Solve for Slope & Prandtl - Meyer angle

$$\theta_3 = \frac{(\theta_1+\nu_1)+(\theta_2-\nu_2)}{2} = \frac{(K_-)_1+(K_+)_2}{2}$$

$$\nu_3 = \frac{(\theta_1+\nu_1)-(\theta_2-\nu_2)}{2} = \frac{(K_-)_1-(K_+)_2}{2}$$

$$\rightarrow \quad (K_-)_1 = \theta_1+\nu_1 \qquad (K_+)_2 = \theta_2-\nu_2$$

Solve for Mach

$$M_3 \rightarrow solve\left[\nu_3 = \sqrt{\frac{\gamma+1}{\gamma-1}}\tan^{-1}\sqrt{\frac{\gamma-1}{\gamma+1}\left(M_3{}^2-1\right)} - \sqrt{\ }\ \tan^{-1}\sqrt{\left(M_3{}^2-1\right)}\right]$$

Solve for Slope of characteristic lines

$$slope\{C_-\} = \frac{(\theta_1-\mu_1)+(\theta_3-\mu_3)}{2}$$

$$slope\{C_+\} = \frac{(\theta_2+\mu_2)+(\theta_3+\mu_3)}{2}$$

Use definition of Slopes to calculate x_3, y_3

$$\frac{y_3-y_1}{x_3-x_1} = \tan\left[slope\{C_-\}\right]$$

$$\frac{y_3-y_2}{x_3-x_2} = \tan\left[slope\{C+\}\right]$$

Solve for y_3

$$y_3 = (x_3-x_1)\cdot\tan\left[slope\{C_-\}\right]+y_1$$

$$y_3 = (x_3-x_2)\cdot\tan\left[slope\{C_+\}\right]+y_2$$

$$\rightarrow \quad y_3 = x_3\cdot\tan\left[slope\{C_-\}\right]+\left(y_1-x_1\cdot\tan\left[slope\{C_-\}\right]\right)$$

$$y_3 = x_3\cdot\tan\left[slope\{C_+\}\right]+\left(y_2-x_2\cdot\tan\left[slope\{C_+\}\right]\right)$$

Eliminate y_3

$$x_3\cdot\tan\left[slope\{C_-\}\right]+\left(y_1-x_1\cdot\tan\left[slope\{C_-\}\right]\right) = x_3\cdot\tan\left[slope\{C_+\}\right]+\left(y_2-x_2\cdot\tan\left[slope\{C_+\}\right]\right)$$

Solve for x_3

$$x_3\cdot\left\{\tan\left[slope\{C_-\}\right]-\tan\left[slope\{C_+\}\right]\right\} = \left(y_2-x_2\cdot\tan\left[slope\{C_+\}\right]\right)-\left(y_1-x_1\cdot\tan\left[slope\{C_-\}\right]\right)$$

$$x_3 = \frac{\left(y_2-x_2\cdot\tan\left[slope\{C_+\}\right]\right)-\left(y_1-x_1\cdot\tan\left[slope\{C_-\}\right]\right)}{\left\{\tan\left[slope\{C_-\}\right]-\tan\left[slope\{C_+\}\right]\right\}} = \frac{x_1\cdot\tan\left[slope\{C_-\}\right]-x_2\cdot\tan\left[slope\{C_+\}\right]+(y_2-y_1)}{\left\{\tan\left[slope\{C_-\}\right]-\tan\left[slope\{C_+\}\right]\right\}}$$

Solve for y_3

$$y_3 = x_3\cdot\tan\left[slope\{C_-\}\right]+\left(y_1-x_1\cdot\tan\left[slope\{C_-\}\right]\right)$$

$$y_3 = x_3\cdot\tan\left[slope\{C_+\}\right]+\left(y_2-x_2\cdot\tan\left[slope\{C_+\}\right]\right)$$

$$y_3 = \frac{1}{2}\left[x_3\cdot\left(\tan\left[slope\{C_-\}\right]+\tan\left[slope\{C_+\}\right]\right)+(y_1+y_2)-\left(x_1\cdot\tan\left[slope\{C_-\}\right]+x_2\cdot\tan\left[slope\{C_+\}\right]\right)\right]$$

Substitute for x_3

$$y_3 = \frac{1}{2}\left[\frac{x_1\cdot\tan\left[slope\{C_-\}\right]-x_2\cdot\tan\left[slope\{C_+\}\right]+(y_2-y_1)}{\left\{\tan\left[slope\{C_-\}\right]-\tan\left[slope\{C_+\}\right]\right\}}\cdot\left(\tan\left[slope\{C_-\}\right]+\tan\left[slope\{C_+\}\right]\right)-\left\{x_1\cdot\tan\left[slope\{C_-\}\right]+x_2\cdot\tan\left[slope\{C_+\}\right]-(y_1+y_2)\right\}\right] =$$

$$\frac{1}{2}\left[\frac{\left\{x_1\cdot\tan\left[slope\{C_-\}\right]-x_2\cdot\tan\left[slope\{C_+\}\right]+(y_2-y_1)\right\}\cdot\left(\tan\left[slope\{C_-\}\right]+\tan\left[slope\{C_+\}\right]\right)-\left\{x_1\cdot\tan\left[slope\{C_-\}\right]+x_2\cdot\tan\left[slope\{C_+\}\right]-(y_1+y_2)\right\}\cdot\left\{\tan\left[slope\{C_-\}\right]-\tan\left[slope\{C_+\}\right]\right\}}{\left\{\tan\left[slope\{C_-\}\right]-\tan\left[slope\{C_+\}\right]\right\}}\right] =$$

$$\frac{1}{2}\left[\frac{\left\{2\cdot x_1\cdot\tan\left[slope\{C_-\}\right]\tan\left[slope\{C_+\}\right]-2\cdot x_2\cdot\tan\left[slope\{C_+\}\right]\tan\left[slope\{C_-\}\right]+\left(2\cdot y_2\tan\left[slope\{C_-\}\right]-2\cdot y_1\tan\left[slope\{C_=\}\right]\right)\right\}}{\left\{\tan\left[slope\{C_-\}\right]-\tan\left[slope\{C_+\}\right]\right\}}\right]$$

Simplify

$$y_3 = \frac{\tan\left[slope\{C_-\}\right]\cdot\tan\left[slope\{C_+\}\right]\cdot(x_1-x_2)+\tan\left[slope\{C_-\}\right]\cdot y_2-\tan\left[slope\{C_+\}\right]\cdot y_1}{\tan\left[slope\{C_-\}\right]-\tan\left[slope\{C_+\}\right]}$$

Summary

$$x_3 = \frac{x_1\cdot\tan\left[slope\{C_-\}\right]-x_2\cdot\tan\left[slope\{C_+\}\right]+(y_2-y_1)}{\left\{\tan\left[slope\{C_-\}\right]-\tan\left[slope\{C_+\}\right]\right\}}$$

$$y_3 = \frac{\tan\left[slope\{C_-\}\right]\cdot\tan\left[slope\{C_+\}\right]\cdot(x_1-x_2)+\tan\left[slope\{C_-\}\right]\cdot y_2-\tan\left[slope\{C_+\}\right]\cdot y_1}{\tan\left[slope\{C_-\}\right]-\tan\left[slope\{C_+\}\right]}$$