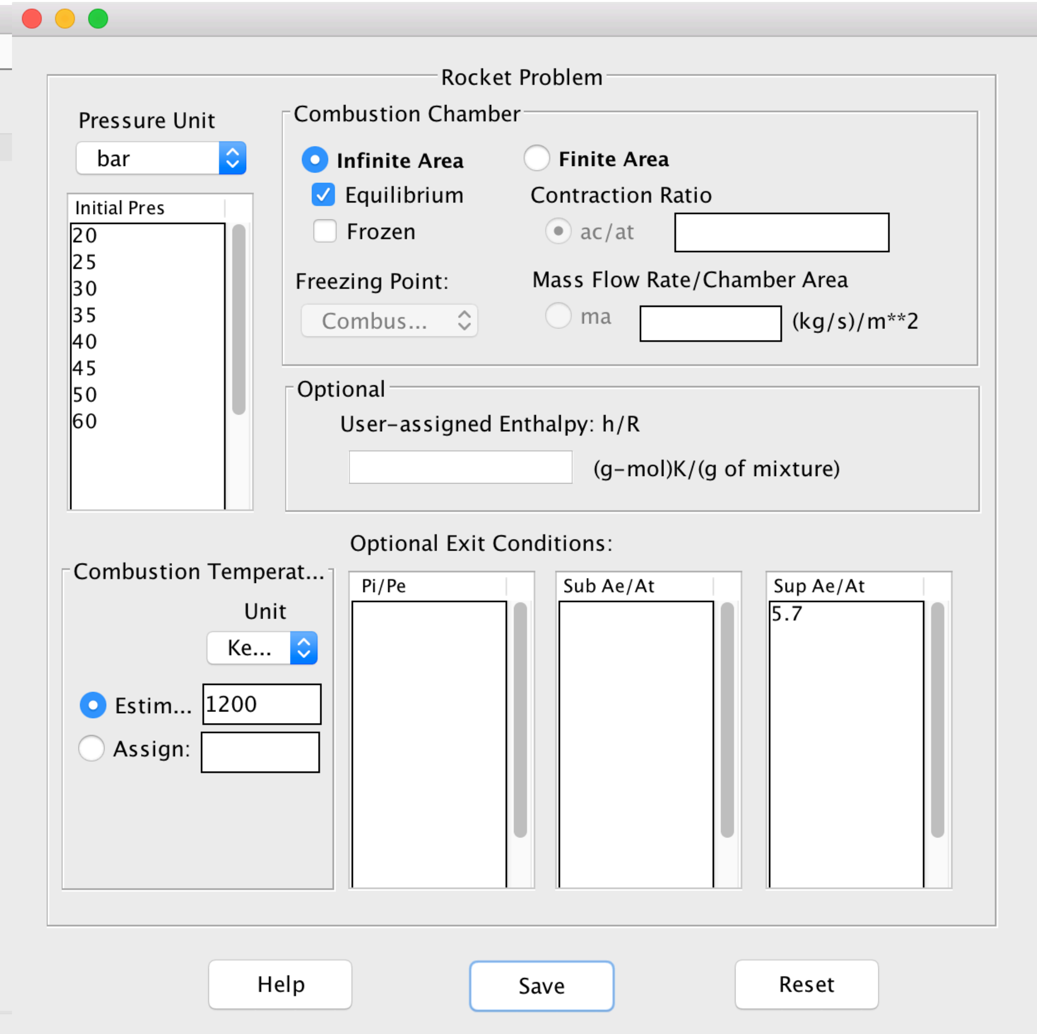
$$\frac{1}{O/F} = \frac{1}{\frac{\%H_2O_2}{100}} - 1 = \frac{1 - \frac{\%H_2O_2}{100}}{\frac{\%H_2O_2}{100}} \rightarrow \boxed{(O/F) = \frac{\frac{\%H_2O_2}{100}}{1 - \frac{\%H_2O_2}{100}}}$$

$$(O/F)_{90\%H_2O_2} = \frac{0.9}{1 - 0.9} = 9:1$$

Hints and Suggestions *(3)*

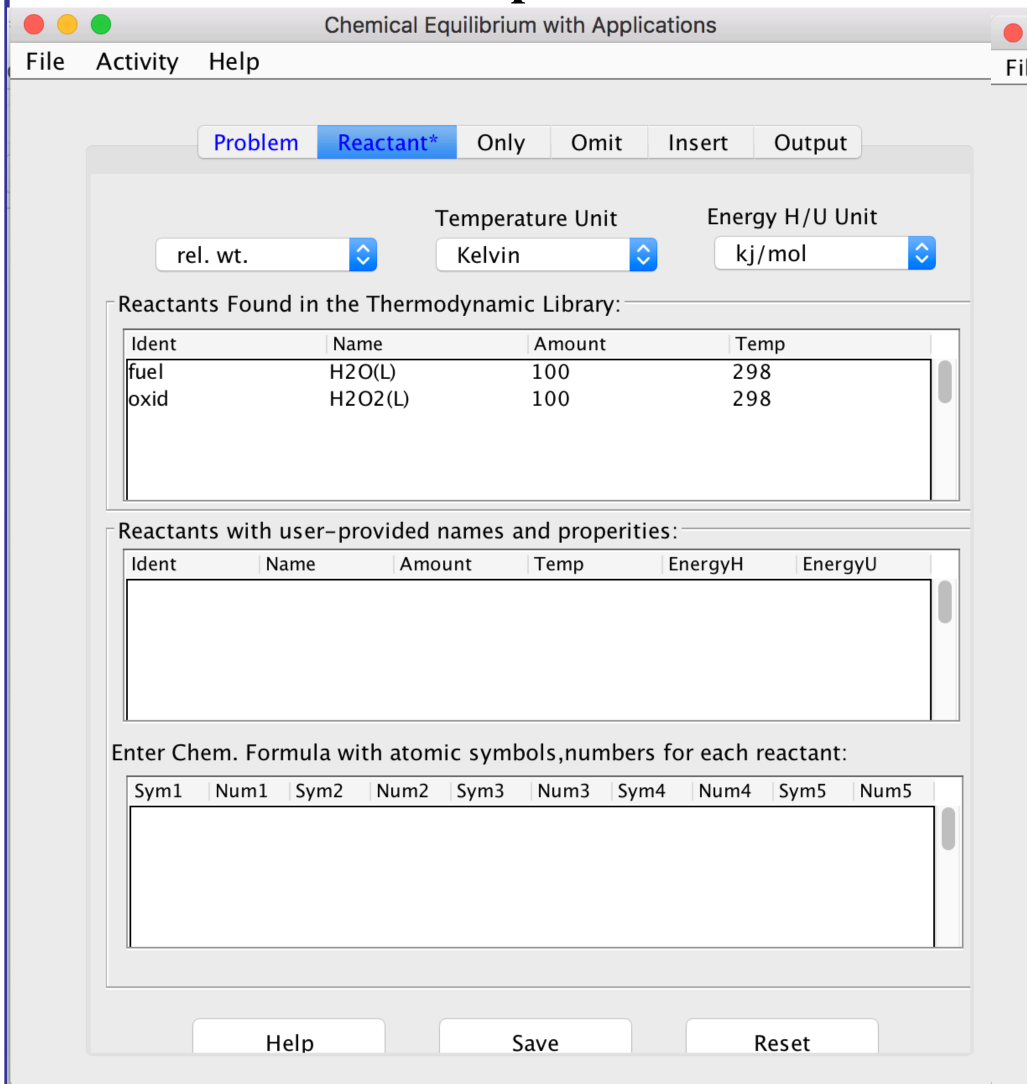
- **CEA Problem Setup**

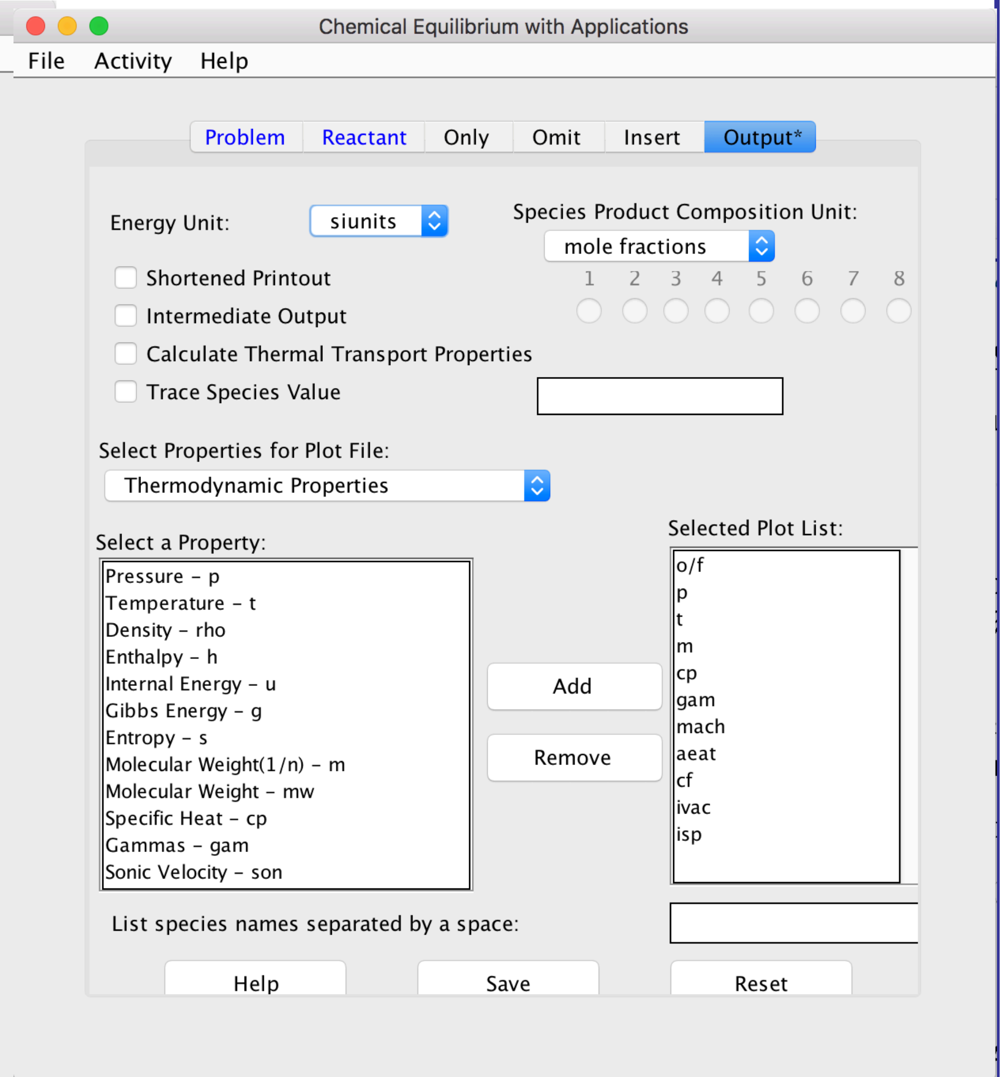




Hints and Suggestions *(4)*

- **CEA Reactant Setup**





Hints and Suggestions *(5)*

- **Example .out file Output**

Peroxide/ABS combustion example

	REACTANT	WT FRACTION (SEE NOTE)	ENERGY KJ/KG-MOL	TEMP K
OXIDANT	H2O2(L)	0.6750000	-188686.700	288.000
OXIDANT	O2	0.2500000	-297.948	288.000
OXIDANT	H2O(L)	0.0750000	-286594.763	288.000
FUEL	Acrylonitrile	0.2840000	98310.000	288.000
FUEL	Styrene	0.3050000	63310.000	288.000
FUEL	Butadiene	0.4110000	32000.000	288.000

O/F= 2.00000 %FUEL= 33.333333 R,EQ.RATIO= 1.936098 PHI,EQ.RATIO= 2.569687

	CHAMBER	THROAT	EXIT
Pinf/P	1.0000	1.8011	11.616
P, BAR	3.4474	1.9140	0.29677
T, K	2144.43	1911.94	1313.11
RHO, KG/CU M	3.5538-1	2.2142-1	4.9996-2
H, KJ/KG	-2974.98	-3514.24	-4857.97
U, KJ/KG	-3945.03	-4378.70	-5451.56
G, KJ/KG	-31505.4	-28951.5	-22328.2
S, KJ/(KG)(K)	13.3044	13.3044	13.3044
M, (1/n)	18.380	18.389	18.393
(dLV/dLP)t	-1.00034	-1.00010	-1.00000
(dLV/dLT)p	1.0089	1.0027	1.0000
Cp, KJ/(KG)(K)	2.3845	2.2891	2.2456
GAMMAs	1.2388	1.2477	1.2521
SON VEL,M/SEC	1096.2	1038.5	862.1
MACH NUMBER	0.000	1.000	2.251

TRANSPORT PROPERTIES (GASES ONLY)
CONDUCTIVITY IN UNITS OF MILLIWATTS/(CM)(K)

VISC,MILLIPOISE	0.71571	0.66042	0.50939

WITH EQUILIBRIUM REACTIONS

Cp, KJ/(KG)(K)	2.3845	2.2891	2.2456
CONDUCTIVITY	3.2251	2.7104	2.0151
PRANDTL NUMBER	0.5292	0.5578	0.5677

WITH FROZEN REACTIONS

Cp, KJ/(KG)(K)	2.2445	2.2020	2.0606
CONDUCTIVITY	2.8049	2.5405	1.8847
PRANDTL NUMBER	0.5727	0.5724	0.5569

PERFORMANCE PARAMETERS

Ae/At	1.0000	2.3700
CSTAR, M/SEC	1499.2	1499.2
CF	0.6927	1.2944
Ivac, M/SEC	1870.9	2246.5
Isp, M/SEC	1038.5	1940.6

"Optimal" Values at exit pressure

Hints and Suggestions *(6)*

- **CEA Performance Parameters**

$$\rightarrow c^* = \frac{\sqrt{\gamma R_g T_0}}{\gamma \sqrt{\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}}} = \frac{c_0}{\gamma \sqrt{\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}}} = \frac{\sqrt{\gamma R_u}}{\gamma \sqrt{\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}}} \sqrt{\frac{T_o}{M_W}}$$

$$C_F = \frac{Thrust}{P_0 \cdot A^*}$$

$$\left(T_0\right)^*_{throat} = T^* \cdot \left(\frac{\gamma^* + 1}{2}\right)$$

$$I_{sp} = \frac{1}{g_0} c^* \cdot C_F$$

$$\left(P_0\right)^*_{throat} = p^* \cdot \left(\frac{\gamma^* + 1}{2}\right)^{\frac{\gamma^*}{\gamma^* - 1}}$$

Hints and Suggestions *(7)*

- **Thrust vs Massflow**

$$\rightarrow F_{Thrust} = C_F \cdot P_0 \cdot A^* = \left[\left(\frac{1+\cos\theta_{exit}}{2}\right)\cdot\left(C_F\right)_{opt} + \left(\frac{p_{exit}-p_\infty}{P_0}\right)\cdot\left(\frac{A_{exit}}{A^*}\right)\right]\cdot P_0 \cdot A^*$$

$$\rightarrow \dot{m}_{exit} = \dot{m}_{throat} = P_0 \cdot A^* \cdot \sqrt{\frac{\gamma^*}{R_g{}^* \cdot T_0{}^*} \cdot \left(\frac{2}{\gamma^*+1}\right)^{\frac{\gamma^*+1}{\gamma^*-1}}} \rightarrow \left[\begin{array}{c} T_0{}^* = \left(\frac{\gamma^*+1}{2}\right)\cdot T^* \\ P_0 \approx P_0{}^* \\ R_g{}^* = R_u / M_w^* \end{array}\right]$$

$$\rightarrow \dot{m}_{exit} = F_{Thrust} \cdot \frac{\sqrt{\frac{\gamma^*}{R_g{}^* \cdot T_0{}^*} \cdot \left(\frac{2}{\gamma^*+1}\right)^{\frac{\gamma^*+1}{\gamma^*-1}}}}{\left[\left(\frac{1+\cos\theta_{exit}}{2}\right)\cdot\left(C_F\right)_{opt} + \left(\frac{p_{exit}-p_\infty}{P_0}\right)\cdot\left(\frac{A_{exit}}{A^*}\right)\right]}$$

Hints and Suggestions *(8)*

- **Thrust vs Massflow**

```
Pin =   675.0 PSIA
CASE =

            REACTANT                    WT FRACTION     ENERGY      TEMP
                                         (SEE NOTE)   KJ/KG-MOL       K
FUEL        H2O(L)                      1.0000000   -285841.390   298.000
OXIDANT     H2O2(L)                     1.0000000   -187793.400   298.000

O/F=    9.00000  %FUEL= 10.000000  R,EQ.RATIO= 0.547468  PHI,EQ.RATIO= 0.000000

                CHAMBER   THROAT     EXIT
Pinf/P            1.0000   1.8187   47.964
P, BAR            46.540   25.590  0.97030
T, K             1029.36   906.30   420.76
RHO, KG/CU M    1.2020 1 7.5064 0 6.1308-1
H, KJ/KG        -6555.52 -6773.05 -7559.20
U, KJ/KG        -6942.71 -7113.95 -7717.47
G, KJ/KG        -16341.7 -15389.3 -11559.4
S, KJ/(KG)(K)     9.5071   9.5071   9.5071

M, (1/n)          22.105   22.105   22.105
MW, MOL WT        22.105   22.105   22.105
(dLV/dLP)t      -1.00000 -1.00000 -1.00000
(dLV/dLT)p        1.0000   1.0000   1.0000
Cp, KJ/(KG)(K)    1.7972   1.7380   1.5036
GAMMAs            1.2647   1.2762   1.3336
SON VEL,M/SEC      699.8    659.6    459.4
MACH NUMBER        0.000    1.000    3.084

PERFORMANCE PARAMETERS

Ae/At                      1.0000   5.7000
CSTAR, M/SEC                940.0    940.0
CF                         0.7017   1.5073
Ivac, M/SEC                1176.4   1528.5
Isp, M/SEC                  659.6   1416.8
```

$$\rightarrow \dot{m}_{exit} = F_{Thrust} \cdot \frac{\sqrt{\frac{\gamma^*}{R_g^* \cdot T_0^*} \cdot \left(\frac{2}{\gamma^*+1}\right)^{\frac{\gamma^*+1}{\gamma^*-1}}}}{\left[\left(\frac{1+\cos\theta_{exit}}{2}\right)\cdot \left(C_F\right)_{opt} + \left(\frac{p_{exit}-p_\infty}{P_0}\right)\cdot\left(\frac{A_{exit}}{A^*}\right)\right]}$$

Linear Relationship!

Hints and Suggestions *(9)*

- **For Hover Control Schedule**

$$@\ t=0 \rightarrow \begin{array}{l} M_0 = 400_{kg} \\ F_0 = g \cdot M_0 = 3112.68_N \end{array}$$

- Use *massflow vs EGC thrust schedule* from Step 3
- Integrate total massflow as a function of time to get propellant consumption
- Update Thrust to match current vehicle mass

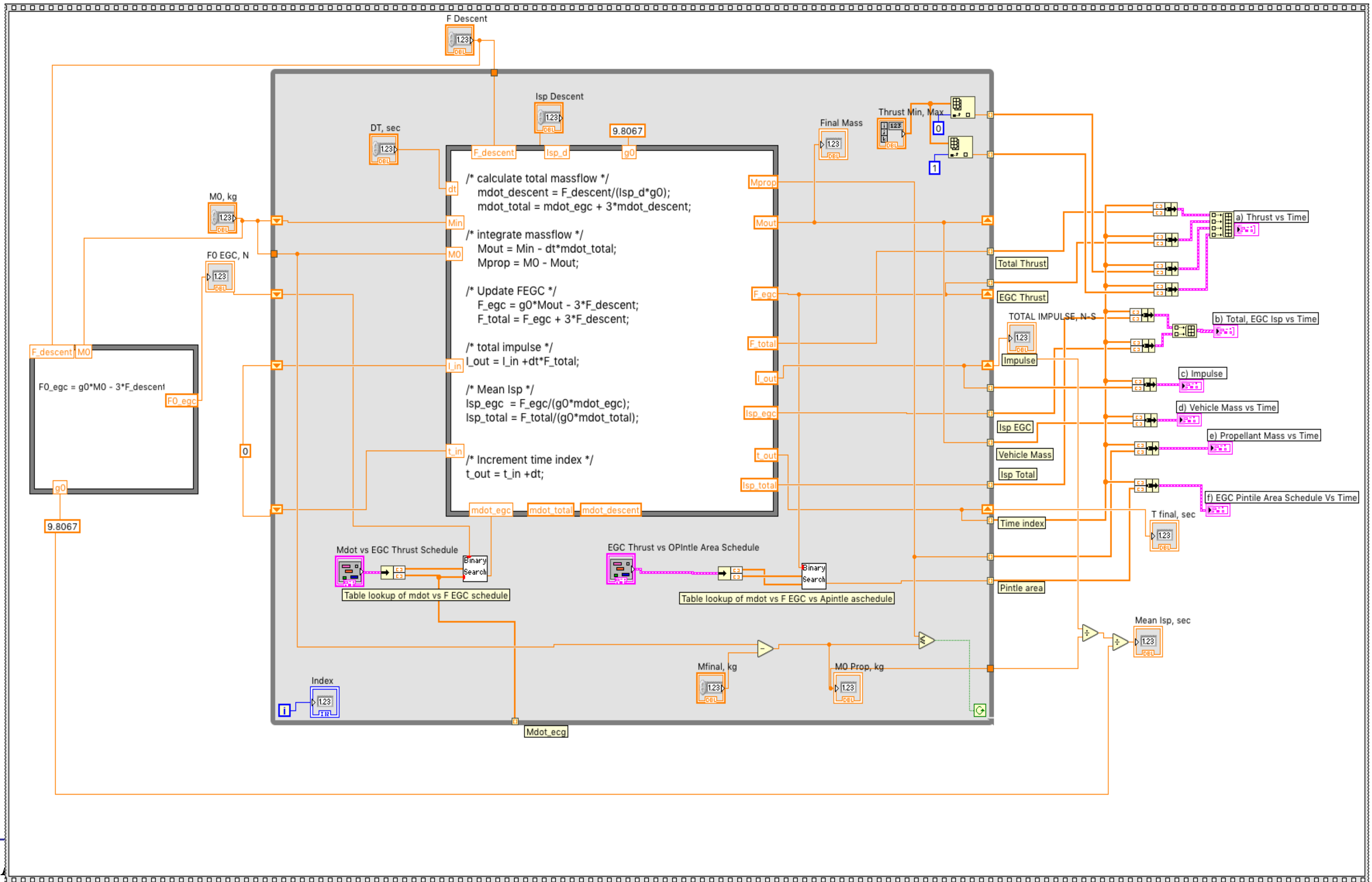
$$M_{(t)} = M_0 - \int_0^t \dot{m} \cdot dt = M_0 - \int_0^t \dot{m}_{EGC} \cdot dt - 3 \cdot \dot{m}_{descent} \cdot t$$ 29

$$\dot{m}_{descent} \approx \frac{F_{descent}}{I_{sp_{opt}} \cdot g_0} \rightarrow F_{descent} = 270_N$$

$$F_{EGC(t)} = g \cdot \left(M_0 - \int_0^t \dot{m}_{EGC} \cdot dt - 3 \cdot \frac{\dot{m}_{descent}}{1000} \cdot t \right) - 3 \cdot F_{descent}$$

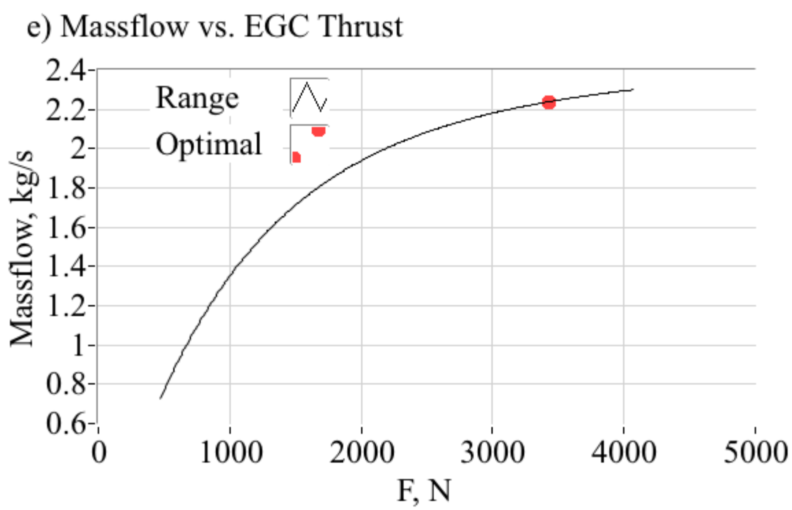
Hints and Suggestions *(10)*

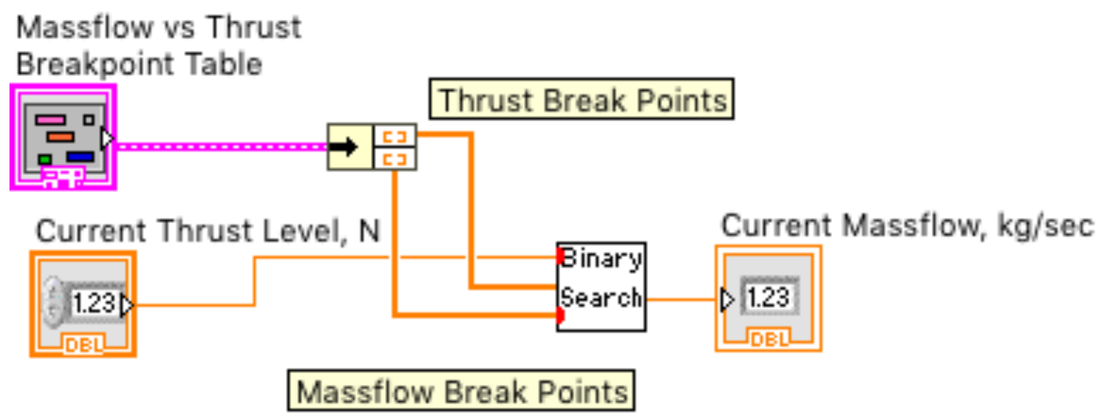
- **Block Diagram For Hover Control Schedule**



Hints and Suggestions *(11)*

- **Example Interpolation of Breakpoint Table**





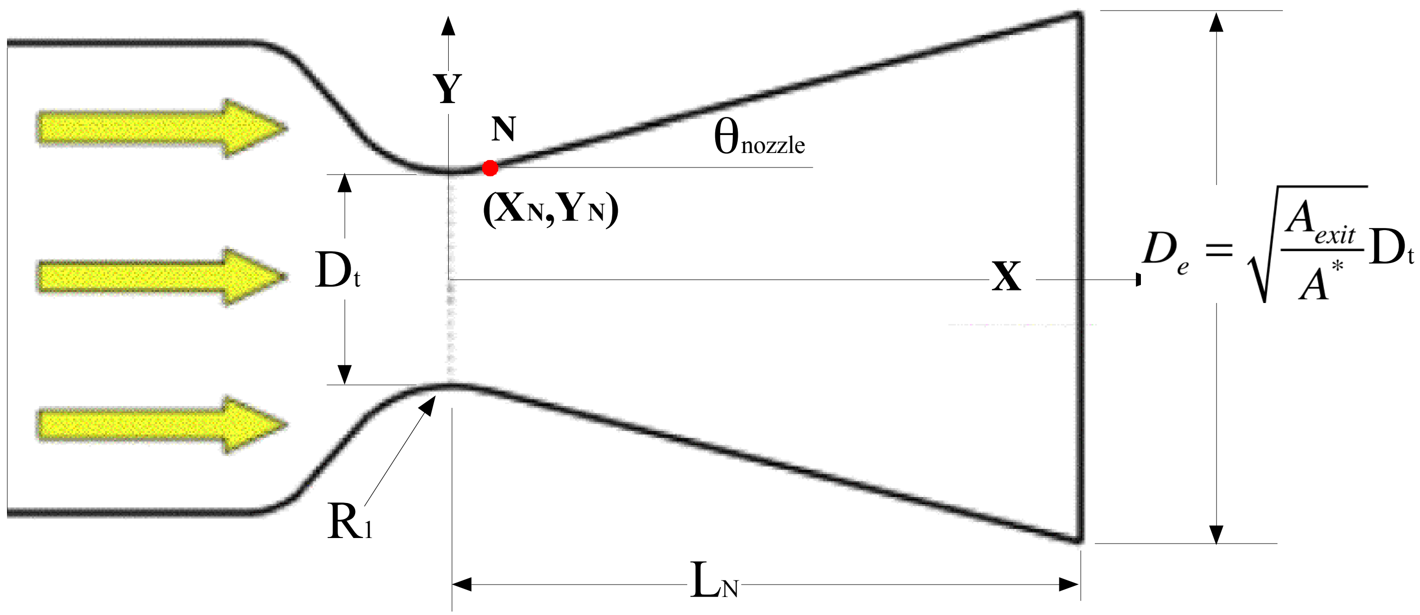

```
X1=XBRK(NMIN)
X2=XBRK(NMAX)
Y1=YBRK(NMIN)
Y2=YBRK(NMAX)

IF(X2.NE.X1)
THEN
SLOPE=(Y2-Y1)/(X2-X1)
Y=(X-X1)*SLOPE+Y1
ELSE
Y=Y1       ENDIF
```

Python Program for Binary Search (Recursive and Iterative)

https://www.geeksforgeeks.org/python-program-for-binary-search/

Hints and Suggestions *(12)*

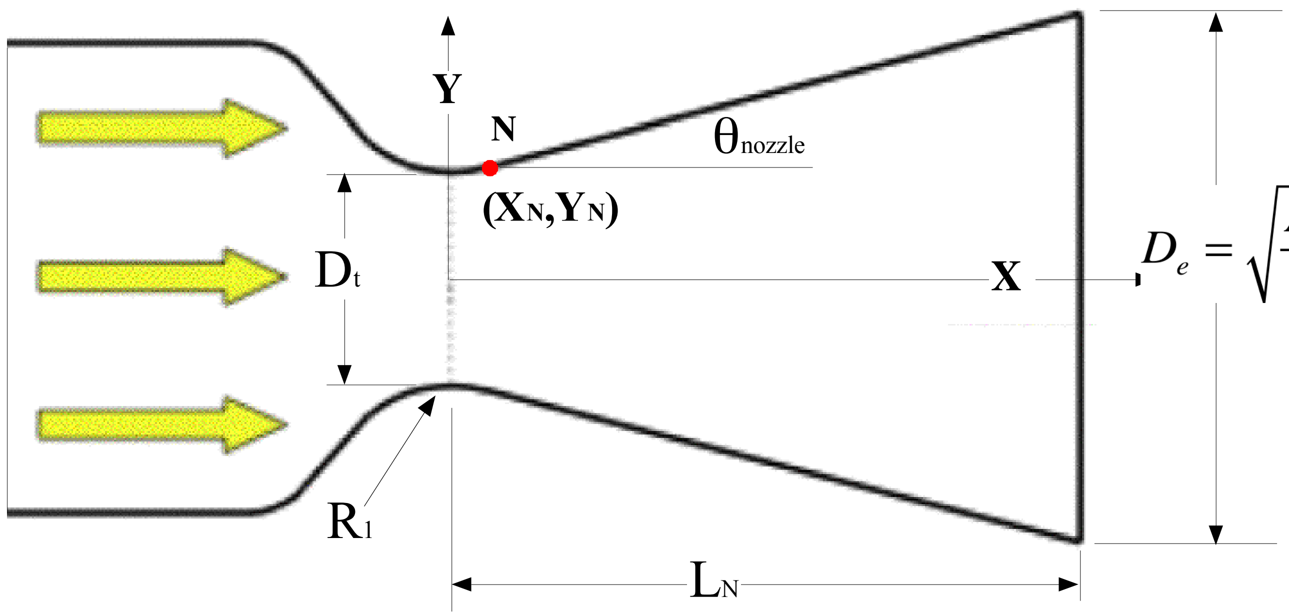
- **Conical Nozzle**



$$L_N = \frac{\frac{1}{2}\left[\sqrt{\frac{A_{exit}}{A^*}} - 1\right] D_{throat} + R_1\left[\frac{1}{\cos(\theta_{nozzle})} - 1\right]}{\tan(\theta_{nozzle})}$$

Hints and Suggestions *(13)*

- **Minimum Length Conical Nozzle**



$$D_e = \sqrt{\frac{A_{exit}}{A^*}} D_t$$

$$\rightarrow \frac{3}{2}\theta_{w_{\max}} = \frac{\nu_{exit}}{2}$$

$$\nu(M_{exit}) = \sqrt{\frac{\gamma+1}{\gamma-1}} \tan^{-1}\left\{\sqrt{\frac{\gamma-1}{\gamma+1}\left(M_{exit}^{\ 2}-1\right)}\right\} - \tan^{-1}\sqrt{M_{exit}^{\ 2}-1}$$

$$\boxed{\rightarrow \theta_{w_{\max}} = \frac{2}{3}\frac{\nu_{exit}}{2} = \frac{\nu_{exit}}{3}}$$