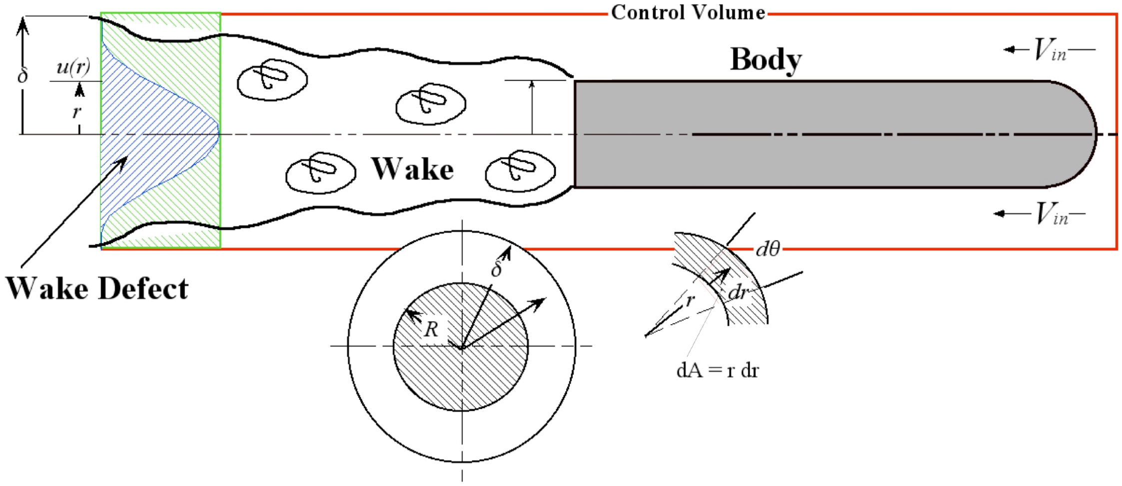
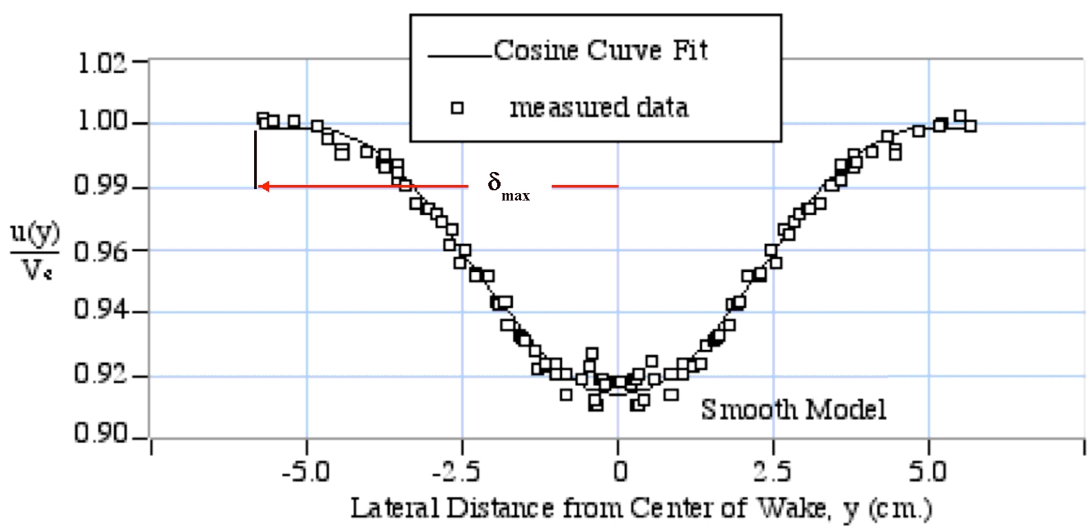
Drag Calculations Using Wake Survey Method



Lateral "Slice" Through Wake Field



Conservation of Mass, non-symmetric Flow Field

$$\dot{m}_{in} = \dot{m}_{out} \Rightarrow \int_0^{2\pi}\int_0^{\delta_{max}} \rho V_{in} r \cdot dr \cdot d\theta = \int_0^{2\pi}\int_0^{\delta_{max}} \rho \cdot u(r) \cdot r \cdot dr \cdot d\theta \Rightarrow$$

$$2\pi \cdot V_{in} \cdot \rho \left[\frac{r^2}{2} \Big|_0^{\delta_{max}}\right] = \rho \cdot \int_0^{2\pi}\int_0^{\delta_{max}} u(r) \cdot r \cdot dr \cdot d\theta \qquad (1$$

$$\frac{\delta_{max}{}^2}{2} = \left(\frac{1}{2\pi}\right) \int_0^{2\pi}\int_0^{\delta_{max}} \left(\frac{u(r)}{V_{in}}\right) \cdot r \cdot dr \cdot d\theta$$

Conservation of Momentum, Axisymmetric Flow Field

$$dD(r) = \dot{m}V_{in} - \dot{m}u(r) = \rho V_{in}\left(r \cdot dr \cdot d\theta\right)V_{in} - \rho \cdot u(r)\left(r \cdot dr \cdot d\theta\right)u(r)$$

$$\Rightarrow \boxed{dD(r) = \rho V_{in}^{2}\left[1 - \left(\frac{u(r)}{V_{in}}\right)^{2}\right]\left(r \cdot dr \cdot d\theta\right)} \qquad (2$$

Integrating Equation 2 to Give the Total Drag (Momentum Defect) Across the Wake

$$D = \int_0^{2\pi}\int_0^{\delta_{\max}} \rho V_{in}^{2}\left[1 - \left(\frac{u(r)}{V_{in}}\right)^{2}\right]\left(r \cdot dr \cdot d\theta\right) =$$

$$2\pi \cdot \rho V_{in}^{2}\left[\frac{\delta_{\max}^{2}}{2} - \frac{1}{2\pi}\int_0^{2\pi}\left[\int_0^{\delta_{\max}}\left[\left(\frac{u(r)}{V_{in}}\right)^{2}\right]\left(r \cdot dr\right)\right]d\theta\right] \qquad (3)$$

But from equation 1

$$\frac{\delta_{\max}^{2}}{2} = \left(\frac{1}{2\pi}\right)\int_0^{2\pi}\int_0^{\delta_{\max}}\left(\frac{u(r)}{V_{in}}\right) \cdot r \cdot dr \cdot d\theta$$

Substituting Eq. 1) into Eq. 3)

$$D = 2\pi \cdot \rho V_{in}^{2}\left[\left(\frac{1}{2\pi}\right)\int_0^{2\pi}\int_0^{\delta_{\max}}\left(\frac{u(r)}{V_{in}}\right) \cdot r \cdot dr \cdot d\theta - \frac{1}{2\pi}\int_0^{2\pi}\left[\int_0^{\delta_{\max}}\left(\frac{u(r)}{V_{in}}\right)^{2}\left(r \cdot dr\right)\right]d\theta\right] =$$

$$\rho V_{in}^{2}\left[\int_0^{2\pi}\left[\int_0^{\delta_{\max}}\left(\frac{u(r)}{V_{in}}\right)\left[1 - \left(\frac{u(r)}{V_{in}}\right)\right]\left(r \cdot dr\right)\right]d\theta\right] \qquad (4$$

Compute the Drag Coefficient

$$C_D = \frac{D}{\frac{1}{2}\rho V_{in}^{2} \cdot A_{ref}} = \frac{D}{\frac{1}{2}\rho V_{in}^{2} \cdot \frac{\pi D_{ref}^{2}}{4}} = \frac{\rho V_{in}^{2}\left[\int_0^{2\pi}\left[\int_0^{\delta_{\max}}\left(\frac{u(r)}{V_{in}}\right)\left[1 - \left(\frac{u(r)}{V_{in}}\right)\right]\left(r \cdot dr\right)\right]d\theta\right]}{\frac{1}{2}\rho V_{in}^{2} \cdot \frac{\pi D_{ref}^{2}}{4}} = \qquad (5$$

$$\boxed{\rightarrow C_D = \frac{8}{\pi D_{ref}^{2}}\left[\int_0^{2\pi}\left[\int_0^{\delta_{\max}}\left(\frac{u(r)}{V_{in}}\right)\left[1 - \left(\frac{u(r)}{V_{in}}\right)\right]\left(r \cdot dr\right)\right]d\theta\right]}$$

How “compressible” will our wind tunnel tests be?

… Our tunnel can generate max speed of ~30 m/sec (58 kts) ….

$$M_{\max} = \frac{V_{\max}}{\sqrt{\gamma \cdot R_g \cdot T}} = \frac{30 \frac{m}{\sec}}{\sqrt{1.4 \cdot 287.056 \frac{j}{kg-K} \cdot 297 K}} = 0.0868 \quad (6$$

Compressible Bernoulli Equation

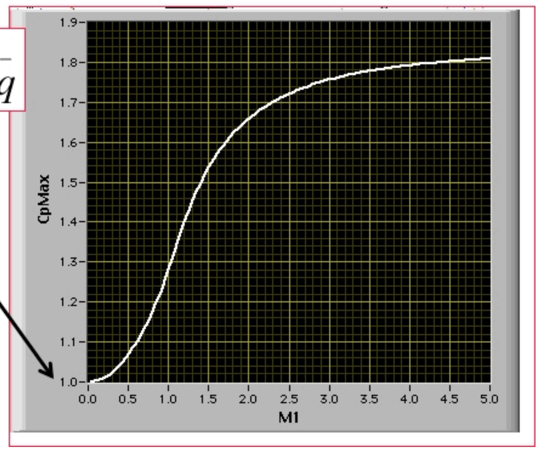
$$P_0 = p_\infty + Cp_{max}\,\bar{q}$$

• M --> 0

(incompressible flow)

Cp_{max} -->1

$$\left(P_0 = p_\infty + \bar{q}\right)_{|M=0}$$



Essentially incompressible!

What dynamic pressure do we expect?

$$\left.\begin{array}{l} M \approx 0.0868 \\ p_{ambient} \approx 86_{kPa} \end{array}\right| \rightarrow \bar{q} \approx \frac{\gamma}{2} \cdot p_{ambient} \cdot M^2 = 453.6_{pascals} (0.0658_{psid},\ 1.82_{\frac{in}{H_2O}})$$

From Bernoulli …for incompressible flow …

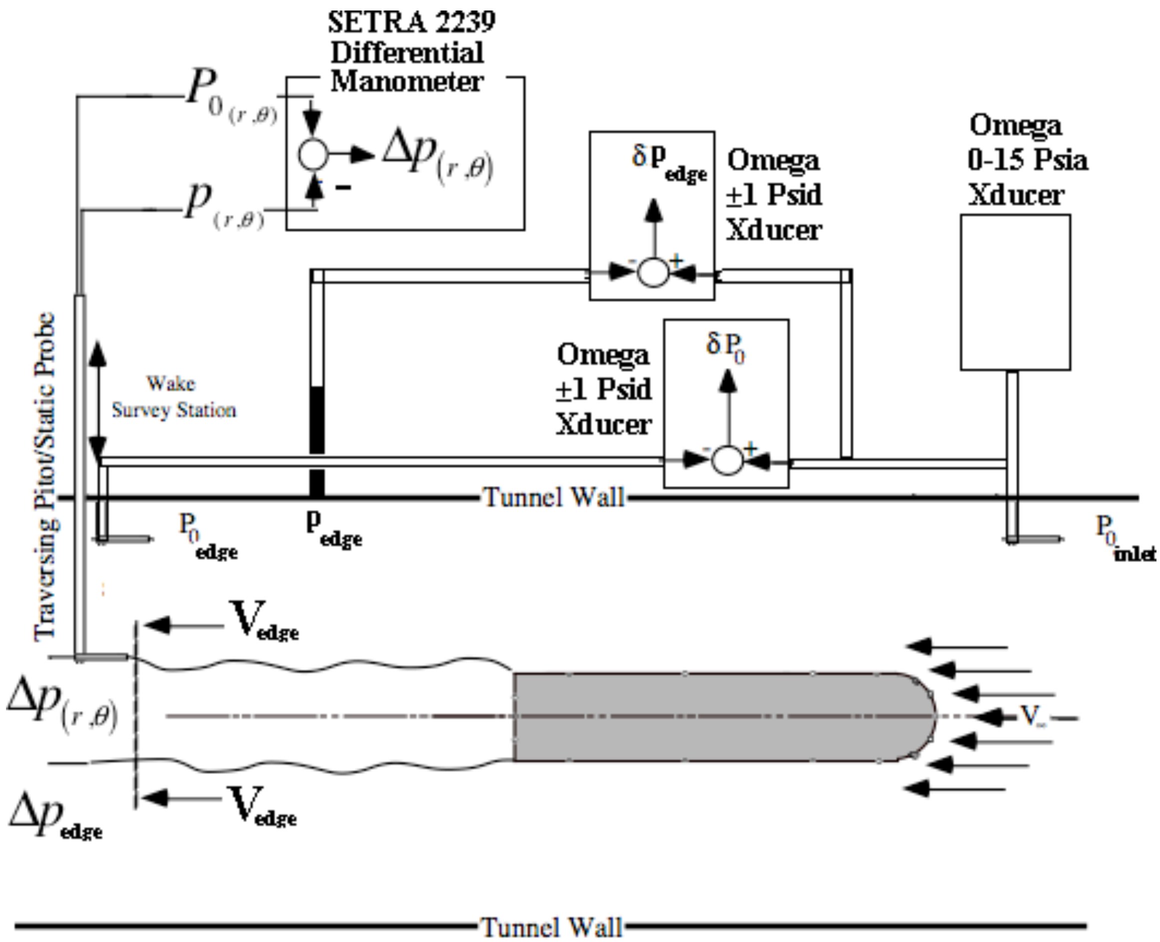
$$Bernoulli \rightarrow P_0 = p_\varsigma + \frac{1}{2}\rho u_\varsigma^{\,2} \rightarrow \begin{array}{l} u_{r,\theta} = \sqrt{2\left[\frac{P_0 - p_{r,\theta}}{\rho}\right]} \\ V_{edge} = \sqrt{2\left[\frac{P_0 - p_{in}}{\rho}\right]} \end{array} \rightarrow \frac{u_{r,\theta}}{V_{edge}} = \sqrt{\frac{\Delta p_{r,\theta}}{\Delta p_{edge}}} \quad (7$$

$$C_D = \left(\frac{8}{\pi \cdot D^2_{ref}}\right) \cdot \left\{ \int_0^{2\pi} \left[\int_0^{\delta_{\max}} \left(\frac{u_{(r,\theta)}}{V_{edge}}\right) \cdot \left(1 - \frac{u_{(r,\theta)}}{V_{edge}}\right) \cdot r \cdot dr \right] \cdot d\theta \right\}$$

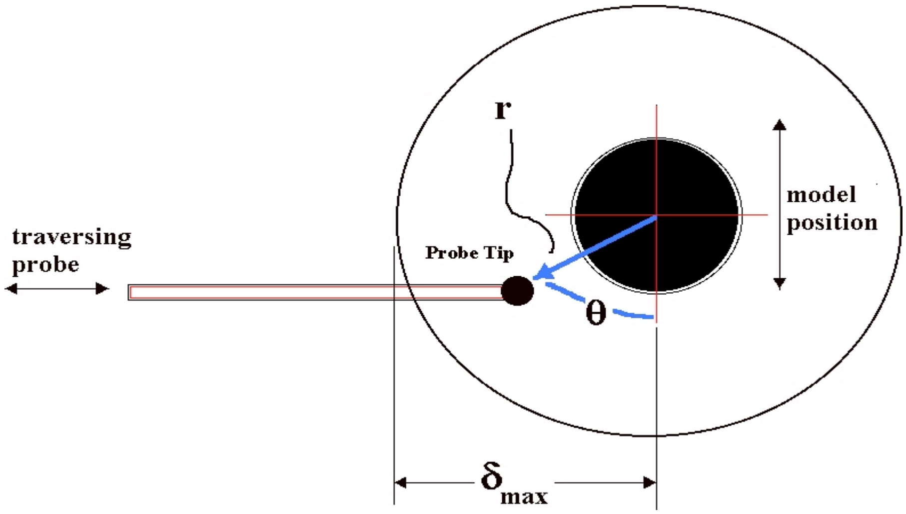
$$\rightarrow \left(\frac{u_{(r,\theta)}}{V_{edge}}\right) = \sqrt{\frac{\Delta p_{(r,\theta)}}{\Delta p_{edge}}} = \sqrt{\frac{(P_0 - p)_{(r,\theta)}}{(P_0 - p)_{(edge)}}}$$

$$\Rightarrow \boxed{C_D = \left(\frac{8}{\pi \cdot D^2_{ref}}\right) \cdot \left\{ \int_0^{2\pi} \left[\int_0^{\delta_{\max}} \left(\sqrt{\frac{\Delta p_{(r,\theta)}}{\Delta p_{edge}}}\right) \cdot \left(1 - \sqrt{\frac{\Delta p_{(r,\theta)}}{\Delta p_{edge}}}\right) \cdot r \cdot dr \right] \cdot d\theta \right\}} \quad (8$$

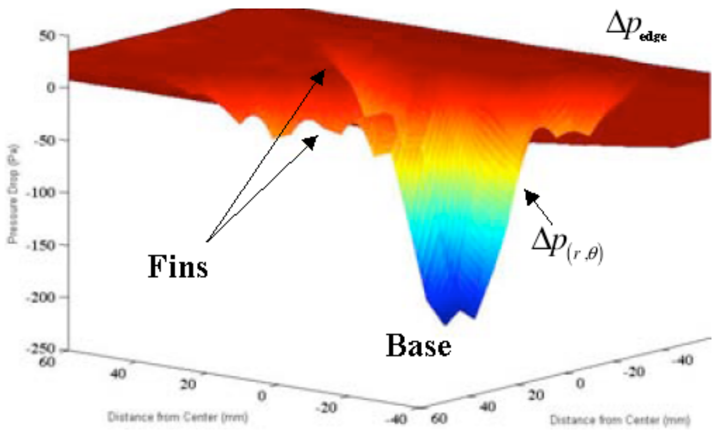
Test Schematic



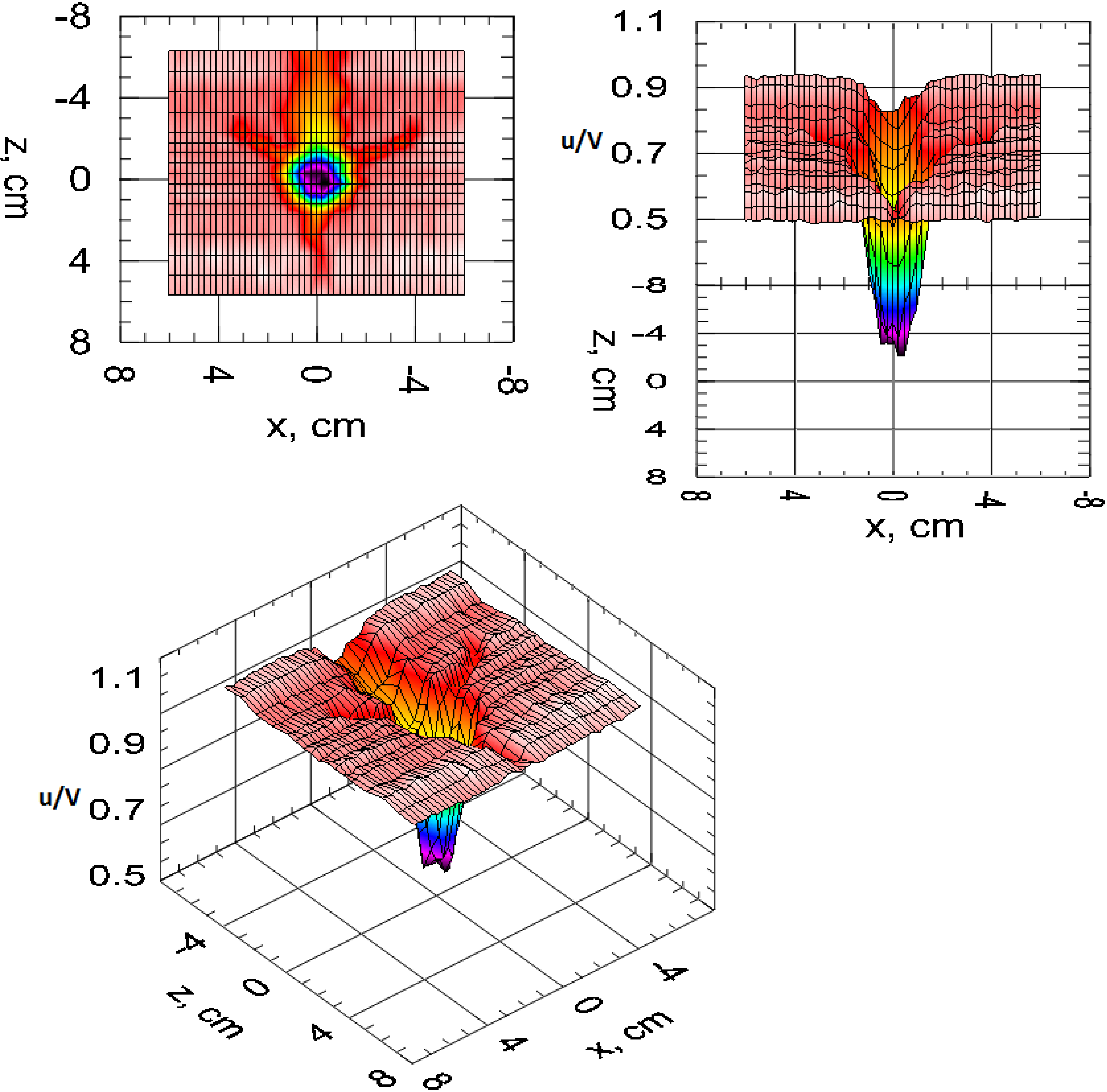
Schematic of Wake Survey Measurements



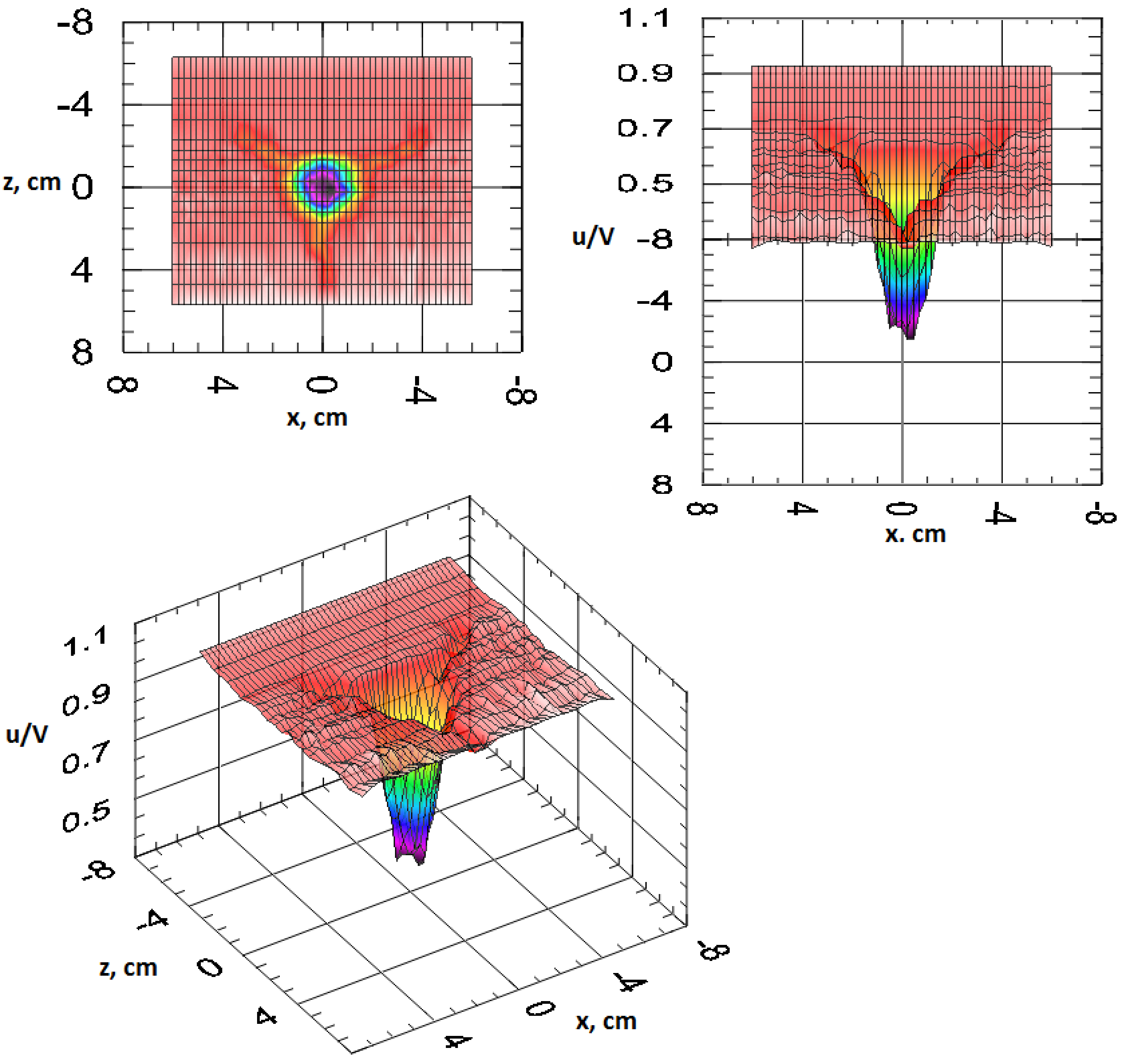
Probe Coordinates



3-D View of Wake Survey Data



z, cm
x, cm
u/V
z, cm
x, cm
u/V
z, cm
x, cm



-8
-4
z, cm 0
4
8
8
4
0
-4
-8
x, cm
1.1
0.9
0.7
0.5
u/V -8
-4
0
4
8
8
4
0
-4
-8
x. cm
1.1
0.9
u/V
0.7
0.5
-8
-4
z, cm
0
4
4
0
-4
-8
x, cm