

Given  $\rightarrow v_M \rightarrow$  Solve for M

$$v_M = \sqrt{\frac{\gamma+1}{\gamma-1}} \tan^{-1} \sqrt{\frac{\gamma-1}{\gamma+1} \cdot (M^2 - 1)} - \sqrt{M^2 - 1} \rightarrow \text{Derivative} \rightarrow \frac{d}{dM} v(M) = \frac{1}{M} \cdot \left( \frac{\sqrt{M^2 - 1}}{1 + \frac{\gamma-1}{2} \cdot M^2} \right)$$

Assume Starting Solution  $\rightarrow M_{(j)}$

Expand in Taylor's Series

$$v_M = v(M_{(j)}) + \frac{d}{dM} v(M) \Big|_{M_{(j)}} \cdot (M - M_{(j)}) + \text{H.O. Terms} \rightarrow \text{Solve for M}$$

$$M = M_{(j)} + \frac{v_M - v(M_{(j)})}{\frac{d}{dM} v(M) \Big|_{M_{(j)}}} + \frac{\text{H.O. Terms}}{\frac{d}{dM} v(M) \Big|_{M_{(j)}}}$$

Truncate after first order  $\rightarrow M_{(j+1)}$  Approximates M

$$v(M_{(j)}) = \sqrt{\frac{\gamma+1}{\gamma-1}} \tan^{-1} \sqrt{\frac{\gamma-1}{\gamma+1} \cdot (M_{(j)}^2 - 1)} - \sqrt{M_{(j)}^2 - 1}$$
$$M_{(j+1)} = M_{(j)} + \frac{v_M - v(M_{(j)})}{\frac{d}{dM} v(M) \Big|_{M_{(j)}}} \rightarrow \frac{d}{dM} v(M) \Big|_{M_{(j)}} = \frac{1}{M_{(j)}} \cdot \left( \frac{\sqrt{M_{(j)}^2 - 1}}{1 + \frac{\gamma-1}{2} \cdot M_{(j)}^2} \right)$$

Drop from Loop when

$$\left| \frac{v_M - v(M_{(j+1)})}{v_M} \right| < \varepsilon$$