Longitudinal Vehicle (Pitch) Dynamics, Static and Dynamic Stability

Sellers: Chapter 12
Vehicle Stability

(a) Equilibrium flight.
- Lift = weight
- Thrust = drag
- No net moments

(b) Statically unstable airplane.
- Equilibrium
- Disturbed moments increase disturbed condition
- Statically unstable divergent
- No moments - airplane holds disturbed condition

(c) Neutral static stability.
- Equilibrium
- Disturbed
Vehicle Stability (2)

If center of gravity (cg) is forward of the (cp), vehicle responds to a disturbance by producing aerodynamic moment that returns Angle of attack of vehicle towards angle that existed prior to the disturbance.

If CG is behind the center of pressure, vehicle will respond to a disturbance by producing an aerodynamic moment that continues to drive angle of attack further away from starting position.
Aerodynamic Lift, Drag, and Pitching Moment Can be thought of acting at a single point ... the Center of Pressure (cp) of the vehicle.

Sometimes (and not quite correctly) referred to as the Aerodynamic Center (AC).

For our purposes, AC and CP are synonymous.
Vehicle Stability, Rocket Flight Example

During rocket flight small wind gusts or thrust offsets can cause the rocket to "wobble", or change its attitude in flight.

Rocket rotates about its center of gravity \((cg)\)

Lift and drag both act through the center of pressure \((cp)\) of the rocket

When \(cp\) is behind \(cg\), aerodynamic forces provide a “restoring force” ... rocket is said to be “statically stable”

When \(cp\) ahead of \(cg\), aerodynamic forces provide a “destabilizing force” ... rocket is said to be “unstable”

*Condition for a statically for a stable rocket is that center of pressure must be located behind the center of gravity.*
Vehicle Stability, Rocket Flight Example

During flight small wind gusts or thrust offsets cause the rocket to "wobble" ... change attitude

Rocket rotates about center of gravity \((cg)\)

Lift and drag both act through center of pressure \((cp)\)

When \(cp\) is behind \(cg\), aerodynamic forces provide a "restoring force" ... rocket is said to be "statically stable"

When \(cp\) ahead of \(cg\), aerodynamic forces provide a "destabilizing force" ... rocket is said to be "unstable"

Condition for a statically for a stable rocket is that center of pressure must be located behind longitudinal center of gravity.

*MAE 5930, Rocket Systems Design*
Vehicle Stability, Rocket Flight Example

Weather Vane Analogy

More wind resistance at this end
Vane turns into wind
This axis is the point that the vane turns around.
Less wind resistance at this end

More area to push against here
Rocket turns into wind
Center of Gravity
Center of Pressure

The point around which this rocket will turn is called the Center of Gravity.
The point where the wind pressure balances out is called the Center of Pressure.
Vehicle Stability, Rocket Flight Example

For stable flight, center of gravity must be above center of pressure.

To improve stability, add weight to the nose, or increase fin area.

Adding weight to the nose, or making the rocket longer moves the center of gravity forward.
Static Margin and Pitching Moment

*Static margin* is a concept used to characterize the static stability and controllability of aircraft and missiles.

In aircraft analysis, static margin is defined as the non-dimensional distance between center of gravity and aerodynamic center of the aircraft.

In missile analysis, static margin is defined as non-dimensional distance between center of gravity and the center of pressure.

Stability requires that the pitching moment about the rotation point, $C_m$, become negative as we increase $C_L$:

$$\frac{\partial C_m}{\partial C_L} < 0 \quad \rightarrow \quad C_m = C_{m_0} - \frac{x}{c} C_L \quad \rightarrow \quad \frac{\partial C_{m_{c.g.}}}{\partial C_L} = -\frac{x}{c} = -\text{static margin}$$
Static Margin and Pitching Moment (2)

For a Rocket Static margin is the distance between the CG and the CP; divided by body tube diameter.

\[
\frac{\partial C_{m,c.g.}}{\partial C_L} = -\frac{x}{c} = -\text{static margin}
\]

\(x\) is the distance from the system's aerodynamic center to the c.g.

When the CG is ahead of NP the weight tends to correct the upset = Stable

When the CG is behind NP the weight worsens the upset = Unstable
Calculating the Static Margin

• Key to calculating static margin is to estimate location of longitudinal center of pressure at low angles of attack

• Barrowman equations provide simple, accurate technique for Axi-symmetric rockets

• cg is measured as the longitudinal balance point of the rocket.

• As a rule of thumb, CP distance should be aft of the CG by at least one rocket diameter. -- "One Caliber stability".
Calculating the Static Margin

Parameter Definitions

\[ L_N = \text{length of nose} \]
\[ d = \text{diameter at base of nose} \]
\[ d_F = \text{diameter at front of transition} \]
\[ d_R = \text{diameter at rear of transition} \]
\[ L_T = \text{length of transition} \]
\[ X_P = \text{distance from tip of nose to front of transition} \]
\[ C_R = \text{fin root chord} \]
\[ C_T = \text{fin tip chord} \]
\[ S = \text{fin semispan} \]
\[ L_F = \text{length of fin mid-chord line} \]
\[ R = \text{radius of body at aft end} \]
\[ X_R = \text{distance between fin root leading edge and fin tip leading edge parallel to body} \]
\[ X_B = \text{distance from nose tip to fin root chord leading edge} \]
\[ N = \text{number of fins} \]
Calculating the Static Margin (3)

$X_N = \text{center of pressure location for nose section}$

$X_T = \text{center of pressure location for transitions}$

$(CN)_N = \text{normal force coefficient for nose}$

$(CN)_T = \text{normal force coefficient for transition}$

**Nose Cone Terms**

- $(C_{N'})_N = 2$
- For Cone: $X_N = 0.666L_N$
- For Ogive: $X_N = 0.466L_N$

**Conical Transition Terms**

$$(C_N)_T = 2 \left[ \left( \frac{d_R}{d} \right)^2 - \left( \frac{d_F}{d} \right)^2 \right] \Rightarrow X_T = X_p + \frac{L_T}{3} \left[ 1 - \frac{d_F}{d_R} \right]$$
Calculating the Static Margin (4)

\[ X_F = \text{center of pressure location for fin groups} \]

\[ (CN)F = \text{Normal force coefficient for fin group} \]

\[
(CN)_F = \left[ \frac{1 + \frac{R}{S+R}}{1 + \sqrt{1 + \left( \frac{2L_F}{C_R + C_T} \right)^2}} \right] 4N \left( \frac{S}{d} \right)^2
\]

\[ X_F = X_B + \frac{X_R (C_R + 2C_T)}{3(C_R + C_T)} + \frac{1}{6} \left( C_R + C_T \right) - \frac{(C_R C_T)}{(C_R + C_T)} \]

Static Margin \( (X_{sm}) = \)

\[ (X_{cp} - X_{cg})/D_{max} \]

\( (CN)R = \text{total normal force coefficient} \)

\( X \text{ – measured aft from nose of vehicle} \)
Example: Stable Static Margin Vehicle

Ogive Nose

L_N = 20 cm
d = 10 cm
d_F = 10 cm
d_R = 8 cm
L_T = 5 cm
X_P = 30 cm
C_R = 10 cm
C_T = 5 cm
S = 10 cm
L_F = 12 cm
R = 4 cm
X_R = 16 cm
X_B = 50 cm
N = 3

Output parameters

Nosecone
- CNN
- XN, cm
- XF, cm

Fins
- CNF
- 6.12587
- 56.9921

Conical Transition
- CNT
- -0.72
- XT, cm
- 32.4074

Total
- CNR
- 7.40587
- Xcp, cm
- 46.5081

Static Margin
- 1.85081

CG, cm from nose
- 28

Chimaera
Example: Unstable Static Margin Calculation

Ogive Nose

**Constant diameter tube**
(No transition section)

<table>
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<tr>
<th>Parameter</th>
<th>Value</th>
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<tbody>
<tr>
<td>$L_N$</td>
<td>12.5 cm</td>
</tr>
<tr>
<td>$d$</td>
<td>5.54 cm</td>
</tr>
<tr>
<td>$d_F$</td>
<td>5.54 cm</td>
</tr>
<tr>
<td>$d_R$</td>
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<td>$L_T$</td>
<td>0 cm</td>
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<td>27 cm</td>
</tr>
<tr>
<td>$N$</td>
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</tr>
</tbody>
</table>

Static Margin: -0.04189
Effect of Static Margin on Launch Conditions

\[ C_m = (C_m)_{\alpha=0} - \left( \frac{x_{cp} - x_{cg}}{D_{max}} \right) \cdot C_L \]

\[ \rightarrow \frac{x_{cp} - x_{cg}}{D_{max}} = "static\ margin" \]

Static Stability \[ \rightarrow \frac{\partial C_m}{\partial C_L} < 0 \approx - \left( \frac{x_{cp} - x_{cg}}{D_{max}} \right) \]

\[ 1 \leq Desired\ Static\ Margin \leq 2 \]

\[ (C_m)_{\alpha=0} \approx 0 \ldots for\ axi - symmetric\ rocket \]
Effect of Static Margin on Launch Conditions

Wind Relative Velocity:

\[ W_{vert} = V_{rail} \cdot \sin \gamma_{rail} \]
\[ W_{hor} = V_{rail} \cdot \cos \gamma_{rail} + V_{wind} \]
\[ \rightarrow \gamma_{wind} = \tan^{-1} \left( \frac{V_{rail} \cdot \sin \gamma_{rail}}{V_{rail} \cdot \cos \gamma_{rail} + V_{wind}} \right) \]

\[ \gamma_{wind} = \tan^{-1} \left( \frac{\sin \gamma_{rail}}{\cos \gamma_{rail} + \frac{V_{wind}}{V_{rail}}} \right) \]

\[ \alpha_{rail} = \gamma_{rail} - \gamma_{wind} \]
Effect of Static Margin on Launch Conditions (3)

In cross wind Launch, angle of attack is NOT zero.
Effect of Static Margin on Launch Conditions (4)

High Static Stability produces Large Initial Pitching Moment

\[
\begin{align*}
(C_m)_{\text{launch}} &= (C_m)_{\alpha=0} - \left( \frac{x_{cp} - x_{cg}}{D_{\text{max}}} \right) \cdot C_L \approx \\
&- \left( \frac{x_{cp} - x_{cg}}{D_{\text{max}}} \right) \cdot \frac{\partial C_L}{\partial \alpha} \cdot \alpha_{\text{rail}}
\end{align*}
\]

\[\gamma_{\text{wind}} = \tan^{-1} \left( \frac{\sin \gamma_{\text{rail}}}{\cos \gamma_{\text{rail}} + \frac{V_{\text{wind}}}{V_{\text{rail}}}} \right)\]

\[\alpha_{\text{rail}} = \gamma_{\text{rail}} - \gamma_{\text{wind}}\]
Effect of Static Margin on Launch Conditions (5)

\[ \theta_t \approx \gamma_{rail} - \tan^{-1}\left( \frac{\sin \gamma_{rail}}{\cos \gamma_{rail} + \frac{V_{wind}}{V_{max}}} \right) \]

**Example:**

\[ \gamma_{rail} = 85^\circ \]
\[ V_{wind} = 5 \text{ m/s} \]
\[ V_{max} = 185 \text{ m/s} \]

\[ 85 - \frac{180}{\pi} \cdot \tan^{-1}\left( \frac{\sin \left( \frac{\pi}{180} \cdot 85 \right)}{\cos \left( \frac{\pi}{180} \cdot 85 \right) + \frac{5}{185}} \right) \geq 1.54^\circ \]

**Effective Launch Angle \leq 83.46^\circ**
Static Versus Dynamic Stability

(a) Equilibrium flight.

Lift = weight
Thrust = drag
No net moments

(b) Statically unstable airplane.

Disturbed moments increase disturbed condition

(c) Neutral static stability.

No moments - airplane holds disturbed condition
Static Versus Dynamic Stability

(a) Statically and dynamically stable.

(b) Statically stable; neutral dynamic stability.

(c) Statically stable; dynamically unstable.
Static Versus Dynamic Stability

Positive: Quick to return, hard to displace

Negative: Quick to displace, hard to return

Neutral: Stays put

Positive: Slower to return, easier to displace

Negative: Slower to displace, easier to return

Dynamic Stability

Displacement vs. Time

Positive Static Stability

Positive Static Stability and Positive Dynamic Stability

Positive Static Stability and Neutral Dynamic Stability

Positive Static Stability and Negative Dynamic Stability

Equilibrium

Amplitude

Period

Time

Time to subside

Period / Time to subside = damping ratio, $\zeta$
Static Versus Dynamic Stability (4)
Simplified Pitch Axis-Rotational Dynamics

Neglect Cross Products of Inertia

\[
\begin{bmatrix}
I_x & -I_{xy} & -I_{xz} \\
-I_{xy} & I_y & -I_{yz} \\
-I_{xz} & -I_{yz} & I_z
\end{bmatrix} \begin{bmatrix}
p \\
q \\
r
\end{bmatrix} = \begin{bmatrix}
q \cdot r (I_y - I_z) + (q^2 - r^2)I_{yz} + p \cdot q (I_{xz}) - r \cdot p (I_{xy}) \\
r \cdot p (I_z - I_x) + (r^2 - p^2)I_{xz} + q \cdot r (I_{xy}) - p \cdot q (I_{yz}) \\
p \cdot q (I_x - I_y) + (p^2 - q^2)I_{xy} + r \cdot p (I_{yz}) - q \cdot r (I_{xz})
\end{bmatrix} + \begin{bmatrix}
M_x \\
M_y \\
M_z
\end{bmatrix}
\]

\[\dot{q} = \dot{\theta} = \frac{M_y}{I_y} + r \cdot p \frac{(I_z - I_x)}{I_y}\]

Forcing moment

Second order Disturbance torque
(neglected when r, p are small)
Assume “wings level”.. That is $\phi \approx 0$

$$\begin{bmatrix}
\dot{\phi} \\
\dot{\theta} \\
\dot{\psi}
\end{bmatrix} =
\begin{bmatrix}
1 & \sin \phi \tan \theta & \cos \phi \tan \theta \\
0 & \cos \phi & -\sin \phi \\
0 & \sin \phi & \cos \phi
\end{bmatrix}
\begin{bmatrix}
p \\
q \\
r
\end{bmatrix}
\rightarrow
\begin{bmatrix}
\dot{\theta} = q \\
\ddot{\theta} = \dot{q}
\end{bmatrix}$$
Simplified Pitch Axis-Rotational Dynamics (3)

Neglecting Disturbance torques

\[ \ddot{\theta} = \dot{q} = \frac{M_y}{I_y} \rightarrow M_y \approx "pitching\ moment" \]

"pitching moment coefficient" \( \equiv C_m = \frac{M_y}{\bar{q} \cdot A_{ref} \cdot c_{ref}} \)

\[ \bar{q} = \left( \frac{1}{2} \cdot \rho \cdot V^2 \right) \]

\[ A_{ref} = \frac{\pi}{4} \cdot D_{ref}^2 \]

"reference length" \( \rightarrow c_{ref} = D_{\text{max}} \)

Check Units

\[ \frac{\bar{q} \cdot A_{ref} \cdot c_{ref}}{\frac{N_t}{m^2 \cdot m^2 \cdot m}} \sim \frac{I_y}{\frac{kg \cdot m^2}{kg \cdot m^2}} \sim \left( \frac{kg \cdot m}{sec^2 \cdot m \cdot m \cdot m} \right) \cdot \frac{1}{kg \cdot m^2} \sim \frac{1}{sec^2} \]
Gravity acts at CG and
Cannot induce pitching moment

\[
\dot{q} = \frac{\overline{q} \cdot A_{ref} \cdot c_{ref}}{I_y} \cdot C_m
\]

Depends on angle of attack

\[
\dot{q} = C_{m_0}
\]

\[
C_L
\]

\[
C_D
\]

\[
\alpha
\]
Simplified Pitch Axis-Rotational Dynamics (4)

\[ \ddot{\theta} = \dot{q} = \frac{\bar{q} \cdot A_{\text{ref}} \cdot c_{\text{ref}}}{I_y} \cdot C_m \]

Depends on angle of attack + Control inputs

It can also be shown that … (NASA RP-1168 pp 10-22) that for angle of attack

\[ \dot{\alpha} = -\frac{\bar{q} \cdot A_{\text{ref}}}{m \cdot V} \cdot C_L + q + \frac{g_{(r)}}{V} \cos(\theta - \alpha) - \frac{F_{\text{thrust}}}{m \cdot V} \sin \alpha \]
Simplified Pitch Axis-Rotational Dynamics \(^{(5)}\)

Collected, simplified pitch dynamics equations

\[
\begin{bmatrix}
\dot{\alpha} \\
\dot{q} \\
\dot{\theta}
\end{bmatrix} = \begin{bmatrix}
-\frac{\bar{q} \cdot A_{\text{ref}}}{m \cdot V} \cdot C_L + q + \frac{g_{(r)}}{V} \cos(\theta - \alpha) - \frac{F_{\text{thrust}}}{m \cdot V} \sin \alpha \\
\bar{q} \cdot A_{\text{ref}} \cdot c_{\text{ref}} \cdot \frac{C_m}{I_y} \\
q
\end{bmatrix}
\]
Collected Longitudinal Equations of Motion

\[
\begin{bmatrix}
\dot{V}_r \\
\dot{V}_v \\
\dot{r} \\
\dot{\gamma} \\
\dot{x} \\
\dot{\alpha} \\
\dot{q} \\
\dot{\theta} \\
\dot{m}
\end{bmatrix} =
\begin{bmatrix}
V_r \\
V_v \\
\frac{V_r}{r} \\
\frac{\bar{q} \cdot A_{ref} \cdot C_L}{m \cdot V} + q + \frac{g_{(h)}}{V} \cos(\gamma) - \frac{F_{\text{thrust}} \sin \theta}{m \cdot V} \\
\frac{\bar{q} \cdot A_{ref} \cdot c_{ref} \cdot C_m}{I_y} \\
q \\
- \frac{F_{\text{thrust}}}{g_0 \cdot I_{sp}} \\
\end{bmatrix}
\]

How do we control this “mess” .. With \( F_{\text{thrust}}, C_m, C_L, \text{and } C_D \)
Fixed Wing Aircraft Controls

-- Stick Roll Control (ailerons)

-- Stick Pitch Control (elevator)

-- Pedal Yaw Control (Rudder)

-- Throttle (Thrust)

Roll Control From Ailerons (rudder “coordinates” turn)

Pitch Control From Elevator (throttle “coordinates” climb)

Yaw Control From Rudder (stick trims roll)

Launch (Rocket) Controls

- Movable Fins
- Gimbaled Thrust
- Vernier Rocket
- Thrust Vane
Spacecraft Reaction Controls (RCS)

• The spacecraft propulsion system provides controlled impulse for:
  – Orbit insertion and transfers
  – Orbit maintenance (station keeping)
  – Attitude Control

• Propulsion Types
  – Cold gas, monopropellant, bipropellants, ion

Thruster rockets apply force at some distance away from center of mass, causing a torque that rotates the spacecraft.
Pitching Moment Control of Vehicle

\[
\begin{pmatrix}
\dot{\alpha} \\
\dot{\beta}
\end{pmatrix}
= \frac{-\bar{q} \cdot A_{\text{ref}} \cdot C_L}{m \cdot V} + q + \frac{g_{(r)}}{V} \cos(\theta - \alpha) - \frac{F_{\text{thrust}} \sin \alpha}{m \cdot V} + \frac{\bar{q} \cdot A_{\text{ref}} \cdot c_{\text{ref}} \cdot C_m}{I_y} \cdot C_m(\alpha) + C_m(\delta)
\]

\[M_y = \frac{\bar{q} \cdot A_{\text{ref}} \cdot c_{\text{ref}}}{I_y} \cdot C_m(\alpha) + C_m(\delta)\]

depends on \(\alpha\)......depends on control "\(\delta\)"
Vehicle “Aerodynamic” Pitching Moment

\[ C_m(\alpha) = C_{m_0} + \left( \frac{X_{cg} - X_{cp}}{c_{ref}} \right) (C_L \cdot \cos \alpha + C_D \cdot \sin \alpha) = C_{m_0} - X_{sm} \left( C_L \cdot \cos \alpha + C_D \cdot \sin \alpha \right) \]

Small \( \alpha \rightarrow C_m(\alpha) = C_{m_0} - X_{sm} (C_L + C_D \alpha) \rightarrow \frac{\partial C_m}{\partial \alpha} = -X_{sm} \left( \frac{\partial C_L}{\partial \alpha} + \frac{\partial C_D}{\partial \alpha} \alpha + C_D \right) \)

Linearized \( \rightarrow C_m(\alpha) = C_{m_0} - X_{sm} \left( \frac{\partial C_L}{\partial \alpha} + C_D \right) \cdot \alpha \equiv C_{m_0} + \frac{\partial C_m}{\partial \alpha} \cdot \alpha \)

\[ \frac{\partial C_m}{\partial \alpha} \rightarrow "C_{m_\alpha}" \]

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Vehicle “Aerodynamic” Pitching Moment (2)

\[
\frac{\partial C_m}{\partial \alpha} \rightarrow "C_{m\alpha}" \quad \text{...Ideally...}
\]

\[
C_{m\alpha} = -X_{sm} \left( \frac{\partial C_L}{\partial \alpha} + C_D \right) \rightarrow \begin{bmatrix} \frac{\partial C_L}{\partial \alpha} \\ C_D \end{bmatrix} > 0 \rightarrow X_{sm} > 0 \quad \text{static stability} \quad \rightarrow \quad C_{m\alpha} < 0 \quad \text{static stability}
\]
Linear Aerodynamic Model

Lift Coefficient \( \rightarrow C_L = C_{L_0} + C_{L_\alpha} \cdot \alpha + C_{L_\delta} \cdot \delta \)

Drag Coefficient \( \rightarrow C_D = C_{D_0} + C_{D_\alpha} \cdot \alpha + C_{D_\delta} \cdot \delta \)

Pitching Moment Coefficient \( \rightarrow C_m = C_{m_0} + C_{m_\alpha} \cdot \alpha + C_{m_\delta} \cdot \delta \)

For Rocket (Typically) \( \rightarrow \begin{bmatrix} C_{L_0} & C_{m_0} \end{bmatrix} \approx 0 \)

\( C_{m_0} \neq 0 \)

\( \delta \rightarrow \text{Longitudinal Control Actuator} \)
Linear Aerodynamic Model (2)

\[ C_L = C_{L0} + C_{L\alpha} \cdot \alpha \]
\[ C_D = C_{D0} + C_{D\alpha} \cdot \alpha \]

For “Thin Airfoil” ….

\[ C_D \approx C_{D0} + \frac{C_L^2}{\pi \cdot \varepsilon \cdot AR} \]
\[ 0.85 < \varepsilon < 0.95 \]

\[ C_{D0} = \text{parasite drag} \]
\[ \varepsilon = \text{Oswald Efficiency Factor} \]
\[ \text{AR} \rightarrow \text{Aspect Ratio} = \frac{b^2}{A_{ref}} \]

Supersonic flow also has “wave drag” … But we won’t worry about that here

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Examples of Aerodynamic Coefficients

**HL-20**

- **Lift Coefficient**
- **Pitching Moment Coefficient**
- **Drag Coefficient**

Legend:
- $\Delta \alpha = -30^\circ$
- $\Delta \alpha = -15^\circ$
- $\Delta \alpha = 0^\circ$
- $\Delta \alpha = 15^\circ$
- $\Delta \alpha = 30^\circ$

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Examples of Aerodynamic Coefficients (2)

HL-20

Legend:

- $\Delta_{EL} + \Delta_{BF} = -30^\circ$
- $\Delta_{EL} + \Delta_{BF} = -15^\circ$
- $\Delta_{EL} + \Delta_{BF} = 0^\circ$
- $\Delta_{EL} + \Delta_{BF} = 15^\circ$
- $\Delta_{EL} + \Delta_{BF} = 30^\circ$

Effect of Symmetric Elevon and Body Flap deflections on Pitching Moment Coefficient

Minimum trim $\alpha$

Maximum trim $\alpha$
Examples of Aerodynamic Coefficients (3)

**a) Effect of Symmetric Body Flap on Lift Coefficient**
- Legend: 
  - \( \Delta_w = -15^\circ \)
  - \( \Delta_w = 0^\circ \)
  - \( \Delta_w = 15^\circ \)
  - \( \Delta_w = 30^\circ \)
  - \( \Delta_w = 45^\circ \)

**b) Effect of Symmetric Body Flap on Drag Coefficient**
- Legend: 
  - \( \Delta_w = -15^\circ \)
  - \( \Delta_w = 0^\circ \)
  - \( \Delta_w = 15^\circ \)
  - \( \Delta_w = 30^\circ \)
  - \( \Delta_w = 45^\circ \)

- Trim Flaps

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Examples of Aerodynamic Coefficients (3)

“Pike”

Even $X_{sm} > 0$ (static stability)
At low angle of attack

“strong stability” region limited to very low angle of attack range
Questions??