Chapter 6
Theoretical Rocket Performance

Before the publication of Gordon and McBride (1988), the Chemical Equilibrium Calculations (CEC) computer program described in Gordon and McBride (1976) could calculate theoretical rocket performance only for an infinite-area combustion chamber (IAC). Calculation of rocket performance for a finite-area combustor (FAC), presented in Gordon and McBride (1988), was added as an option to the Chemical Equilibrium with Transport Properties (CET) program in 1988.

Figure 6.1 presents schematic cross sections of FAC and IAC rocket engines. Various points at which calculations are made in the CEA program to obtain rocket performance are indicated in these figures. Combustion and throat parameters are always calculated first automatically. For the IAC model, only one combustion point is calculated, namely, at infinite area (inf). However, for the FAC model, two combustion points are calculated, namely, at the combustion chamber inlet (or equivalently at the injector face, inj) and at the combustor end, c. In addition to these two combustion points for the FAC, a combustion calculation for an infinite-area combustor, indicated in figure 6.1(a) by the dashed line, is also made. The results at this fictitious point are used as an aid in an iteration procedure to obtain combustor end conditions, as discussed in section 6.4. In addition, the pressure at this point is used in calculating c^* (see section 6.2.6). Throat conditions are indicated by the subscript t; other nozzle exit conditions, either subsonic or supersonic, are indicated by the subscript e. Nozzle conditions are assigned as an option and may be in the form of assigned area ratios, pressure ratios, or both.

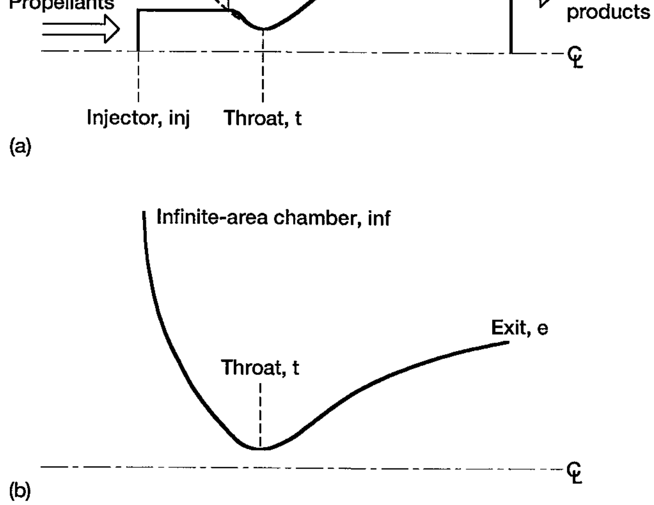

Figure 6.1.—Schematics of rocket combustion chamber cross sections. (a) Finite-area combustion chamber. (b) Infinite-area combustion chamber.

6.1 Assumptions

The calculation of theoretical rocket performance involves a number of assumptions. For the same propellant and operating conditions theoretical performance can vary depending on which assumptions are used. For this report most of the assumptions are the same for both the IAC and FAC models. These assumptions are one-dimensional form of the continuity, energy, and momentum equations; zero velocity at the combustion chamber inlet; complete combustion; adiabatic combustion; isentropic expansion in the nozzle; homogeneous mixing; ideal-gas law; and zero temperature lags and zero velocity lags between condensed and gaseous species. The chamber in the FAC model is assumed to have a constant cross-sectional area. In this chamber combustion is a nonisentropic, irreversible process. During the burning process part of the energy released is used to raise the entropy, and the pressure drops. Expansion in the nozzle is assumed to be isentropic.

Combustion conditions are obtained with the assumption of chemical equilibrium of the combustion products. For the IAC model the CEA program provides the option of calculating either equilibrium or frozen theoretical rocket performance. Equilibrium performance is based on the assumption of instantaneous chemical equilibrium during expansion in the nozzle. Frozen performance is based on the assumption that composition remains frozen at the combustion composition during expansion. For the FAC model only equilibrium performance is permitted.

Assuming the same velocity (either zero or otherwise) at the combustion chamber inlet, identical thermodynamic results are obtained for the combustion inlet condition for both the IAC and FAC models.

6.2 Parameters

6.2.1 Conservation Equations

Rocket performance, as well as other fluid dynamic problems in the program, is based on the following conservation equations, which are consistent with the assumptions previously discussed:

(1) Continuity:

$$\rho_2 A_2 u_2 = \rho_1 A_1 u_1 \tag{6.1}$$

(2) Momentum:

$$P_2 + \rho_2 u_2^2 = P_1 + \rho_1 u_1^2 \tag{6.2}$$

(3) Energy:

$$h_2 + \frac{u_2^2}{2} = h_1 + \frac{u_1^2}{2} \tag{6.3}$$

Equation (6.1) describes the condition of constant mass flow rate, which will be given the symbol $\dot{m}$; that is,

$$\dot{m} = \rho A u \tag{6.4}$$

Equation (6.2) applies only for constant-area, one-dimensional flow.

6.2.2 Velocity of Flow

The combustion chamber inlet is indicated by the subscripts inf for the IAC model and inj for the FAC model. Then using these subscripts instead of 1 and using e instead of 2 in equation (6.3) and assuming the velocity at the combustion chamber inlet to be negligible relative to the exit velocity result in equation (6.3) becoming

$$u_e = \begin{cases} \sqrt{2(h_{\text{inf}} - h_e)} & \text{for IAC model} \\ \sqrt{2(h_{\text{inj}} - h_e)} & \text{for FAC model} \end{cases} \tag{6.5}$$

where h is in units of joules per kilogram and u is in units of meters per second.

6.2.3 Force

From the momentum principle of fluid mechanics the external force on a body in a steadily flowing fluid is due to the change of momentum of the fluid and to the increase in pressure forces acting on the body. For rocket applications this is expressed as

$$F = \frac{\dot{m} u_e}{g_c} + (P_e - P_a) A_e \tag{6.6}$$

The conversion factor g_c has been introduced to allow for various units. For some commonly used systems of units, such as the cgs system or the International System (Goldman and Bell, 1986), $g_c = 1$. However, in the English Technical System, commonly used by engineers,

$$g_c = 32.1740 \text{ (lbm/lbf)(ft/s}^2)$$

6.2.4 Specific Impulse

Specific impulse is defined as force per unit mass flow rate. From equation (6.6)

$$I = \frac{F}{\dot{m}} = \frac{u_e}{g_c} + \frac{(P_e - P_a) A_e}{\dot{m}} \tag{6.7}$$

In rocket literature specific impulse is often expressed in English Technical System units of pounds force per pound mass per second. However, for those systems of units previously mentioned for which $g_c = 1$, I is both dimensionally and numerically equal to velocity.

In this report when the exit pressure is equal to the ambient pressure, specific impulse will be given the symbol I_{sp}. From equation (6.7)

$$I_{sp} = \frac{u_e}{g_c} \tag{6.8}$$

When the ambient pressure is assumed to be zero (vacuum conditions), specific impulse will be given the symbol I_{vac}. From equations (6.7) and (6.8)

$$I_{vac} = I_{sp} + \frac{P_e A_e}{\dot{m}} \tag{6.9}$$

6.2.5 Mach Number

Mach number is defined as the ratio of velocity of flow to velocity of sound:

$$\mathscr{M} = \frac{u}{a} \tag{6.10}$$

Velocity of flow is given by equation (6.5). Velocity of sound is given by equation (2.74) (or eq. (3.10)).

6.2.6 Characteristic Velocity

Characteristic velocity is given the symbol c^* and is defined as

$$c^* = \frac{P_{inf} A_t g_c}{\dot{m}} \tag{6.11}$$

6.2.7 Area per Unit Mass Flow Rate

From equation (6.4)

$$\frac{A}{\dot{m}} = \frac{1}{\rho u_e} \tag{6.12}$$

The terms ρ and u_e are obtained from equations (2.1b) and (6.5), respectively.

6.2.8 Coefficient of Thrust

The coefficient of thrust is defined in terms of previously defined parameters

$$C_F = \frac{u}{c^*} \tag{6.13}$$

6.2.9 Area Ratio

The ratio of area at any exit nozzle station to area at the throat is obtained from the values of equation (6.12) at these two points

$$\frac{A_e}{A_t} = \frac{(A/\dot{m})_e}{(A/\dot{m})_t} \tag{6.14}$$

6.3 Procedure for Obtaining Equilibrium Rocket Performance for IAC Model

The procedure for obtaining equilibrium performance differs somewhat for the IAC and FAC models. The principal difference is due to only one combustion point being required for the IAC model (point inf in fig. 6.1(b)) but two combustion points being required for the FAC model, namely, at the inlet and exit of the finite chamber (points inj and c in fig. 6.1(a)). The procedure for the IAC model is discussed first, inasmuch as it is somewhat simpler.

For the IAC model the procedure consists of first determining combustion properties and then determining exhaust properties at the throat and at other assigned stations, if any, in the nozzle exit. Combustion and throat conditions are always obtained first automatically by the CEA program. To obtain other nozzle conditions (either subsonic or supersonic), the desired points must be specified as part of the input in the form of assigned area ratios or pressure ratios.

For the FAC model the procedure involves first determining combustion properties at the combustor inlet. This station is also referred to as the "injector (inj) station." Conditions at the end of the combustor c and at the throat are then both determined by means of an iteration loop that also includes the fictitious point inf