UtahState UNIVERSITY

Mechanical & Aerospace Engineering

Rocket Science 101:
Basic Concepts and Definitions

Newton's Laws as Applied to "Rocket Science"

... its not just a job ... its an adventure

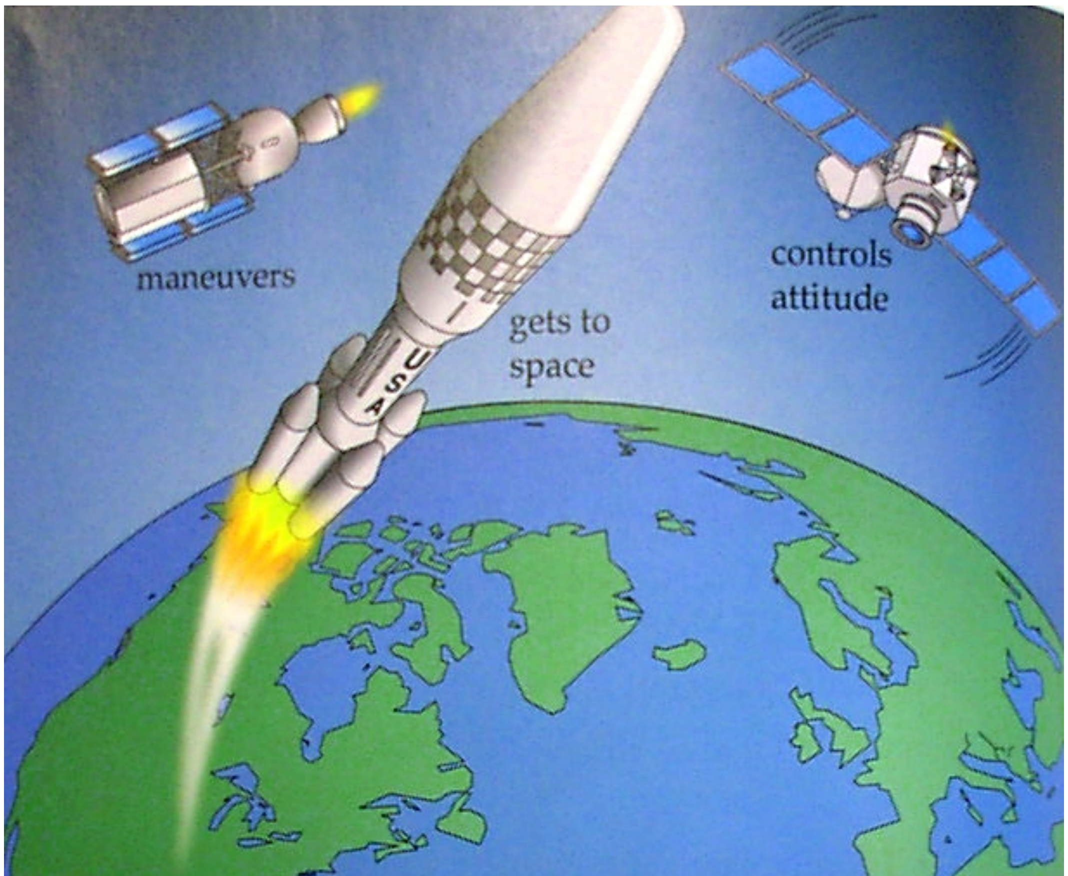
- How Does a Rocket Work?

Sellers: Chapter 14

MAE 5930, Rocket Systems Design

CHIMAERA

What does a rocket "do"?

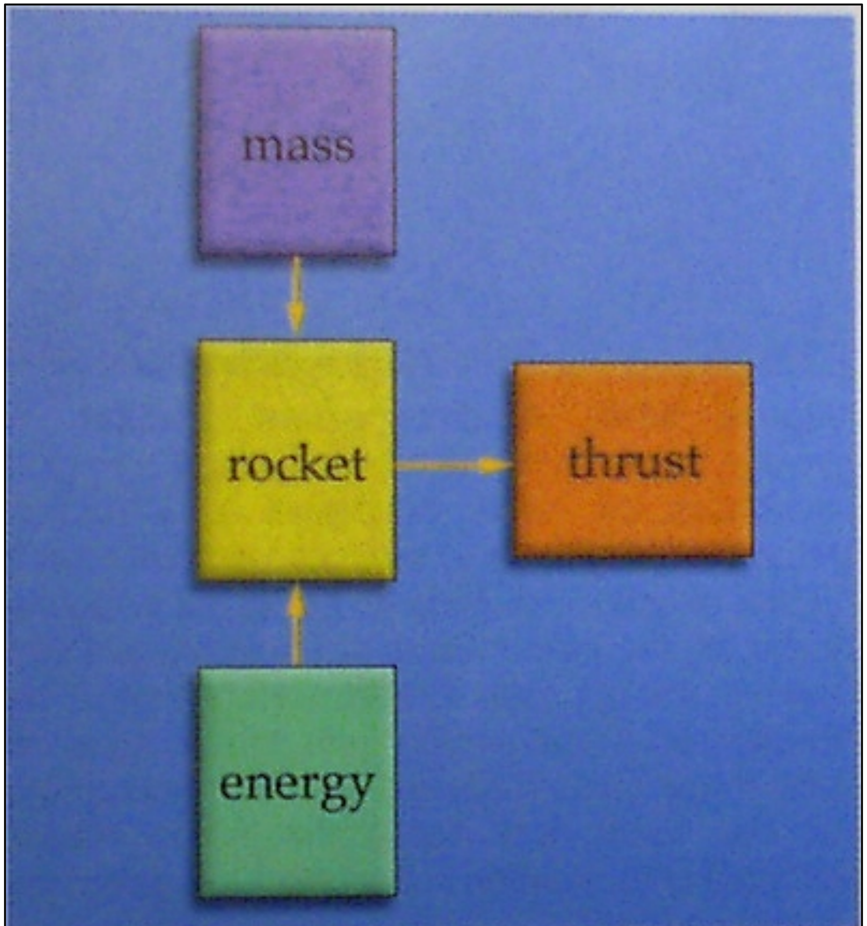


Rockets take spacecraft to orbit

Move them around in space, and

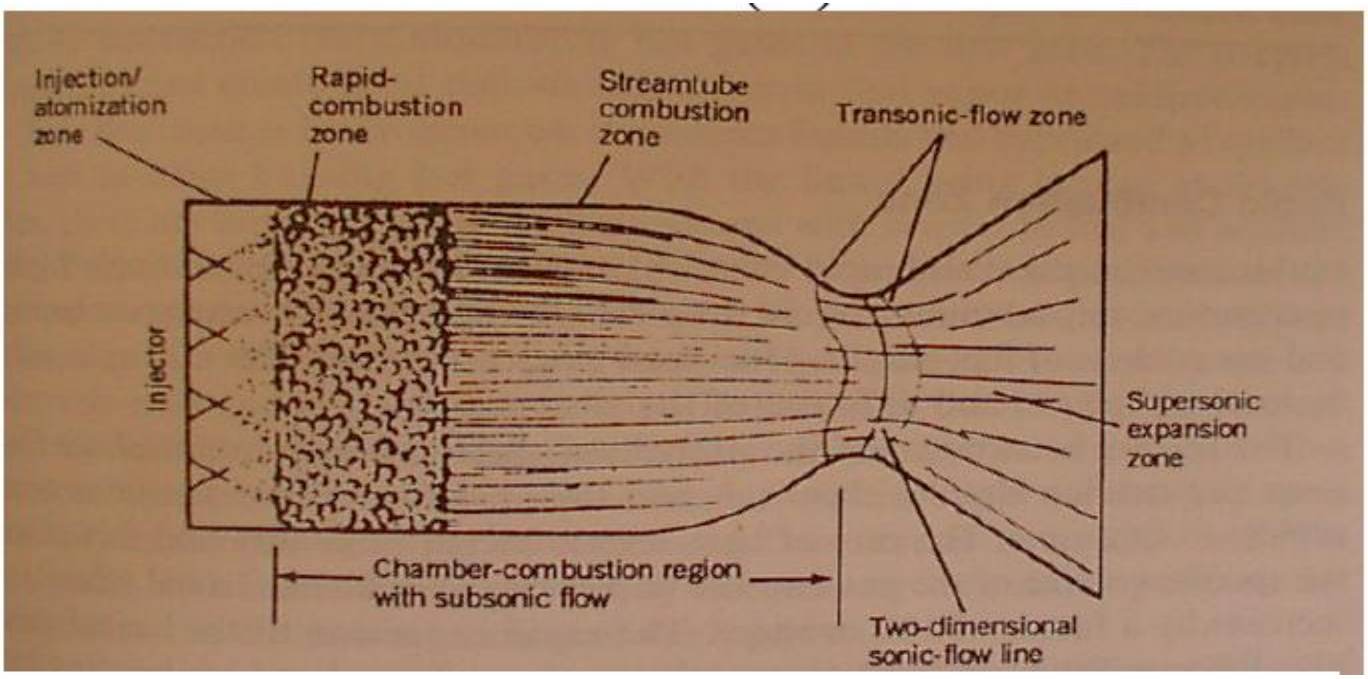
Slow them down for atmospheric reentry

Basics



Rocket's basic function is to take mass, add energy, and convert that to thrust.

Basics (2)



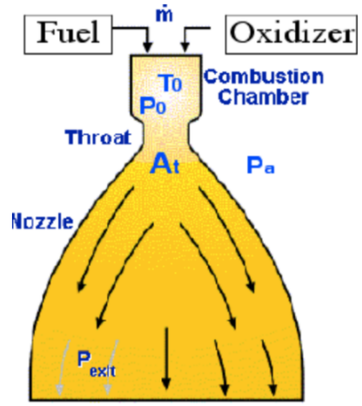
Combustion is an exothermic chemical reaction. Often an external heat source is required (igniter) to supply the necessary energy to a threshold level where combustion is self sustaining

Propellants that combust spontaneously are referred to as *Hypergolic*

Basics (2)

- Combustion Produces High temperature gaseous By-products
- Gases Escape Through Nozzle Throat
- Nozzle Throat Chokes (maximum mass flow)
- Since Gases cannot escape as fast as they are produced
  … Pressure builds up
- As Pressure Builds .. Choking mass flow grows
- Eventually Steady State Condition is reached

What is a NOZZLE? (1)



- FUNCTION of rocket nozzle is to convert thermal energy in propellants into kinetic energy as efficiently as possible

- Nozzle is substantial part of the total engine mass.

- Many of the historical data suggest that 50% of solid rocket failures stemmed from nozzle problems.

The design of the nozzle must trade off:

1. Nozzle size (needed to get better performance) against nozzle weight penalty.

2. Complexity of the shape for shock-free performance vs. cost of fabrication

Newton

Newton's First Law

- An object at rest stays at rest unless acted on by an external force

Concept of *inertia* ... the resistance to changes in motion

"No forces here!"

- An object in motion, stays in motion in a straight line unless acted on by an external force

"No forces here!"

Newton

Newton's Second Law

"The acceleration produced by a force is directly proportional to the force and inversely proportional to the mass which is being accelerated"

Newton's Second law $\quad \bar{F} = m\,\bar{a} = m\dfrac{d\bar{V}}{dt}$

- **But what happens when the mass is no longer constant?**

- **Newton recognized that the early formulation of second law was incomplete and modified the formulation accordingly**

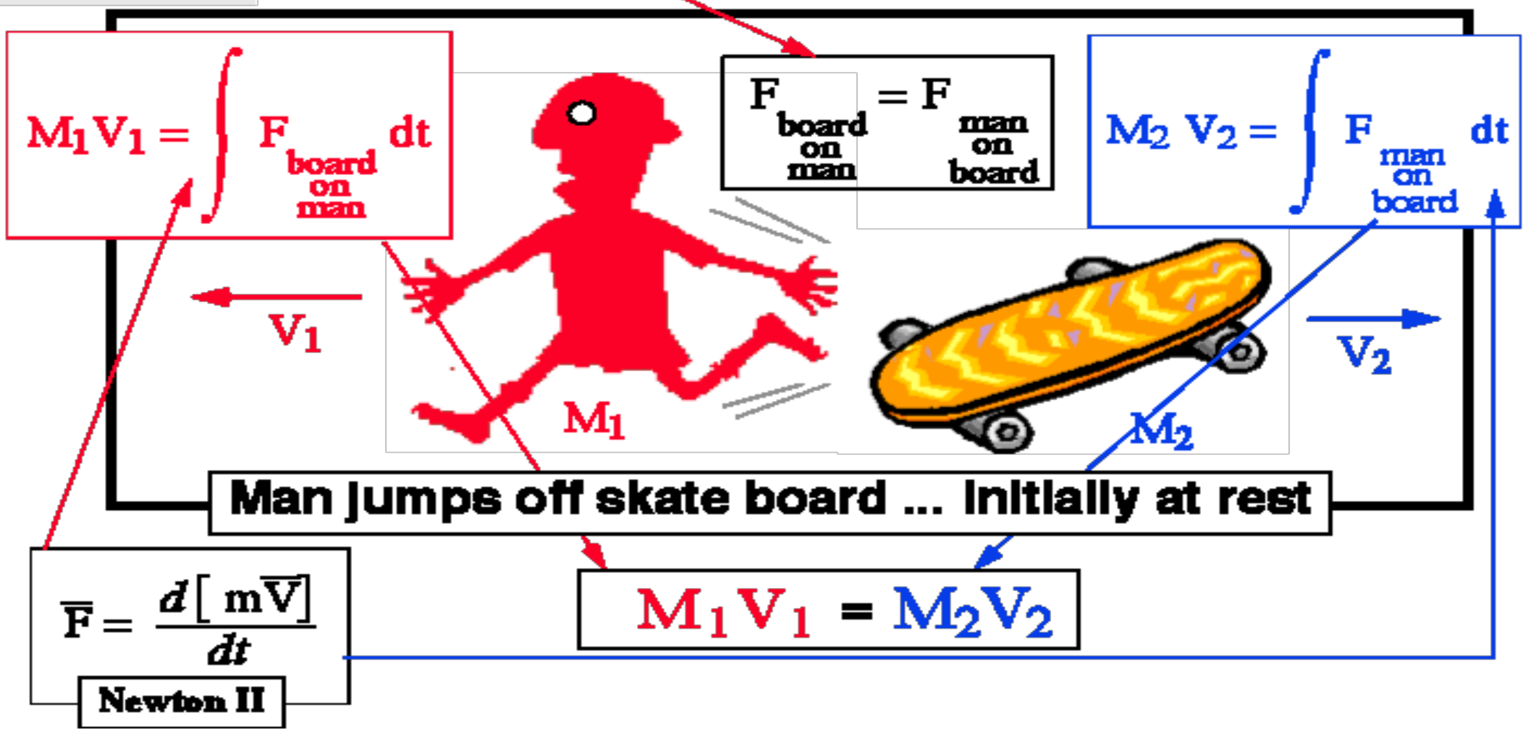
$$\bar{F} = \frac{d\left[m\bar{V}\right]}{dt} = \frac{d\left[\bar{P}\right]}{dt} \Rightarrow \bar{P} \equiv m\bar{V} \left[\begin{matrix}\text{"momentum} \\ \text{vector"}\end{matrix}\right]$$

Conservation of Momentum

-

Newton's Third Law =
Conservation of momentum

- *For every action, there is an equal and oppposite RE-action*

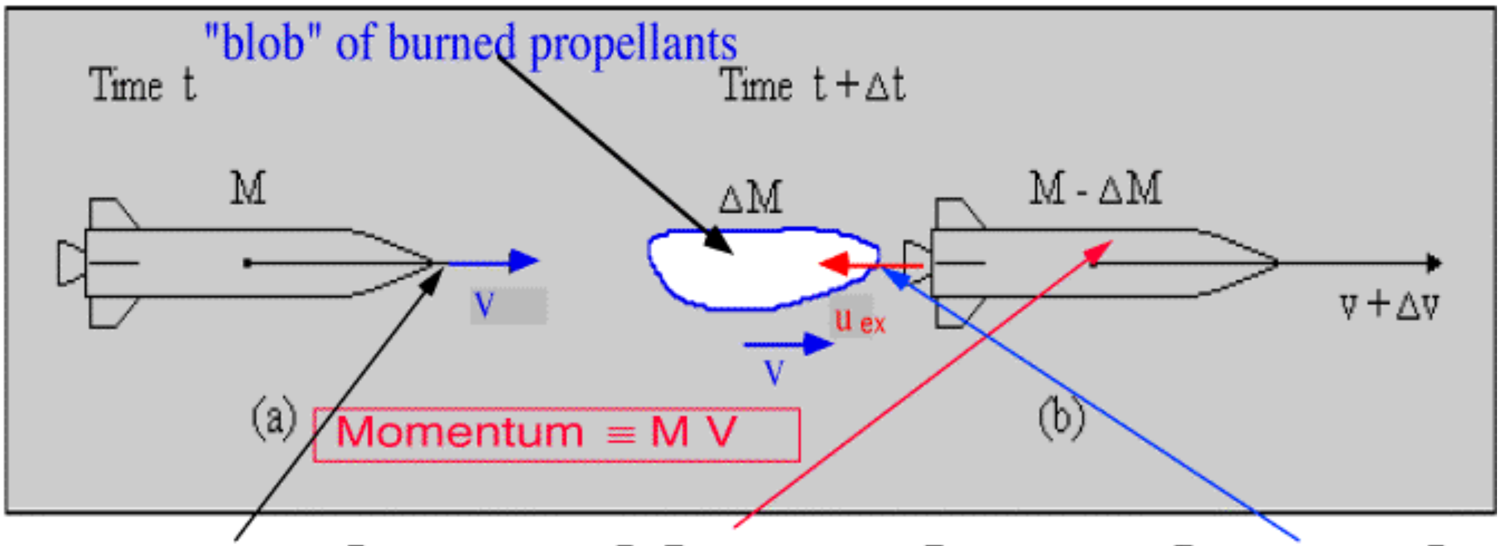


Newton

Newton's Third Law =

Conservation of momentum

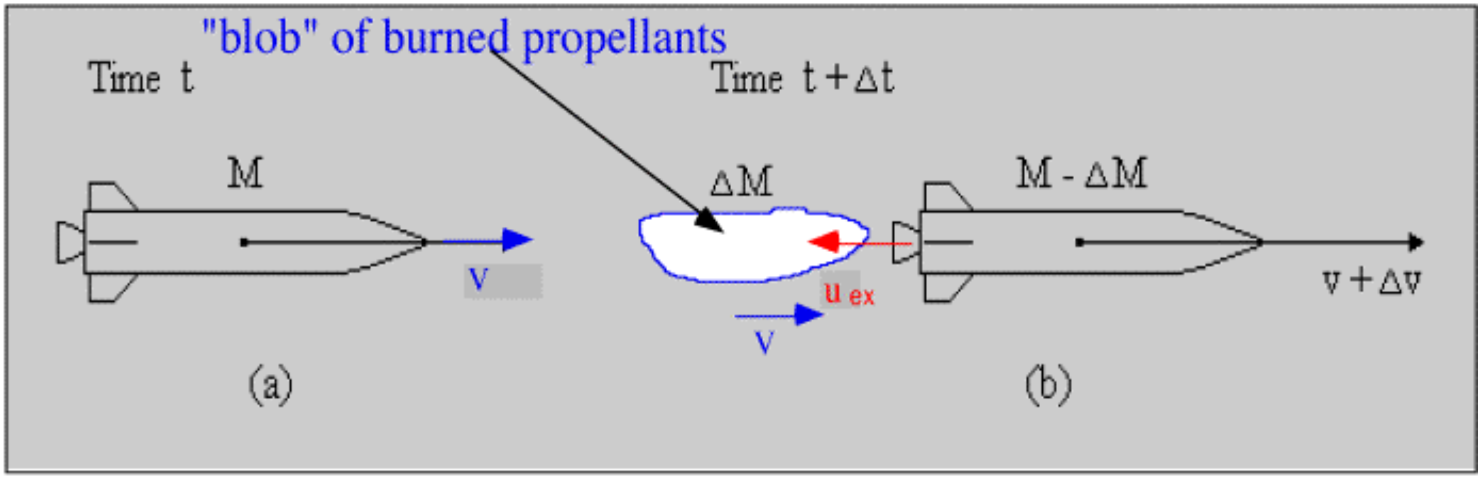
Look at a rocket in horizontal flight



$$M V = [M - \Delta M][V + \Delta V] + \Delta M [V - U_{ex}]$$

$$\sum \overline{F}_{external} = \frac{d\overline{P}}{dt} = \frac{d[M\overline{V}]}{dt}$$

Newton's Third Law (cont'd)



$$M\,V = \quad [M - \Delta M]\,[V + \Delta V] + \Delta M\,[V - U_{ex}]$$

$$\Downarrow$$

$$\underline{M\,V} = \underline{M\,V} - \underline{\Delta M\,V} + M\,\Delta V - \underline{\Delta M\,\Delta V} + \underline{\Delta M\,V} - \Delta M\,U_{ex}$$

$$\Downarrow$$

$$M\,\Delta V = \Delta M\,U_{ex} + \Delta M\,\Delta V$$

Newton's Third Law (cont'd)

- Dividing by Δt and evaluating limit $\{\Delta M, \Delta V, \Delta t\} \to 0$

$$\left[\frac{M\dfrac{\Delta V}{\Delta t} = \dfrac{\Delta M}{\Delta t}U_{ex} + \Delta M\dfrac{\Delta V}{\Delta t}}{\lim\{\Delta M, \Delta V, \Delta t\} \Rightarrow 0}\right] =$$

We shrink time
As small as possible

$$\Downarrow$$

Engine massflow

$$F = M\frac{dV}{dt} = \frac{dM}{dt}U_{ex}$$

Reaction Force on Rocket

Engine thrust equation

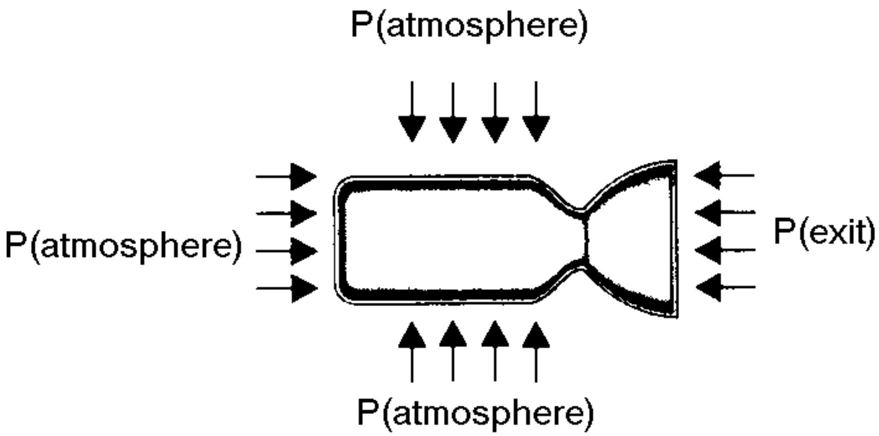
Thrust Components

Momentum Thrust: $\dot{P}_{rocket} = \dot{m} V_{exit}$

- $\dot{P}_{rocket}$ **is the time rate of change of momentum of the rocket (*Newtons*)**
- $\dot{m}$ **is the mass flow rate of the exhaust products (*kg/s*)**
- V_{exit} **is the exit velocity of the exhaust products (*m/s*)**

Pressure Thrust:

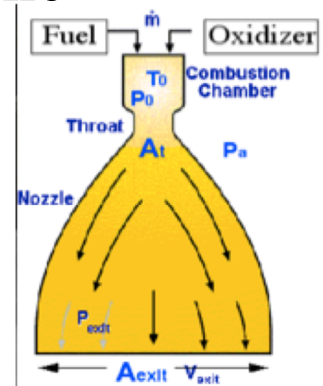
- **Pressure is identical from all directions except for the Area of the exit nozzle. This pressure difference produces a thrust (which may be negative.)**

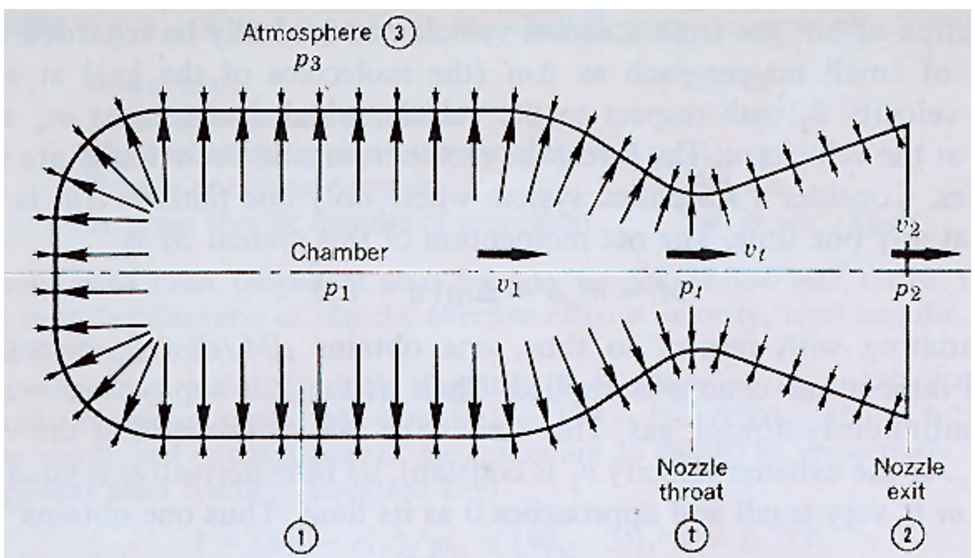


Collected Rocket Thrust Equation

$$F_{thrust} = \dot{m} V_{exit} + A_{exit}\left(P_{exit} - P_{atmosphere}\right)$$

- **Total thrust must be greater than the weight, or the rocket will not fly.**
- **V_{exit} and P_{exit} are related (inversely)**
- **Ideal thrust is achieved when $P_{exit} = P_{atmosphere}$**

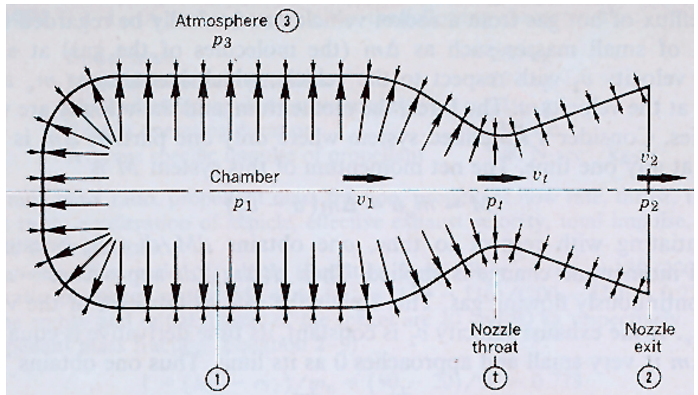




- Thrust + Oxidizer enters combustion chamber at ~0 velocity, … combustion adds thermal energy … High Chamber pressure accelerates flow through Nozzle

•*Resultant pressure forces produce thrust*

Rocket Thrust Equation (2)



What Causes Thrust?

a) Increase in momentum of the propellant fluid (momentum thrust)

b) Pressure at the exit plane being higher than the outside pressure (pressure thrust).

Where does the thrust act?

In the rocket engine, the force is felt on the nozzle and the combustor walls, and is transmitted through the engine mountings to the rest of the vehicle.

Specific Impulse

- Specific Impulse is a scalable characterization of a rocket's ability to deliver a certain *(specific)* impulse for a given weight of propellant

$$I_{sp} = \frac{\mathrm{I}_{mpulse}}{g_0 M_{propellant}} = \frac{\int_0^t F_{thrust} dt}{g_0 \int_0^t \dot{m}_{propellant}\, dt} \rightarrow g_0 = 9.806 \frac{m}{\sec^2} (mks)$$

- *Total specific impulse*

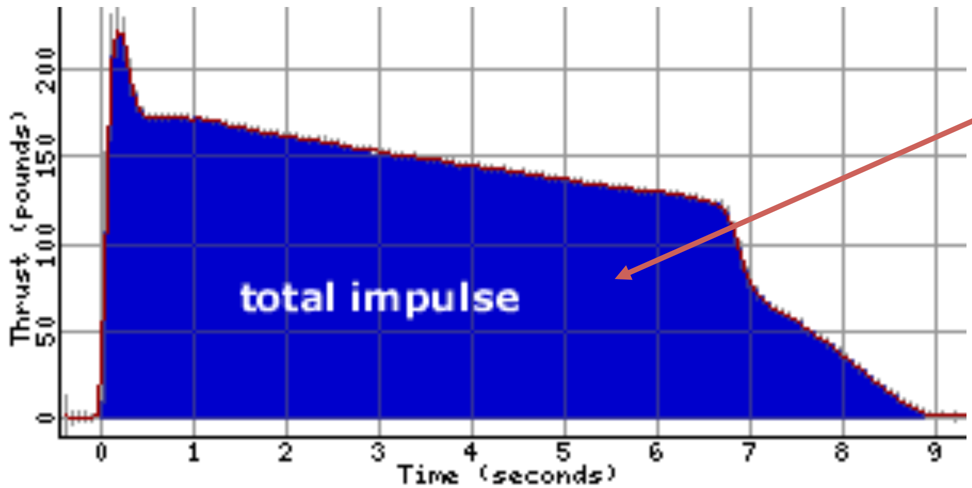
- At a constant altitude, with constant mass flow through engine

$$I_{sp} = \frac{\mathrm{I}_{mpulse}}{g_0 M_{propellant}} = \frac{\int_0^t F_{thrust} dt}{g_0 \int_0^t \dot{m}_{propellant}\, dt} = \frac{F_{thrust}}{g_0 \dot{m}_{propellant}}$$

- *Instantaneous specific impulse*

Specific Impulse (2)

- Look at total impulse for a rocket



$$\frac{\int_0^t F_{thrust}\,dt}{\quad}$$

$$I_{sp} = \frac{\mathrm{I}_{mpulse}}{g_0 M_{propellant}} = \frac{\int_0^t F_{thrust}\,dt}{g_0 \int_0^t \dot{m}_{propellant}\,dt}$$

Specific Impulse (2)

- Historically, I_{sp} was measured in units of *seconds*

$$I_{sp} = \frac{|\bar{F}|}{\dot{m}_p} \Rightarrow \text{(English Units)} \frac{\text{lbf}}{\text{lbm/sec}} \approx \text{seconds, right?}$$

Wrong! *lbms* are not a fundamental unit for mass

(Slugs are the fundamental english unit of mass)

$$I_{sp} = \frac{|\bar{F}|}{\dot{m}_p} \Rightarrow \text{(MKS units)} \frac{\text{Nt}}{\text{kg/sec}} \approx \frac{\text{kg-m/sec}^2}{\text{kg/sec}} \approx \frac{\text{m}}{\text{sec}}$$

- Since most engine manufacturers still give I_{sp} in *seconds* -- we correct for this by letting

$$I_{sp} \equiv \frac{|\bar{F}|}{g_0\, \dot{m}_p} \Rightarrow g_0 \approx 9.81 \frac{\text{m}}{\text{sec}^2} \text{ [acceleration of gravity at sea level]}$$

English Units -- use *slugs* not *lbms* !

$$\text{(MKS units)} \frac{\dfrac{\text{Nt}}{\text{kg/sec}}}{\dfrac{\text{m}}{\text{sec}^2}} \approx \frac{\dfrac{\text{kg-m/sec}^2}{\text{kg/sec}}}{\dfrac{\text{m}}{\text{sec}^2}} \approx \text{sec}$$

Specific Impulse *(3)*

$$I_{sp} = \frac{1}{g_0}\frac{F_{thrust}}{\dot{m}_{propellant}} \rightarrow \dot{m}_e \equiv \dot{m}_{propellant} \rightarrow$$

$$I_{sp} = \frac{1}{g_0}\left[V_e + \frac{p_e A_e - p_\infty A_e}{\dot{m}_e}\right] \equiv \frac{C_e}{g_0}$$

Effective exhaust velocity $C_e \equiv V_{exit} + \frac{A_{exit}}{\dot{m}}(P_{exit} - P_{atmosphere})$

Specific Impulse (4)

- Example

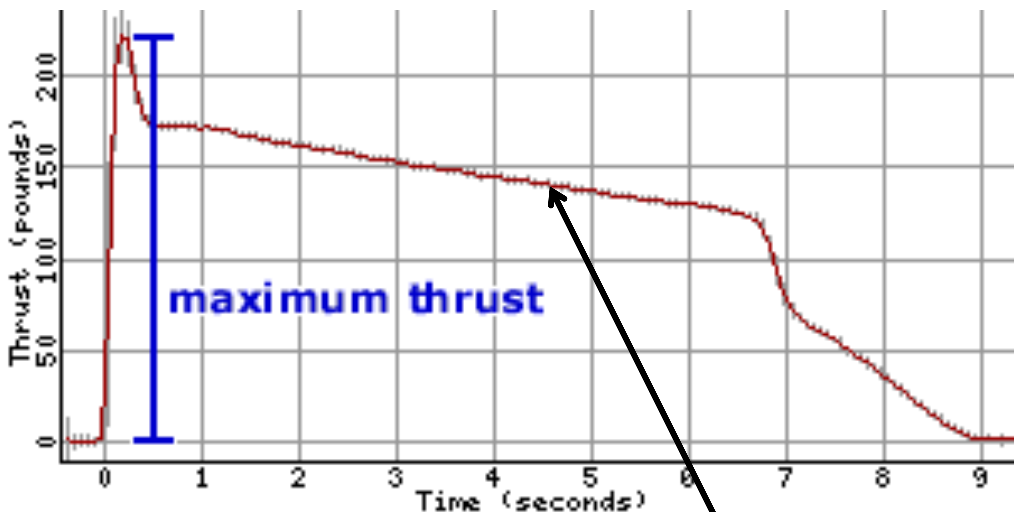
- Aparticular engine with a specific impulse of 300 sec. will produce one pound (force!) of thrust for 300 seconds -- or

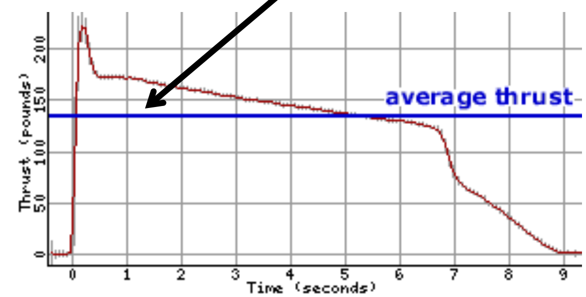
- Another engine with a specific impulse of 300 sec. may produce 300 pounds (force!) of thrust for 1 second

Specific Impulse (cont'd)

$$\frac{\int_0^t F_{thrust}\,dt}{g_0 \int_0^t \dot{m}_{propellant}\,dt}$$

- Look at instantaneous impulse for a rocket





Instantaneous

$$I_{sp} = \frac{1}{g_0}\frac{F_{thrust}}{\dot{m}_{propellant}}$$

- *Not necessarily the same*

Example 2

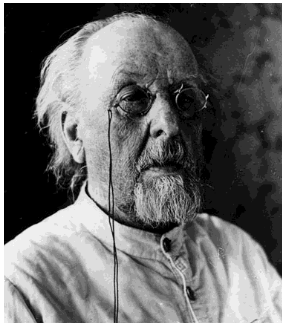
- **A man is sitting in a rowboat throwing bricks over the stern. Each brick weighs 5 lbs, he is throwing six bricks per minute, at a velocity of 32 fps. What is his thrust and I_{sp}?**

$$F = \dot{m}_{propellant} C_e = \frac{6_{bricks}}{1\,\text{min}} \times 5\,\frac{lbm}{brick} \times 32\,\frac{ft}{\sec} \times \frac{1\,\text{min}}{60\,\sec} =$$

$$\frac{6 \times 5 \times 32}{60} \frac{lbm - ft}{\sec^2} \ldots ooops \ldots need \ldots . g_c$$

$$F = \frac{6 \times 5 \times 32}{60} \frac{lbm - ft}{\sec^2} \times \frac{1}{32.1742 \dfrac{lbm - ft}{lbf - \sec^2}} =$$

$$I_{sp} = \frac{F}{\dot{m}_{propellant} g_0} = \frac{0.497\,lbf \times 32.1742 \dfrac{ft}{\sec^2}}{\dfrac{6_{bricks}}{1\,\text{min}} \times 5\,\dfrac{lbm}{brick} \times \dfrac{1\,\text{min}}{60\,\sec} \times 32.1742 \dfrac{lbm - ft}{lbf - \sec^2}} = 0.994\,\sec$$



How Much Fuel? "The Rocket Equation"

Conservation of momentum leads to the so-called rocket equation, which trades off exhaust velocity with payload fraction. Based on the assumption of short impulses with coast phases between them, it applies to chemical and nuclear-thermal rockets. First derived by Konstantin Tsiolkowsky in 1895 for straight-line rocket motion with constant exhaust velocity, it is also valid for elliptical trajectories with only initial and final impulses.

Re-visit

Newton's Third Law (cont'd)

- Dividing by Δt and evaluating limit {ΔM , ΔV , Δt} -> 0

$$\left[\frac{M\frac{\Delta V}{\Delta t} = \frac{\Delta M}{\Delta t}U_{ex} + \Delta M\frac{\Delta V}{\Delta t}}{\lim\{\Delta M,\ \Delta V,\ \Delta t\} \Rightarrow 0}\right] =$$

$$\Downarrow$$

$$M\frac{dV}{dt} = \frac{dM}{dt}U_{ex} = \dot{m}_{propellant}c_e$$

Rocket Equation (2)

$$M\frac{dV}{dt} = \dot{m}_{propellant} C_e = g_0 I_{sp} \dot{m}_{propellant} \rightarrow \dot{m}_{propellant} = -\frac{dM}{dt}$$

M -> rocket mass

$$\frac{dV}{dt} = g_0 I_{sp} \frac{-\frac{dM}{dt}}{M} \rightarrow \boxed{dV = -g_0 I_{sp} \frac{dM}{M}}$$

- Assuming constant I_{sp} and burn rate …. integrating over a burn time t_{burn}

$$V_{final} - V_0 = -g_0 I_{sp} \ln\left[M_{final}\right] + g_0 I_{sp} \ln\left[M_0\right] = g_0 I_{sp} \ln\left[\frac{M_0}{M_{final}}\right]$$

$$V_{final} = V_0 + g_0 I_{sp} \ln\left[\frac{M_0}{M_{final}}\right]$$

Rocket Equation (3)

$$M\frac{dV}{dt} = \dot{m}_{propellant} C_e = g_0 I_{sp} \dot{m}_{propellant} \rightarrow \dot{m}_{propellant} = -\frac{dM}{dt}$$

M -> rocket mass

$$\frac{dV}{dt} = g_0 I_{sp} \frac{-\frac{dM}{dt}}{M} \rightarrow \boxed{dV = -g_0 I_{sp} \frac{dM}{M}}$$

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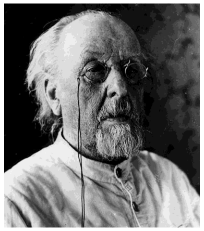
$$V_{final} = V_0 + g_0 I_{sp} \ln\left[\frac{M_0}{M_{final}}\right]$$



Anatomy of the Rocket Equation … Its all about ΔV

- Consider a rocket burn of duration t_{burn}

Tsiolkowsky's Rocket Equation

Initial Mass

$$V_{final} = V_0 + g_0 I_{sp} \ln\left[\frac{M_0}{M_{final}}\right]$$

Final Mass

Initial Velocity

$$\Delta V = V_{final} - V_0$$

Final Velocity

Consumed propellant

$$M_0 = M_{final} + m_{propellant}$$

Rocket Equation (5)

- Or rewriting

$$M_0 = M_{final} + m_{propellant}$$

$$\Delta V = V_{final} - V_0$$

$$\Delta V = g_0 I_{sp} \ln\left[1 + \frac{m_{propellant}}{M_{final}}\right] = g_0 I_{sp} \ln\left[1 + P_{mf}\right]$$

$$P_{mf} = \text{"propellant mass fraction"}$$

$$\frac{m_{propellant}}{M_{final} + m_{propellant}}$$

Is also called propellant mass Fraction or "load mass fraction" L_{mf}

"Propellant Mass Fraction"

- How do we compute the amount of propellant required?

$$\frac{M_0}{M_{final}} = \frac{M_{dry} + M_{payload} + M_{fuel + oxidizer}}{M_{dry} + M_{payload}} = 1 + P_{mf}$$

$$\Downarrow$$

Load mass fraction

$$L_{mf} \equiv \frac{M_{fuel + oxidizer}}{M_{dry} + M_{payload} + M_{fuel + oxidizer}} = \frac{P_{mf}}{1 + P_{mf}}$$

Better to work with

Propellant mass fraction

$$P_{mf} \equiv \frac{M_{fuel + oxidizer}}{M_{dry} + M_{payload}}$$

$$\Delta V_{burn} = g_0 I_{sp} \ln\left[\left(\frac{M_{initial}}{M_{final}}\right)_{burn}\right] = g_0 I_{sp} \ln\left[(1 + P_{mf})_{burn}\right]$$

Relating Delta V delivered by a rocket burn to propellant Mass fraction

Propellant Budgeting Equation

- **Solving for** P_{mf}

$$\left(P_{mf}\right)_{burn} = e^{\left[\frac{\Delta V_{burn}}{g_0 I_{sp}}\right]} - 1$$

- **Mass of Fuel and oxidizer required for a burn to give a specified ΔV**

$$M_{fuel + oxidizer} = \left[M_{dry} + M_{payload}\right]\left[e^{\left[\frac{\Delta V_{burn}}{g_0 I_{sp}}\right]} - 1\right]$$

Propellant Budgeting Equation (2)

$$V = g_0 \cdot I_{sp} \cdot \ln\left(\frac{M_{initial}}{M_{final}}\right) \rightarrow \frac{M_{final}}{M_{initial}} = e^{-\left(\frac{\Delta V}{g_0 \cdot I_{sp}}\right)}$$

$$\frac{M_{initial} - M_{propellant}}{M_{initial}} = e^{-\left(\frac{\Delta V}{g_0 \cdot I_{sp}}\right)} \quad 1 - \frac{M_{propellant}}{M_{initial}} = e^{-\left(\frac{\Delta V}{g_0 \cdot I_{sp}}\right)}$$

$$L_{mf} \equiv \frac{M_{propellant}}{M_{initial}} = \frac{M_{propellant}}{M_{final} + M_{initial}} 1 - e^{-\left(\frac{\Delta V}{g_0 \cdot I_{sp}}\right)}$$

Ramifications of "the Rocket Equation"

- **Any increase in ΔV must come from increasing I_{sp} or P_{mf}**

  -- First case (I_{sp}) requires adopting a more efficient propulsion system

  --Second case (mass fraction) requires reduction of the structural mass or reduced payload (for same vehicle weight)

  -- Can't just add more propellant -- because that means bigger tanks and the dry weight rises proportionately

- **Reducing payload to obtain more ΔV is a bad-tradeoff**

Ramifications of "the Rocket Equation"

Specific Impulse (revisited)

- For chemical Rockets, I_{sp} depends on the type of fuel/oxydizer used

Vacuum Isp

Fuel	Oxidzer	Isp (s)
Liquid propellants		
Hydrogen (LH2)	Oxygen (LOX)	450
Kerosene (RP-4)	Oxygen (LOX)	280
Monomethyl hydrazine	Nitrogen Tetraoxide	310
Solid propellants		
Powered Al	Ammonium Perchlorate	270

Specific Impulse (revisited)

450 sec is "best you can get" with chemical rockets

ΔV for a Vertically Accelerating Vehicle

- **Rocket Equation originally derived for straight and level travel**
- **What happens for vertically climbing rocket ?**

For example .. Look at a hovering vehicle … Lunar Lander
During hover, change in velocity is zero .. So according to …

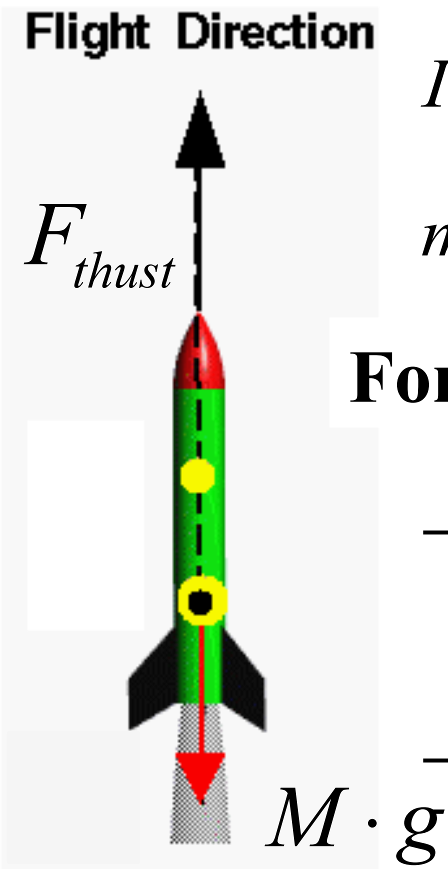
$$\Delta V = g_0 I_{sp} \cdot \ln\left[1 + \frac{m_{propellant}}{M_{dry}}\right]$$

$$\rightarrow \left| \begin{array}{l} \Delta V = 0 \\ \rightarrow \ln\left[1 + \frac{m_{propellant}}{M_{dry}}\right] \end{array} \right. = 0 \rightarrow \boxed{m_{propellant} = 0}$$

We burn no gas! Of course this result is absurd! Need to account for ***"gravity losses"***

ΔV for a Vertically Accelerating Vehicle (2)

- Look at 1-D equations of motion for vertically accelerating rocket
  - *Ignore Aerodynamic Drag, assume constant g_0*



Instantaneously :

$$m\frac{dV}{dt} = F_{thrust} - m \cdot g_0 \rightarrow \frac{dV}{dt} = \frac{F_{thrust}}{m} - g_0$$

For constant thrust m(t) $= M_{initial} - \dot{m}\ t$

$$\rightarrow \frac{dV}{dt} = \frac{F_{thrust}}{\left(M_{initial} - \dot{m} \cdot t\right)} - g_0$$

$$\rightarrow V_{t_{burn}} = V_0 + \int_0^{t_{burn}} \frac{F_{thrust}}{\left(M_{initial} - \dot{m} \cdot \tau\right)} d\tau - g_0 \cdot t_{burn}$$

Velocity @ Motor Burnout

ΔV for a Vertically Accelerating Vehicle (3)

$$\int_0^{t_{burn}} \frac{d\tau}{\left(M_{initial} - \dot{m}\cdot\tau\right)} = \frac{1}{\dot{m}} \int_{t=0}^{t=t_{burn}} \frac{-du}{u} = \frac{1}{\dot{m}}\left[\ln\left(M_0\right) - \ln\left(M_0 - \dot{m}\cdot t_{burn}\right)\right]$$

$$\rightarrow \boxed{\begin{array}{c} M_0 = M_{initial} \\ M_{initial} - \dot{m}\cdot t_{burn} = M_{final} \end{array}}$$

•Evaluate Integral

$$\int_0^{t_{burn}} \frac{d\tau}{\left(M_{initial} - \dot{m}\cdot\tau\right)} = \frac{1}{\dot{m}}\left[\ln\left(M_{initial}\right) - \ln\left(M_{final}\right)\right] = \frac{1}{\dot{m}}\left[\ln\left(\frac{M_{initial}}{M_{final}}\right)\right]$$

$\rightarrow$ *Substitute back in*

$$\boxed{\Delta V_{t_{burn}} = V_{t_{burn}} - V_0 = \frac{F_{thrust}}{\dot{m}}\left[\ln\left(\frac{M_{initial}}{M_{final}}\right)\right] - g_0\cdot t_{burn}}$$

ΔV for a Vertically Accelerating Vehicle (4)

• *Apply earlier fundamental definitions*

$$\Delta V_{t_{burn}} = \frac{F_{thrust}}{\dot{m}}\left[\ln\left(\frac{M_{initial}}{M_{final}}\right)\right] - g_0 \cdot t_{burn}$$

$$\frac{M_{initial}}{M_{final}} = \frac{M_{final} + m_{propellant}}{M_{final}} = 1 + P_{mf}$$

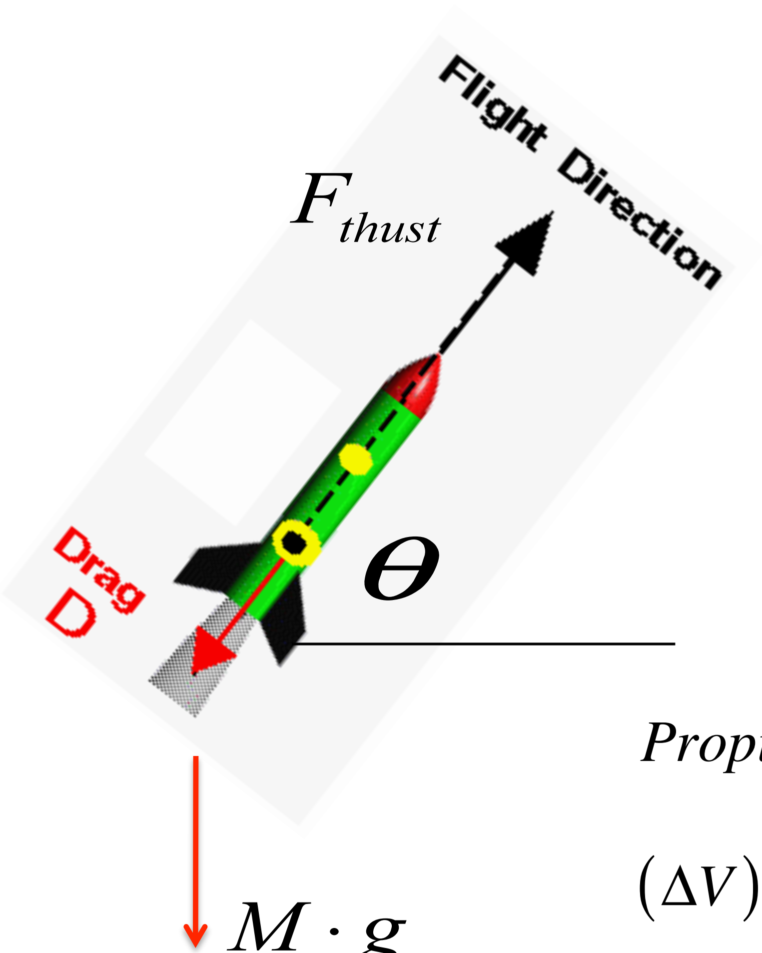
$$\frac{F_{thrust}}{\dot{m}} = g_0 \cdot I_{sp} \qquad t_{burn} \approx \frac{g_0 \cdot I_{sp} \cdot m_{propellant}}{F_{thrust}}$$

$$\boxed{\Delta V_{t_{burn}} = g_0 \cdot I_{sp} \cdot \left[\ln\left(1 + P_{mf}\right)\right] - g_0 \cdot t_{burn}}$$

"gravity loss" for Vertical acceleration

Or More Generally …..

$$\theta \rightarrow "Pitch\ Angle"$$



Applies for rocket accelerating along an ARBITRARY path

$$(\Delta V)_{t_{burn}} = g_0 \cdot I_{sp} \cdot \ln\left(1 + P_{mf}\right) - \int_0^{T_{burn}} g(t) \cdot \sin\theta \cdot dt$$

Propulsive ΔV loss from acting against gravity….

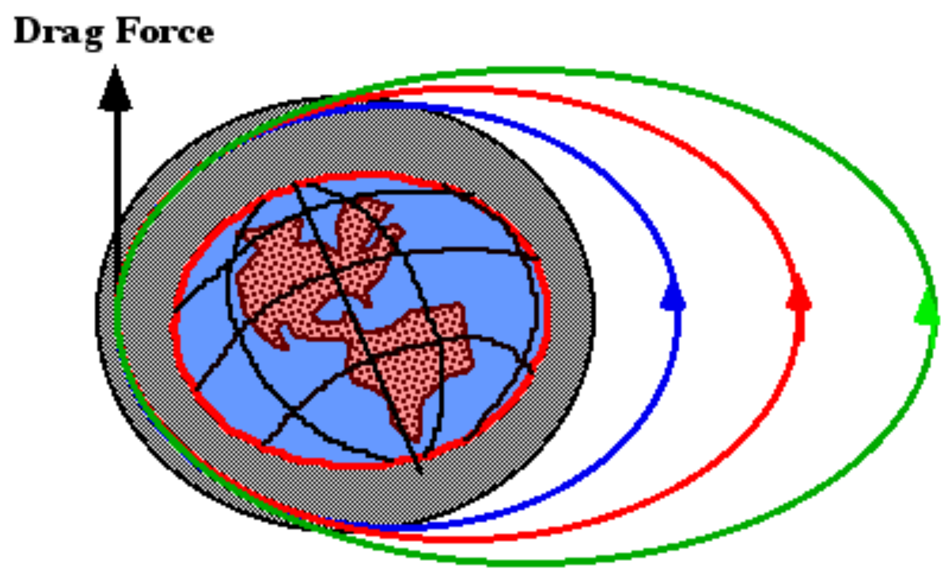
$$(\Delta V)_{\substack{gravity\\loss}} = \int_0^{T_{burn}} g(t) \cdot \sin\theta \cdot dt$$

Drag Losses

- Any Orbiting Object with a perigee altitude less than 600 km willl experience the effects of the Earths' outer atmosphere

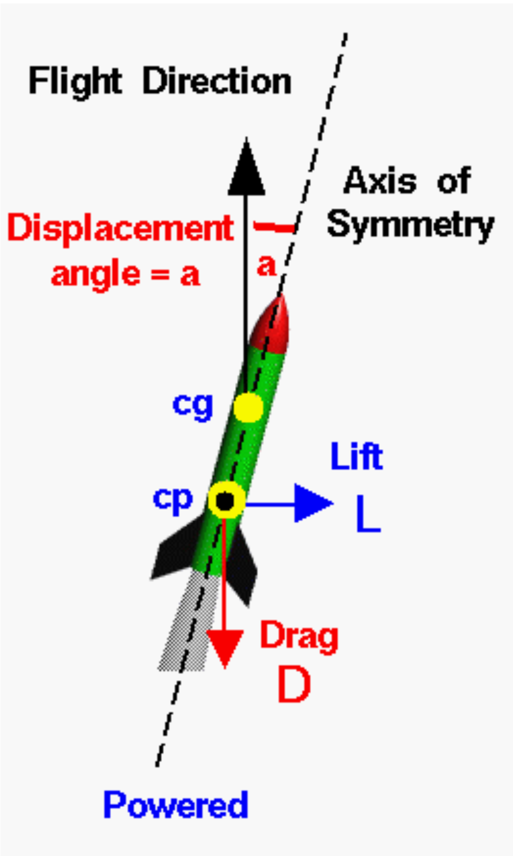
- Resulting *Drag* is a *non-conservative* force, and as such will remove energy from the orbit

- Energy Loss will cause orbit to decay



… and
This decay
Also applies
To launch
trajectory

Aerodynamic Forces Acting on Rocket



- •Lift – acts perpendicular to flight path
- •Drag – acts along flight path
- •Thrust – acts along longitudinal axis of rocket

$$C_L = \frac{L_{ift}}{\frac{1}{2}\cdot\rho\cdot V^2\cdot A_{ref}}$$

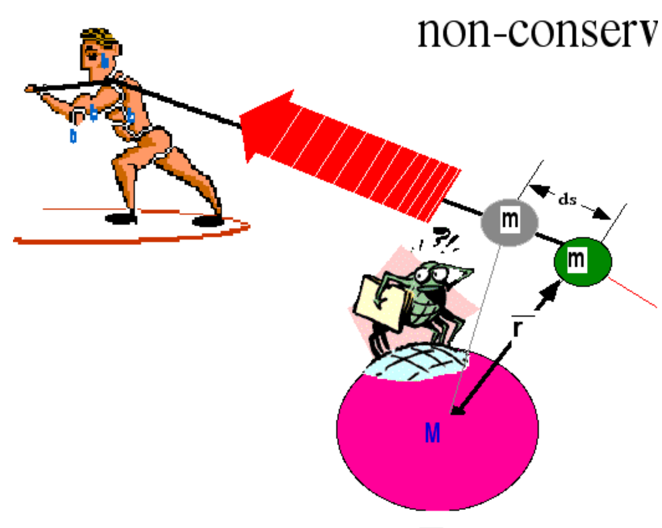
$$C_D = \frac{D_{rag}}{\frac{1}{2}\cdot\rho\cdot V^2\cdot A_{ref}} \rightarrow \frac{1}{2}\cdot\rho_{(h)}\cdot V^2 = \bar{q} \rightarrow "DyamicPressure"$$

$$A_{ref} \equiv reference\ area \rightarrow$$ Typically based on planform or maximum cross section

Drag Losses (2)

$$\Delta E_{\substack{non- \\ conservative}} = \int_{path} \vec{F}_{\substack{non \\ -conservative}} \cdot d\vec{s} = \int_{path} \vec{F}_{\substack{non \\ -conservative}} \cdot \frac{d\vec{s}}{dt} \cdot dt$$

$$\int_{t} \vec{F}_{\substack{non \\ -conservative}} \cdot \vec{V} \cdot dt \approx \frac{\Delta V_{loss}^{\ 2}}{2} \cdot M$$



•**Lift – acts perpendicular to flight path**
.. *Cannot effect energy level of rocket*

•**Gravity – acts downward *(conservative)***
… cannot effect energy level of rocket

Drag Losses (3)

$$\frac{\Delta E_{drag}}{M} = \frac{1}{2}\cdot\left(\Delta V_{drag}\right)^2 = \int_0^t \frac{D_{rag}\cdot V}{M}dt$$

$\rightarrow$ *equivalent specif ic energy loss due to dra*

$$\rightarrow \Delta V_{drag} = \sqrt{2\cdot\int_0^t \frac{D_{rag}\cdot V}{M}dt}$$

For constant C_D, M

$$\rightarrow \Delta V_{drag} = \sqrt{2\cdot\int_0^t \frac{1}{2}\frac{C_D\cdot A_{ref}\cdot\rho\cdot V^3}{M}dt} = \sqrt{\frac{C_D\cdot A_{ref}}{M}\int_0^t \rho\cdot V^3 dt}$$

$$\beta = \frac{M}{C_D\cdot A_{ref}} \rightarrow \text{"Ballistic Coefficient"}$$

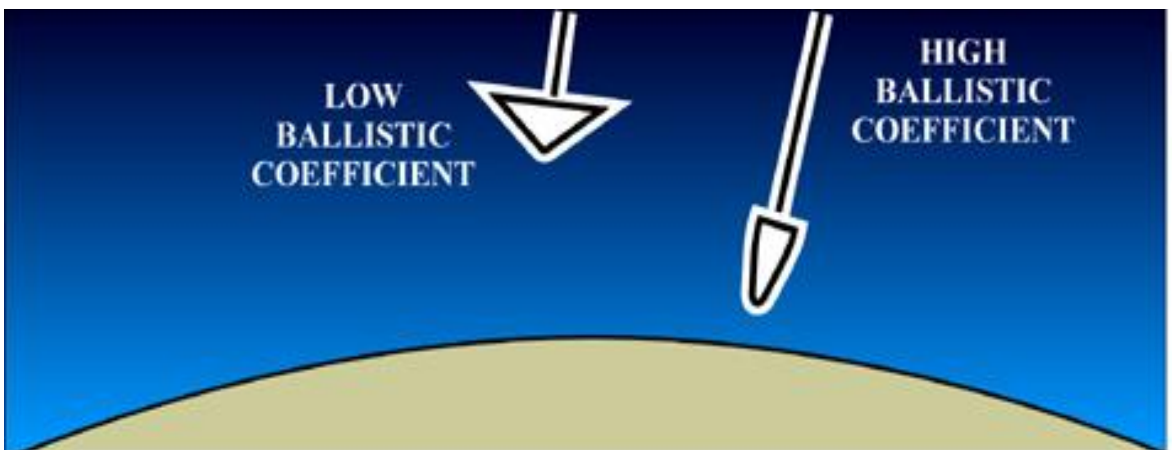
Drag Losses (4)

$$D_{rag} = C_D A_{ref} \frac{1}{2} \rho V^2 \rightarrow \Delta V_{drag} = \sqrt{A_{ref} \int_0^t \frac{C_D \rho V^3}{m} dt} = \sqrt{\int_0^t \frac{\rho V^3}{\beta} dt}$$

- Aerodynamic drag/mass inertial effects can be incorporated into a single parameter .... ***Ballistic Coefficient (*** β ***)***

- …..measure of a projectile's ability to coast. … $\beta = M/C_D A_{ref}$
… M is the projectile's mass and … $C_D A_{ref}$ is the drag *form factor*.

- At given velocity and air density drag deceleration inversely proportional to β



Low Ballistic Coefficients Dissipate More Energy Due to drag

Summary

•Rocket Thrust Equation

$$F_{thrust} = \dot{m} V_{exit} + A_{exit}\left(P_{exit} - P_{atmosphere}\right)$$

•Specific Impulse .. Total …. Instantaneous)

$$I_{sp} = \frac{\mathrm{I}_{mpulse}}{g_0 M_{propellant}} = \frac{\int_0^t F_{thrust}\,dt}{g_0 \int_0^t \dot{m}_{propellant}\,dt} \cong \frac{F_{thrust}}{g_0\,\dot{m}_{propellant}}$$

•Rocket Equation

$$\Delta V_{burn} = g_0 \cdot I_{sp} \cdot \ln\left(\frac{M_{initial}}{M_{final}}\right)$$

Summary (2)

•*Propellant Mass Budget Equation*

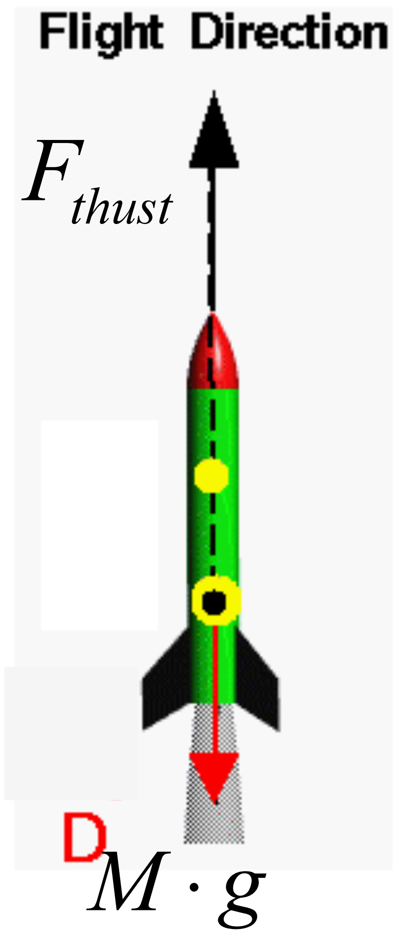
$$M_{\substack{\text{fuel}\\+\text{oxidizer}}} = \left[M_{dry} + M_{payload}\right]\left[e^{\left[\frac{\Delta V_{burn}}{g_0 I_{sp}}\right]} - 1\right]$$

•*Load Mass Fraction*

$$L_{mf} \equiv \frac{M_{propellant}}{M_{initial}} = \frac{M_{propellant}}{M_{final} + M_{initial}} 1 - e^{-\left(\frac{\Delta V}{g_0 \cdot I_{sp}}\right)}$$

Summary (3)

ΔV for a Vertically Accelerating Vehicle (7)



$$\Delta V_{t_{burn}} = g_0 \cdot I_{sp} \left[\ln\left(1 + P_{mf}\right)\right] - g_0 \cdot t_{burn}$$

$\rightarrow$

$$P_{mf} = \frac{M_{propellant}}{M_{f\,inal}}$$

$$t_{burn} = \frac{g_0 \cdot I_{sp} \cdot m_{propellamt}}{F_{thrust}}$$

Summary (4)

Available ΔV

$$\Delta V_{t_{burn}} = g_0 \cdot I_{sp} \left[\ln\left(1 + P_{mf}\right) \right] -$$

$$\text{"combustion } \Delta V\text{"}$$

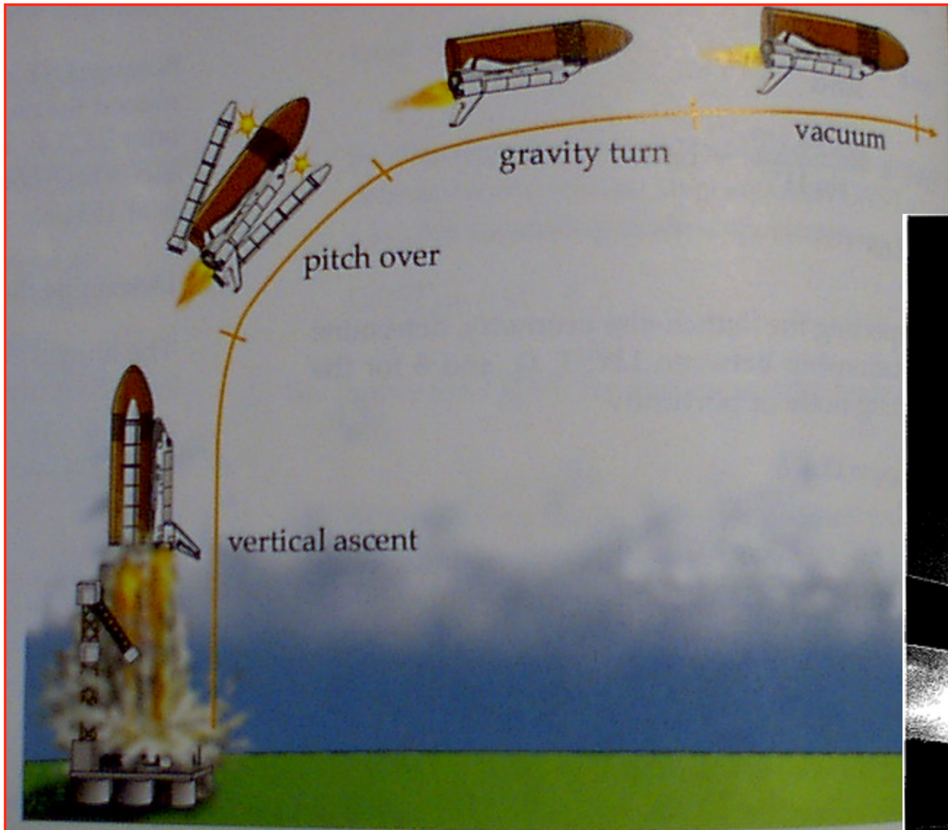
$$\int_0^{t_{burn}} g(t) \cdot \sin\theta dt - \sqrt{\int_0^{t_{burn}} \frac{\rho V^3}{\beta} dt}$$

$$\text{"gravity loss"} \qquad \text{"drag loss"}$$

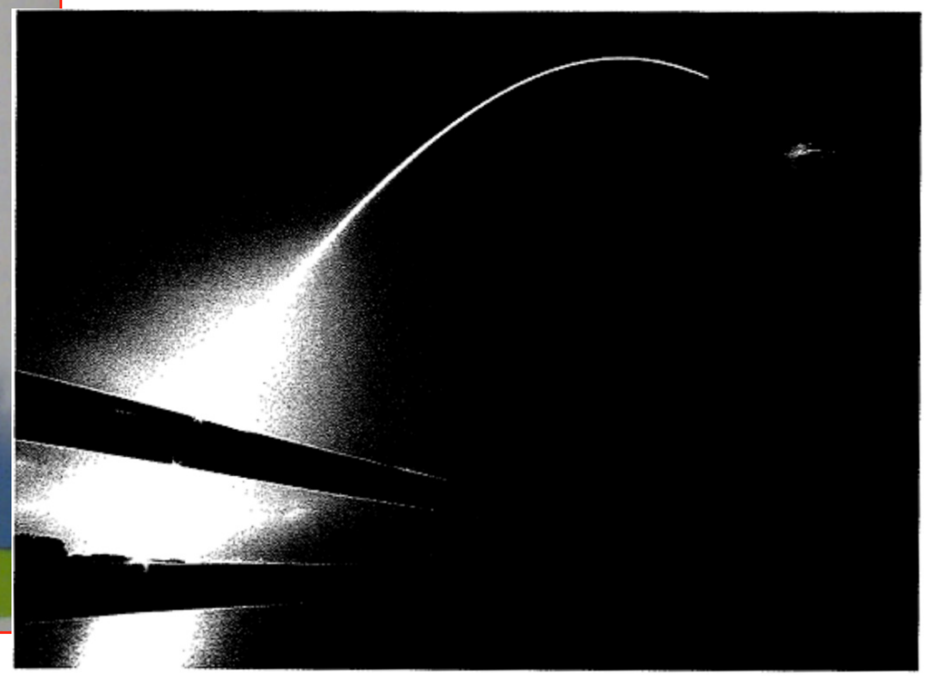
$$\beta = \frac{M}{C_D \cdot A_{ref}}$$

Path dependent velocity losses

Why Does this "Turn" Occur at Launch?



Phases of Launch Vehicle Ascent. During ascent a launch vehicle goes through four phases—vertical ascent, pitch over, gravity turn, and vacuum.



Gravity-turn maneuver of an ascending Delta II rocket with Messenger spacecraft on August 3, 2004.

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