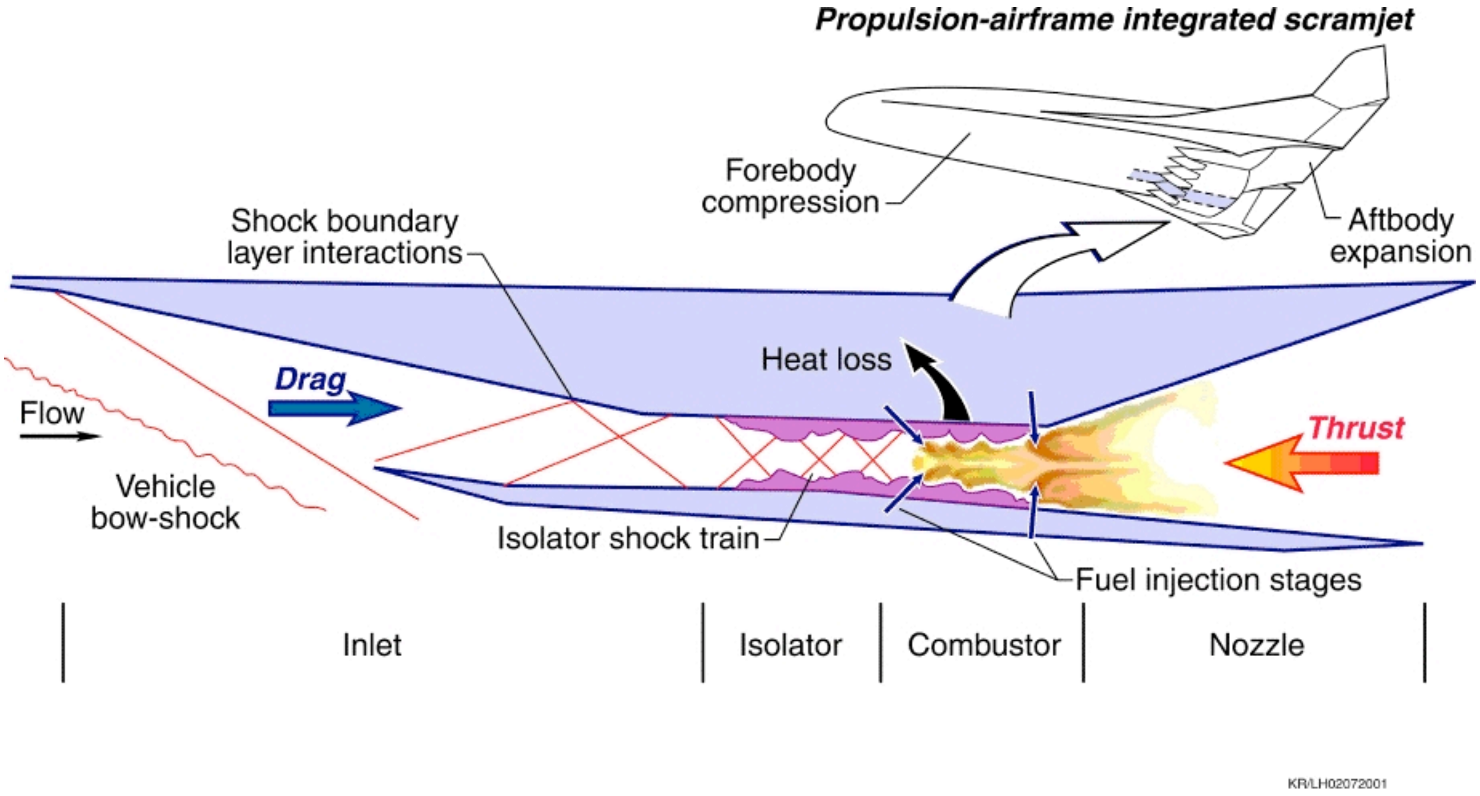
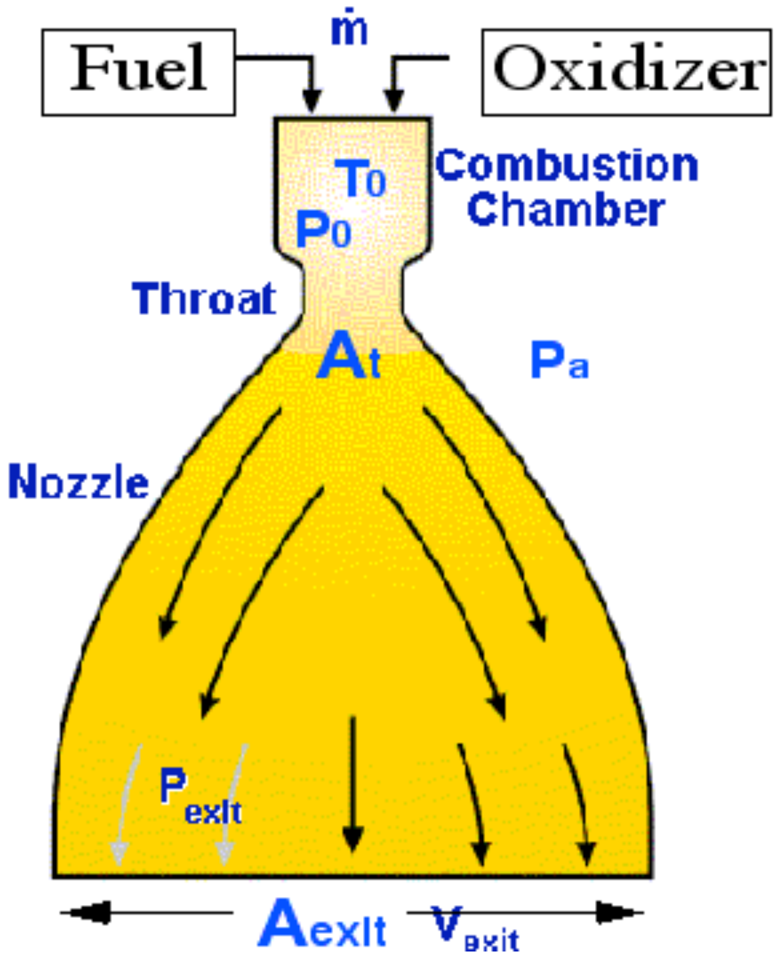
# *Section 9.2*

# Introduction to High Speed Airbreathing Propulsion Systems





# About the Rocket Motor?




- **Propellants combine and burn in combustion chamber**

- Oxidizer … "liquid air"
Have to take our own along
So that the engine can breathe
At high altitudes

- How Much Oxidizer?
… depending op chosen propellants
… 4-9 times as much
As the fuel we carry!

# Flight Without Oxidizer, I

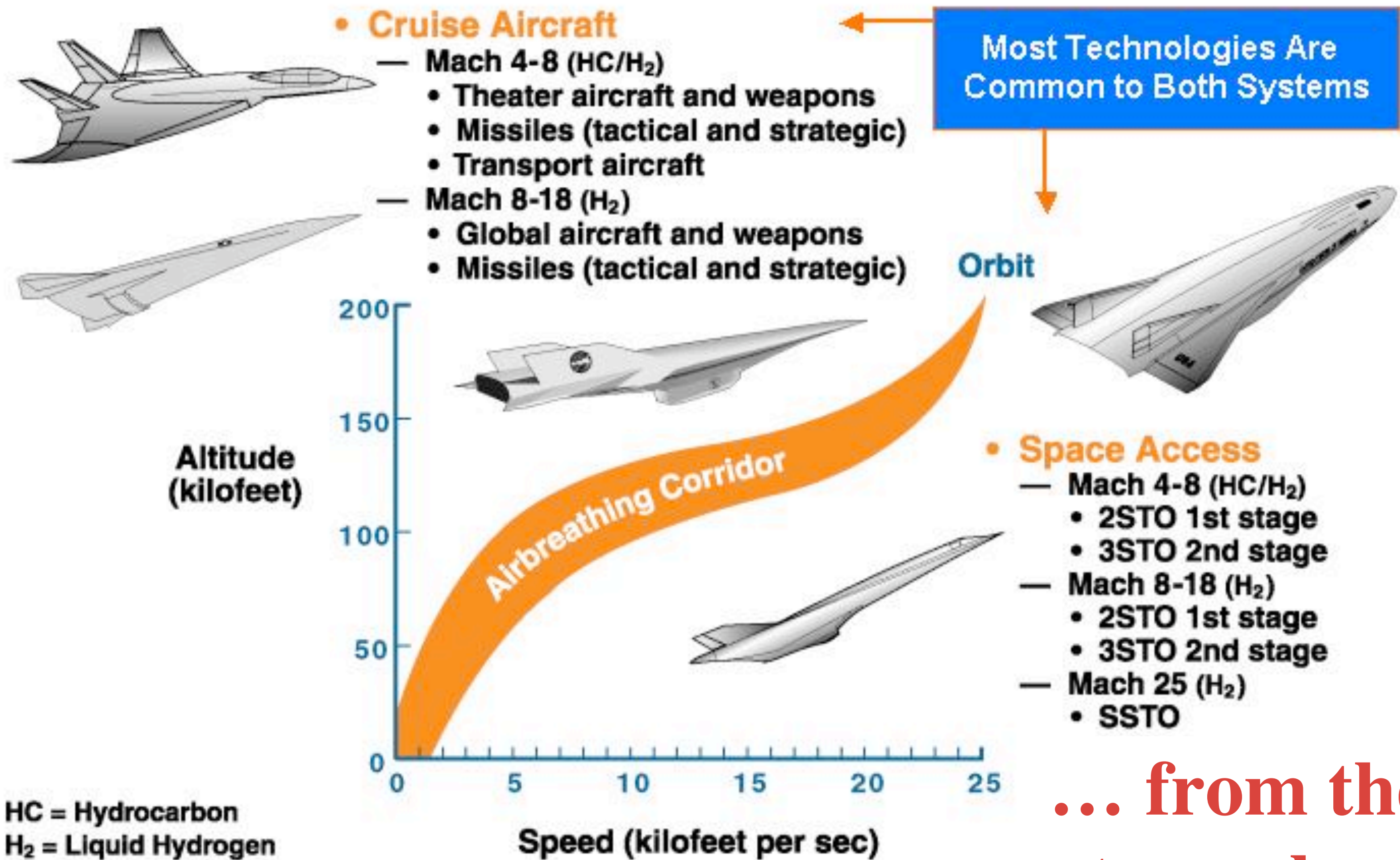
What if we could get enough oxygen from ambient air?


What happens to our Isp?

$$I_{sp} = \frac{F}{g_o \, \dot{m}} = \frac{F}{g_o \left[\dot{m}_{fuel} + \dot{m}_{ox}\right]} \approx \frac{F}{g_o \left[\dot{m}_{fuel} + 6\, \dot{m}_{fuel}\right]} = \frac{F}{7\, g_o \left[\dot{m}_{fuel}\right]}$$

OK .. Lets loose the Oxidizer … and our Isp goes up by a factor Of 7! … but where do we the the Oxidizer for combustion? ….

# POTENTIAL AIR-BREATHING HYPERSONIC VEHICLE APPLICATIONS AND THEIR FLIGHT ENVELOPES




… from the atmosphere

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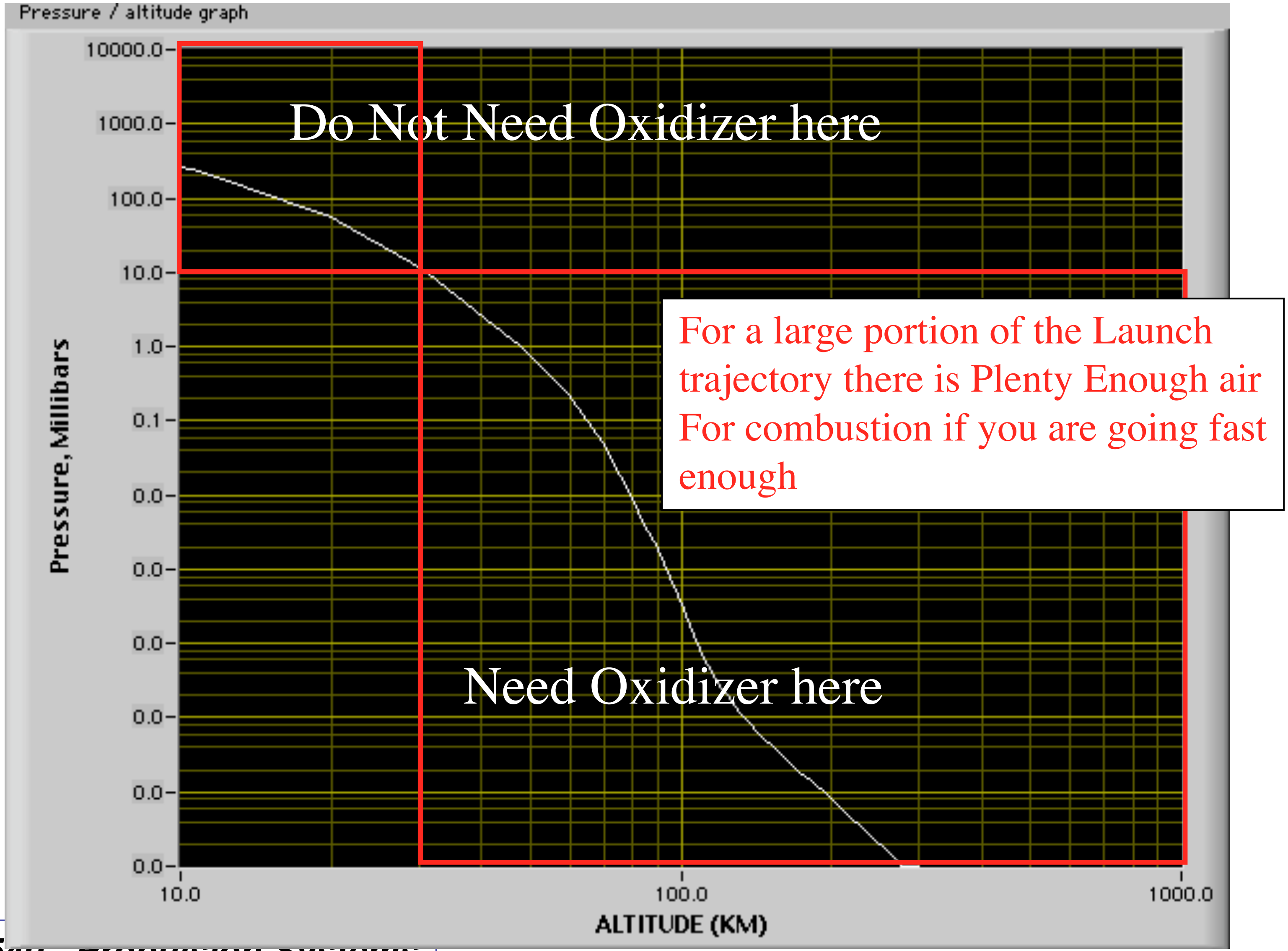
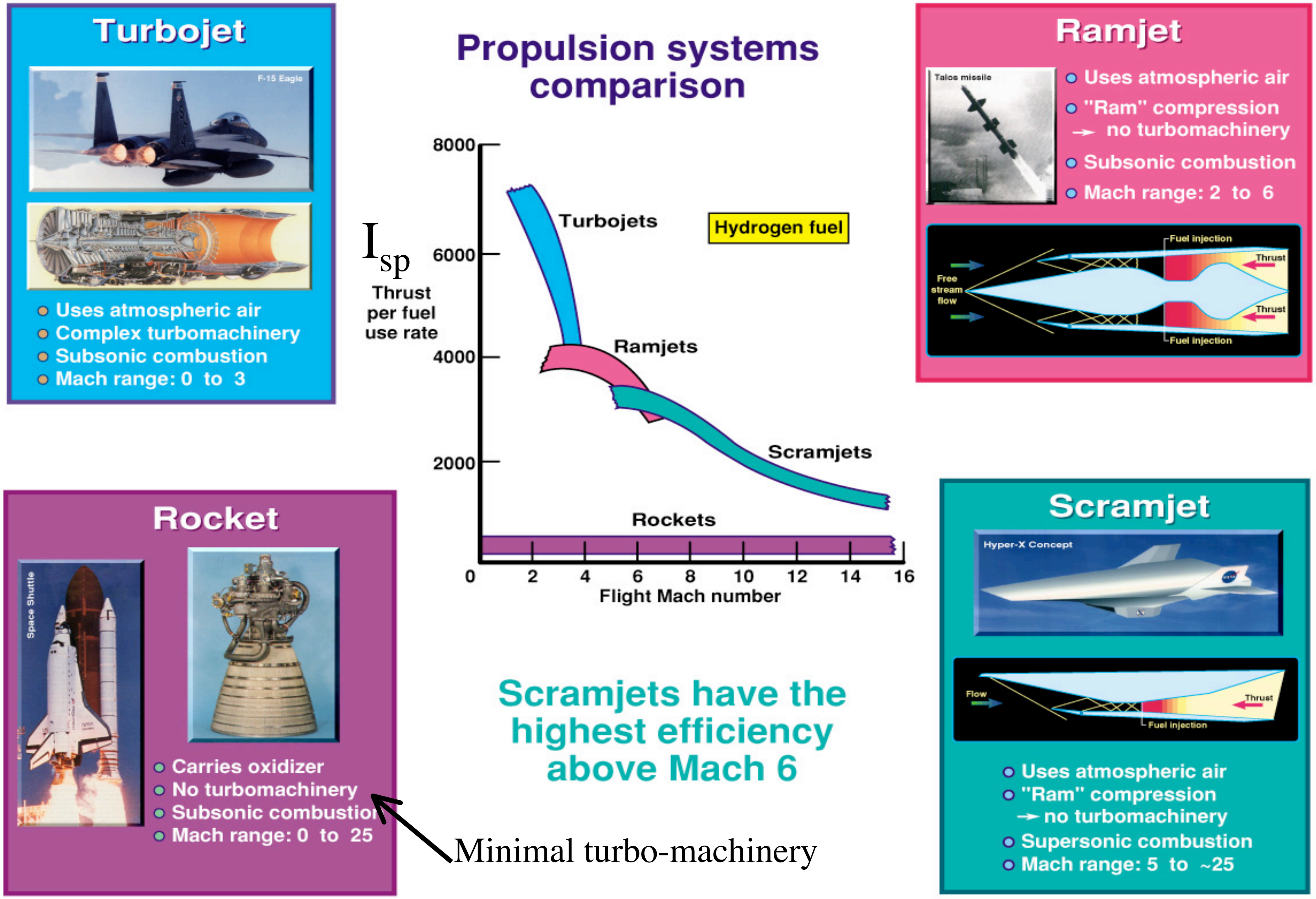
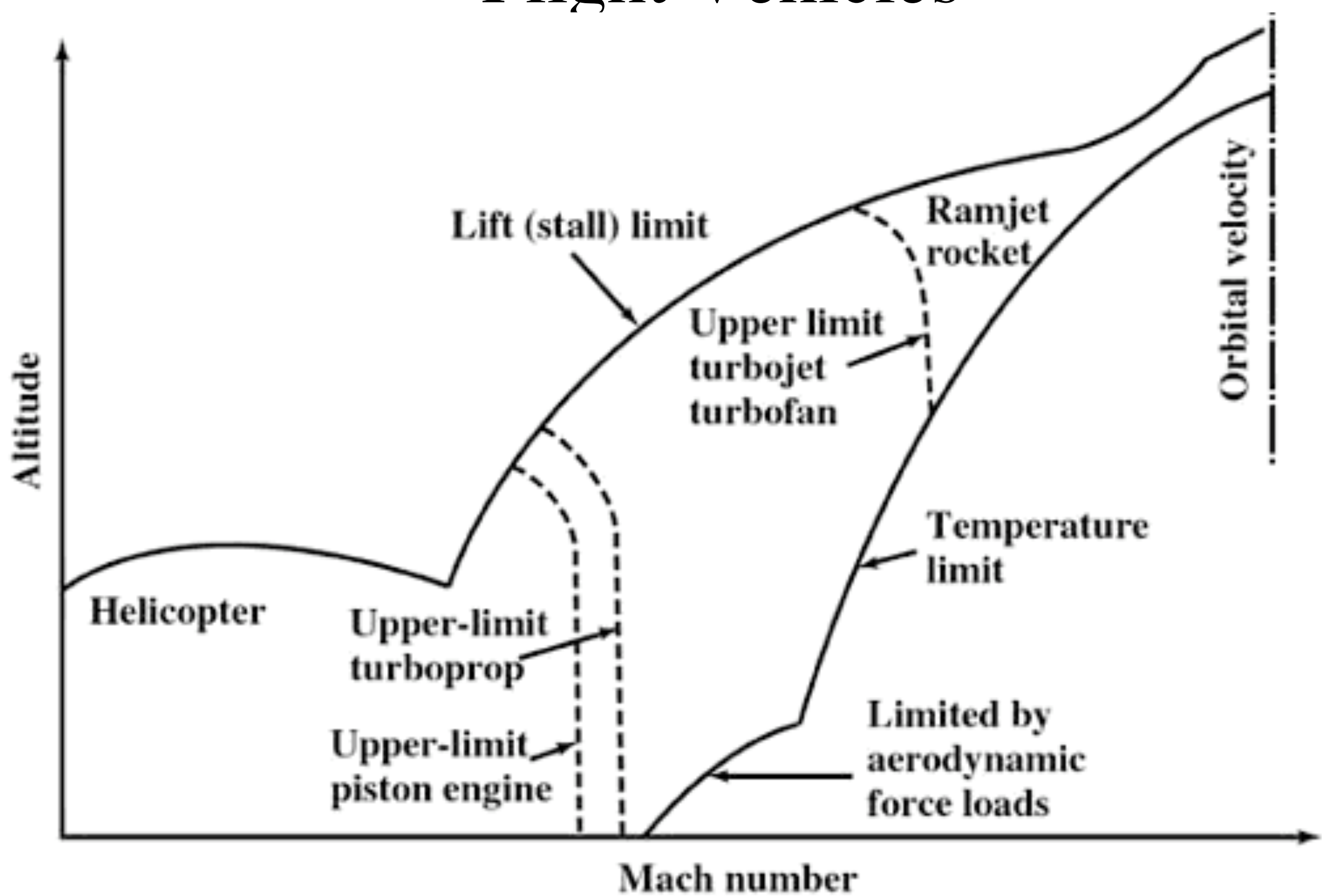
# Flight Without Oxidizer, II



# Fuel Efficiencies of Various High Speed Propulsion Systems



# Operational Flight Envelope for Various Flight Vehicles

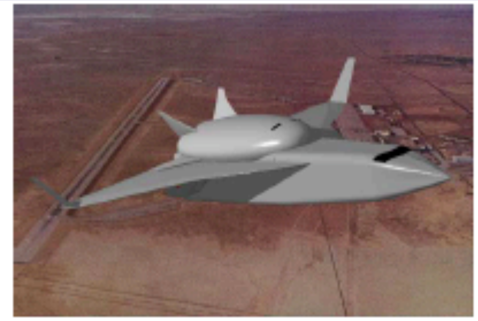


# Operations Payoffs for Airbreathing Launch

- Decreased gross lift-off weight, resulting in smaller facilities and easier handling
- Wider range of emergency landing sites for intact abort
- Powered flyback/go-around & more margin at reduced power
- Self-ferry & taxi capabilities
- Greatly expanded launch windows (double or triple)
- Rapid orbital rendezvous (up to three times faster than rockets)
- Wider array of landing sites from orbit, with 2,000-mile cross range and increased range
- Reduced sensitivity to weight growth

# Applications

*Space Access*



*Weapons*

RLV (Affordable, timely access to space)

Hypersonic Cruiser (Global Reach/Attack)

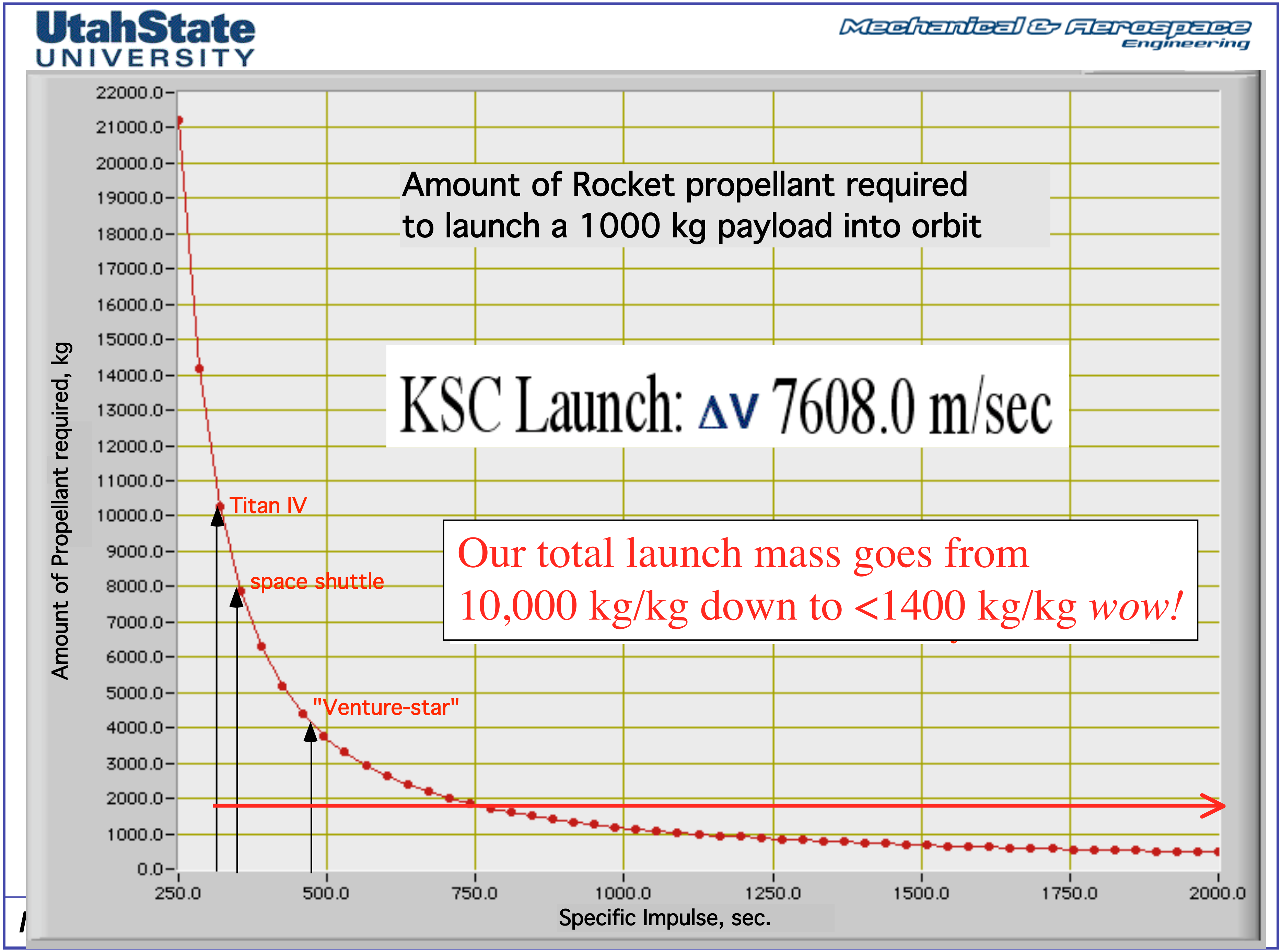
**Far-Term**

Hypersonic Missile (Time-critical targets)

**Mid-Term**

**Near-Term**

**Pursue Stepping-Stone Approach**



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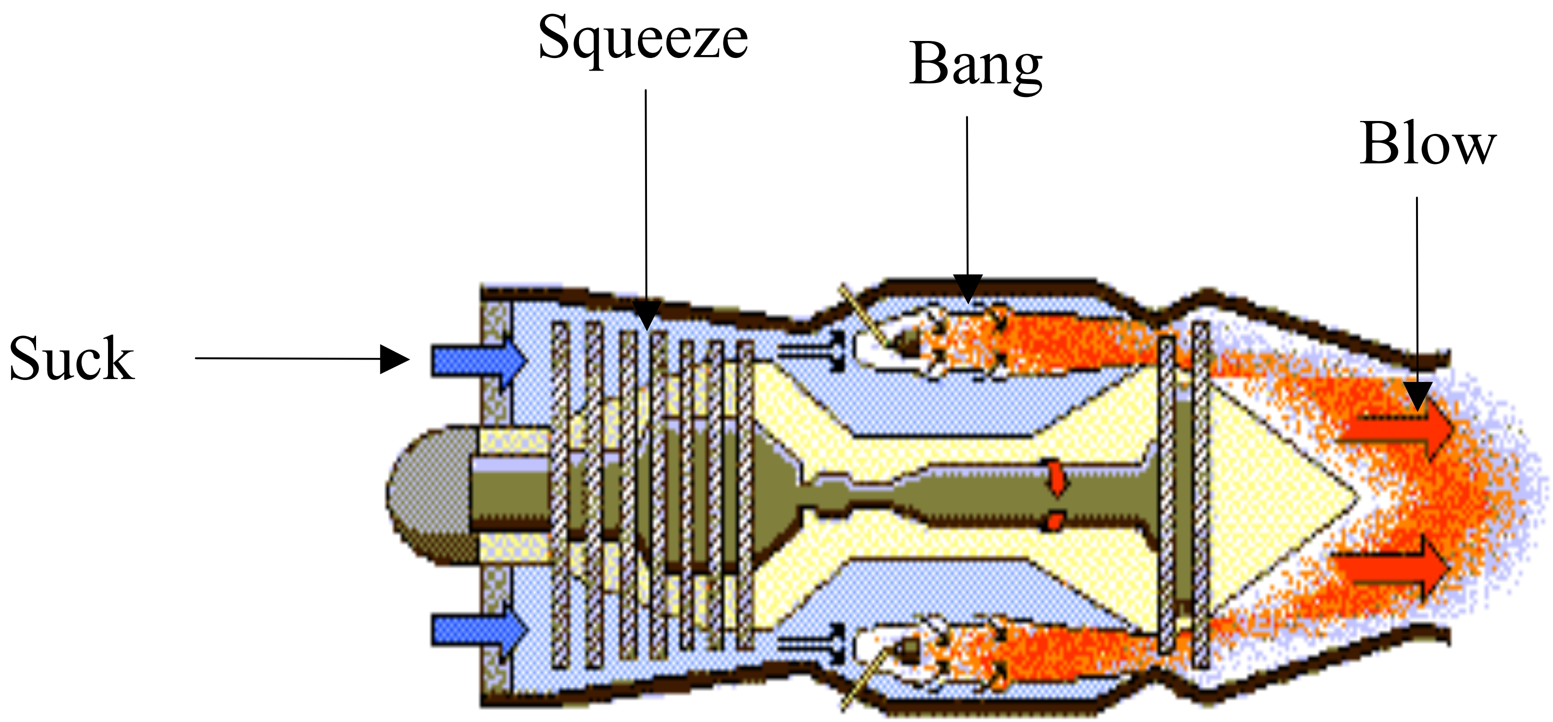
Amount of Rocket propellant required
to launch a 1000 kg payload into orbit
KSC Launch: ΔV 7608.0 m/sec
Our total launch mass goes from
10,000 kg/kg down to <1400 kg/kg wow!
Titan IV
space shuttle
"Venture-star"
Amount of Propellant required, kg
Specific Impulse, sec.
22000.0
21000.0
20000.0
19000.0
18000.0
17000.0
16000.0
15000.0
14000.0
13000.0
12000.0
11000.0
10000.0
9000.0
8000.0
7000.0
6000.0
5000.0
4000.0
3000.0
2000.0
1000.0
0.0
250.0
500.0
750.0
1000.0
1250.0
1500.0
1750.0
2000.0

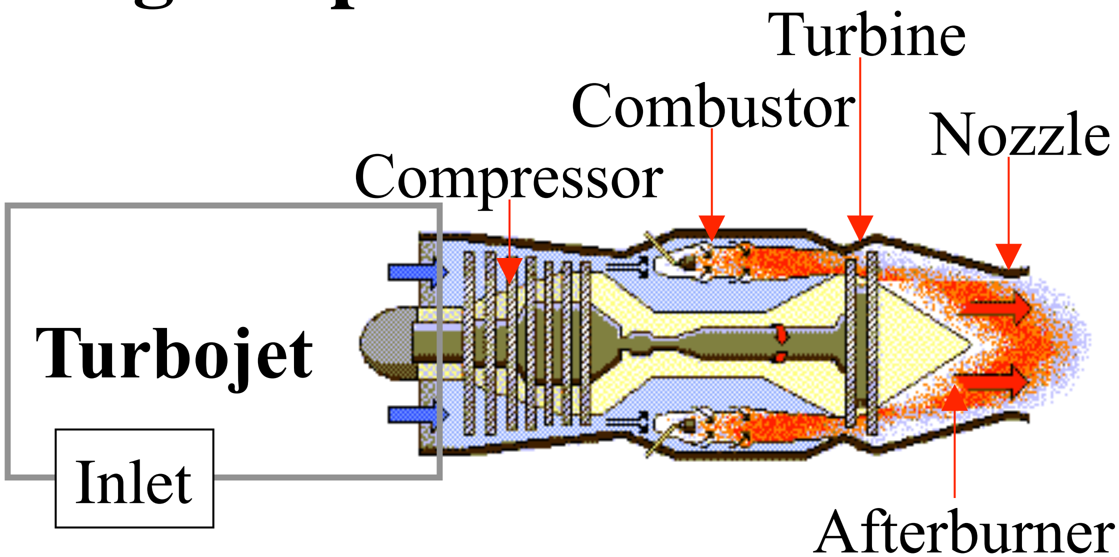
# Airbreathing Propulsion Basics

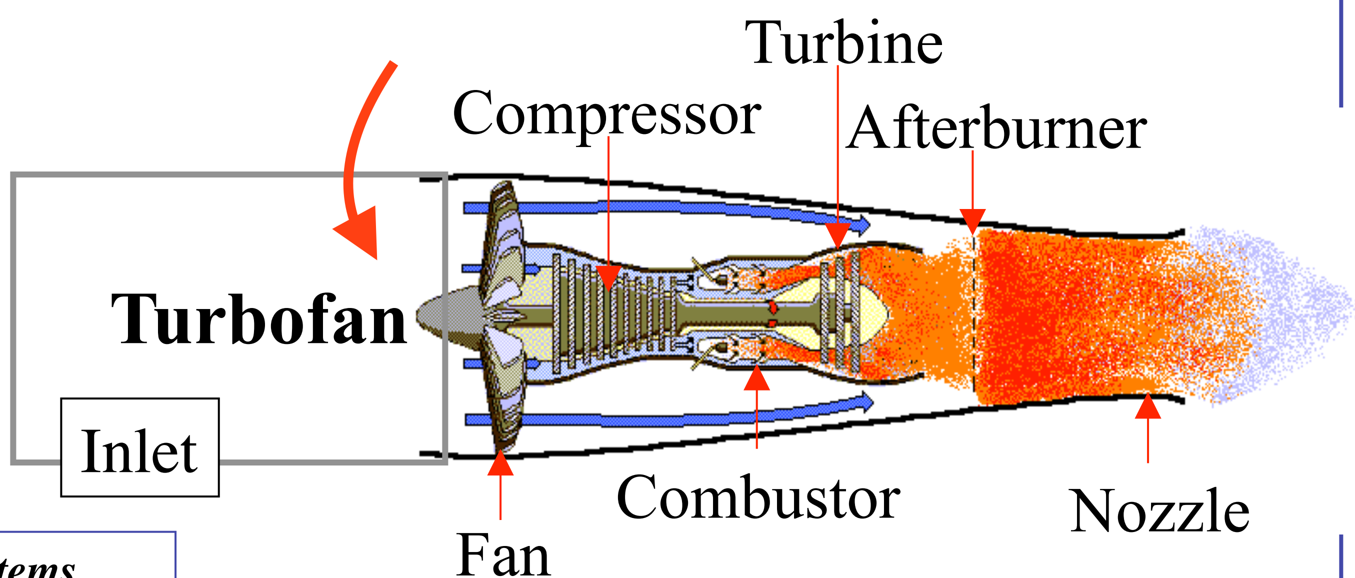


# Airbreathing Propulsion Basics (cont'd)

• Turbojet/Turbo fan engines Combustion Cycle steps

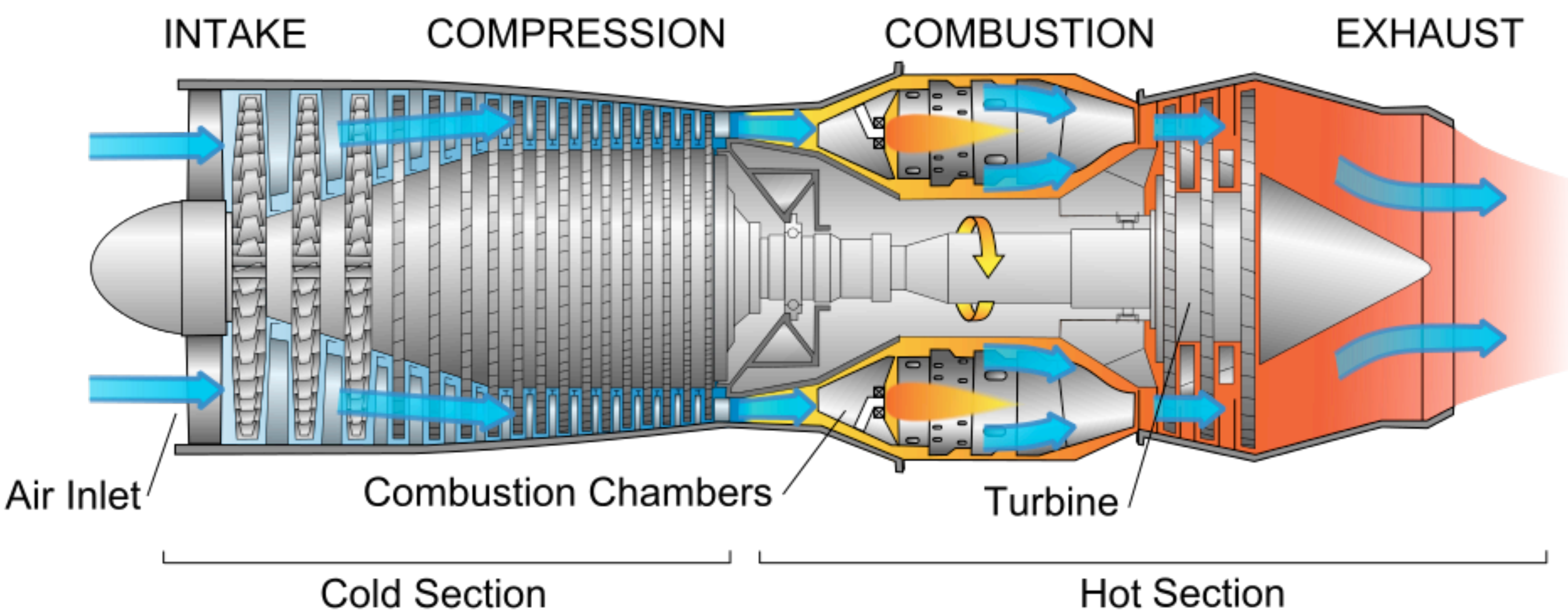
Compression and Power extraction steps use Turbo-machinery To augment cycle



# Airbreathing Propulsion Basics (cont'd)

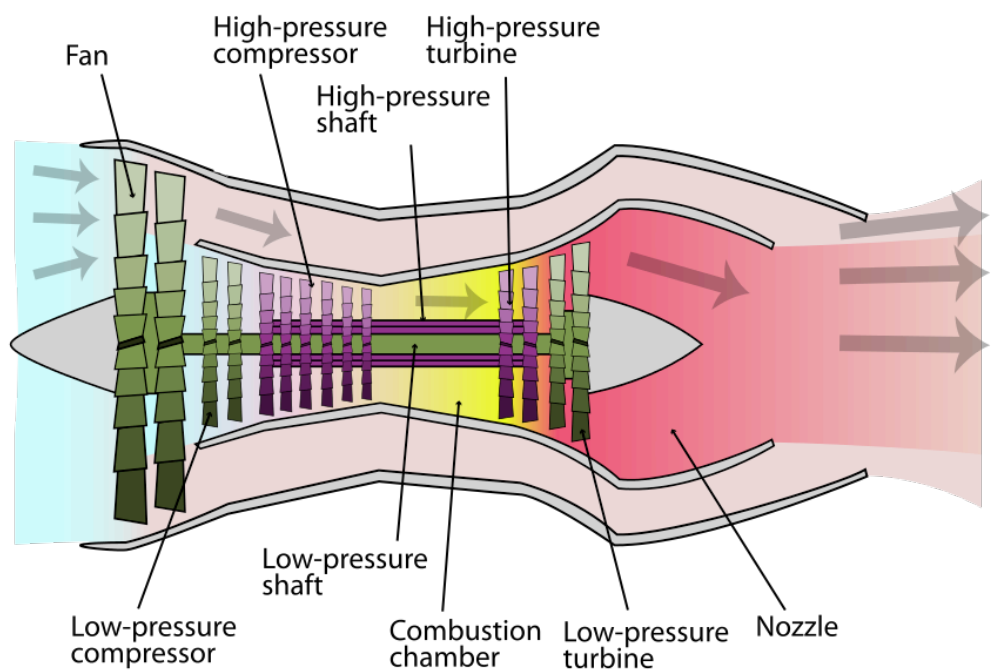
***turbojet.*** Simple turbine engine that produces all of its thrust from the exhaust from the turbine section. However, because all of the air is passing through the whole turbine, all of it must burn fuel.



# Airbreathing Propulsion Basics (cont'd)

**Turbofan.** Turbine primarily drives a fan at the front of the engine. Most engines drive the fan directly from the turbine. Part of the air enters the turbine section of the engine, and the rest is bypassed around the engine. In high-bypass engines, most of the air only goes through the fan and bypasses the rest of the engine and providing most of the thrust.

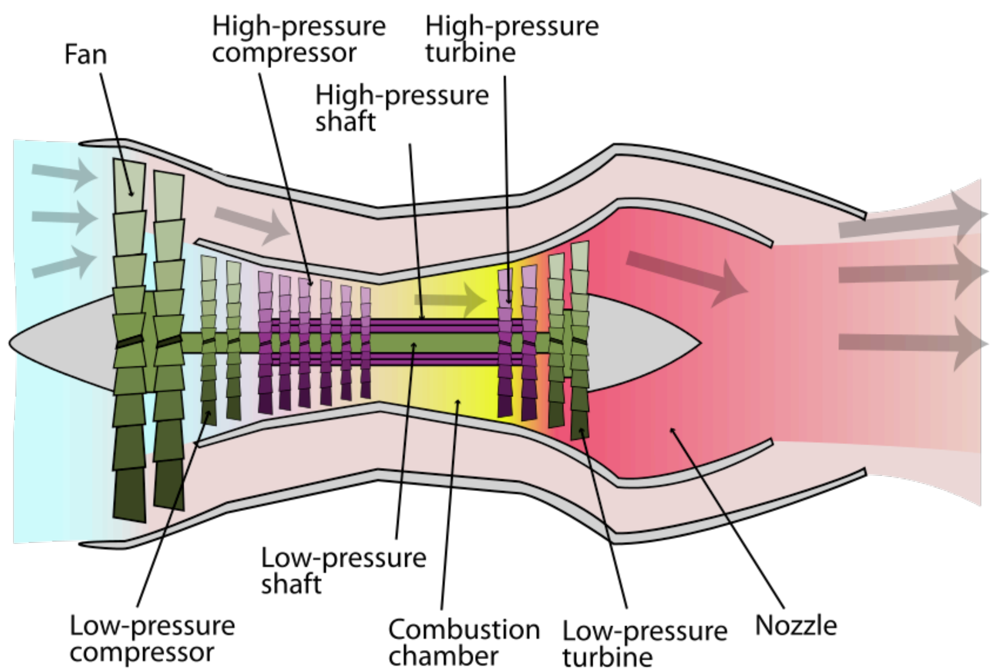
A turbofan thus can be thought of as a turbojet being used to drive a ducted fan, with both of those contributing to the thrust. The ratio of the mass-flow of air bypassing the engine core compared to the mass-flow of air passing through the core is referred to as the bypass ratio.



# Airbreathing Propulsion Basics (cont'd)

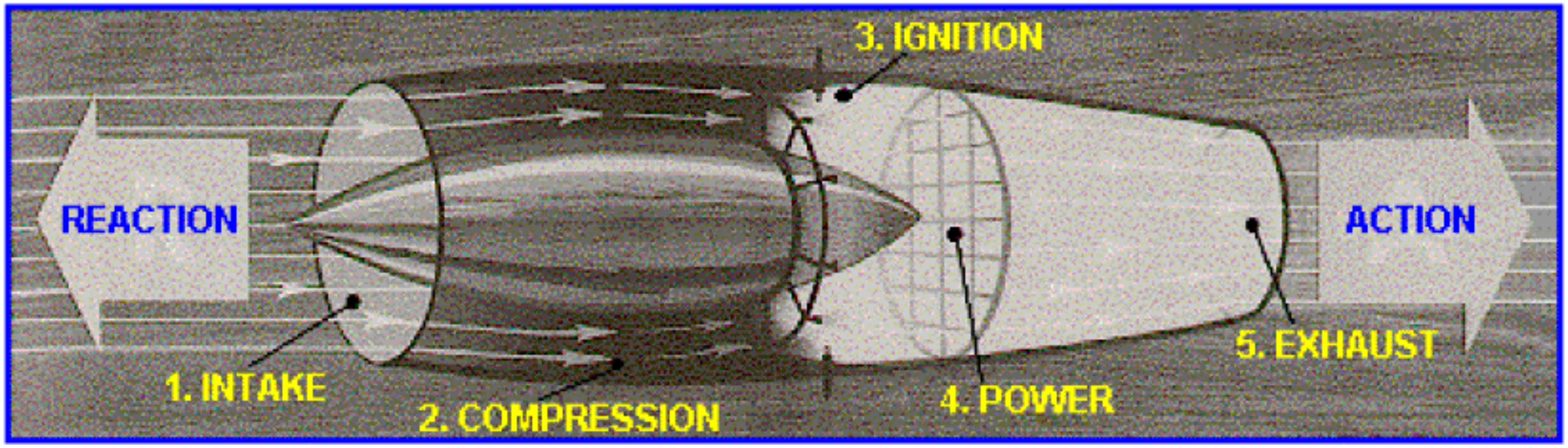
Turbofans have a number of benefits over simple turbojets. It is more energy efficient to accelerate a large air mass by a small amount, than a small air mass by a lot. A turbofan effectively increases the volume of air propelled with a corresponding decrease in average exhaust gas velocity.

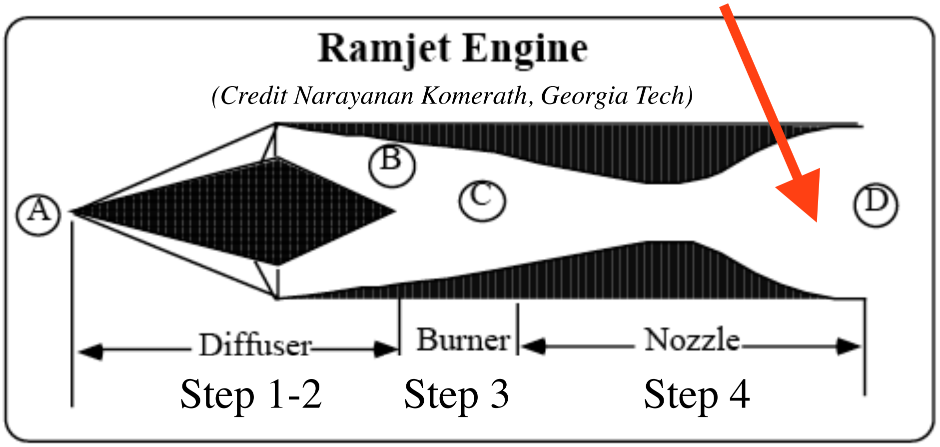
Larger, slower fan, positioned upwind of the compressor pushes air (relatively) slowly through both the core and the bypass. The air that makes it to the core is compressed and undergoes combustion. The air that makes it to the bypass is ducted around the core, and mixes with the exhaust gases when it exits the back of the engine.



# Airbreathing (Jet) Propulsion Basics (cont'd)

- Ramjet Engine



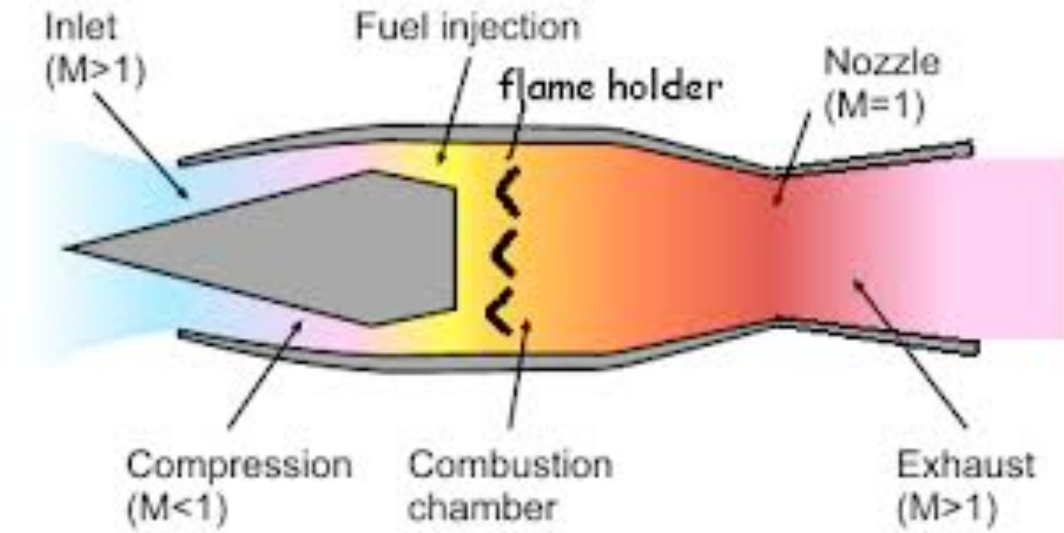

- Simplest of all Jet Engine= Concepts

- Compression and Power Extraction steps Performed passively

- Requires High Speed inlet flow to operate

# Airbreathing (Jet) Propulsion Basics (cont'd)

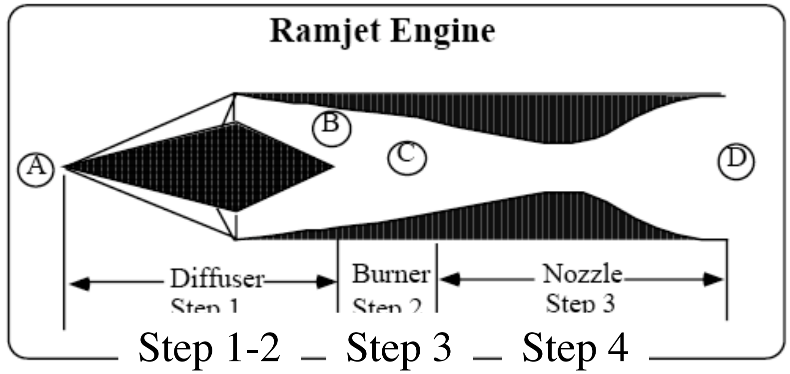
• Ramjet Engine




Ramjets cannot produce thrust at zero airspeed; they cannot move an aircraft from a standstill. A ramjet powered vehicle, therefore, requires an assisted take-off like a rocket assist to accelerate it to a speed where it begins to produce thrust.

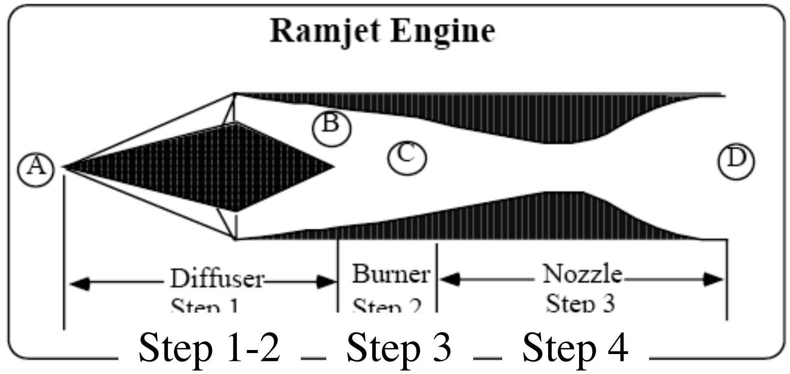
https://www.youtube.com/watch?v=1DG4f2umclk

# Ideal Ramjet Cycle Analysis

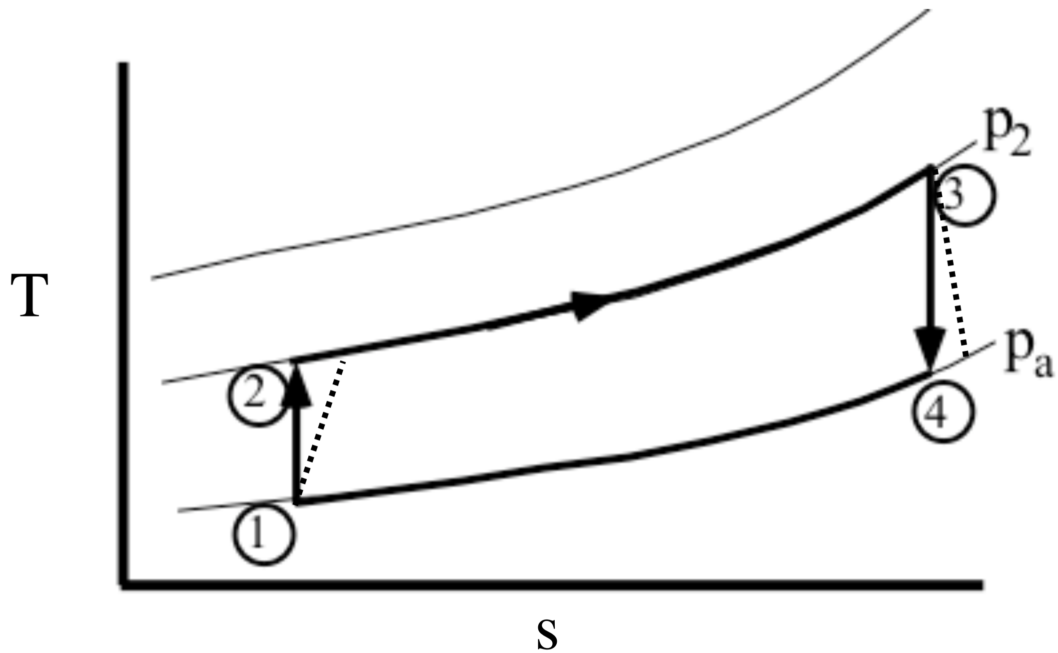



| Region | Process | Ideal Behavior | Real Behavior |
|---|---|---|---|
| A to 1(inlet) | Isentropic flow | $P_0,T_0$ constant | $P_0$ drop |
| 1-2 (diffuser) | Adiabatic Compression | $P,T$ increase<br>$P_0$ drop | $P_0$ drop |
| 2-3 (burner) | Heat Addition | $P_0$ constant, $T_0$<br>$s$ Increase $\Delta s = \left(\frac{\Delta q}{T}\right)_{rev} > 0$ | $P_0$ drop |
| 3-4 (nozzle) | Isentropic expansion | $T_0,P_0$ constant<br>$\Delta s > \Delta s_{rev}$ | $s$ Increase<br>$T_0$ drop |

# Ideal Ramjet Cycle Analysis



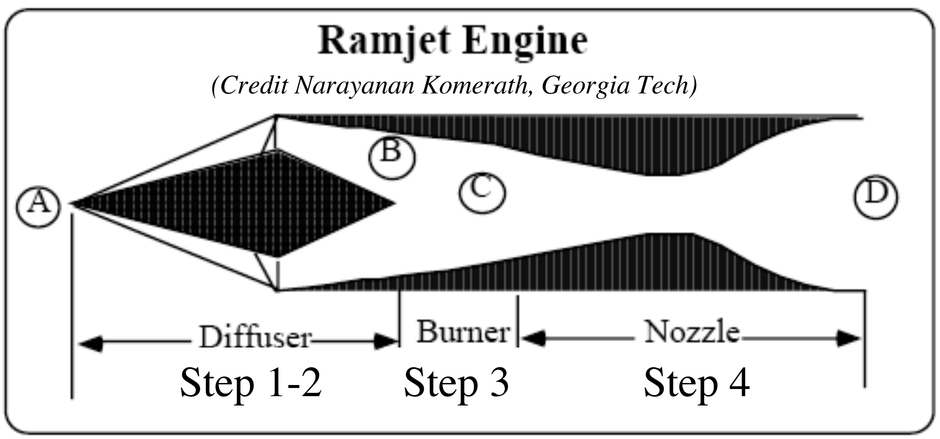

T-s Diagram



# Thermodynamic Efficiency of Ideal Ramjet

- Net Work Available --> work perform by system in step 4 minus work required for step 1-2

- Net heat input --> heat input during step 3 (combustion) - heat lost in exhaust plume

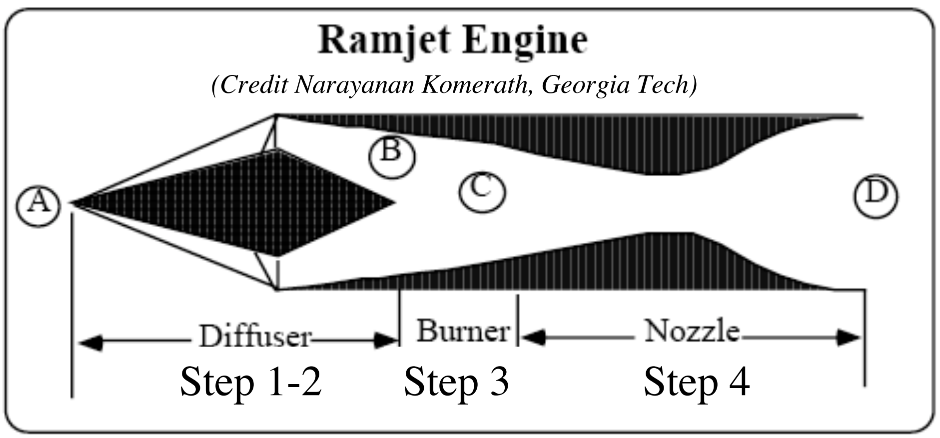


# Thermodynamic Efficiency of Ideal Ramjet (cont'd)

- Ideal Cycle Efficiency($\eta$) = (Net work output)/(Net heat input}

$$\frac{Net\ Work}{\dot{m}} = \left(h_A - h_B\right) + \left(h_C - h_D\right)$$

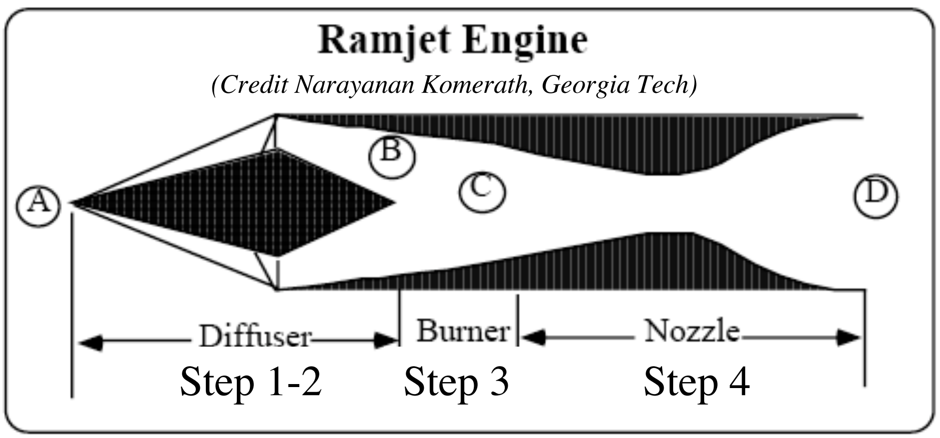
$$\frac{Net\ Heat\ Input}{\dot{m}} = \left(h_C - h_B\right)$$



# Thermodynamic Efficiency of Ideal Ramjet (cont'd)

- Ideal Cycle Efficiency = (Net work output)/(Net heat input}

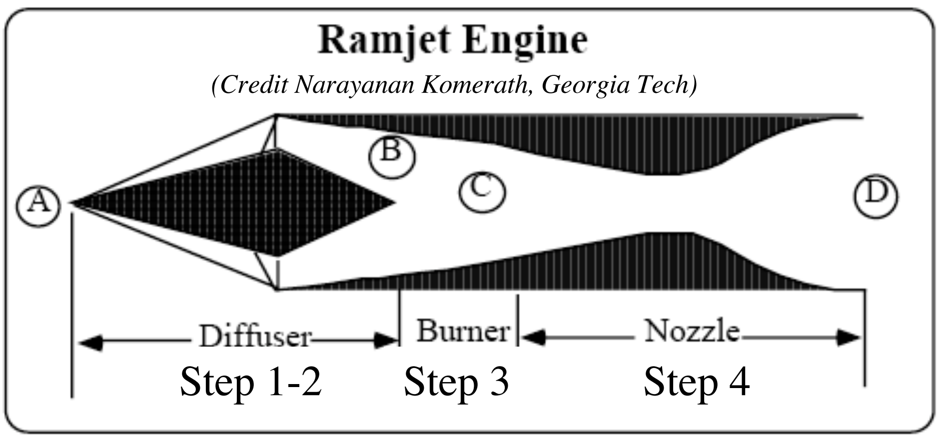
$$\eta = \frac{\left(\dfrac{Net\ Work}{\dot{m}}\right)}{\left(\dfrac{Net\ Heat\ Input}{\dot{m}}\right)} = \frac{(h_A - h_B) + (h_C - h_D)}{(h_C - h_B)} = 1 - \frac{(h_D - h_A)}{(h_C - h_B)}$$



# Thermodynamic Efficiency of Ideal Ramjet (cont'd)

- Assume … $Cp_{air} \sim Cp_{products}$

$$\eta = 1 - \frac{\left(C_{p_{products}} T_D - C_{p_{air}} T_A\right)}{\left(C_{p_{air}} T_C - C_{p_{products}} T_B\right)} = 1 - \frac{\left(T_D - \frac{C_{p_{air}}}{C_{p_{products}}} T_A\right)}{\left(\frac{C_{p_{air}}}{C_{p_{products}}} T_C - T_B\right)}$$



# Thermodynamic Efficiency of Ideal Ramjet (cont'd)

- For simplicity … let … $Cp_{air} \sim Cp_{products}$

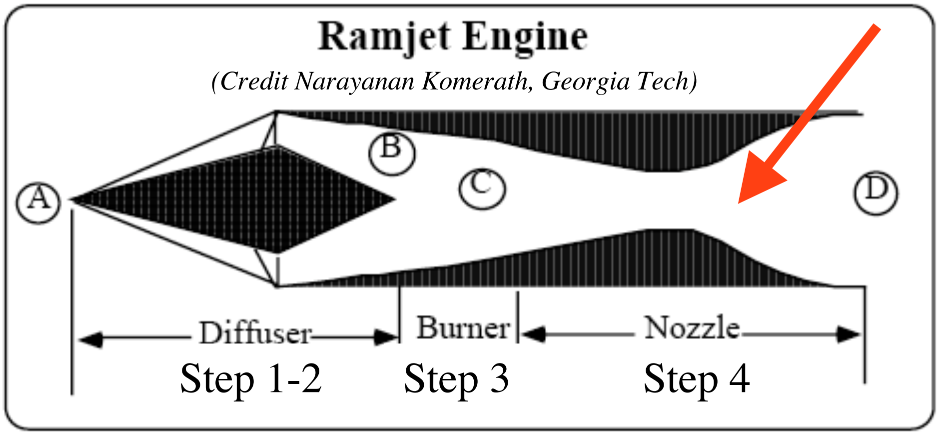
$$\eta \approx 1 - \frac{\left(T_D - \frac{C_{p\ air}}{C_{p\ products}} T_A\right)}{\left(\frac{C_{p\ air}}{C_{p\ products}} T_C - T_B\right)} = 1 - \frac{(T_D - T_A)}{(T_C - T_B)}$$

$$= 1 - \frac{\left(\frac{T_D}{T_C} - \frac{T_A}{T_C}\right)}{\left(1 - \frac{T_B}{T_C}\right)} = 1 - \frac{\left(\frac{T_D}{T_C} - \frac{T_A}{T_B}\frac{T_B}{T_C}\right)}{\left(1 - \frac{T_B}{T_C}\right)}$$

# Thermodynamic Efficiency of Ideal Ramjet (cont'd)

- From C-->D flow is isentropic …

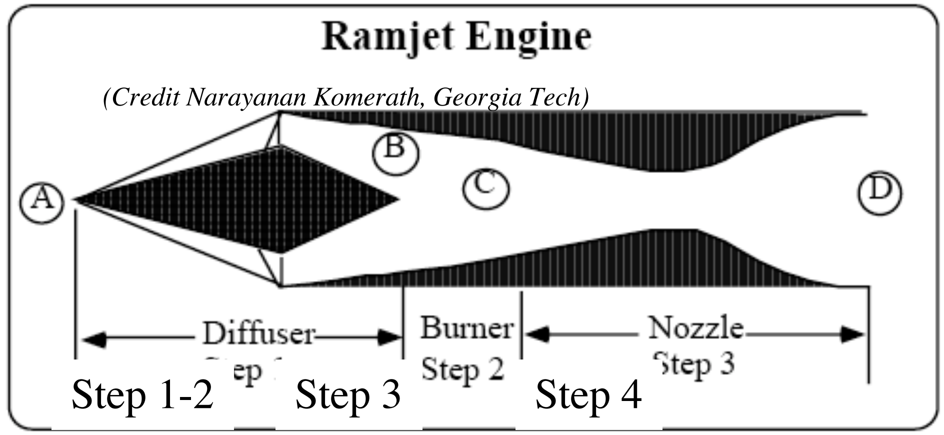
$$\rightarrow \frac{T_D}{T_C} = \left(\frac{P_D}{P_C}\right)^{\frac{\gamma-1}{\gamma}}$$




$$\eta = 1 - \frac{\left(\frac{T_D}{T_C} - \frac{T_A}{T_B}\frac{T_B}{T_C}\right)}{\left(1 - \frac{T_B}{T_C}\right)} = 1 - \frac{\left(\left(\frac{P_D}{P_C}\right)^{\frac{\gamma-1}{\gamma}} - \frac{T_A}{T_B}\frac{T_B}{T_C}\right)}{\left(1 - \frac{T_B}{T_C}\right)}$$

# Thermodynamic Efficiency of Ideal Ramjet (cont'd)

- Adiabatic compression across diffuser

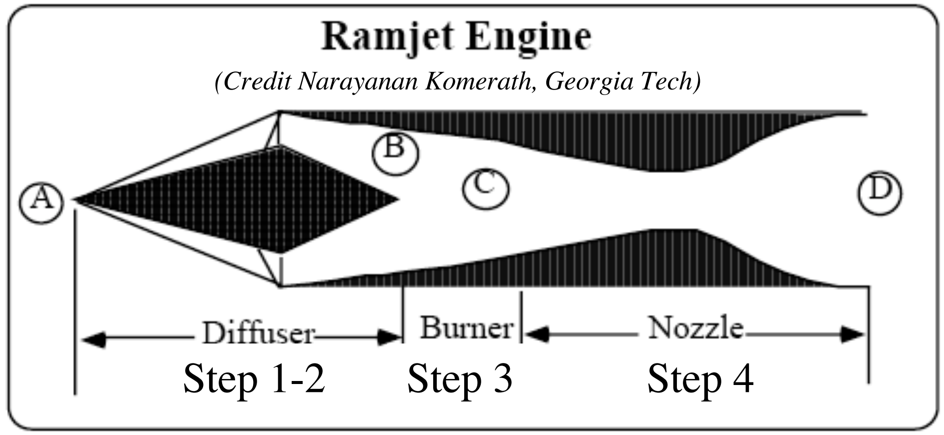



$$\left(\frac{P_A}{P_B}\right) = \left(\frac{P_A}{P_{0_A}} \times \frac{P_{0_A}}{P_{0_B}} \times \frac{P_{0_B}}{P_B}\right) =$$

$$\left(\frac{T_A}{T_{0_A}}\right)^{\frac{\gamma}{\gamma-1}} \left(\frac{T_{0_B}}{T_B}\right)^{\frac{\gamma}{\gamma-1}} \frac{P_{0_A}}{P_{0_B}} \rightarrow T_{0_A} = T_{0_B} \rightarrow \frac{T_A}{T_B} = \left(\frac{P_A}{P_B}\right)^{\frac{\gamma-1}{\gamma}} \left(\frac{P_{0_B}}{P_{0_A}}\right)^{\frac{\gamma-1}{\gamma}}$$

# Thermodynamic Efficiency of Ideal Ramjet (cont'd)

- Sub into efficiency equation

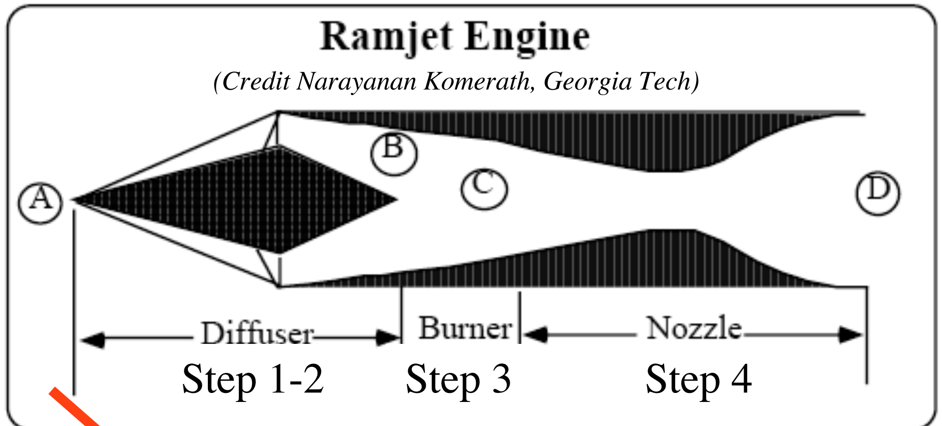



$$\eta = 1 - \frac{\left( \left(\frac{P_D}{P_C}\right)^{\frac{\gamma-1}{\gamma}} - \left(\frac{P_A}{P_B}\right)^{\frac{\gamma-1}{\gamma}} \left(\frac{P_{0_B}}{P_{0_A}}\right)^{\frac{\gamma-1}{\gamma}} \frac{T_B}{T_C} \right)}{\left(1 - \frac{T_B}{T_C}\right)}$$

# Thermodynamic Efficiency of Ideal Ramjet (cont'd)

$$\text{ideal nozzle} \rightarrow P_A = P_D$$

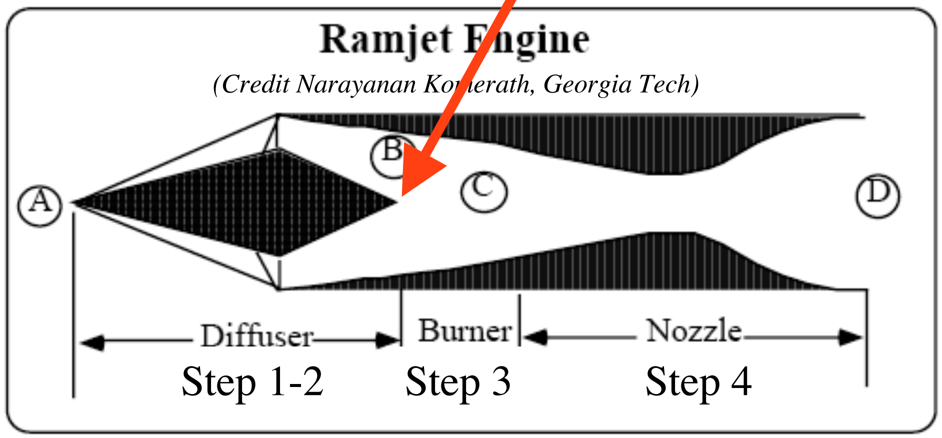
$$\text{ideal burner} \rightarrow P_B = P_C$$




$$\eta = 1 - \left(\frac{P_A}{P_B}\right)^{\frac{\gamma-1}{\gamma}} \frac{\left(1 - \left(\frac{P_{0_B}}{P_{0_A}}\right)^{\frac{\gamma-1}{\gamma}} \frac{T_B}{T_C}\right)}{\left(1 - \frac{T_B}{T_C}\right)} = 1 - \left(\frac{P_A}{P_B}\right)^{\frac{\gamma-1}{\gamma}} \frac{\left(T_C - \left(\frac{P_{0_B}}{P_{0_A}}\right)^{\frac{\gamma-1}{\gamma}} T_B\right)}{\left(T_C - T_B\right)}$$

# Thermodynamic Efficiency of Ideal Ramjet (cont'd)

$$\eta = 1 - \left(\frac{P_A}{P_B}\right)^{\frac{\gamma-1}{\gamma}} \frac{\left(T_C - \left(\frac{P_{0_B}}{P_{0_A}}\right)^{\frac{\gamma-1}{\gamma}} T_B\right)}{\left(T_C - T_B\right)}$$




i) As engine pressure ratio, $P_B/P_A$, goes up … *$\eta$ goes up*

ii) As combustor temperature difference $T_C$-$T_B$ goes up … *$\eta$ goes up*

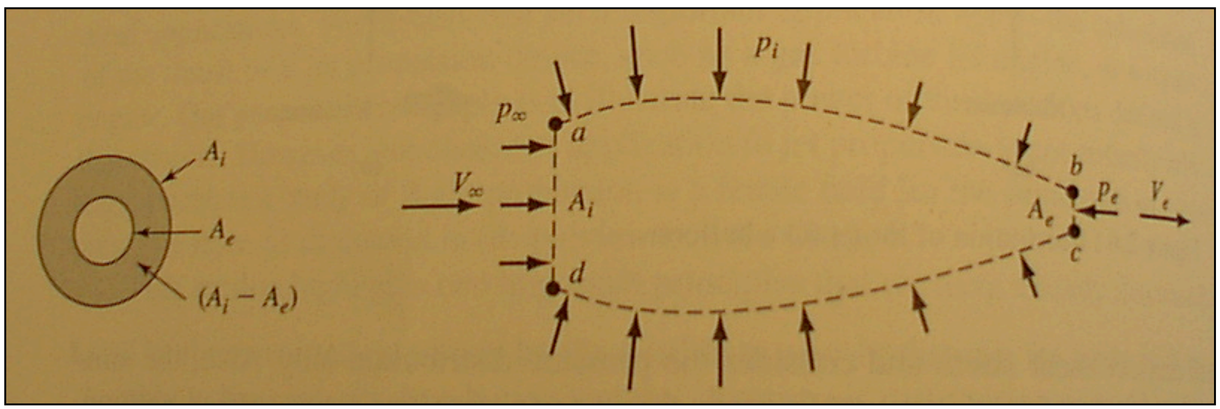
iii) As inlet total pressure ratio ($P_{0_B}/P_{0_A}$) goes down … (stagnation pressure loss goes up) … *$\eta$ goes down*

# Why is total pressure recovery important?

- Choked Nozzle Throat massflow

$$\frac{\dot{m}}{A^*} = \sqrt{\frac{\gamma}{R_g}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}}\frac{p_0}{\sqrt{T_0}}$$

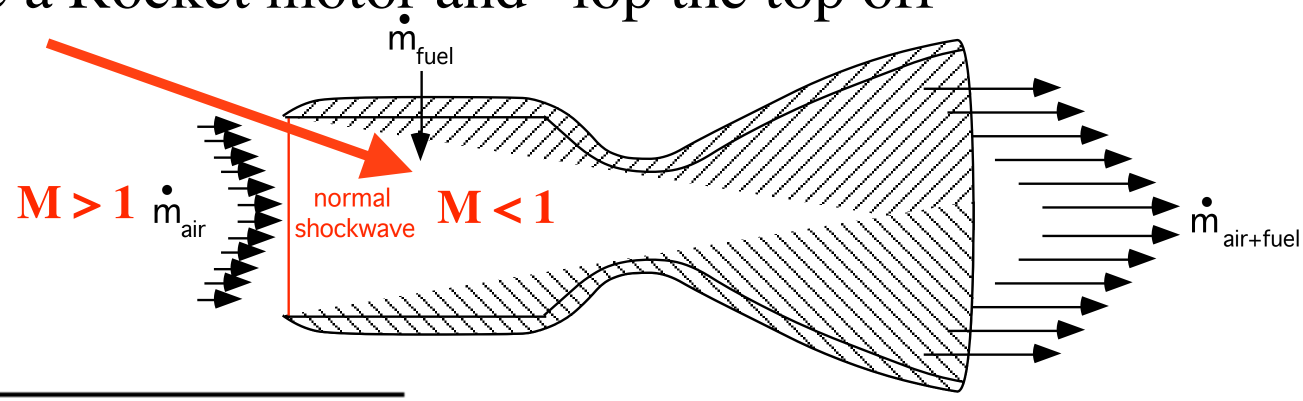
- Thrust is proportional to engine massflow

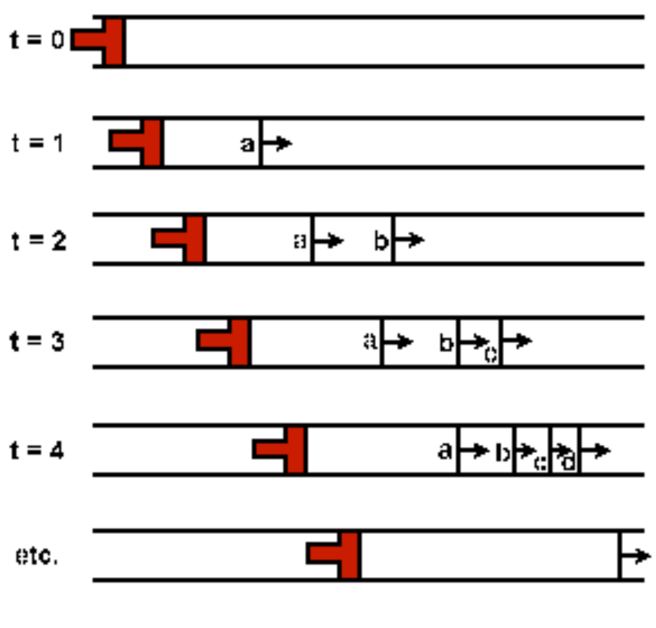



$$Thrust = \dot{m}_e V_e - \dot{m}_i V_i + \left(p_e A_e - p_\infty A_e\right)$$

# Ideal Ramjet: *Inlet and Diffuser*

- Take a Rocket motor and "lop the top off"

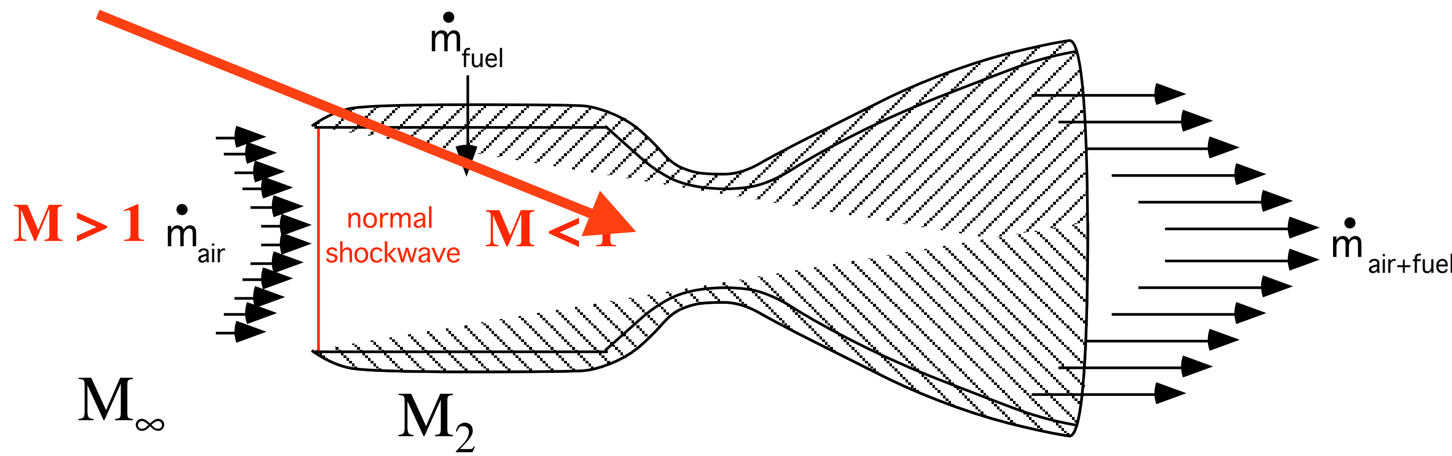



- Works Ok for subsonic, but for supersonic flow … can't cram enough air down the tube

- Result is a *normal shock wave* at the inlet lip

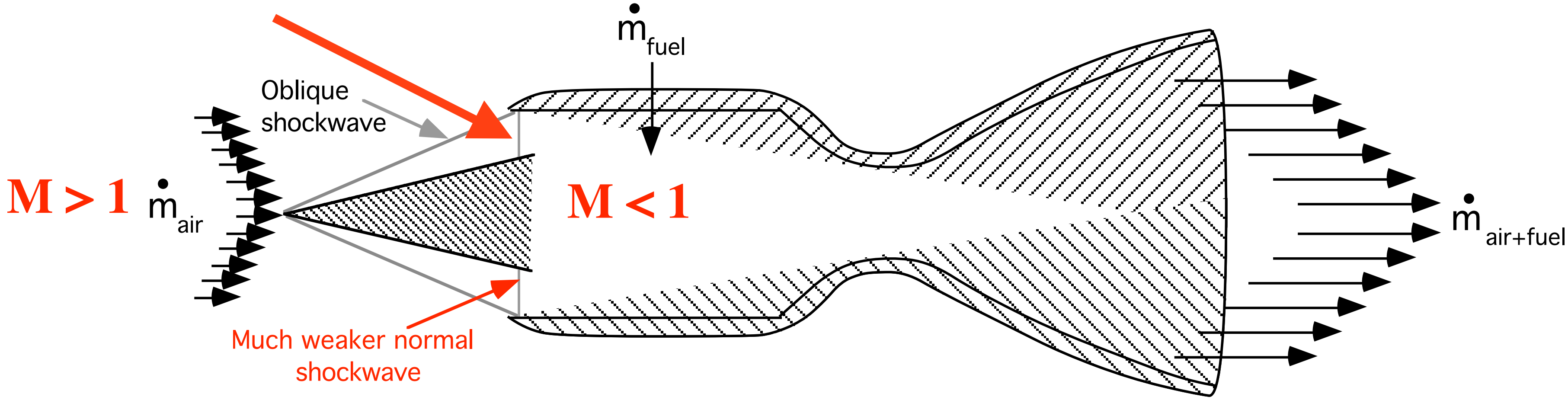
# Ideal Ramjet: *Inlet and Diffuser (cont'd)*

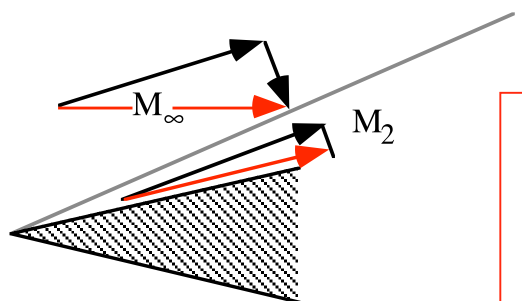



- Mechanical Energy is Dissipated into Heat
- Huge Loss in Momentum

# Ideal Ramjet: *Inlet and Diffuser (cont'd)*

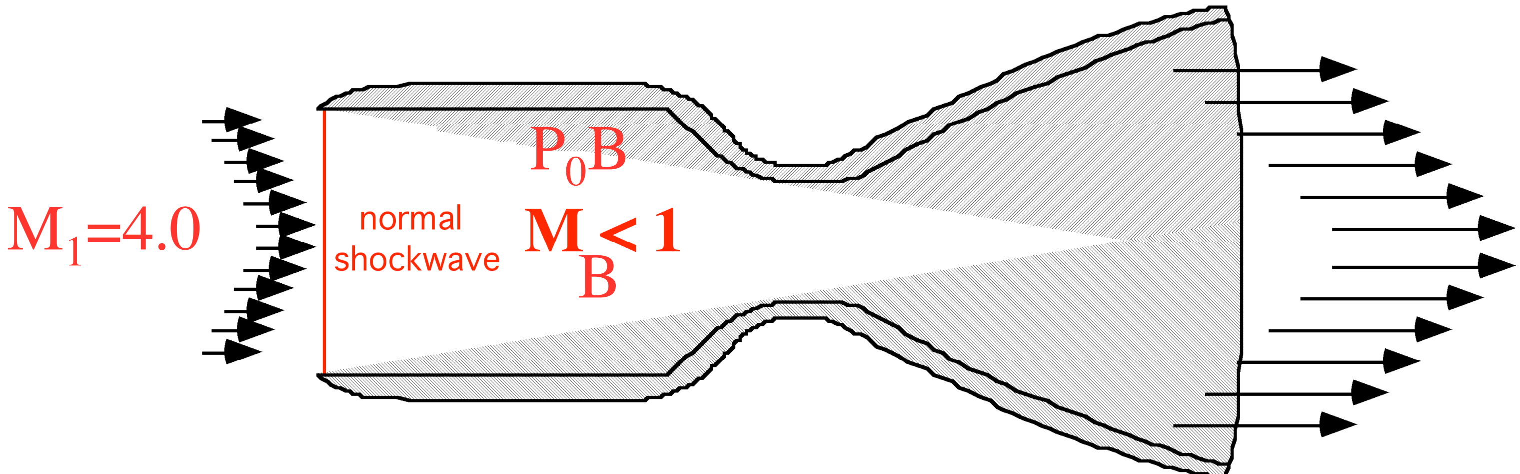
- So … we put a spike in front of the inlet



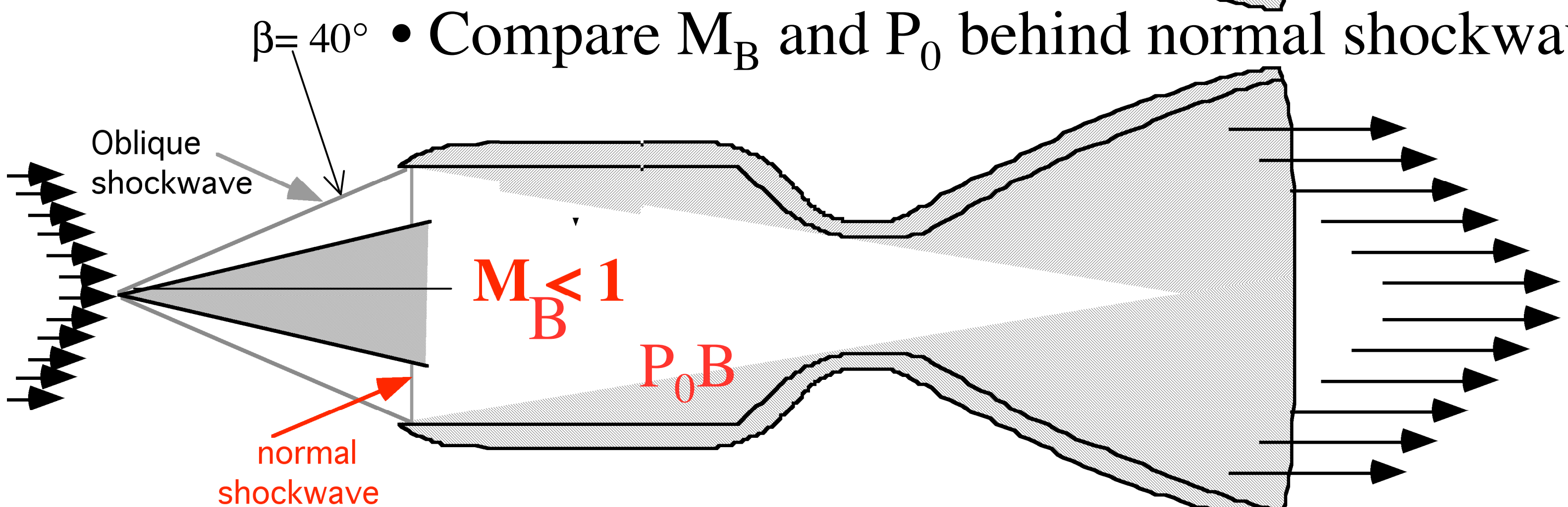




- How does this spike Help?
- By forming an Oblique Shock wave ahead of the inlet

# 2-D Ramjet Inlet Example

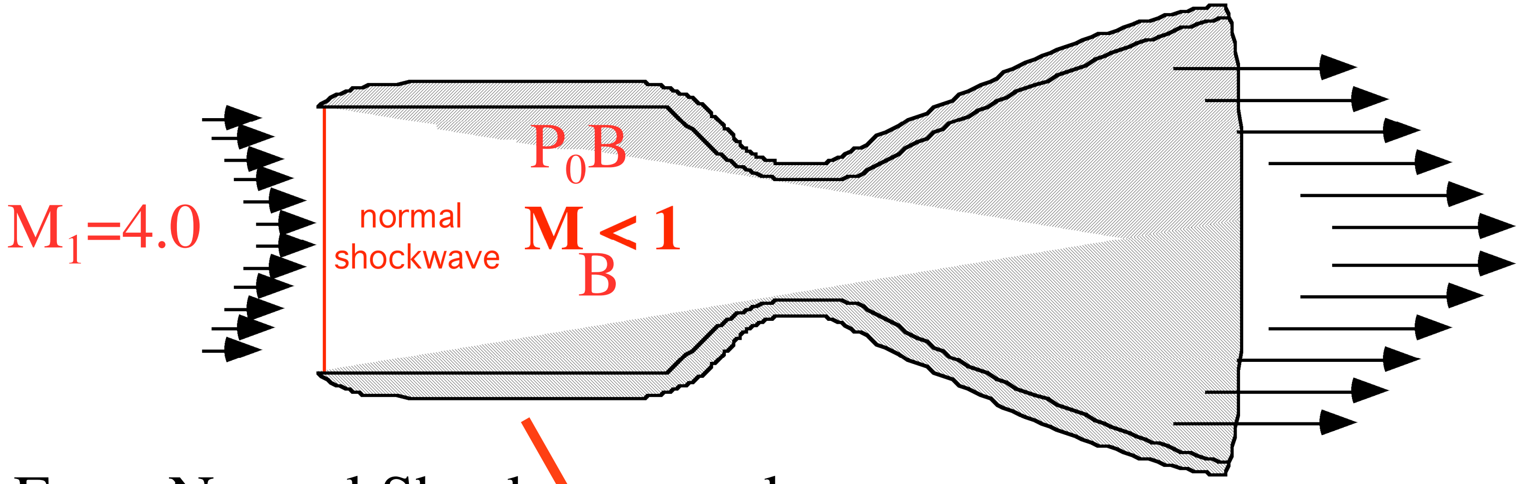



- Compare $M_B$ and $P_0$ behind normal shockwaves




Assume γ=1.4

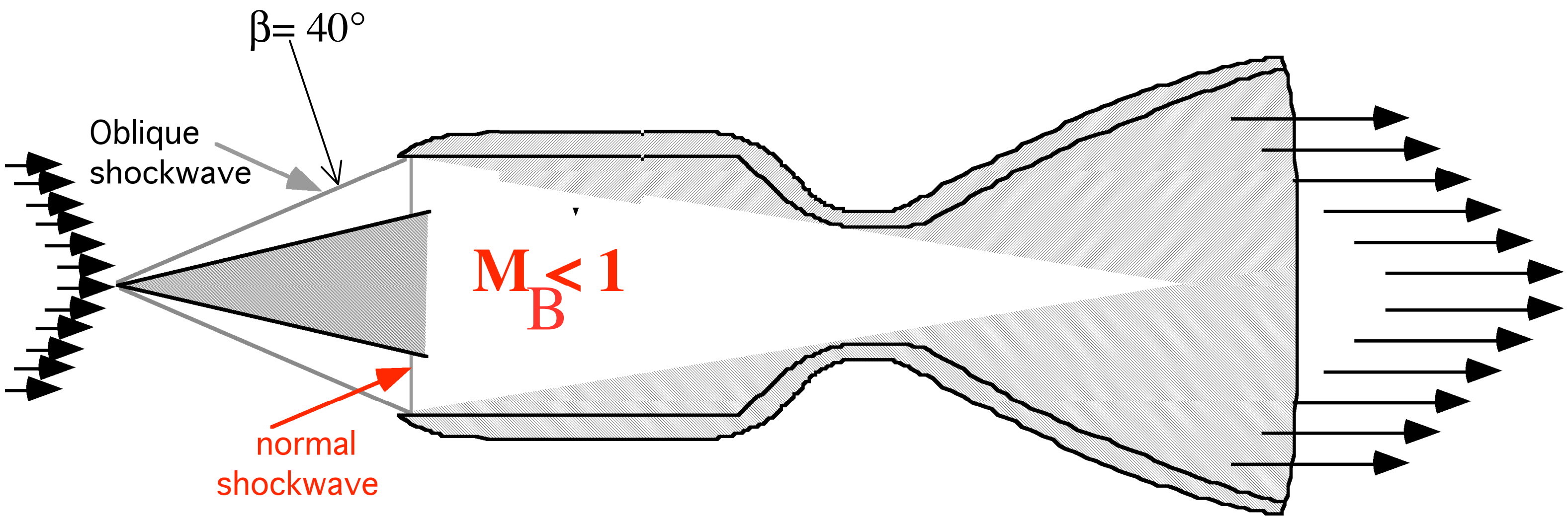
# 2-D Ramjet Inlet Example (cont'd)




- From Normal Shock wave solver

$$M_{\infty} \xrightarrow{normal..shock} M_B = 0.434959 \Rightarrow \begin{bmatrix} \dfrac{P_{0_B}}{P_{0_\infty}} = 0.1388 \\ \dfrac{p_B}{p_\infty} = 18.5 \end{bmatrix}$$

# 2-D Ramjet Inlet Example (cont'd)



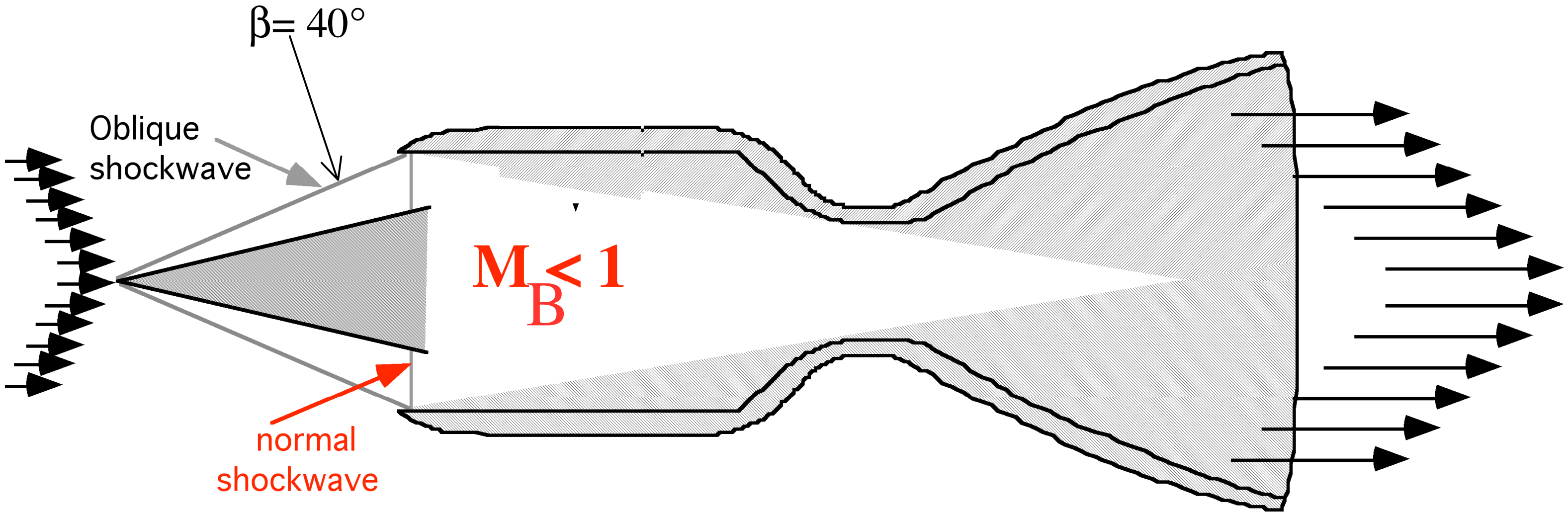

- Across Oblique Shock wave

- $M_{1n} = M_1 \sin \beta_1 = 4 \sin\left(\frac{\pi}{180} 40\right) = 2.571 \longrightarrow M_2 n = 0.5064$

$$\tan(\theta) = \frac{2\left\{M_1^{\,2}\sin^2(\beta) - 1\right\}}{\tan(\beta)\left[2 + M_1^{\,2}\left[\gamma + \cos(2\beta)\right]\right]} \rightarrow \frac{180}{\pi}\,\mathrm{atan}\left(\frac{2\left(4^2\sin^2\left(\frac{\pi}{180}40\right) - 1\right)}{\left(\tan\left(\frac{\pi}{180}40\right)\right)\left(2 + 4^2\left(1.4 + \cos\left(\frac{\pi}{180}2\cdot 40\right)\right)\right)}\right)$$

$$\boxed{\theta = 26.2^\circ}$$

# 2-D Ramjet Inlet Example (cont'd)




- Across Oblique Shock wave

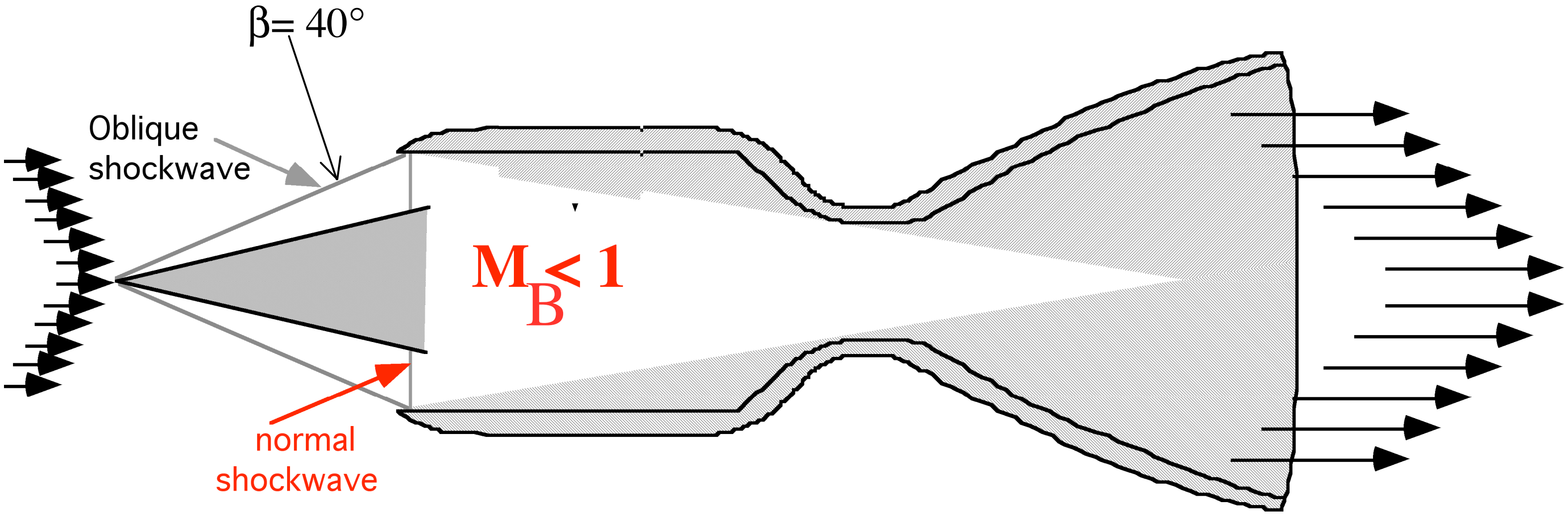
$$M_2 n = 0.5064 \rightarrow \boxed{M_2 = \frac{M_2 n}{\sin(\beta_1 - \theta)} = \frac{0.5064}{\sin\left(\frac{\pi}{180}(40 - 26.2)\right)} = 2.123}$$

$$P_{02}/P_{01} = 0.4711$$

UtahState UNIVERSITY

Mechanical & Aerospace Engineering

# 2-D Ramjet Inlet Example (cont'd)



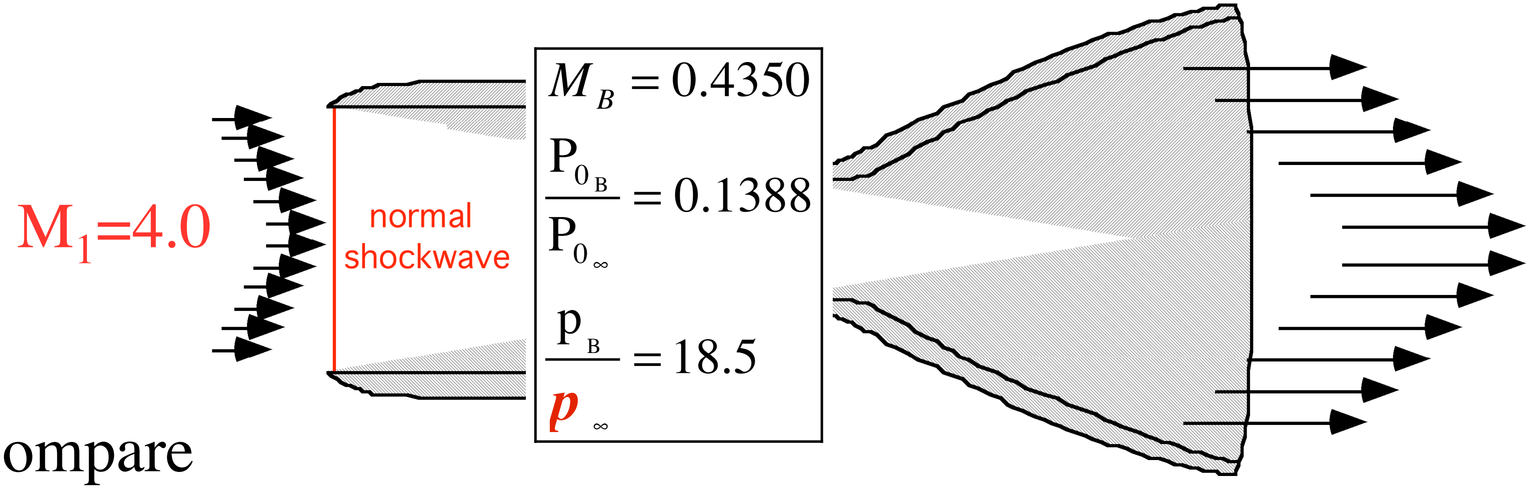

- Across Normal Shock wave *(behind oblique Shock)*

$$M_2 = 2.123 \xrightarrow{normal..shock} M_B = 0.557853 \Rightarrow$$

$$\frac{P_{0_B}}{P_{0_2}} = 0.663531 \Rightarrow \frac{P_{0_B}}{P_{0_\infty}} = \frac{P_{0_B}}{P_{0_2}} \frac{P_{0_2}}{P_{0_\infty}} = (0.663531)(0.4711) = 0.3126$$

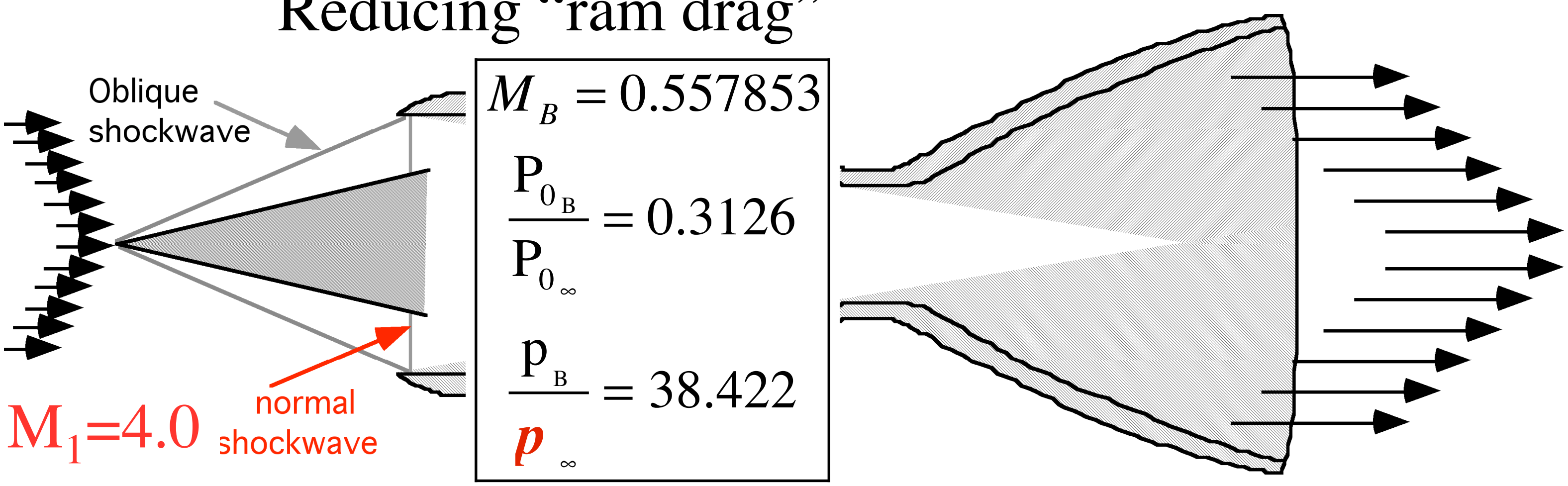
$$\frac{p_B}{p_\infty} = \frac{p_B}{P_{0_B}} \times \frac{P_{0_B}}{P_{0_\infty}} \times \frac{P_{0_\infty}}{p_\infty} = \frac{0.3126\left(\left(1+\frac{1.4-1}{2}4^2\right)^{\frac{1.4}{(1.4-1)}}\right)}{\left(\left(1+\frac{1.4-1}{2}0.557853^2\right)^{\frac{1.4}{(1.4-1)}}\right)} = 38.422$$

# 2-D Ramjet Inlet Example (cont'd)




$$M_B = 0.4350 \qquad \frac{P_{0_B}}{P_{0_\infty}} = 0.1388 \qquad \frac{p_B}{p_\infty} = 18.5$$

- Compare
  - Spike aids in increasing Total Pressure recovery Reducing "ram drag"




$$M_B = 0.557853 \qquad \frac{P_{0_B}}{P_{0_\infty}} = 0.3126 \qquad \frac{p_B}{p_\infty} = 38.422$$

# 2-D Ramjet Inlet Example (cont'd)

- … Continuing example … **Incoming Air to Ramjet**

- Molecular weight = 28.96443 kg/kg-mole
- $\gamma$ = 1.40
- $R_g$ = 287.056 J/°K-(kg)
- $T_\infty$ = 216.65 °K
- $p_\infty$ = 19.330 kPa
- Combustor $\dot{q} = \dot{Q}/\dot{m} =$ 500 kJ/kg

- *Assume that mass of added fuel is negligible, exhaust and $\gamma$, $R_g$ are the same*

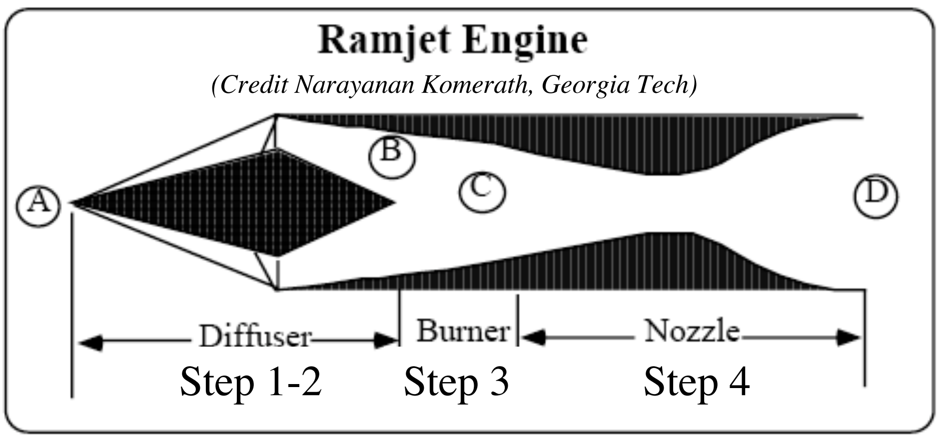
# 2-D Ramjet Inlet Example (cont'd)

- Compute free stream stagnation temperature

$$T_{0_\infty} = T_\infty\left[1+\frac{\gamma-1}{2}M_\infty{}^2\right] = 216.65\left(1+\frac{1.4-1}{2}4^2\right) = 909.93^\circ K$$

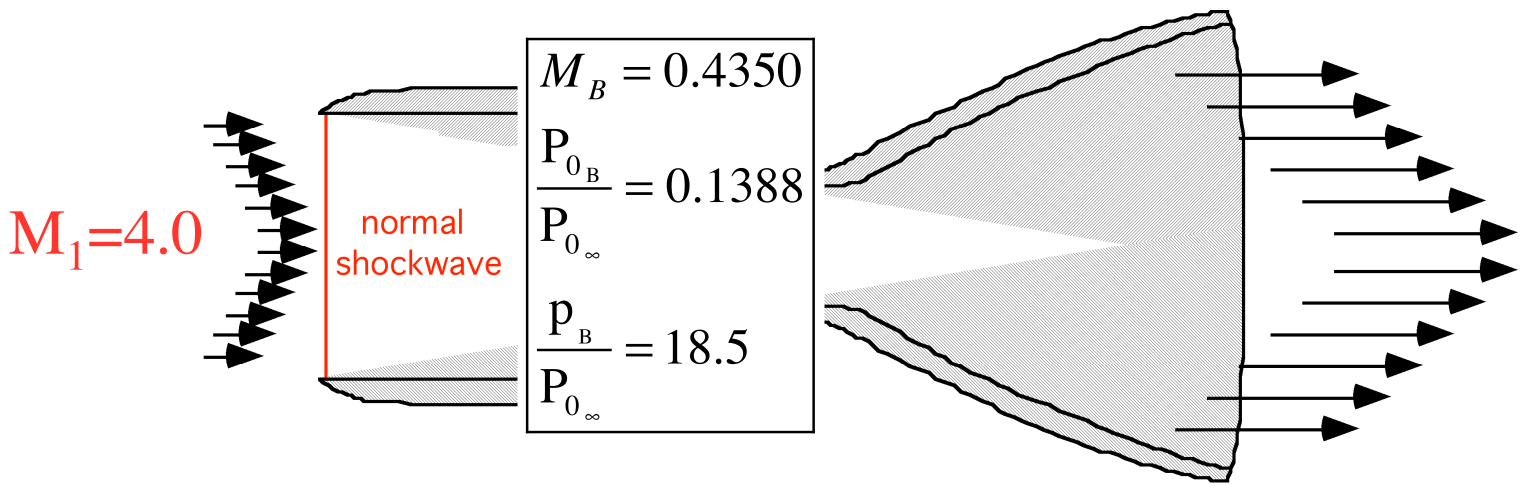
- Compute BURNER stagnation temperature

$$T_{0_C} = \frac{q + c_p T_{0_\infty}}{c_p} = \frac{500\cdot 10^3 + 1004.696\,(909.93)}{1004.696} = 1407.6\ ^\circ K$$



# 2-D Ramjet Inlet Example (cont'd)

- Compute efficiency …. Normal shock inlet



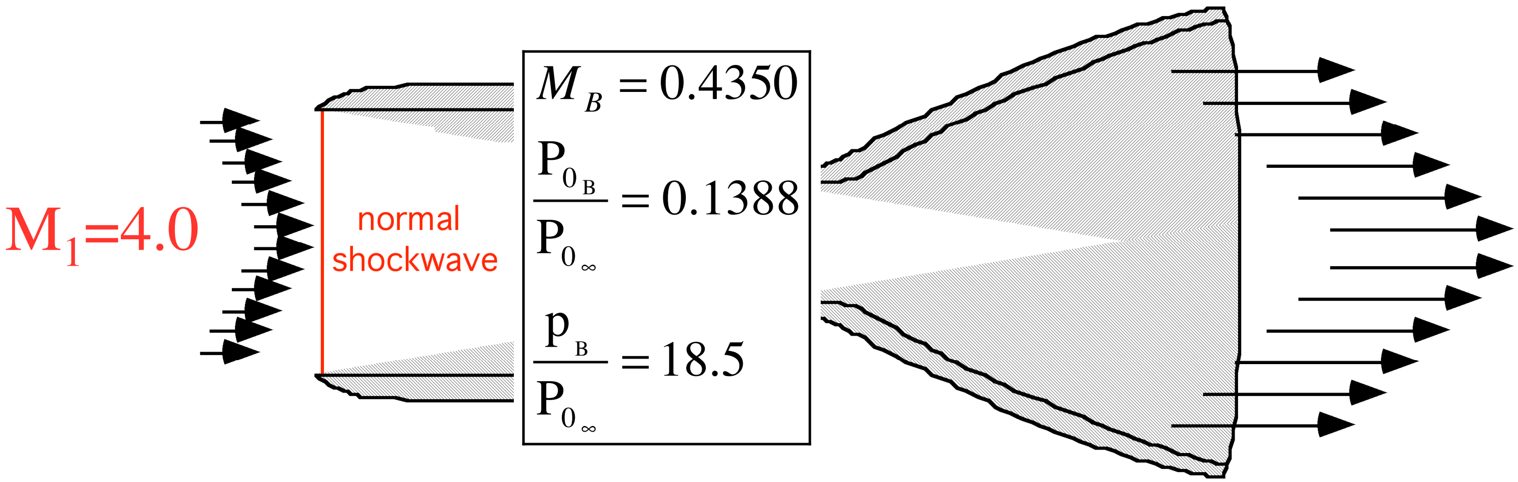

- Compute $T_C$, $T_B$

$$T_{\underset{B}{\cancel{C}}} = \frac{909.93}{\left(1 + \frac{1.4-1}{2} 0.435^2\right)} = 867.75°K$$

$$T_{\underset{C}{\cancel{B}}} = \frac{1407.6}{\left(1 + \frac{1.4-1}{2} 0.435^2\right)} = 1356.3°K$$

# 2-D Ramjet Inlet Example (cont'd)

- Compute efficiency …. Normal shock inlet




- Compute $T_C$, $T_B$

$T_{\cancel{C}\,B}$ = 867.75°K

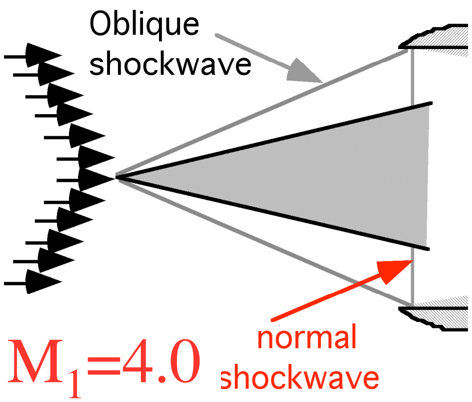
$T_{\cancel{B}\,C}$ = 1356.3°K

$$\eta = 1 - \left(\frac{P_A}{P_B}\right)^{\frac{\gamma-1}{\gamma}} \frac{\left(T_C - \left(\frac{P_{0_B}}{P_{0_A}}\right)^{\frac{\gamma-1}{\gamma}} T_B\right)}{\left(T_C - T_B\right)} =$$

$$1 - \frac{18.5^{\frac{-(1.4-1)}{1.4}}\left(1356.3 - \left(0.1388^{\frac{(1.4-1)}{1.4}}\right)867.75\right)}{(1356.3 - 867.75)} = 0.2328$$

# 2-D Ramjet Inlet Example (cont'd)

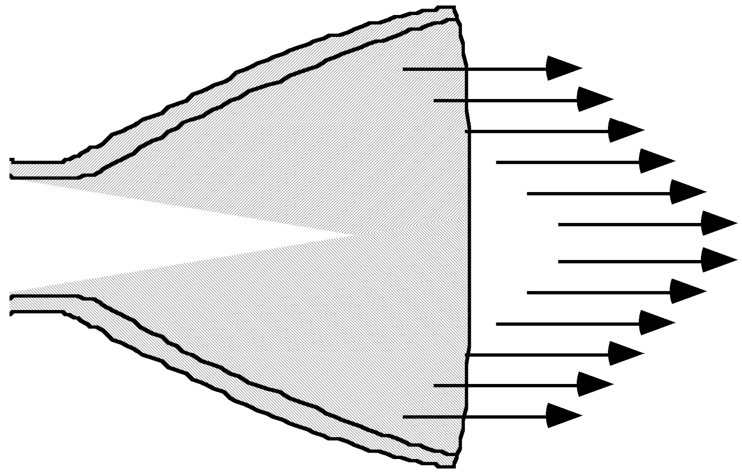
- Compute efficiency …. Oblique shock inlet




$$M_B = 0.557853$$

$$\frac{P_{0_B}}{P_{0_\infty}} = 0.3126$$

$$\frac{p_B}{P_{0_\infty}} = 38.422$$



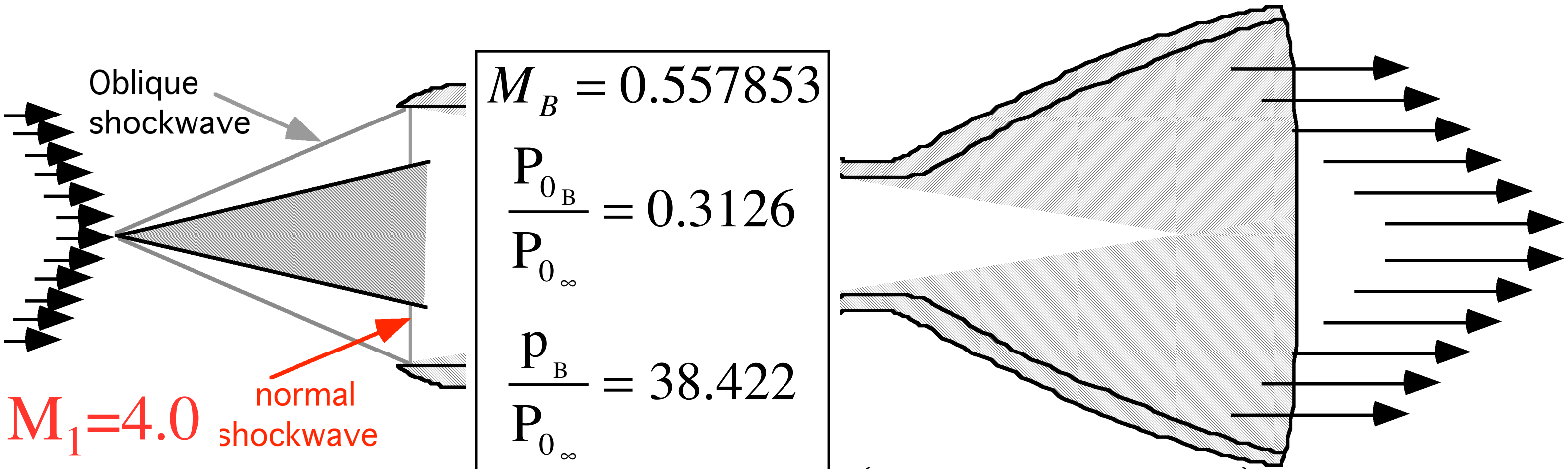
- Compute $T_C$, $T_B$

$$T_{\cancel{C}B} = \frac{909.93}{\left(1 + \frac{1.4-1}{2} 0.557853^2\right)} = 856.61°K$$

$$T_{\cancel{B}C} = \frac{1407.6}{\left(1 + \frac{1.4-1}{2} 0.557853^2\right)} = 1325.1°K$$

# 2-D Ramjet Inlet Example (cont'd)

- Compute efficiency …. Oblique shock inlet




$$M_B = 0.557853$$

$$\frac{P_{0_B}}{P_{0_\infty}} = 0.3126$$

$$\frac{p_B}{P_{0_\infty}} = 38.422$$

- Compute $T_C$, $T_B$

$T_{\cancel{C}B} =$ 856.61°K

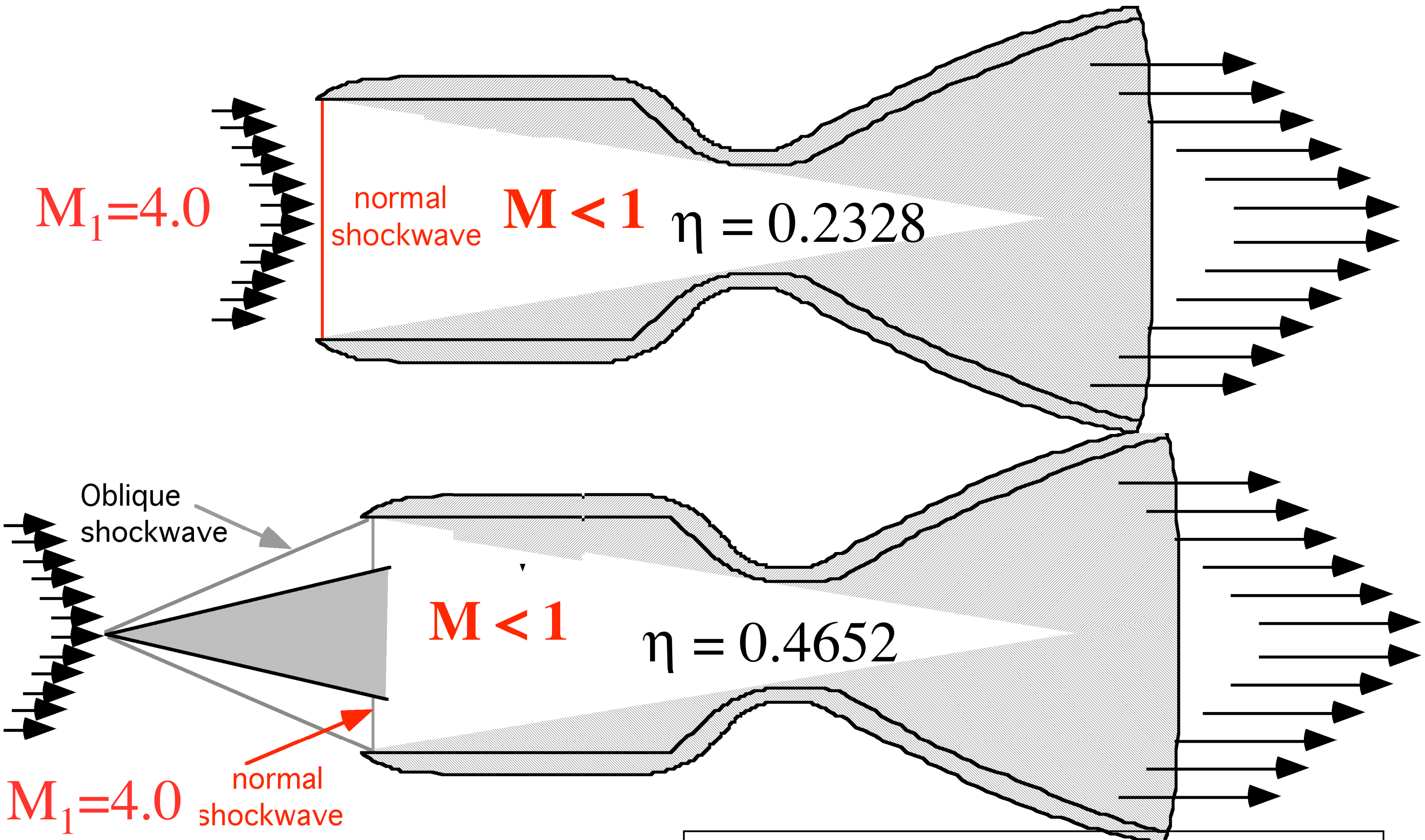
$T_{\cancel{B}C} =$ 1325.1°K

$$\eta = 1 - \left(\frac{P_A}{P_B}\right)^{\frac{\gamma-1}{\gamma}} \frac{\left(T_C - \left(\frac{P_{0_B}}{P_{0_A}}\right)^{\frac{\gamma-1}{\gamma}} T_B\right)}{\left(T_C - T_B\right)} =$$

$$1 - \frac{38.422^{\frac{-(1.4-1)}{1.4}}\left(1325.1 - \left(0.3126^{\frac{(1.4-1)}{1.4}}\right)856.61\right)}{(1325.1 - 856.61)} = 0.4652$$

# 2-D Ramjet Inlet Example (cont'd)

- Compute efficiency .... Oblique shock inlet

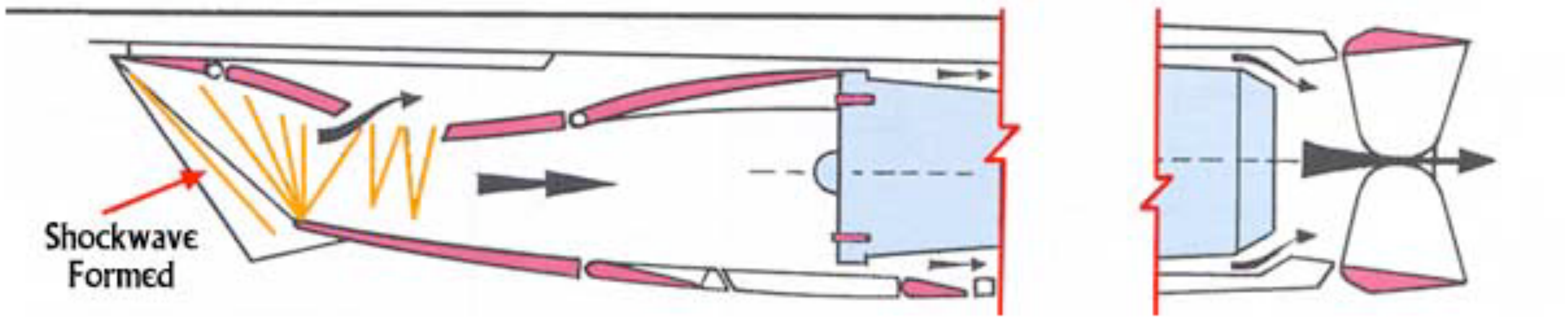



200% increase in efficiency!

# Supersonic Inlet: Condorde Inlet Design



- Mach 2 Cruise




Credit: http://www.concordesst.com/powerplant.html

# Conical versus 2-D Inlets




# "Starting" a Constant Geometry Ramjet Inlet

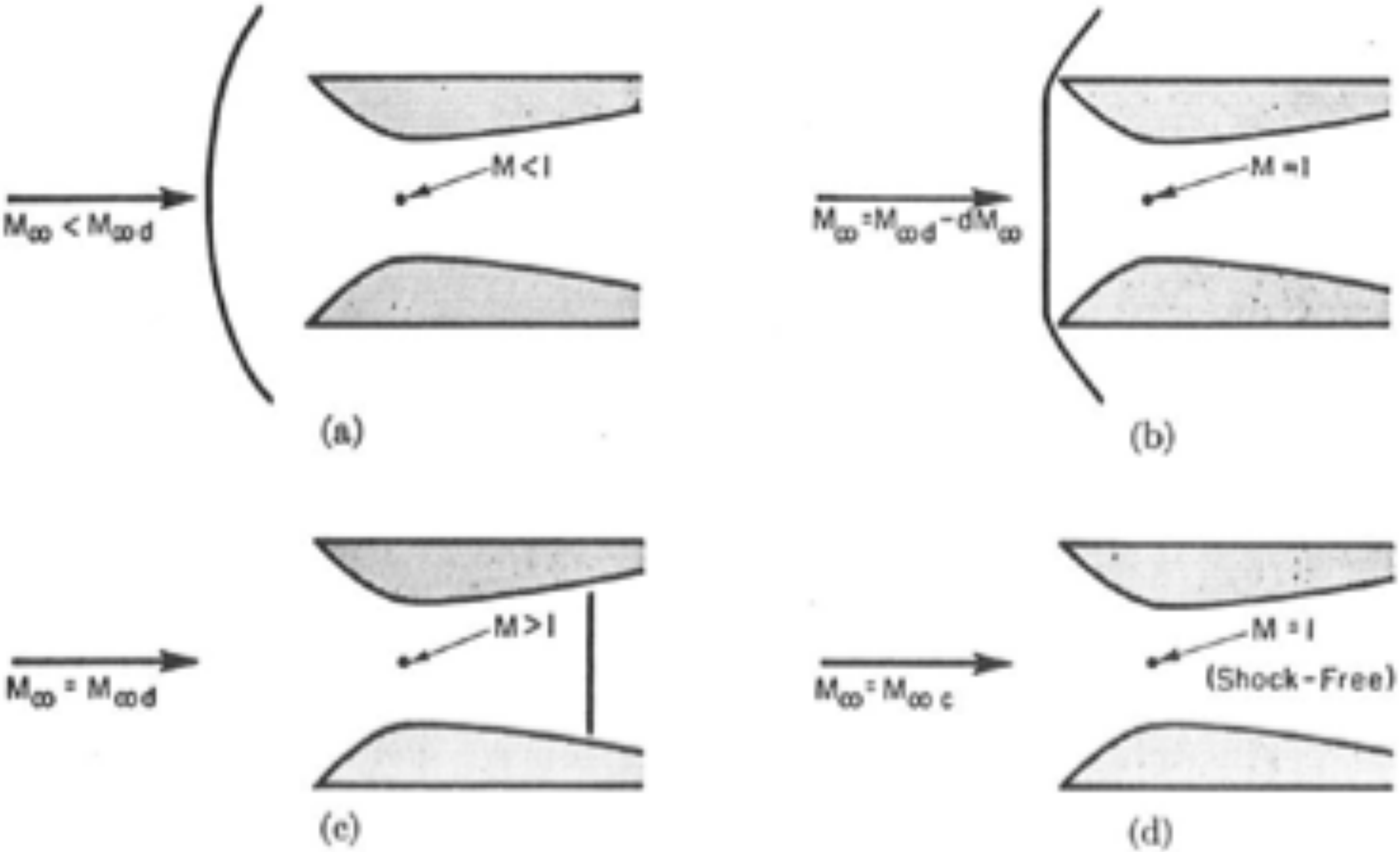
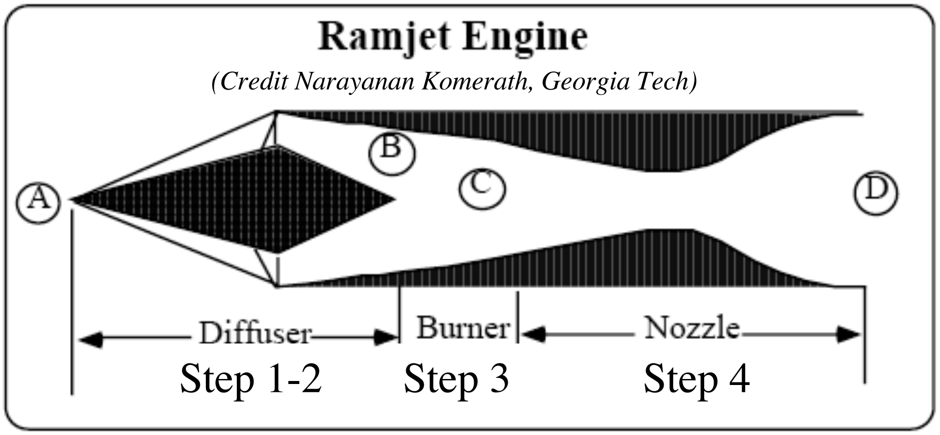



FIG. 5.33. Overspeed starting of fixed-geometry supersonic inlet, designed for free-stream Mach Number $M_{\infty d}$, and having contraction ratio $(A_2/A_1)_c$.

# Air to Fuel Ratio Computations



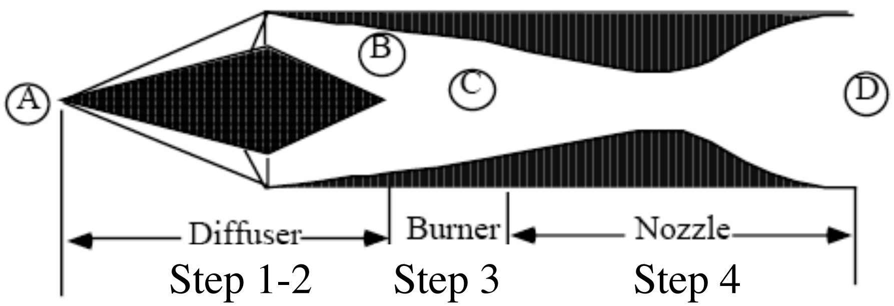

- Stagnation enthalpy of the air and burned products leaving the combustor equals the enthalpy of the air entering the combustor + heat released by chemical reaction

- Sensible enthalpy of fuel assumed to be negligible when compared to heat of reaction

# Fuel to Air Ratio Computations (cont'd)

**Ramjet Engine**

*(Credit Narayanan Komerath, Georgia Tech)*




$$\dot{m}_{air}(1+f)C_{p_3}T_{0_3} = \dot{m}_{air}C_{p_2}T_{0_2} + \dot{m}_f q_r$$

$$= \dot{m}_{air}C_{p_2}T_{0_2} + f\,\dot{m}_{air}q_r$$

*Solve for f*

$$f = \frac{C_{p_3}T_{0_3} - C_{p_2}T_{0_2}}{\left(q_r - C_{p_3}T_{0_3}\right)} = \frac{\dfrac{C_{p_3}T_{0_3}}{C_{p_2}T_{0_2}} - 1}{\left(\dfrac{q_r}{C_{p_2}T_{0_2}} - \dfrac{C_{p_3}T_{0_3}}{C_{p_2}T_{0_2}}\right)}$$

$T_{0_2}$ -- stagnation temperature entering burner

$T_{0_3}$ -- stagnation temperature exiting burner

$f$ -- fuel to air ratio

$C_{p_2}$ -- specific heat of air entering burner

$C_{p_3}$ -- specific heat of combustion products

$q_r$ -- heat of reaction

# Equivalence Ratio versus Mixture Ratio

• The equivalence ratio is used to characterize the mixture ratio Of airbreathing engines … *analogous to mixture ratio*

• The *equivalence ratio*, $\Phi$ , is defined as the ratio of the actual fuel-air ratio to the stoichiometric fuel-air ratio.

… $\Phi > 1$ ---> a rich mixture
… $\Phi < 1$ ---> lean mixture

• For $\Phi = 1$, no oxygen is left in exhaust products … combustion is called *stoichiometric*

$$\Phi \equiv \frac{\left[\frac{\dot{m}_{fuel}}{\dot{m}_{air}}\right]_{actual}}{\left[\frac{\dot{m}_{fuel}}{\dot{m}_{air}}\right]_{stoich}} = \frac{[f]_{actual}}{[f]_{stoich}}$$

# Equivalence Ratio versus Mixture Ratio

**(cont'd)**

- Consider combustion of $H_2$ fuel with air … what is Stoich fuel ratio (define only water as product of combustion)

$$2H_2 + O_2 \rightleftarrows H_2O$$

$$\left(\frac{\dot{m}H_2}{\dot{m}O_2}\right)_{stoich} = \frac{2\frac{kg}{kg-mol}\times 2_{kg-mol}}{32\frac{kg}{kg-mol}\times 1_{kg-mol}} = \frac{1}{8} \rightarrow (f)_{stoich} = \left(\frac{\dot{m}H_2}{\dot{m}_{air}}\right)_{stoich} = \left(\frac{\dot{m}H_2}{\dot{m}O_2}\right)_{stoich} \times \frac{0.21\dot{m}O_2}{\dot{m}_{air}} = 0.02625$$

$\boxed{\text{Air/Fuel=38.09}}$

- Assumed equilibrium chemistry is described by reaction above with only water vapor as combustion product, + left over atmospheric nitrogen

**Let's do the real flame temperature calculation**

# CEA Calculation

- Actual Equilibrium Calculations, Gaseous $H_2$, air, $\phi$=1, $p_c$= 1000 kPa (145 psi) TB=856.6 °K (earlier ramjet problem)**(After compression)_**

Assumed air composition (dry air)

| | |
|---|---|
| Nitrogen | 79.00 |
| Oxygen | 21.00 |

Combustion Product MOLE FRACTIONS

| | Burner | M=1 |
|---|---|---|
| H2O | 0.31761 | 0.31761 |
| N2 | 0.68239 | 0.68239 |

**Frozen Flow Calculation**

Combustion properties

| | Burner | M=1 |
|---|---|---|
| T, K | $\boxed{2826.42}$ | 2521.42 |
| H, KJ/KG | 785.57 | 260.70 |
| U, KJ/KG | -160.58 | -583.35 |
| G, KJ/KG | -28431.7 | -25803.7 |
| M, (1/n) | 24.838 | 24.838 |
| GAMMAs | 1.2395 | 1.2437 |

O/F= 38.09

# CEA Calculation, more realistic reaction

- Actual Equilibrium Calculations, Gaseous $H_2$, air, $\phi$=1, $p_c$= 1000 kPa (145 psi) TB=856.6 °K (earlier ramjet problem) **(After compression)**

| Gas | % of Earth Atmosphere at sea level (dry air) |
|---|---|
| Nitrogen | 78.08 |
| Oxygen | 20.95 |
| Argon | 0.94 |
| Carbon dioxide | 0.03 |

**Equilibrium reactions**
**Reduces flame temperature**
**Compared to previous**

**Equilibrium Flow Calculation**

Combustion properties

| | Burner | M=1 |
|---|---|---|
| T, K | 2662.82 | 2446.29 |
| H, KJ/KG | 779.69 | 288.07 |
| U, KJ/KG | -120.74 | -534.64 |
| G, KJ/KG | -26714.1 | -24970.0 |
| M, (1/n) | 24.588 | 24.723 |
| GAMMAs | 1.1814 | 1.1951 |

O/F= 37.88393

Combustion Product
MOLE FRACTIONS

| | Burner | M=1 |
|---|---|---|
| Ar | 0.00564 | 0.00567 |
| CO | 0.00005 | 0.00003 |
| CO2 | 0.00012 | 0.00014 |
| H | 0.00205 | 0.00086 |
| H2 | 0.01718 | 0.01062 |
| H2O | 0.29080 | 0.30174 |
| NO | 0.00402 | 0.00229 |
| N2 | 0.66569 | 0.67022 |
| O | 0.00065 | 0.00025 |
| OH | 0.00936 | 0.00523 |
| O2 | 0.00445 | 0.00296 |

# CEA Calculations (cont'd)

- Actual Equilibrium Calculations, Gaseous $H_2$, air, $\phi$=2, $p_c$= 1000 kPa (145 psi) TB=856.6 °K (earlier ramjet problem) **(rich mixture)**

| Gas | % of Earth Atmosphere at sea level (dry air) |
|---|---|
| Nitrogen | 78.08 |
| Oxygen | 20.95 |
| Argon | 0.94 |
| Carbon dioxide | 0.03 |

**Reduced flame temperature**
**Compared to previous**
**But molecular weight is lower**

Combustion Product
MOLE FRACTIONS

| | Burner | M=1 |
|---|---|---|
| Ar | 0.00433 | 0.00433 |
| CO | 0.00011 | 0.00010 |
| CO2 | 0.00002 | 0.00002 |
| H | 0.00274 | 0.00091 |
| H2 | 0.23958 | 0.24051 |
| H2O | 0.24021 | 0.24097 |
| NH3 | 0.00001 | 0.00001 |
| NO | 0.00004 | 0.00001 |
| N2 | 0.51239 | 0.51300 |
| OH | 0.00057 | 0.00013 |

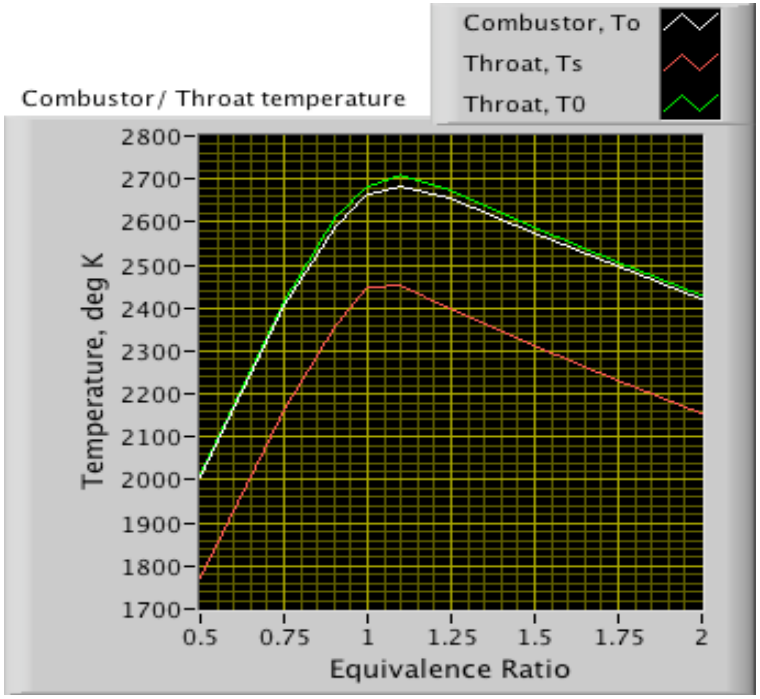
Combustion properties

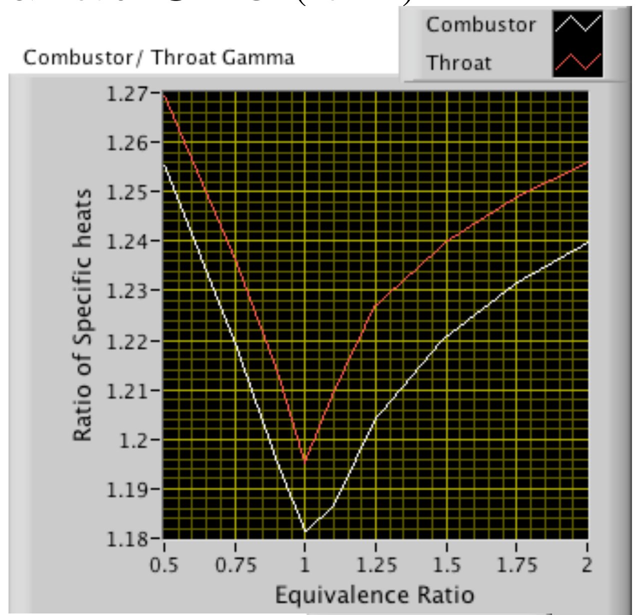
| | Burner | M=1 |
|---|---|---|
| T, K | 2422.36 | 2156.10 |
| H, KJ/KG | 963.90 | 382.97 |
| U, KJ/KG | -76.701 | -542.18 |
| G, KJ/KG | -28674.1 | -25997.3 |
| M, (1/n) | 19.355 | 19.377 |
| GAMMAs | 1.2399 | 1.2559 |

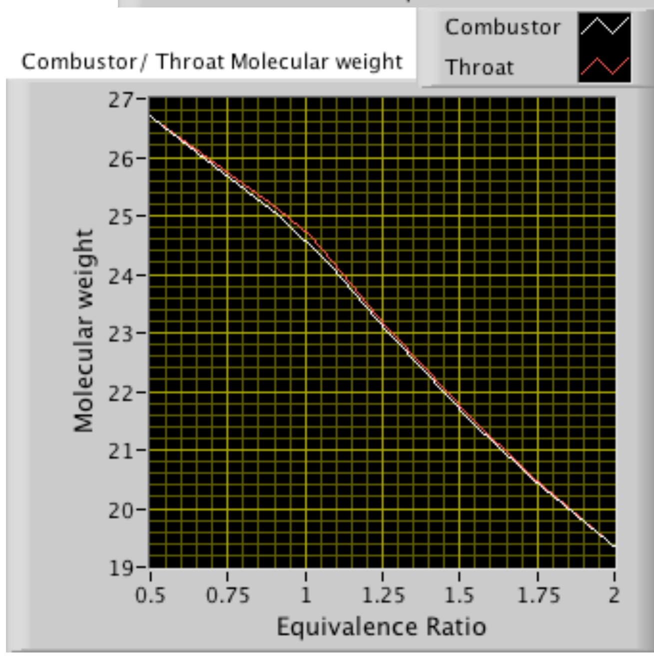
O/F= 37.88393

# CEA Calculations (cont'd)



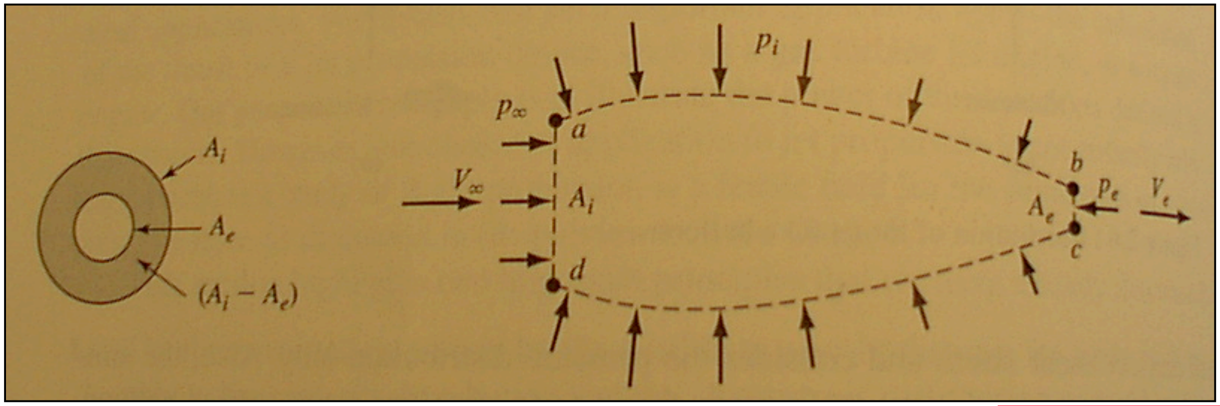


# Specific Thrust of Air Breathing Engine

- Analogous to specific impulse




$$Thrust = \dot{m}_e V_e - \dot{m}_i V_i + \left(p_e A_e - p_\infty A_e\right)$$

Cruise design condition
When $p_e = p_\infty$

$$\left(\frac{F_{thrust}}{\dot{m}_f}\right)_{opt} = \frac{\left[\dot{m}_f + \dot{m}_{air}\right] V_e - \dot{m}_{air} V_\infty}{\dot{m}_f} = \left[1 + \frac{1}{f}\right] V_e - \frac{1}{f} V_\infty = V_e + \frac{1}{f}\left(V_e - V_\infty\right)$$

# Specific Thrust of Air Breathing Engine (cont'd)

- Re writing in terms of mach number

$$\left(\frac{F_{thrust}}{\dot{m}_f}\right)_{opt} = \left(\frac{f+1}{f}\right)V_e - \frac{1}{f}V_\infty = \left(\frac{f+1}{f}\right)\sqrt{\gamma_e R_{g_e} T_e}M_e - \frac{1}{f}\left(\sqrt{\gamma_\infty R_{g_\infty} T_\infty}M_\infty\right) =$$

$$\left(\frac{f+1}{f}\right)\sqrt{\gamma_e R_{g_e} T_{o_e}}\sqrt{\frac{T_e}{T_{o_e}}}M_e - \frac{1}{f}\left(\sqrt{\gamma_\infty R_{g_\infty} T_{0_\infty}}\sqrt{\frac{T_\infty}{T_{0_\infty}}}M_\infty\right) =$$

$$\left(\frac{f+1}{f}\right)\sqrt{\gamma_e R_{g_e} T_{o_e}}\frac{M_e}{\sqrt{1+\frac{\gamma_e-1}{2}M_e^{\,2}}} - \frac{1}{f}\left(\sqrt{\gamma_\infty R_{g_\infty} T_{0_\infty}}\frac{M_\infty}{\sqrt{1+\frac{\gamma_\infty-1}{2}M_\infty^{\,2}}}\right)$$

# Specific Thrust of Air Breathing Engine (cont'd)

- Re writing in terms of mach number

$$\left(\frac{F_{thrust}}{\dot{m}_f}\right)_{opt} = \left(\frac{f+1}{f}\right)\sqrt{\gamma_e R_{g_e} T_{o_e}}\,\frac{M_e}{\sqrt{1+\frac{\gamma_e-1}{2}M_e^{\;2}}} - \frac{1}{f}\left(\sqrt{\gamma_\infty R_{g_\infty} T_{0_\infty}}\,\frac{M_\infty}{\sqrt{1+\frac{\gamma_\infty-1}{2}M_\infty^{\;2}}}\right)$$

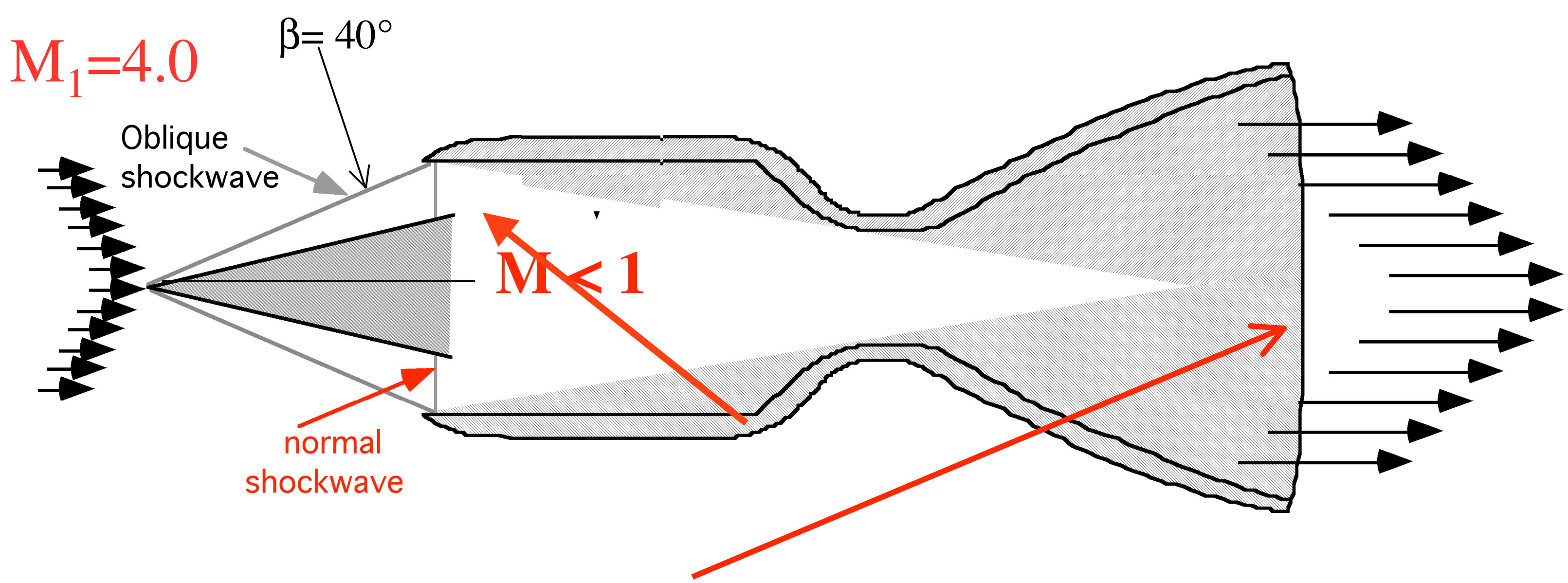
Function of Nozzle geometry

$f = \Phi f_{stoich}$

Function of combustion

- Equivalence ratio, engine pressure ratio, nozzle drive process

# Revisit Earlier Ramjet problem




- Compute Specific Thrust for Φ=1, gaseous $H_2$ fuel
- Nozzle Optimized for operation at ~10 km altitude ($p_\infty = p_e$= 26.43 kPa) …$p_{burner}$ ~ 1015.5 kPa
- Assume frozen chemistry at burner

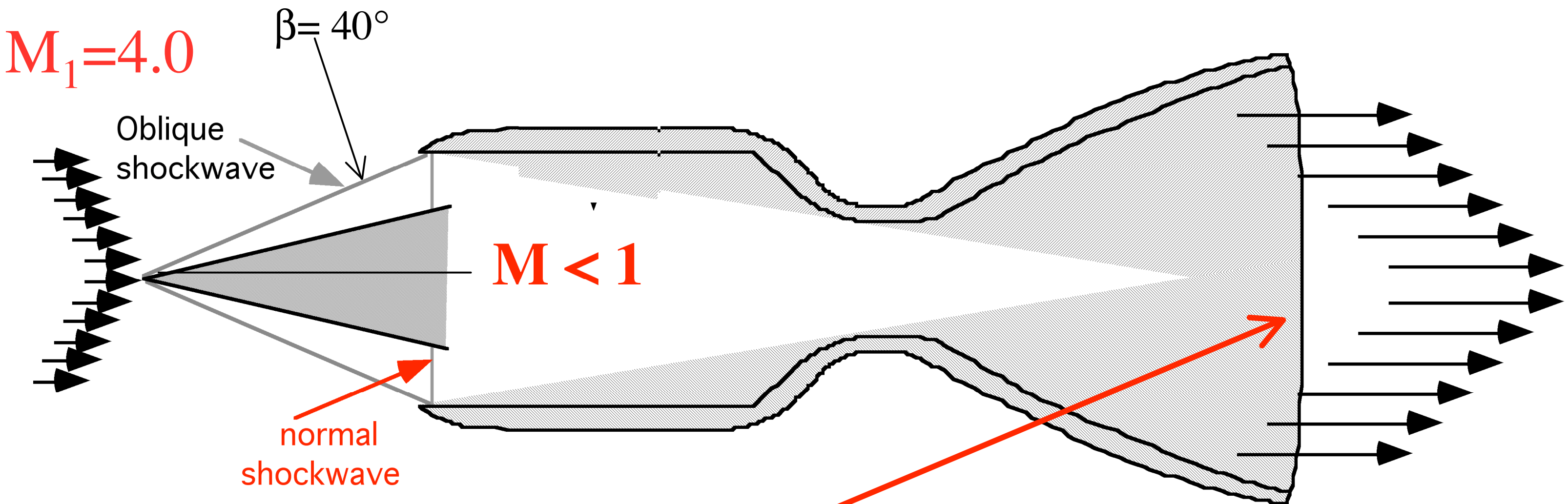
$$M_B = 0.557853$$

$$\frac{P_{0_B}}{P_{0_\infty}} = 0.3126$$

$$\frac{p_B}{P_{0_\infty}} = 38.422$$

# Revisit Earlier Ramjet problem (cont'd)

= 1253 kPa




- Calculate exit stagnation pressure:

$$P_{0_\infty} = 26.4\left(1 + \frac{1.4-1}{2}4^2\right)^{\left(\frac{1.4}{1.4-1}\right)} = 4008.5 \text{ kpa}$$

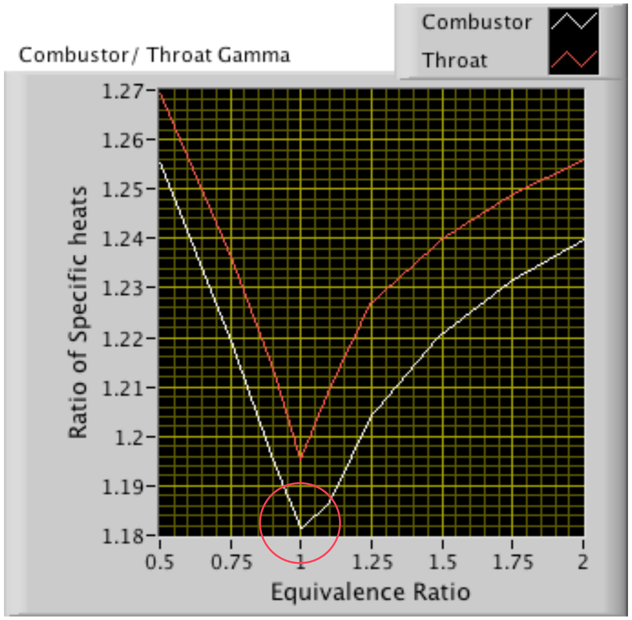
$$P_{0_e} = 0.3126\left(26.4\left(1 + \frac{1.4-1}{2}4^2\right)^{\left(\frac{1.4}{1.4-1}\right)}\right) = 1253.0 \text{ kPa}$$

$$M_B = 0.557853$$

$$\frac{P_{0_B}}{P_{0_\infty}} = 0.3126$$

$$\frac{p_B}{P_{0_\infty}} = 38.422$$

# Revisit Earlier Ramjet problem (cont'd)




From earlier CEA calculations

$\gamma_e \sim 1.1814$ ---> $\Phi$=1, gaseous $H_2$ fuel,
$p_{burner} \sim 1000$ kPa

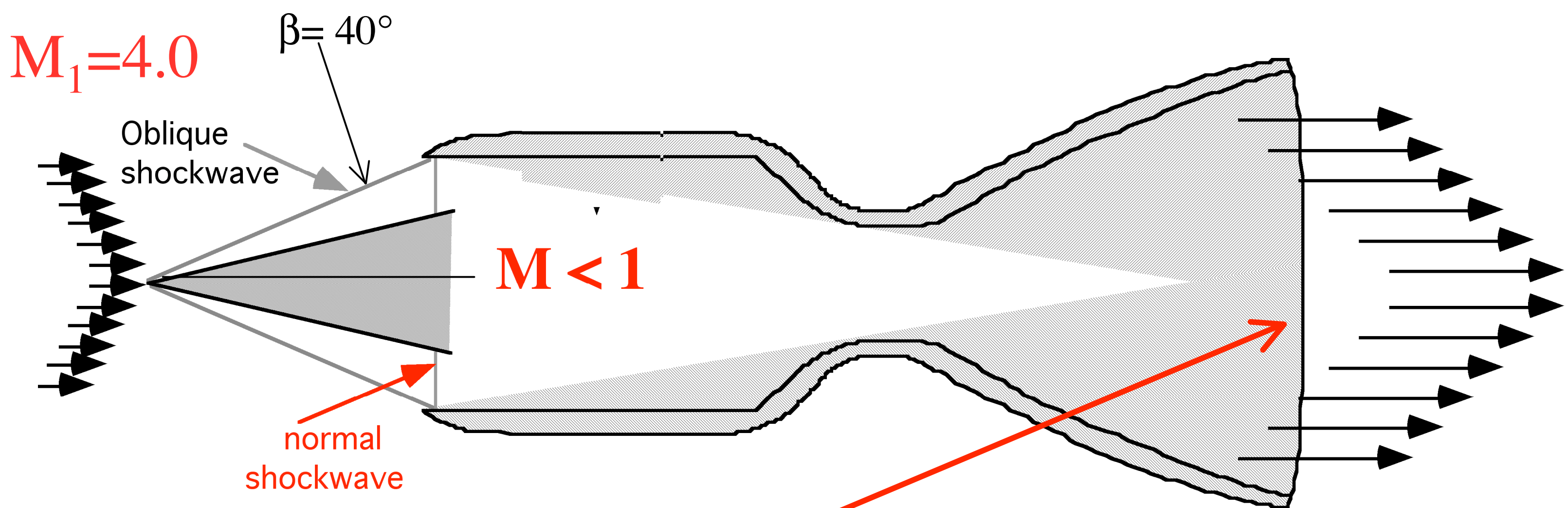
- Calculate exit $M_e$

$$M_e = \sqrt{\frac{2}{\gamma_e - 1}\left[\left(\frac{P_{0_e}}{p_e}\right)^{\frac{\gamma_e - 1}{\gamma_e}} - 1\right]}$$

$$\left(\frac{2}{1.1814 - 1}\left(\left(\frac{1253}{26.4}\right)^{\frac{1.1814-1}{1.1814}} - 1\right)\right)^{0.5} = 2.986$$

$\boxed{A_e/A^* = 7}$

# Revisit Earlier Ramjet problem (cont'd)



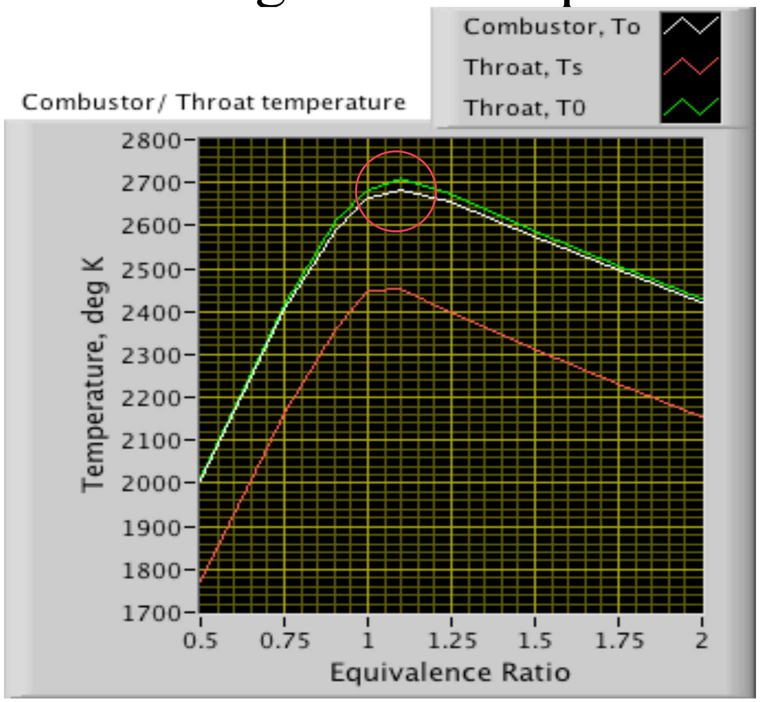

- Assume $A_e/A^* = 7.0$

- Compute free stream stagnation temperature

$$T_{0_\infty} = T_\infty\left[1 + \frac{\gamma - 1}{2} M_\infty^{\ 2}\right] = 216.65\left(1 + \frac{1.4 - 1}{2} 4^2\right) = 909.93^\circ\mathrm{K}$$

# Revisit Earlier Ramjet problem (cont'd)

- Exit stagnation temperature … From earlier CEA calculations




$$T_{burner} = 2662.82\ ^\circ K$$

- From inlet earlier problem

$$\boxed{\begin{array}{l} M_B = 0.557853 \\ \dfrac{P_{0_B}}{P_{0_\infty}} = 0.3126 \\ \dfrac{p_B}{P_{0_\infty}} = 38.422 \end{array}}$$

$$T_{0_{burner}} =$$

$$2662.82\left(1 + \frac{1.1814 - 1}{2} 0.55785^2\right) = 2738.2\ ^\circ K$$

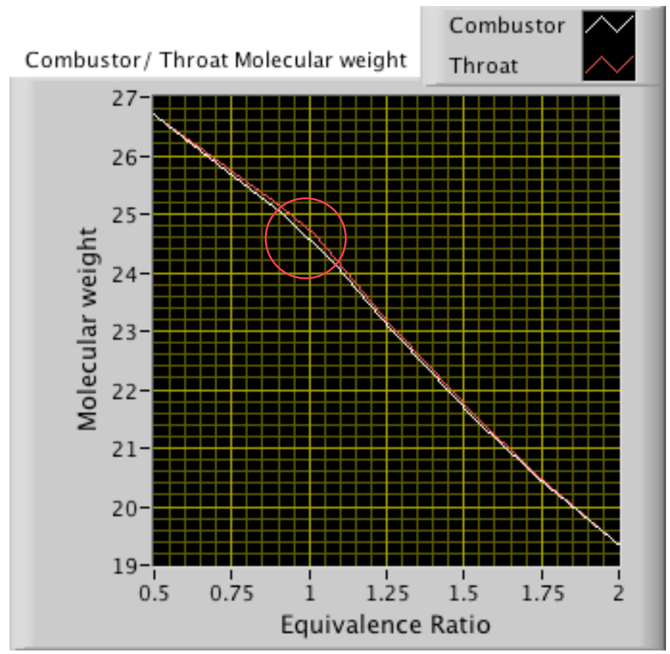
# Revisit Earlier Ramjet problem (cont'd)

- Calculate specific thrust

$$\left(\frac{F_{thrust}}{\dot{m}_f}\right)_{opt} = \left(\frac{f+1}{f}\right)\sqrt{\gamma_e R_{g_e} T_{o_e}}\frac{M_e}{\sqrt{1+\frac{\gamma_e-1}{2}M_e^{\;2}}} - \frac{1}{f}\left(\sqrt{\gamma_\infty R_{g_\infty} T_{0_\infty}}\frac{M_\infty}{\sqrt{1+\frac{\gamma_\infty-1}{2}M_\infty^{\;2}}}\right)$$

$$f = \Phi f_{stoich} = (1)\frac{1}{37.88393} = 0.0265$$

$M_W = 24.6 \text{ --> } R_g = 337.98$ J/kg-degK



# Revisit Earlier Ramjet problem (cont'd)

- Calculate specific thrust

$$\left(\frac{F_{thrust}}{\dot{m}_f}\right)_{opt} = \left(\frac{f+1}{f}\right)\sqrt{\gamma_e R_{g_e} T_{o_e}}\frac{M_e}{\sqrt{1+\frac{\gamma_e-1}{2}M_e^{\,2}}} - \frac{1}{f}\left(\sqrt{\gamma_\infty R_{g_\infty} T_{0_\infty}}\frac{M_\infty}{\sqrt{1+\frac{\gamma_\infty-1}{2}M_\infty^{\,2}}}\right)$$

$$\left(\frac{0.0265+1}{0.0265}\right)(1.1814\cdot 337.98\cdot 2738.2)^{0.5}\frac{(2.986)}{\left(1+\frac{1.1814-1}{2}(2.986)^2\right)^{0.5}} -$$

$$\left(\frac{1}{0.0265}\right)(1.4\cdot 287\cdot 909.93)^{0.5}\frac{4}{\left(1+\frac{1.4-1}{2}(4)^2\right)^{0.5}} = 45397 \text{ Nt/kg/sec}$$

… convert to seconds $\frac{1}{g_0}\left(\frac{F_{thrust}}{\dot{m}_f}\right)_{opt} = \frac{89928.2835-44534.69}{9.8077} = 4628.4 \text{ sec}$

**Wow!**

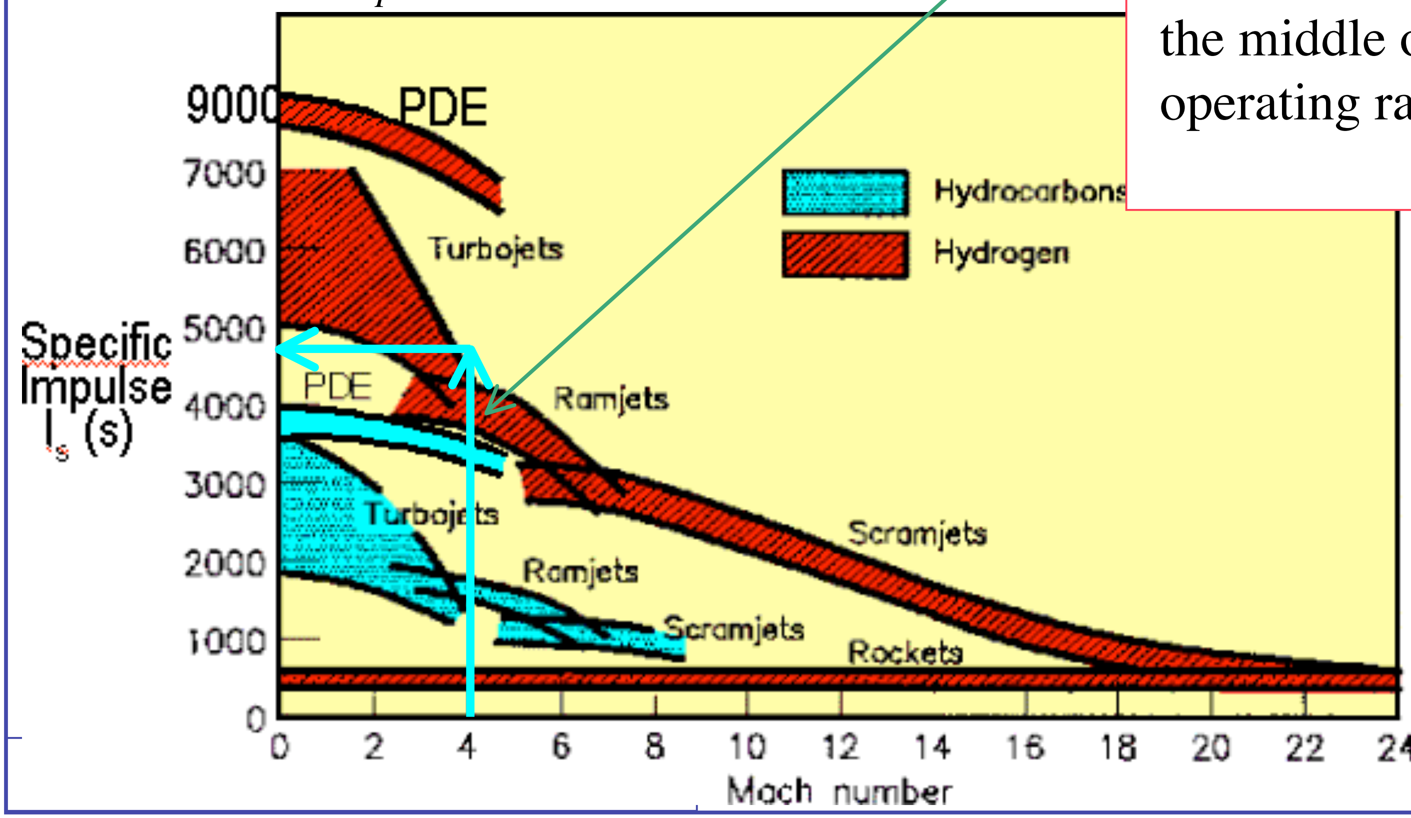
# Revisit Earlier Ramjet problem (cont'd)

- Calculate specific thrust

$$\frac{1}{g_0}\left(\frac{F_{thrust}}{\dot{m}_f}\right)_{opt} = 4628.4 \text{ sec}$$

- Idealized analysis … with a 2-D inlet

- If we consider losses of [redacted] 10% then we are right in the middle of the predicted operating range



# What is the thermodynamic efficiency

$$\eta = 1 - \left(\frac{P_A}{P_B}\right)^{\frac{\gamma-1}{\gamma}} \frac{\left(T_C - \left(\frac{P_{0_B}}{P_{0_A}}\right)^{\frac{\gamma-1}{\gamma}} T_B\right)}{\left(T_C - T_B\right)} =$$

$$1 - \frac{38.422^{\frac{-(1.4-1)}{1.4}}\left(2662.82 - \left(0.3126^{\frac{(1.4-1)}{1.4}}\right)856.61\right)}{(2662.82 - 856.61)} = 0.600$$

**Not Bad!**