Using Method of Characteristics for Aerospike Nozzle Contour Design

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Apply Method of Characteristics to Aerospike Nozzle

FORTRAN PROGRAM FOR PLUG NOZZLE DESIGN
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Apply Method of Characteristics to Aerospike Nozzle (1)

\[ v_{\text{throat}} = \sqrt{\frac{\gamma + 1}{\gamma - 1}} \tan^{-1} \left\{ \frac{\sqrt{\gamma - 1}}{\sqrt{\gamma + 1} (1^2 - 1)} \right\} - \tan^{-1} \sqrt{1^2 - 1} = 0 \]

\[
\frac{A_{\text{exit}}}{A^*} = \frac{1}{M_{\text{exit}}} \left[ \left( \frac{2}{\gamma + 1} \right) \left( 1 + \frac{\gamma - 1}{2} M_{\text{exit}}^2 \right) \right]^{\frac{\gamma + 1}{2(\gamma - 1)}}
\]
Apply Method of Characteristics to Aerospike Nozzle (2)

\[ \theta_{\text{throat}} + v_{\text{throat}} = \theta_x + v_x = \theta_{\text{exit}} + v_{\text{exit}} \]

\[ \delta = 90^\circ - \theta_{\text{throat}} = 90^\circ - \left( \theta_{\text{exit}} + v_{\text{exit}} - v_{\text{throat}} \right) \rightarrow \]

\[ \rightarrow v_{\text{throat}} = 0, \quad \theta_{\text{exit}} = 0 \]

Throat Exit Angle Fixed by Spike
Design expansion ratio

\[ \delta = 90^\circ - \left( v_{\text{exit}} \right) = \]

\[ 90^\circ - \left[ (M_{\text{exit}}) = \frac{\gamma + 1}{\gamma - 1} \tan^{-1} \left\{ \frac{\gamma - 1}{\gamma + 1} \left( M_{\text{exit}}^2 - 1 \right) \right\} - \tan^{-1} \sqrt{M_{\text{exit}}^2 - 1} \right] \]
Apply Method of Characteristics to Aerospike Nozzle (3)

\[ \theta_x + v_x = v_{exit} \]

\[ \theta_x = \left[ \sqrt{\frac{\gamma + 1}{\gamma - 1}} \tan^{-1} \left\{ \sqrt{\frac{\gamma - 1}{\gamma + 1}} \left( M_{exit}^2 - 1 \right) \right\} - \tan^{-1} \sqrt{M_{exit}^2 - 1} \right] - \left[ \sqrt{\frac{\gamma + 1}{\gamma - 1}} \tan^{-1} \left\{ \sqrt{\frac{\gamma - 1}{\gamma + 1}} \left( M_x^2 - 1 \right) \right\} - \tan^{-1} \sqrt{M_x^2 - 1} \right] \]

Along spike surface
Plug Geometry

At position \( X \)

\[
A_x \sin \phi_x = \pi \left( R_e^2 - R_x^2 \right)
\]

\[
\phi_x = \theta_x + \mu_x \quad \Rightarrow \quad \phi_x = v_e - v_x + \mu_x
\]

Prandtl-Meyer Expansion

Wave through Point \( x \)

\[ A_x \sin \phi_x \]

\[ A_x \sin(\mu_x) \]

\[ A_x = \pi \left[ \frac{R_e^2 - R_x^2}{\sin(v_e - v_x + \mu_x)} \right] \]
Apply Continuity equation

\[ \dot{m} = \rho_x A_x V_x \sin(\mu_x) = \rho_t A_t V_t \]
Apply Method of Characteristics to Aerospike Nozzle (5)

\[ A_x = \frac{\rho_t A_t}{\rho_x \frac{V_x}{V_t} \sin(\mu_x)} = \frac{P_t A_t}{T_t \frac{P_x}{V_t} \frac{V_x}{V_t} \sin(\mu_x)} \]

\[ \frac{P_t A_t}{P_x \sqrt{\frac{T_t}{T_x}} \sqrt{\frac{T_t}{T_x}} \frac{V_x}{V_t} \sin(\mu_x)} = \frac{P_t A_t}{P_x \sqrt{\frac{T_t}{T_x}} \frac{V_x}{V_t} \sqrt{\frac{T_t}{T_x}} \sin(\mu_x)} = \frac{P_t A_t}{P_x \sqrt{\frac{T_t}{T_x}} \frac{M_x}{M_t} \sin(\mu_x)} \]

• Divide by throat area

\[ A_x = \frac{P_0 \sqrt{\frac{T_x}{T_0}}}{P_x \sqrt{\frac{T_t}{T_0}} \frac{M_x}{M_t} \sin(\mu_x)} = \frac{1 + \frac{\gamma - 1}{2} M_x^2}{\left[1 + \frac{\gamma - 1}{2} M_x^2\right]^{\gamma - 1} \cdot \left[1 + \frac{\gamma - 1}{2} M_t^2\right]^{-1/2} \cdot \frac{1}{M_x} \cdot \sin(\mu_x)} \]
Apply Method of Characteristics to Aerospike Nozzle \( (6) \)

- Simplifying

\[
\frac{A_x}{A_t} = \frac{1 + \frac{\gamma - 1}{2} M_x^2}{\left[1 + \frac{\gamma - 1}{2} M_t^2\right]^{\frac{\gamma+1}{2(\gamma-1)}}} \cdot \frac{1}{M_t} \cdot \sin(\mu_x)
\]

\[
\frac{A_t}{A^*} = \left[\frac{2}{\gamma + 1} \cdot \left(1 + \frac{\gamma - 1}{2} M_x^2\right)\right]^{\frac{\gamma+1}{2(\gamma-1)}} \cdot \frac{1}{M_t} \cdot \sin(\mu_x)
\]

\[
\frac{A_x}{A^*} \cdot \frac{A_t}{A^*} = \left(\frac{2}{\gamma + 1}\right) \cdot \left(1 + \frac{\gamma - 1}{2} M_t^2\right) \cdot \left(\frac{2}{\gamma + 1} \cdot \left(1 + \frac{\gamma - 1}{2} M_x^2\right)\right)^{\frac{\gamma+1}{2(\gamma-1)}} \cdot \frac{1}{M_x} \cdot \sin(\mu_x)
\]
Apply Method of Characteristics to Aerospike Nozzle (7)

• Simplifying again

\[
A_x = A^* \frac{\left[ \left( \frac{2}{\gamma + 1} \right) \cdot \left( 1 + \frac{\gamma - 1}{2} M_x^2 \right) \right]^{\gamma+1} \cdot \frac{1}{M_x}}{\sin(\mu_x)} \rightarrow \sin(\mu_x) = \frac{1}{M_x}
\]

\[
A_x = A^* \left[ \left( \frac{2}{\gamma + 1} \right) \cdot \left( 1 + \frac{\gamma - 1}{2} M_x^2 \right) \right]^{\gamma+1} = \frac{A_e}{\varepsilon} \left[ \left( \frac{2}{\gamma + 1} \right) \cdot \left( 1 + \frac{\gamma - 1}{2} M_x^2 \right) \right]^{\gamma+1}
\]

\[
\frac{\pi R_e^2}{\varepsilon} \left[ \left( \frac{2}{\gamma + 1} \right) \cdot \left( 1 + \frac{\gamma - 1}{2} M_x^2 \right) \right]^{\gamma+1}
\]

thus

\[
A_x = \frac{\pi \left( R_e^2 - R_x^2 \right)}{\sin(\nu_e - \nu_x + \mu_x)} = \frac{\pi R_e^2}{\varepsilon} \left[ \left( \frac{2}{\gamma + 1} \right) \cdot \left( 1 + \frac{\gamma - 1}{2} M_x^2 \right) \right]
\]
Apply Method of Characteristics to Aerospike Nozzle \((8)\)

- Solve for \(R_x\)

\[
1 - \left( \frac{R_x}{R_e} \right)^2 = \frac{\sin(v_e - v_x + \mu_x)}{\varepsilon} \left[ \left( \frac{2}{(\gamma + 1)} \right) \cdot \left( 1 + \frac{\gamma - 1}{2} M_x^2 \right) \right]^{\frac{\gamma + 1}{2(\gamma - 1)}}
\]

\[
\rightarrow \quad \frac{R_x}{R_e} = \sqrt{1 - \frac{\sin(v_e - v_x + \mu_x)}{\varepsilon} \left[ \left( \frac{2}{(\gamma + 1)} \right) \cdot \left( 1 + \frac{\gamma - 1}{2} M_x^2 \right) \right]^{\frac{\gamma + 1}{2(\gamma - 1)}}}
\]
Apply Method of Characteristics to Aerospike Nozzle (9)

- and since by geometry of the surface

\[ \tan \phi_x = \frac{R - R_x}{X_x} \rightarrow \phi_x = \nu_e - \nu_x + \mu_x \rightarrow \]

\[ X_x = \frac{R_{\text{exit}} - R_x}{\tan(\nu_e - \nu_x + \mu_x)} \]

\[ R_x = R_{\text{exit}} \sqrt{1 - \frac{\sin(\nu_e - \nu_x + \mu_x)}{\varepsilon} \left[ \left( \frac{2}{\gamma + 1} \right) \left( 1 + \frac{\gamma - 1}{2} M_x^2 \right) \right]^{\gamma+1}}^{2(\gamma-1)} \]

\[ \sin(\mu_x) = \frac{1}{M_x} \]

- These equations define the isentropic spike profile
Aerospike Contour Computational Algorithm

• Step up in $M$ from

\[ M = 1 \ldots M_{\text{exit}} \]

\[
\nu_{\text{exit}} = \sqrt{\frac{\gamma + 1}{\gamma - 1}} \tan^{-1} \left[ \sqrt{\frac{\gamma - 1}{\gamma + 1}} (M_{\text{exit}}^2 - 1) \right] - \tan^{-1} \left[ \sqrt{M_{\text{exit}}^2 - 1} \right]
\]

\[
\nu_x = \sqrt{\frac{\gamma + 1}{\gamma - 1}} \tan^{-1} \left[ \sqrt{\frac{\gamma - 1}{\gamma + 1}} (M_x^2 - 1) \right] - \tan^{-1} \left[ \sqrt{M_x^2 - 1} \right]
\]

\[
\mu_x = \sin^{-1} M_x
\]

\[
R_x = R_{\text{exit}} \sqrt{1 - \left( \frac{\sin(\nu_{\text{exit}} - \nu_x + \mu_x)}{\varepsilon} \right) \left[ \left( \frac{2}{\gamma + 1} \left( 1 + \frac{\gamma - 1}{2} M_x^2 \right) \right)^{\frac{\gamma + 1}{2(\gamma - 1)}} \right]}
\]

\[
X_x = \frac{R_{\text{exit}} - R_x}{\tan(\nu_{\text{exit}} - \nu_x + \mu_x)}
\]
Apply Method of Characteristics to Aerospike Nozzle \(^{(10)}\)

Exit pressure expands to ambient condition
Apply Method of Characteristics to Aerospike Nozzle (11)

\[ \mu_x = \sin^{-1}\left( \frac{1}{M_x} \right) \]

“Mach lines” converge on cowl lip
Thrust Calculation

• This algorithm works for both Design and Off-design configuration where Altitude Greater than Design Condition (8)

\[ F_{total} = F_{throttle} + F_{spike} \]

\[ F_{throttle} = \left[ \dot{m} \cdot V_{throttle} + (p_{throttle} - p_\infty) \cdot A^* \right] \cdot \sin \delta_{throttle} \]

\[ \rightarrow F_{throttle} = \left[ \dot{m} \cdot \sqrt{\gamma \cdot R_g \cdot T^*} + (p^* - p_\infty) \cdot A^* \right] \cdot \sin \delta_{throttle} \]

\[ T^* = \frac{T_0}{(\gamma + 1)/2} \rightarrow p^* = \frac{P_0}{\left[(\gamma + 1)/2\right]^{\gamma/\gamma-1}} \]
Thrust Calculation

- Impulse Thrust at Throat exit

\[ F_{\text{throat}} = \left[ \dot{m} \cdot V_{\text{throat}} + (p_{\text{throat}} - p_\infty) \cdot A^* \right] \cdot \sin \delta_{\text{throat}} \]

\[ \rightarrow F_{\text{throat}} = \left[ \dot{m} \cdot \sqrt{\gamma \cdot R_g \cdot T^*} + (p^* - p_\infty) \cdot A^* \right] \cdot \sin \delta_{\text{throat}} \]

\[ T^* = \frac{T_0}{(\gamma+1)/2} \rightarrow p^* = \frac{P_0}{\left[ (\gamma+1)/2 \right]^{\gamma/(\gamma-1)}} \]
Thrust Calculation \((2)\)

Calculate ramp pressure force

\[
\delta F_j = \left( \frac{p_j + p_{j+1}}{2} - p_\infty \right) \cdot dA_j \cdot \sin \theta
\]

From geometry

\[
dA_j = \frac{(X_{j+1} - X_j)}{\cos \theta_j} \cdot 2\pi \cdot \left( \frac{R_j + R_{j+1}}{2} \right)
\]

Substitute

\[
\delta F_j = \left( \frac{p_j + p_{j+1}}{2} - p_\infty \right) \cdot dA_j \cdot \sin \theta = \left( \frac{p_j + p_{j+1}}{2} - p_\infty \right) \frac{(X_{j+1} - X_j)}{\cos \theta_j} \pi \cdot (R_j + R_{j+1}) \cdot \sin \theta_j
\]

From Geometry

\[
\left( X_{j+1} - X_j \right) \cdot \tan \theta_j = \left( R_j - R_{j+1} \right)
\]
Thrust Calculation \((3)\)

\[
(X_{j+1} - X_j) \cdot \tan \theta_j = (R_j - R_{j+1})
\]

→ Difference of squares

\[
\delta F_j = \left( \frac{p_j + p_{j+1}}{2} - p_\infty \right) \cdot \pi \cdot (R_j - R_{j+1})(R_j + R_{j+1}) = \left( \frac{p_j + p_{j+1}}{2} - p_\infty \right) \cdot \pi \cdot (R_j^2 - R_{j+1}^2)
\]

\[
F_{ramp} = \sum_{j=0}^{N} \left( \frac{p_j + p_{j+1}}{2} - p_\infty \right) \cdot \pi \cdot (R_j^2 - R_{j+1}^2)
\]
Ramp Pressure Thrust

\[
F_{ramp} = \sum_{j=0}^{N} \left( \frac{p_j + p_{j+1}}{2} - p_\infty \right) \cdot \pi \cdot (R_j^2 - R_{j+1}^2)
\]
Collected Thrust Calculation

**Lift off**

**Vacuum (Space)**

\[
F_{\text{ramp}} = \sum_{j=0}^{N} \left( \frac{p_j + p_{j+1}}{2} - p_\infty \right) \cdot \pi \cdot \left( R_j^2 - R_{j+1}^2 \right)
\]

**Thrust from throat exit**

\[
F_{\text{throat}} = \left[ \dot{m} \cdot V_{\text{throat}} + \left( p_{\text{throat}} - p_\infty \right) \cdot A^* \right] \cdot \sin \delta_{\text{throat}}
\]

\[
\rightarrow F_{\text{throat}} = \left[ \dot{m} \cdot \sqrt{\gamma \cdot R_g \cdot T^* + \left( p^* - p_\infty \right) \cdot A^*} \right] \cdot \sin \delta_{\text{throat}}
\]

\[
T^* = \frac{T_0}{(\gamma + 1)/2} \quad \Rightarrow \quad p^* = \frac{P_0}{\left[ (\gamma + 1)/2 \right]^{\gamma}}
\]
Off Design Operation

\[ \theta_{\text{expansion}} = \begin{cases} > 0 & \text{At Higher Altitude} \\ = 0 & \text{At Design} \\ < 0 & \text{At Lower Altitude} \end{cases} \]
• Shadowgraph flow visualization of an ideal isentropic spike at
  (a) low altitude and
  (b) high altitude conditions

[from Tomita et al, 1998]

Credit: Aerospace web
Off Design Operation (1)
Altitude Greater than Design Condition

Calculating the Off-design conditions

- Throat exit expands to ambient conditions’ due to unconstrained flow

- Use Isentropic Flow laws to calculate effective expansion Mach number .. as flow “turns corner”

\[ M_{\text{expansion}} = \sqrt{\frac{2}{\gamma - 1}} \left[ \left( \frac{P_0}{P_{\text{amb}}} \right)^{\frac{\gamma - 1}{\gamma}} - 1 \right] \]
Off Design Operation (2)
Altitude Greater than Design Condition

- calculate turning angle

\[ M_{\text{expansion}} = \sqrt{\frac{2}{\gamma - 1}} \left[ \left( \frac{P_0}{P_{\text{amb}}} \right)^{\frac{\gamma - 1}{\gamma}} - 1 \right] \]

\[ \theta_{\text{expansion}} = \theta_{\text{expansion}} + \left( \theta_{\text{expansion}} - \theta_{\text{throat}} \right) \]

\[ \theta_{\text{expansion}} = \nu_{\text{expansion}} + \theta_{\text{throat}} \]

\[ \theta_{\text{expansion}} = \nu_{\text{expansion}} + \delta_{\text{throat}} - 90^0 \]

\[ \theta_{\text{expansion}} = \nu_{\text{expansion}} + \left( 90^0 - \nu_{\text{exit}} \right) - 90^0 \]

\[ \theta_{\text{expansion}} = \nu_{\text{expansion}} - \nu_{\text{exit}} \]

@ design condition
Off Design Operation
Altitude Greater than Design Condition (3)

\[ \theta_{\text{expansion}} = \nu_{\text{expansion}} - (\nu_{\text{exit}})_{\text{design condition}} \]

\[ \nu_{\text{expansion}} > (\nu_{\text{exit}})_{\text{design condition}} \]
Off Design Operation  Altitude Greater than Design Condition (4)

“Left” characteristics lines … intersect expansion slip line

\[
\begin{bmatrix}
\theta \\
M \\
\nu_{\text{expansion}}
\end{bmatrix}
\rightarrow \theta_x - \nu_x = \theta_{\text{expansion}} - \nu_{\text{expansion}}
\]

\[
\rightarrow \begin{bmatrix} \nu_x \end{bmatrix} = \begin{bmatrix} \nu_{\text{expansion}} \end{bmatrix} - \begin{bmatrix} \theta_{\text{expansion}} \end{bmatrix} + \begin{bmatrix} \theta_x \end{bmatrix}
\]

What about spike surface Mach numbers?

Expansion slip line
Off Design Operation Altitude Greater than Design Condition (5)

“Left” characteristics lines … Intersect expansion line

\[ \theta_x - \nu_x = \theta_{\text{expansion}} - \nu_{\text{expansion}} \]

\[ \nu_x = \nu_{\text{expansion}} - \theta_{\text{expansion}} + \theta_x \]

\[ \nu_x = \nu_{\text{expansion}} - \left( \nu_{\text{expansion}} - \left( \nu_{\text{exit}} \right)_{\text{design condition}} \right) + \theta_x \]

\[ \nu_x = \left( \nu_{\text{exit}} \right)_{\text{design condition}} + \theta_x \]

… Which is our original spike contour prescription!

See … Slide 5
Off Design Operation  Altitude Greater than Design Condition

…Ramp Surface Pressure and Mach Numbers Unaffected by nozzle operating at higher-than-design altitude
Off-Design Algorithm Summary

Altitude Greater than Design Condition (6)

\[
\begin{align*}
 P_0 &= \text{operating chamber pressure} \\
 p_{\text{amb}} &= \text{ambient pressure at operating altitude}
\end{align*}
\]

… Expansion Line Mach Number and Flow Angle

\[
M_{\text{expansion}} = \sqrt{\left(\frac{2}{\gamma - 1}\right) \left(\frac{P_0}{p_{\text{amb}}}\right)^{(\gamma - 1)/\gamma} - 1}
\]

\[
\theta_{\text{expansion}} = \nu_{\text{expansion}} - \left(\nu_{\text{exit}}\right)_{\text{design condition}}
\]

\[
\nu_x = \nu_{\text{expansion}} - \theta_{\text{expansion}} + \theta_x \rightarrow \begin{bmatrix} M_x \\ p_x \end{bmatrix}
\]
Aerospike Nozzle Endo-Atmospheric Compensation (2)

High Altitude Aerodynamics

Pressure, Mach Distribution on Spike is Unchanged

- Thruster flow discharges to ramp
- Expansion waves turn flow axially
- No compression waves exist - all flow turning done by expansion waves
- Nozzle behaves like a bell
Altitude Greater than Design Condition (7)

\[ \nu_x = (\nu_{exit})_{design} + \theta_x \]

Mach Number on Spike

Mach Number Along Spike is Unaltered From Design Condition
Altitude Greater than Design Condition (8)

Significant gain in pressure thrust
Altitude Greater than Design Condition (9)

Over-expanded Conditions
Altitude Greater than Design Condition (10)

- Mach Numbers on Spike Surface Unaffected
Off Design Operation
@ Altitude Lower than Design Condition

External Pressure Compresses Flow Field Resulting in Higher Spike Pressures

What about spike surface Mach numbers?

\[ M = M_{\text{expansion}} \]

Expansion slip line

What about spike surface Mach numbers?
Off Design Operation
Altitude Lower than Design Condition (2)

- At below design pressure ratio, the flow in the plug nozzle is radically different from that in a conventional nozzle. The expansion occurring at the cowl-lip would proceed only up to the ambient pressure $p_a$ and not all the way down to the design exit pressure $p_e$. 
Off Design Operation
Altitude Lower than Design Condition (2)

\( p_\infty > p_\infty,\text{design} \) (overexpanded)
- \( p_{\text{spike}} = p_\infty \) before plug ends
- weak shocks and expansions downstream

Average Nozzle Pressure greater Than Freestream, No suction effects or separation like on conventional nozzle
Aerospike Nozzle Endo-Atmospheric Compensation

Low Altitude Aerodynamics

- Thruster flow discharges to ramp
- Expansion waves turn flow axially
- Ramp curves, turns flow axially (at low altitudes)
- Turning causes compression wave from (1) to (2) - nozzle pressure increases
- Compression wave reflects off boundary causing expansion waves
- Flow crosses expansion waves in (2) - nozzle pressure decreases
- Ramp continues to curve and turn flow
- Process repeats (2) to (3)

Averaged ramp pressure > $P_\infty$

... no losses or separation of flow, even with high expansion ratio nozzle
Wave Reflection Rules for Solid and Free Boundaries

1. Waves Incident on a Solid Boundary Reflect in a Like manner;
   Compression wave reflects as compression wave, expansion wave reflects as expansion wave

2. Waves Incident on a Free Boundary Reflect in an Opposite manner;
   Compression wave reflects as expansion wave, expansion wave reflects as compression wave

• Anderson, Chapter 4 pp. 152-164
Wave reflections from solid/free boundary

- Solid Boundary

- Free Boundary

Shock Wave Incident on Constant Pressure Boundary
Wave reflections from a free boundary

Shock Wave Incident on Constant Pressure Boundary
Wave reflections from a free boundary

(2) $p_1 < p_\infty$

(2) $p_2 < p_1$

(2) $p_3 = p_\infty > p_2$

Reflection of an expansion
Wave Incident on Constant Pressure Boundary

Coalesced Compression Wave
Off Design Operation

Flow phenomena of a plug nozzle with full length at different pressure ratios $p_c/p_{amb}$, off-design (top, bottom) and design (center) pressure ratio.
Off Design Operation

At Higher Altitude

\[ \nu_{\text{expansion}} > \nu_{\text{exit}} \rightarrow \theta_{\text{expansion}} > 0 \]

At Design

\[ \nu_{\text{expansion}} = \nu_{\text{exit}} \rightarrow \theta_{\text{expansion}} = 0 \]

At Lower Altitude

\[ \nu_{\text{expansion}} < \nu_{\text{exit}} \rightarrow \theta_{\text{expansion}} < 0 \]
Aerospike Nozzle With Shock Diamonds
Aeropike Nozzle Endo-Atmospheric Compensation (3)

SlipStream effects

- Air streaming over cowl lowers local pressure - \( P_{\text{Local}} < P_{\infty} \)
- Exhaust plume expands beyond still air case
- Expansion and compression wave systems move aft from still air case

- Resulting recompression Delays Nozzle separation

Averaged ramp pressure > \( P_{\infty} \)
... no losses or separation of flow, even with high expansion ratio nozzle
Compare Aerospike to Minimum Length Nozzle with Same Expansion Ratio

**Minimum Length Nozzle 4.546 cm** (including convergent section)

**Full Conical Aerospike Nozzle 8.03 cm length**
Compare Aerospike to Minimum Length Nozzle with Same Expansion Ratio

At design condition

Superior Aerospike Performance at Design Condition
Compare Aerospike to Minimum Length Nozzle with Same Expansion Ratio

Even more improved aerospike performance at Altitude
Compare Aerospike to Minimum Length Nozzle .... (4)

4.546 cm length

8.03 cm length

Spike wins on Everything Except length
Effects of Spike Truncation

NASA DFRC (Trong Bui)

Example Base Integration

• Long Beach State
  (Eric Besnard)
Effects of Spike Truncation (2)

Flow phenomena of a plug nozzle truncated central body (right column) at different pressure ratios $pc / pam b$, off-design (top, bottom) and design (center) pressure ratio.

Separated Base Area

(low pressure produces drag)
Effects of Spike Truncation (2)

*Pressure Ramp Thrust Terminates at Spike Truncation Point*
Effects of Spike Truncation

- Primary Loss is base drag
Effects of Spike Truncation (3)

Lift off

Vacuum (Space)

\[ F_{\text{base}} = \left( p_{\text{base}} - p_{\infty} \right) \cdot \pi \cdot R_{\text{base}}^2 \]

\[ \nu_{\text{base}} = \nu_{\text{trun}} + \left( 90^\circ - \theta_{\text{trun}} \right) \rightarrow M_{\text{base}} \rightarrow p_{\text{base}} \]
Effects of Spike Truncation (7)

Effects are not as dramatic as one would think!

... At higher altitudes truncation hurts you less
terms of base pressure insensitivity to ambient pressure - can even occur when a closed recirculation bubble forms downstream of the base.

Fig. 5.2: Normalised base pressure versus PR. Experimental data from a linear aerospikke nozzle cold gas model \(^2\) compared with Hagemann's assumption \(^8\).

![Graph showing base pressure versus PR]

Fig. 5.3: Subsonic open wake flow pattern downstream of a truncated plug nozzle base.

Furthermore, it is also possible that the separated inner shear layer does not reattach itself downstream of the base surface. This could be the case of a truncated plug nozzle with a large base surface comparatively to the annular section of the incoming supersonic jet, see Fig. 5.3. In such a base flow pattern, there is a **subsonic open wake** - really opened this time in terms of fluid mechanics-, and the base pressure is close to the ambient pressure. Thus, in that case, the problem is not to determine the base pressure, but to predict at which value of \(p_b/p_a\) the transition 'opening-closing' of the recirculation bubble occurs.

If the base flow pattern is a closed recirculation bubble, whatever the plug nozzle wake regime - closed or open-, then the determination of the base pressure in supersonic regime is submitted to the same flow physics, namely the physics of the **Supersonic Turbulent Flow Reattachment**. Motivated by the base drag prediction not only for truncated plug nozzles but mainly for projectile and missile applications, the supersonic base flow physics have been extensively investigated in the world since the 50's. Thanks to many investigations performed downstream of the base of two-dimensional backward-facing steps, it has been derived analytical, pure-empirical and theorectico-empirical models as those presented below.

**BASE PRESSURE PREDICTION**

**Pure empirical relationships**

Fick et al.\(^{20}\) have evaluated empirical relations of the base pressure versus constant incoming Mach number \(M_e\) and specific heat ratio. A comparison with the few available measurements showed that the two empirical relations issued from Ref. 30, see Eqs (5.1) and (5.2) below, failed to produce reliable results.

\[
P_b = \frac{0.846 p_e}{M_e^3} \quad (5.1)
\]

\[
p_b = p_e \left( 1 - 0.715 \gamma M_e^{2.3} - 0.92 M_e^2 - 0.03 \right) \quad (5.2)
\]

A slightly better agreement was found\(^{20}\) notably for 12-16% plug lengths if it was assumed that the base pressure results from a very simple averaging between pressure \(p_e\) at the truncated nozzle exit and pressure \(p_d\) at the exit of the hypothetical design full-length plug, as written below:

\[
p_b = k(p_e + p_d) \quad \text{with } k = 0.5 \quad (5.3)
\]

When applied to linear aerospikke nozzles, it was found\(^{21}\) that the constant \(k\) had to be changed from 0.5 to 0.3 according to measured data.

Derived from cylinders and cones, an original empirical base-pressure model\(^1\) has been changed in Ref. 20 by setting an exponent which should take into account the negative angle of the flow incoming the base region. For cold flow tests, and setting the exponent at 0.35, agreement with measured base pressure has been attainable with the 'conical-approximation' equation below,

\[
P_b = p_e \left( 0.025 + \frac{0.906}{1 + \frac{\gamma - 1}{2} M_e^2} \right)^{0.35} \quad (5.4)
\]

The empirical model derived in Re. 20 from a cylinder embedded in supersonic flow also gives a good agreement if Mach number \(M_e\) and a sonic pressure ratio are introduced, thus we obtain the "cylindrical-approximation" equation below,

\[
P_b = p_e M_e \left( \frac{2}{\gamma + 1} \right)^{\frac{\gamma}{\gamma - 1}} \left( 0.05 + \frac{0.967}{1 + \frac{\gamma - 1}{2} M_e^2} \right) \quad (5.5)
\]

Onofri et al.\(^{32}\) have also evaluated Eqs. (5.4) and (5.5), and another plug nozzle base model proposed by
Calculations Using Prandtl/Meyer Expansion Theory

Base drag coefficient vs. Plug percent truncation for chamber pressures between 2 and 15 atm, at sea level

Percent Truncation, \((1-L)/L_{\text{max}}\)

Base Drag Coefficient, \(C_{Fb}\)

Percentage of Ideal Length, \(L/L_{\text{max}}\)

Thanks Matthew Wilson!
Calculations Using Prandtl/Meyer Expansion Theory

Base drag coefficient vs. Plug percent truncation for chamber pressures between 2 and 15 atm, at 10 km

Percent Truncation, \((1-L)/L_{\text{max}}\)

Base Drag Coefficient, \(C_{\text{fb}}\)

Percent of Ideal Length, \(L/L_{\text{max}}\)

Thanks Matthew Wilson!
Truncated Spike Comparison to Minimum Length Nozzle

Spike Truncated at Length of Minimum Length Conventional Nozzle

“Break Even Length”

“Still Significant Net Gain from Spike”
Truncated Spike Comparison to Minimum Length Nozzle

Spike is Significantly Shorter Than Minimum Length Conventional Nozzle

"Break Even Performance"

"Only 1/3rd of Min. Nozzle length"
thrust vector is concerned, can be shown to be additive. A typical set of data showing the variation of effective thrust vector angle with overpressure in the higher pressure combustor cell is shown in Fig. 9.

Application to Liquid Propellant Engine Design

The plug nozzle can be combined with annular combustors to form attractive liquid propellant propulsion system configurations. Fig. 10 represents an example of a plug-type turbopumped liquid propellant engine. The combustors are located around the base of the nozzle, and, in this particular unit, the turbine exhaust is brought out through the center of the nozzle. Such engines can be designed extremely short and compact to produce a well-packaged propulsion system. In fact, studies indicate that for identical thrust levels, the plug-type engines would be about half the length of conventional configurations as shown in Fig. 11 for a 1,500,000-lb thrust level. This makes them attractive not only for first-stage booster applications, but also for medium thrust size upper-stage propulsion systems. Similarly, the weight of the plug-type engine compares favorably with that of the conventional unit.

Application to Solid Rocket Engines

The problems and advantages of plug nozzle application to solid rocket engines differ considerably from those of liquid engines. Not all of the major advantages of liquid propellant plug nozzle engines such as thrust vector control, scaling, combustion stability and thrust structure simplification can be directly transposed to solid engine application. Gains unique to the application of plug nozzles to solid engines exist, but are generally not as compelling. Consequently, the advisability of converting to the plug from the cluster of four DeLaval nozzles typically employed in modern solid propellant rockets is not particularly clear-cut and depends to a large extent on the specific missile application intended.

Fig. 6 Schematic illustration of pressure distribution along a fixed plug nozzle at various operating pressure ratios

Fig. 7 Theoretical loss in performance caused by replacement of lower plug

Fig. 8 Plug profiles for isentropic expansion and for shortened versions terminating with conical contours

Fig. 9 Effect of per cent overpressure on effective thrust vector angle

Fig. 10 Plug nozzle engine assembly

Fig. 11 Size of plug nozzle engine compared with that of conventional engine
Very High Expansion Ratio Spike Nozzles

- Characteristic Line “bends backward” near sonic line (cowl exit)
- Cowl exit momentum-thrust is negative!
- Very inefficient approach
Better Approach is to use supersonic cowl nozzle and position so exit mach and angle lines along spike characteristic line.
Very High Expansion Ratio Spike Nozzles (3)
Very High Expansion Ratio Spike Nozzles (4)

- Axi-symmetric supersonic cowl design
Very High Expansion Ratio Spike Nozzles (5)

Figure 1 - Size comparison of a recently studied NOFB monopropellant lunar lander ascent engine for Altair program using a conventional bell nozzle vs. a 12-clustered aerospike plug. Far right is the original 15,000 lbf H_2/O_2 aerospike plug engine developed and tested by Rocketdyne in 1974 [2,4].