Project 2 ***(16 pts)***

Build Unsteady Model of "Pike" .. Use Integrator of your choice

Part 1, Cylindrical Port (7 points total)

Calculate ***and Plot vs time*** ***1 Point each for Cylindrical Port***

Chamber pressure profile

Regression rate profile

Massflow rate (compare to choking massflow) ***show both plots***

6 pts *Mass depletion vs time*

Thrust Profile

Calculate and Show:

Effective Mean Specific Impulse

Allow:

St. Robert's Parameter Input

Variable Step Size

Variable Thermodynamic Properties ***(as inputs to the problem)***

Erosive burn model for cylindrical port (Not Bates grain)

1 additional Point for Correct Erosive Burn Plots (All of Those Plots Listed Above)

UtahState UNIVERSITY

Mechanical & Aerospace Engineering

Project 2 (2)

Part 1, cylindrical port

Fuel Grain Geometry

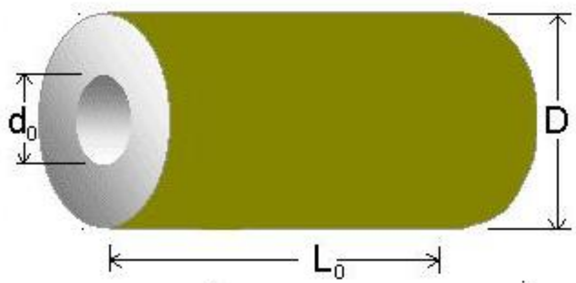
$L_0 = 35$ cm
$D_0 = 6.6$ cm
$D_0 = 3$ cm
$\rho_{propellant} = 1260$ kg/M^3

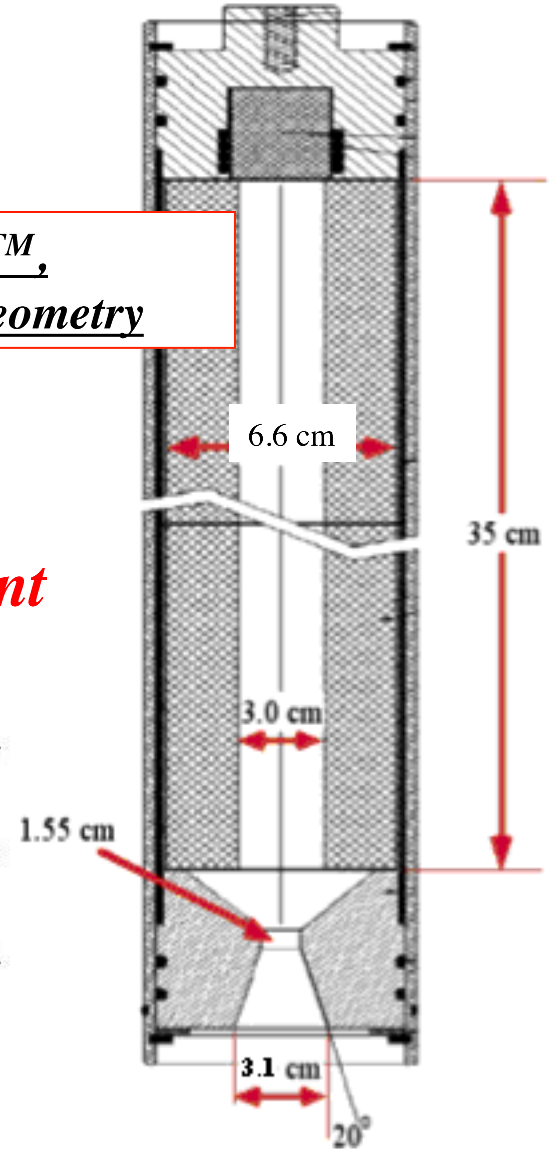
Nozzle Geometry

$A^* = 1.887$ cm^2
$A_{exit}/A^* = 4.0$
$\theta_{exit} = 20$ deg.

Animal Works™, L700 Motor Geometry

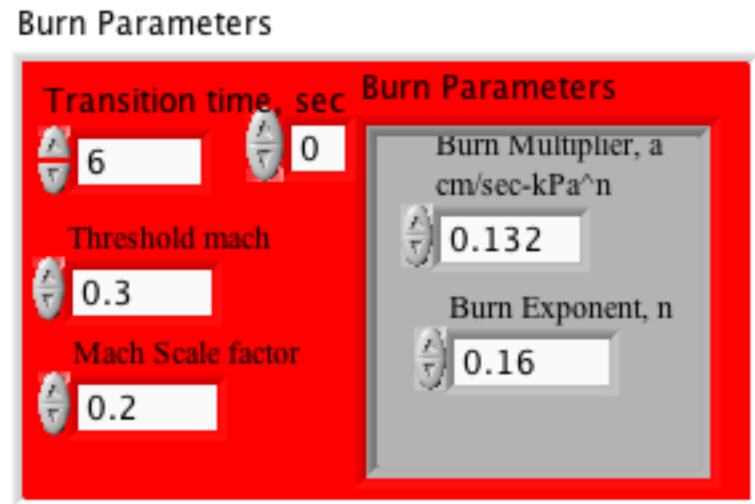
Single propellant segment





A***ssume ends are burn inhibited***

Project 2 (3)



Combustion Gas Properties

γ = 1.18

M_W = 23 *kg/kg-mol*

T_0= 2900 K

Burn Parameters

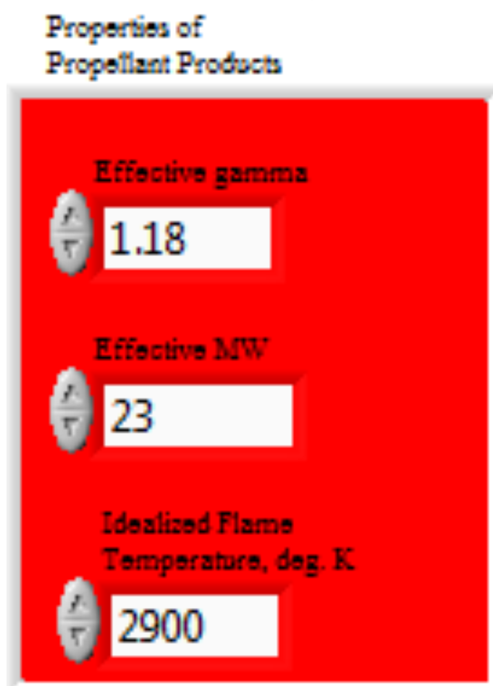
a= 0.132 *cm/(sec-kPa ^{n})*

n=0.16

M^{crit} = 0.3

k = 0.2

(cylindrical port only)



Project 2 (4)

Part 2, Bates Grain (Repeat Calculations, Plots of part 1)

(6 pts, 1 Point for each correct plot)

Fuel Grain Geometry

$L_0 = 35$ cm (3 x 11.667 cm)

$D_0 = 6.6$ cm

$D_0 = 3$ cm

$\rho_{propellant} = 1260$ kg/M^3

Animal Works™, L700 Motor Geometry

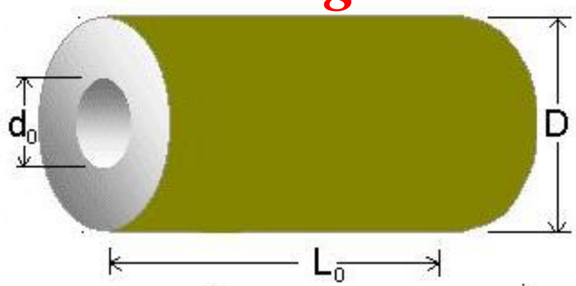
Repeat results Using bates grain With 3 segments

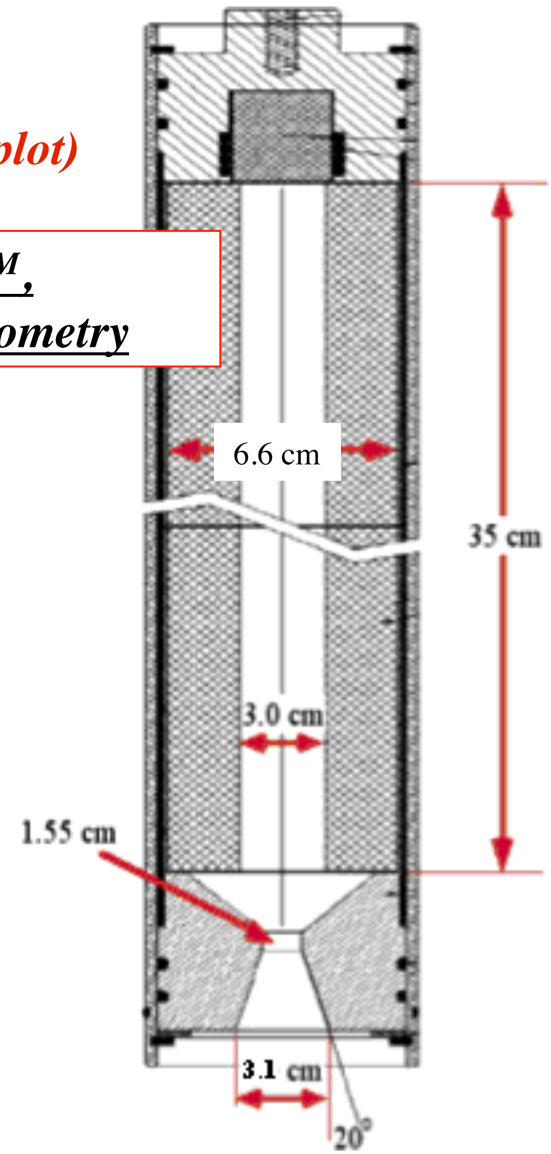
Nozzle Geometry

$A^* = 1.887$ cm ^{2}

$A_{exit}/A^* = 4.0$

$\theta_{exit} = 20$ deg.





Assume ends are not! burn inhibited

Project 2 (5)

Combustion Gas Properties

$\gamma = 1.18$

$M_W = 23$ kg/kg-mol

$T_0 = 2900$ K

Burn Parameters

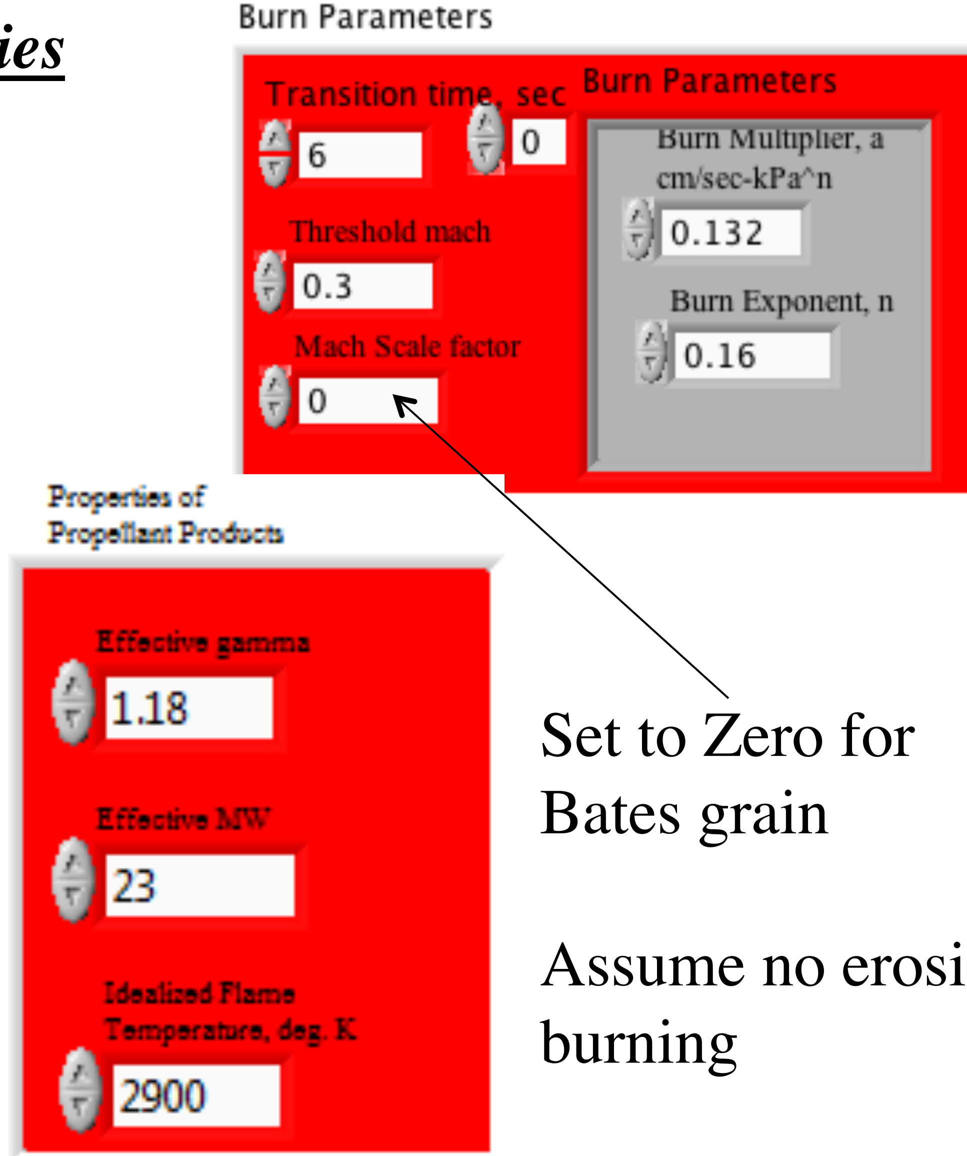
$a = 0.132$ cm/(sec-kPa ^{n})

$n = 0.16$

$M^{crit} = 0.3$

$k = 0.0$

(Bates grain only)



Set to Zero for Bates grain

Assume no erosive' burning

Project 2 (6)

Part 3, Sensitivity Analysis (2 Pts)

Examine sensitivity of calculations to burn rate parameters, ***{a, n}***

Critical Mach number (for erosion) … cylindrical port

Only, Assume bates grain does not burn erosively

Cylindrical and Bates Grain, Show Chamber Pressure and Thrust Plots

What is the effect of Flame temperature (T_0)

Plot Regression rate versus Chamber pressure

Prepare report stating your results and conclusions

1-Point for comprehensivness and neatness

Cylindrical Port Hints

Integrate Ballistic Equation to Calculate Chamber pressure (P0), Regression rate, and Port Radius

$$\begin{bmatrix} \dot{P}_0 \\ \dot{r} \end{bmatrix} = \begin{bmatrix} \left(\frac{A_{burn} \cdot \dot{r}}{V_c}\right) \cdot \left(\rho_{propellant} \cdot R_g \cdot T_0 - P_0\right) - \left(\frac{A^*}{V_c}\right) \cdot P_0 \cdot \sqrt{\gamma \cdot R_g \cdot T_0 \cdot \left(\frac{2}{\gamma+1}\right)^{\left(\frac{\gamma+1}{\gamma-1}\right)}} \\ a \cdot P_0^n \text{ ... or ... } a \cdot P_0^n \cdot 1 + k \cdot \left(\frac{M_{port}}{M_{crit}}\right) \Big/ (1+k) \end{bmatrix}$$

Assume $\rightarrow$

Initial Conditions:

$$\begin{bmatrix} P_{ambient} = 86_{kPa} \text{ (USU Altitude)} \\ P_0(o) = 2 \cdot P_{ambient} \\ r(0) = r_{port} \text{ (3 cm)} \end{bmatrix}$$

Port Geometry:

$\rightarrow$ *Cylindrical Port :*

$$A_{burn} = 2 \cdot \pi \cdot r \cdot L_{port}$$

$$V_c = \pi \cdot r^2 \cdot L_{port}$$

$$\begin{bmatrix} r \equiv \text{Port Radius} \\ L_{port} \equiv \text{Port Length} \end{bmatrix}$$

Calculate Massflows:

choking massflow $\rightarrow \dot{m}_{throat} = A^* \cdot \frac{P_0}{\sqrt{T_0}} \cdot \sqrt{\frac{\gamma}{R_g} \cdot \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}}$

Propellant massflow due to fuel pyrolysis $\rightarrow \dot{m}_{prop} = \rho_p \cdot A_{burn} \cdot \dot{r} \rightarrow$

$$\begin{bmatrix} \dot{r} = a \cdot P_0^n \text{ (Non-erosive Burn)} \\ \dot{r} = a \cdot P_0^n \cdot \dfrac{1 + k \cdot \left(\frac{M_{port}}{M_{crit}}\right)}{1+k} \text{ (Erosive Burn)} \end{bmatrix}$$

Calculate Consumed Propellant Mass:

$$M_{prop}(t) = \int_0^t \dot{m}_{prop}(\tau) \cdot d\tau$$

$$M_{prop}(t) \geq \left(M_{prop}\right)_{total} \rightarrow \dot{r} = 0$$

Calculate Port Mach Number:

$$\frac{A_{port}}{A^*} = \left(\pi \cdot r^2\right) = \frac{1}{M_{port}} \cdot \left[\left(\frac{2}{\gamma+1}\right) \cdot \left(1 + \frac{\gamma-1}{2} M_{port}^{\ 2}\right)\right]^{\frac{1}{2}\left(\frac{\gamma+1}{\gamma-1}\right)}$$

$\rightarrow$ *subsonic Mach Solution*

UtahState UNIVERSITY

Mechanical & Aerospace Engineering

Bates Grain Hints

Integrate Ballistic Equation to Calculate Chamber pressure (P0), Regression rate, and Port Radius

$$\begin{bmatrix} \dot{P}_0 \\ \dot{r} \end{bmatrix} = \begin{bmatrix} \left(\frac{A_{burn} \cdot \dot{r}}{V_c}\right) \cdot \left(\rho_{propellant} \cdot R_g \cdot T_0 - P_0\right) - \left(\frac{A^*}{V_c}\right) \cdot P_0 \cdot \sqrt{\gamma \cdot R_g \cdot T_0 \cdot \left(\frac{2}{\gamma+1}\right)^{\left(\frac{\gamma+1}{\gamma-1}\right)}} \\ a \cdot P_0^n \end{bmatrix}$$

Initial Conditions:

$$\text{Assume} \rightarrow \begin{bmatrix} P_{ambient} = 86_{kPa} (\text{USU Altitude}) \\ P_0(o) = 2 \cdot P_{ambient} \\ r(0) = r_{port} (3\ cm) \end{bmatrix}$$

Calculate Massflows:

$$\text{choking massflow} \rightarrow \dot{m}_{throat} = A^* \cdot \frac{P_0}{\sqrt{T_0}} \cdot \sqrt{\frac{\gamma}{R_g} \cdot \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}}$$

$$\text{Propellant massflow due to fuel pyrolysis} \rightarrow \dot{m}_{prop} = \rho_p \cdot A_{burn} \cdot \dot{r} \rightarrow$$

Port Geometry:

$$A_{burn} = N \cdot \pi \cdot \left\{ \left[\frac{D_0^2 - (d_0 + 2 \cdot s)^2}{2}\right] + (L_0 - 2 \cdot s) \cdot (d_0 + 2 \cdot s) \right\}$$

$$V_c = \frac{N \cdot \pi}{4} \cdot \left\{ (d_0 + 2 \cdot s)^2 \cdot (L_0 - 2 \cdot s) + D_0^2 \cdot 2s \right\}$$

Do NOT! Use Erosive Burning for Bates Grain

Calculate Consumed Propellant Mass:

$$M_{prop}(t) = \int_0^t \dot{m}_{prop}(\tau) \cdot d\tau$$

Thrust, I_{sp} Calculations, Hints

For the nozzle exit

$$\frac{A_{exit}}{A^*} = \frac{1}{M_{exit}} \cdot \left[\left(\frac{2}{\gamma+1}\right)\cdot\left(1+\frac{\gamma-1}{2}M_{exit}{}^2\right)\right]^{\frac{1}{2}\left(\frac{\gamma+1}{\gamma-1}\right)} \rightarrow \text{supersonic}$$

$$\rightarrow \quad T_{exit} = \frac{T_0}{1+\frac{\gamma+1}{2}M^2_{exit}}$$

$$\rightarrow V_{exit} = M_{exit} \cdot \sqrt{\gamma \cdot R_g \cdot T_{exit}}$$

at each time point....

$$\dot{m}_{throat} = \dot{m}_{exit} = A^* \cdot P_0 \cdot \sqrt{\frac{\gamma}{R_g \cdot T_0} \cdot \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}}$$

$$p_{exit} = \frac{P_0}{\left(1+\frac{\gamma+1}{2}M^2_{exit}\right)^{\frac{\gamma}{\gamma-1}}}$$

Effective Specific Impulse

$$\left(I_{sp}\right)_{eff} = \frac{I_{mpulse}\left(t_{burn}\right)}{g_0 \cdot M_{prop}\left(t_{butn}\right)}$$

$$\rightarrow F = \lambda \cdot \dot{m}_{exit} \cdot V_{exit} + \left(p_{exit} - p_{\infty}\right) \cdot A_{exit}$$

$$I_{mpulse}\left(t\right) = \int_0^t F\left(\tau\right) \cdot d\tau$$

Careful with Units!

$$\begin{bmatrix} \dot{P}_0 \\ \dot{r} \end{bmatrix} = \left[\left(\frac{A_{burn}\cdot \dot{r}}{V_c}\right)\cdot\left(\rho_{propellant}\cdot R_g \cdot T_0 - P_0\right) - \left(\frac{A^*}{V_c}\right)\cdot P_0 \cdot \sqrt{\gamma \cdot R_g \cdot T_0 \cdot \left(\frac{2}{\gamma+1}\right)^{\left(\frac{\gamma+1}{\gamma-1}\right)}} \right]$$

$$a \cdot P_0^n$$

$$\begin{bmatrix} a = 0.132 \, \frac{cm}{\sec - kPa^n} \\ 0.16 \end{bmatrix} \rightarrow$$

$$\dot{r} = a \cdot P_0^n \rightarrow \begin{bmatrix} P_0 \sim kPa \\ \dot{r} \sim cm \end{bmatrix}$$

$$\dot{r} = \left(\frac{a}{100 \, \frac{cm}{m}}\right) \cdot \left(\frac{P_0}{1000 \, \frac{pa}{kPa}}\right)^n \rightarrow \begin{bmatrix} P_0 \sim Pa \\ \dot{r} \sim m \end{bmatrix}$$

$\rho_{propellant} \cdot R_g \cdot T_0$ →Must have same Units as P_0!

$$\frac{A^*}{V_c} \cdot \sqrt{\gamma \cdot R_g \cdot T_0} \sim seconds!$$

State Equation Formulation of Problem

$$\begin{bmatrix} \dot{P}_0 \\ \dot{r} \end{bmatrix} = \begin{bmatrix} \left(\frac{A_{burn} \cdot \dot{r}}{V_c}\right) \cdot \left(\rho_{propellant} \cdot R_g \cdot T_0 - P_0\right) - \left(\frac{A^*}{V_c}\right) \cdot P_0 \cdot \sqrt{\gamma \cdot R_g \cdot T_0 \cdot \left(\frac{2}{\gamma+1}\right)^{\left(\frac{\gamma+1}{\gamma-1}\right)}} \\ a \cdot P_0^n \end{bmatrix}$$

Careful with Units! **Careful with Units!**

$$\begin{bmatrix} P_0 \\ r \end{bmatrix}_{t=0} = \begin{bmatrix} P_{ambient} \\ \frac{d_0}{2} \end{bmatrix} \rightarrow \boxed{s(t) = \int_0^t \dot{r} \cdot dt \approx 2 \cdot r(t) - d_0}$$

$$\begin{bmatrix} k \equiv \text{Erosion Constant (GRAIN DEPENDENT)} \\ M_{crit} \equiv \text{Critical Port Mach Number} \end{bmatrix}$$

$\rightarrow$ *State Equations for Erosive Burning :*

$$\begin{bmatrix} \dot{P}_0 \\ \dot{r} \end{bmatrix} = \begin{bmatrix} \left(\frac{A_{burn} \cdot \dot{r}}{V_c}\right) \cdot \left(\rho_{propellant} \cdot R_g \cdot T_0 - P_0\right) - \left(\frac{A^*}{V_c}\right) \cdot P_0 \cdot \sqrt{\gamma \cdot R_g \cdot T_0 \cdot \left(\frac{2}{\gamma+1}\right)^{\left(\frac{\gamma+1}{\gamma-1}\right)}} \\ \left(1 + k \cdot \frac{M_{port}}{M_{crit}}\right) \cdot a \cdot P_0^n / \left(1+k\right) \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ r \end{bmatrix}_{t=0} = \begin{bmatrix} P_{ambient} \\ \frac{d_0}{2} \end{bmatrix} \rightarrow \boxed{s(t) = \int_0^t \dot{r} \cdot dt \approx 2 \cdot r(t) - d_0}$$

State Equation Formulation of Problem (2)

$\rightarrow$ *Cylindrical Port:*

$$A_{burn} = 2 \cdot \pi \cdot r \cdot L_{port}$$
$$V_c = \pi \cdot r^2 \cdot L_{port}$$

$$\rightarrow \begin{bmatrix} r \equiv Port\ Radius \\ L_{port} \equiv Port\ Length \end{bmatrix}$$

$\rightarrow$ *Bates Grain:*

$$A_{burn} = N \cdot \pi \cdot \left\{ \left[\frac{D_0^2 - (d_0 + 2 \cdot s)^2}{2} \right] + (L_0 - 2 \cdot s) \cdot (d_0 + 2 \cdot s) \right\}$$

$$V_c = \frac{N \cdot \pi}{4} \cdot \left\{ (d_0 + 2 \cdot s)^2 \cdot (L_0 - 2 \cdot s) + D_0^2 \cdot 2s \right\}$$

Do NOT! Use Erosive Burning for Bates Grain

State Equation Formulation of Problem (3)

Calculating Chamber Mach Number

Erosive Burning

$$\rightarrow \frac{V_c / L_{port}}{A^*} = \frac{1}{M_{port}} \cdot \left[\left(\frac{2}{\gamma + 1} \right) \cdot \left(1 + \left(\frac{\gamma - 1}{2} \right) \cdot M_{port}^2 \right) \right]^{\left(\frac{\gamma+1}{2 \cdot (\gamma - 1)} \right)}.$$

$$\ldots \textit{Subsonic Branch Solution}!$$

Do NOT! Use Erosive Burning for Bates Grain

Cylindrical Port: Decoupled Model

- Use Trapezoidal rule or Runge-Kutta to integrate

$$\frac{\partial P_0}{\partial t} = \frac{A_{burn} a P_o^{\,n}}{V_c}\left[\rho_p R_g T_0 - P_0\right] - P_0\left[\frac{A^*}{V_c}\sqrt{\gamma R_g T_0\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}}\right]$$

- Recursive propagation of chamber diameter

$$R_{burn_{k+1}} = R_{i_{initial}} + \int_0^{(k+1)\Delta t} \dot{r}dt = R_{i_{initial}} + \int_0^{(k)\Delta t} \dot{r}dt + \int_{(k)\Delta t}^{(k+1)\Delta t} \dot{r}dt \rightarrow$$

$$R_{burn_{k+1}} = R_{burn_k} + \int_{(k)\Delta t}^{(k+1)\Delta t} \dot{r}dt \approx R_{burn_k} + \dot{r}\,\Delta t = R_{burn_k} + a P_{o_k}^{\,n}\,\Delta t$$

Bates grain Port: Decoupled Model

- Use Trapezoidal rule or Runge-Kutta to integrate

$$\frac{\partial P_0}{\partial t} = \frac{A_{burn} a P_o^{\,n}}{V_c}\left[\rho_p R_g T_0 - P_0\right] - P_0\left[\frac{A^*}{V_c}\sqrt{\gamma R_g T_0\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}}\right]$$

$$\dot{r} = a\cdot P_o^{\,n}$$

$$s_{regression} = \int_t \dot{r}\cdot dt \;\rightarrow$$

$$\left(A_{burn}\right)_{total} = N\cdot\pi\cdot\left[\frac{\left(D_0^{\,2} - \left(d_0 + 2\cdot s\right)^2\right)}{2} + \left(L_0 - 2\cdot s\right)\cdot\left(d_0 + 2\cdot s\right)\right]$$

$$\left(V_{ol}\right)_{total} = \frac{N\cdot\pi}{4}\left[\left(d_0 + 2\cdot s\right)^2\cdot\left(L_0 - 2\cdot s\right) + D_0^{\,2}\cdot\left(2\cdot s\right)\right]$$