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FORTRAN PROGRAM FOR PLUG NOZZLE DESIGN

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FORTRAN PROGRAM FOR PLUG NOZZLE DESIGN

By

Che-Ching Lee* and Donald D. Thompson

ABSTRACT

Two FORTRAN computer programs for the design of pure external and internal-external expansion plug nozzles are described. The program output includes the contour of the nozzle and various performance parameters. This design method is based on simple wave flow concepts.

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* Research Laboratories, Brown Engineering Co., Inc.

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LIST OF SYMBOLS

A	Surface area of Prandtl-Meyer expansion wave after it is revolved about the plug axis
a	Length of triangle side
b	Length of triangle side
C_f	Thrust coefficient
F	Thrust
f	Function
g	Constant of proportionality in Newton's second law
h_t	Width of throat gap on pure external expansion plug nozzle
I_{sp}	Specific impulse
L	Chord length of internal circular arc contour
M	Mach number
M^*	Ratio of local velocity to velocity at sonic flow conditions
$\dot{m}$	Mass flow rate
N	Number of contour points computed on pure external expansion plug nozzle
N_1	Number of internal contour points computed on internal-external expansion plug nozzle
N_2	Number of external contour points computed on internal-external expansion plug nozzle
n	Any number of the series 0, 1, 2, n
P	Static pressure
X	Axial distance from lip of shroud



Greek Symbols

β	Central angle between the radius to point p and that to any point x, on the internal circular arc contour of an internal-external expansion plug nozzle
γ	Ratio of specific heats
Δ	Small increment
δ	Angle between plug axis and sonic line on pure external expansion plug nozzle
ϵ	Expansion ratio
ϕ	Angle between plug axis and Prandtl-Meyer expansion wave
μ	Mach angle
ν	Prandtl-Meyer turning or expansion angle
ϕ	Mass density
θ	Flow angle measured from plug axis
ψ	Slope of chord of internal circular arc contour

Subscripts:

c	Chamber condition
e	Exit conditions or condition of lip of shroud
ei	Condition at end of internal expansion
R	Radius from plug axis
R_r	Radius of internal circular arc contour
T	Temperature
V	Velocity on expansion wave through the point indicated by subscript



FORTTRAN SYMBOLS

CFL	$C_{F\text{ opt}}$	Optimum thrust coefficient
DELTA	δ	Angle between plug axis and sonic line on pure external expansion plug nozzle
G	g	Constant of proportionality in Newton's second law
GAMA	γ	Ratio of specific heats
GAM(I)	γ	Ratio of specific heats in thermodynamic table
HT	h_t/R_e	Ratio of throat gap to the radius at the shroud on pure external plug nozzle
HM(I)	M	Mach number in thermodynamic table
NT		Number of thermodynamic data
PAPC	P_a/P_c	Ratio of atmospheric pressure to chamber pressure
PXPC	P_x/P_c	Ratio of pressure at point x to chamber pressure
PEIPC	P_{ei}/P_c	Ratio of pressure at end of internal expansion to chamber pressure
PHT	ϕ	Angle of sonic surface to plug axis
R	R	Gas constant
RM	M_e	Exit Mach number
RMEI	M_{ei}	Mach number at end of internal expansion
RRRE	R_r/R_e	Radius of internal circular arc contour to shroud radius



FORTRAN SYMBOLS (CONCLUDED)

RXRE	R_x	Ratio of radius of point x to radius of shroud
SUMCG	C_{Fvacx}	Cumulative vacuum thrust coefficient
SUMIM	I_s	Cumulative specific impulse
SUMVA	I_{svac}	Cumulative vacuum specific impulse
TE	T_e	Exit temperature
VE	ν	Exit Prandtl-Meyer turning angle
XP	ϵ	Expansion ratio
XM	M_x	Mach number at the contour
XXRE	X_x/R_e	Ratio of x co-ordinate of point x to radius of shroud



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ACKNOWLEDGEMENT

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DESIGN THEORY

Design of External Expansion Plug Nozzles

In one-dimensional isentropic supersonic flow, an area ratio based on throat area can be written as follows:

$$\frac{A}{A^*} = \epsilon = \frac{1}{M} \left[\left(\frac{2}{\gamma + 1} \right) \left(1 + \frac{\gamma - 1}{2} M^2 \right) \right]^{\frac{\gamma + 1}{2(\gamma - 1)}} \quad (1)$$

where ϵ is defined as an expansion ratio. By rearranging equation (1), a function can be obtained to calculate exit Mach number:

$$f(M_e) = M_e \epsilon - \left[\frac{2 + (\gamma - 1) M_e^2}{\gamma + 1} \right]^{\frac{\gamma + 1}{2(\gamma - 1)}} \quad (2)$$

Expanding this function in a Taylor's series:

$$f(M_e + \Delta M) = f(M_e) + f'(M_e) \Delta M + f''(M_e) \frac{\Delta M^2}{2!} + \dots + f^n(M_e) \frac{\Delta M^n}{n!} + \dots \quad (3)$$

where:

$$f'(M_e) = \epsilon - M_e \left[\frac{2 + (\gamma - 1) M_e^2}{\gamma + 1} \right]^{\frac{3 - \gamma}{2(\gamma - 1)}}$$

Truncate equation (3) at the first two terms, and assume a value M_{est} for M_e , and solve for a ΔM :

$$\Delta M_o = - \frac{f(M_{est_o})}{f'(M_{est_o})} \quad (4)$$

A new approximation for M_e is:

$$M_{est_1} = M_{est_o} + \Delta M_o \quad (5)$$

By carrying on this process until ΔM is within the desired limit, the exit Mach number can be obtained.

From the Prandtl-Meyer relation, a total flow turning angle can be calculated by using the following equation:

$$\nu_e = \left(\frac{\gamma + 1}{\gamma - 1} \right)^{\frac{1}{2}} \tan^{-1} \left[\frac{\gamma - 1}{\gamma + 1} (M_e^2 - 1) \right]^{\frac{1}{2}} - \tan^{-1} (M_e^2 - 1)^{\frac{1}{2}} \quad (6)$$

From the geometry of **FIG 1**, the following relations of throat gap can be obtained:

$$a = h_t \cos \delta \quad (7)$$

$$b = h_t \sin \delta \quad (8)$$

$$R_t = R_e - h_t \sin \delta \quad (9)$$

$$A_t = \pi (R_e - R_t) \left[a^2 + (R_e - R_t)^2 \right]^{\frac{1}{2}} \quad (10)$$

$$= \pi h_t (2 R_e - h_t \sin \delta)$$

or

$$\frac{\pi R_e^2}{\epsilon} = \pi h_t (2 R_e - h_t \sin \delta) \quad (11)$$

Solving the dimensionless parameter, h_t/R_e , in (11),

$$\frac{h_t}{R_e} = \frac{\epsilon - [\epsilon'(\epsilon - \sin \delta)]^{\frac{1}{2}}}{\epsilon \sin \delta} \quad (12)$$

The optimum thrust coefficient, $C_{F_{opt}}$, can be calculated from the following equation.

$$C_{F_{opt}} = \frac{\dot{m} V_e}{P_e A_t} = \frac{(\rho_t A_t V_t) V_e}{P_e A_t} = \frac{\rho_t V_t V_e}{P_e} = \frac{\rho_t V_t^2 M_e^*}{P_e} \quad (13)$$

*By the definition of the velocity of sound in a perfect gas

$$V_t = \frac{\gamma P_t}{\rho_t} \quad (14)$$



Equation (13) can be reduced to:

$$C_{F_{opt}} = \gamma M_e^* \left(\frac{P_t}{P_e} \right) \cdot$$

$$= \gamma M_e \left(\frac{2}{\gamma+1} \right)^{\frac{\gamma+1}{2(\gamma-1)}} \left(1 + \frac{\gamma-1}{2} M_e^2 \right)^{-\frac{1}{2}} \quad (15)$$

The following procedure of calculation is for determining the plug contour. The Mach number on the plug surface is increased from $M_x = 1$ at the throat to $M_x = M_e$ at the tip by regular increments M_{in} .

$$M_{in} = \frac{M_e - 1}{N} \quad (16)$$

$$M_x = 1 + X M_{in} \quad (17)$$

The area of the revolved expansion wave is given by:

$$A_x = \pi (R_e - R_x) \left[X_x^2 + (R_e - R_x)^2 \right]^{\frac{1}{2}} \quad (18)$$

From the geometry of FIG 2

$$\tan \phi_x = \frac{R_e - R_x}{X_x} \quad (19)$$

Solving equations (18) and (19), one obtains:

$$A_x = \frac{\pi (R_e^2 - R_x^2)}{\sin \phi_x} \quad (20)$$

From the geometry of FIG 2

$$\phi_x = \nu_e - \nu_x + \mu_x \quad (21)$$

Substitute equation (21) into (20),

$$A_x = \frac{\pi (R_e^2 - R_x^2)}{\sin (\nu_e - \nu_x + \mu_x)} \quad (22)$$

1

(23)

The mass flow through the throat is:

(24)

The mass flow through these two sections should be equal; therefore, A_x can be determined as follows:

(25)

Equations (20) and (25) are then solved for R_x :

(26)

Once R_x value is determined, X_x can be calculated by using equation (19).

The pressure ratio at point X can be calculated by using the following relationship:

(27)

The cumulative thrust is made up of the momentum flux and the pressure thrust at the throat surface plus the pressure integral down the plug to the point in question

(28)

$$I_s = \frac{\dot{m}_t V_t \sin \delta}{\dot{m}_t} + \frac{(P_t - P_a) A_t \sin \delta}{\rho_t A_t V_t} + \int \frac{(P_x - P_a)}{\rho_t A_t V_t} \quad (29)$$

$$= V_t \sin \delta + \frac{V_t \sin \delta (P_t - P_a)}{\rho_t \frac{\gamma P_t}{\rho_t}} + \frac{P_e V_t}{\rho_t \frac{\gamma P_t}{\rho_t}} \int \frac{(P_x - P_a)}{P_e} \frac{dA}{A_t} \quad (30)$$

$$= V_t \sin \delta + \frac{V_t \sin \delta}{\gamma} \left[1 - \left(\frac{P_a}{P_e} \right) \left(\frac{P_e}{P_t} \right) \right] + \frac{V_t}{\gamma} \left(\frac{P_e}{P_t} \right)$$

$$\int \frac{(P_x - P_a)}{P_e} \frac{dA}{A_t}$$

$$I_s = V_t \sin \delta \left\{ 1 + \frac{1}{\gamma} \left[1 - \left(\frac{\gamma+1}{2} \right)^{\frac{\gamma}{\gamma-1}} \left(\frac{P_a}{P_e} \right) \right] \right\} \quad (31)$$

$$+ \frac{V_t}{\gamma} \left(\frac{\gamma+1}{2} \right)^{\frac{\gamma}{\gamma-1}} \sum_{n=1}^N \frac{\epsilon}{2} \left[\left(\frac{P_x - P_a}{P_e} \right)_{n-1} + \left(\frac{P_x - P_a}{P_e} \right)_n \right]$$

$$\left[\left(\frac{R_x}{R_e} \right)_{n-1}^2 - \left(\frac{R_x}{R_e} \right)_n^2 \right]$$

$$\begin{aligned} \text{The vacuum thrust coefficient is:} \\ C_{F_{\text{vac}}} &= \frac{\dot{M}_t V_t \sin \delta}{P_e A_t} + \frac{P_t A_t \sin \delta}{P_e A_t} + \int \frac{P_x}{P_e} \frac{dA}{A_t} \\ &= \left(\frac{\rho_t A_t V_t^2 \sin \delta}{P_e A_t} \right) + \frac{P_t}{P_e} \sin \delta + \int \frac{P_x}{P_e} \frac{dA}{A_x} \\ &= \left(\frac{\rho_t \frac{\gamma P_t}{\rho_t} \sin \delta}{P_e} \right) + \left(\frac{2}{\gamma + 1} \right) \frac{\gamma}{\gamma - 1} + \int \frac{P_x}{P_e} \frac{dA}{A_x} \end{aligned} \quad (32)$$

(equation 32 continued)

$$= \left(\frac{2}{\gamma + 1} \right)^{\frac{\gamma}{\gamma - 1}} (\gamma + 1) \sin \delta + \sum_{n=1}^N \frac{\epsilon}{2} \left[\left(\frac{P_x}{P_e} \right)_{n-1} + \left(\frac{P_x}{P_e} \right)_n \right] \left[\left(\frac{R_x}{R_e} \right)_{n-1}^2 - \left(\frac{R_x}{R_e} \right)_n^2 \right].$$



$$\frac{X_p}{R_e} = \frac{\frac{R_p}{R_e} - 1}{\tan \phi_{ei}} \quad (38)$$

The first part of the contour calculation is similar to those of the previous section:

$$M_{in} = \frac{M_{ei} - 1}{N_1} \quad (39)$$

$$M_x = 1 + xM_{in} \quad (40)$$

The Prandtl-Meyer angle at any location is now calculated from:

$$\nu_x = \left(\frac{\gamma+1}{\gamma-1} \right)^{\frac{1}{2}} \tan^{-1} \left[\frac{\gamma-1}{\gamma+1} (M_x^2 - 1) \right]^{\frac{1}{2}} - \tan^{-1} (M_x^2 - 1)^{\frac{1}{2}} \quad (41)$$

The flow of point P is assumed to be perpendicular to the radius of the circular arc contour. The central angle, B_x , can be obtained from:

$$B_x = \phi_t - 90^\circ - \nu_x + |\theta_e| \quad (42)$$

The chord length is equal to:

$$\frac{L_x}{R_e} = 2 \frac{R_r}{R_e} \sin \frac{1}{2} \beta_x \quad (43)$$

From the geometry of FIG 3:

$$\psi_{\mathbf{x}} = 180 - \phi_t + \nu_{\mathbf{x}} - \frac{180 - \beta_{\mathbf{x}}}{2} \quad (44)$$

The co-ordinates of the point X_1 can be determined from the following equations:

$$\frac{R_{x_1}}{R_e} = \frac{R_p}{R_e} + \frac{L_x}{R_e} \sin \psi_x \quad \text{and} \quad (45)$$

$$\frac{X_{x1}}{R_e} = \frac{X_p}{R_e} - \frac{L_x}{R_e} \cos \psi_x \quad (46)$$



The derivation of the calculation of point X_2 is similar to that used in equation (26).

$$\frac{R_{x_2}}{R_e} = \left\{ \left(\frac{R_{x_1}}{R} \right)^2 + \left[\left(\frac{2}{\gamma+1} \right) \left(1 + \frac{\gamma-1}{2} M_x^2 \right) \right]^{\frac{\gamma+1}{2(\gamma-1)}} \sin \phi_x \right\}^{\frac{1}{2}} \quad (47)$$

$$\frac{X_{x_2}}{R_e} = \frac{X_{x_1}}{R_e} + \frac{\frac{R_{x_2}}{R_e} - \frac{R_{x_1}}{R_e}}{\tan \phi_x} \quad (48)$$

where:

$$\phi_x = 2 \nu_{ei} - \nu_e - \nu_x + \mu_x \quad (49)$$

Equation (27) can be used to calculate the pressure ratio at points X_1 and X_2 . When M_x has been incremented from $M_x=1$ to $M_x=M_{ei}$, the design of the internal portion of the nozzle is complete.

The external portion can be designed using the following relations:

(1) The last expansion wave from the initial circular arc contour at point P is a member of a family of left running waves and intersects the lip of the shroud as shown in FIG 4.

(2) The remaining expansion to the exit Mach number occurs about the lip of the shroud and is made up of a family of right-running expansion waves.

(3) Flow properties on the first of the right-running wave are equal to those on the last left-running wave.

(4) The external contour is determined in the same manner as for a pure external expansion nozzle.

$$F_x = m_t V_q \cos \theta_q + (P_q - P_a) A_q \sin \phi_q + \int (P_x - P_a) dA \quad (50)$$

$$I_{sp} = \frac{F_x}{\dot{m}_t}$$

$$\begin{aligned}
 &= V_q \cos \theta_q + \frac{(P_q - P_a)}{\rho_q V_q} \sin \phi_q + \int \frac{(P_x - P_a)}{\rho_t A_t V_t} dA \\
 &= V_q \left\{ \cos \theta_q + \frac{1}{\gamma} \left[1 - \left(\frac{P_a}{P_e} \right) \left(1 + \frac{\gamma-1}{2} M_q^2 \right)^{\frac{\gamma}{\gamma-1}} \frac{\sin \phi_q}{M_q^2} \right] \right\}
 \end{aligned} \tag{51}$$

$$+ \frac{V_t}{\gamma} \left(\frac{\gamma+1}{2} \right)^{\frac{\gamma}{\gamma-1}} N_2 \sum_{n=1}^{\infty} \frac{\varepsilon}{2} \left[\left(\frac{P_x - P_a}{P_e} \right) + \left(\frac{P_x - P_a}{P_e} \right)_n \right] \left[\left(\frac{R_x}{R_e} \right)_{n-1}^2 - \left(\frac{R_x}{R_e} \right)_n^2 \right]$$

$$V_q = \left[KR \frac{T_e \left(1 + \frac{\gamma-1}{2} M_e^2 \right)}{1 + \frac{\gamma-1}{2} M_q^2} \right]^{\frac{1}{2}}$$
$$V_t = \left[KR \frac{T_e (1 + \frac{\gamma-1}{2} M_e^2)}{\frac{\gamma+1}{2}} \right]^{\frac{1}{2}}.$$

The cumulative vacuum thrust coefficient can be calculated as follows:

$$\begin{aligned}
 C_{Fvac_x} &= \frac{\gamma P_t V_q \cos \theta_q}{P_e V_t} + \varepsilon \frac{P_q}{P_e} \left[1 - \left(\frac{R_q}{R_e} \right)^2 \right] + \int \frac{P_x dA}{P_e A_t} \\
 &= \gamma \left(\frac{P_t}{P_e} \right) \left(\frac{V_q}{V_t} \right) \cos \theta_q + \varepsilon \frac{P_q}{P_e} \left[1 - \left(\frac{R_q}{R_e} \right)^2 \right] \\
 &\quad + \sum_{n=1}^{N_2} \frac{\varepsilon}{2} \left[\left(\frac{P_x}{P_e} \right)_{n-1} + \left(\frac{P_x}{P_e} \right)_n \right] \left[\left(\frac{R_x}{R_e} \right)_{n-1}^2 - \left(\frac{R_x}{R_e} \right)_n^2 \right] \quad (52)
 \end{aligned}$$

THE FORTRAN PROGRAMS

Outline of External Expansion Plug Nozzle Design

This program was used to compute the example of design nozzle contours shown in FIG 5. The vacuum thrust coefficient and vacuum specific impulse distributions along the plug axis are shown in FIG 6 and FIG 7.

- INPUT:
- (1) Estimated exit Mach number (obtained from isentropic flow tables based on the expansion ratio and the ratio of specific heats)
 - (2) Expansion ratio
 - (3) Number of contour points
 - (4) Gas constant
 - (5) Exit temperature
 - (6) Atmosphere pressure ratio
 - (7) Constant of proportionality in Newton's second law
 - (8) Ratio of specific heats (constant or variable)
- OUTPUT:
- (1) Angle between plug axis and sonic line
 - (2) Width of throat gap
 - (3) Optimum thrust coefficient
 - (4) Mach number distribution
 - (5) Co-ordinates of plug contour
 - (6) Pressure ratio at each point
 - (7) Cumulative vacuum thrust coefficient
 - (8) Cumulative specific impulse
 - (9) Cumulative vacuum specific impulse



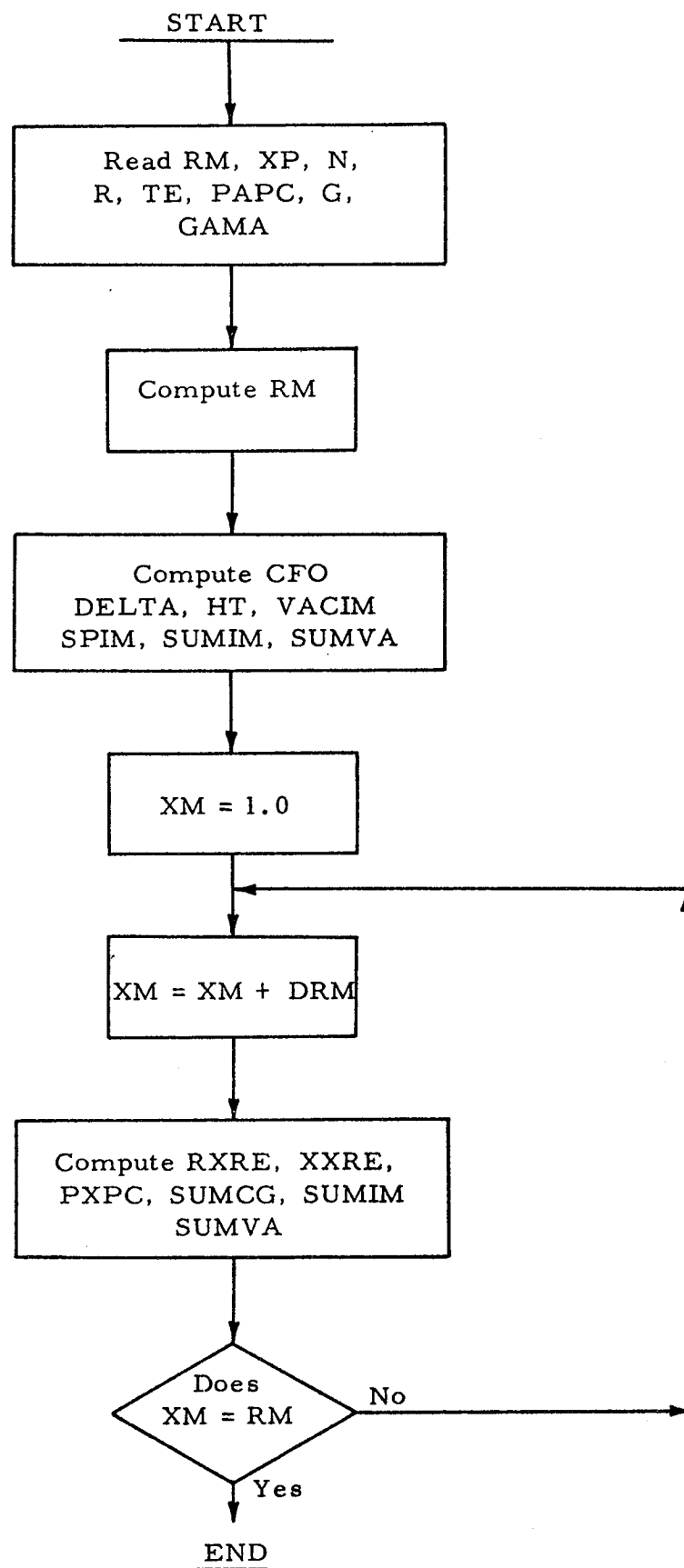
Outline of Internal-External Expansion Plug Nozzle Design

This program has been used to compute a few examples. The results of design nozzle contour are shown in FIG 8. The vacuum thrust coefficient and vacuum specific impulse distributions along the plug axis are shown in FIG 9 and 10 respectively.

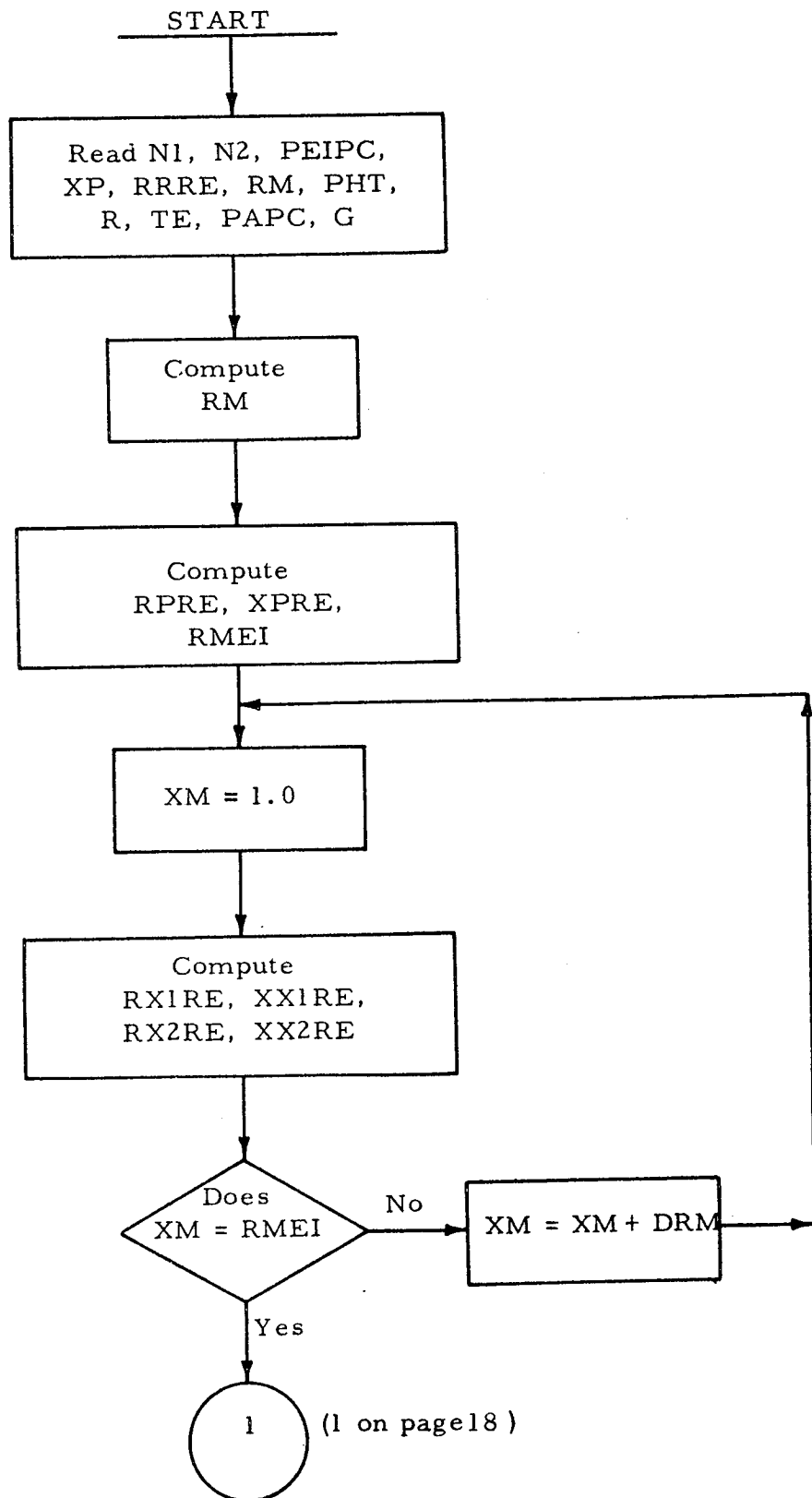
- INPUT:
- (1) Number of internal contour points and external contour points
 - (2) Pressure ratio at end of internal expansion
 - (3) Expansion ratio
 - (4) Radius of internal circular arc contour
 - (5) Estimated Mach number
 - (6) Angle between plug axis and Prandtl-Meyer expansion wave at throat
 - (7) Gas constant
 - (8) Exist temperature
 - (9) Atmosphere pressure ratio
 - (10) Constant of proportionality in Newton's second law
 - (11) Ratio of specific heats (constant or variable)
- OUTPUT:
- (1) Mach number distribution
 - (2) Co-ordinates of nozzle contour
 - (3) Pressure ratio at each point
 - (4) Cumulative vacuum thrust coefficient
 - (5) Cumulative specific impulse
 - (6) Cumulative vacuum specific impulse

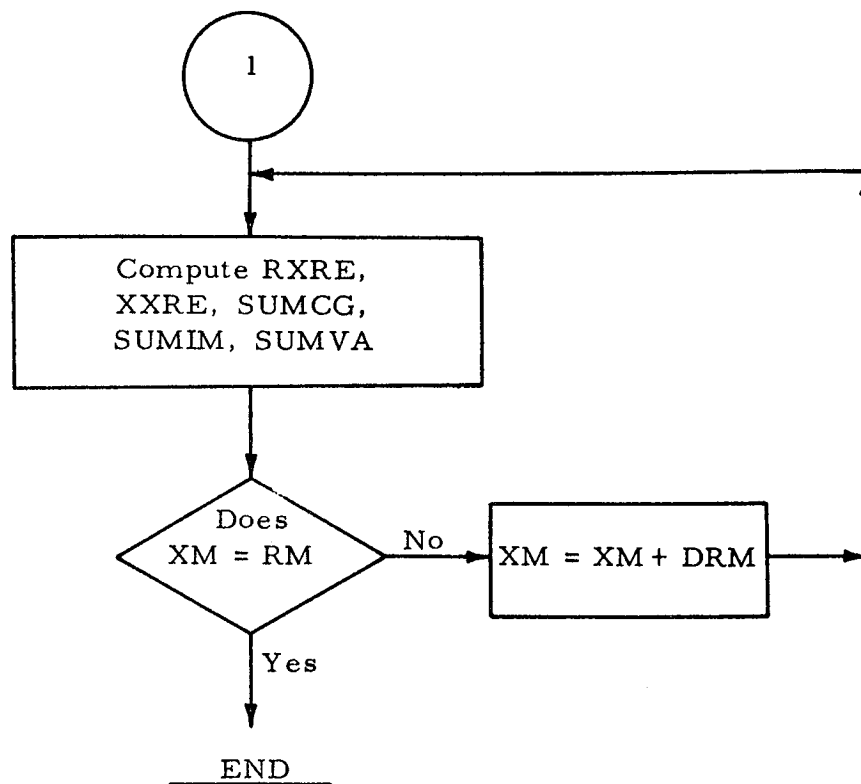


FLOW CHART OF EXTERNAL EXPANSION PLUG NOZZLES DESIGN



FLOW CHART OF INTERNAL-EXTERNAL
EXPANSION PLUG NOZZLES DESIGN





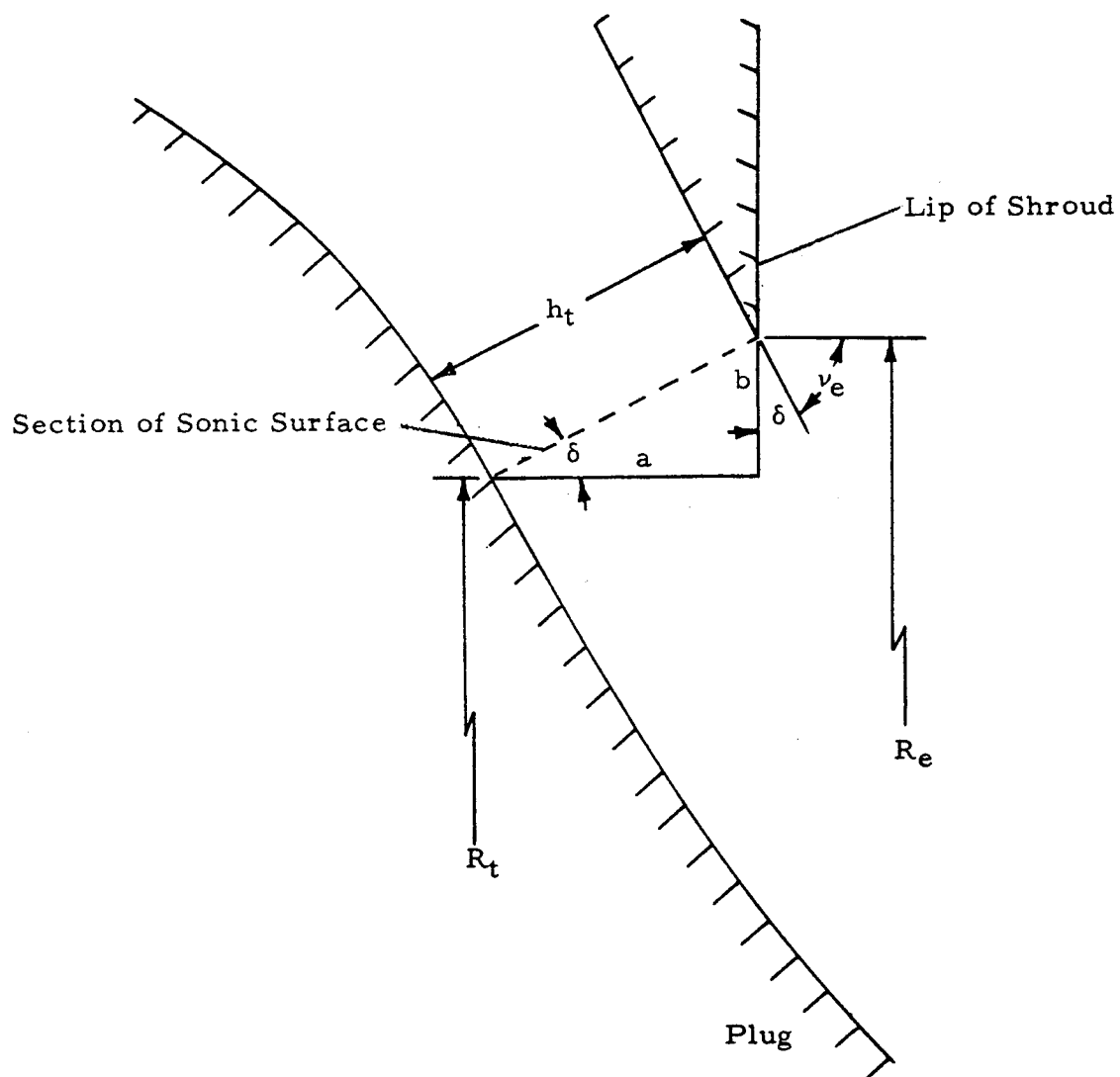


FIG 1 - External Expansion Plug Nozzle Throat Configuration

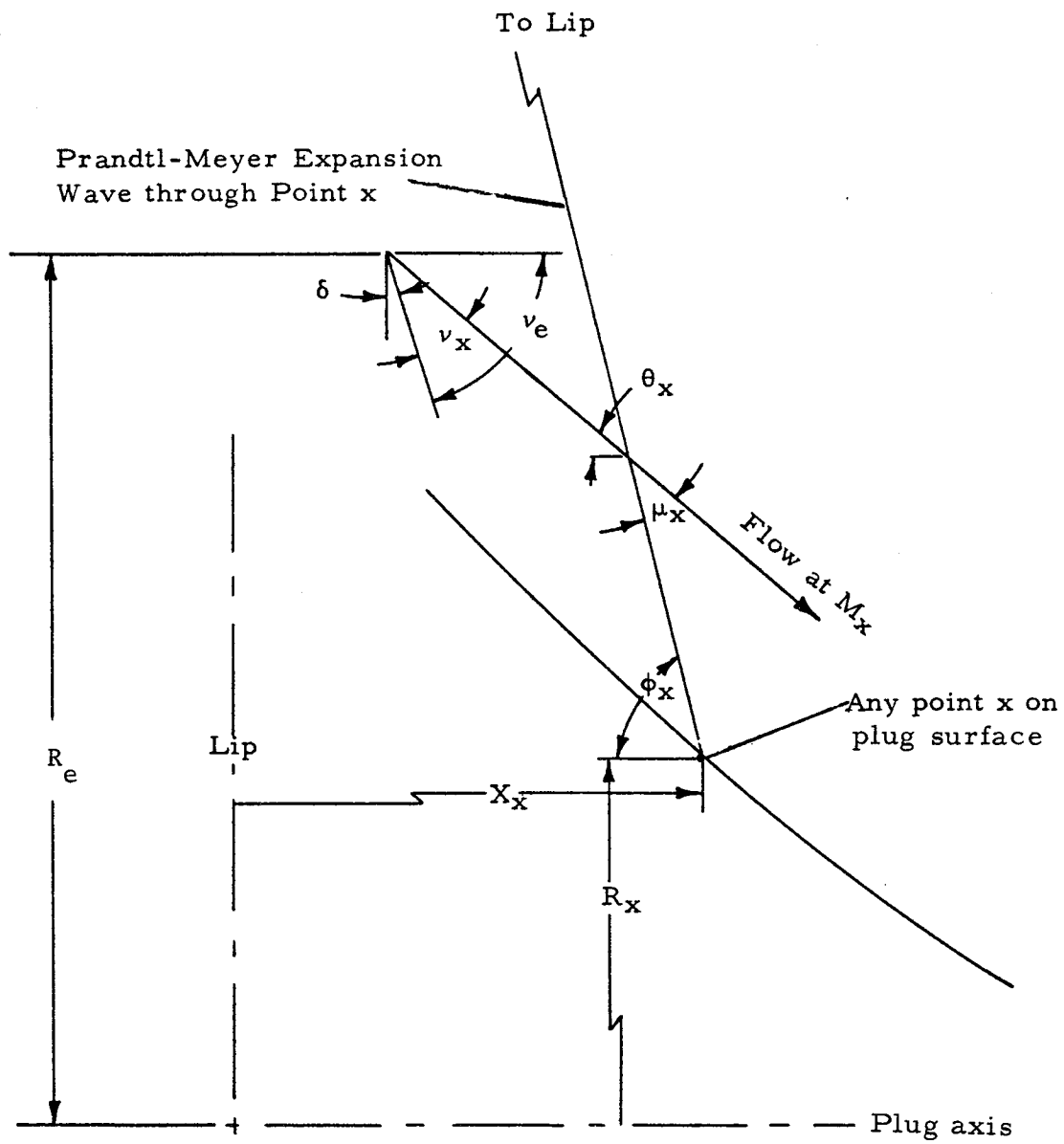


FIG 2 - External Expansion Plug Nozzle

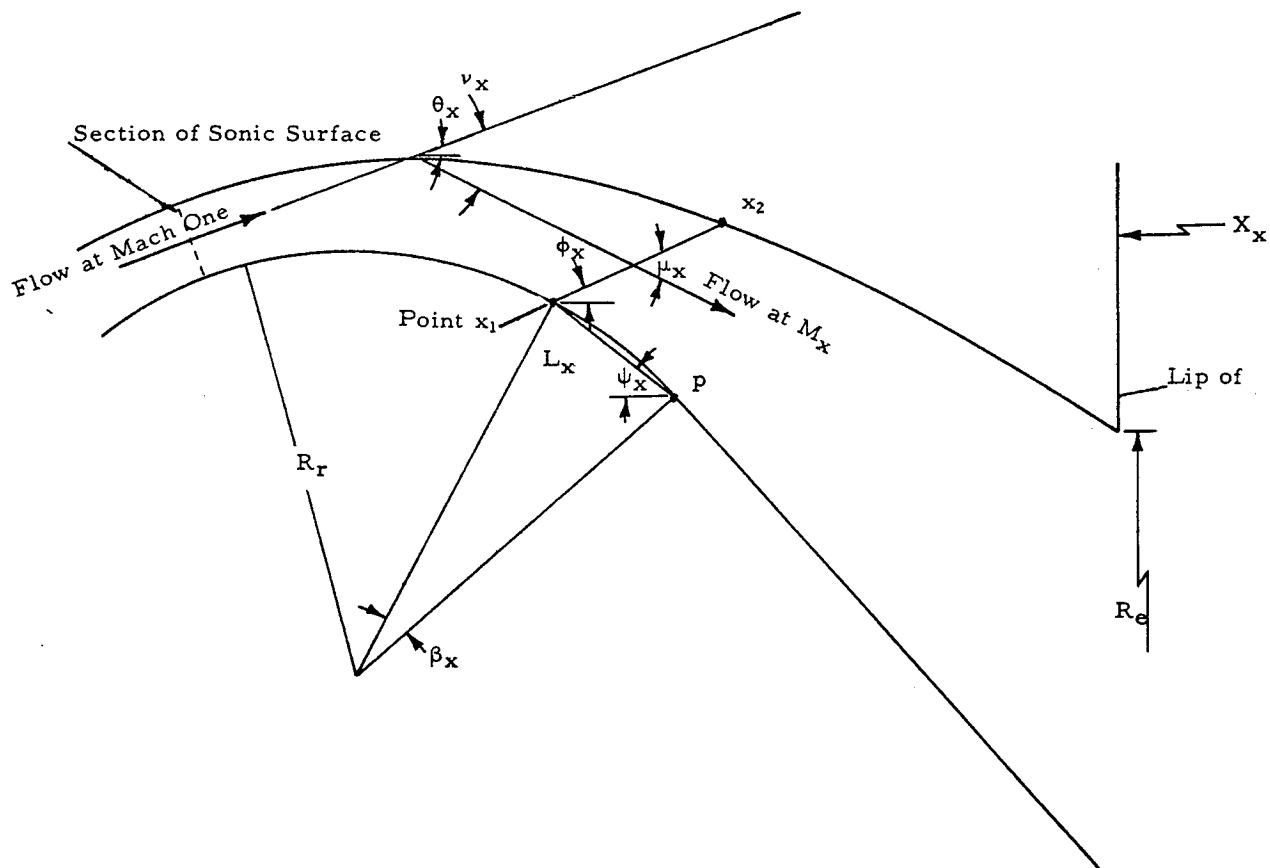


FIG 3 - Internal Portion of Internal-External Expansion Plug Nozzle

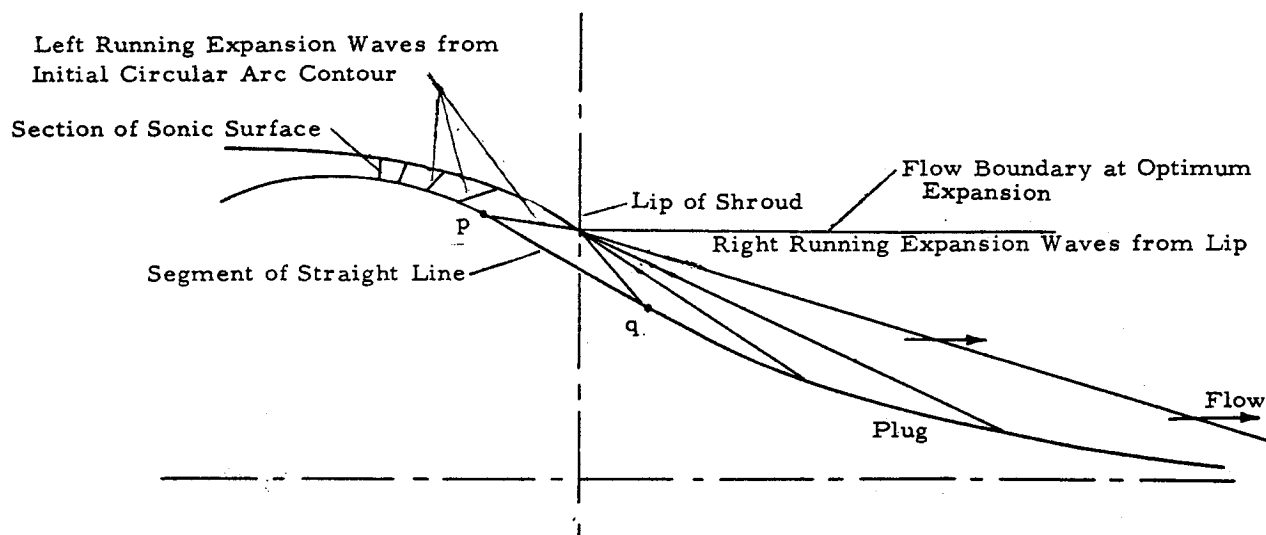
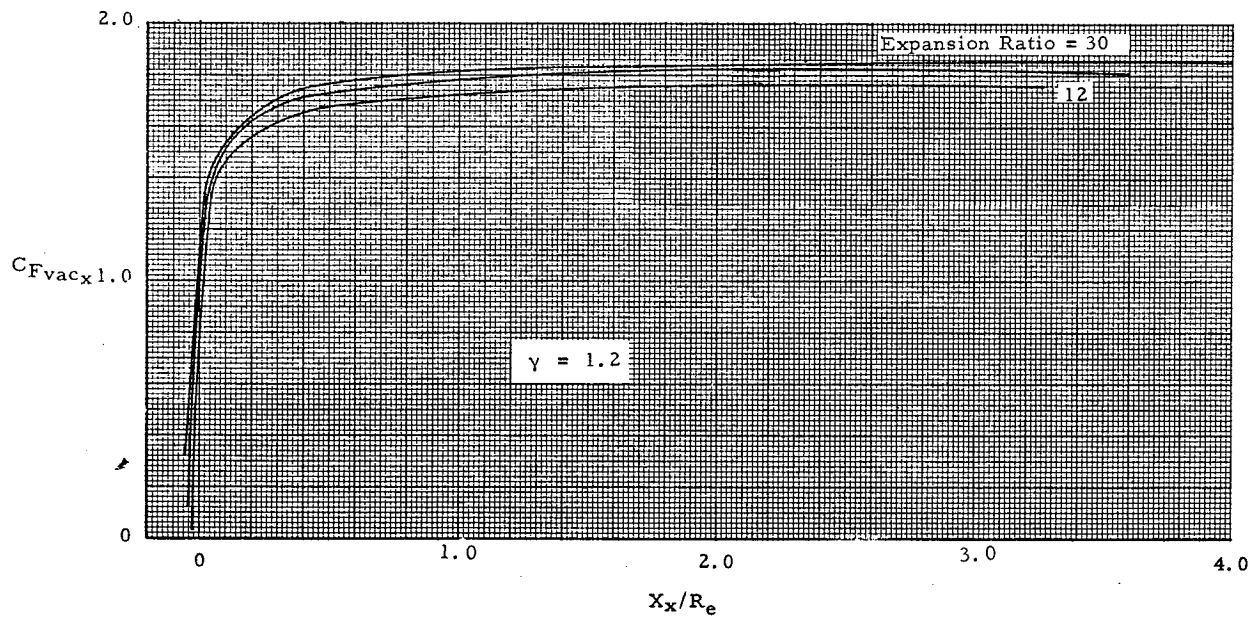
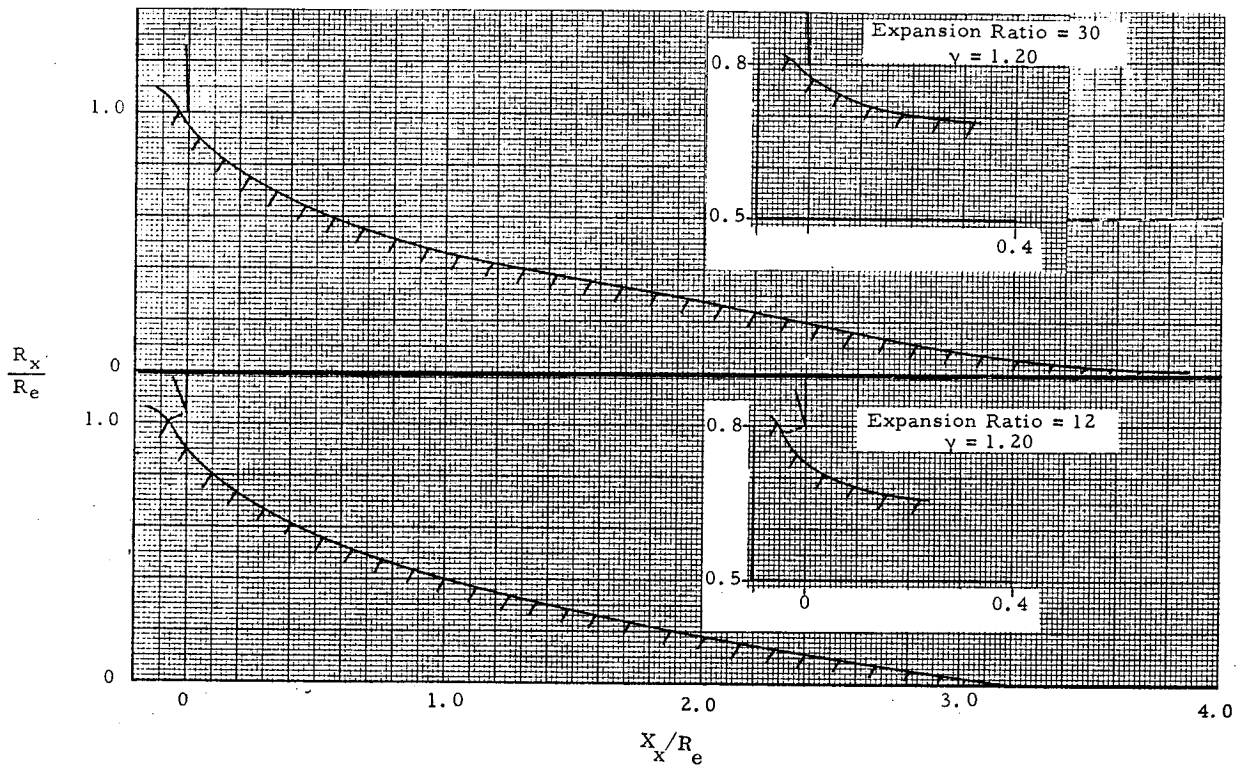


FIG 4 - Internal-External Expansion Plug Nozzle



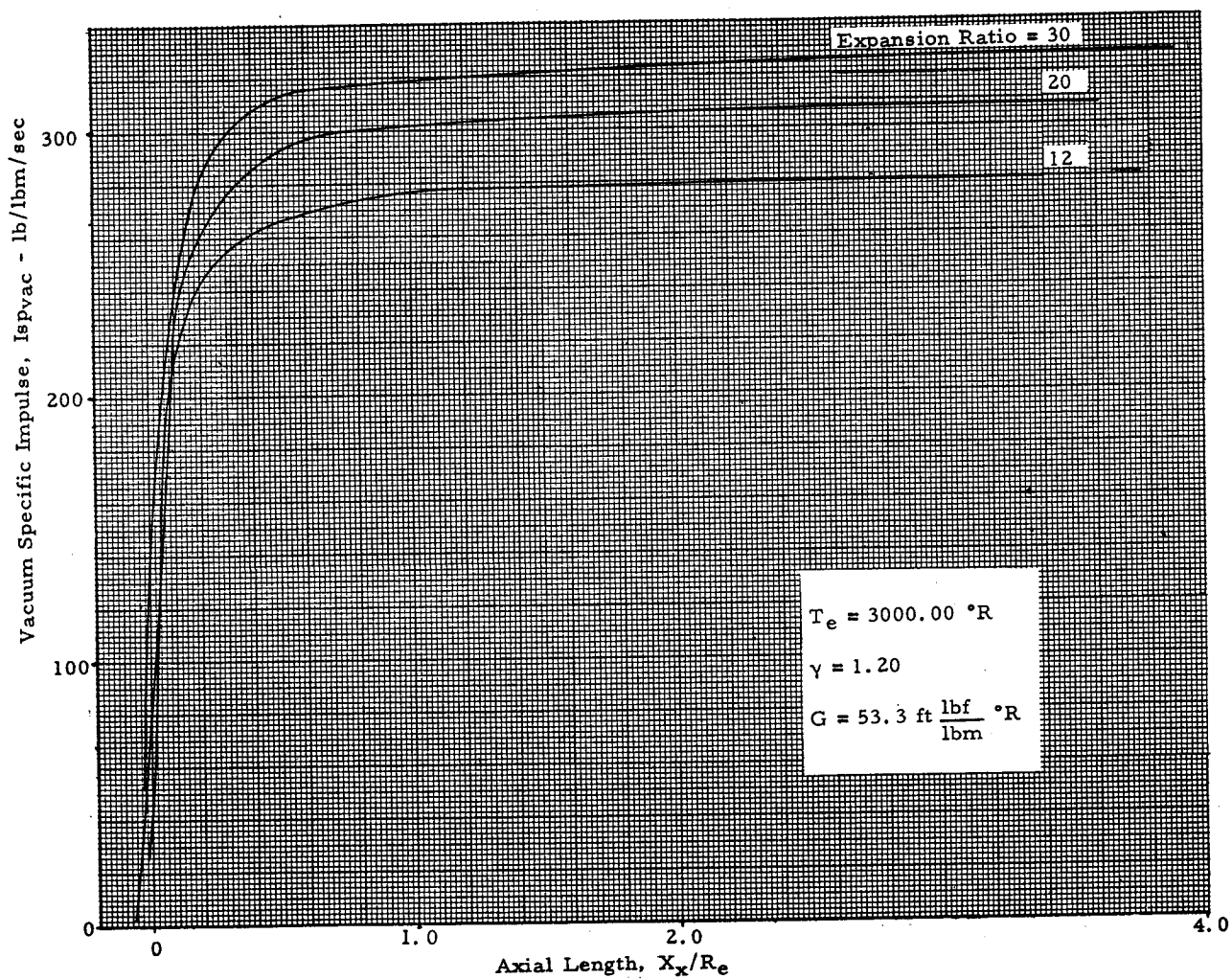
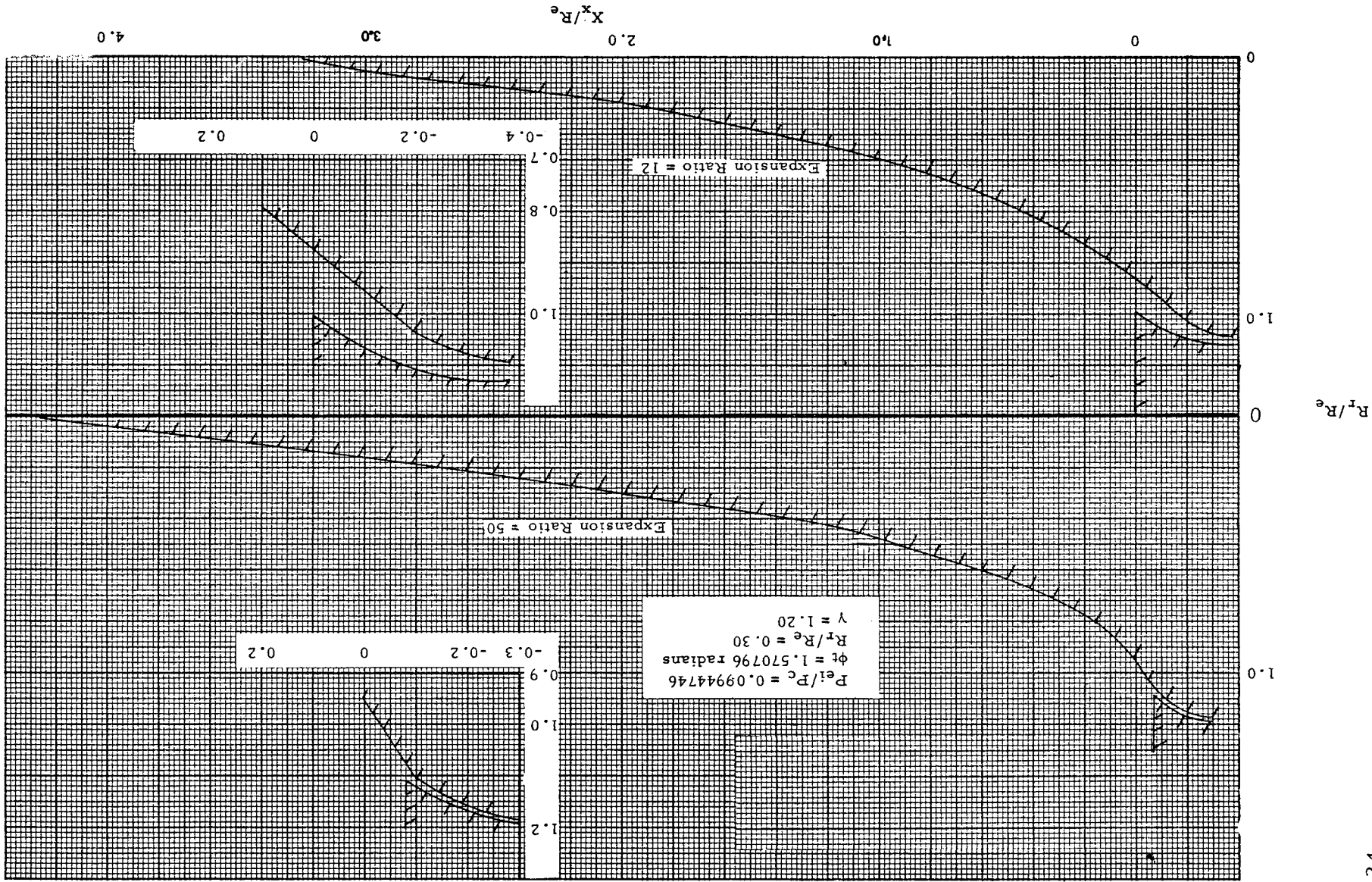


FIG 7 - Vacuum Specific Impulse Distribution Along the Axis of External Expansion Plug Nozzles

FIG 8 Internal - External Expansion Plug Nozzle Contours



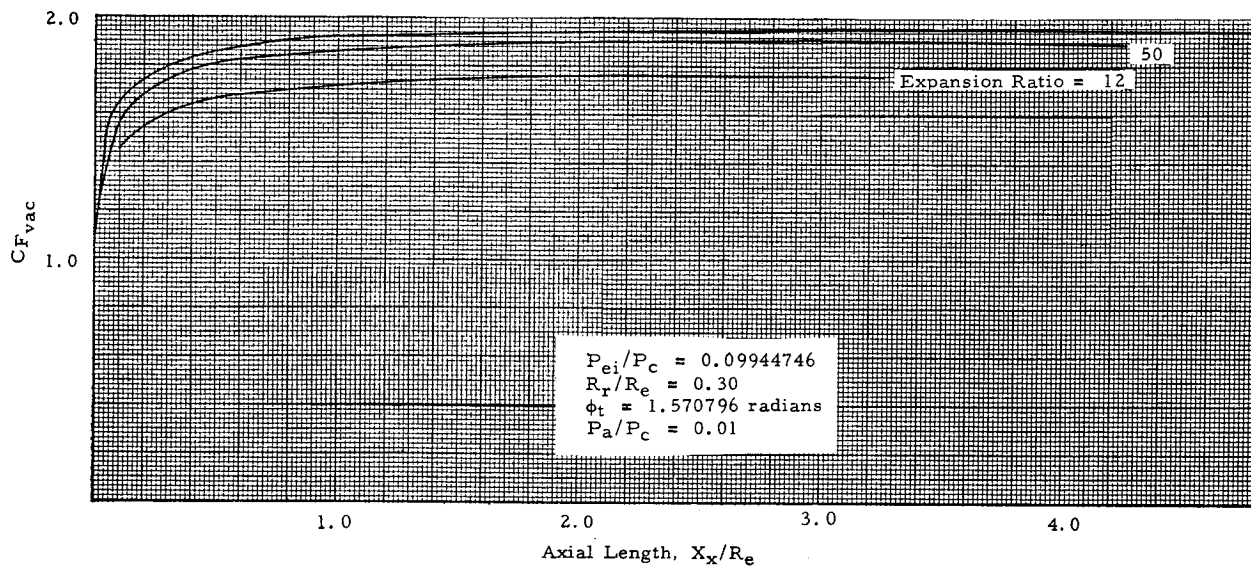


FIG 9 - Vacuum Thrust Coefficient Distribution Along the Axis of Internal - External Expansion Plug Nozzles

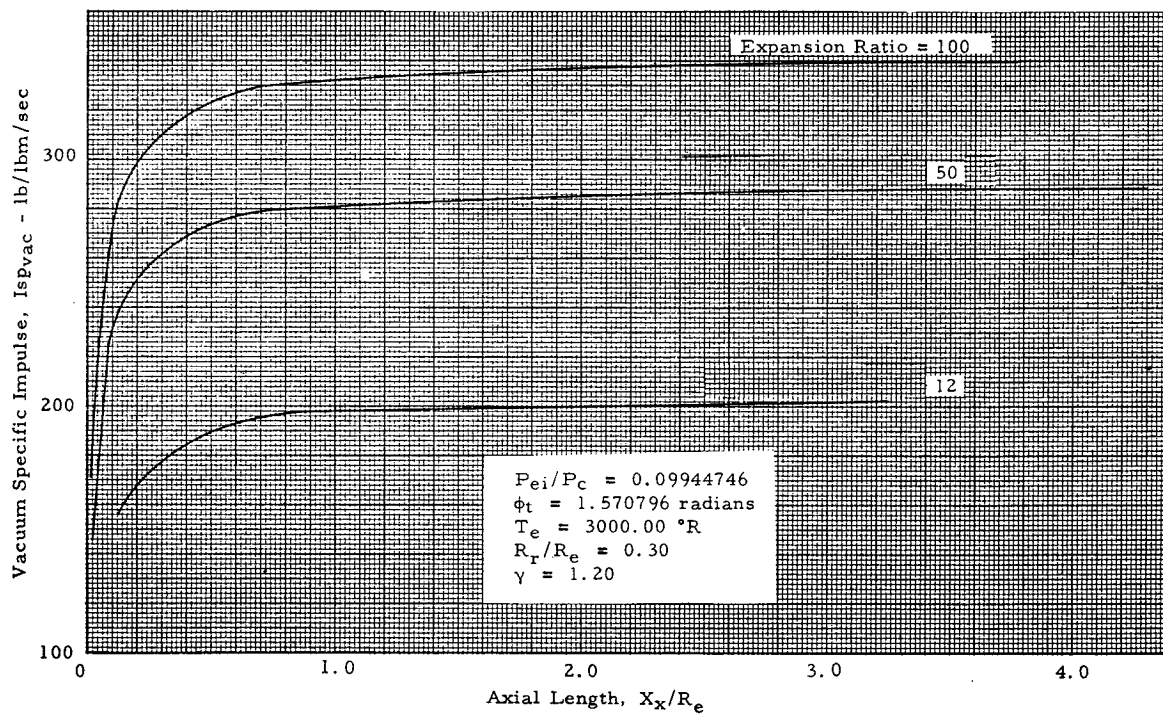


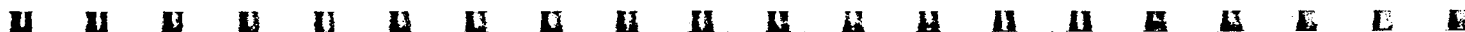
FIG 10 - Vacuum Specific Impulse Distribution Along the Axis of Internal-External Expansion Plug Nozzles

APPENDIX I

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DESIGN OF EXTERNAL EXPANSION PLUG NOZZLES

27



LIST OF FORTRAN PROGRAM

DESIGN OF EXTERNAL EXPANSION PLUG NOZZLES

```

53 FORMAT(1HK,5HPA/PC,1X,1H=,E14.7)
    IF(GAS)4,4,10
10 READ3,GAMA
    3 FORMAT(F5.0)
    PRINT 45
45 FORMAT(1HK,5HUSING,1X,5HIDEAL,1X,3HGAS)
    PRINT49,GAMA
49 FORMAT(1HK,1X,4HGAMA,1X,1H=,F5.2)
    GO TO 9
C    REAL GAS HAS TO INPUT NT VALUES OF THERMODYNAMIC DATA
    4 READ5,NT
    5 FORMAT(I4)
    PRINT 46
46 FORMAT(1HK,5HUSING,1X,4HREAL,1X,3HGAS)
    DO8I=1,NT
    READ6,HM(I),GAM(I)
    6 FORMAT(2F10.7)
    8 CONTINUE
    9 CONTINUE
13 IF(GAS)30,31,31

```


LIST OF FORTRAN PROGRAM

DESIGN OF EXTERNAL EXPANSION PLUG NOZZLES

```

30 DO34J=1,NT
    I=J
    IF(RM-HM(J))32,33,34
34 CONTINUE
33 GAMA=GAM(I)
    GO TO 31
32 GAMA=GAM(I-1)+(RM-HM(I-1))*(GAM(I)-GAM(I-1))/(HM(I)-HM(I-1))
31 FME=(2.0+(GAMA-1.0)*RM*RM)/(GAMA+1.0)
    COM=(GAMA+1.0)/(2.0*(GAMA-1.0))
    FME=RM*XP-FME**COM
    FPM=(2.0+(GAMA-1.0)*RM*RM)/(GAMA+1.0)
    COM=(3.0-GAMA)/(2.0*(GAMA-1.0))
    FPM=XP-RM*(FPM**COM)
    DM=-FME/FPM
    RM=RM+DM
    DM=ABSF(DM)
    IF(DM-0.00001)12,12,13
12 CONTINUE
    A=SQRTF((GAMA-1.0)*(RM*RM-1.0)/(GAMA+1.0))
    B=SQRTF((GAMA+1.0)/(GAMA-1.0))

```

LIST OF FORTRAN PROGRAM

DESIGN OF EXTERNAL EXPANSION PLUG NOZZLES

```

C=SQRTF(RM*RM-1.0)
C=ATANF(C)
VE=B*ATANF(A)-C
DELTA=1.570796-VE
SUMCG=((2.0/(GAMA+1.0))**(GAMA/(GAMA-1.0)))*(GAMA+1.0)*SINF(DELTA)
HT=(XP-SQRTF(XP*(XP-SINF(DELTA))))/(XP*SINF(DELTA))
A1=(GAMA+1.0)/(2.0*(GAMA-1.0))
B1=SQRTF(1.0+0.5*(GAMA-1.0)*RM*RM)
CFD=GAMA*RM*((2.0/(GAMA+1.0))**A1)/B1
PCPT=(0.5*(GAMA+1.0))**(GAMA/(GAMA-1.0))
VT=1.0+0.5*(GAMA-1.0)*RM*RM
VT=GAMA*R*TE*VT/(0.5*(GAMA+1.0))
VT=VT*G
VT=SQRTF(VT)
SPIM=(1.0-PCPT*PAPC)/GAMA
SPIM=1.0+SPIM
SUMIM=VT*SINF(DELTA)*SPIM/G
VACIM=1.0+1.0/GAMA
SUMVA=VT*SINF(DELTA)*VACIM/G
PRINT 15

```

LIST OF FORTRAN PROGRAM

DESIGN OF EXTERNAL EXPANSION PLUG NOZZLES

```

15 FORMAT(1HK,5HDELTA,9X,5HHT/RE,9X,5HCFOPT)
   PRINT 16,DELTA,HT,CFO
16 FORMAT(1HK,3E 14.7)
   XN=N
   DRM=(RM-1.0)/XN
   XM=1.0
   K=1
   RXRE=1.0-HT*SINF(DELTA)
   XXRE=(-HT)*COSF(DELTA)
   IF(GAS)38,39,39
38 DO 37J=1,NJ
   I=J
   IF(XM-HM(J))35,36,37
37 CONTINUE
36 GAMA=GAM(I)
   GO TO 39
35 GAMA=GAM(I-1)+(XM-HM(I-1))*(GAM(I)-GAM(I-1))/(HM(I)-HM(I-1))
39 A3=(-GAMA)/(GAMA-1.0)
   PXPC=(1.0+0.5*(GAMA-1.0)*XM*XM)**A3
   PRINT17

```

DESIGN OF EXTERNAL EXPANSION PLUG NOZZLES

32

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LIST OF FORTRAN PROGRAM

DESIGN OF EXTERNAL EXPANSION PLUG NOZZLES

```

B2=(2.0/(GAMA+1.0))*(1.0+0.5*(GAMA-1.0)*XM*XM)
RXRE= 1.0-(B2**A2)*SINF(VE-VX+UX)/XP
RXRE=SQRTF(RXRE)
52 XXRE=(1.0-RXRE)*COSF(VE-VX+UX)/SINF(VE-VX+UX)
A3=(-GAMA)/(GAMA-1.0)
PXPC=(1.0+0.5*(GAMA-1.0)*XM*XM)**A3
SUMCG=SUMCG+0.5*XP*(PRO+PXPC)*(RX0*RX0-RXRE*RXRE)
CO=PCPT*VT*XP/(G*GAMA)
SUMIM=SUMIM+0.5*CO*(PRO+PXPC-2.0*PAPC)*(RX0*RX0-RXRE*RXRE)
SUMVA=SUMVA+0.5*CO*(PRO+PXPC)*(RX0*RX0-RXRE*RXRE)
22 PRINT18,XM,RXRE,XXRE,PXPC,SUMCG,SUMIM,SUMVA
18 FORMAT(1HK,7E14.7)
IF(K-N)19,19,20
19 XM=XM+DRM
PRO=PXPC
RX0=RXRE
GO TO 14
20 PRINT21
21 FORMAT(1HK, 8HEXTERNAL,1X,9HEXPANSION,1X,6HNOZZLE,1X,7HCONTOUR)
GO TO 101

```

LIST OF FORTRAN PROGRAM

DESIGN OF EXTERNAL EXPANSION PLUG NOZZLES

END



APPENDIX II

U U U U U U U U U U U U U U U U U U

LIST OF FORTRAN PROGRAM

DESIGN OF INTERNAL-EXTERNAL EXPANSION PLUG NOZZLE

```
C    DESIGN OF INTERNAL-EXTERNAL EXPANSION PLUG NOZZLES
C    INPUT GAS=+1.0 WHEN DEALING WITH IDEAL GAS
C    INPUT GAS=-1.0 WHEN DEALING WITH REAL GAS
      DIMENSION HM(30),GAM(30)
101 READ(5,11) N1,N2
      11 FORMAT(2I4)
      READ(5,1) GAS,PEIPC,XP,RRRE,RM,PHT
      1 FORMAT(6F10.0)
      WRITE (6,52) PEIPC
52 FORMAT(1H1,6HPEI/PC,1X,1H=,E14.7)
      WRITE (6,53) XP
53 FORMAT(1H0,9HEXPANSION,1X,5HRATIO,1X,1H=,F10.5)
      WRITE (6,54) RRRE
54 FORMAT(1H0,5HRR/RE,1X,1H=,E14.7)
      WRITE (6,55) RM
55 FORMAT(1H0,8HESTIMATE,1X,4HMACH,1X,6HNUMBER,1X,1H=,E14.7)
      WRITE (6,56) PHT
56 FORMAT(1H0,3HPHT,1X,1H=,E14.7)
      READ(5,66) R,TE,PAPC,G
      66 FORMAT(4F10.0)
```


LIST OF FORTRAN PROGRAM

DESIGN OF INTERNAL-EXTERNAL EXPANSION PLUG NOZZLE

```

WRITE(6,67) TE
67 FORMAT(1H0,4HEXIT,1X,11HTEMPERATURE,1X,1H=,E14.7)
WRITE(6,68) PAPC
68 FORMAT(1H0,5HPA/PC,1X,1H=,E14.7)
IF(GAS)4,4,2
2 READ(5,3) GAMA
3 FORMAT(F5.0)
WRITE (6,57)
57 FORMAT(1H0,5HUSING,1X,5HIDEAL,1X,3HGAS)
GO TO 8
4 READ(5,5) NT
5 FORMAT(I4)
DO7I=1,NT
READ(5,6) HM(I),GAM(I)
6 FORMAT(2F10.7)
7 CONTINUE
WRITE(6,51)
51 FORMAT(1H0,5HUSING,1X,4HREAL,1X,3HGAS)
8 CONTINUE
34 IF(GAS)30,9,9

```



LIST OF FORTRAN PROGRAM

DESIGN OF INTERNAL-EXTERNAL EXPANSION PLUG NOZZLE

```

30 DO 31 J = 1, NT
    I=J
    IF(RM-HM(J))33,32,31
31 CONTINUE
32 GAMA=GAM(I)
    GO TO 9
33 GAMA=GAM(I-1)+(RM-HM(I-1))*(GAM(I)-GAM(I-1))/(HM(I)-HM(I-1))
    9 FME=(2.0+(GAMA-1.0)*RM*RM)/(GAMA+1.0)
    COM=(GAMA+1.0)/(2.0*(GAMA-1.0))
    FME=RM*XP-FME**COM
    FPM=(2.0+(GAMA-1.0)*RM*RM)/(GAMA+1.0)
    COM=(3.0-GAMA)/(2.0*(GAMA-1.0))
    FPM=XP-RM*(FPM**COM)
    DM=-FME/FPM
    RM=RM+DM
    DM = ABS(DM)
    IF(DM-0.00001)10,10,34
10 CONTINUE
    A=(GAMA-1.0)*(RM*RM-1.0)/(GAMA+1.0)
    A=SQRT (A)

```

LIST OF FORTRAN PROGRAM

DESIGN OF INTERNAL-EXTERNAL EXPANSION PLUG NOZZLE

```

A=ATAN (A)
B=SQRT ((GAMA+1.0)/(GAMA-1.0))
C=SQRT (RM*RM-1.0)
C=ATAN (C)
VE=B*A-C
RMEI=PEIPC**((1.0-GAMA)/GAMA)
RMEI=(2.0/(GAMA-1.0))*(RMEI-1.0)
RMEI=SQRT (RMEI)
IF(GAS)36,35,35
36 DO 37 J = 1, NT
   I=J
   IF(RMEI-HM(J))39,38,37
37 CONTINUE
38 GAMA=GAM(I)
   GO TO 35
39 GAMA=GAM(I-1)+(RM-HM(I-1))*(GAM(I)-GAM(I-1))/(HM(I)-HM(I-1))
35 A=(GAMA-1.0)*(RMEI*RMEI-1.0)/(GAMA+1.0)
   A=SQRT (A)
   A=ATAN (A)
   B=SQRT ((GAMA+1.0)/(GAMA-1.0))

```

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LIST OF FORTRAN PROGRAM

DESIGN OF INTERNAL-EXTERNAL EXPANSION PLUG NOZZLE

```

C=SQRT (RMEI*RMEI-1.0)
C=ATAN (C)
VEI=B*A-C
Y=1.0/RMEI
UEI=ATAN (Y/SQRT (1.0-Y*Y))
THEI=VEI-VE
PHEI=THEI+UEI
C  CALCULATE THE ORIGIN OF THE LAST INTERNAL EXPANSION WAVE
A1=(2.0/(GAMA+1.0))*(1.0+0.5*(GAMA-1.0)*RMEI*RMEI)
A1=A1*((GAMA+1.0)/(2.0*(GAMA-1.0)))
B1=SIN (PHEI)
RPRE=SQRT (1.0-A1*B1/XP)
XP=(RPRE-1.0)*COS (PHEI)/SIN (PHEI)
XN1=N1
DRM=(RMEI-1.0)/XN1
K=0
XM=1.0
WRITE(6,17)
170FORMAT(1H0,4HMACH,10X,6HRX1/RE,8X,6HXX1/RE,8X,6HRX2/RE,8X,6HXX2/RE
1,8X,5HPX/PC)

```

LIST OF FORTRAN PROGRAM

DESIGN OF INTERNAL-EXTERNAL EXPANSION PLUG NOZZLE

```

44 IF(GAS)40,12,12
40 DO 43 J = 1, NT
    I=J
    IF(XM-HM(J))41,42,43
43 CONTINUE
42 GAMA=GAM(I)
    GO TO 12
41 GAMA=GAM(I-1)+(XM-HM(I-1))*(GAM(I)-GAM(I-1))/(HM(I)-HM(I-1))
12 A=(GAMA-1.0)*(XM*XM-1.0)/(GAMA+1.0)
    A=SQRT (A)
    A=ATAN (A)
    B=SQRT ((GAMA+1.0)/(GAMA-1.0))
    C=SQRT (XM*XM-1.0)
    C=ATAN (C)
    VX=A*B-C
    BX=PHT-1.570796-VX+ABS (THEI)
    XLRE=2.0*RRRE*SIN (0.5*BX)
    PSI=3.1416-PHT+VX-0.5*(3.1416-BX)
    RX1RE=RPRE+XLRE*SIN (PSI)
    XX1RE=XPRE-XLRE*COS (PSI)

```

DESIGN OF INTERNAL-EXTERNAL EXPANSION PLUG NOZZLE

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LIST OF FORTRAN PROGRAM

DESIGN OF INTERNAL-EXTERNAL EXPANSION PLUG NOZZLE

C DESIGN OF EXTERNAL CONTOUR

```
WRITE(6,18)
```

```
180FORMAT(1H0,4HMACH,10X,5HRX/RE,9X,5HXX/RE,9X,5HPX/PC,9X,5HCFVAC
```

```
19X,3HSP.,1X,7HIMPULSE,2X,4HVAC.,1X,7HIMPULSE)
```

```
UX=ATAN (1.0/(XM*SQRT (1.0-(1.0/XM)**2)))
```

```
A=SQRT ((GAMA-1.0)*(XM*XM-1.0)/(GAMA+1.0))
```

```
A=ATAN (A)
```

```
B=SQRT ((GAMA+1.0)/(GAMA-1.0))
```

```
C=SQRT (XM*XM-1.0)
```

```
C=ATAN (C)
```

```
VX=B*A-C
```

```
RXRE=(2.0/(GAMA+1.0))*(1.0+0.5*(GAMA-1.0)*XM*XM)
```

```
RXRE=RXRE**((GAMA+1.0)/(2.0*(GAMA-1.0)))
```

```
RXRE=1.0-RXRE*SIN (VE-VX+UX)/XP
```

```
RXRE=SQRT (RXRE)
```

```
XXRE=(1.0-RXRE)*COS (VE-VX+UX)/SIN (VE-VX+UX)
```

```
C1=(2.0/(GAMA+1.0))*(GAMA/(GAMA-1.0))
```

```
C2=SQRT ((0.5*(GAMA+1.0)*XM*XM)/(1.0+0.5*(GAMA-1.0)*XM*XM))
```

```
SUMCG=GAMA*C1*C2*COS (THEI)+XP*PXPC*(1.0-RXRE*RXRE)
```

```
VT=TE*(1.0+0.5*(GAMA-1.0)*RM*RM)
```

LIST OF FORTRAN PROGRAM

DESIGN OF INTERNAL-EXTERNAL EXPANSION PLUG NOZZLE

```

VT=GAMA*G*R*VT/(0.5*(GAMA+1.0))
VT=SQRT (VT)
VQ=TE*(1.0+0.5*(GAMA-1.0)*RM*RM)
VC=1.0+0.5*(GAMA-1.0)*XM*XM
VQ=GAMA*R*G*VQ/VC
VQ=SQRT (VQ)
A=1.0+0.5*(GAMA-1.0)*XM*XM
B=-GAMA/(GAMA-1.0)
A=A**B
C=1.0-PAPC*A*SIN (PHEI)/(XM*XM)
D=COS (THEI)+C/GAMA
SUMIM=VQ*D/G
CO=COS (THEI)+1.0/GAMA
SUMVA=VQ*CO/G
WRITE(6,19)XM,RXRE,XXRE,PXPC,SUMCG,SUMIM,SUMVA
19 FORMAT(1H0,7E14.7)

K1=1
XN2=N2
DRM=(RM-XM)/XN2
XM=XM+DRM

```


LIST OF FORTRAN PROGRAM

DESIGN OF INTERNAL-EXTERNAL EXPANSION PLUG NOZZLE

```

PRD=PXPC
RXD=RXRE
50 UX=ATAN (1.0/(XM*SQRT (1.0-(1.0/XM)**2)))
    IF(GAS)46,45,45
46 DO 49 J = 1, NT
    I=J
    IF(XM-HM(J))47,48,49
49 CONTINUE
48 GAMA=GAM(I)
    GO TO 45
47 GAMA=GAM(I-1)+(XM-HM(I-1))*(GAM(9)-GAM(I-1))/(HM(I)-HM(I-1))
45 A=SQRT ((GAMA-1.0)*(XM*XM-1.0)/(GAMA+1.0))
    A=ATAN (A)
    B=SQRT ((GAMA+1.0)/(GAMA-1.0))
    C=SQRT (XM*XM-1.0)
    C=ATAN (C)
    VX=B*A-C
    RXRE= (2.0/(GAMA+1.0))*(1.0+0.5*(GAMA-1.0)*XM*XM)
    RXRE=RXRE**((GAMA+1.0)*0.5/(GAMA-1.0))
    RXRE=SQRT (1.0-RXRE*SIN (VE-VX+UX)/XP)

```

U U U U U U U U U U U U U U U U U U

LIST OF FORTRAN PROGRAM

DESIGN OF INTERNAL-EXTERNAL EXPANSION PLUG NOZZLE

```

65 XXRE=(1.0-RXRE)*COS (VE-VX+UX)/SIN (VE-VX+UX)
    PXPC=(1.0+0.5*(GAMA-1.0)*XM*XM)**(-GAMA/(GAMA-1.0))
    SUMCG=SUMCG+0.5*XP*(PRO+PXPC)*(RXO*RXO-RXRE*RXRE)
    A=GAMA/(GAMA-1.0)
    A=(0.5*(GAMA+1.0))**A
    A=A*VT/(GAMA*G)
    B=0.5*A*XP
    SUMIM=SUMIM+B*(PRO+PXPC-2.0*PAPC)*(RXO*RXO-RXRE*RXRE)
    SUMVA=SUMVA+B*(PRO+PXPC)*(RXO*RXO-RXRE*RXRE)
    WRITE(6,21)XM,RXRE,XXRE,PXPC,SUMCG,SUMIM,SUMVA
21  FORMAT(1H0,7E14.7)
    IF(K1-N2)22,23,23
22  XM=XM+DRM
    PRO=PXPC
    RXO=RXRE
    K1=K1+1
    GO TO 50
23  WRITE(6,24)
240FORMAT(1H0,8HEXTERNAL,1X,7HPORTION,1X,2HOF,1X,3HTHE,1X,6HNOZZLE,1X
    1,2HIS,1X,8HCOMPLETE)

```

LIST OF FORTRAN PROGRAM

DESIGN OF INTERNAL-EXTERNAL EXPANSION PLUG NOZZLE

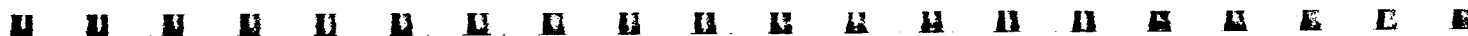
GO TO 101

END



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