Major Project 1 … 16 *Points Total!*

- **Look at problem of transferring satellite to MEO (GPS) from Initial LEO Orbit**
  - Code Continuous Thrust Example
  - A_{LEO} ~ 8530 km
  - A_{MEO} ~ 13,200 km
  - Isp ~ 2000 sec
  - Thrust ~ 10 Nt

… Implement both Trapezoidal and Runge-Kutta Integration schemes

… compare algorithm performance as Time interval becomes progressively larger

- Include a *Thrust Termination* Criterion which puts you in the proper final transfer orbit (apogee tangent to desired MEO Orbit)
- Calculate Impulse ΔV and Propellant Mass Required to Circularize Final Orbit

for continuous thrust problem .. assume final delta V is delivered impulsively with Apogee Kick Motor Isp = 270 sec ….. Ignore atmospheric drag

a.1) Continuous Small Thrust, Simulation *(6 pts)*

Terminate thrust when

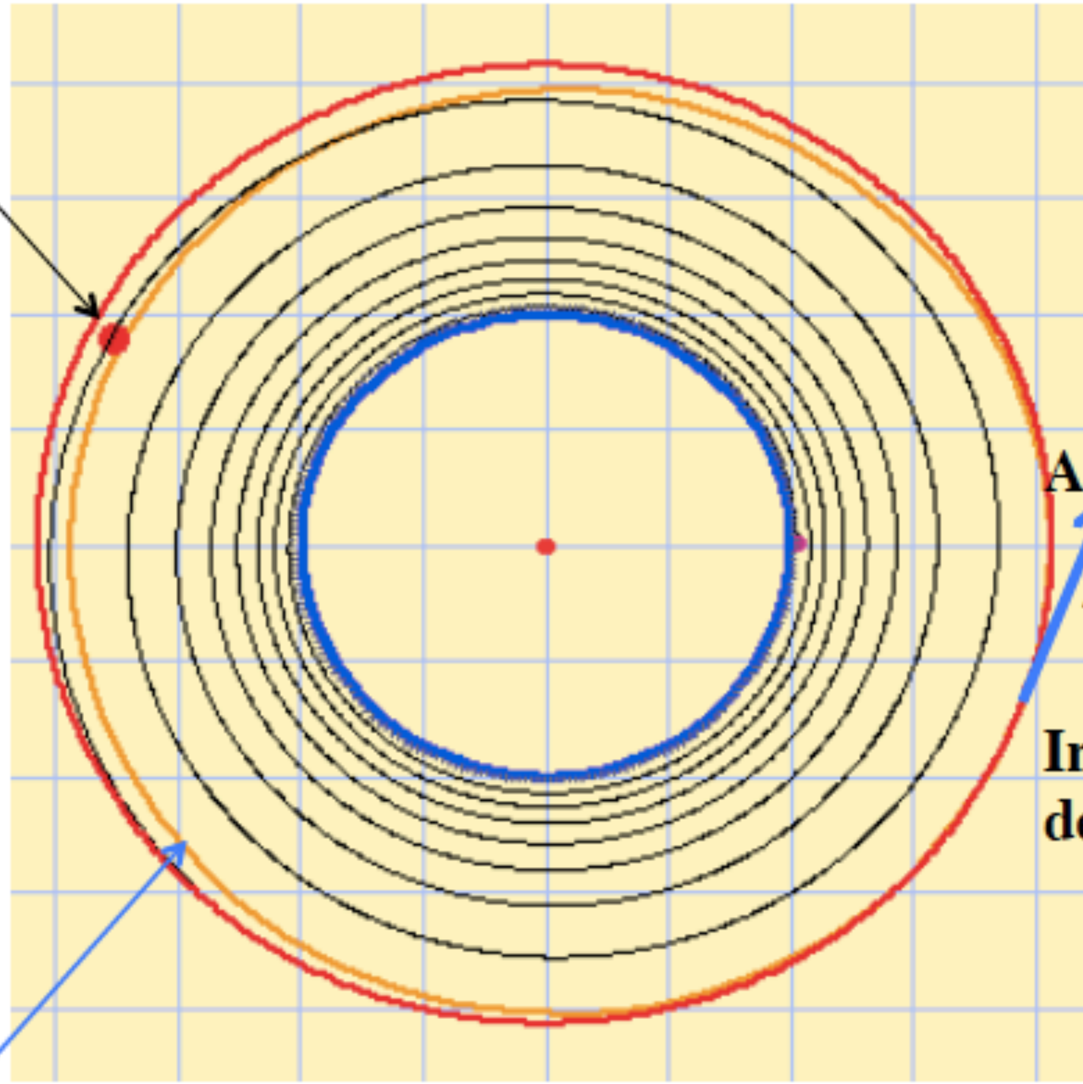
$$R_{apogee} = a \cdot (1+e)$$
$$= 13{,}200\,km$$

Calculate:

1) Propellant mass req.
For continuous transfer

2) Propellant mass req.
For kick delta *V (impulsive)*
(orbit circularization)

3) Final mass = 1000 kg



a.2) Continuous Small Thrust *(1 pt)*

… Implement *both* Trapezoidal and Runge-Kutta Integration schemes

… Assume continuous thrust transfer to transfer orbit apogee using EP device, final orbit insertion using high thrust kick motor

… compare algorithm performance as Time interval ΔT becomes progressively larger

… Is there a point where algorithm blows up?

a.3) Continuous Small Thrust, Simulation to Hohmann Transfer Comparisons (*1 pt*)

… compare continuous thrust propellant mass calculations against Hohmann transfer calculations .. Assuming impulsively delivered Delta V for each burn

Burn 1: Isp = 2000 sec

Burn 2: Isp = 270 sec

… what can you conclude about the accuracy of the rocket equations and the impulsive Delta V assumption when applied to a long duration non-impulsive burn?

a.3) Impulsive Hohmann Transfer …

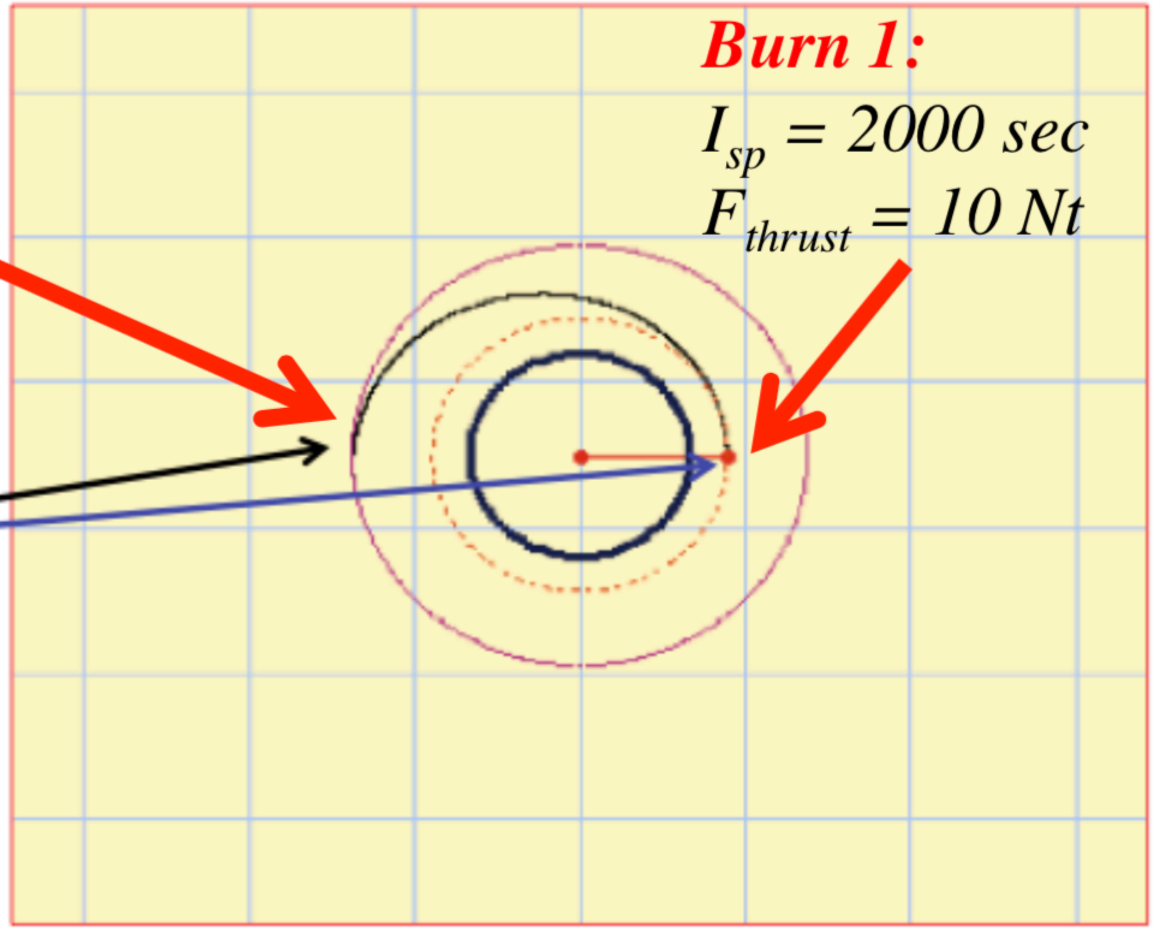
Burn 2

Hohmann Transfer:

I_{sp}=270 sec

F_{thrust}=2000 Nt

- ***Impulsive Burn Calculations***



- Calculate ΔV and Consumed Mass, Including Kick

Impulsive Hohmann Transfer (cont'd)

.... Small thrust transfer comparison ....

- Compare to continuous thrust transfer

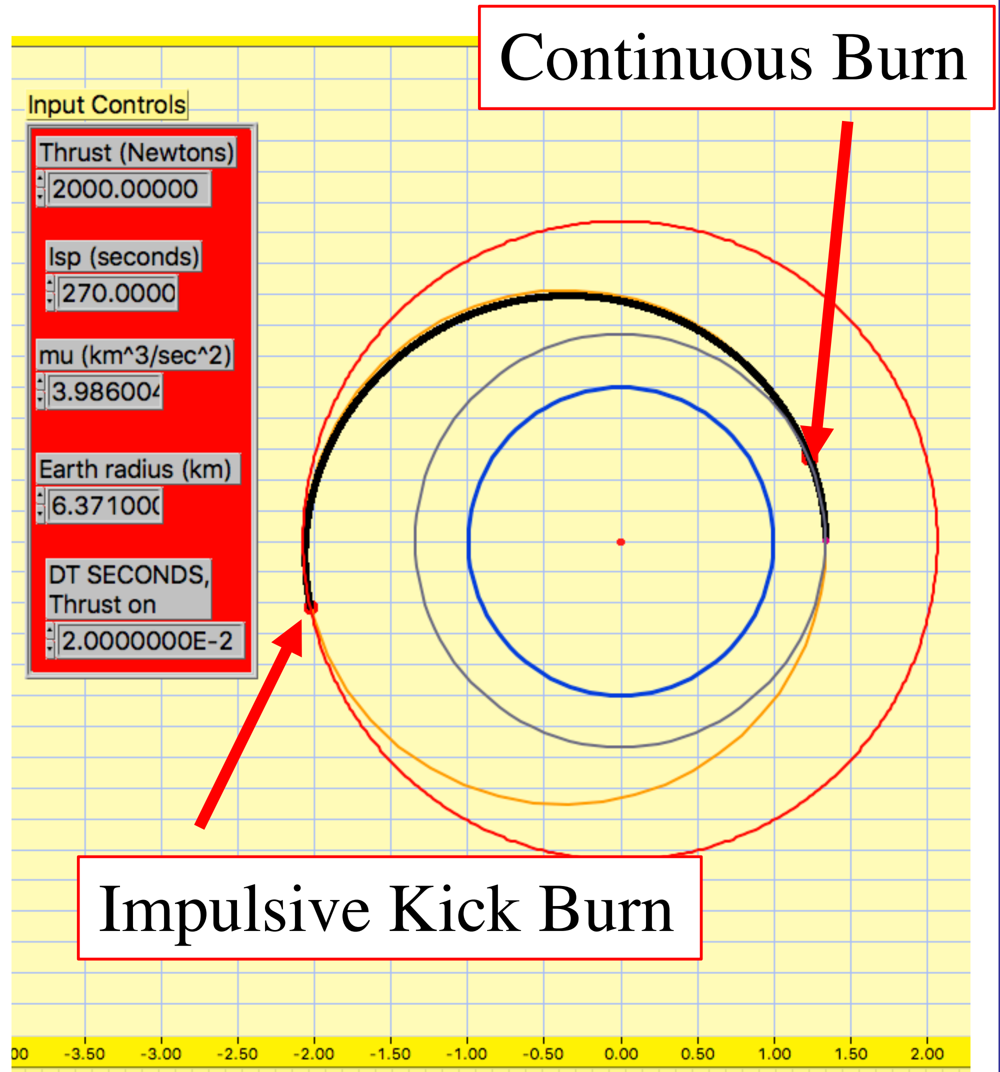
	Propellant Mass kg **Continuous Thrust Transfer**	**Propellant Mass kg** **Impulsive Thrust Transfer**
Burn 1 I_{sp} = 2000 sec F = 10 N		
Burn 2 I_{sp} = 270 sec F = 2000 N		
Total		

Part b.1) Continuous Large Thrust Analysis *(3 pts)*

- F = 2000 N, Isp = 270 sec
- *Continuous First Burn*
- Terminate Thrust

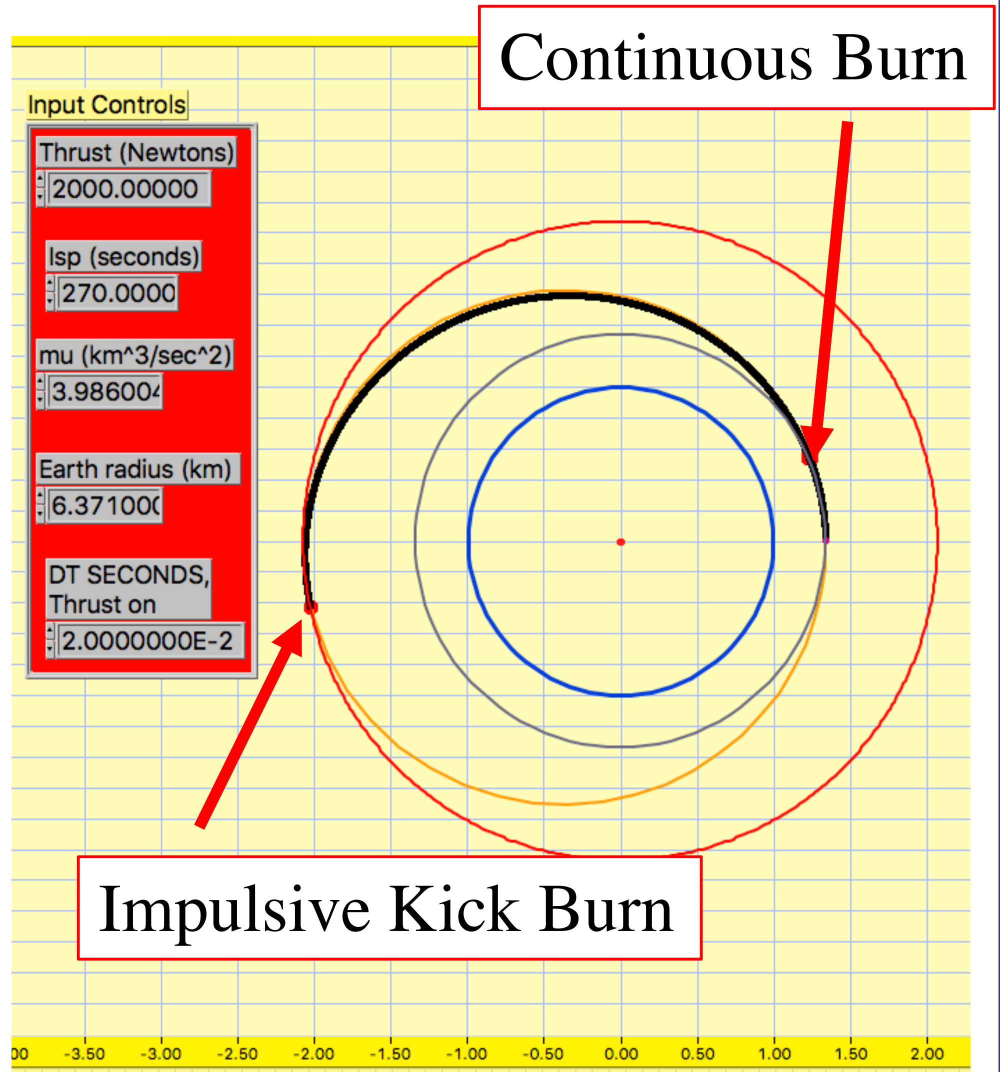
When $a \bullet (1+e) \sim 13{,}200$ *km*

- •*Impulsive Second (Kick) Burn*
- Required M_{final} = 1000 kg



Part b.1) Continuous Large Thrust Analysis

- → Calculate
- Propellant Mass Required for *Continuous* Transfer Burn
- ΔV, Propellant Mass Required for *Impulsive* Final Kick Burn
- M_{final} = 1000 kg



Part b.2) Large Thrust Hohmann Transfer Analysis ***(1 pt)***

… compare continuous thrust propellant mass and ΔV Calculations against Hohmann transfer calculations .. Assuming impulsively delivered Delta V for each burn

Burn 1: Isp = 270 sec

Burn 2: Isp = 270 sec

… what can you conclude about the accuracy of the rocket equations and the impulsive Delta V assumption when applied to a long Short-Duration (compared to orbit Period), Non-impulsive burn

Part b.2) Large Thrust Hohmann Transfer Analysis

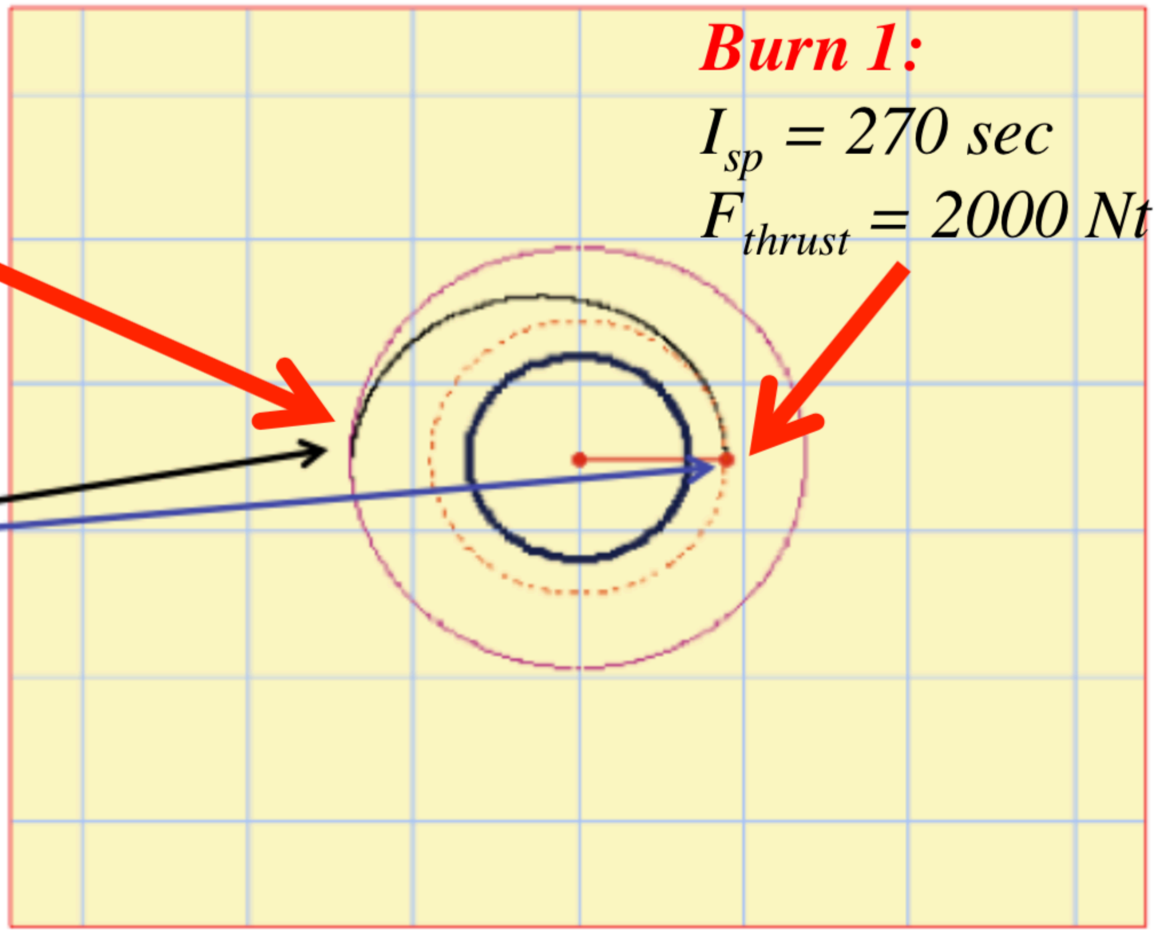
Burn 2

Hohmann Transfer:

I_{sp}=270 sec

F_{thrust}=2000 Nt

- ***Impulsive Burn Calculations***



Part b) Large Thrust Hohmann Transfer Analysis

- Compare to one-burn continuous thrust transfer

	Propellant Mass kg Continuous Thrust Transfer	Propellant Mass kg Impulsive Thrust Transfer
Burn 1 I_{sp} = 270 sec F = 2000 N		
Burn 2 I_{sp} = 270 sec F = 2000 N		266.80 kg
Total		

• Part c.1) Two-Burn Grand Challenge (2 pts)

- **Assume BOTH burns are performed non-impulsively**
Terminate burn thrust when

Terminate Burn 1 Thrust When

$$R_{apogee} = a \bullet (1+e) \sim 13{,}200\ km$$

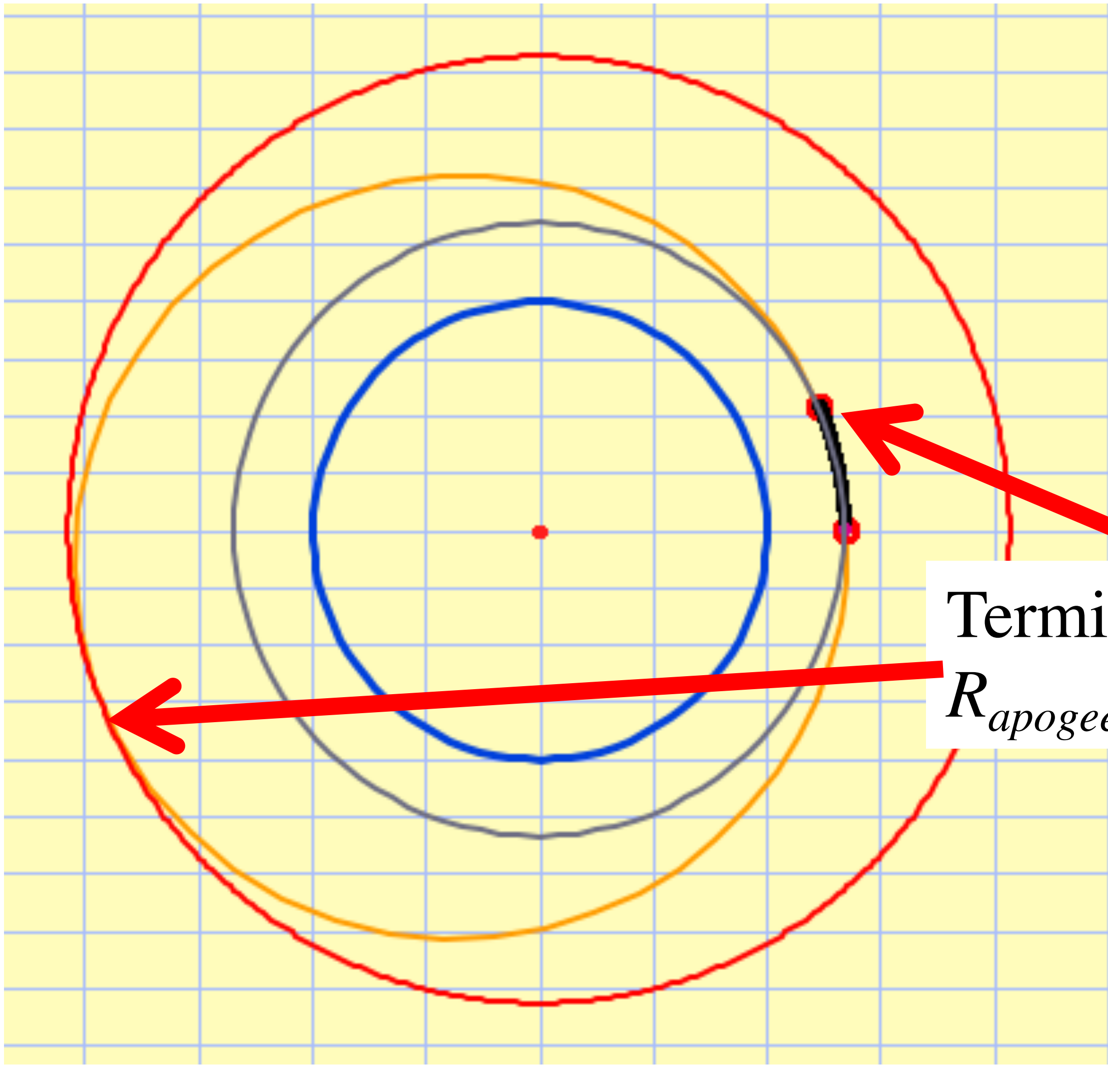
- **You decide when and how long to initiate the second burn to circularize the orbit**

- **Assume for large thrust …. 2000 Nt thrust (both burns) … Isp = 270 sec**

- **Calculate required propellant mass for Burn1, Burn2 (and Total)**

- **Use integrator of your choice … calculate actual delivered Delta V**
Based on consumed mass … using rocket equation

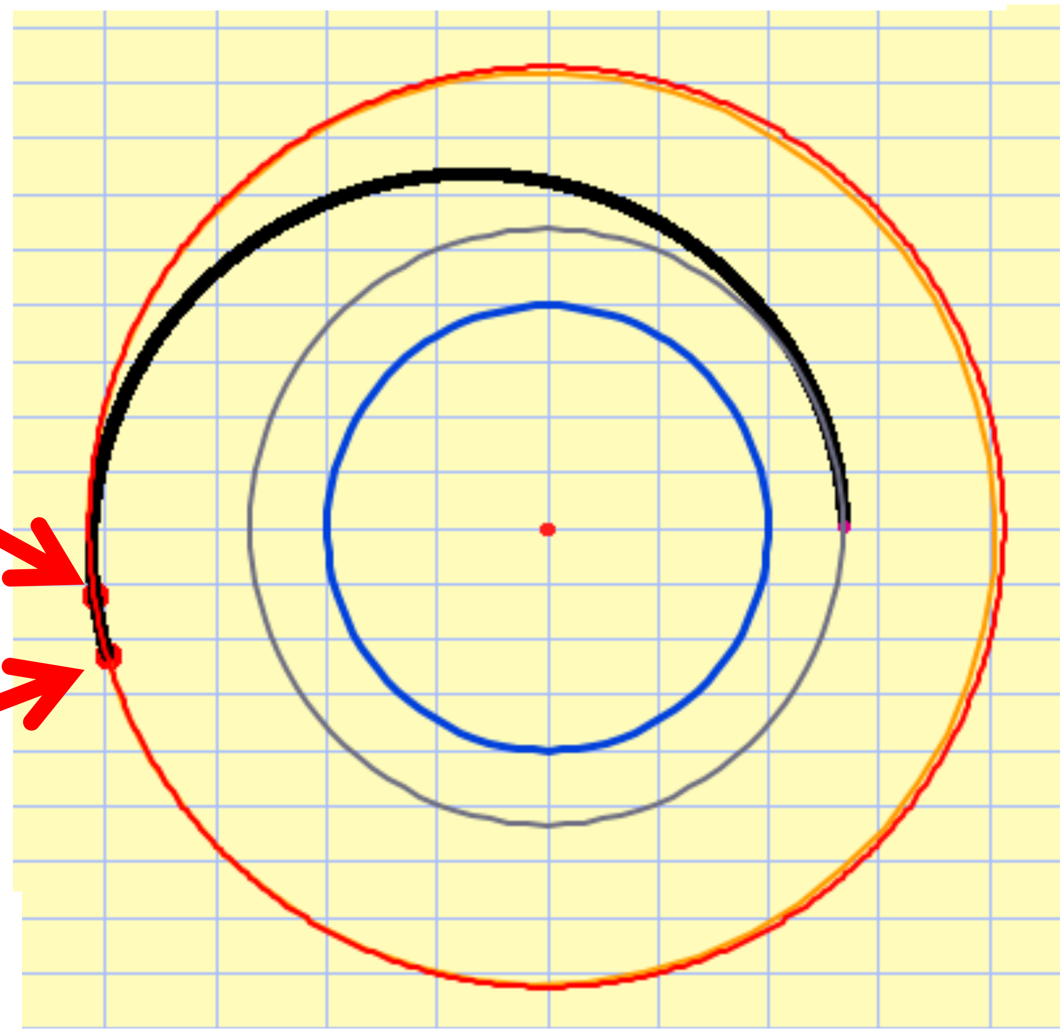
• Part c.1) Grand Challenge



Terminate Burn 1 Thrust When
$R_{apogee} = a \cdot (1+e) \sim 13{,}200\ km$

• Part c.1) Two-Burn Grand Challenge

• You Decide When (or Where) to Initiate Second Burn



• Terminate Burn 2 Thrust Final Orbit $R_{apogee} \sim 13{,}200\ km$

- Continuous Large Thrust Problem

c.2) Final Summary (1 pt)

… compare Hohmann Transfer for 2000 Nt Rocket (assuming impulsive thrust) Versus 2000 Nt rocket with Non Impulsive Thrust …. Also compare consumed masses to High I_{sp} Continuous Thrust transfer

… what can you conclude about the accuracy of the rocket equation and the impulsive Delta V assumption when applied to a short duration non-impulsive burn?

… what can you conclude about the effect of I_{sp} on required propellant mass?

Impulsive Hohmann Transfer (cont'd)

.... Large thrust transfer comparison ....

- Compare to two-burn continuous thrust transfer

	Propellant Mass kg **Continuous Thrust Transfer**	**Propellant Mass kg** **Impulsive Thrust Transfer**
Burn 1 I_{sp} = 270 sec F = 2000 N		
Burn 2 I_{sp} = 270 sec F = 2000 N		
Total		

+ 1 Point for neatness and completeness of presentation/report

Collected Equations, Ballistic Trajectory

$$\begin{bmatrix} \dot{V}_r \\ \dot{V}_v \\ \dot{r} \\ \dot{v} \\ \dot{m} \end{bmatrix} = \begin{bmatrix} \frac{V_v^{\ 2}}{r} - \frac{\mu}{r^2} + \left[\frac{F_{thrust}}{m} - \frac{\rho V_\infty^{\ 2}}{2\beta}\right]\sin(\gamma) \\ -\frac{V_r V_v}{r} + \left[\frac{F_{thrust}}{m} - \frac{\rho V_\infty^{\ 2}}{2\beta}\right]\cos(\gamma) \\ V_r \\ \frac{V_v}{r} \\ -\frac{F_{thrust}}{g_0 I_{sp}} \end{bmatrix}_{\alpha=0}$$

$$\gamma = \tan^{-1}\left[\frac{V_r}{V_v}\right]$$

Ignore Drag

$$\beta = \sim\infty$$

$$\dot{X} = f\left[X, F_{thrust}\right]$$

Project Hints (1)

Position within initial orbit:

$$\begin{bmatrix} r \\ \nu \end{bmatrix}_0 = \begin{bmatrix} \dfrac{a_0\left(1-e_0^{\,2}\right)}{1+e_0\cos\left(\nu_0\right)} \\ \nu_0 \end{bmatrix} \rightarrow \begin{bmatrix} \text{circular orbit} \rightarrow e_0 = 0 \\ \text{can assume} \rightarrow \nu_0 = 0 \rightarrow a_0 = r_0 \end{bmatrix}$$

Angular velocity within initial orbit:

$$\omega_0 = \frac{\sqrt{\mu}\left[1+e_0\cos\left(\nu_0\right)\right]^2}{\left[a_0\left(1-e_0^{\,2}\right)\right]^{3/2}} \rightarrow \begin{bmatrix} \text{circular orbit} \rightarrow e_0 = 0 \\ \text{can assume} \rightarrow \nu_0 = 0, a_0 = r_0 \end{bmatrix}$$

$$\omega_0 = \frac{\sqrt{\mu}\left[1+e_0\cos\left(\nu_0\right)\right]^2}{\left[a_0\left(1-e_0^{\,2}\right)\right]^{3/2}} = \frac{1}{r_0}\sqrt{\frac{\mu}{r_0}}$$

Project Hints (2)

Linear Velocity within initial orbit:

$$\begin{bmatrix} V_r \\ V_v \end{bmatrix}_0 = r_0\omega_0 \begin{bmatrix} \dfrac{e_0 \sin\left[v_o\right]}{\left[1+e_0\cos\left(v_0\right)\right]} \\ 1 \end{bmatrix} \rightarrow \begin{bmatrix} \text{circular orbit} \rightarrow e_0 = 0 \\ \text{can assume} \rightarrow v_0 = 0, a_0 = r_0 \end{bmatrix}$$

$$\begin{bmatrix} V_r \\ V_v \end{bmatrix}_0 = \begin{bmatrix} 0 \\ r_0\omega_0 \end{bmatrix} = \begin{bmatrix} 0 \\ \sqrt{\dfrac{\mu}{r_0}} \end{bmatrix}$$

Project Hints (3)

Instantaneous $_{\text{(no-nonconservative foreces acting)}}$ Keplerian orbit $\rightarrow$ $given: \begin{bmatrix} V_r \\ V_\nu \end{bmatrix}, \begin{bmatrix} r \\ \nu \end{bmatrix}$

$$a = \frac{\mu}{\left[\frac{2\mu}{r} - \left[V_r^2 + V_\nu^2\right]\right]}$$

$$e = \frac{r}{\mu}\sqrt{\left(V_\nu^2 - \frac{\mu}{r}\right)^2 + \left(V_r V_\nu\right)^2}$$

$$r_{perigee} = a(1-e)$$

$$r_{apogee} = a(1+e)$$