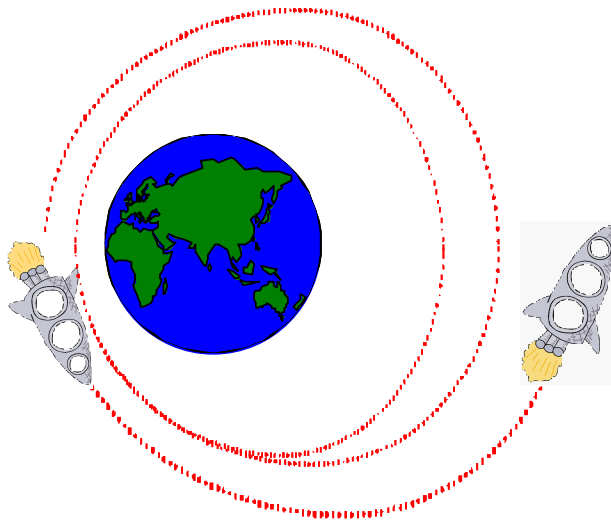


# Longitudinal Vehicle (Pitch) Dynamics, Static and Dynamic Stability

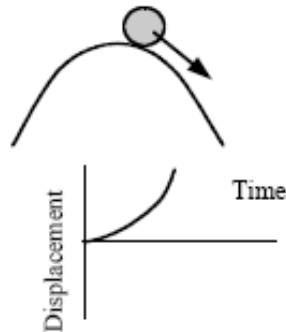
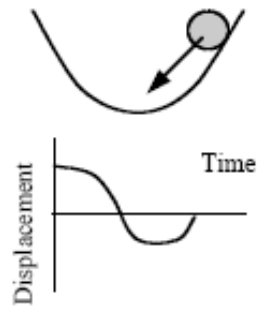


Sellers: Chapter 12

# Vehicle Stability

Positive: Quick to return, hard to displace

Negative: Quick to displace, hard to return

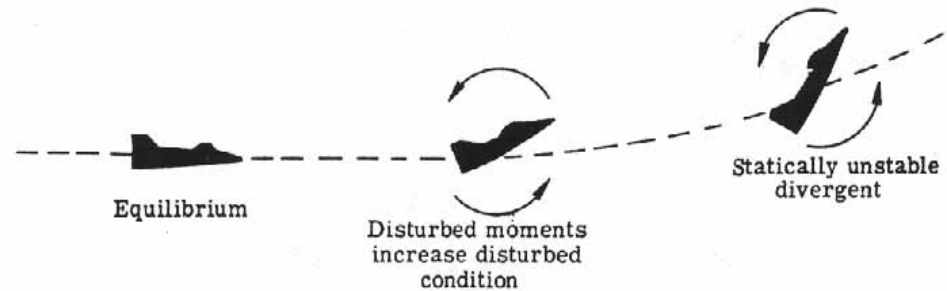
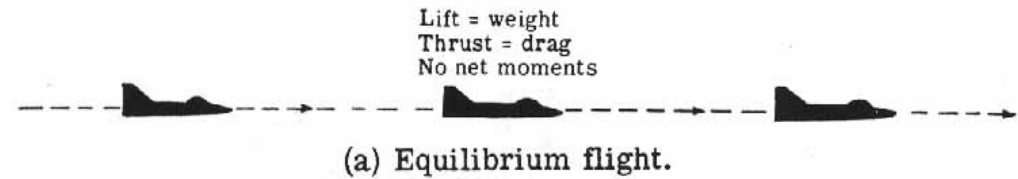
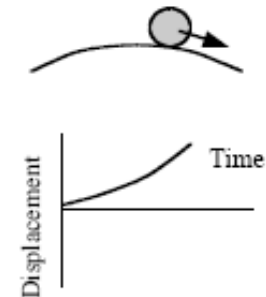
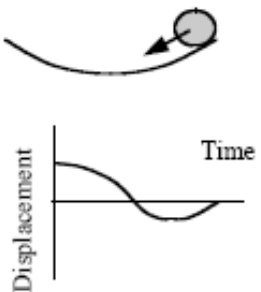


Neutral: Stays put

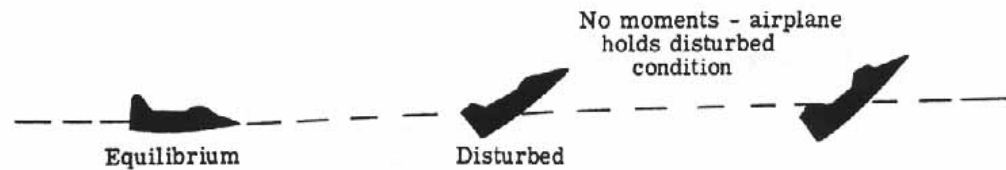


Positive: Slower to return, easier to displace

Negative: Slower to displace, easier to return



(b) Statically unstable airplane.

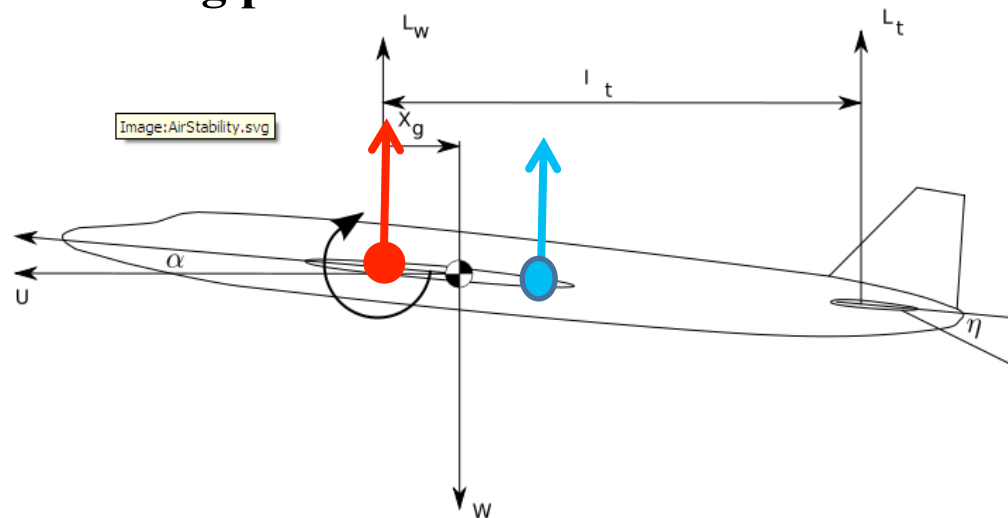


(c) Neutral static stability.

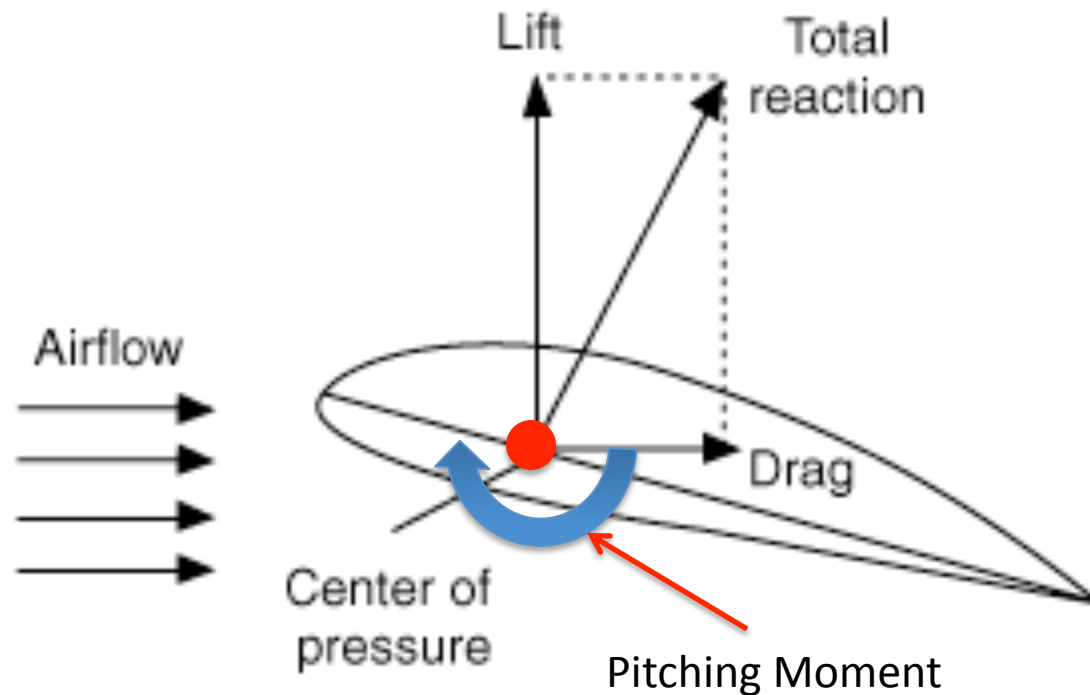
## Vehicle Stability (2)

If center of gravity ( $cg$ ) is forward of the ( $cp$ ), vehicle responds to a disturbance by producing aerodynamic moment that returns Angle of attack of vehicle towards angle that existed prior to the disturbance.

If CG is behind the center of pressure, vehicle will respond to a disturbance by producing an aerodynamic moment that continues to drive angle of attack further away from starting position.



# Center of Pressure



- Aerodynamic *Lift, Drag, and Pitching Moment* Can be thought of acting at a single point ... the **Center of Pressure (*cp*)** of the vehicle

- Sometimes (*and not quite correctly*) referred to as the **Aerodynamic Center (AC)**

- For our purposed AC and CP are synonymous**

# Vehicle Stability, Rocket Flight Example

During rocket flight small wind gusts or thrust offsets can cause the rocket to "wobble", or change its attitude in flight.

Rocket rotates about its center of gravity ( $cg$ )

Lift and drag both act through the center of pressure ( $cp$ ) of the rocket

When  $cp$  is behind  $cg$ , aerodynamic forces provide a "restoring force" ... rocket is said to be "statically stable"

When  $cp$  ahead of  $cg$ , aerodynamic forces provide a "destabilizing force" ... rocket is said to be "unstable"

*Condition for a statically for a stable rocket is that center of pressure must be located behind the center of gravity.*

## Vehicle Stability, Rocket Flight Example (2)

During flight small wind gusts or thrust offsets cause the rocket to "wobble" ... change attitude

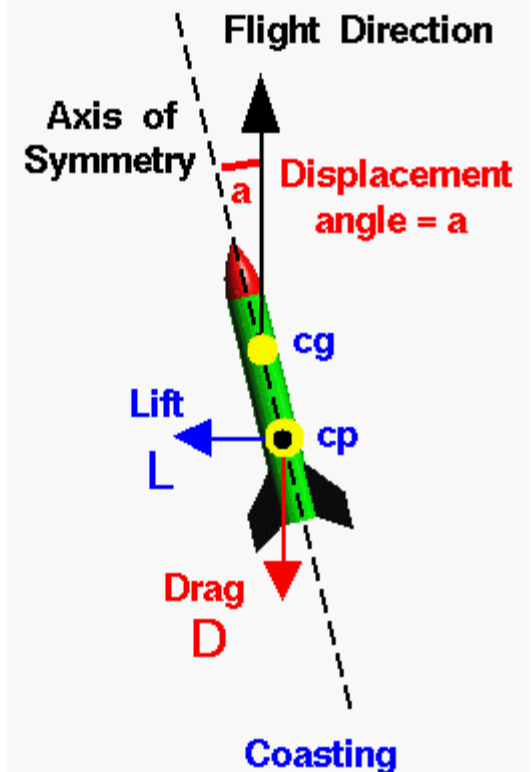
Rocket rotates about center of gravity ( $cg$ )

Lift and drag both act through center of pressure ( $cp$ )

When  $cp$  is behind  $cg$ , aerodynamic forces provide a "restoring force" ... rocket is said to be "statically stable"

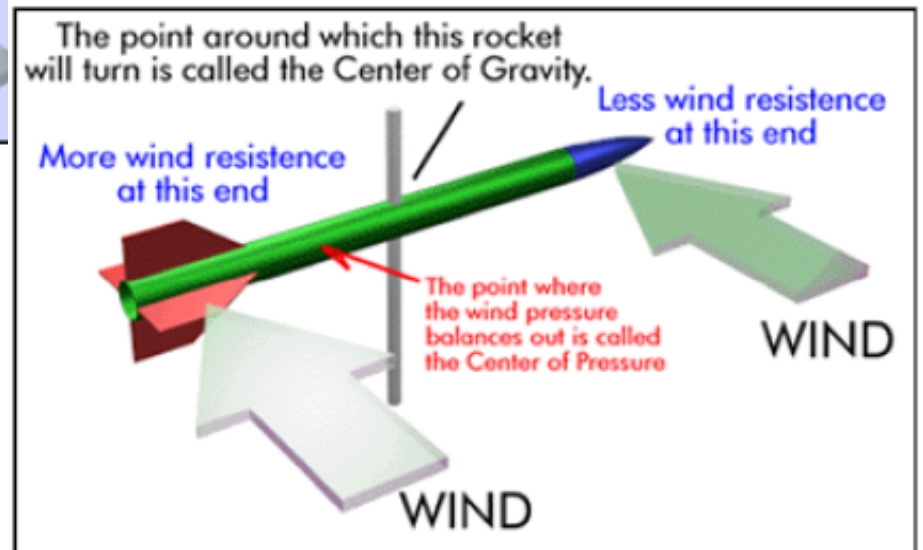
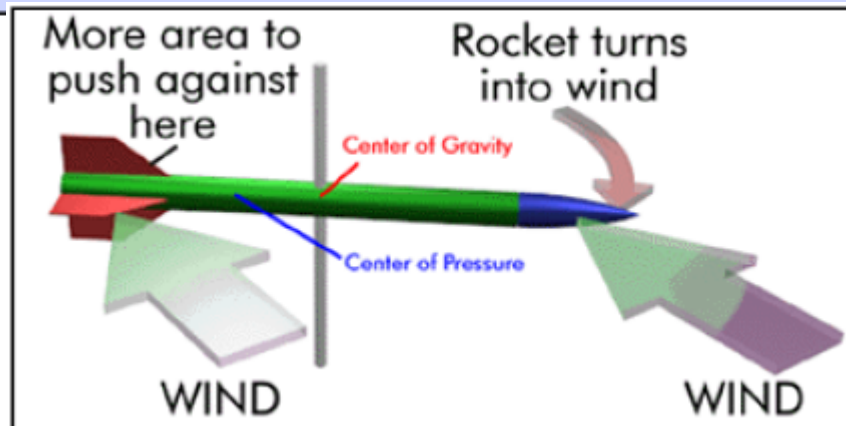
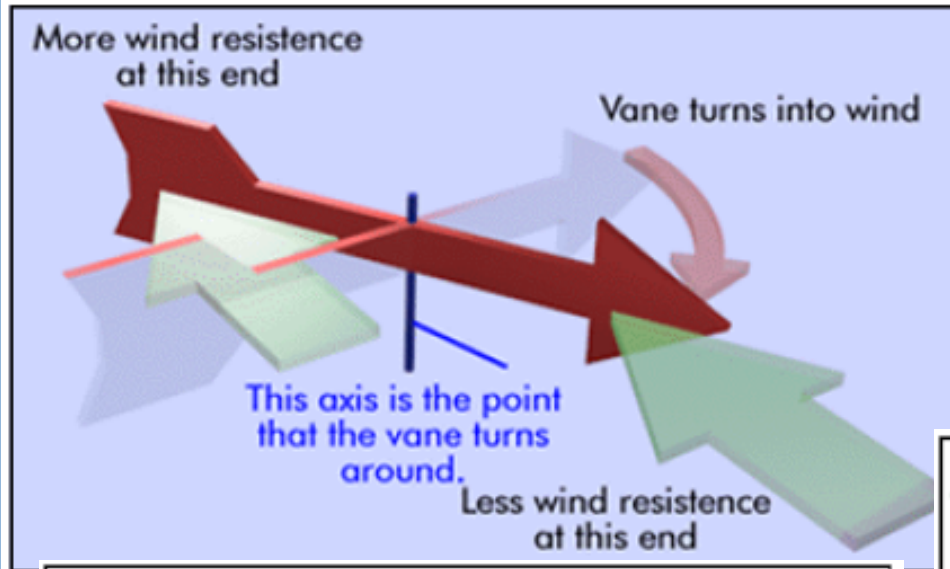
When  $cp$  ahead of  $cg$ , aerodynamic forces provide a "destabilizing force" ... rocket is said to be "unstable"

*Condition for a statically for a stable rocket is that center of pressure must be located behind longitudinal center of gravity.*

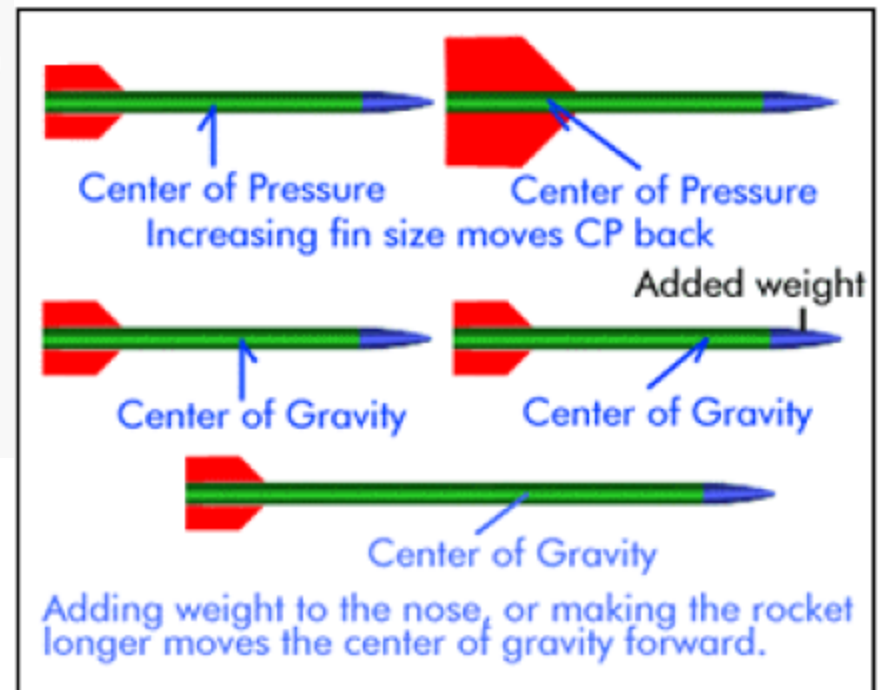
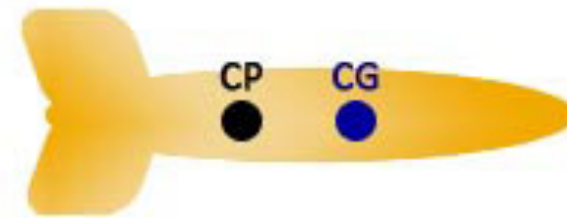
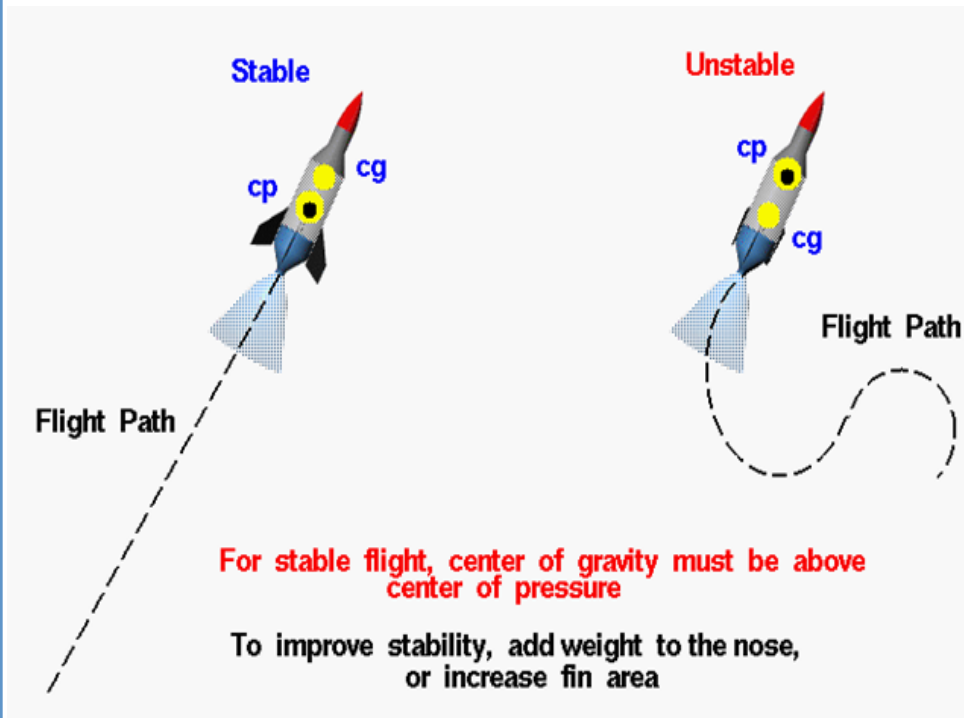


# Vehicle Stability, Rocket Flight Example (3)

## Weather Vane Analogy



# Vehicle Stability, Rocket Flight Example (4)





# Static Margin and Pitching Moment

*Static margin* is a concept used to characterize the static stability and controllability of aircraft and missiles.

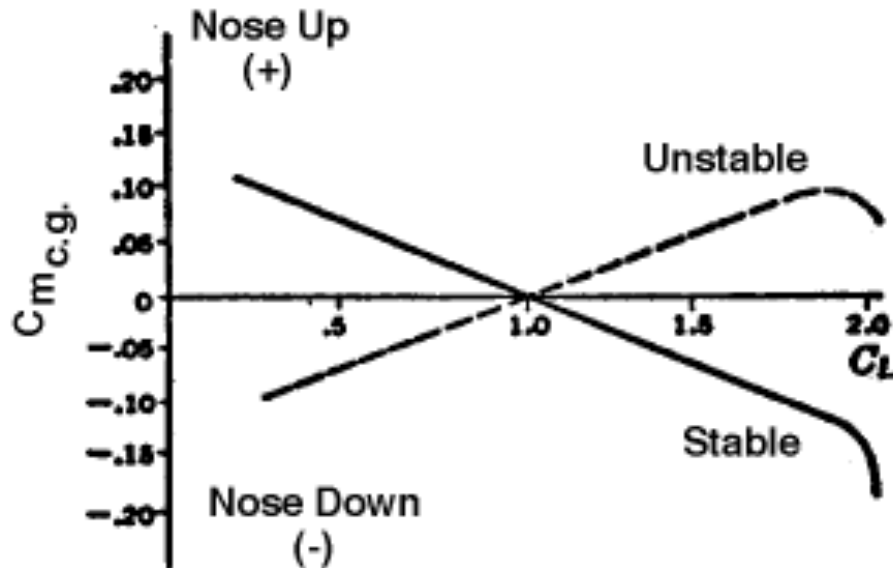
In aircraft analysis, static margin is defined as the non-dimensional distance between center of gravity and aerodynamic center of the aircraft.

In missile analysis, static margin is defined as non-dimensional distance between center of gravity and the center of pressure.

Stability requires that the pitching moment about the rotation point,  $C_m$ , become negative as we increase  $C_L$ :

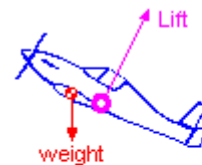
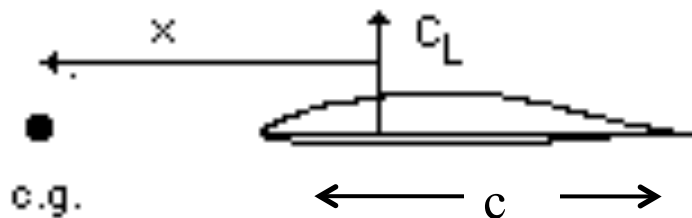
$$\frac{\partial C_m}{\partial C_L} < 0 \rightarrow C_m = C_{m_0} - \frac{x}{c} C_L \rightarrow \frac{\partial C_{m_{c.g.}}}{\partial C_L} = -\frac{x}{c} = \text{-static margin}$$

# Static Margin and Pitching Moment (2)

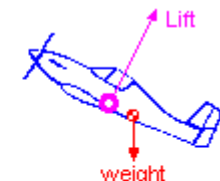


$$\frac{\partial C_{m_{c.g.}}}{\partial C_L} = -\frac{x}{c} = -\text{static margin}$$

*For a Rocket Static margin is the distance between the CG and the CP; divided by body tube diameter.*



When the CG is ahead of NP the weight tends to correct the upset = Stable



When the CG is behind NP the weight worsens the upset = Unstable

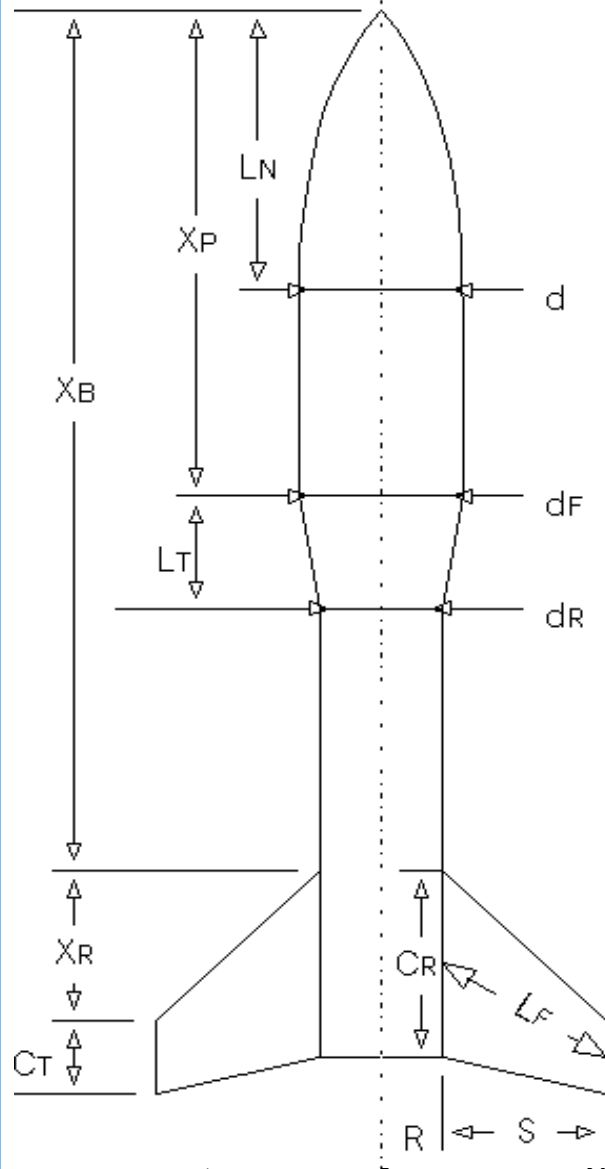
x is the distance from the system's aerodynamic center to the c.g..

# Calculating the Static Margin

- Key to calculating static margin is to estimate location of longitudinal center of pressure at low angles of attack
- [Barrowman equations](#) provide simple, accurate technique for Axi-symmetric rockets
- cg is measured as the longitudinal balance point of the rocket.
- As a rule of thumb, CP distance should be aft of the CG by at least one rocket diameter. -- "One Caliber stability".

# Calculating the Static Margin (2)

## Parameter Definitions



$L_N$  = length of nose

$d$  = diameter at base of nose

$d_F$  = diameter at front of transition

$d_R$  = diameter at rear of transition

$L_T$  = length of transition

$X_P$  = distance from tip of nose to front of transition

$C_R$  = fin root chord

$C_T$  = fin tip chord

$S$  = fin semispan

$L_F$  = length of fin mid-chord line

$R$  = radius of body at aft end

$X_R$  = distance between fin root leading edge and fin tip leading edge parallel to body

$X_B$  = distance from nose tip to fin root chord leading edge

$N$  = number of fins

# Calculating the Static Margin (3)

$X_N$  = center of pressure location for nose section

## Nose Cone Terms

$X_T$  = center of pressure location for transitions

- $(C_N)_N = 2$

- For Cone:  $X_N = 0.666L_N$

- For Ogive:  $X_N = 0.466L_N$

$(C_N)_N$  = normal force coefficient for nose

$(C_N)_T$  = normal force coefficient for transition

## Conical Transition Terms

$$(C_N)_T = 2 \left[ \left( \frac{d_R}{d} \right)^2 - \left( \frac{d_F}{d} \right)^2 \right] \rightarrow X_T = X_P + \frac{L_T}{3} \left[ 1 + \frac{1 - \frac{d_F}{d_R}}{1 - \left( \frac{d_F}{d_R} \right)^2} \right]$$

# Calculating the Static Margin (4)

## Fin Terms

$X_F$  = center of pressure location for fin groups

$(C_N)_F$  = Normal force coefficient for fin group

$$(C_N)_F = \left[ 1 + \frac{R}{S+R} \right] \left[ \frac{4N \left( \frac{S}{d} \right)^2}{1 + \sqrt{1 + \left( \frac{2L_F}{C_R + C_T} \right)^2}} \right] \longrightarrow X_F = X_B + \frac{X_R (C_R + 2C_T)}{3 (C_R + C_T)} + \frac{1}{6} \left[ (C_R + C_T) - \frac{(C_R C_T)}{(C_R + C_T)} \right]$$

## Finding the Center of Pressure

- Sum up coefficients:

$$(C_N)_R = (C_N)_N + (C_N)_T + (C_N)_F$$

- Find CP Distance from Nose Tip:

$$X_{cp} = \frac{(C_N)_N X_N + (C_N)_T X_T + (C_N)_F X_F}{(C_N)_R}$$

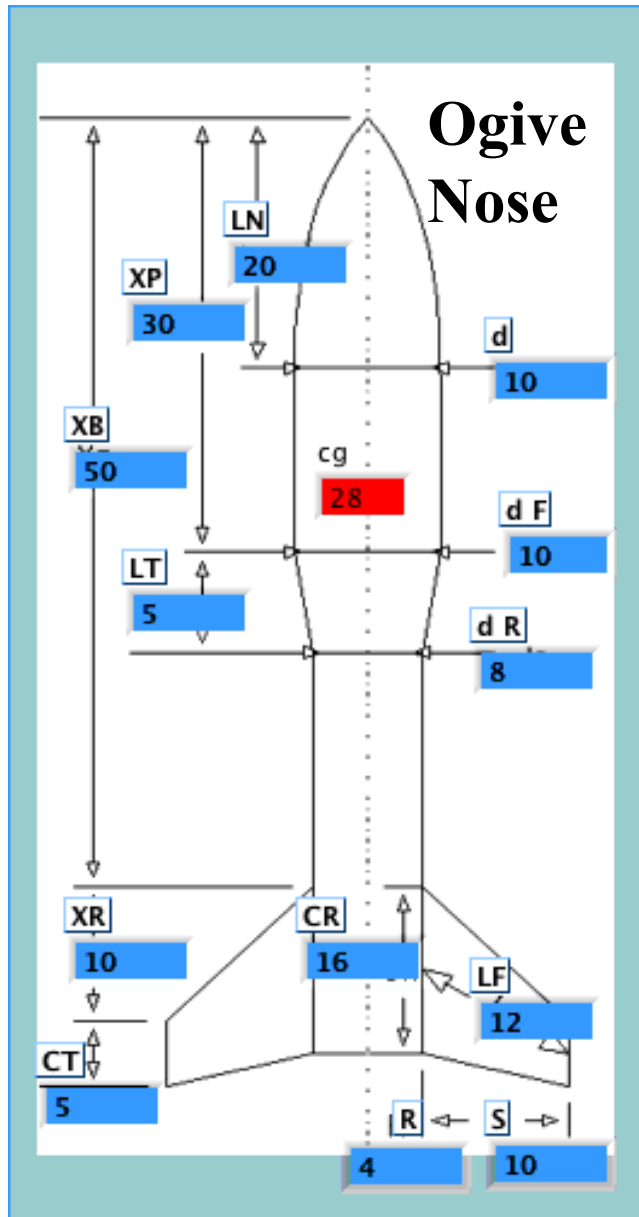
Static Margin ( $X_{sm}$ ) =

$$(X_{cp} - X_{cg}) / D_{max}$$

$(C_N)_R$  = total normal force coefficient

$X$  – measured aft from  
nose of vehicle

# Example: Stable Static Margin Vehicle



$$L_N = 20 \text{ cm}$$

$$d = 10 \text{ cm}$$

$$d_F = 10 \text{ cm}$$

$$d_R = 8 \text{ cm}$$

$$L_T = 5 \text{ cm}$$

$$X_P = 30 \text{ cm}$$

$$C_R = 10 \text{ cm}$$

$$C_T = 5 \text{ cm}$$

$$S = 10 \text{ cm}$$

$$L_F = 12 \text{ cm}$$

$$R = 4 \text{ cm}$$

$$X_R = 16 \text{ cm}$$

$$X_B = 50 \text{ cm}$$

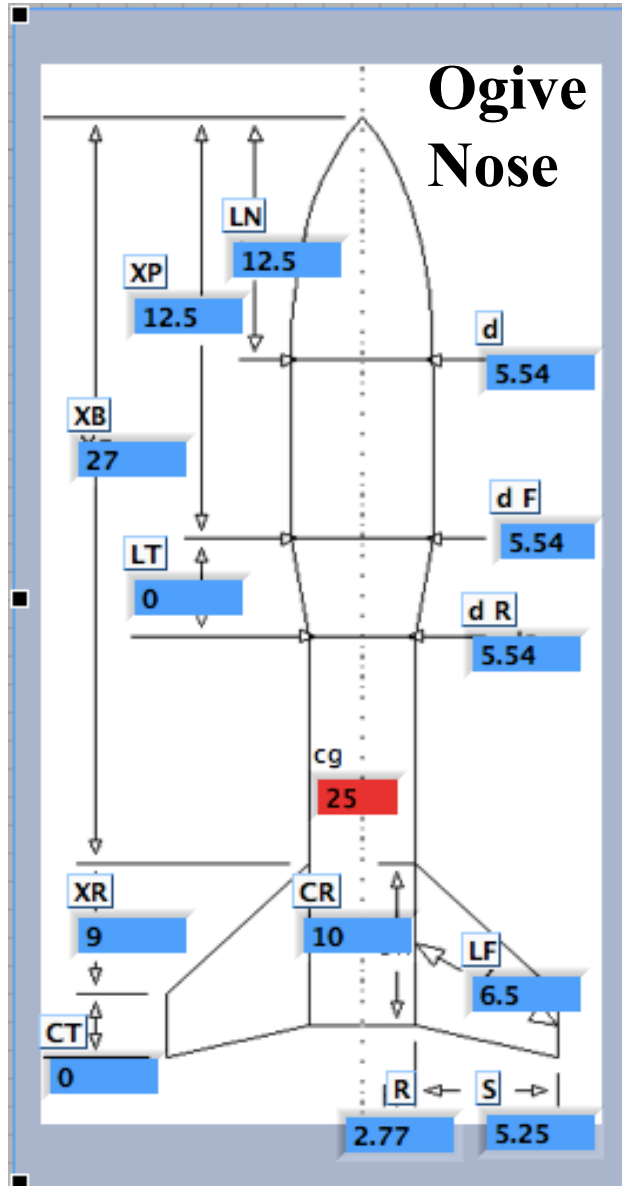
$$N = 3$$

Output parameters

Nosecone		Fins		Static Margin
CNN	2	CNF	6.12587	
XN, cm	9.32	XF, cm	56.9921	
Conical Transition		Total		
CNT	-0.72	CNR	7.40587	
XT, cm	32.4074	Xcp, cm	46.5081	

CG, cm from nose  
28

# Example: Unstable Static Margin Calculation



- $L_N = 12.5 \text{ cm}$
- $d = 5.54 \text{ cm}$
- $d_F = 5.54 \text{ cm}$
- $d_R = 5.54 \text{ cm}$
- $L_T = 0 \text{ cm}$
- $X_P = 12.5 \text{ cm}$
- $C_R = 10 \text{ cm}$
- $C_T = 0 \text{ cm}$
- $S = 5.25 \text{ cm}$
- $L_F = 6.5 \text{ cm}$
- $R = 2.77 \text{ cm}$
- $X_R = 9 \text{ cm}$
- $X_B = 27 \text{ cm}$
- $N = 3$

Constant diameter tube  
(No transition section)

Output parameters

Nosecone		Fins	
CNN	2	CNF	5.49166
XN, cm	5.825	XF, cm	31.6667
Conical Transition		Total	
CNT	0	CNR	7.49166
XT, cm	12.5	Xcp, cm	24.7679

Static Margin: -0.04189

CG, cm from nose

25



# Effect of Static Margin on Launch Conditions

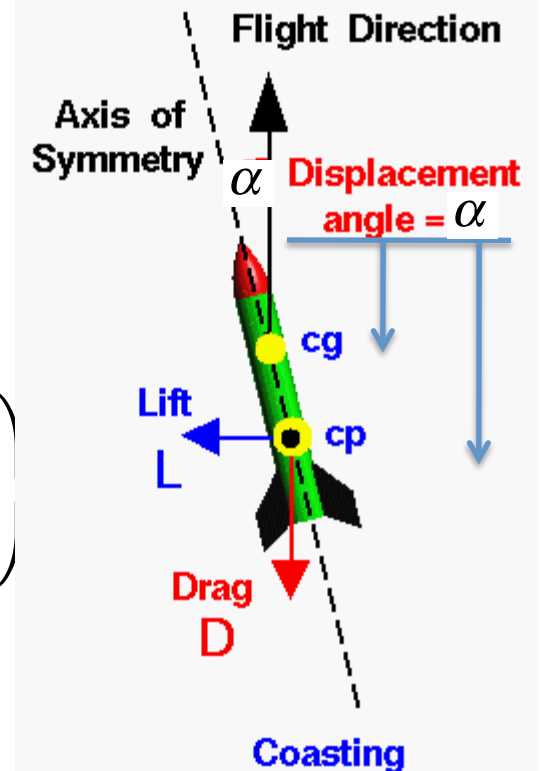
$$C_m = (C_m)_{\alpha=0} - \left( \frac{x_{cp} - x_{cg}}{D_{max}} \right) \cdot C_L$$

$$\rightarrow \frac{x_{cp} - x_{cg}}{D_{max}} = \text{"static margin"}$$

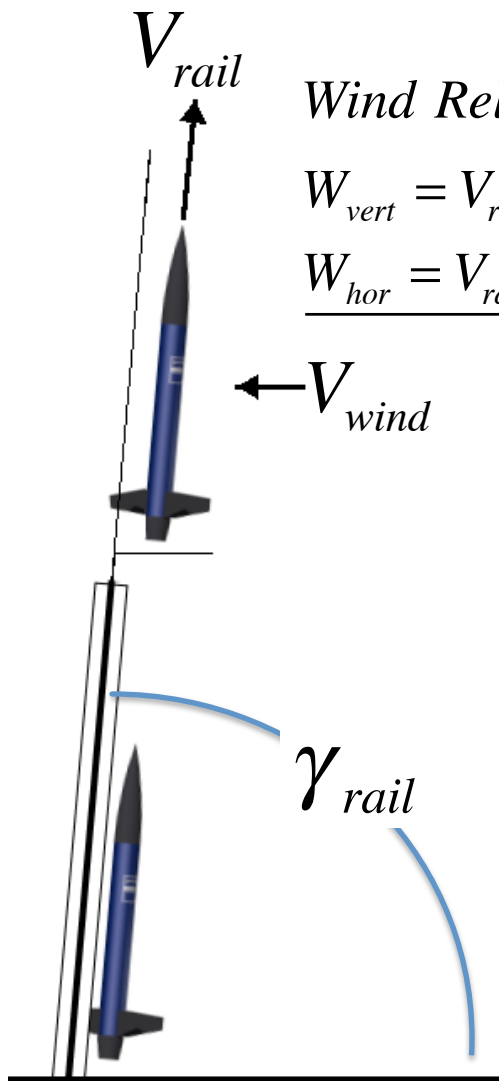
$$\text{Static Stability} \rightarrow \frac{\partial C_m}{\partial C_L} < 0 \approx - \left( \frac{x_{cp} - x_{cg}}{D_{max}} \right)$$

$$1 \leq \text{Desired Static Margin} \leq 2$$

$(C_m)_{\alpha=0} \approx 0$ ....for axi – symmetric rocket



# Effect of Static Margin on Launch Conditions (2)



Wind Relative Velocity :

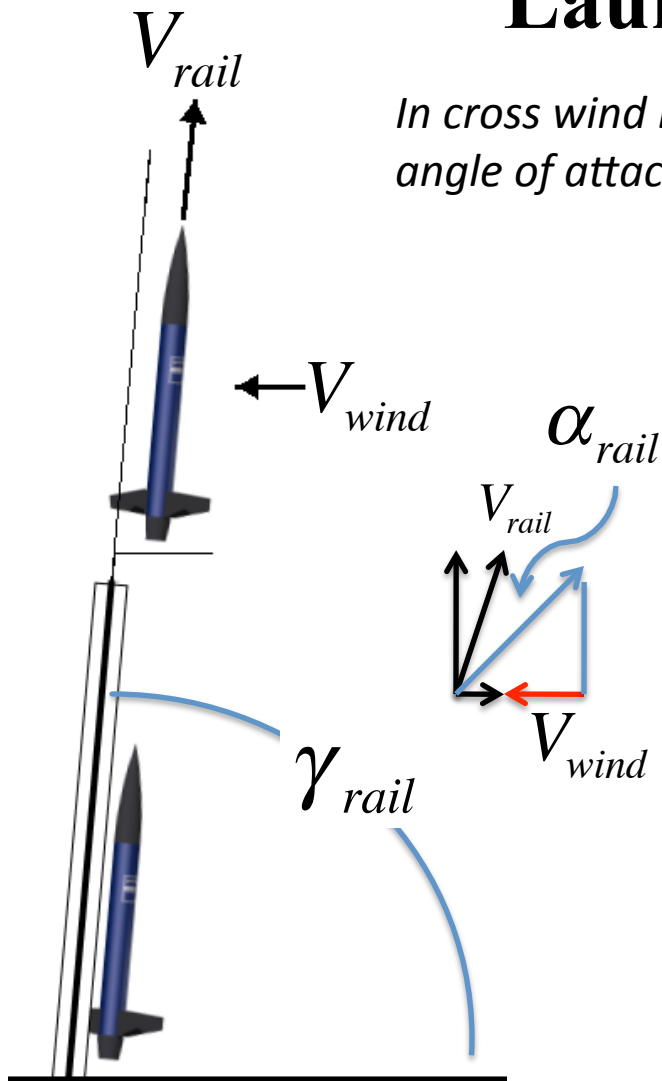
$$\left. \begin{aligned} W_{vert} &= V_{rail} \cdot \sin \gamma_{rail} \\ W_{hor} &= V_{rail} \cdot \cos \gamma_{rail} + V_{wind} \end{aligned} \right\} \rightarrow \gamma_{wind} = \tan^{-1} \left( \frac{V_{rail} \cdot \sin \gamma_{rail}}{V_{rail} \cdot \cos \gamma_{rail} + V_{wind}} \right)$$

$$\gamma_{wind} = \tan^{-1} \left( \frac{\sin \gamma_{rail}}{\cos \gamma_{rail} + \frac{V_{wind}}{V_{rail}}} \right)$$

$$\alpha_{rail} = \gamma_{rail} - \gamma_{wind}$$

# Effect of Static Margin on Launch Conditions (3)

In cross wind Launch, angle of attack is NOT! zero



Vwind/Vrail

0

0

0.1

0.2

0.5

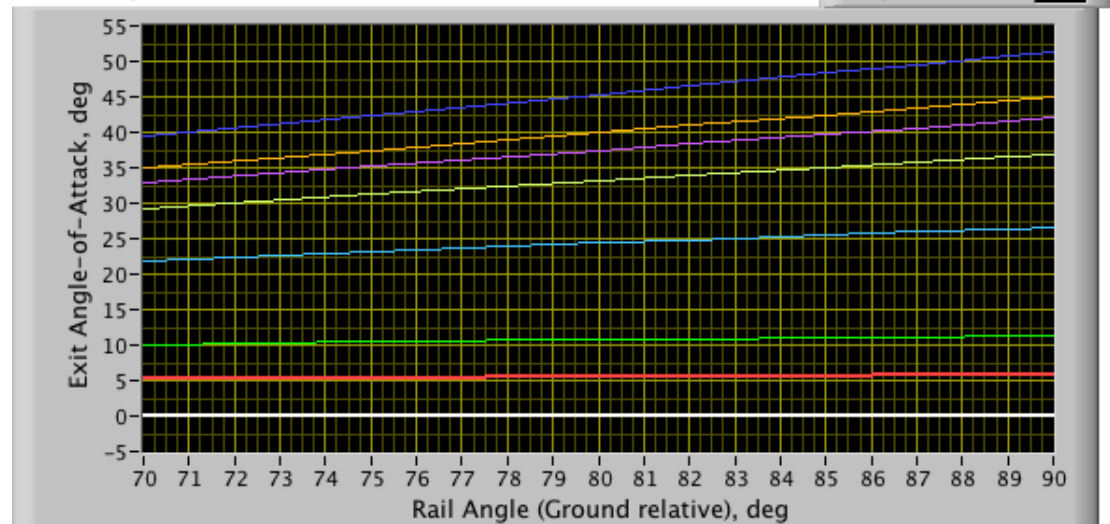
0.75

0.9

1

1.25

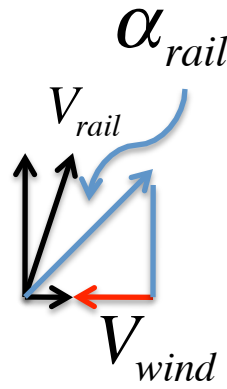
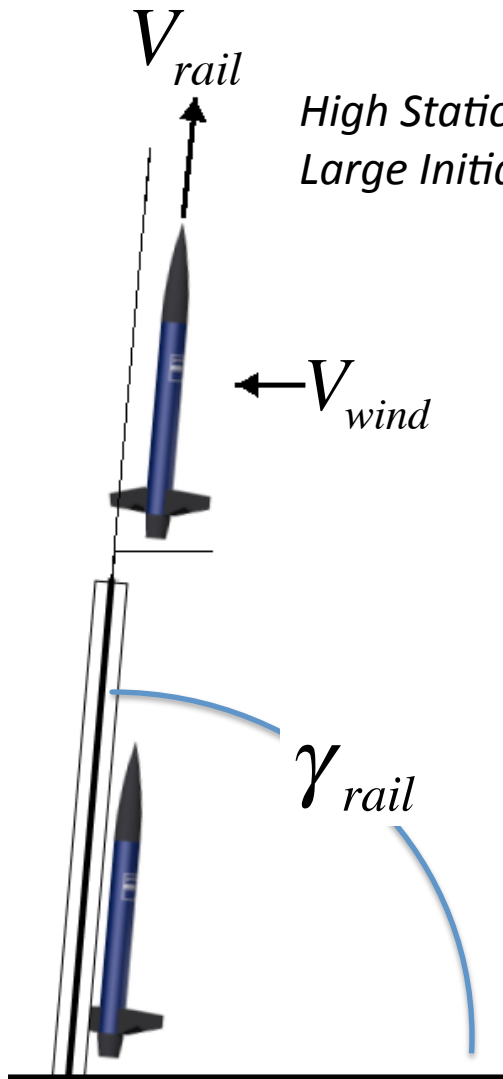
Rail Exit Angles



# Effect of Static Margin on Launch Conditions (4)

High Static Stability produces Large Initial Pitching Moment

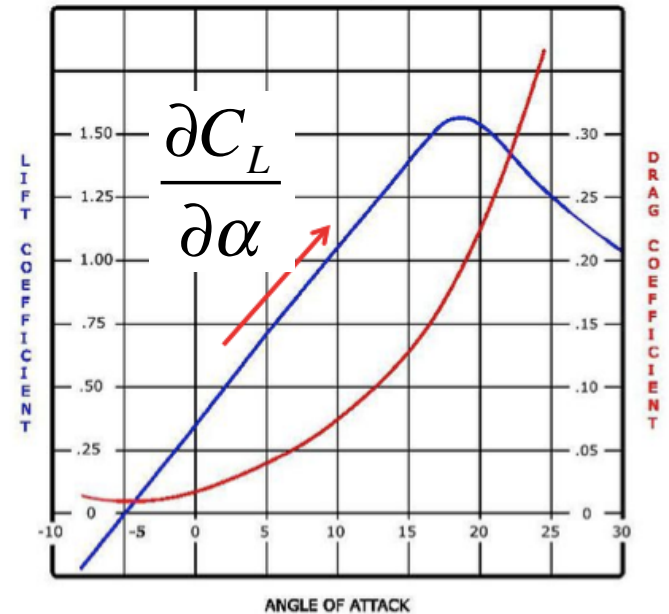
$$(C_m)_{launch} = (C_m)_{\alpha=0} - \left( \frac{x_{cp} - x_{cg}}{D_{max}} \right) \cdot C_L \approx - \left( \frac{x_{cp} - x_{cg}}{D_{max}} \right) \cdot \frac{\partial C_L}{\partial \alpha} \cdot \alpha_{rail}$$



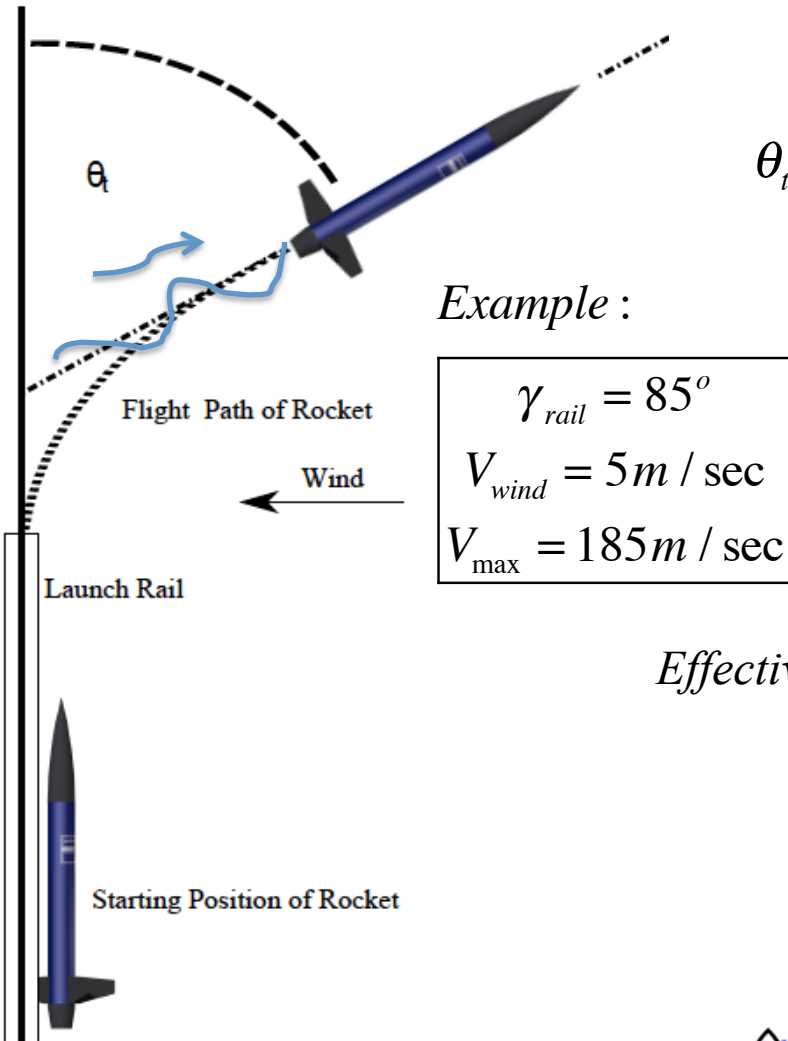
$$C_{L\alpha} \equiv \frac{\partial C_L}{\partial \alpha}$$

$$\gamma_{wind} = \tan^{-1} \left( \frac{\sin \gamma_{rail}}{\cos \gamma_{rail} + \frac{V_{wind}}{V_{rail}}} \right)$$

$$\alpha_{rail} = \gamma_{rail} - \gamma_{wind}$$



# Effect of Static Margin on Launch Conditions (5)



Example :

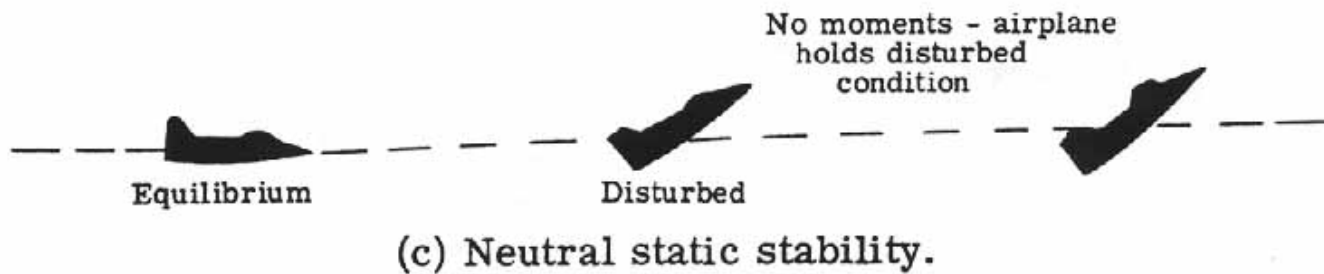
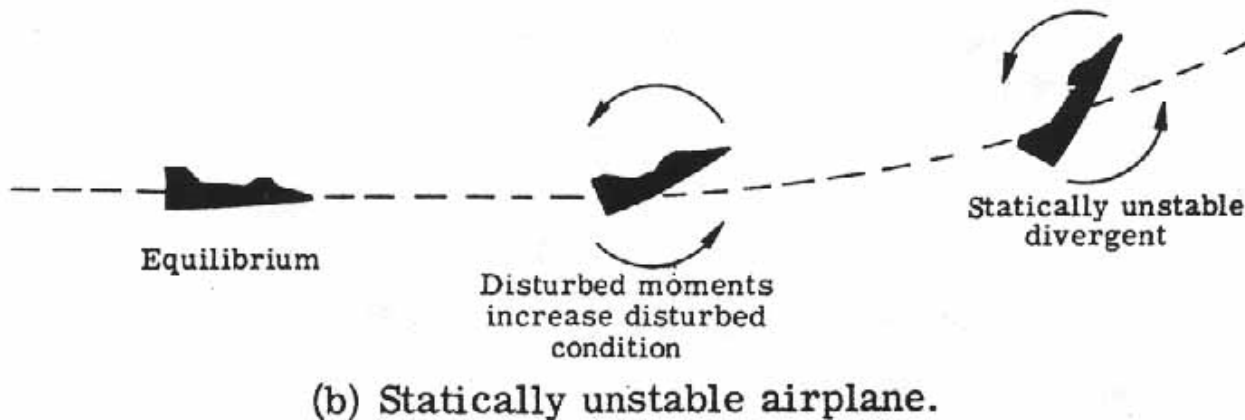
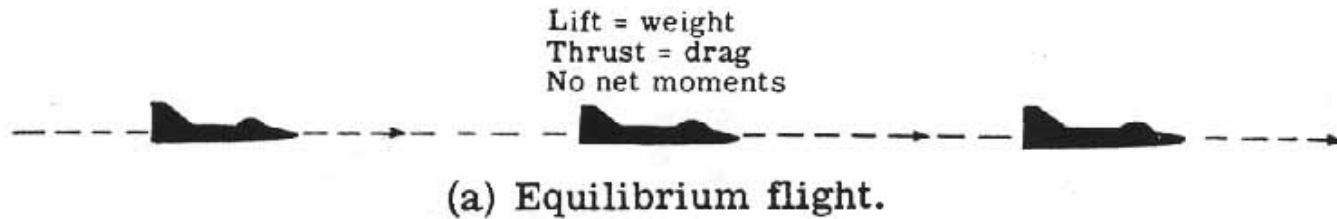
$$\begin{aligned} \gamma_{rail} &= 85^\circ \\ V_{wind} &= 5 \text{ m / sec} \\ V_{max} &= 185 \text{ m / sec} \end{aligned}$$

$$\theta_t \approx \gamma_{rail} - \tan^{-1} \left( \frac{\sin \gamma_{rail}}{\cos \gamma_{rail} + \frac{V_{wind}}{V_{max}}} \right)$$

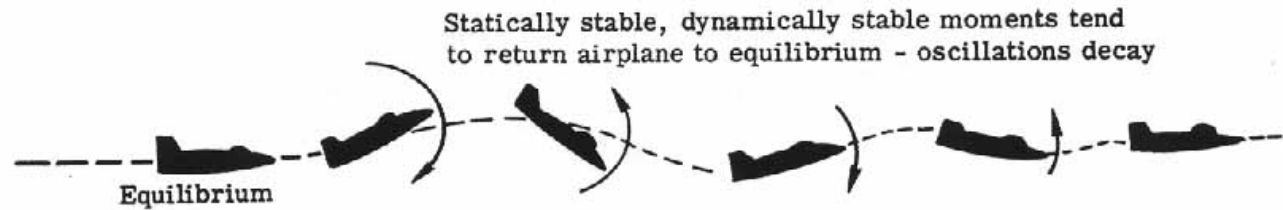
$$85 - \frac{180}{\pi} \operatorname{atan} \left( \frac{\sin \left( \frac{\pi}{180} 85 \right)}{\cos \left( \frac{\pi}{180} 85 \right) + \frac{5}{185}} \right) \geq 1.54^\circ$$

Effective Launch Angle  $\leq 83.46^\circ$

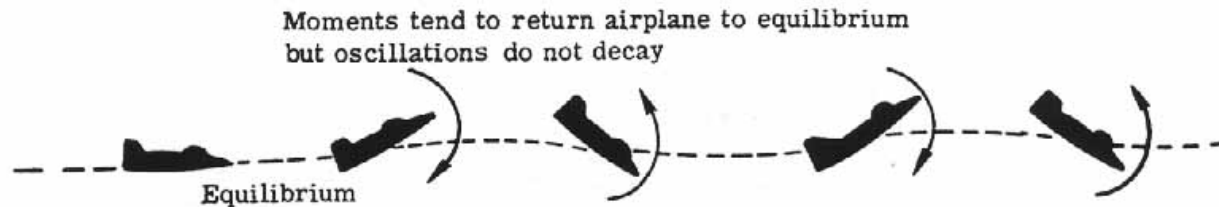
# Static Versus Dynamic Stability



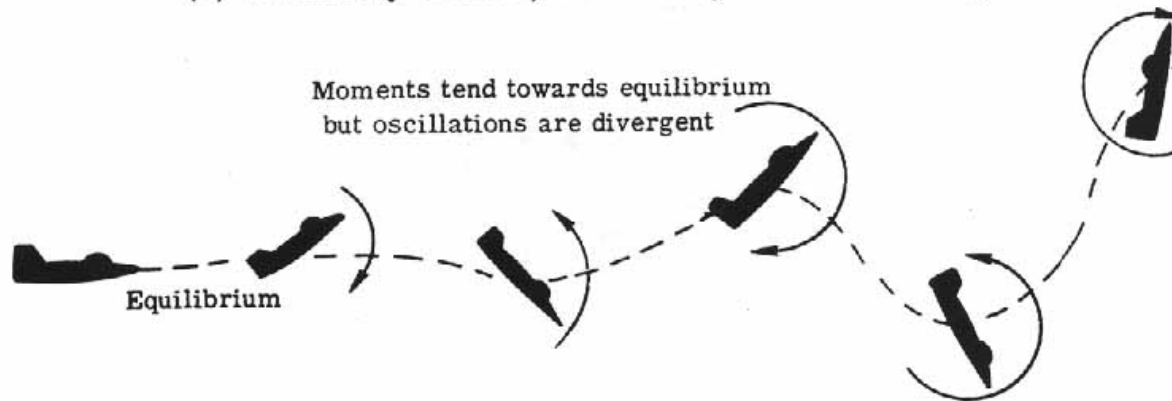
# Static Versus Dynamic Stability (2)



(a) Statically and dynamically stable.



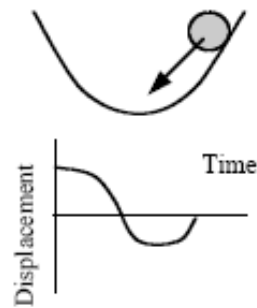
(b) Statically stable; neutral dynamic stability.



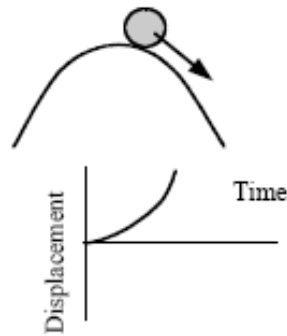
(c) Statically stable; dynamically unstable.

# Static Versus Dynamic Stability (3)

Positive: Quick to return, hard to displace



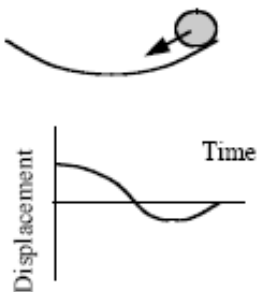
Negative: Quick to displace, hard to return



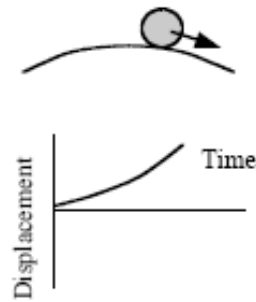
Neutral: Stays put



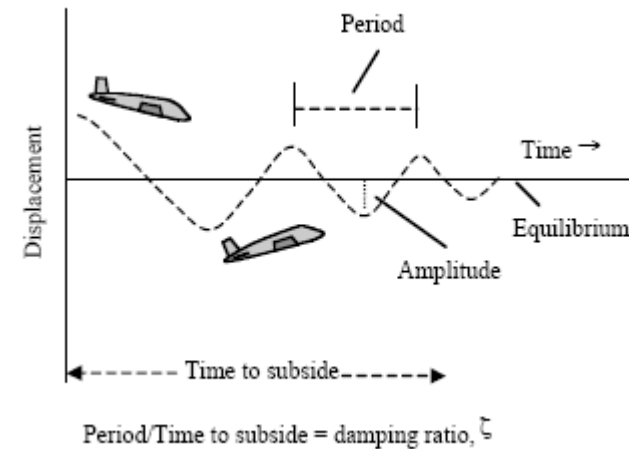
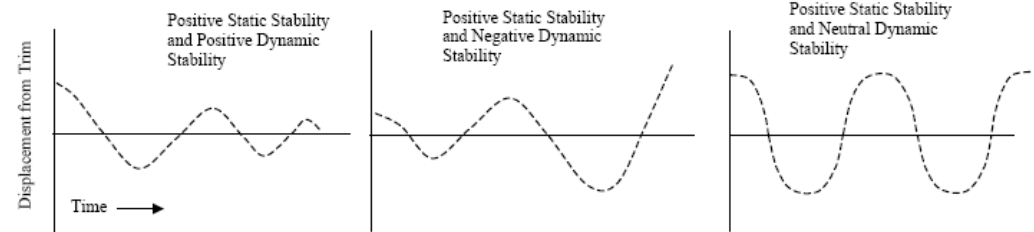
Positive: Slower to return, easier to displace



Negative: Slower to displace, easier to return

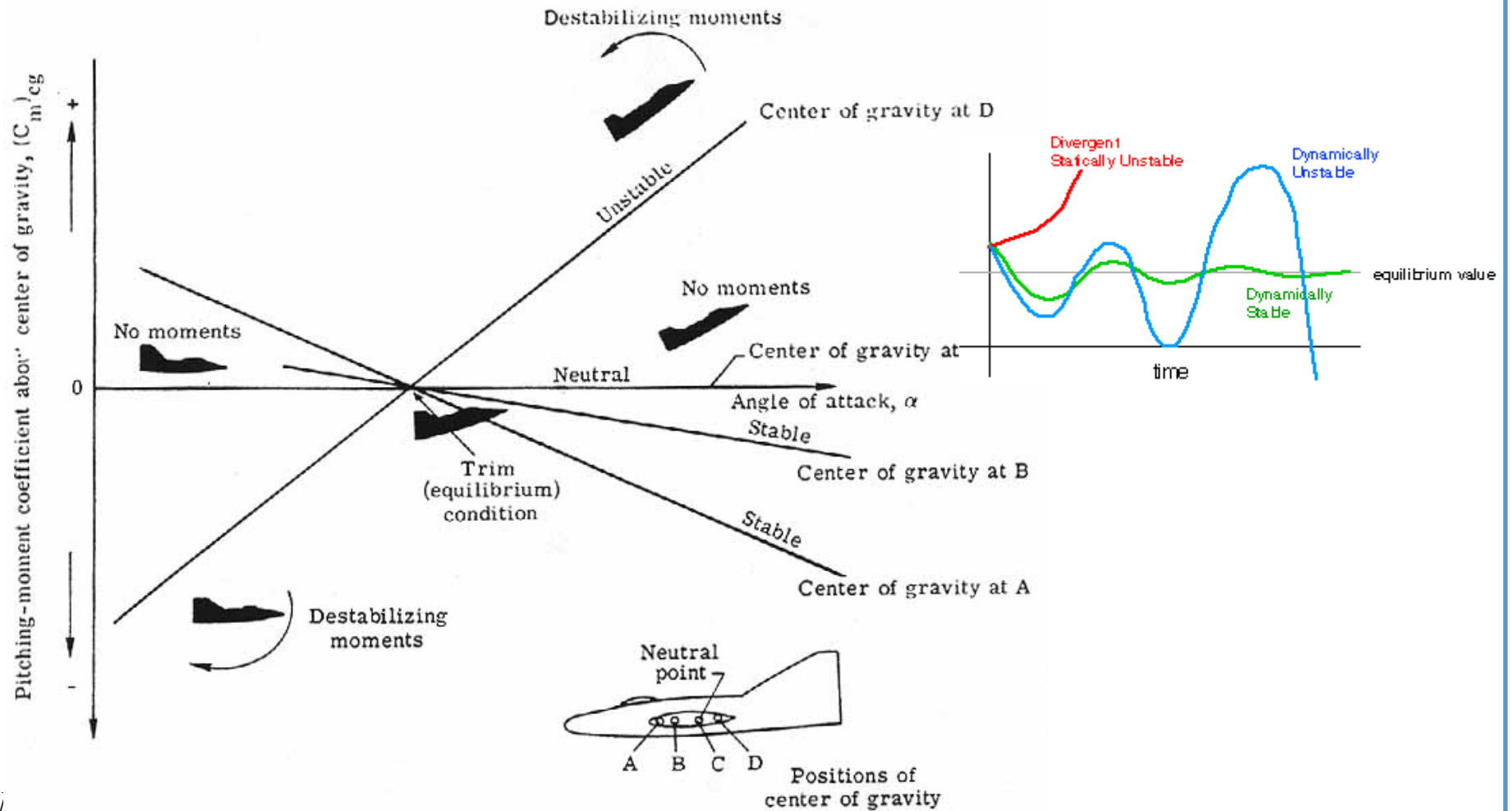


## Dynamic Stability





# Static Versus Dynamic Stability (4)



# Simplified Pitch Axis-Rotational Dynamics

*Neglect Cross Products of Inertia*

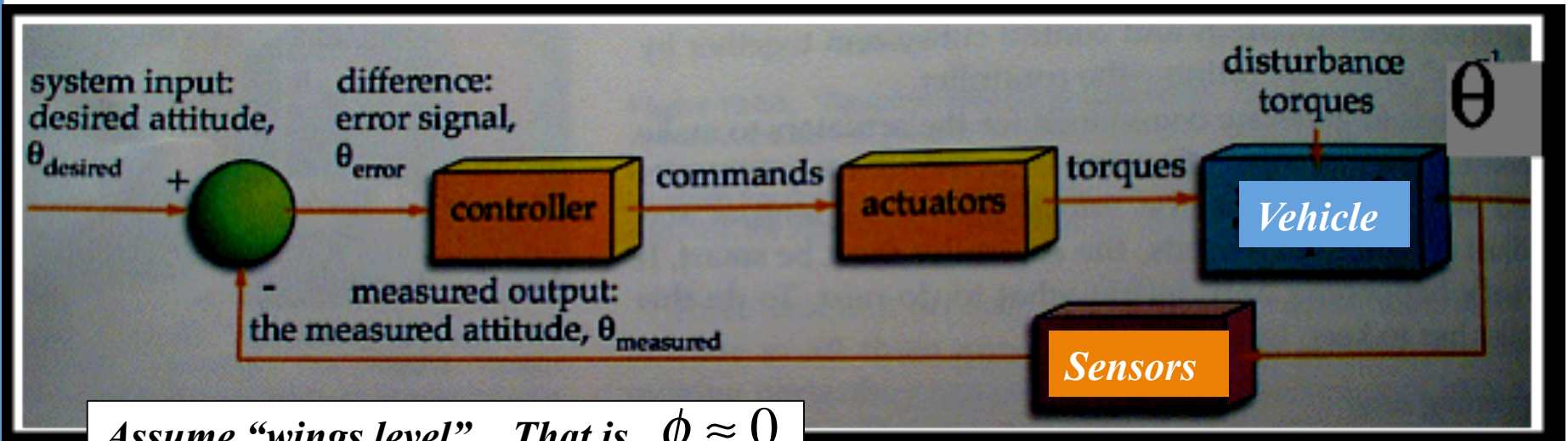
$$\begin{bmatrix} I_x & -I_{xy} & -I_{xz} \\ -I_{xy} & I_y & -I_{yz} \\ -I_{xz} & -I_{yz} & I_z \end{bmatrix} \cdot \begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \begin{pmatrix} q \cdot r (I_y - I_z) + (q^2 - r^2) I_{yz} + p \cdot q (I_{xz}) - r \cdot p (I_{xy}) \\ r \cdot p (I_z - I_x) + (r^2 - p^2) I_{xz} + q \cdot r (I_{xy}) - p \cdot q (I_{yz}) \\ p \cdot q (I_x - I_y) + (p^2 - q^2) I_{xy} + r \cdot p (I_{yz}) - q \cdot r (I_{xz}) \end{pmatrix} + \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix}$$

$$\rightarrow \dot{q} = \ddot{\theta} = \frac{M_y}{I_y} + r \cdot p \frac{(I_z - I_x)}{I_y}$$

*Forcing moment*

*Second order Disturbance torque  
(neglected when r, p are small)*

# Simplified Pitch Axis-Rotational Dynamics (2)



Assume "wings level" .. That is  $\phi \approx 0$

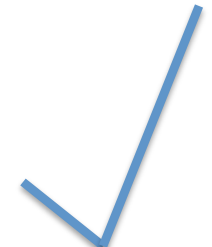
$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{pmatrix} 1 & \sin \phi \tan \theta & \cos \phi \tan \theta \\ 0 & \cos \phi & -\sin \phi \\ 0 & \frac{\sin \phi}{\cos \theta} & \frac{\cos \phi}{\cos \theta} \end{pmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \rightarrow \begin{bmatrix} \dot{\theta} = q \\ \ddot{\theta} = \dot{q} \end{bmatrix}$$

# Simplified Pitch Axis-Rotational Dynamics (3)

Neglecting Disturbance torques

$$\ddot{\theta} = \dot{q} = \frac{M_y}{I_y} \rightarrow M_y \approx \text{"pitching moment"}$$

"pitching moment coefficient"  $\equiv C_m = \frac{M_y}{\bar{q} \cdot A_{ref} \cdot c_{ref}}$



Check Units

$$\bar{q} = \left( \frac{1}{2} \cdot \rho \cdot V^2 \right)$$

$$A_{ref} = \frac{\pi}{4} \cdot D_{ref}^2$$

"reference length"  $\rightarrow c_{ref} = D_{max}$

$$\frac{\bar{q} \cdot A_{ref} \cdot c_{ref}}{I_y} \sim \frac{\frac{Nt}{m^2} \cdot m^2 \cdot m}{kg \cdot m^2} \sim \left( \frac{kg \cdot m}{sec^2 m^2} \cdot m^2 \cdot m \right) \cdot \frac{1}{kg \cdot m^2} \sim \frac{1}{sec^2}$$

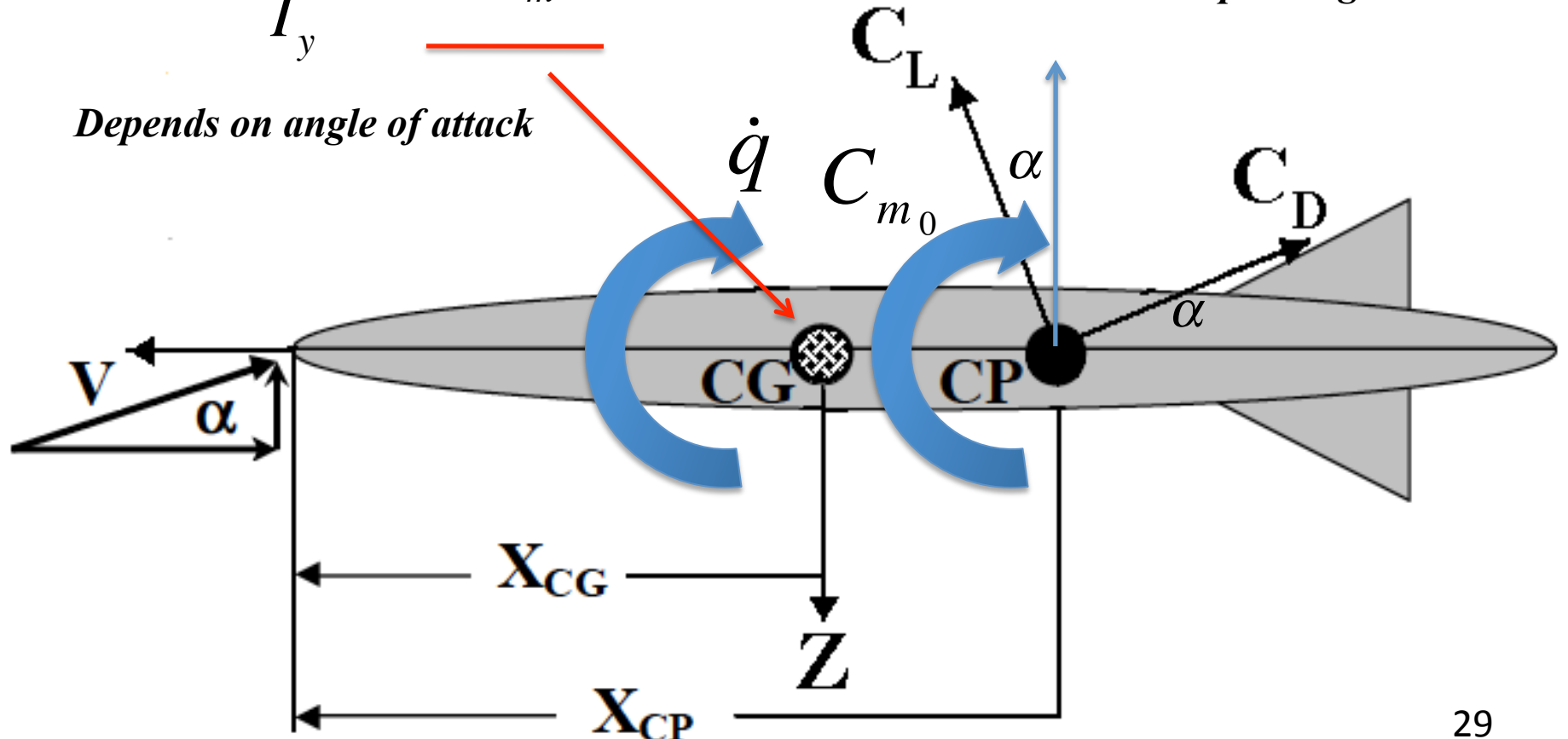
# Simplified Pitch Axis-Rotational Dynamics (4)

*Pitching moment about cg*

$$\dot{q} = \frac{\bar{q} \cdot A_{ref} \cdot c_{ref}}{I_y} \cdot C_m$$

*Depends on angle of attack*

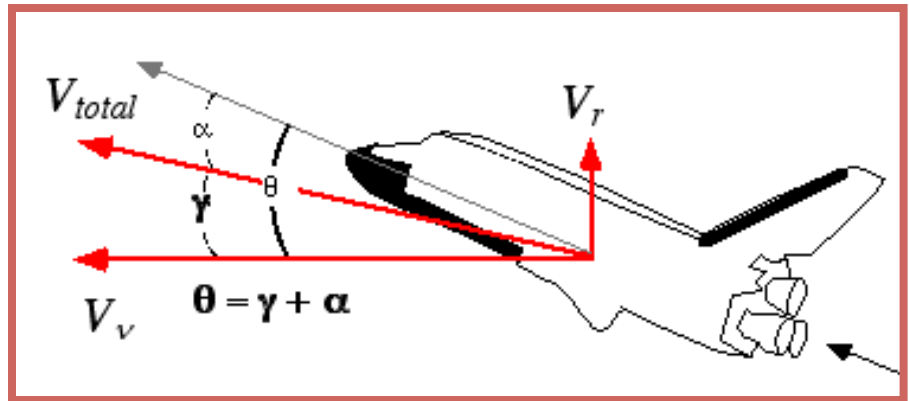
*Gravity acts at CG and  
Cannot induce pitching moment*



# Simplified Pitch Axis-Rotational Dynamics (4)

$$\ddot{\theta} = \dot{q} = \frac{\bar{q} \cdot A_{ref} \cdot C_{ref}}{I_y} \cdot C_m$$

*Depends on angle of attack +  
Control inputs*



It can also be shown that ... (NASA RP-1168 pp 10-22) that for angle of attack

$$\dot{\alpha} = -\frac{\bar{q} \cdot A_{ref}}{m \cdot V} \cdot C_L + q + \frac{g_{(r)}}{V} \cos(\theta - \alpha) - \frac{F_{thrust} \sin \alpha}{m \cdot V}$$

# Simplified Pitch Axis-Rotational Dynamics (5)

Collected, simplified pitch dynamics equations

$$\begin{bmatrix} \dot{\alpha} \\ \dot{q} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} -\frac{\bar{q} \cdot A_{ref}}{m \cdot V} \cdot C_L + q + \frac{g_{(r)}}{V} \cos(\theta - \alpha) - \frac{F_{thrust} \sin \alpha}{m \cdot V} \\ \frac{\bar{q} \cdot A_{ref} \cdot c_{ref}}{I_y} \cdot C_m \\ q \end{bmatrix}$$

# Collected Longitudinal Equations of Motion

$$\begin{bmatrix} \dot{V}_r \\ \dot{V}_v \\ \dot{r} \\ \dot{v} \\ \dot{x} \\ \dot{\alpha} \\ \dot{q} \\ \dot{\theta} \\ \dot{m} \end{bmatrix} = \begin{bmatrix} \left(\frac{V_v^2}{r}\right) + \left(\frac{\bar{q} \cdot A_{ref}}{m}\right) \cdot (C_L \cos \gamma - C_D \sin \gamma) + \left(\frac{F_{thrust} \sin \theta}{m}\right) - g_{(r)} \\ -\left(\frac{V_r \cdot V_v}{r}\right) - \left(\frac{\bar{q} \cdot A_{ref}}{m}\right) \cdot (C_L \sin \gamma + C_D \sin \gamma) + \left(\frac{F_{thrust} \cos \theta}{m}\right) \\ V_r \\ \frac{V_v}{r} \\ V_v \\ -\frac{\bar{q} \cdot A_{ref}}{m \cdot V} \cdot C_L + q + \frac{g_{(h)}}{V} \cos(\gamma) - \frac{F_{thrust} \sin \alpha}{m \cdot V} \\ \frac{\bar{q} \cdot A_{ref} \cdot c_{ref}}{I_y} \cdot C_m \\ q \\ -\frac{F_{thrust}}{g_0 \cdot I_{sp}} \end{bmatrix}$$

How do we control this “mess”  
.. With “ $F_{thrust}$ ,  $C_m$ ,  $C_L$ , and  $C_D$ ”

$$\begin{aligned}
 g_{(r)} &= \frac{\mu}{r^2} \\
 \gamma &= \tan^{-1} \frac{V_r}{V_v} = \theta - \alpha \\
 \dot{x} &= r \cdot \dot{v} \\
 V &= \sqrt{V_r^2 + V_v^2} \\
 \bar{q} &= \frac{1}{2} \cdot \rho_{(h)} \cdot V^2 \\
 h &= r - R_{earth}
 \end{aligned}$$



# Fixed Wing Aircraft Controls

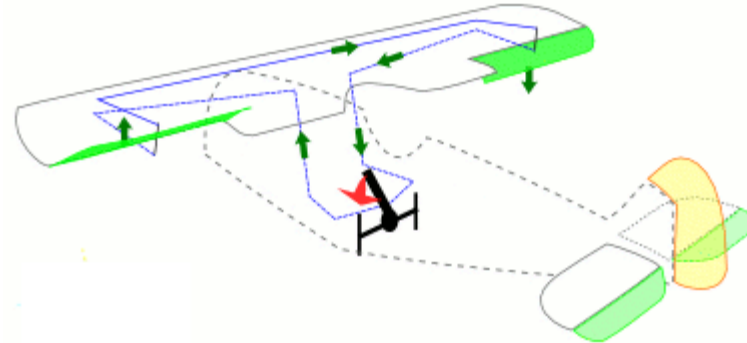
-- Stick Roll Control (ailerons)

-- Stick Pitch Control (elevator)

-- Pedal Yaw Control (Rudder)

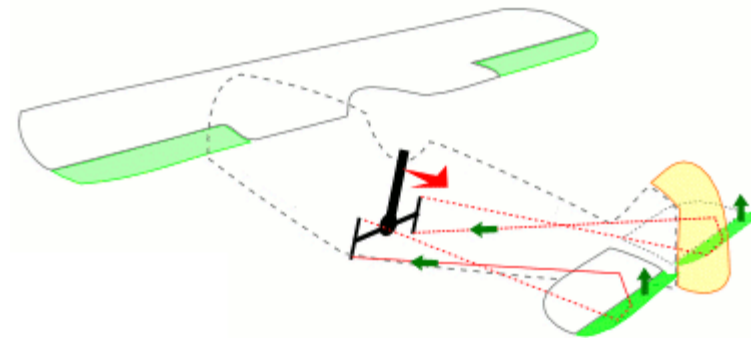
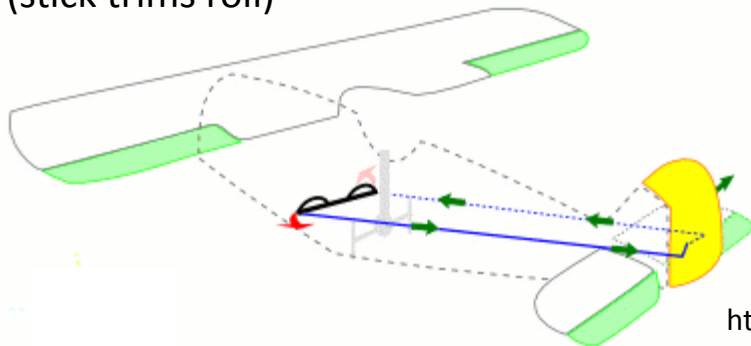
-- Throttle (Thrust)

Roll Control From Ailerons (rudder “coordinates” turn)



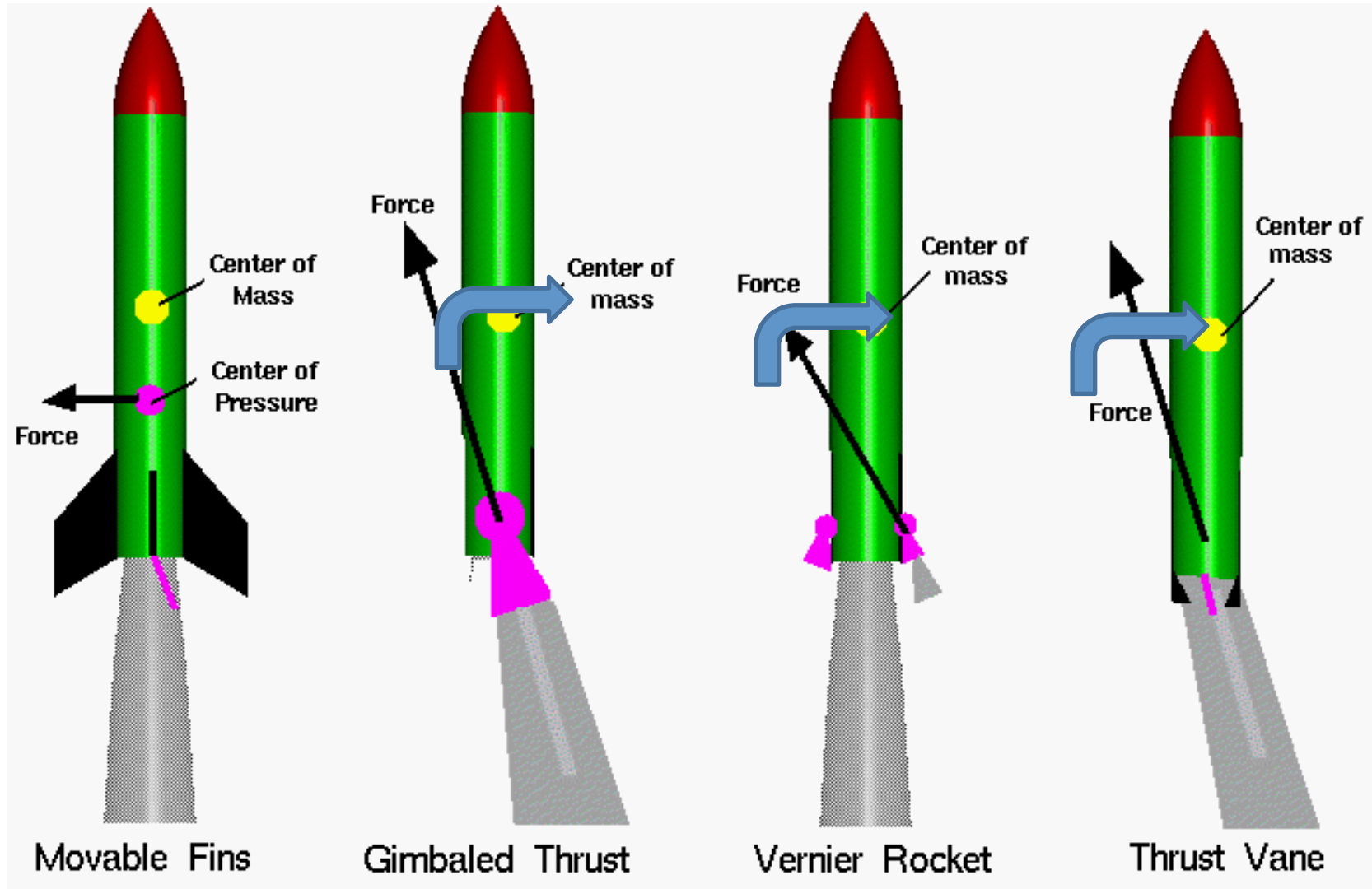
*Pitch Control From Elevator (throttle “coordinates” climb)*

Yaw Control From Rudder  
(stick trims roll)



<http://en.wikipedia.org/wiki/File:ControlSurfaces.gif>

# Launch (Rocket) Controls



# Spacecraft Reaction Controls (RCS)

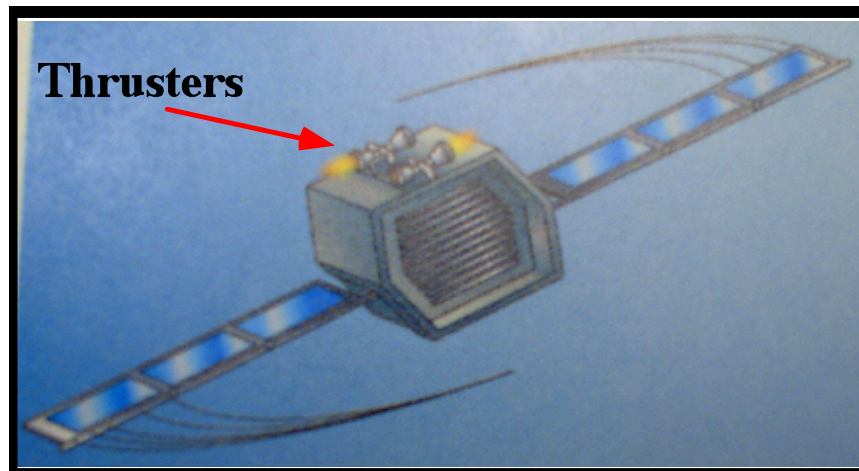
- The spacecraft propulsion system provides controlled impulse for:

- Orbit insertion and transfers
- Orbit maintenance (station keeping)
- Attitude Control

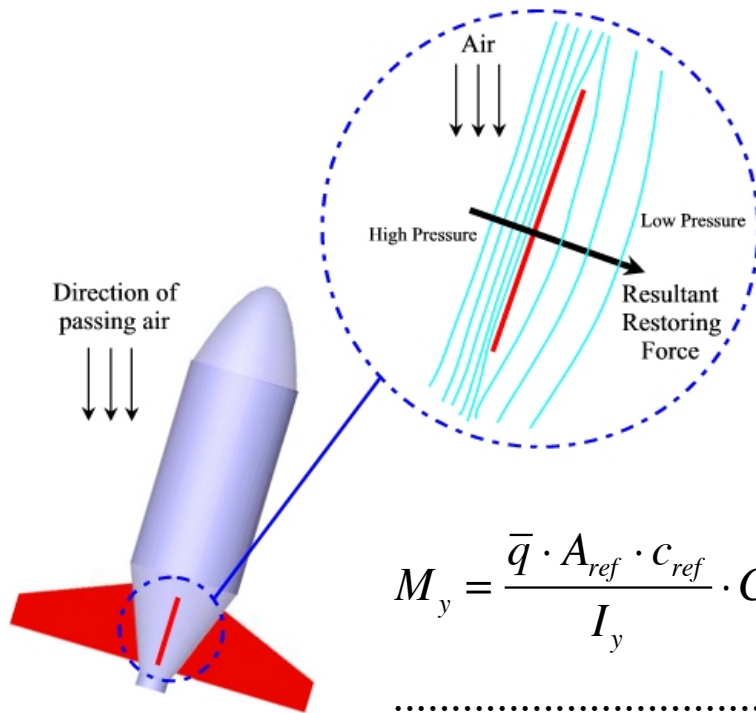
**Thruster rockets apply force at some distance away from center of mass, causing a torque that rotates the spacecraft**

- Propulsion Types

- Cold gas, monopropellant, bipropellants, ion



# Pitching Moment Control of Vehicle

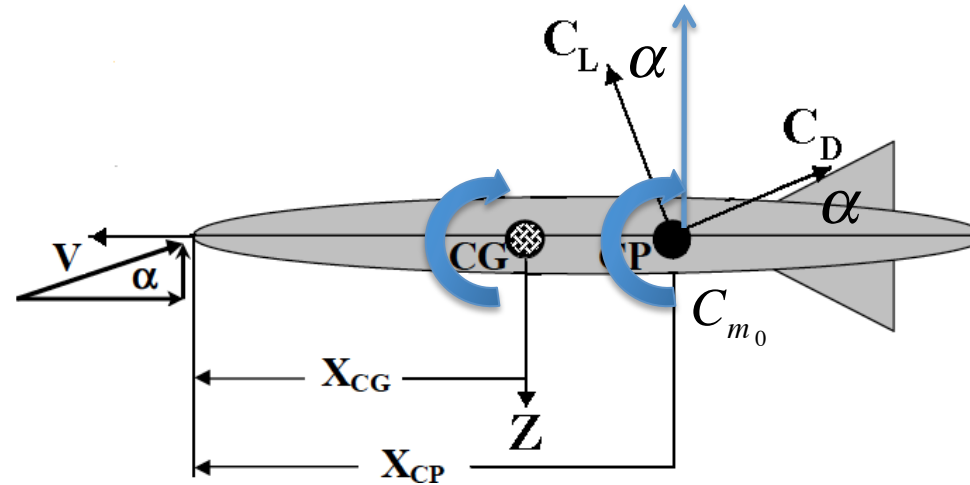


$$\begin{bmatrix} \dot{\alpha} \\ \dot{q} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} -\frac{\bar{q} \cdot A_{ref}}{m \cdot V} \cdot C_L + q + \frac{g(r)}{V} \cos(\theta - \alpha) - \frac{F_{thrust} \sin \alpha}{m \cdot V} \\ \frac{\bar{q} \cdot A_{ref} \cdot c_{ref}}{I_y} \cdot C_m \\ q \end{bmatrix}$$

$$M_y = \frac{\bar{q} \cdot A_{ref} \cdot c_{ref}}{I_y} \cdot C_m = \frac{\bar{q} \cdot A_{ref} \cdot c_{ref}}{I_y} \cdot [C_m(\alpha) + C_m(\delta)]$$

.....depends on  $\alpha$ .....depends on control " $\delta$ "

# Vehicle "Aerodynamic" Pitching Moment



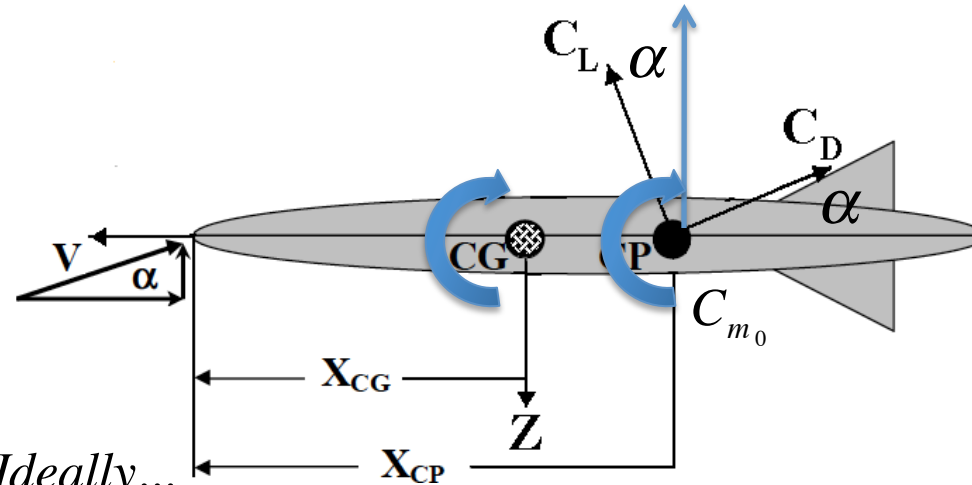
$$C_m(\alpha) = C_{m_0} + \frac{(X_{cg} - X_{cp})}{c_{ref}} (C_L \cdot \cos \alpha + C_D \cdot \sin \alpha) = C_{m_0} - X_{sm} (C_L \cdot \cos \alpha + C_D \cdot \sin \alpha)$$

$$\text{Small } \alpha \rightarrow C_m(\alpha) = C_{m_0} - X_{sm} (C_L + C_D \alpha) \rightarrow \frac{\partial C_m}{\partial \alpha} = -X_{sm} \left( \frac{\partial C_L}{\partial \alpha} + \frac{\partial C_D}{\partial \alpha} \alpha + C_D \right)$$

$$\text{Linearized} \rightarrow C_m(\alpha) = C_{m_0} - X_{sm} \left( \frac{\partial C_L}{\partial \alpha} + C_D \right) \cdot \alpha \equiv C_{m_0} + \frac{\partial C_m}{\partial \alpha} \cdot \alpha$$

$$\frac{\partial C_m}{\partial \alpha} \rightarrow "C_{m_\alpha}"$$

# Vehicle “Aerodynamic” Pitching Moment (2)



$$\frac{\partial C_m}{\partial \alpha} \rightarrow "C_{m_\alpha}" \dots \text{Ideally...}$$

$$C_{m_\alpha} = -X_{sm} \left( \frac{\partial C_L}{\partial \alpha} + C_D \right) \rightarrow \begin{bmatrix} \frac{\partial C_L}{\partial \alpha} \\ C_D \end{bmatrix} > 0 \rightarrow X_{sm} > 0 \dots \begin{array}{l} \text{static} \\ \text{stability} \end{array} \rightarrow \boxed{C_{m_\alpha} < 0 \dots \begin{array}{l} \text{static} \\ \text{stability} \end{array}}$$

## Linear Aerodynamic Model

*Linear Aerodynamic Model*

$$\text{Lift Coefficient} \rightarrow C_L = C_{L_0} + C_{L_\alpha} \cdot \alpha + C_{L_\delta} \cdot \delta$$

$$\text{Drag Coefficient} \rightarrow C_D = C_{D_0} + C_{D_\alpha} \cdot \alpha + C_{D_\delta} \cdot \delta$$

$$\text{Pitching Moment Coefficient} \rightarrow C_m = C_{m_0} + C_{m_\alpha} \cdot \alpha + C_{m_\delta} \cdot \delta$$

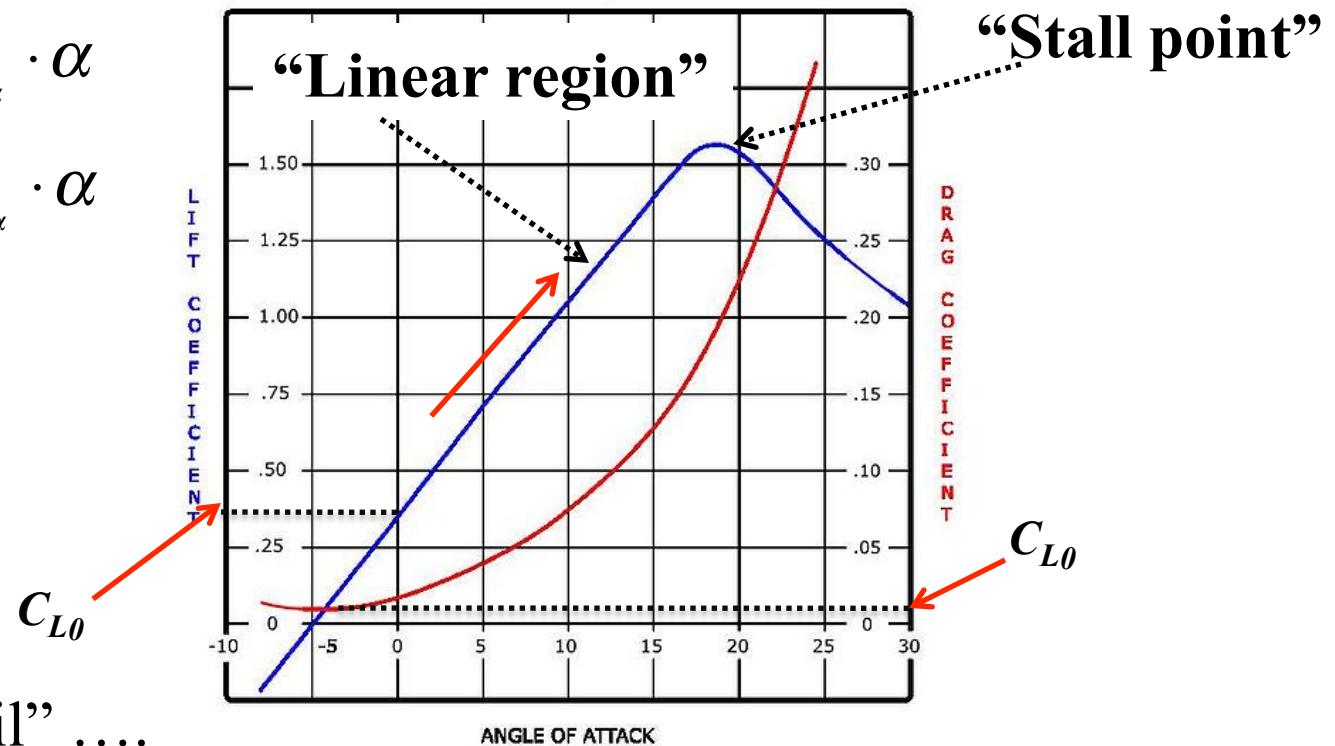
$$\text{For Rocket (Typically)} \rightarrow \left. \begin{array}{l} \{ C_{L_0} \quad C_{m_0} \} \approx 0 \\ C_{m_0} \neq 0 \end{array} \right\}$$

$\delta \rightarrow$  Longitudinal Control Actuator

# Linear Aerodynamic Model (2)

$$C_L = C_{L_0} + C_{L_\alpha} \cdot \alpha$$

$$C_D = C_{D_0} + C_{D_\alpha} \cdot \alpha$$



For “Thin Airfoil” ....

$$C_D \approx C_{D_0} + \frac{C_L^2}{\pi \cdot \epsilon \cdot AR}$$

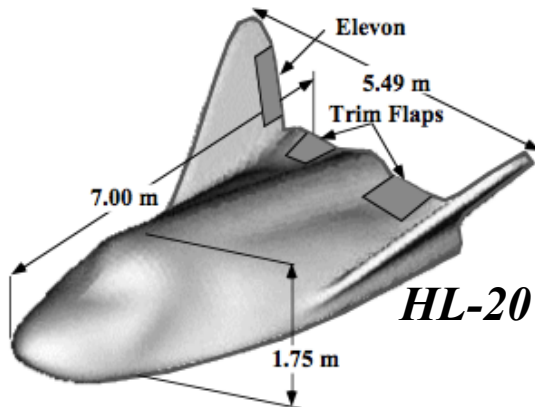
$0.85 < \epsilon < 0.95$

$C_{D_0} = \text{parasite drag}$   
 $\epsilon = \text{Oswald Efficiency Factor}$   
 $AR \rightarrow \text{Aspect Ratio} = b^2 / A_{ref}$

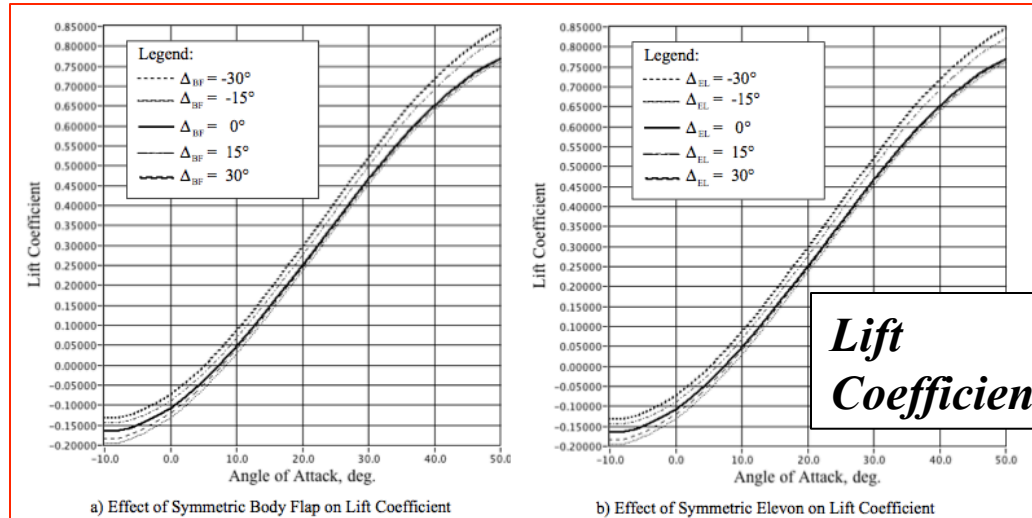
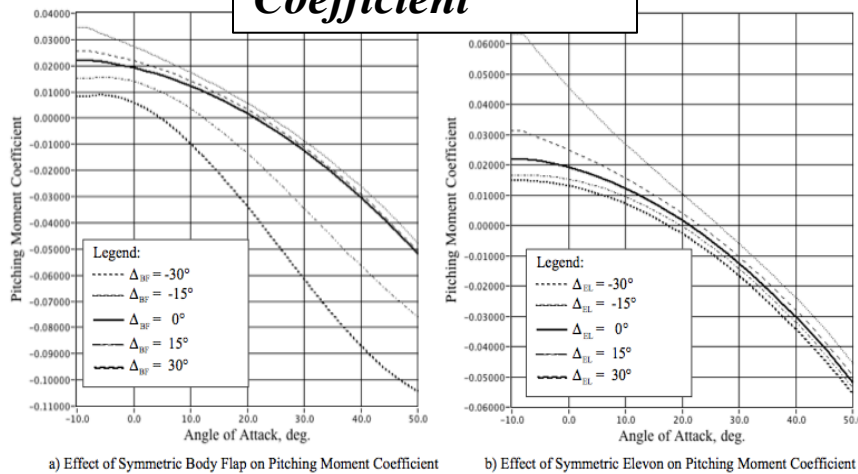
**Supersonic flow also has  
 “wave drag”  
 ... But we won’t worry  
 about that here**



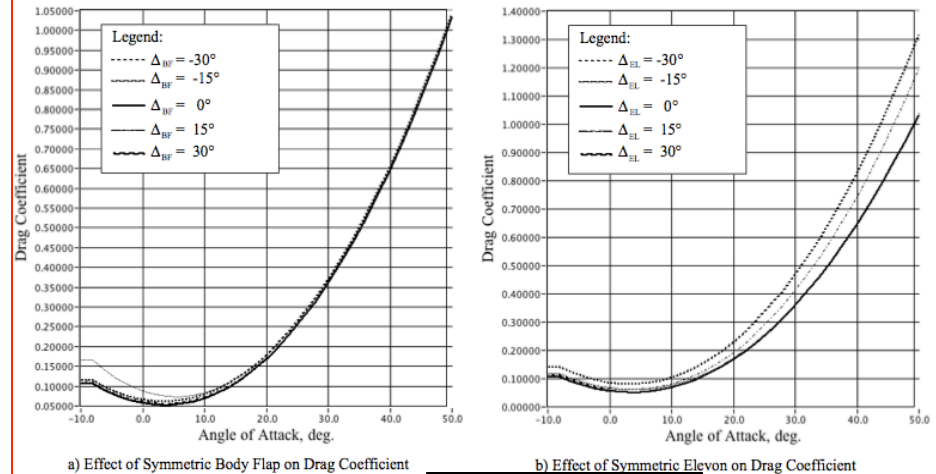
# Examples of Aerodynamic Coefficients



## Pitching Moment Coefficient

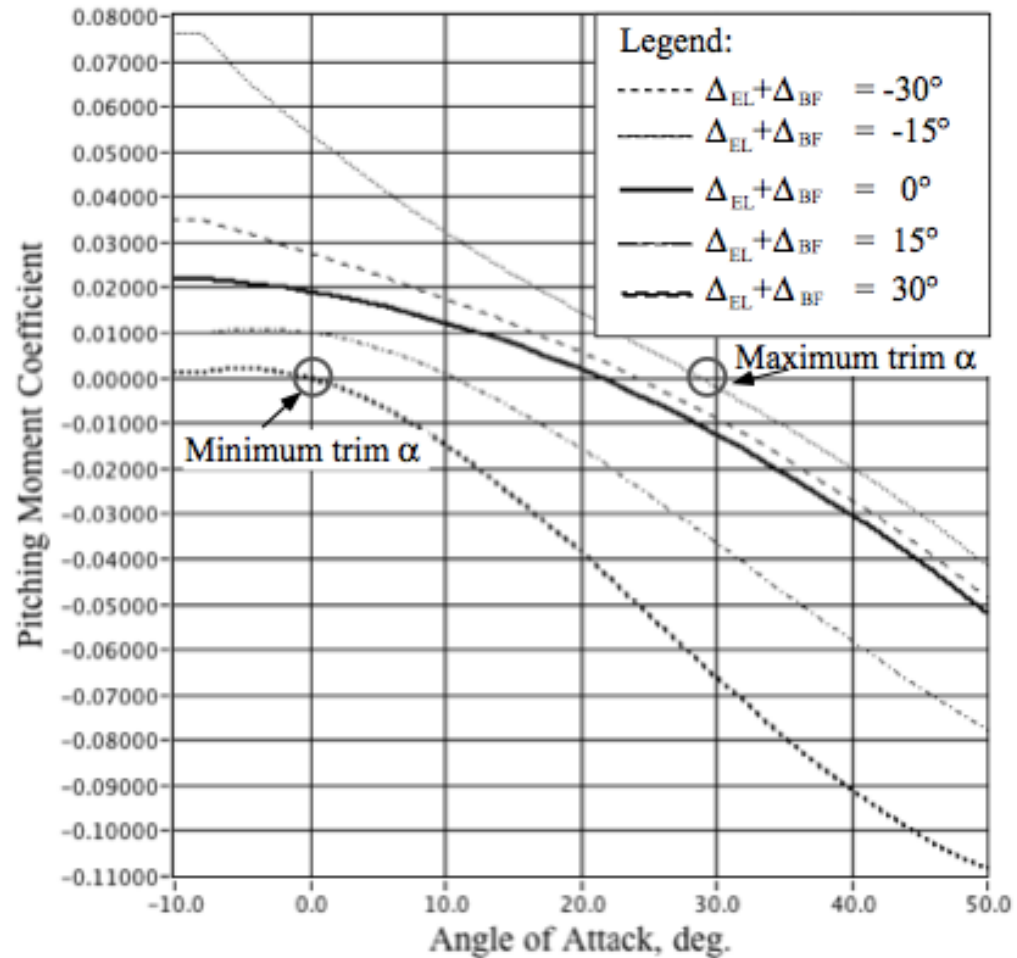
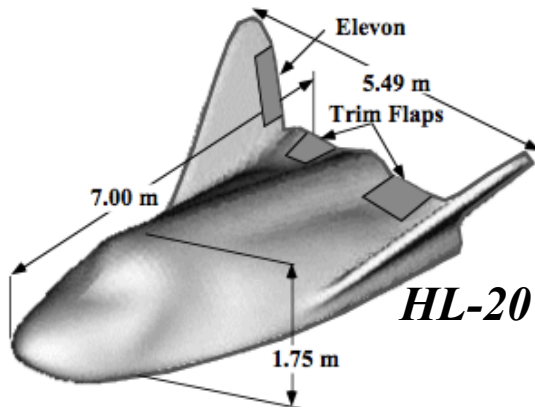


## Lift Coefficient



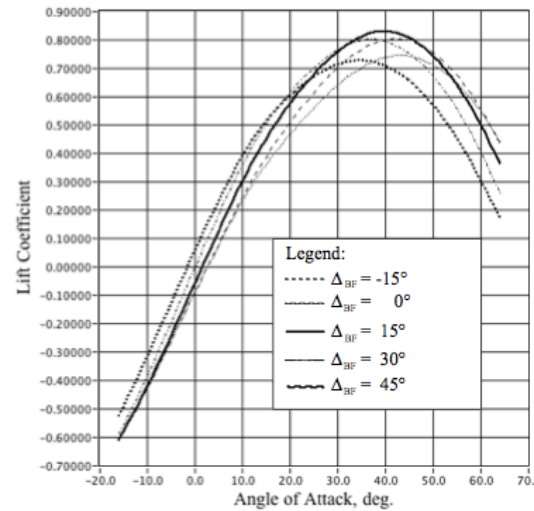
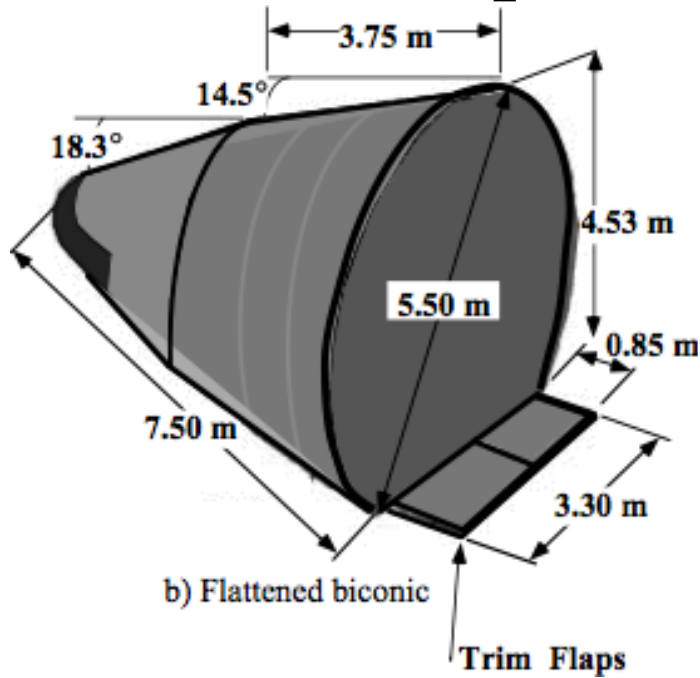
## Drag Coefficient

# Examples of Aerodynamic Coefficients (2)

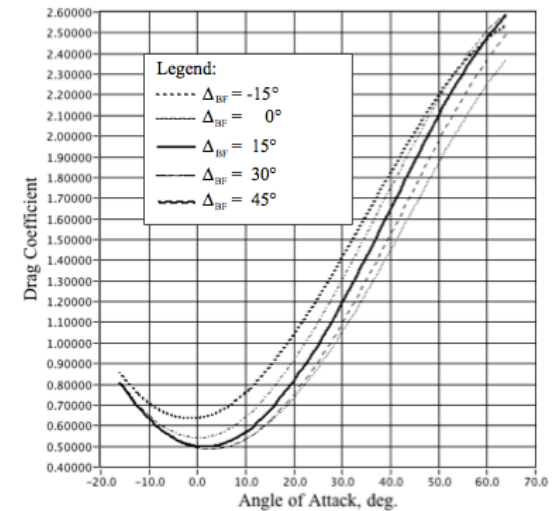


c) Effect of Symmetric Elevon and Body Flap deflections on Pitching Moment Coefficient

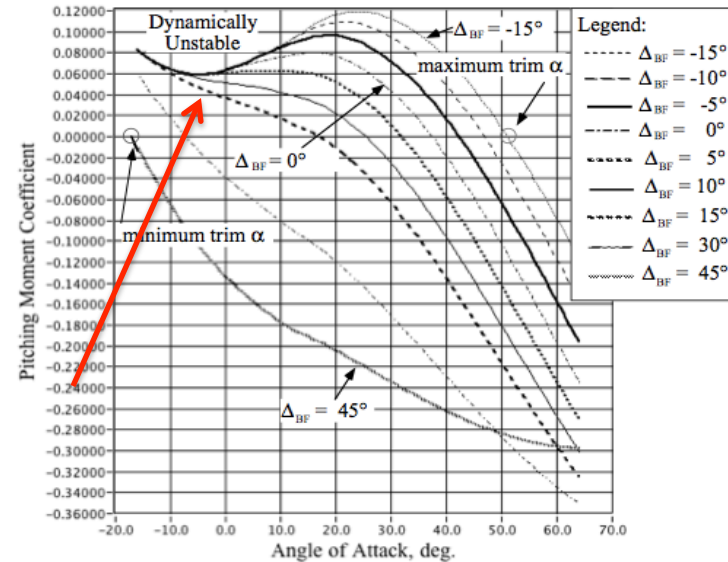
# Examples of Aerodynamic Coefficients (3)



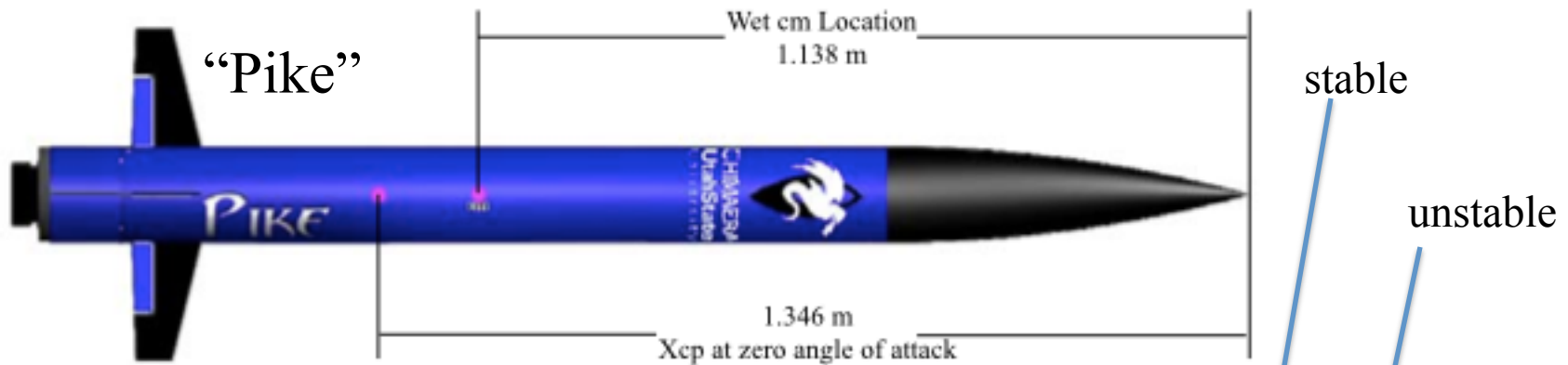
a) Effect of Symmetric Body Flap on Lift Coefficient



b) Effect of Symmetric Body Flap on Drag Coefficient

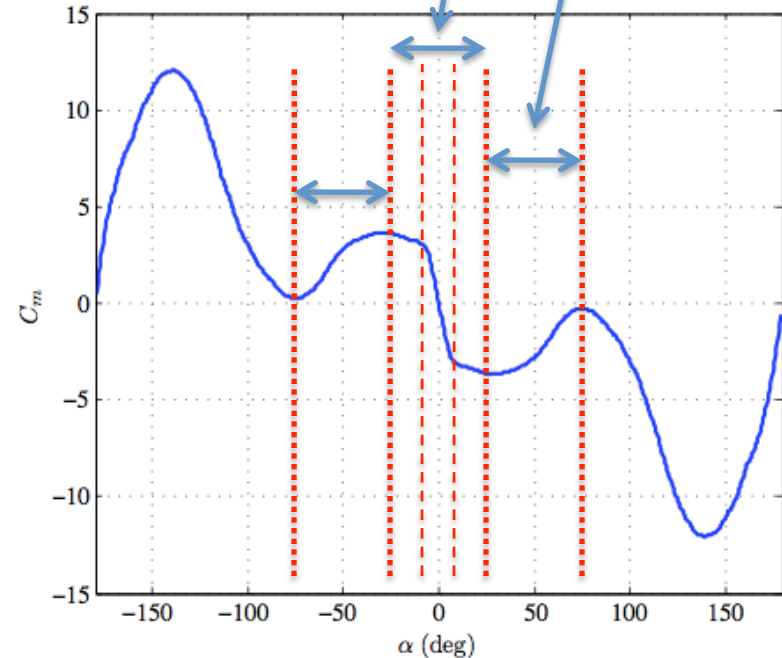


# Examples of Aerodynamic Coefficients (3)



Even  $X_{sm} > 0$  (static stability)  
At low angle of attack

“strong stability” region limited to very low angle of attack range



# Questions??

