Rocket Science 103: Cold-Gas Thruster Analysis

Newton's Laws as Applied to "Rocket Science"

... its not just a job ... its an adventure

Re-calculated performance metric …

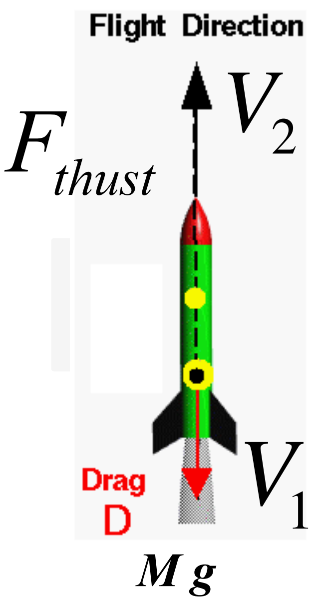
More accurate quantitative potential Altitude change predictions"

RS 103: Rocket Equation Revisited

$$\mathit{Rocket\ Equation} \rightarrow \Delta V = \underbrace{g_0 \cdot I_{sp} \cdot \ln\left(1 + P_{mf}\right)}_{\textbf{Propulsion Energy}} - \underbrace{\int_{t_0}^{t_{burn}} g \cdot \sin\gamma \cdot dt}_{\textbf{Gravity Loss}} - \underbrace{\int_{t_0}^{t_{burn}} \frac{D \cdot V}{M} \cdot dt}_{\textbf{Drag Loss}} =$$

$$\ldots \mathit{assume}\ \Delta V\ \mathit{added}\ \text{"}\mathit{impulsively}\text{"} \rightarrow t_{burn} - t_0 \rightarrow \text{"}\mathit{small}\text{"}$$

$$\underline{\mathit{ignore\ gravity, drag\ losses}} \rightarrow \Delta V = g_0 \cdot I_{sp} \cdot \ln\left(1 + \frac{M_{propellant}}{M_{dry}}\right)$$



Look at change in rocket energy state ….

$$\frac{\Delta K.E.}{M} = \frac{V_2^{\,2} - V_1^{\,2}}{2} \approx \frac{\left(V_2 + V_1\right) \cdot \left(V_2 - V_1\right)}{2} = \frac{\left(V_1 + \Delta V + V_1\right) \cdot \Delta V}{2}$$

$$\frac{\Delta K.E.}{M} = \left(V_1 + \frac{\Delta V}{2}\right) \cdot \Delta V \rightarrow \boxed{\Delta h_{potential} = \left(V_1 + \frac{\Delta V}{2}\right) \cdot \frac{\Delta V}{g}}$$

RS 103: Rocket Equation Revisited

$$Sub - in\ Rocket\ Equation$$

$$\Delta h_{potential} = \left(V_1 + \frac{\Delta V}{2}\right) \cdot \frac{\Delta V}{g} = \left(V_1 + \frac{g_0 \cdot I_{sp} \cdot \ln\left(1 + P_{mf}\right)}{2}\right) \cdot \frac{g_0 \cdot I_{sp} \cdot \ln\left(1 + P_{mf}\right)}{g}$$

$$assume\ \frac{g_0}{g} \approx 1$$

$$\boxed{\Delta h_{potential} = \frac{g_0 \cdot I_{sp}{}^2}{2} \cdot \left[\ln\left(1 + P_{mf}\right)\right]^2 \left[1 + \frac{2 \cdot V_1}{g_0 \cdot I_{sp} \cdot \ln\left(1 + P_{mf}\right)}\right]}$$

How do we estimate specific impulse for the cold-gas thruster? ... Fundamentals!

RS 103: Summary

"Metric" of performance

$$\Delta h_{potential} = \frac{g_0 \cdot I_{sp}^{\;2}}{2} \cdot \left[\ln\left(1 + \frac{M_{propellant}}{M_{dry}}\right)\right]^2 \left[1 + \frac{2 \cdot V_1}{g_0 \cdot I_{sp} \cdot \ln\left(1 + \frac{M_{propellant}}{M_{dry}}\right)}\right]$$

Specific Impulse

$$\left(I_{sp}\right)_{aerospike} = \frac{1}{g_0}\sqrt{\left(\frac{2 \cdot \gamma \cdot R_u}{\gamma - 1}\right)} \cdot \sqrt{\left(\frac{T_0}{M_w}\right)} \sqrt{\frac{\left[\left(\frac{A_{exit}}{A^*}\right)^{\frac{2(\gamma-1)}{(\gamma+1)}} - \left(\frac{2}{\gamma+1}\right)\right]}{\left(\frac{A_{exit}}{A^*}\right)^{\frac{2(\gamma-1)}{(\gamma+1)}}}}$$

Compare CO_2 and HPA as Working Propellant

From section 6.1 notes

Fluid	*Molecular Weight*	*Gamma*	*T_0, K*	*A/A**	*I_{sp}, sec*
CO_2	**44.0**	**1.3**	**288**	**15**	***52.92 sec***
HPA	**28.964**	**1.4**	**288**	**15**	***63.12 sec***

Tank Capacities, Weights

CO_2

$$M_{CO_2} = \frac{20_{oz}}{16_{\frac{oz}{LBM}}} \times \frac{1_{kg}}{2.204_{lbm}} = 0.567_{kg}$$

Part No.	Service Pressure	Diameter	Length	Estimated Weight	CO_2 Capacity	Air Capacity	H_2O Capacity	O-Ring Seat Dia.	Threads
	(psi / bar)	(in / mm)	(in / mm)	(lbs / kgs)	(lbs / kgs)	(cu ft / liters)	(cu in / cc)	(in / mm)	
20 oz.	1800 / 123	3.20 / 81	9.38 / 238	1.5 / .68	1.25 / .57	3.7 / 103	51 / 836	.750 / 19.0	5/8-18UNF-2B

HPA

Paintball Cylinders

Model	Spec	psi	Bar	Volume cu in	Volume liter	OD inches	OD MM	OAL inches	OAL MM	Weight LBS	Weight KG	FiberType
802	DOT / TC	4500	310	90.00	1.47	4.62	117.35	10.00	254.00	3.00	1.36	CAR/GLS

$$M_{HPA} = \rho \cdot V_{tank} = \frac{p}{R_g T} \cdot V_{tank} = \frac{(4500 + 14.7)_{psi} \times \frac{101325}{14.7 \frac{pa}{psi}}}{287.06_{\frac{J}{kg-K}} \, 288_K} \cdot \frac{90_{in^3}}{39.37^3_{\frac{in^3}{m^3}}} = 0.555_{kg}$$

Mass fraction summary

Sample calculation (All masses in kg)

Propellant	$M_{propellant}$	M_{tank}	M_{clean}	$M_{motorcase}$	M_{dry}	Pmf
CO_2 (20 Oz tank)	0.567	0.68	9.25	1 kg	10.93	0.0519
HPA (4500 psig, 90 cu. In tank)	0.555	1.36	9.25	1.kg	11.61	0.0478
CO_2 (24 Oz tank)	0.68	0.90	9.25	1 kg	11.15	0.0610

$$\left(\Delta V\right)_{CO_2} = g_0 \cdot I_{sp} \cdot \ln\left(1 + \frac{M_{propellant}}{M_{dry}}\right) = 9.8067 \cdot 52.92\,(\ln\,(1 + 0.0519)) = 26.26 \text{ m/sec}$$

20 oz

$$\left(\Delta V\right)_{CO_2} = 9.8067 \cdot 52.92\,(\ln\,(1 + 0.061)) = 30.73 \text{ m/sec}$$

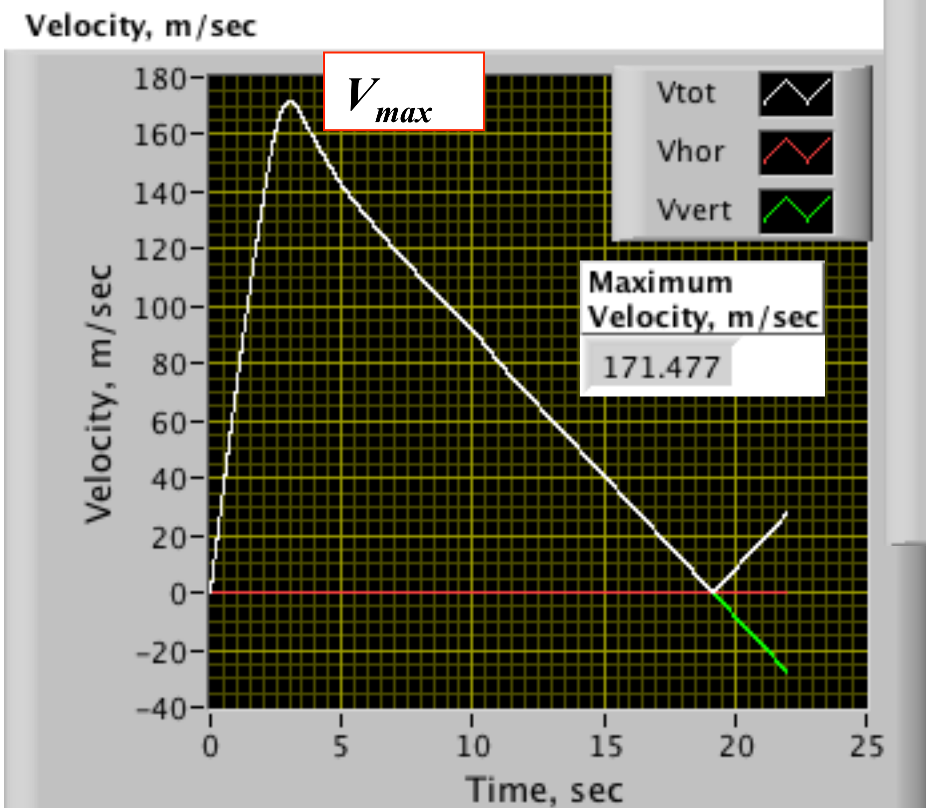
24 oz

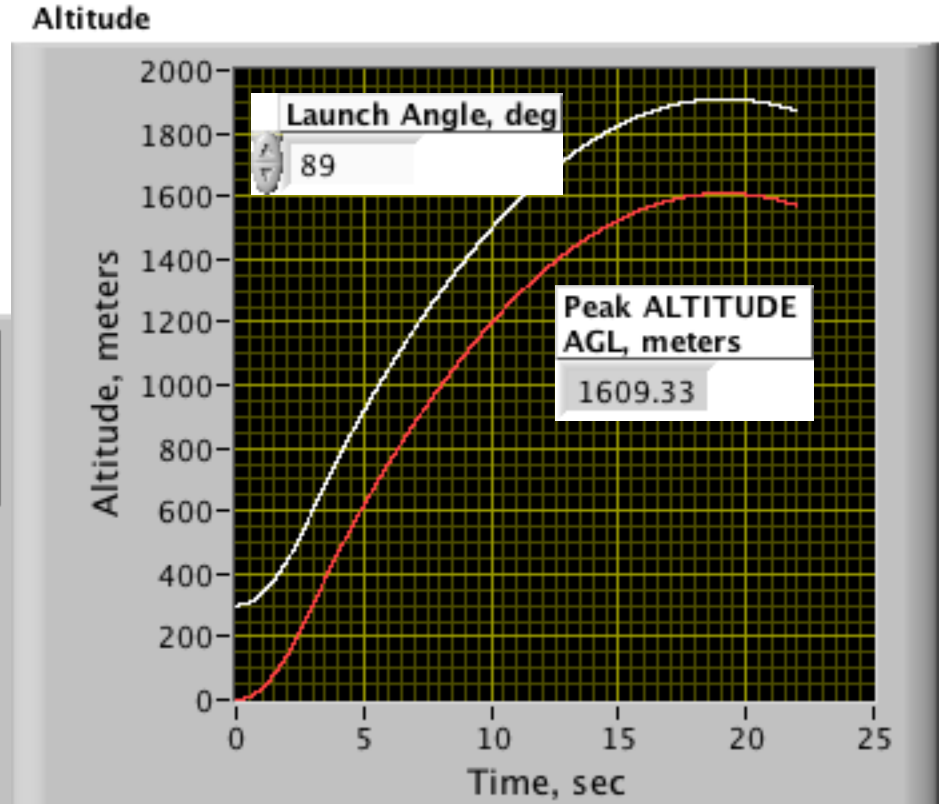
$$\left(\Delta V\right)_{HPA} = 9.8067 \cdot 63.12\,(\ln\,(1 + 0.0478)) = 28.90 \text{ m/sec}$$

90 cu. In.

Calculate Available Augmentation Lift

$$\Delta h_{potential} = \left(V_1 + \frac{\Delta V}{2}\right) \cdot \frac{\Delta V}{g}$$

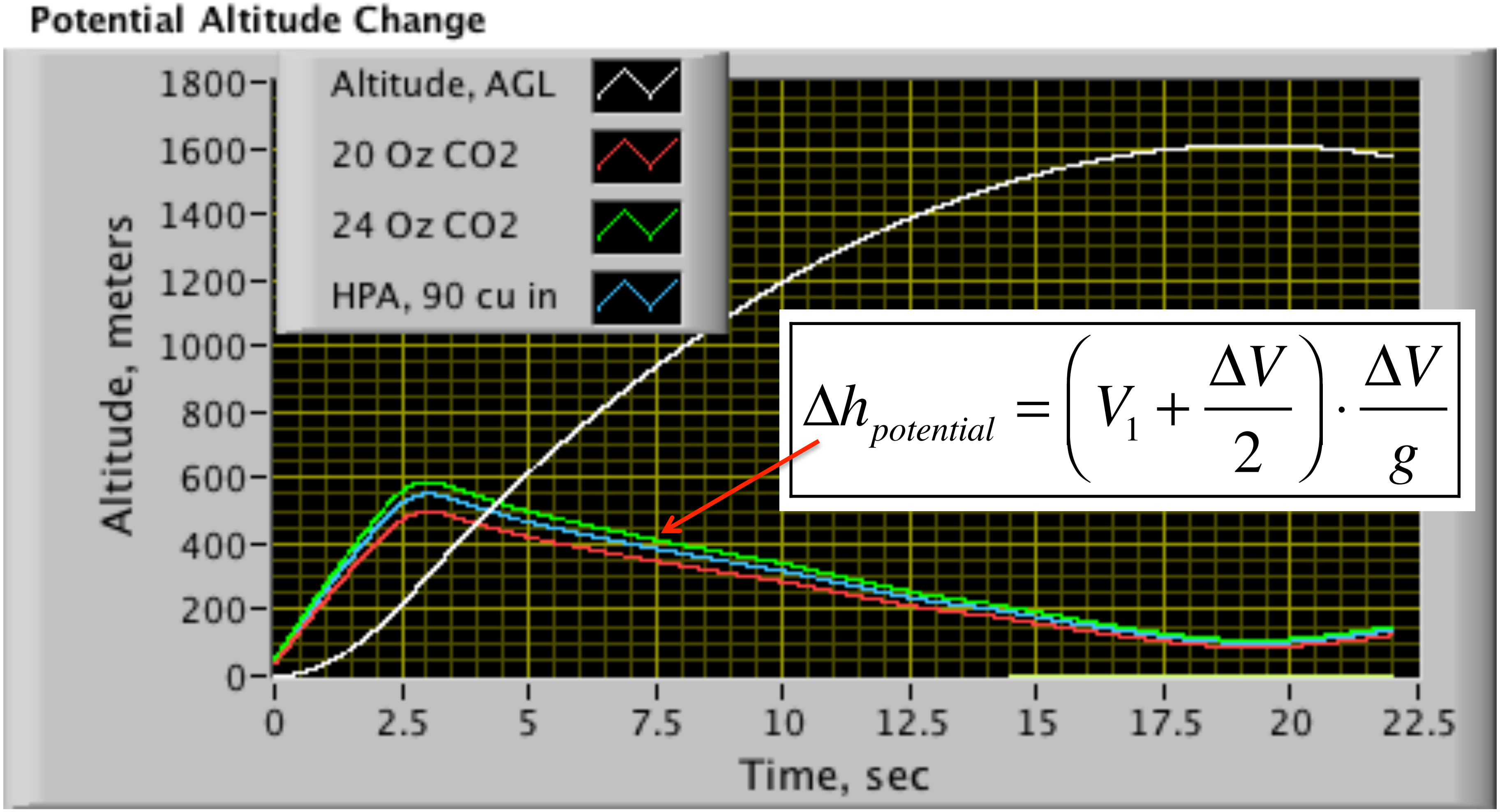




V_{max} → *main motor burnout*

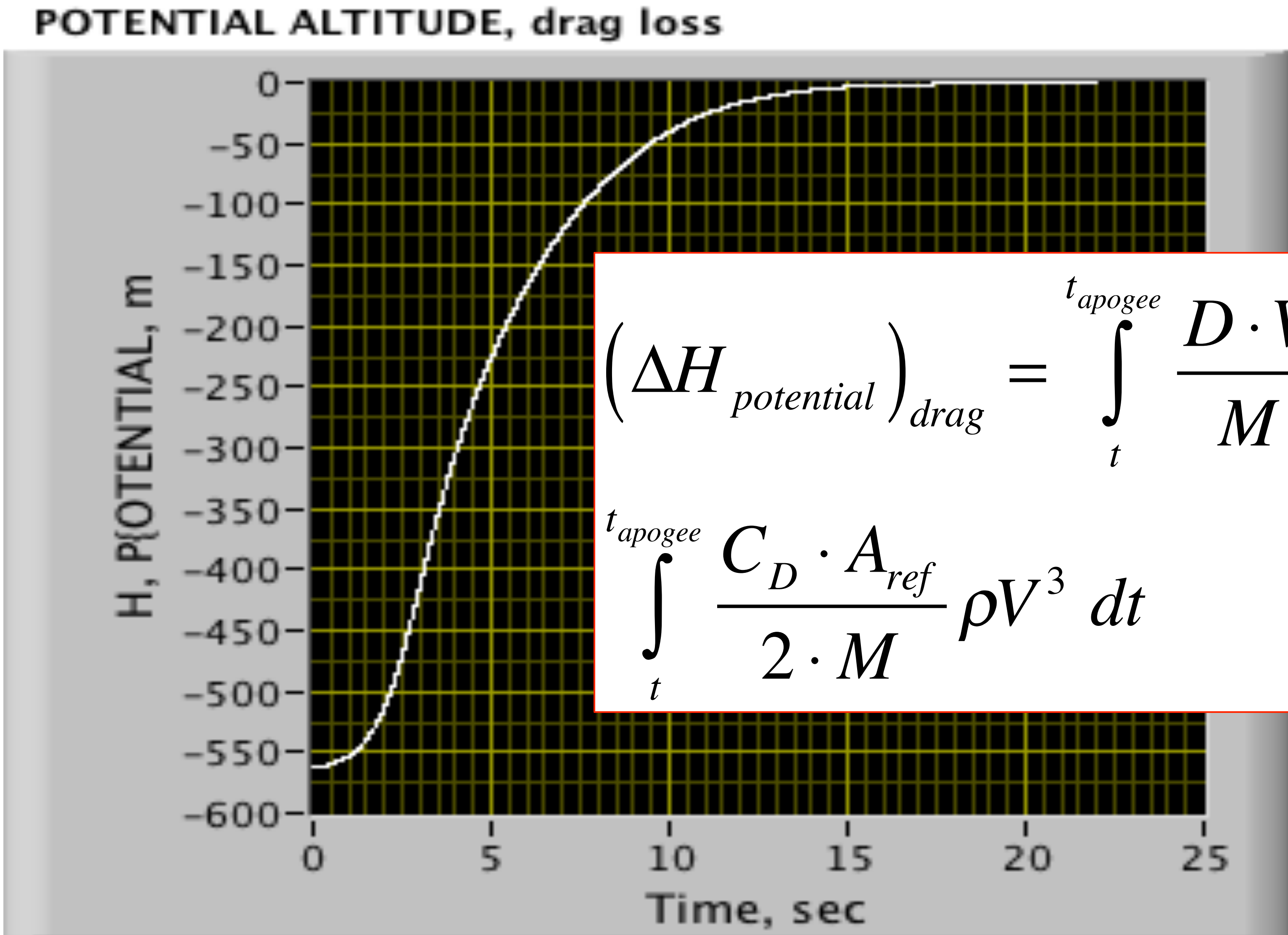
V_{min} → *apogee*

Calculate Available Augmentation Lift (2)



$$\Delta h_{potential} = \left(V_1 + \frac{\Delta V}{2} \right) \cdot \frac{\Delta V}{g}$$

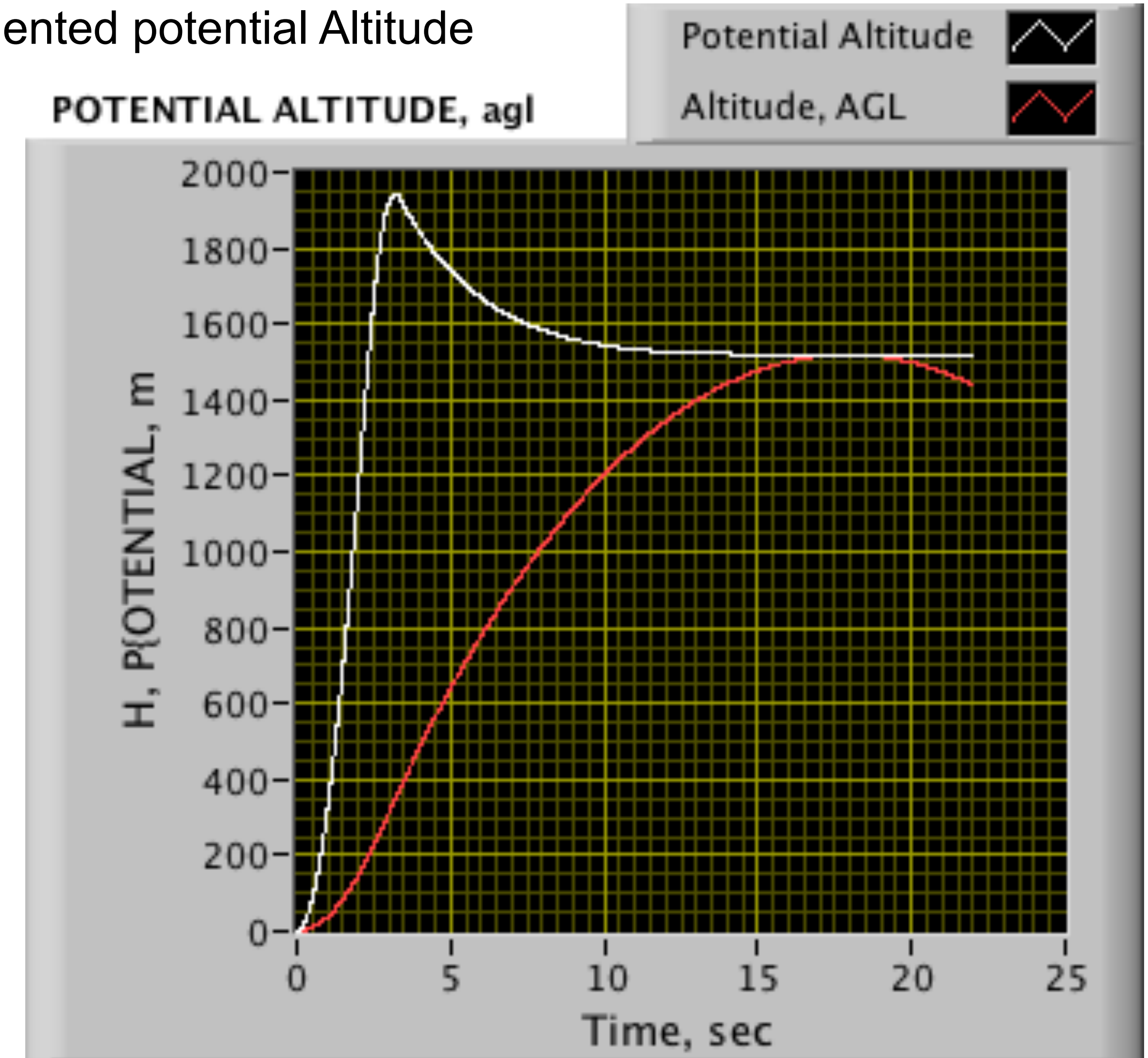
Calculate Drag Loss



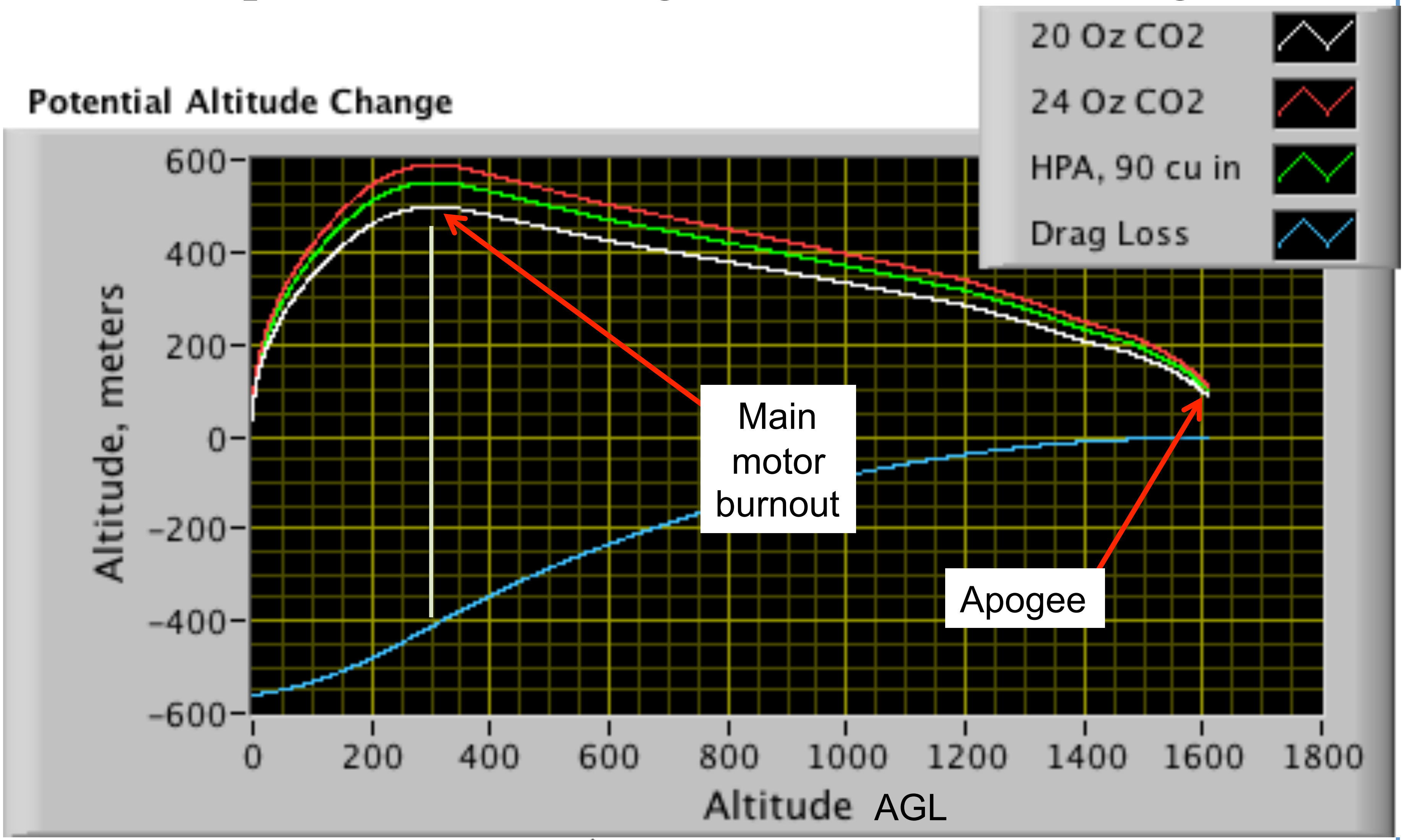
$$\left(\Delta H_{potential}\right)_{drag} = \int_{t}^{t_{apogee}} \frac{D \cdot V}{M} dt = \int_{t}^{t_{apogee}} \frac{C_D \cdot A_{ref}}{2 \cdot M} \rho V^3 \, dt$$

… Drag Loss (2)

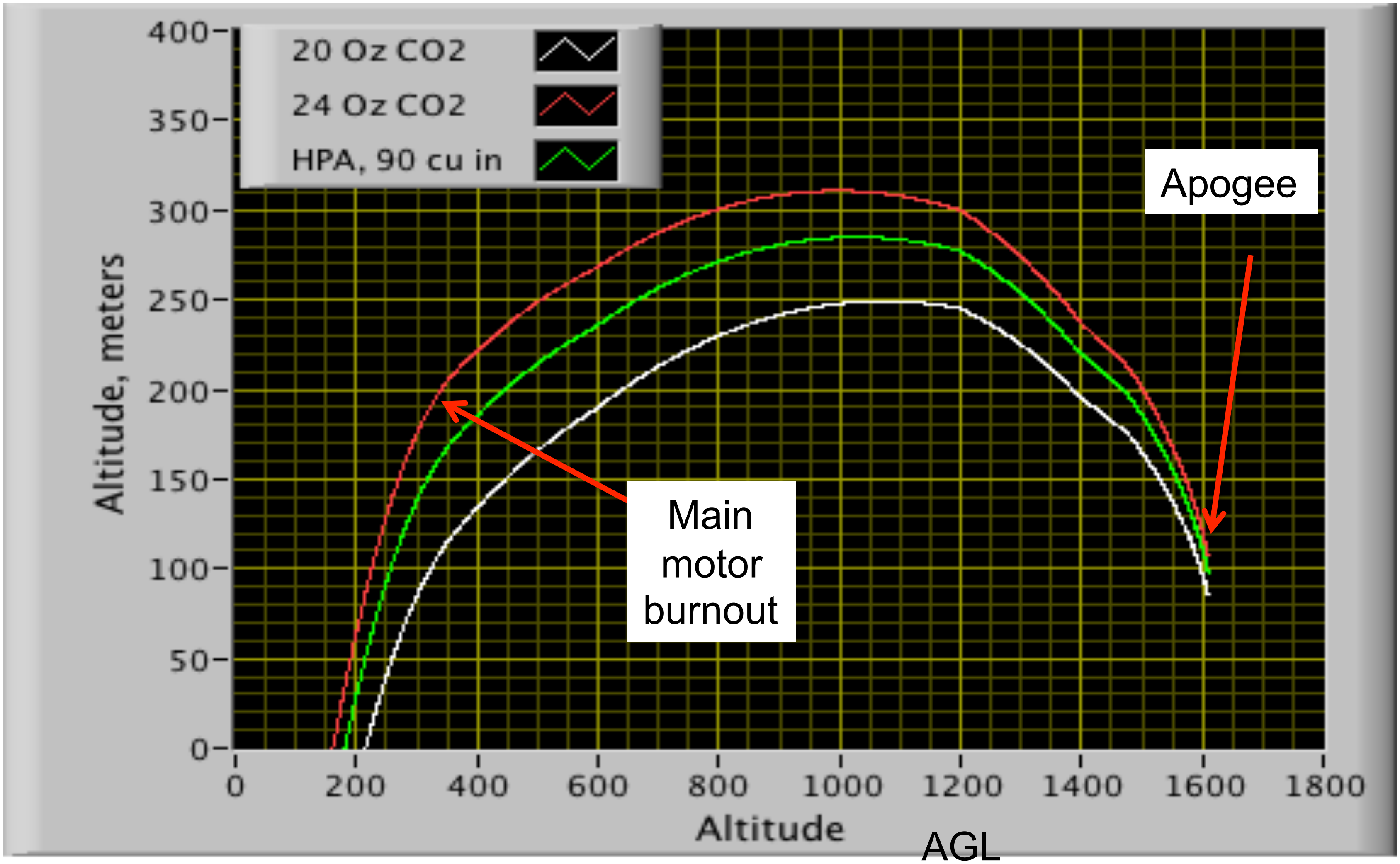
Un-augmented potential Altitude



Compare Available Augmentation Lift to Drag Loss



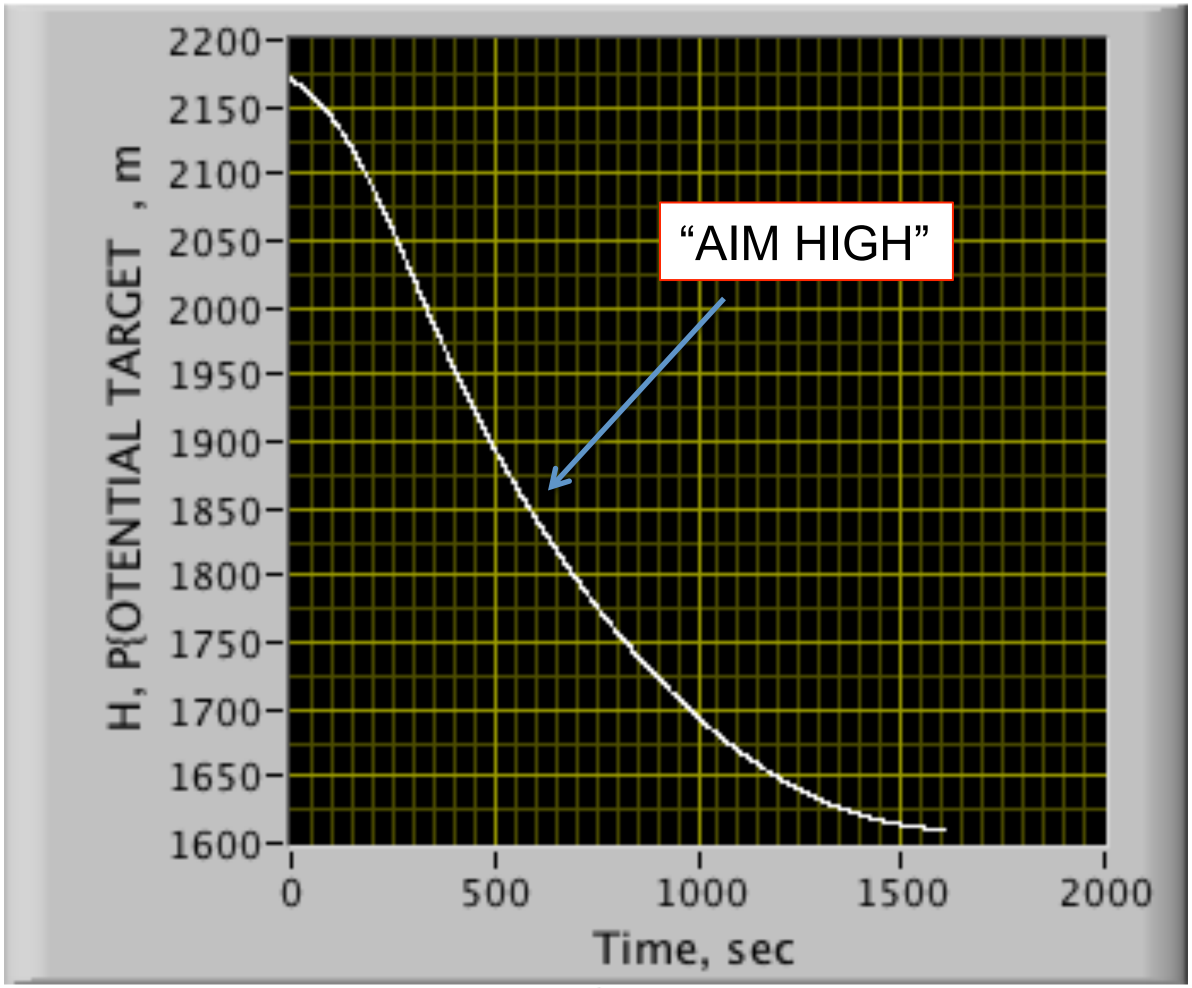
Total Augmentation Lift "Effectiveness Margin"



Potential Altitude Effectiveness "Margin"

"Target Altitude" Schedule

POTENTIAL ALTITUDE, TARGET




Questions??