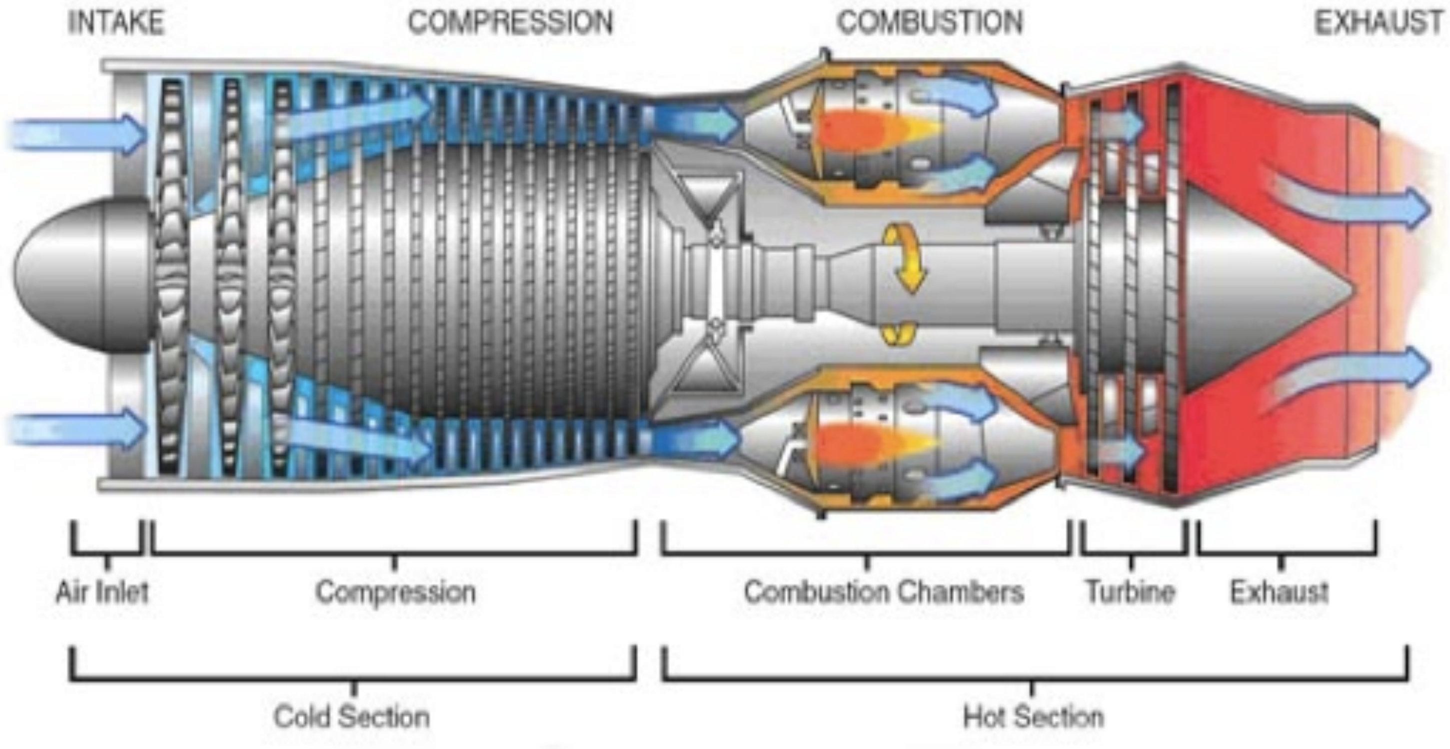
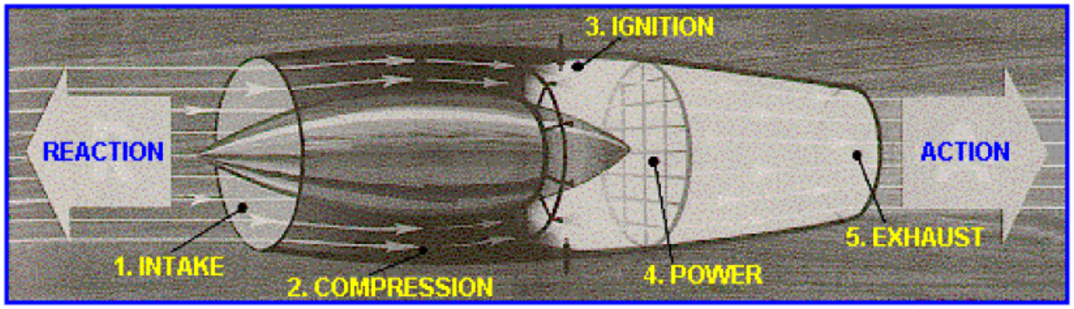
UtahState UNIVERSITY

Mechanical & Aerospace Engineering

Section *9.3:*
Jet Propulsion Basics Revisited

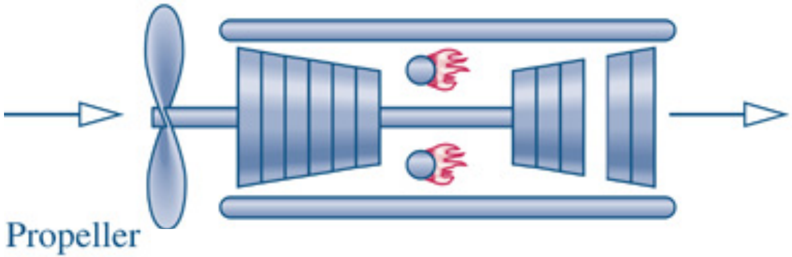


Basic Types of Jet Engines



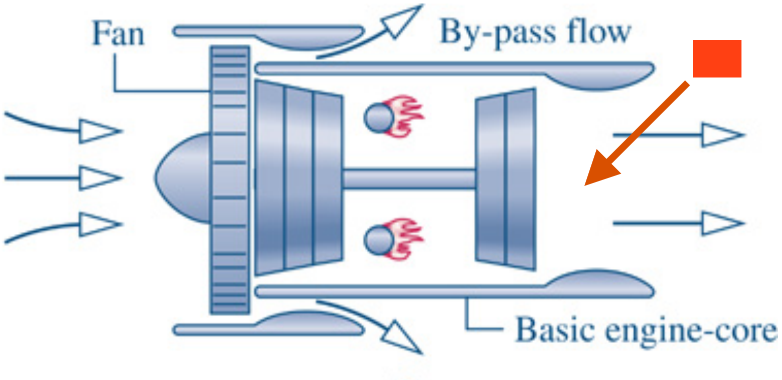
Ramjet

High Speed, Supersonic Propulsion, Passive Compression/Expansion



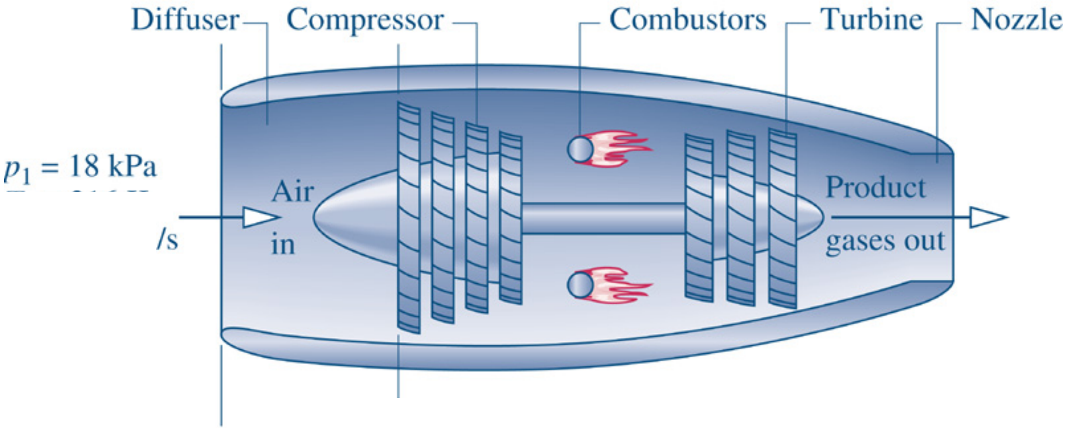
Turboprop

Low to Intermediate Subsonic
Small Commuter Planes



Turbofan

Larger Passenger Airliners
Intermediate Speeds, Subsonic Operation



Turbojet

High Speeds Supersonic or Subsonic Operation

Basic Types of Jet Engines (2)

- **Thrust produced by increasing the kinetic energy of the air in the opposite direction of flight**

- **Slight acceleration of a large mass of air**
  → Engine driving a propeller

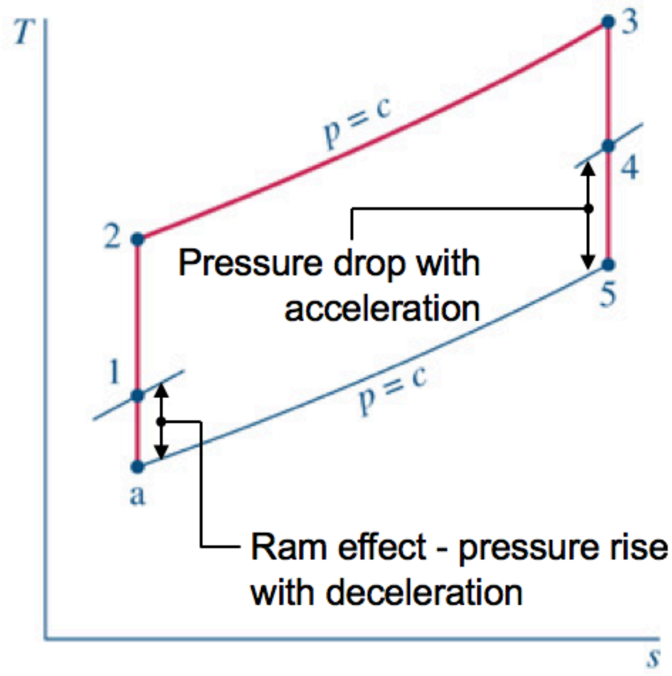
- **Large acceleration of a small mass of air**
  → Turbojet or turbofan engine

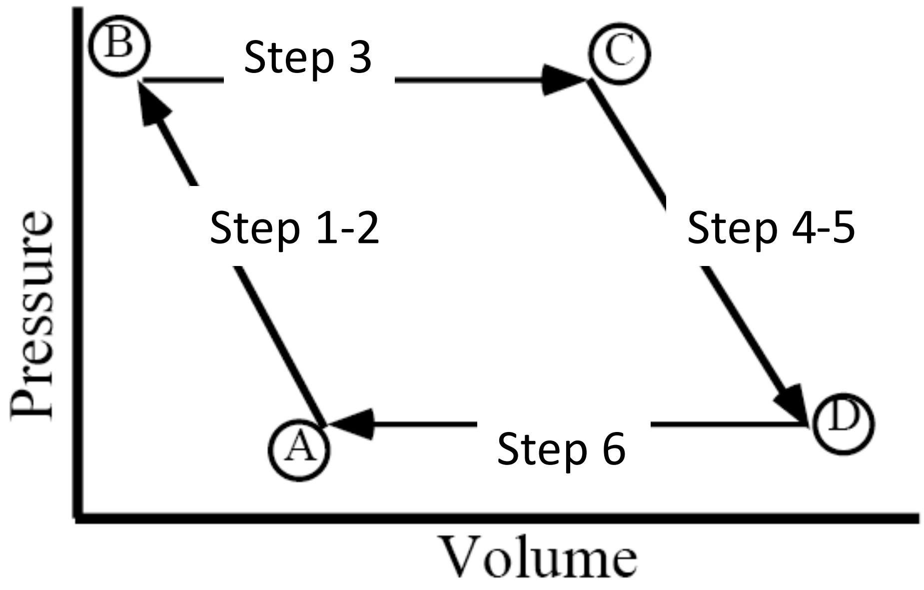
- **Combination of both**
  → Turboprop engine

Brayton Cycle for Jet Propulsion

Step	Process
1) Intake *(suck)*	Isentropic Compression
2) Compress the Air *(squeeze)*	Adiabatic Compression
3) Add heat *(bang)*	Constant Pressure Combustion
4) Extract work *(blow)*	Isentropic Expansion in Nozzle
5) Exhaust	Heat extraction by surroundings

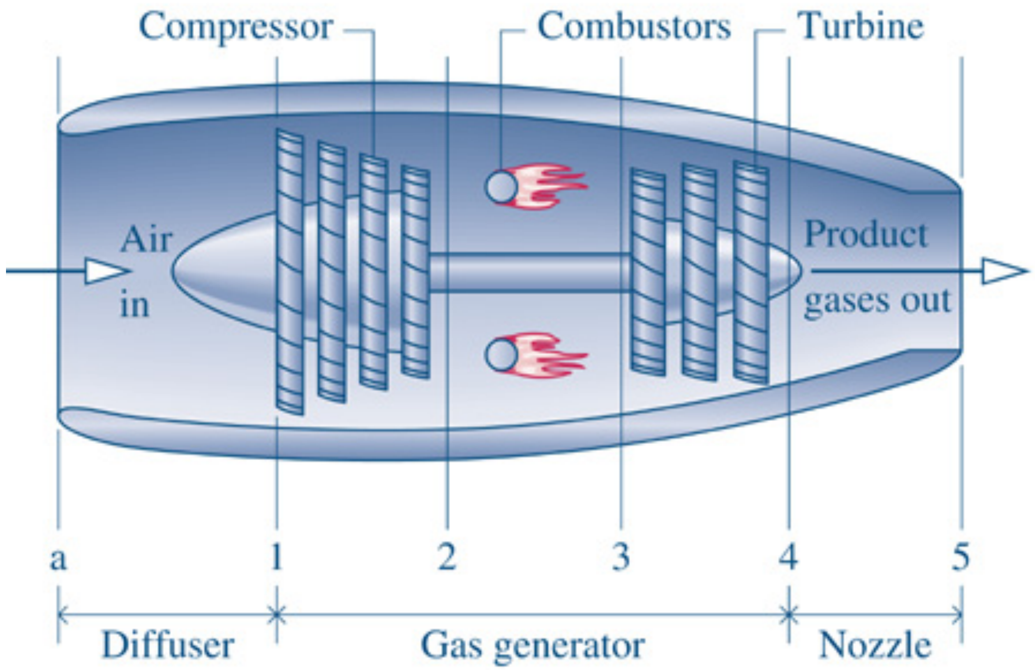
… step 5 above happens In the exhaust plume and has minimal Effect on engine performance

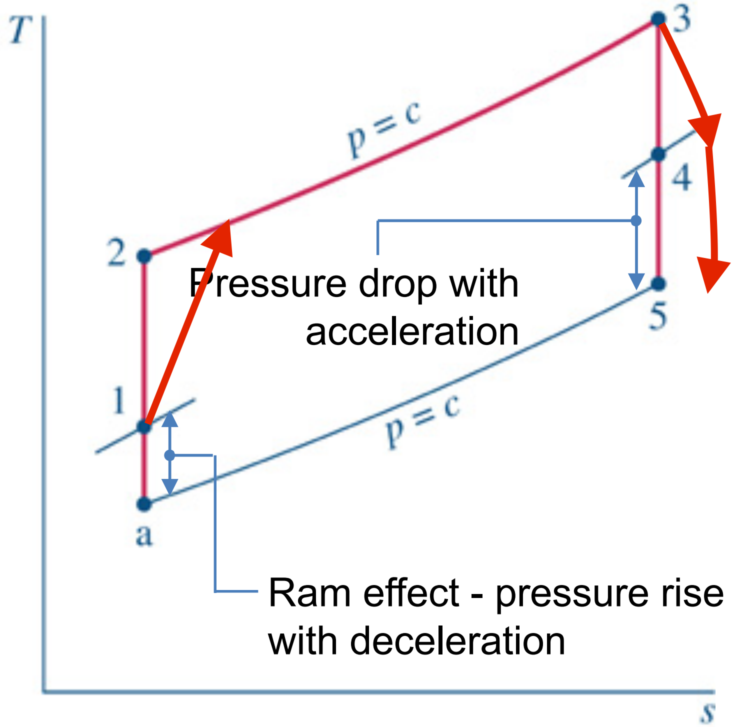




(Credit Narayanan Komerath, Georgia Tech)

Ideal TurboJet Cycle Analysis
Very Similar to Brayton Cycle



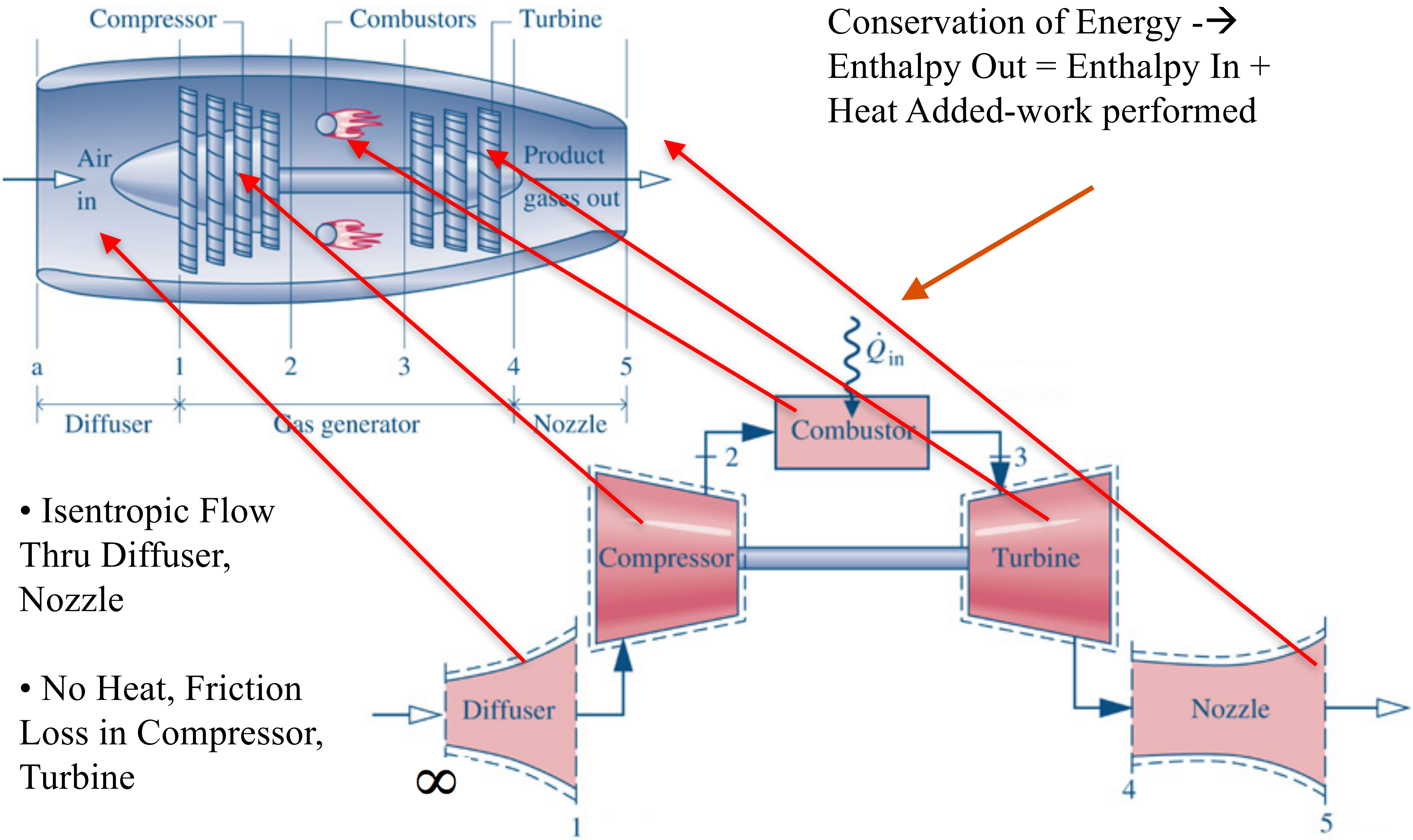


Pressure drop with acceleration

Ram effect - pressure rise with deceleration

a-1 Isentropic increase in pressure (diffuser)
1-2 Isentropic compression (compressor)
2-3 Isobaric heat addition (combustion chamber)
3-4 Isentropic expansion (turbine)
4-5 Isentropic decrease in pressure with an increase in fluid velocity (nozzle)

Idealized Thermodynamic Model

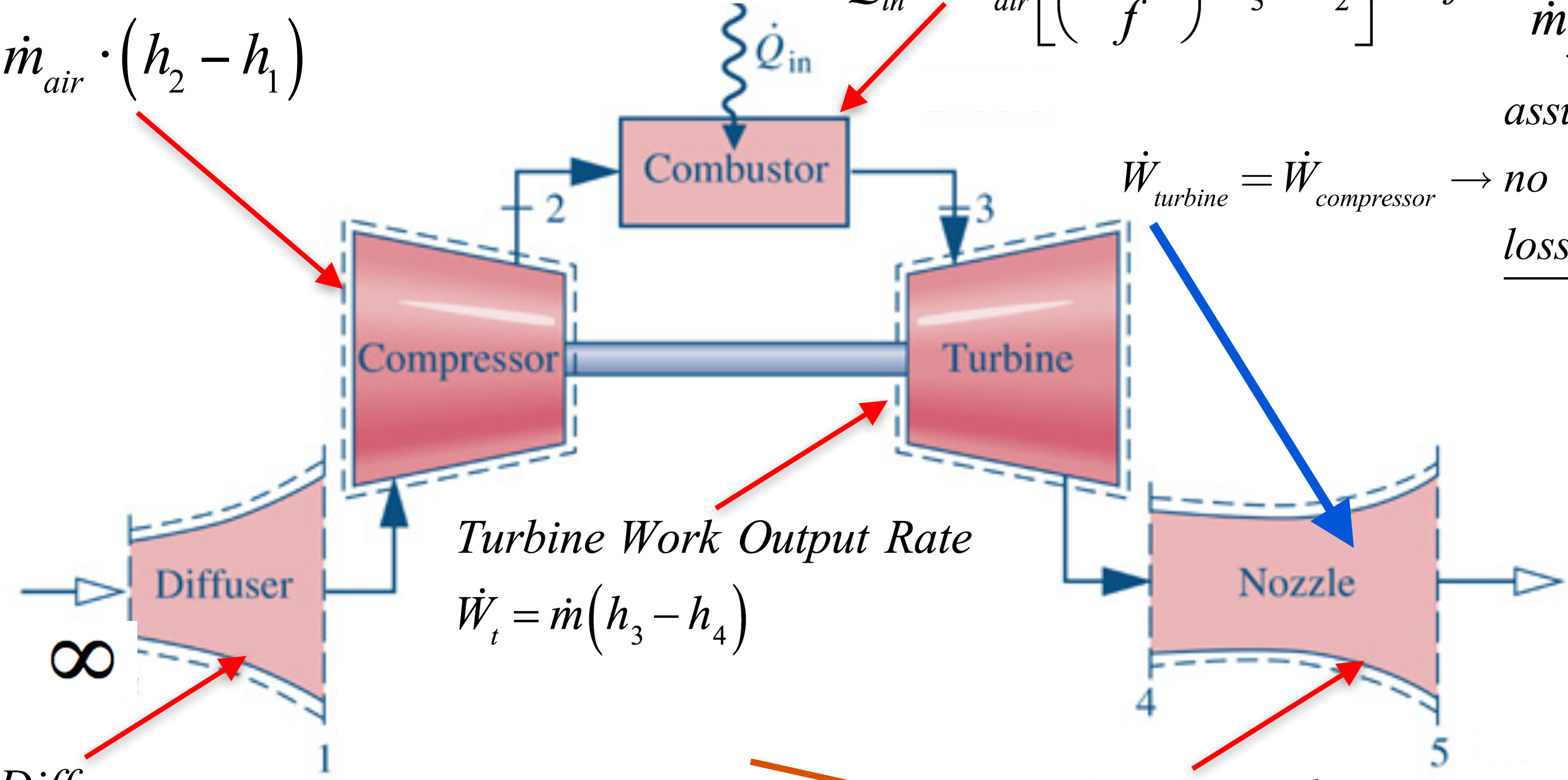


Conservation of Energy -→ Enthalpy Out = Enthalpy In + Heat Added-work performed

- Isentropic Flow Thru Diffuser, Nozzle

- No Heat, Friction Loss in Compressor, Turbine

Idealized Thermodynamic Model (2)



Combustor Heat Input Rate:

$$\dot{Q}_{in} = \dot{m}_{air}\left[\left(\frac{f+1}{f}\right)\cdot h_3 - h_2\right] \rightarrow f = \frac{\dot{m}_{air}}{\dot{m}_{fuel}}$$

Compressor Work Input Rate:

$$\dot{W}_c = \dot{m}_{air}\cdot\left(h_2 - h_1\right)$$

$$\dot{W}_{turbine} = \dot{W}_{compressor} \rightarrow \left.\begin{array}{l} \text{assume} \\ \text{no} \\ \text{losses} \end{array}\right|$$

Turbine Work Output Rate

$$\dot{W}_t = \dot{m}\left(h_3 - h_4\right)$$

Across Diffuser:

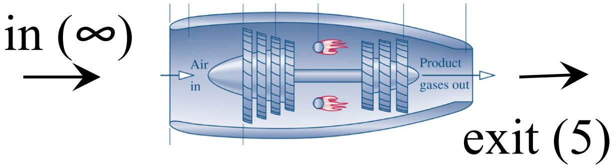
$$h_1 + \frac{1}{2}V_1^2 = h_\infty + \frac{1}{2}V_\infty^2$$

Across Nozzle

$$h_4 + \frac{V_4^2}{2} = h_5 + \frac{V_5^2}{2}$$

Idealized Thermodynamic Model (3)

- **Energy balance** → change in the stagnation enthalpy rate of the gas flow between the exit and entrance of the engine is equal to the added chemical enthalpy rate of the injected fuel flow.



$$\left(\dot{m}_{air}+\dot{m}_{fuel}\right)\cdot h_{0_{exit}} = \dot{m}_{air}\cdot h_{0_\infty}+\dot{m}_{fuel}\cdot h_{fuel}$$

$$h_{0_{exit}} = h_{exit}+\frac{1}{2}V_{exit}^2 \quad , \quad h_{0_\infty}=h_\infty+\frac{1}{2}V_\infty^2$$

- Letting $f=\dot{m}_{air}/\dot{m}_{fuel} \rightarrow h = c_p\cdot T$

$$\left(\frac{f+1}{f}\right)\left(h_{exit}+\frac{1}{2}V_{exit}^2\right)=h_\infty+\frac{1}{2}V_\infty^2+\frac{1}{f}\cdot h_{fuel}=$$

$$\boxed{\left(\frac{f+1}{f}\right)\left(c_{p_{exit}}T_{exit}+\frac{1}{2}V_{exit}^2\right)=c_{p_\infty}h_\infty+\frac{1}{2}V_\infty^2+\frac{1}{f}\cdot h_{fuel}}$$

Idealized Thermodynamic Model (3)

- The high energy content of hydrocarbon fuels is remarkably large and allow extended powered flight to be possible.

A typical value of fuel enthalpy for JP-4 jet fuel is

$$h_f|_{JP-4} = 4.28 \times 10^7 \; J/kg.$$

As a comparison, the enthalpy of Air at sea level static conditions is

$$h|_{Airat288.15K} = C_p T_{SL} = 1005 \times 288.15 = 2.896 \times 10^5 \; J/kg.$$

The ratio is

$$\frac{h_f|_{JP-4}}{h|_{Airat288.15K}} = 148.$$

Credit: B. Cantwell Stanford

Jet Engine Performance Performance Parameters

- **Propulsive Force (Thrust)**
  - – The force resulting from the velocity at the nozzle exit
- **Propulsive Power**
  - – The equivalent power developed by the thrust of the engine
- **Propulsive Efficiency**
  - – Relationship between propulsive power and the rate of kinetic energy production
- **Thermal Efficiency**
  - – Relationship between kinetic energy rate of the system and heat Input the system

Propulsive and Thermal Efficiency of Cycle

Propulsive Efficiency =

Propulsive power

$$\eta_{propulsive} = \frac{\dot{W}_p}{\left(K.E._{exit} - K.E._{\infty}\right)}$$

Kinetic energy production rate

Kinetic energy production rate

Thermal Efficiency = →

$$\eta_{thermal} = \frac{\left(K.E._{exit} - K.E._{\infty}\right)}{\dot{m}_{fuel} \cdot h_{fuel}}$$

Combustion Enthalpy of Fuel

Look a Product of Efficiencies

$$\eta_{propulsive} \times \eta_{thermal} =$$

$$\frac{\dot{W}_p}{\left(K.E._{exit} - K.E._{\infty}\right)} \times \frac{\left(K.E._{exit} - K.E._{\infty}\right)}{\dot{m}_{fuel} \cdot h_{fuel}} = \frac{\dot{W}_p}{\dot{m}_{fuel} \cdot h_{fuel}}$$

Jet Engine Performance – *Propulsive and Thermal Efficiency*

Look a Product of Efficiencies

$$\eta_{propulsive} \times \eta_{propulsive} =$$

$$\frac{\dot{W}_p}{\left(K.E._{exit} - K.E._{\infty}\right)} \times \frac{\left(K.E._{exit} - K.E._{\infty}\right)}{\dot{m}_{fuel} \cdot h_{fuel}} = \frac{\dot{W}_p}{\dot{m}_{fuel} \cdot h_{fuel}}$$

Overall Thermodynamic Cycle Efficiency =

Net Propulsion Power Output/Net Heat Input

$$\eta_{overall} = \eta_{thermal}\,\eta_{propulsive}$$

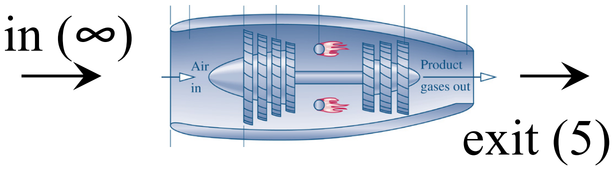
Jet Engine Performance Efficiencies

Propulsive Efficiency

Ratio of Power Developed from Engine (desired beneficial output) Thrust to Change in Kinetic Energy of the Moving Airstream (cost of propulsion)

Thrust Equation:

$$\boxed{\begin{aligned}&F=\left(\dot m_{air}+\dot m_{fuel}\right)\cdot V_{exit}-\dot m_{air}\cdot V_{inlet}+\left(p_{exit}-p_{\infty}\right)\cdot A_{exit}\\&f=\frac{\dot m_{air}}{\dot m_{fuel}}\rightarrow Optimal\ \ Nozzle\rightarrow p_{exit}=p_{\infty}\\&\rightarrow F\approx\dot m_{air}\cdot\left[\left(\frac{f+1}{f}\right)\cdot V_{exit}-V_{\infty}\right]\end{aligned}}$$



$$Optimal\ \ Nozzle\rightarrow p_{exit}=p_{\infty}$$

$$\boxed{\dot W_p=F\cdot V_{aircraft}=\dot m_{air}\cdot\left(\left(\frac{f+1}{f}\right)V_{exit}-V_{\infty}\right)\cdot V_{\infty}}$$

Propulsive Power

The power developed from the thrust of the engine

Jet Engine Performance Efficiencies (2)

Propulsive Efficiency

Ratio of Power Developed from Engine (desired beneficial output) Thrust to Change in Kinetic Energy of the Moving Airstream (cost of propulsion)

Propulsive power

$$\eta_{propulsive} = \frac{\dot{W}_p}{\left(K.E._{exit} - K.E._{\infty}\right)} = \frac{\dot{m}_{air}\cdot\left(\left(\frac{f+1}{f}\right)V_{exit} - V_{\infty}\right)\cdot V_{\infty}}{\dot{m}_{air}\cdot\left(\frac{1}{2}\left(\frac{f+1}{f}\right)V^2_{exit} - \frac{1}{2}V^2_{\infty}\right)}$$

Kinetic energy production rate

assuming $\dot{m}_{air} >> \dot{m}_{fuel} \rightarrow f << 1$

$$\boxed{\eta_{propulsive} = \frac{2\cdot\left(V_{exit} - V_{\infty}\right)\cdot V_{\infty}}{\left(V_{exit} + V_{\infty}\right)\cdot\left(V_{exit} - V_{\infty}\right)} = \frac{2\cdot V_{\infty}}{\left(V_{exit} + V_{\infty}\right)} = \frac{2}{\left(1 + V_{exit}/V_{\infty}\right)}}$$

Maximum propulsive efficiency achieved by generating thrust moving as much air as possible with as little a change in velocity across the engine as possible.

Jet Engine Performance Efficiencies (3)

Thermal Efficiency

The thermal efficiency of a thermodynamic cycle compares work output from cycle to heat added…

Analogously, thermal efficiency of a propulsion cycle directly compares change in gas kinetic energy across engine to energy released through combustion.

$$\rightarrow \quad \eta_{thermal} = \frac{\left(K.E._{exit} - K.E._{\infty}\right)}{\dot{m}_{fuel} \cdot h_{fuel}} =$$

Kinetic energy production rate

Thermal power available from the fuel

$$1 - \frac{\textit{Heat Rejected During Cycle}}{\textit{Heat Input During Cycle}} = \frac{\left(\frac{1}{2}\left(\frac{\cancel{f}+1}{\cancel{f}}\right)V^2{}_{exit} - \frac{1}{2}V^2{}_{\infty}\right)}{\frac{1}{f} \cdot h_{fuel}}$$

Jet Engine Performance Efficiencies (4)

- **Rewriting the expression**

$$\eta_{thermal} = \frac{\left(\frac{1}{2}\left(\frac{f+1}{f}\right)V^2{}_{exit} - \frac{1}{2}V^2{}_{\infty}\right)}{\frac{1}{f}\cdot h_{fuel}} = 1 - \frac{\frac{1}{f}\cdot h_{fuel} + \frac{1}{2}V^2{}_{\infty} - \frac{1}{2}\left(\frac{f+1}{f}\right)V^2{}_{exit}}{\frac{1}{f}\cdot h_{fuel}}$$

- **Rewriting in terms of the gas enthalpies where** $\frac{1}{2}V^2 = h_0 - h$

$$\eta_{thermal} = 1 - \frac{\frac{1}{f}\cdot h_{fuel} + \left(h_{0_\infty} - h_\infty\right) - \left(\frac{f+1}{f}\right)\left(h_{0_{exit}} - h_{exit}\right)}{\frac{1}{f}\cdot h_{fuel}}$$

Jet Engine Performance Efficiencies (5)

- **From Energy Balance** $\frac{1}{f}\cdot h_{fuel} = h_\infty + \frac{1}{2}V_\infty^2 - \left(\frac{f+1}{f}\right)\left(h_{exit} + \frac{1}{2}V_{exit}^2\right)$

- **Substituting and Rearranging**

$$\eta_{thermal} = 1 - \frac{\left(\frac{f+1}{f}\right)(h_{exit}) - h_\infty - \left(\frac{f+1}{f}\right)h_\infty + \left(\frac{f+1}{f}\right)h_\infty}{\frac{1}{f}\cdot h_{fuel}} =$$

$$1 - \frac{\left(\frac{f+1}{f}\right)(h_{exit} - h_\infty) - \left[1 - \left(\frac{f+1}{f}\right)\right]h_\infty}{\frac{1}{f}\cdot h_{fuel}} = 1 - \frac{\left(\frac{f+1}{f}\right)(h_{exit} - h_\infty) - \frac{1}{f}\cdot h_\infty}{\frac{1}{f}\cdot h_{fuel}}$$

Jet Engine Performance Efficiencies (6)

$$\eta_{thermal} = 1 - \frac{\left(\frac{f+1}{f}\right)\left(h_{exit} - h_{\infty}\right) - \frac{1}{f}\cdot h_{\infty}}{\frac{1}{f}\cdot h_{fuel}}$$

$$f = \frac{\dot{m}_{air}}{\dot{m}_{fuel}}$$

$$\rightarrow \boxed{\begin{aligned} &\left(\frac{f+1}{f}\right)\left(h_{exit} - h_{\infty}\right) - \frac{1}{f}\cdot h_{\infty} = Heat\ Rejected\ During\ Cycle \\ &\frac{1}{f}\cdot h_{fuel} = Heat\ Input\ During\ Cycle \end{aligned}}$$

Jet Engine Performance Efficiencies (7)

- Strictly speaking engine is not closed system because of fuel mass addition across the burner.

- Heat rejected by exhaust consists of two distinct parts.

1. Heat rejected by conduction from nozzle flow to the surrounding atmosphere

2. Physical removal from the thermally equilibrated nozzle flow of a portion equal to the added fuel mass flow.

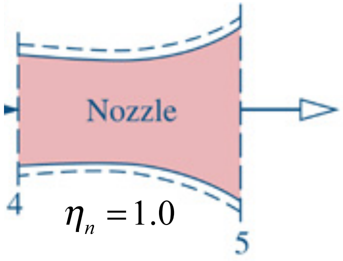
Fuel mass flow carries enthalpy into system by injection/combustion in burner and exhaust fuel mass flow carries ambient enthalpy out mixing with the surroundings.

There is no net mass increase or decrease to the system.

Propulsive and Thermal Efficiency Revisited

@ Cruise Assumed Optimized Nozzle → $p_{exit} = p_\infty$ $\qquad T_{exit} = T_4 \cdot \left(\frac{P_4}{p_{exit}}\right)^{\frac{\gamma-1}{\gamma}}$

$F = \dot{m}\left(V_{exit} - V_\infty\right) \qquad \dot{W}_p = F \cdot V$



Nozzle Enthalpy Balance

$$\dot{m}\left(h_4 + \frac{V_4^2}{2}\right) = \dot{m}\left(h_5 + \frac{V_5^2}{2}\right) = \dot{m}\left(h_{exit} + \frac{V_{exit}^2}{2}\right) \rightarrow V_4 \approx 0 \rightarrow V_{exit} = \sqrt{2\left(h_4 - h_{exit}\right)}$$

$$\eta_{propulsive} = \frac{\dot{W}_p}{\dot{m}_{air}\left(K.E._{exit} - K.E._\infty\right)} \qquad \eta_{thermal} = \frac{\left(K.E._{exit} - K.E._\infty\right)}{\dot{m}_{fuel} \cdot h_{fuel}}$$

$$\eta_{total} = \eta_{prop} \cdot \eta_{thermal} = \frac{F \cdot V_\infty}{\dot{m}_{fuel} \cdot h_{fuel}}$$

Propulsive and Thermal Efficiency Revisited (2)

$$\dot{Q}_{total} = \dot{m}\left(h_{0_3} - h_{02}\right) \qquad \dot{Q}_{\substack{out\\ excess}} = \dot{m}\left(h_{0_{exit}} - h_{0\infty}\right)$$

$$P_{prop} = F_{thrust}\cdot V_{\infty} = \left(\dot{m}\cdot V_{exit} - \dot{m}\cdot V_{\infty}\right)\cdot V_{\infty} = \frac{1}{2}\left(\dot{m}\cdot V^{2}{}_{exit}\right)\left(2\left(\frac{V_{exit}}{V_{\infty}}\right) - 2\left(\frac{V_{exit}}{V_{\infty}}\right)^{2}\right)$$

$$K.E._{net} = \frac{1}{2}\dot{m}\cdot\left(V^{2}{}_{exit} - V^{2}{}_{\infty}\right) = \frac{1}{2}\left(\dot{m}\cdot V^{2}{}_{exit}\right)\cdot\left(1 - \left(\frac{V_{\infty}}{V_{exit}}\right)^{2}\right)$$

$$\eta_{propulsive} = \frac{\dot{W}_{p}}{\left(K.E._{exit} - K.E._{\infty}\right)} = \frac{\frac{1}{2}\left(\dot{m}\cdot V^{2}{}_{exit}\right)\left(2\left(\frac{V_{exit}}{V_{\infty}}\right) - 2\left(\frac{V_{exit}}{V_{\infty}}\right)^{2}\right)}{\frac{1}{2}\left(\dot{m}\cdot V^{2}{}_{exit}\right)\cdot\left(1 - \left(\frac{V_{\infty}}{V_{exit}}\right)^{2}\right)} = \frac{2\left(\left(\frac{V_{exit}}{V_{\infty}}\right) - \left(\frac{V_{exit}}{V_{\infty}}\right)^{2}\right)}{\left(1 - \left(\frac{V_{\infty}}{V_{exit}}\right)^{2}\right)}$$

$$\eta_{thermal} = \frac{\left(K.E._{exit} - K.E._{\infty}\right)}{\dot{m}_{fuel}\cdot h_{fuel}} = \frac{\left(\frac{1}{2}V^{2}{}_{exit}\right)\cdot\left(1 - \left(\frac{V_{\infty}}{V_{exit}}\right)^{2}\right)}{\left(h_{0_3} - h_{02}\right)}$$

Propulsive and Thermal Efficiency Revisited (3)

$$K.E._{\substack{out\\excess}} = K.E._{net} - P_{prop} =$$

$$\frac{1}{2}\left(\dot{m}\cdot V^2{}_{exit}\right)\cdot\left(1-\left(\frac{V_\infty}{V_{exit}}\right)^2\right)-\frac{1}{2}\left(\dot{m}\cdot V^2{}_{exit}\right)\left(2\left(\frac{V_{exit}}{V_\infty}\right)-2\left(\frac{V_{exit}}{V_\infty}\right)^2\right)=$$

$$\frac{1}{2}\dot{m}\cdot V^2{}_{exit}\cdot\left(1-\left(\frac{V_\infty}{V_{exit}}\right)^2-2\left(\frac{V_{exit}}{V_\infty}\right)+2\left(\frac{V_{exit}}{V_\infty}\right)^2\right)=$$

$$\frac{1}{2}\dot{m}\cdot V^2{}_{exit}\cdot\left(1-2\left(\frac{V_{exit}}{V_\infty}\right)+\left(\frac{V_{exit}}{V_\infty}\right)^2\right)=\frac{1}{2}\dot{m}\cdot V^2{}_{exit}\cdot\left(1-\left(\frac{V_{exit}}{V_\infty}\right)\right)$$

Propulsive and Thermal Efficiency Revisited (9)

Summary

$$\eta_{propulsive} = \frac{\dot{W}_p}{\left(K.E._{exit} - K.E._{\infty}\right)} = \frac{2\left(\left(\frac{V_{exit}}{V_{\infty}}\right) - \left(\frac{V_{exit}}{V_{\infty}}\right)^2\right)}{\left(1 - \left(\frac{V_{\infty}}{V_{exit}}\right)^2\right)}$$

$$\eta_{thermal} = \frac{\left(K.E._{exit} - K.E._{\infty}\right)}{\dot{m}_{fuel} \cdot h_{fuel}} = \frac{\left(\frac{1}{2} V^2{}_{exit}\right) \cdot \left(1 - \left(\frac{V_{\infty}}{V_{exit}}\right)^2\right)}{\left(h_{0_3} - h_{02}\right)}$$

$$K.E._{\substack{out \\ excess}} = K.E._{net} - P_{prop} = \frac{1}{2} \dot{m} \cdot V^2{}_{exit} \cdot \left(1 - \left(\frac{V_{exit}}{V_{\infty}}\right)\right)$$

"Equivalence Ratio" and Engine Performance

- Combustion efficiency and stability limits are depending on several parameters : fuel, equivalence ratio, air stagnation pressure and temperature

- The ***equivalence ratio*** is used to characterize the mixture ratio Of airbreathing engines ... *analogous to O/F for rocket propulsion*

- The *equivalence ratio*,Φ , is defined as the ratio of the actual fuel-air ratio to the stoichiometric fuel-air ratio.

- For $\Phi = 1$, no oxygen is left in exhaust produc ... combustion is called *stoichiometric*

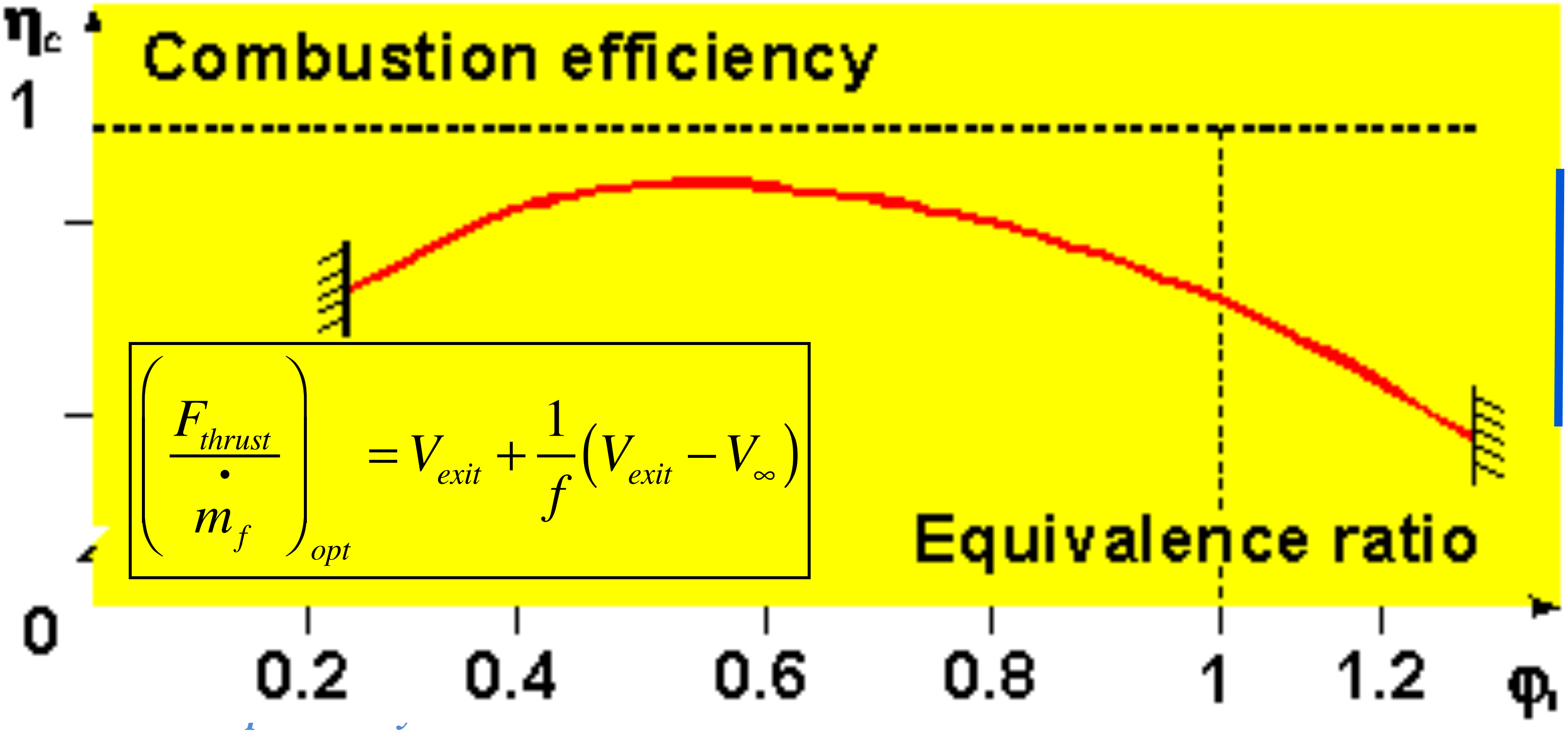
$$\Phi \equiv \frac{\left[\frac{\dot{m}_{fuel}}{\dot{m}_{air}}\right]_{actual}}{\left[\frac{\dot{m}_{fuel}}{\dot{m}_{air}}\right]_{stoich}} = \frac{f_{stoich}}{f_{actual}}$$

… $\Phi > 1$ ---> a rich mixture
… $\Phi < 1$ ---> lean mixture

“Equivalence Ratio” and Engine Performance (2)

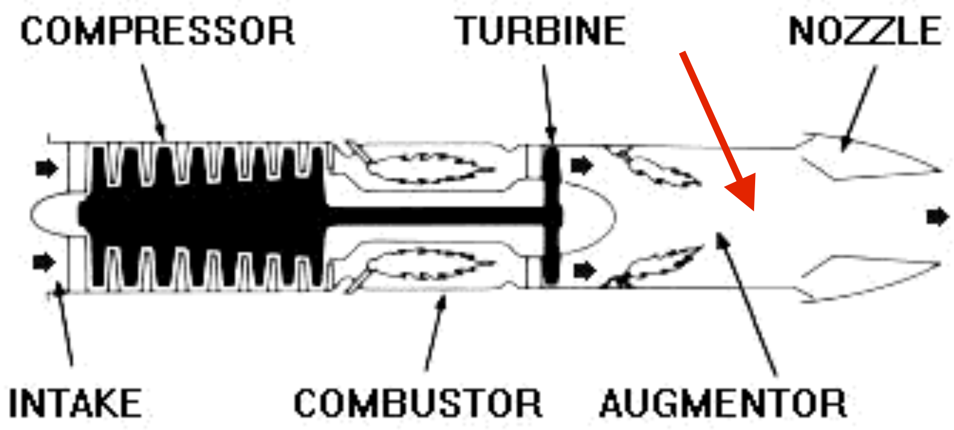
- Unlike Rockets .. Ramjets … and air breathing propulsion systems tend to be more efficient when engine runs leaner than *stoichiometric*
- *Also Thermal Capacity of Turbine Materials Limits Maximum Allowable Combustion Temperature, not Allowing Engine to Run Stoichiometric*

$$\eta_{thermal} = 1 - \frac{(f+1)(h_{exit} - h_\infty) - h_\infty}{\frac{1}{f} \cdot h_{fuel}}$$



$$\left(\frac{F_{thrust}}{\dot{m}_f}\right)_{opt} = V_{exit} + \frac{1}{f}\left(V_{exit} - V_\infty\right)$$

“Equivalence Ratio” and Engine Performance (3)



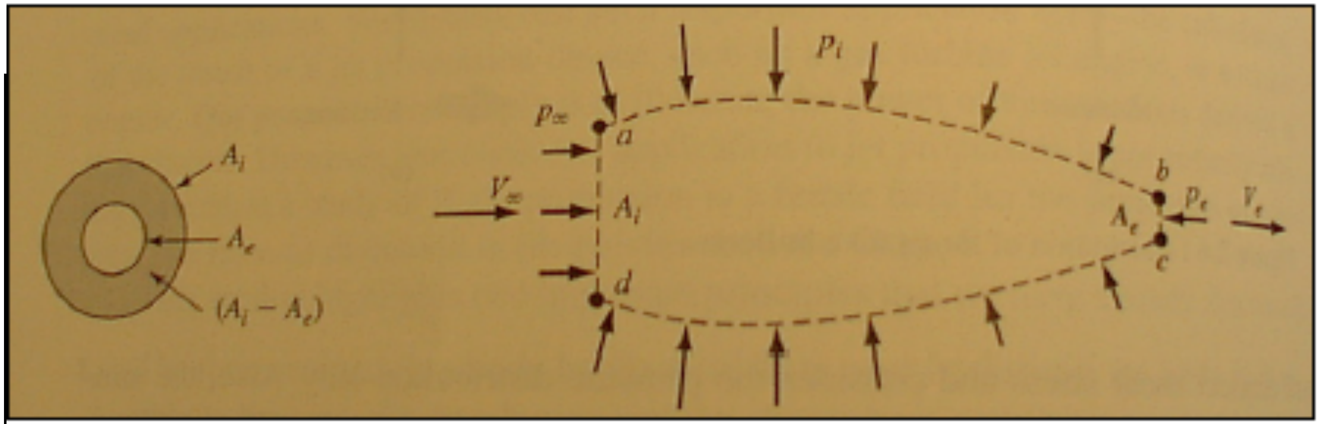
- … that is why afterburners work … left over O_2 after combustion

Additional fuel is introduced into the hot exhaust and burned using excess O_2 from main combustion

- The afterburner increases the temperature of the gas ahead of the nozzle Increases exit velocity

- The result of this increase in temperature is an increase of about 40 percent in thrust at takeoff and a much larger percentage at high speeds

Specific Thrust of Air Breathing Engine



$$\left(\frac{F_{thrust}}{\dot{m}_f}\right)_{net}$$

Analogous to I_{sp}

Net thrust

$$F_{thrust} = \dot{m}_{exit} V_{exit} - \dot{m}_\infty V_\infty + \left(p_{exkit} - p_\infty\right)\cdot A_{exit} \rightarrow$$

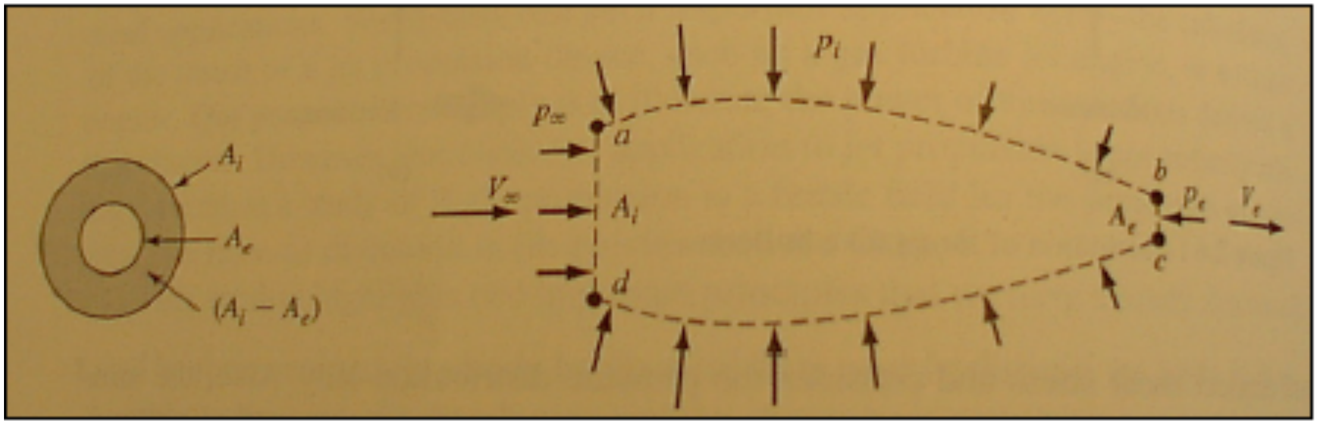
$$\dot{m}_\infty = \dot{m}_{air}$$

$$\dot{m}_{exit} = \dot{m}_{air} + \dot{m}_{fuel}$$

$$f = \frac{\dot{m}_{air}}{\dot{m}_{fuel}}$$

$$F_{thrust} = \dot{m}_{air}\left[\left(\frac{\dot{m}_{air} + \dot{m}_{fuel}}{\dot{m}_{air}}\right)V_{exit} - V_\infty\right] + \left(p_{exit} - p_\infty\right)\cdot A_{exit} = \dot{m}_{air}\left[\left(\frac{1+f}{f}\right)V_e - V_i\right] + \left(p_e - p_\infty\right)\cdot A_e$$

Specific Thrust of Air Breathing Engine (2)



$$Thrust = \dot{m}_e V_e - \dot{m}_i V_i + \left(p_e A_e - p_\infty A_e\right)$$

Cruise design condition
When $p_e = p_\infty$

$$\left(\frac{F_{thrust}}{\dot{m}_f}\right)_{opt} = \frac{\left[\dot{m}_f + \dot{m}_{air}\right]V_{exit} - \dot{m}_{air} V_\infty}{\dot{m}_f} = [f+1]V_{exit} - f \cdot V_\infty = V_{exit} + f \cdot \left(V_{exit} - V_\infty\right)$$

%Ram Drag Reduced at lower air-fuel ratio "f"

$$f = \frac{\dot{m}_{air}}{\dot{m}_{fuel}}$$

Jet Engine Fuel Efficiency Performance Measure

Thrust Specific Fuel Consumption (TSFC) → Inverse of Specific Thrust

- *Measure of fuel economy*
- *Analogous to specific impulse in Rocket Propulsion*

$$TSFC = \frac{\dot{m}_f}{F_{thrust}} \approx \frac{1}{I_{sp} g_0}$$

$$Typical\ Turbojet \approx TSFC = (2-4)_{\frac{lbm}{lbf-hr}}$$

$$SFC|_{JT9D-takeoff} \cong 0.35$$

$$SFC|_{JT9D-cruise} \cong 0.6$$

$$SFC|_{militaryengine} \cong 0.9to1.2$$

$$SFC|_{militaryenginewithafterburning} \cong 2.$$

TSFC generally goes up engine moves from takeoff to cruise, as energy required to produce a thrust goes up with increased percentage of stagnation pressure losses and with increased momentum of incoming air.

Breguet Aircraft Range Equation

- Aviation Analog of “Rocket Equation”
- Assumes Constant Lift-to-Drag (L/D) and Constant Overall Efficiency

$$\eta_{overall} = \eta_{propulsive} \cdot \eta_{propulsive} = \frac{\dot{W}_p}{\dot{m}_{fuel} \cdot h_{fuel}} = \frac{F_{thrust} \cdot V_\infty}{\dot{m}_{fuel} \cdot h_{fuel}}$$

For Fight Optimal Conditions

$$\rightarrow V_\infty = \frac{\eta_{overall} \cdot \dot{m}_{fuel} \cdot h_{fuel}}{F_{thrust}}$$

$$Total\ Range:$$

$$R = \int V_\infty dt = \int \left(\frac{\eta_{overall} \cdot \dot{m}_{fuel} \cdot h_{fuel}}{F_{thrust}} \right) \cdot dt$$

- Fuel mass flow is directly related to the change in aircraft weight

$$\dot{m}_{fuel} = -\frac{1}{g}\frac{dW}{dt}$$

Breguet Aircraft Range Equation (2)

- In equilibrium (cruise) flight Thrust equals drag and aircraft weight equals lift …

$$T = D = L/\left(\frac{L}{D}\right) = W/\left(\frac{L}{D}\right)$$

- Subbing into Range Equation

$$R = \int V_\infty dt = -\int \left(\frac{\eta_{overall}\cdot\frac{1}{g}\frac{dW}{dt}\cdot h_{fuel}}{W/\left(\frac{L}{D}\right)} \right)\cdot dt = -\eta_{overall}\cdot\frac{h_{fuel}}{g}\cdot\left(\frac{L}{D}\right)\cdot\int\left(\frac{dW}{W}\right)$$

- Integration Gives

$$R = -\eta_{overall}\cdot\frac{h_{fuel}}{g}\cdot\left(\frac{L}{D}\right)\cdot\left[\ln\left(W_{final}\right)-\ln\left(W_{initial}\right)\right] = \eta_{overall}\cdot\frac{h_{fuel}}{g}\cdot\left(\frac{L}{D}\right)\cdot\ln\left(\frac{W_{initial}}{W_{final}}\right)$$

$$\boxed{R = \eta_{overall}\cdot\frac{h_{fuel}}{g}\cdot\left(\frac{L}{D}\right)\cdot\ln\left(\frac{W_{initial}}{W_{final}}\right)}$$

Breguet Aircraft Range Equation (3)

$$\boxed{R = \eta_{overall} \cdot \frac{h_{fuel}}{g} \cdot \left(\frac{L}{D}\right) \cdot \ln\left(\frac{W_{initial}}{W_{final}}\right)}$$

- Result highlights the key role played by the engine overall efficiency in available aircraft range.

- Note that as the aircraft burns fuel it must increase altitude to maintain constant L/D , and the required thrust decreases.

Breguet Aircraft Range Equation (4)

- Compare to “Rocket Equation”

$$\boxed{R = \eta_{overall} \cdot \frac{h_{fuel}}{g} \cdot \left(\frac{L}{D}\right) \cdot \ln\left(\frac{W_{initial}}{W_{final}}\right)}$$

$$R = \eta_{overall} \cdot \frac{h_{fuel}}{g} \cdot \left(\frac{L}{D}\right) \cdot \ln\left(\frac{W_{initial}}{W_{final}}\right) = \frac{F_{thrust} \cdot V_{\infty}}{\dot{m}_{fuel} \cdot h_{fuel}} \cdot \frac{h_{fuel}}{g} \cdot \left(\frac{L}{D}\right) \cdot \ln\left(\frac{W_{initial}}{W_{final}}\right) =$$

$$\frac{F_{thrust}}{\dot{m}_{fuel} \cdot g} \cdot \left(\frac{L}{D} \cdot V_{\infty}\right) \cdot \ln\left(\frac{M_{initial}}{M_{final}}\right) = I_{sp} \cdot \left(\frac{L}{D} \cdot V_{\infty}\right) \cdot \ln\left(\frac{M_{initial}}{M_{final}}\right)$$

$$\boxed{\frac{R \cdot g_o}{V_{\infty}} = \left(\frac{L}{D}\right) \cdot g_0 \cdot I_{sp} \cdot \ln\left(\frac{M_{initial}}{M_{final}}\right)}$$

Breguet Aircraft Range Equation (5)

- Breguet Range Equation, Scaled Range Velocity

$$\boxed{\bar{V} \equiv \frac{R\cdot g_o}{V_\infty} = \left(\frac{L}{D}\right)\cdot g_0 \cdot I_{sp} \cdot \ln\left(\frac{M_{initial}}{M_{final}}\right)}$$

- Rocket Equation, Available Propulsion ΔV

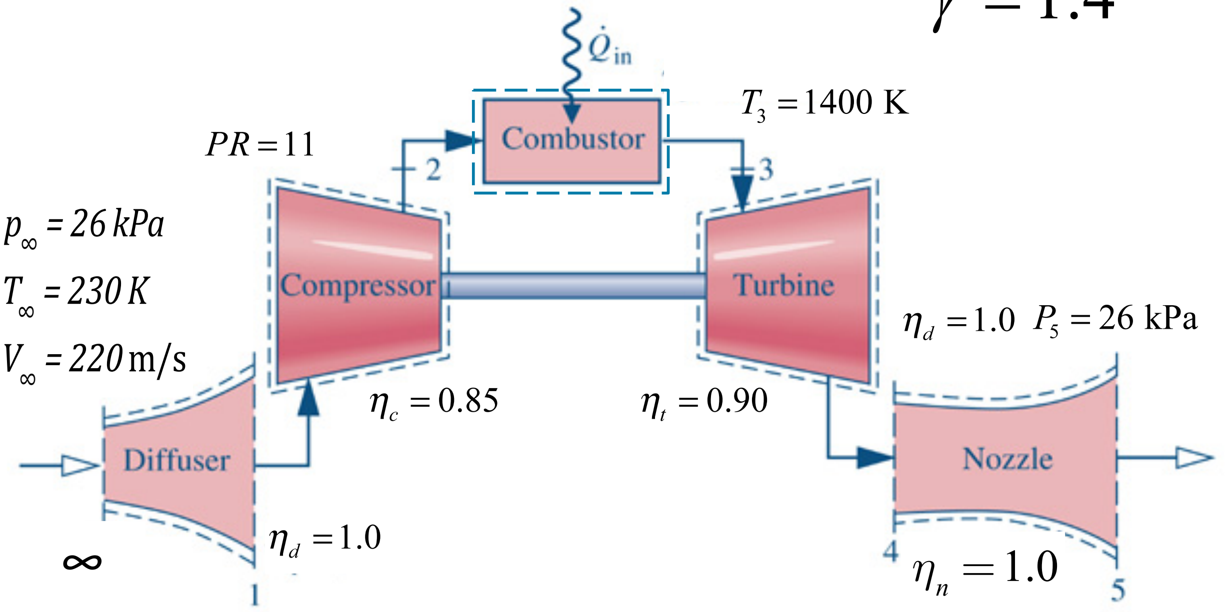
$$\boxed{\Delta V = g_0 \cdot I_{sp} \cdot \ln\left(\frac{M_{initial}}{M_{final}}\right)}$$

Same Basic Physics
Same Basic Solution!

Turbojet Engine, Example Problem

Given: A turbojet engine operating as shown below

$\gamma = 1.4$



- Assume Isentropic Diffuser, Nozzle
- Compressible, Combustor Turbine NOT! Isentropic
- Assume Constant C_p, C_v across cycle
- Air massflow >> fuel massflow

Calculate :

(a) The properties at all the state points in the cycle
(b) The heat transfer rate in the combustion chamber (*kW*)
(c) The velocity at the nozzle exit (*m/s*)
(d) The propulsive force (*lbf*)
(e) The propulsive power developed (*kW*)
(f) Propulsive Efficiency
(g) Thermal Efficiency
(h) Total Efficiency
(i) Draw *T-s* diagram
(j) Draw p-v diagram

Section 4.1 Homework (2)

Given: A turbojet engine operating as shown below

Incoming Air to Turbojet (@ to station *3*)

- Molecular weight = 28.96443 *kg/kg-mole*
- γ = 1.40
- R_g = 287.058 *J/kg-K*
- T_∞ = 230 K
- p_∞ = 26 kPa
- V_∞ = 220 *m/sec*
- Universal Gas Constant: R_u = 8314.4612 *J/kg-K*

Calorically Perfect Gas

$$\gamma = \frac{c_p}{c_v}$$

$$R_g = c_p - c_v$$

$$c_p = \frac{\gamma}{\gamma - 1} \cdot R_g$$

$$c_v = \frac{1}{\gamma - 1} \cdot R_g$$

For …Isentropic Conditions →

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma - 1}{\gamma}}$$

Ideal Gas

$$p = \rho \cdot R_g \cdot T$$

Section 4.1 Homework (3)

Given: Across Components

Isentropic Diffuser

Assume $D_{inlet} = 60.96$ cm (24 in.)
$D_{outlet} = 1.5 \text{ x } D_{inlet}$

$$h_{0_1} \equiv h_1 + \frac{V_1^2}{2} = h_\infty + \frac{V_\infty^2}{2}$$

$$h_{0_1} \approx C_{p1} \cdot T_{0_1}$$

Compressor

- ASSUME COMPRESSOR EXIT MACH ~ 0

$$\eta c = \frac{\text{isentropic power input}}{\text{actual power input}}$$

$$\eta_c = \frac{h_{0_{2|s=0}} - h_{0_1}}{h_{0_2} - h_{0_1}} \rightarrow$$

$$h_{0_1} = C_{p_{air}} \cdot T_{0_1}$$

$$h_{0_2} = C_{p_{air}} \cdot T_{0_1\,actual}$$

$$h_{0_{2|s=0}} = C_{p_{air}} \cdot T_{0_2\,ideal}$$

$$\frac{\dot{w}_c}{\dot{m}} = h_{0_2} - h_{0_1}$$

$$\frac{p_2}{p_1} \approx \frac{P_{0_2}}{P_{0_1}} = 11$$

$$s_2 - s_1 = C_p \ln\left(\frac{T_{2_{actual}}}{T_1}\right) - R_g \ln\left(\frac{p_2}{p_1}\right)$$

$$\frac{h_{0_2|s=0}}{h_{0_1}} = \frac{C_p \cdot T_{0_2|s=0}}{C_p \cdot T_{0_1}} \approx \frac{T_{0_2|s=0}}{T_{0_1}} = \left(\frac{P_{0_2}}{P_{0_1}}\right)^{\frac{\gamma-1}{\gamma}}$$

Section 4.1 Homework (4)

Given: Across Components

Combustor

$$contant\ pressure, \dot{m}_{air} >> \dot{m}_{air}$$

$$C_p, \gamma \sim const,\quad \mathrm{T}_3 = T_{flame} = 1400K$$

$$s_3 - s_2 = C_p \ln\left(\frac{T_{flame}}{T_{2_{actual}}}\right)$$

Assume combustor Inlet/ outlet Mach numbers are essentially zero

$$\frac{p_3}{p_2} \approx \frac{P_{0_3}}{P_{0_2}} = 1$$

Turbine

$$\eta_t = \frac{\text{actual power output}}{\text{isentropic power poutput}}$$

$$\eta_t = \frac{h_{0_3} - h_{0_4}}{h_{03} - h_{0_{4_{s=0}}}} \rightarrow \begin{array}{l} h_{0_3} = C_{p_{air}} \cdot T_{0_3} \\ h_{0_4} = C_{p_{air}} \cdot T_{0_{4actual}} \\ h_{0_{4_{s=0}}} = C_{p_{air}} \cdot T_{0_{4ideal}} \end{array}$$

$$Assume \rightarrow \frac{\dot{w}_t}{\dot{m}} = \frac{\dot{w}_c}{\dot{m}} = h_{0_3} - h_{0_4}$$

Actual !

$$\frac{P_{0_4}}{P_{0_3}} = \left(\frac{T_{0_{4|s=0}}}{T_{0_3}}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{h_{0_3} - \frac{1}{\eta_t} \cdot \frac{\dot{w}}{\dot{m}}}{h_{0_3}}\right)^{\frac{\gamma}{\gamma-1}} = \left(1 - \frac{1}{\eta_t \cdot h_{0_3}} \cdot \frac{\dot{w}}{\dot{m}}\right)^{\frac{\gamma}{\gamma-1}}$$

$$s_4 - s_3 = C_p \ln\left(\frac{T_{0_{4actual}}}{T_{0_3}}\right) - R_g \ln\left(\frac{P_{0_4}}{P_{0_3}}\right)$$

Section 4.1 Homework (5)

Given: Across Components

Nozzle Assumed Optimized Nozzle → $p_{exit} = p_\infty$ $T_{exit} = T_4 \cdot \left(\frac{P_4}{p_{exit}} \right)^{\frac{\gamma-1}{\gamma}}$

$$\dot{m}\left(h_4 + \frac{V_4^2}{2} \right) = \dot{m}\left(h_{exit} + \frac{V_{exit}^2}{2} \right) \rightarrow V_4 \approx 0 \rightarrow V_{exit} = \sqrt{2\left(h_4 - h_{exit}\right)}$$

$$F = \dot{m}\left(V_{exit} - V_\infty\right)$$

$$\dot{W}_p = F \cdot V_\infty$$

$$\eta_{propulsive} = \frac{\dot{W}_p}{\dot{m}_{air}\left(K.E._{exit} - K.E._\infty\right)}$$

$$\eta_{thermal} = \frac{\left(K.E._{exit} - K.E._\infty\right)}{\dot{m}_{fuel} \cdot h_{fuel}}$$

$$\eta_{total} = \eta_{prop} \cdot \eta_{thermal} = \frac{F \cdot V_\infty}{\dot{m}_{fuel} \cdot h_{fuel}}$$

Section 4.1 Homework (8)

Summary

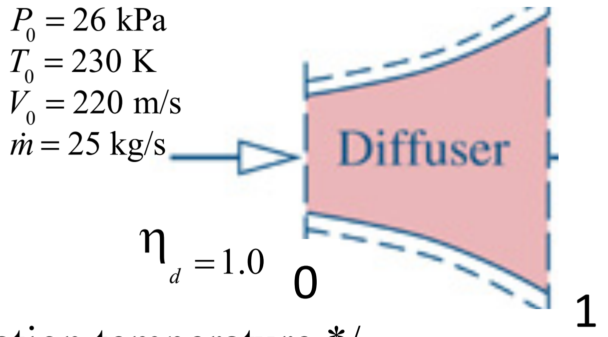
$$\eta_{propulsive} = \frac{\dot{W}_p}{\left(K.E._{exit} - K.E._{\infty}\right)} = \frac{2\left(\left(\frac{V_{exit}}{V_{\infty}}\right) - \left(\frac{V_{exit}}{V_{\infty}}\right)^2\right)}{\left(1 - \left(\frac{V_{\infty}}{V_{exit}}\right)^2\right)}$$

$$\eta_{thermal} = \frac{\left(K.E._{exit} - K.E._{\infty}\right)}{\dot{m}_{fuel} \cdot h_{fuel}} = \frac{\left(\frac{1}{2} V^2{}_{exit}\right) \cdot \left(1 - \left(\frac{V_{\infty}}{V_{exit}}\right)^2\right)}{\left(h_{0_3} - h_{02}\right)}$$

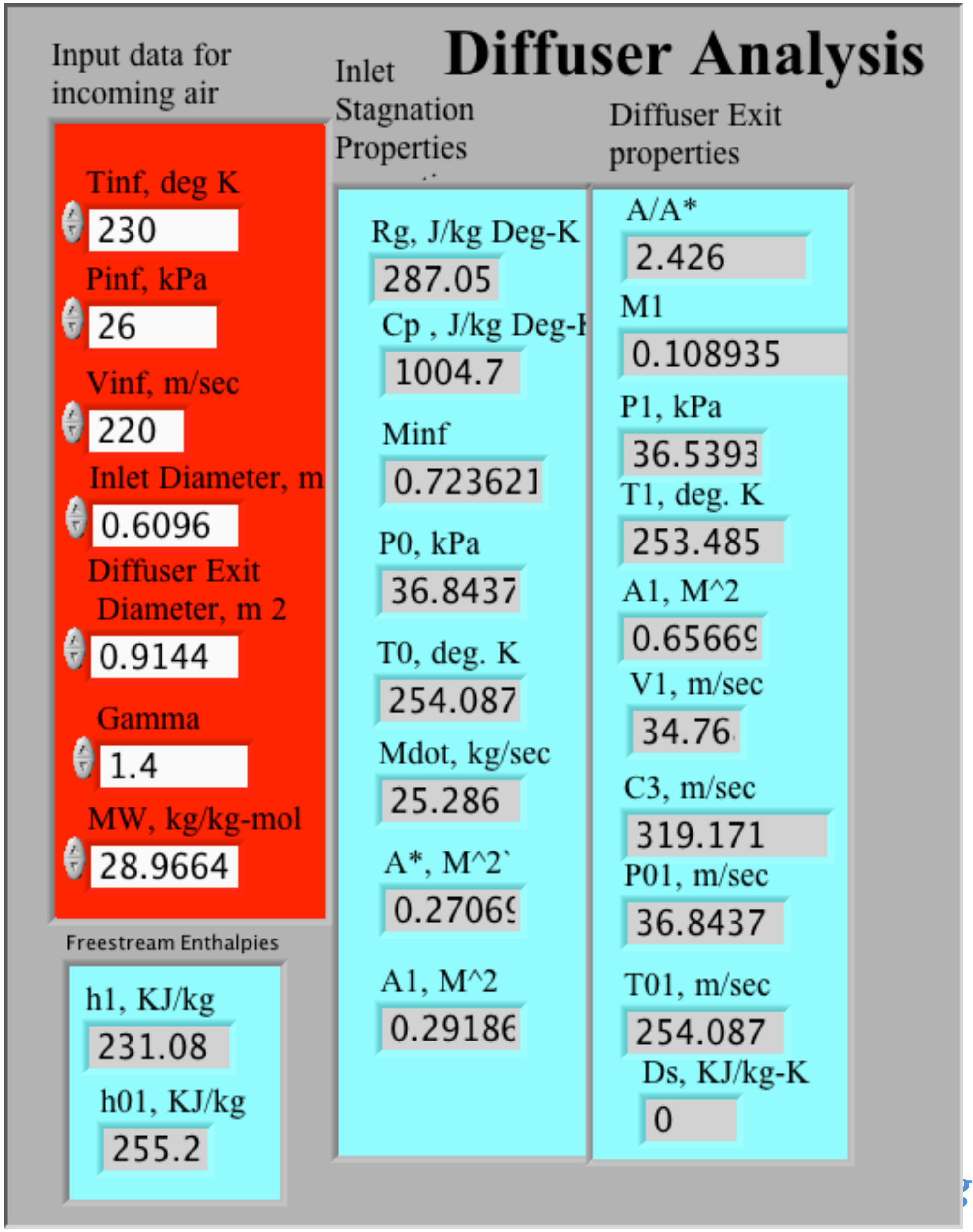
$$K.E._{\substack{out \\ excess}} = K.E._{net} - P_{prop} = \frac{1}{2} \dot{m} \cdot V^2{}_{exit} \cdot \left(1 - \left(\frac{V_{exit}}{V_{\infty}}\right)\right)$$

Problem Solution

Diffuser Analysis



$P_0 = 26$ kPa
$T_0 = 230$ K
$V_0 = 220$ m/s
$\dot{m} = 25$ kg/s
$\eta_d = 1.0$



Diffuser Analysis

Input data for incoming air

- Tinf, deg K: 230
- Pinf, kPa: 26
- Vinf, m/sec: 220
- Inlet Diameter, m: 0.6096
- Diffuser Exit Diameter, m 2: 0.9144
- Gamma: 1.4
- MW, kg/kg-mol: 28.9664

Freestream Enthalpies

- h1, KJ/kg: 231.08
- h01, KJ/kg: 255.2

Inlet Stagnation Properties

- Rg, J/kg Deg-K: 287.05
- Cp , J/kg Deg-: 1004.7
- Minf: 0.723621
- P0, kPa: 36.8437
- T0, deg. K: 254.087
- Mdot, kg/sec: 25.286
- A*, M^2`: 0.27069
- A1, M^2: 0.29186

Diffuser Exit properties

- A/A*: 2.426
- M1: 0.108935
- P1, kPa: 36.5393
- T1, deg. K: 253.485
- A1, M^2: 0.65669
- V1, m/sec: 34.76
- C3, m/sec: 319.171
- P01, m/sec: 36.8437
- T01, m/sec: 254.087
- Ds, KJ/kg-K: 0

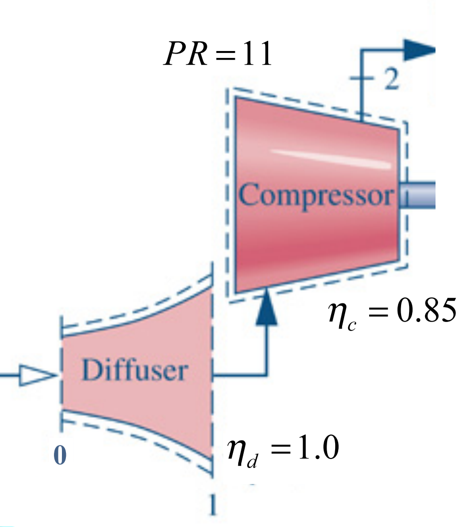
```
/* Calculate stagnation temperature */
T01=T1 + (V1**2)/(2*Cp1);

/* Calculate Mach number */
term2 = sqrt(gamma*Rg1*T1);
Minf = V1/sqrt(gamma*Rg1*T1);

/* Calculate stagnation pressure */
expn = gamma/(gamma-1);
P01 = P1*( 1 + ( ( gamma-1)/2
)*(Minf**2))**(expn);

/* calculate inlet massflow */
A1 = (pi/4)*(D1**2);
mdot = ( (P1*1000)/(Rg1*T1) )*V1*A1;

/* calculate Inlet specific enthalpies *
/h1 = Cp1*T1/1000;
h01 = Cp1*T01/1000;
```

Compressor Analysis

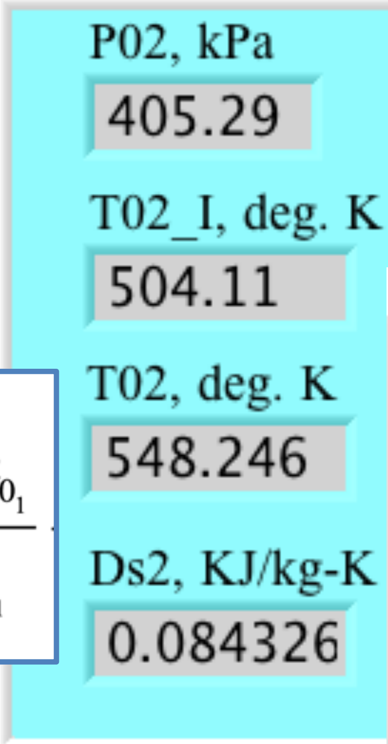
```
/* calculate exit pressure */
p2 = P01*Pr;

/* Ideal (ISENTROPIC) Stagnation Temperature */
expn = (gamma-1)/gamma;
T02_i=  T01*(Pr**expn);

/* Ideal DEMAND stagnation specific enthalpy */
h02_i = Cp1*T02_i/1000;

/* true DEMAND stagnation specific enthalpy */
h02 =h01+(h02_i - h01)/eta;

/* True Stagnation Temperature */
T02 = 1000*h02/Cp1;

/* change in entropy */
DS2 = (Cp1*ln(T02/T01) - Rg*ln(Pr) )/1000;

/* actual compressor work */
Wdot = h02-h01;
```

Assume compressor outlet Mach number is essentially zero

$$\eta_c = \frac{h_{0_{2}|_{s=0}} - h_{0_1}}{h_{0_2} - h_{0_1}}$$



Compressor Exit


P02, kPa
405.29

T02_I, deg. K 2
504.11

T02, deg. K
548.246

Ds2, KJ/kg-K
0.084326


Outputput Enthalpies 2

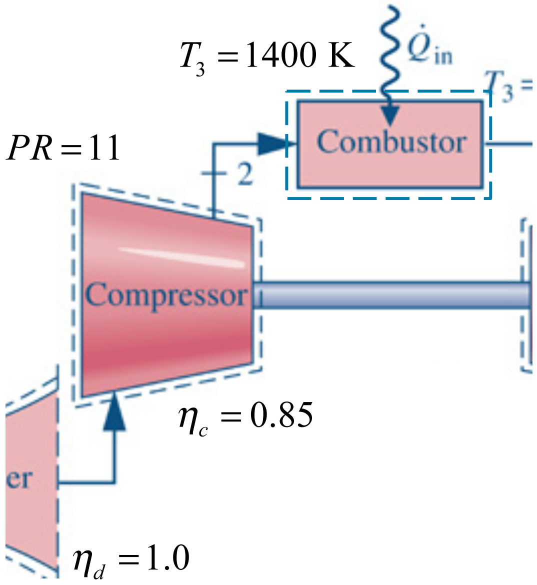
h02_i, KJ/kg
506.4

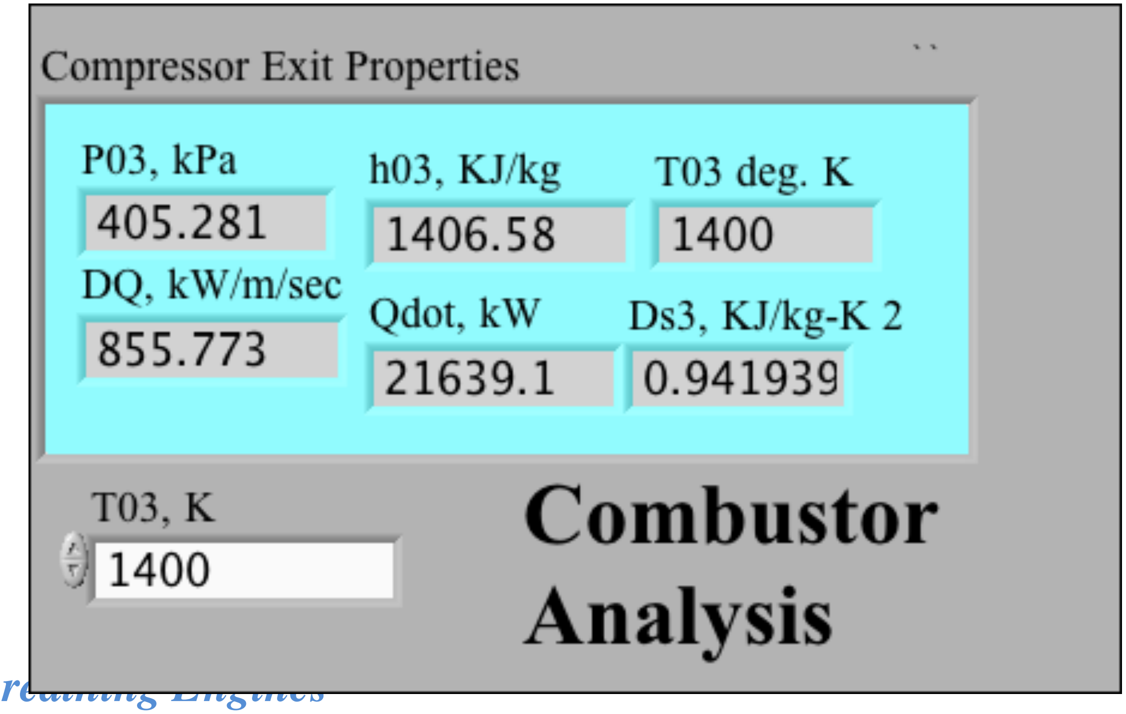
h02, KJ/kg
550.8

Compressor DEMAND specific Power kW/kg/sec
295.523

Combustor Analysis

```
/* calculate outlet enthalpy */
h03 = Cp*T03/1000;

/* calculate heat input
per unit massflow */
DQ = (h03-h02);

/* calculate total heat  input */
qdot = DQ*mdot;

/* calculate change in enthalpy */
DS =
Cp*ln(T03/T02) /1000;
```





Compressor Exit Properties

P03, kPa: 405.281
h03, KJ/kg: 1406.58
T03 deg. K: 1400
DQ, kW/m/sec: 855.773
Qdot, kW: 21639.1
Ds3, KJ/kg-K 2: 0.941939

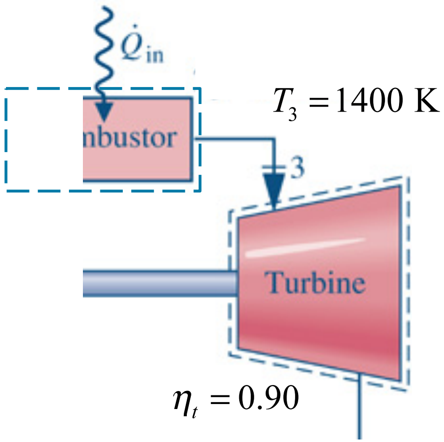
T03, K: 1400

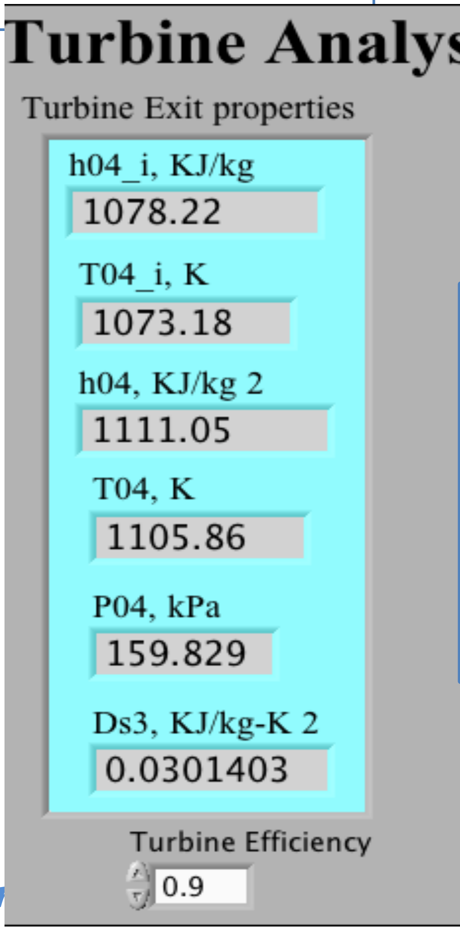
Combustor Analysis

Turbine Analysis

```
/* calculate idealized REQUIRED
Output enthalpy */
h04_i= h03-(Wdot)/eta;
T04_i = 1000*h04_i/Cp;

/* calculate actual REQUIRED output
enthalpy from turbine */
h04= h03-Wdot;

/* calculate output stagnation temperature */
T04=T03+(h04-h03)/(Cp/1000);

expn = gamma/(gamma-1);
P04=P03*( ( h04_i/h03)**expn);

/* change in entropy */
DS = (Cp*ln(h04/h03) - Rg*ln( P04/P03  ) )/1000;
```

$$\eta_t = \frac{h_{0_3} - h_{0_4}}{h_{03} - h_{0_{4_{s=0}}}}$$





Turbine Analysis

Turbine Exit properties

h04_i, KJ/kg: 1078.22

T04_i, K: 1073.18

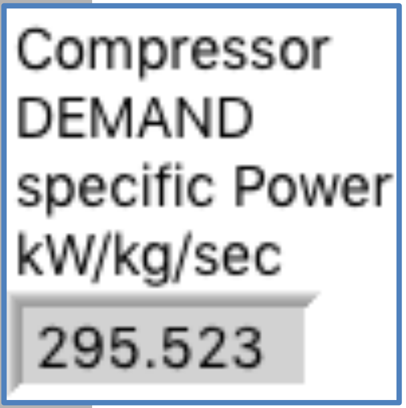
h04, KJ/kg 2: 1111.05

T04, K: 1105.86

P04, kPa: 159.829

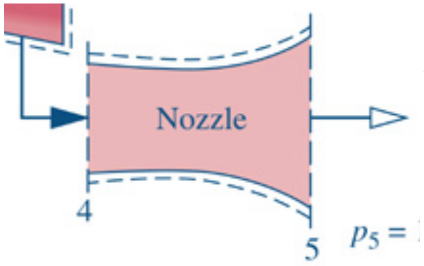
Ds3, KJ/kg-K 2: 0.0301403

Turbine Efficiency: 0.9



Compressor DEMAND specific Power kW/kg/sec: 295.523

Nozzle Analysis



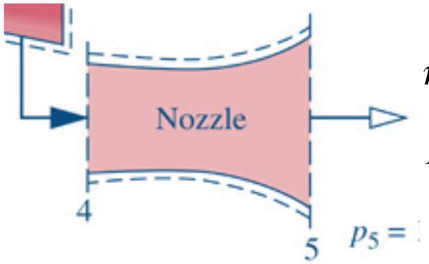
$\eta_n = 1.0$

$P_5 = 26$ kPa

```
/* calculate exit temperature */
expn = (gamma-1)/gamma;
Pratio = P0/pinf;
Texit = T4*( (1/Pratio) **expn );
hexit = Cp*Texit/1000.;

/* calculate exit velocity */
Vexit = sqrt( 2*( h04*1000- Cp*Texit ) );
h0exit = hexit+0.5*(Vexit**2);

/* calculate exit sonic velocity.Mach */
Cexit = sqrt(gamma*Rg*Texit);
Mexit1 = Vexit/Cexit;

/* calculate output mach */
expn = (gamma-1)/gamma;
Pratio = P0/pinf;
Mach =sqrt( ( Pratio**expn - 1)*(2/(gamma-1) ) );

/* Calculate Thrust */
Thrust = mdot*(Vexit-Vinf)/1000;

/* Propulsive Power */
PF = Thrust*Vinf;
```

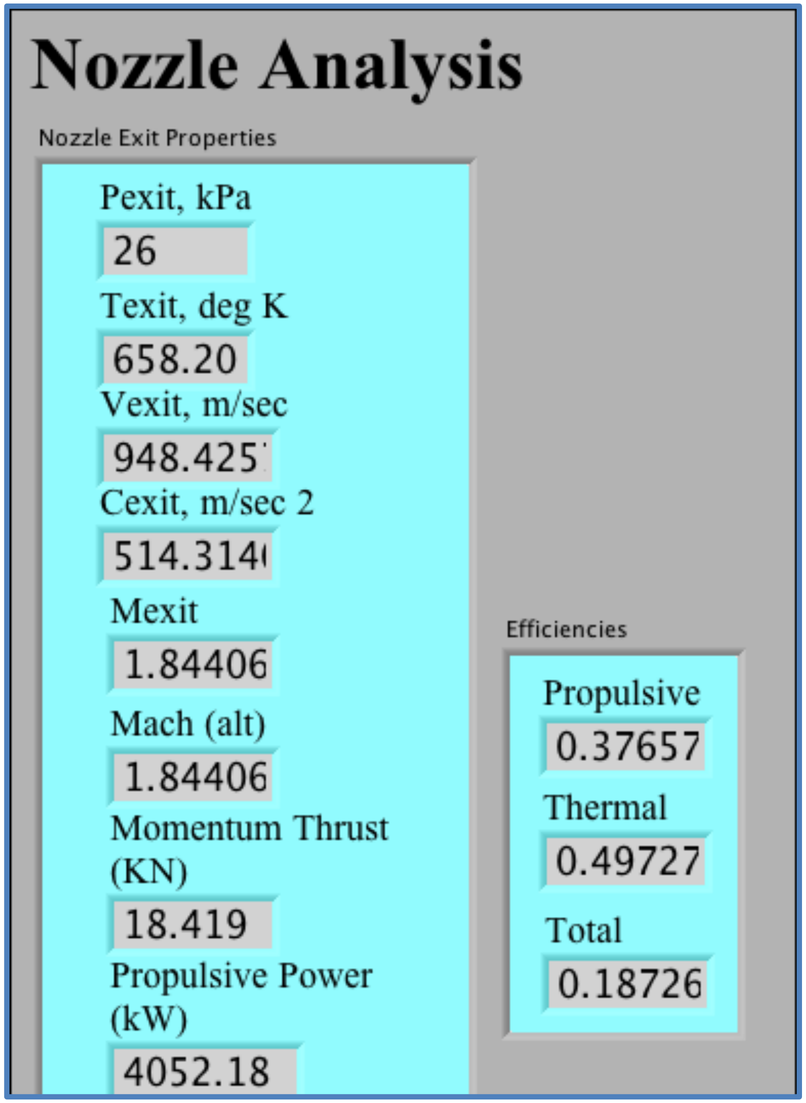
```
/* Net kinetic energy rate leaving engine */
DKE = 0.001*mdot*( Vexit**2 - Vinf**2)/2.0;

/* propulsive efficiency */
Peff = PF/DKE;

/* shed excess heat */
Qdotout=mdot*( Cp*Texit  -1000*h1)/1000;

/* shed excess kinetic energy */
ShedKE = DKE-PF;

/* Thermal efficiency */
Teff = 0.0005*(Vexit**2)*
(1- ( Vinf/Vexit)**2)/(h03-h02);

/* Total imported energy */
TE = mdot*(h03-h02);
```

Nozzle Analysis (2)



$\eta_n = 1.0$

$P_5 = 26$ kPa



Nozzle Analysis

Nozzle Exit Properties

Pexit, kPa
26

Texit, deg K
658.20

Vexit, m/sec
948.425

Cexit, m/sec 2
514.314

Mexit
1.84406

Mach (alt)
1.84406

Momentum Thrust (KN)
18.419

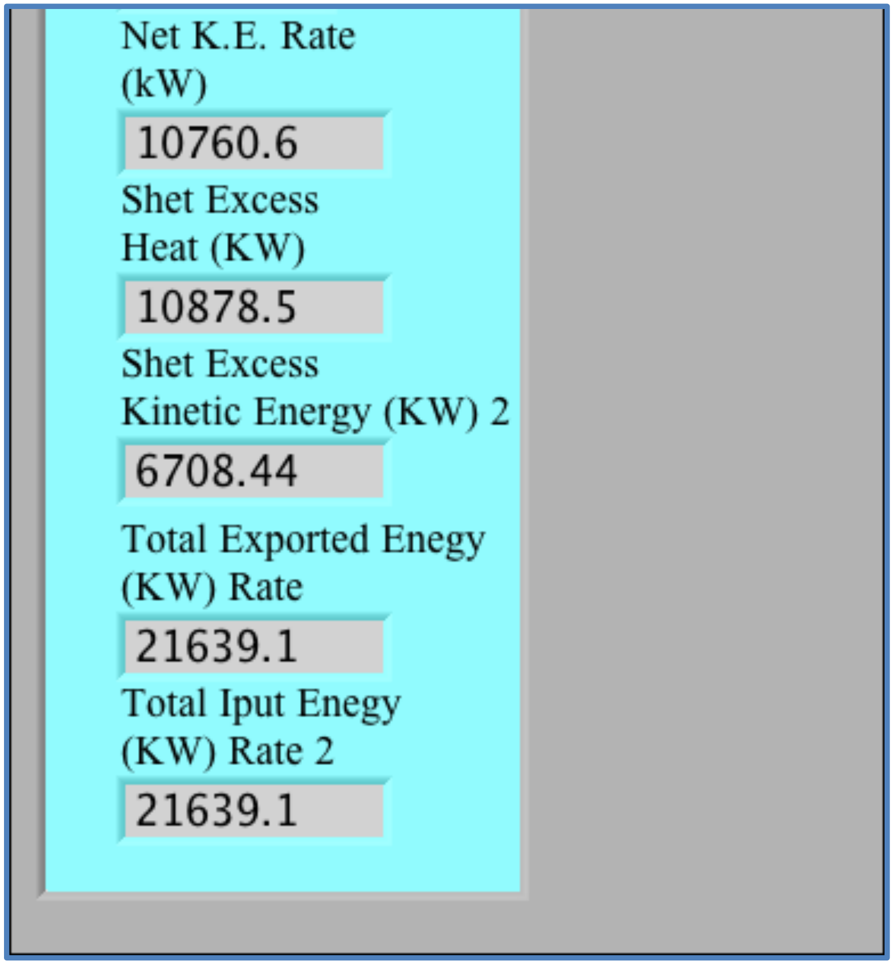
Propulsive Power (kW)
4052.18

Efficiencies

Propulsive
0.37657

Thermal
0.49727

Total
0.18726



Net K.E. Rate (kW)
10760.6

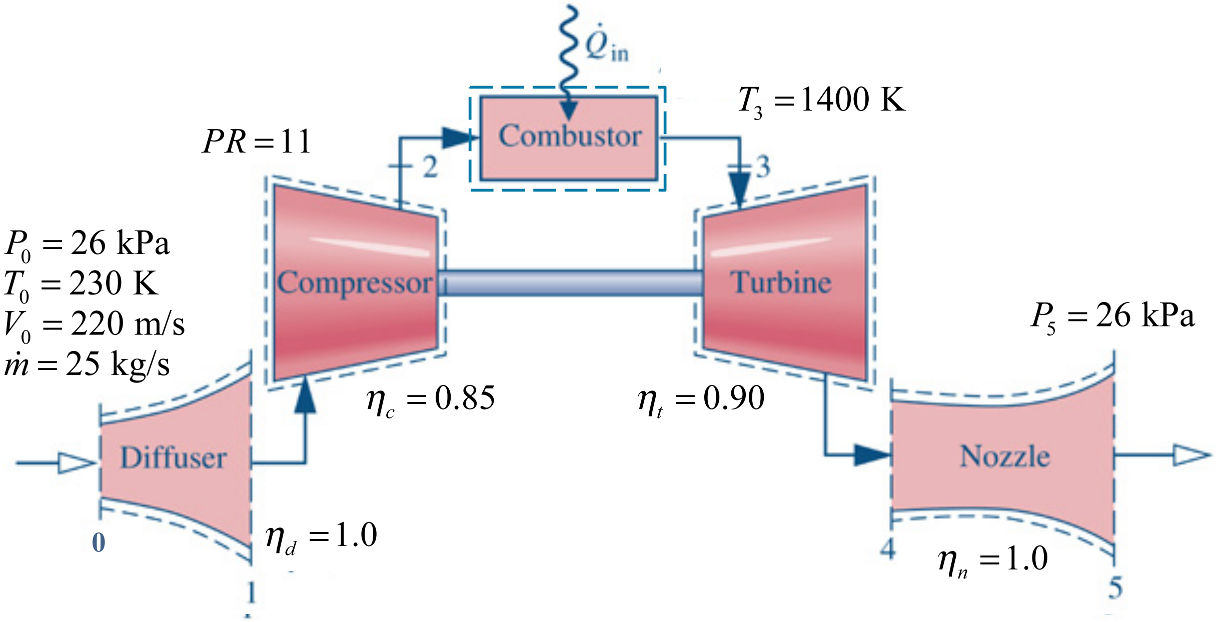
Shet Excess Heat (KW)
10878.5

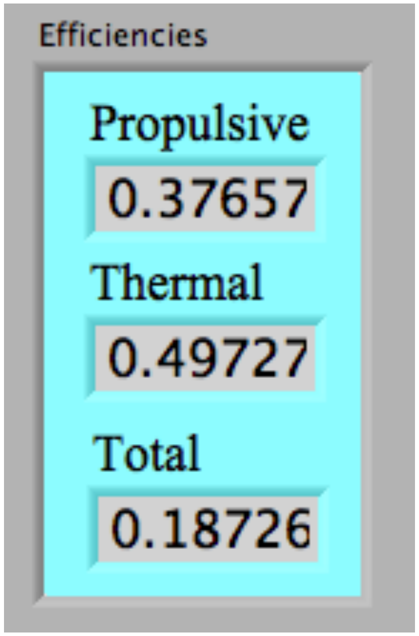
Shet Excess Kinetic Energy (KW) 2
6708.44

Total Exported Enegy (KW) Rate
21639.1

Total Iput Enegy (KW) Rate 2
21639.1

End-to-End State Table



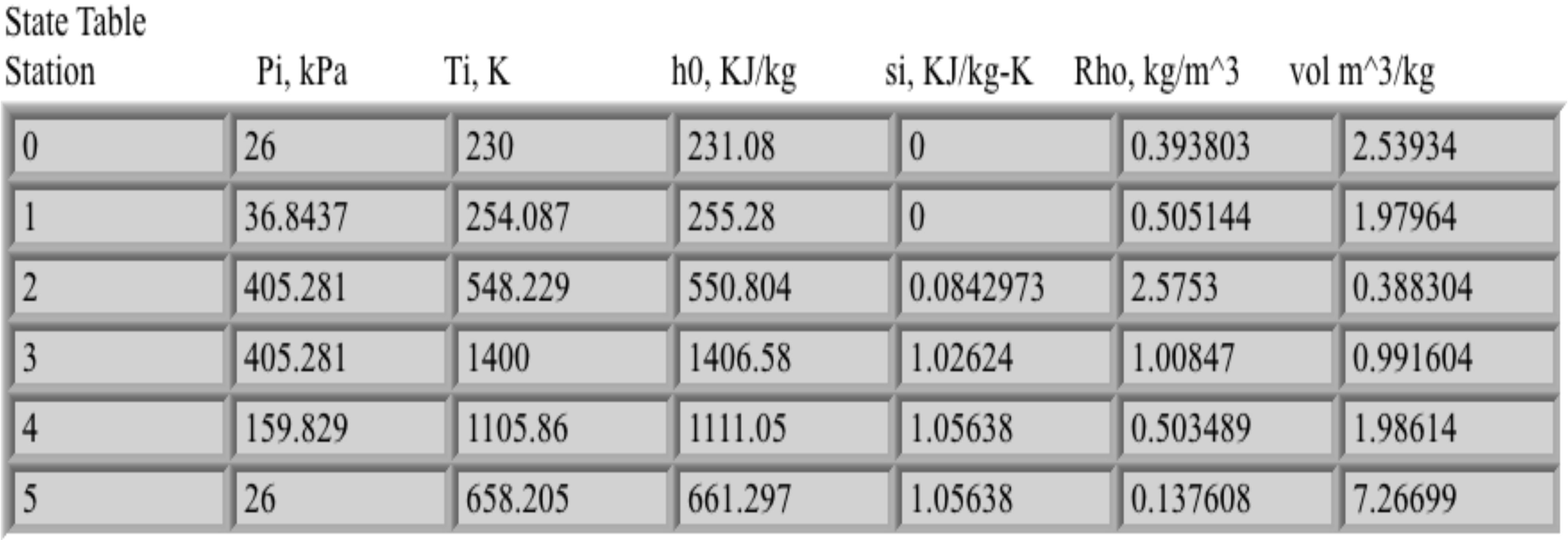


Efficiencies

Propulsive
0.37657

Thermal
0.49727

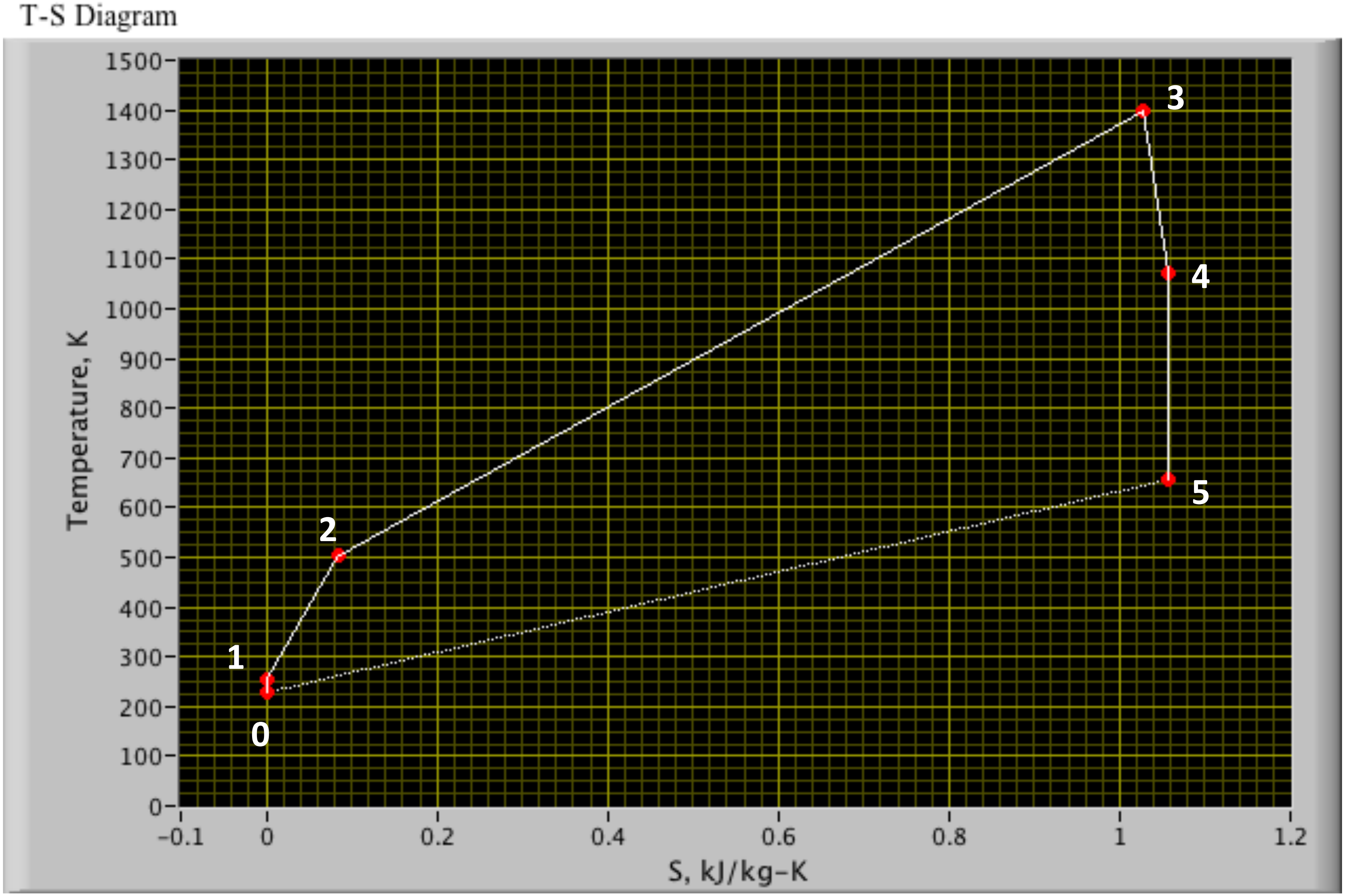
Total
0.18726



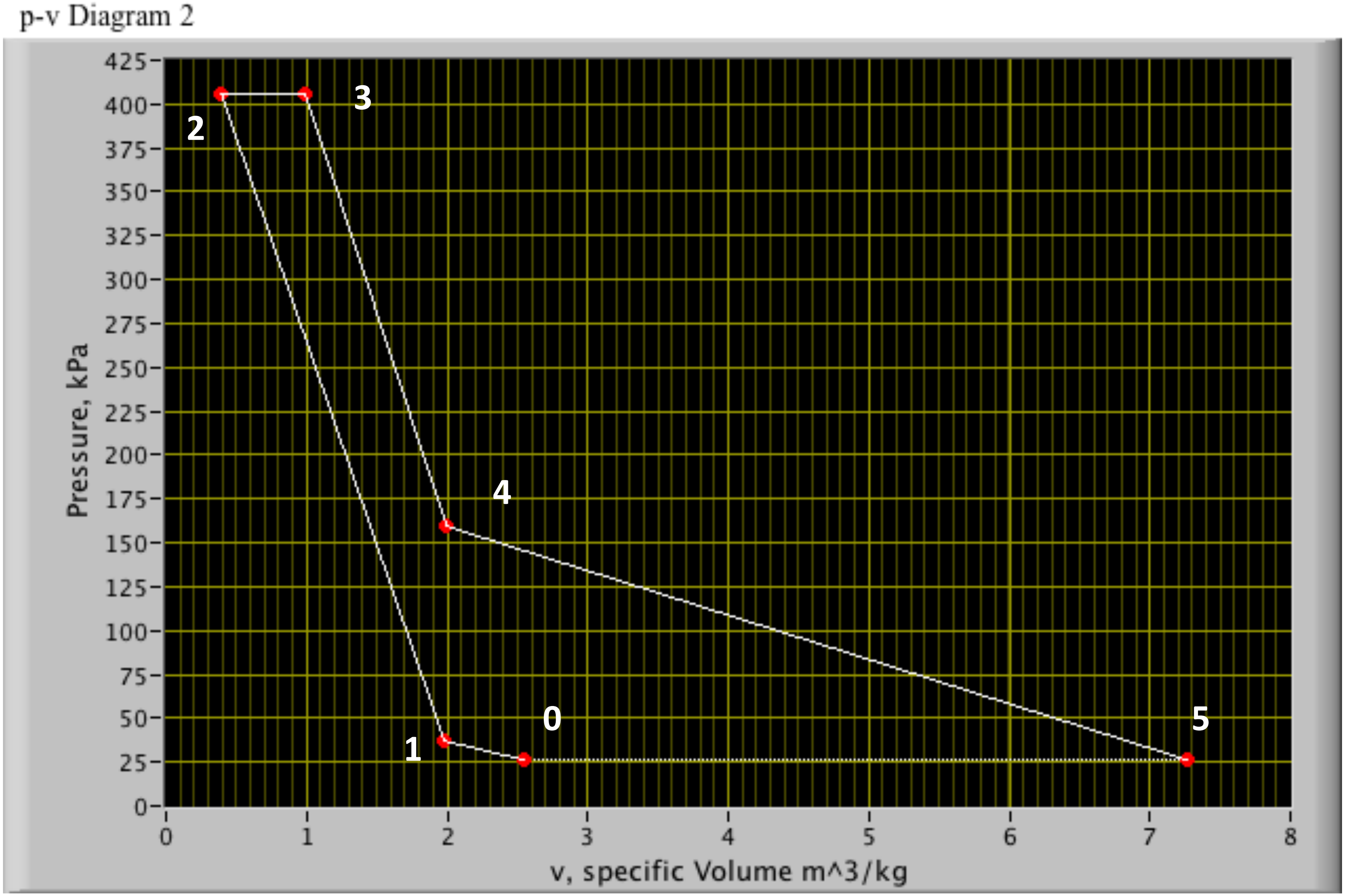
State Table

Station	Pi, kPa	Ti, K	h0, KJ/kg	si, KJ/kg-K	Rho, kg/m^3	vol m^3/kg
0	26	230	231.08	0	0.393803	2.53934
1	36.8437	254.087	255.28	0	0.505144	1.97964
2	405.281	548.229	550.804	0.0842973	2.5753	0.388304
3	405.281	1400	1406.58	1.02624	1.00847	0.991604
4	159.829	1105.86	1111.05	1.05638	0.503489	1.98614
5	26	658.205	661.297	1.05638	0.137608	7.26699

T-S Diagram


T-S Diagram
Temperature, K
S, kJ/kg-K
1500
1400
1300
1200
1100
1000
900
800
700
600
500
400
300
200
100
0
-0.1
0
0.2
0.4
0.6
0.8
1
1.2
0
1
2
3
4
5

P-v Diagram


p-v Diagram 2
Pressure, kPa
425
400
375
350
325
300
275
250
225
200
175
150
125
100
75
50
25
0
v, specific Volume m^3/kg
0
1
2
3
4
5
6
7
8
2
3
4
1
0
5

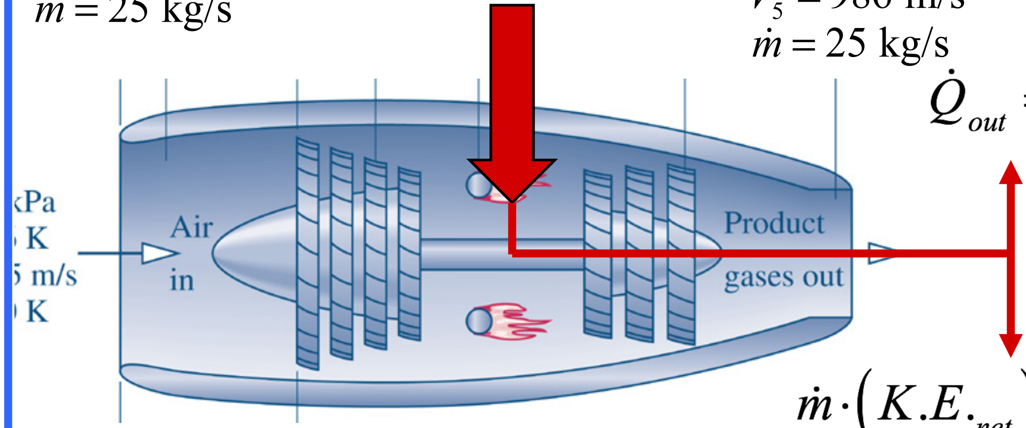
Energy Decomposition

How is the energy input to this engine distributed?

$P_0 = 26\ \text{kPa}$
$T_0 = 230\ \text{K}$
$V_0 = 220\ \text{m/s}$
$\dot{m} = 25\ \text{kg/s}$

$\dot{Q}_{in} = 21{,}639.1\ \text{kW}$

$P_5 = 26\ \text{kPa}$
$T_5 = 719.5\ \text{K}$
$V_5 = 986\ \text{m/s}$
$\dot{m} = 25\ \text{kg/s}$



excess thermal energy transfer

$$\dot{Q}_{out} = \dot{m}\cdot\left(h_{exit} - h_{\infty}\right) = 10{,}878.5\ \text{kW} \quad (50.3\%)$$

kinetic energy production rate

$$\dot{m}\cdot\left(K.E._{net}\right) = \frac{\dot{m}}{2}\left(V_{exit}^2 - V_{\infty}^2\right) = 10{,}760.6\ \text{kW} \quad (49.7\%)$$

$$\dot{m}\cdot\left(K.E._{excess}\right) = 6708.4\ \text{kW} \quad (62.3\%)$$

$$\dot{W}_{prop} = 4{,}052.2\ \text{kW} \quad (37.7\%)$$

Excess Enthalpy Transfer Rate	*Thrust Power Output*	*Excess K.E. Lost*	*Total Heat Input*
10878.5 +	4052.2 +	6708.4	= 21639.1 *KW*

Questions??