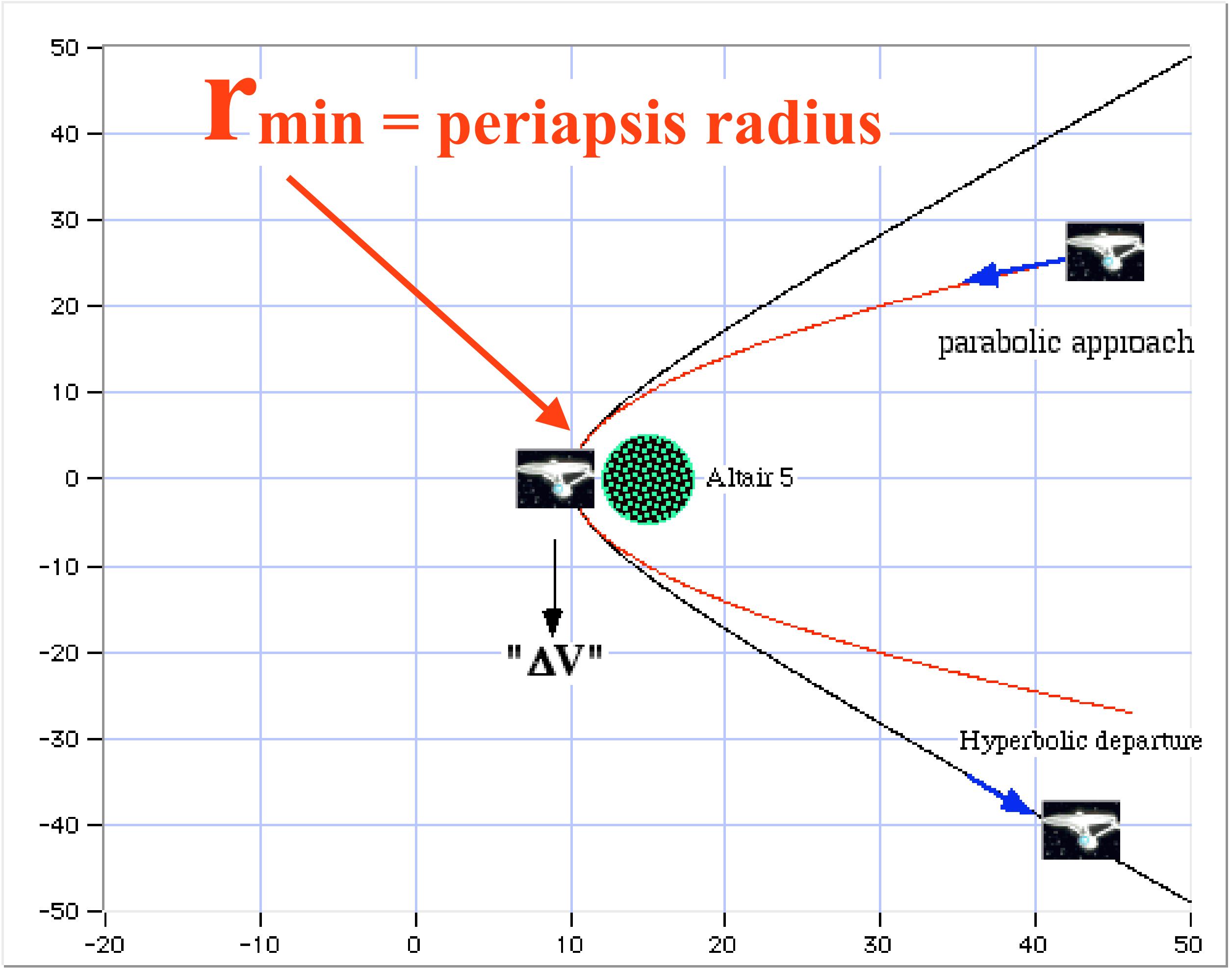
Homework *4* Part a

Parabolic and Hyperbolic Trajectories



Homework 4a

Parabolic and Hyperbolic Trajectories (cont'd)

- *United Federation of Planets* starship *Excelsior* approaches *Klingon* outpost *Altair 5* on a covert retaliatory bombing mission

- A cloaking device uses enormous energy & *Warp drive* is non-operational with the cloak engaged

- All maneuvering must be done on *impulse power* alone

- The *Excelsior* uses a gravity assisted *parabolic* approach trajectory to *Altair 5* in order to save on waning impulse power and insure a stealthy approach

Homework 4a

Parabolic and Hyperbolic Trajectories (cont'd)

- After dropping photo-torpedos, Captain Checkov wants to get out the *sphere of influence* (SOI) of*Altair 5* as fast as possible without being spotted

- The *Excelsio*r has enough impulse power left for *one big burn* before, having to recharge the *dilithium crystals*

- The best way to "get out of town fast" is to fire impulse engines at closest approach to Altair 5 -- taking advantage of the gravity assist to give the highest approach speed without using impulse power and then use impulse power to depart on a hyperbolic trajectory at angle of 45 degrees

- What is the "*Delta-V*" required to depart on a *Hyperbolic* trajectory with an asymptotic departure angle of 45 degrees

Homework: 4a

Parabolic and Hyperbolic Trajectories (cont'd)

- **Hint 1: For a Parabolic trajectory**

r is measured from the parabolic *focus* to the location of the *Excelsior*

- **Hint 2: For a Hyperbolic trajectory**

r is measured from the *right (perifocus) focus* to the location of the *Excelsior*

r_{min} = periapsis radius

Homework: 4a

Parabolic and Hyperbolic Trajectories (concluded)

- **Hint 3: For a Parabolic to Hyperbolic trajectory transfer**

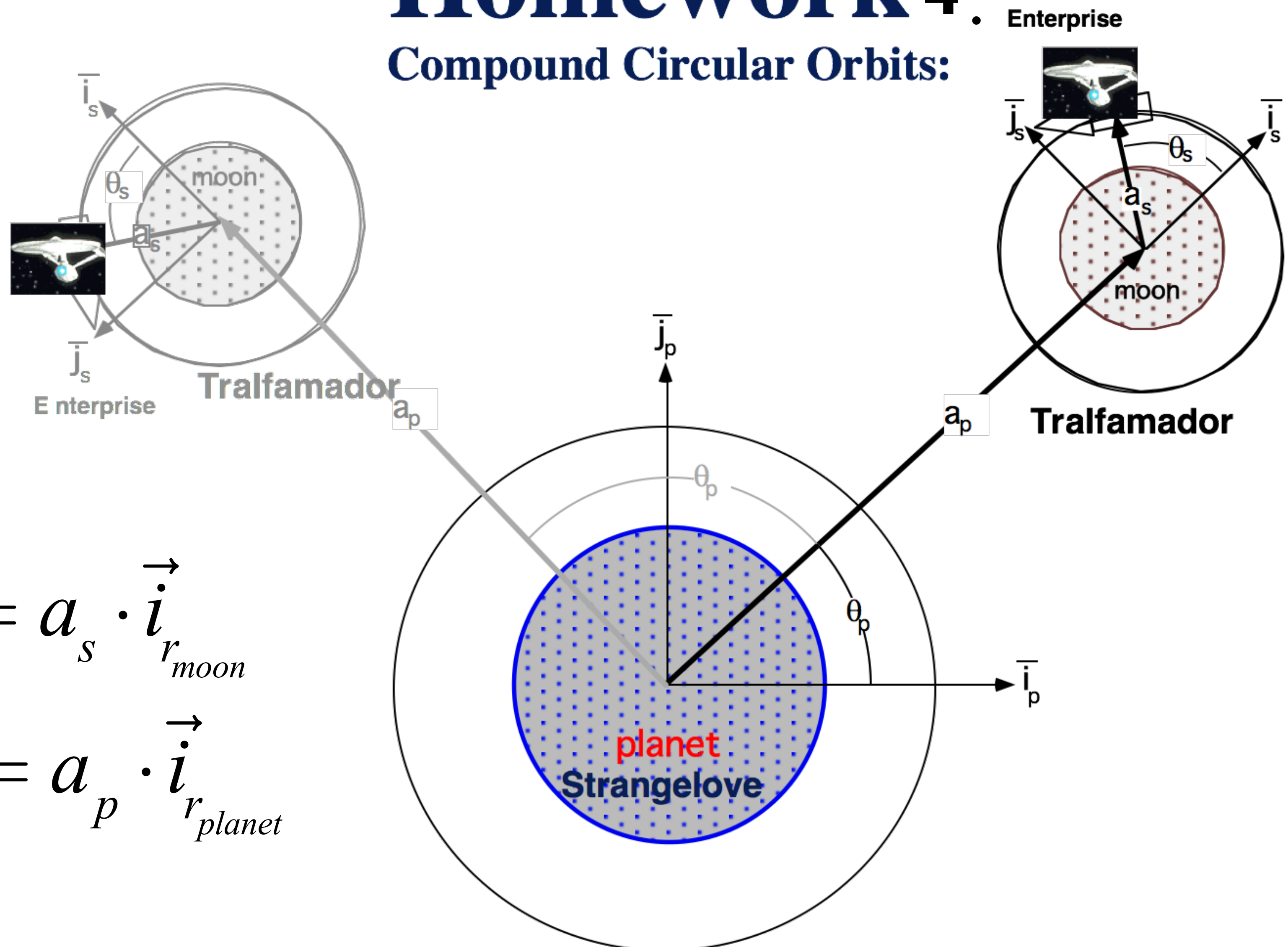
$$"\Delta V" = V_h - V_p = V_p \left[\frac{V_h}{V_p} - 1 \right]$$

- **Hint 4: At closest apprach, the distance from the *parabolic focus* to the *Excelsior* must equal the distance from the *Hyperbolic right focus* to the *Excelsior***

- **Your answer should be expressed in terms μ and r_{min} (closest approach distance)**

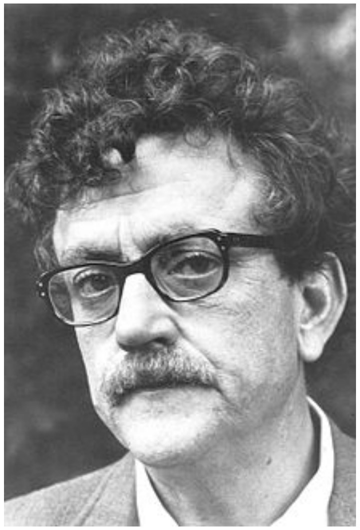
Homework 4: Part b

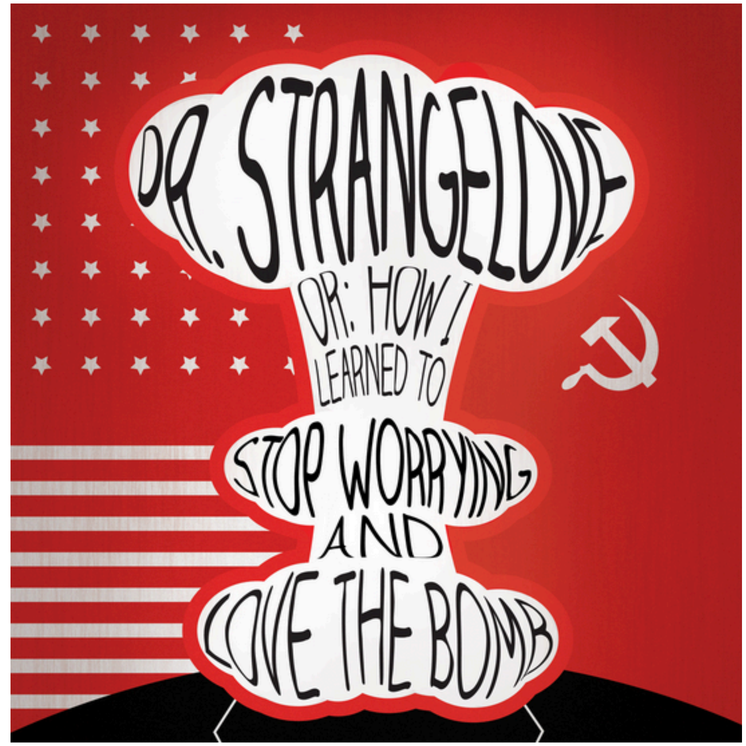
Compound Circular Orbits:



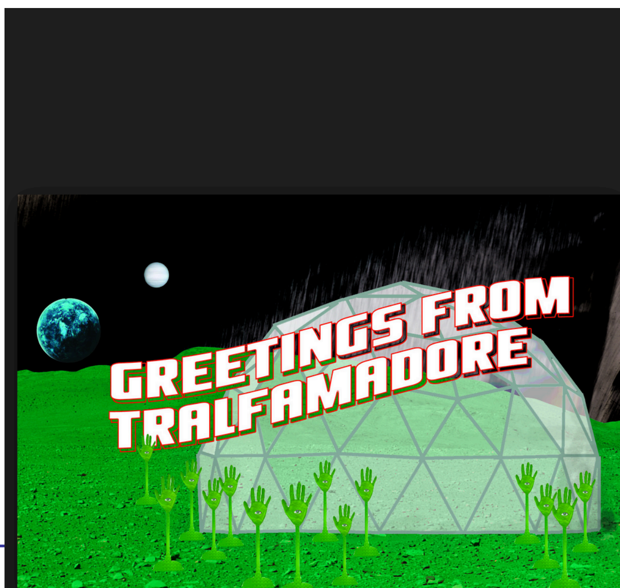
$$\vec{R}_s = a_s \cdot \vec{i}_{r_{moon}}$$

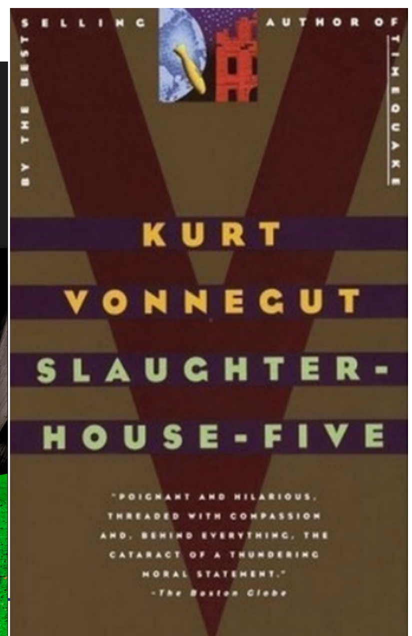
$$\vec{R}_p = a_p \cdot \vec{i}_{r_{planet}}$$





Kurt Vonnegut





Homework: Compound Orbits 4b

(cont'd)

- Starship *Enterprise* orbits alien moon *Tralfamador* in a circular orbit of radius a_s
- Moon orbits alien planet *Strangelove* in circular orbit with radius a_p
- Alien GPS system orbiting moon gives position relative to *Tralfamadorian* -fixed coordinate system.
- Due to gravitational damping *Tralfamador*, always keeps the same face directed towards *Strangelove*

UtahState UNIVERSITY

Homework: 4b Compound Orbits

(cont'd)

- **Compute the position vector of the *Enterprise relative to Strangelove* ... in the *Strangeloveian* -fixed coordinate system --** $\bar{R}_{sp}$

- **Solution should have a_s, a_p, θ_s, θ_p as parameters**

Hint 1 : $\bar{i}_s = \left(\bar{i}_r\right)_{planet}$

$\bar{j}_s = \left(\bar{i}_\theta\right)_{planet}$

Hint 2 : $\cos[a+b] = \cos[a]\cos[b] - \sin[a]\sin[b]$

$\sin[a+b] = \sin[a]\cos[b] + \cos[a]\sin[b]$

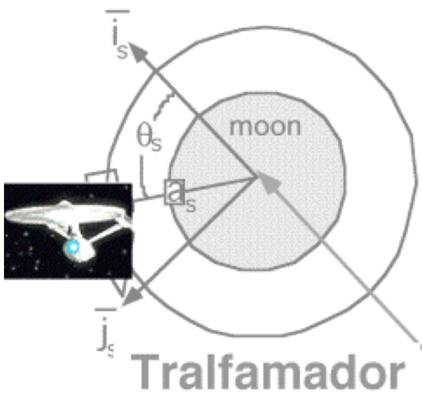
Hint 3 : $\vec{R}_{sp} = \vec{R}_s + \vec{R}_p$

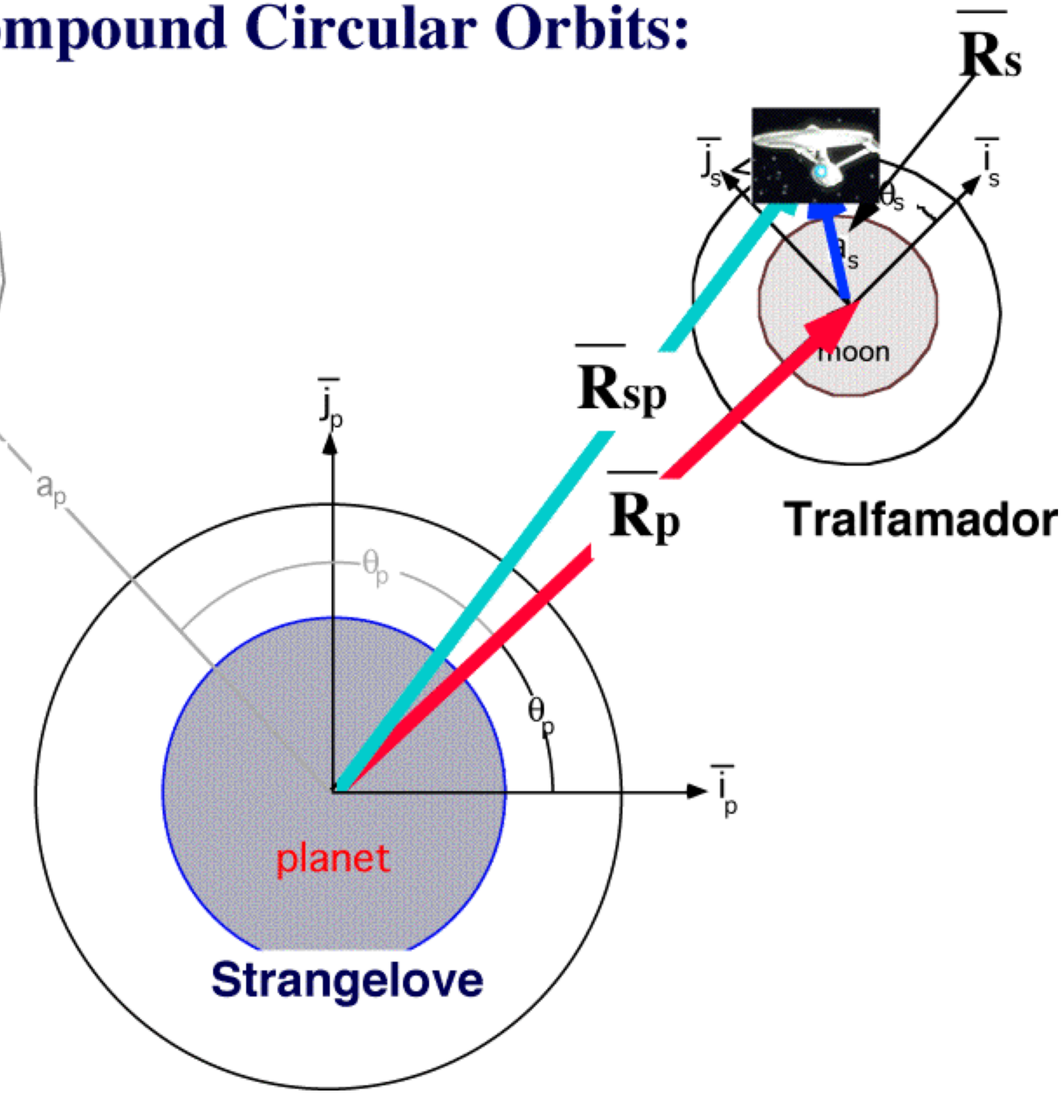
$$\vec{R}_s = a_s \cdot \vec{i}_{r_{moon}}$$

$$\vec{R}_p = a_p \cdot \vec{i}_{r_{planet}}$$

Homework: 4b

Compound Circular Orbits:





$$\vec{R}_s = a_s \cdot \vec{i}_{r_{moon}}$$

$$\vec{R}_p = a_p \cdot \vec{i}_{r_{planet}}$$

Homework: 4b Compound Orbits

continued

- **Compute the velocity vector of the *Enterprise relative to Strangelove* ... in the *Strangeloveian* -fixed coordinate system.**

Hint 4 :

$$\overline{V}_{sp} = \frac{d}{dt}\left[\overline{R}_{sp}\right] = \frac{d}{dt}\left[\overline{R}_s + \overline{R}_p\right]$$

Text

$$\omega_s \equiv \frac{d}{dt}\left[\theta_s\right] \qquad \omega_p \equiv \frac{d}{dt}\left[\theta_p\right]$$

Homework: 4b

(concluded)

- Givens

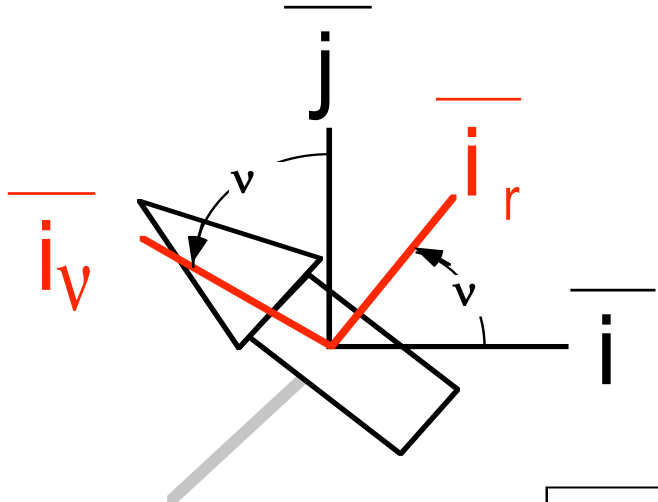
Parameter	Orbit Radius	Planetary Mass
Tralfamador	5000 km	$0.1_{\oplus Mass}$
Strangelove	250,000 km (also look at 25,000 km)	$1.5_{\oplus Mass}$

- Plot the Enterprise Position and Velocity Components in Strangelovian Coordinates $\left\{\vec{i}_p, \vec{j}_p\right\}$, as a function of time

- Show at least 1 complete period

- Assume Initial $\left\{\theta_p, \theta_s\right\} = 0$

Coordinate Transformations:



{i, j} fixed in space

Transform ⇒ polar ↑ inertial

$$\bar{i} = \bar{i}_r \cos[\nu] - \bar{i}_\nu \sin[\nu]$$
$$\bar{j} = \bar{i}_r \sin[\nu] + \bar{i}_\nu \cos[\nu]$$

Transform ⇒ inertial ↑ polar

$$\bar{i}_r = \bar{i} \cos[\nu] + \bar{j} \sin[\nu]$$
$$\bar{i}_\nu = -\bar{i} \sin[\nu] + \bar{j} \cos[\nu]$$