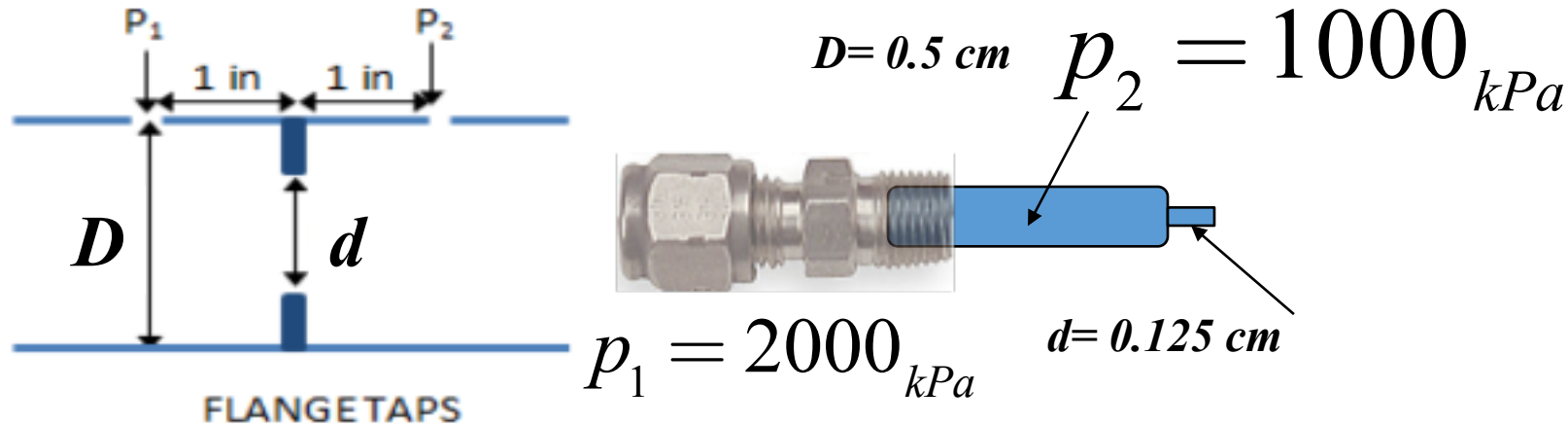


Example Incompressible Injector Calculation

Injector Geometry Model



$$\tilde{P}_{0_c} = p_2$$

Initial Chamber Pressure Guess Downstream of Injector

Gamma, Ox
1.4

Incompressible
Injector Properties 2

Injector Port
Diameter, cm (d)

0.125

Injector Tube
Diameter, cm (D)

0.5

Upstream Pressure
(P1), kPa

2000

Down Stream Pressure
(P1), kPa

1000

Initial Cv Value

1

Tinlet, C

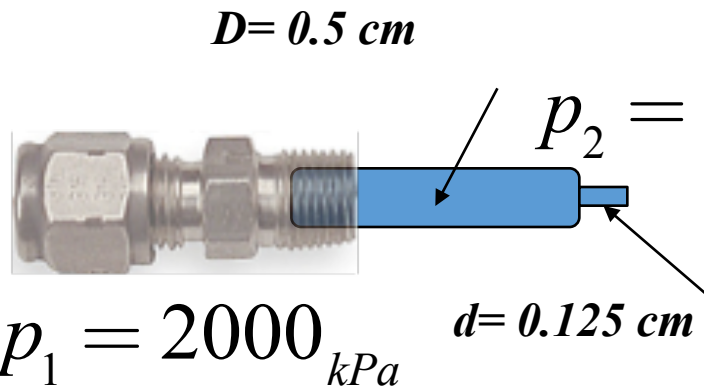
20

MW, kg/kg-mol

32

Incompressible Massflow Calculation

Injector Geometry Model



$$\rho_{inc} = \frac{P_1}{R_g \cdot T_1} = \frac{2000 \frac{1000}{8314.4612}}{32 (20 + 273.15)} = 26.2577 \text{ kg / m}^3$$

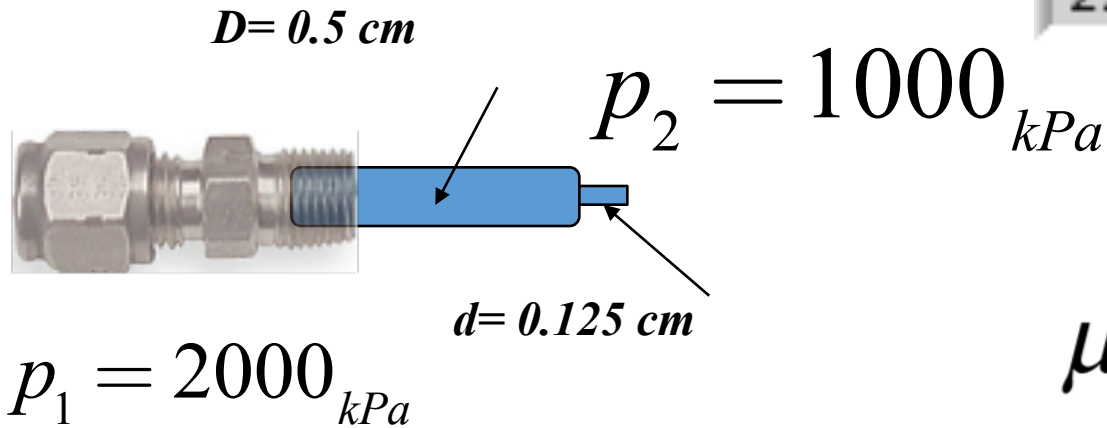
• Assume Initial Cv=1

$$\dot{m} = \left(\frac{Cd \cdot A_2}{\sqrt{1 - \left(\frac{A_2}{A_1}\right)^2}} \right) \sqrt{2 \cdot \rho \cdot (p_1 - p_2)} = \frac{\left(\frac{0.125^2 \frac{2\pi}{4}}{10000} \right) (2 \cdot 26.2577 (2000 - 1000) 1000)^{0.5} 1000}{\left(1 - \left(\frac{0.125}{0.5}\right)^4 \right)^{0.5}} = 8.91052 \text{ g / sec}$$

Calculate GOX Viscosity

Southerland's Law for GOX VISCOSITY

Injector Geometry Model



$\mu_0, \text{ Nt-sec/m}^2$

2.018E-5

$C_s, \text{ deg K}$

127

$T_0, \text{ deg K}$

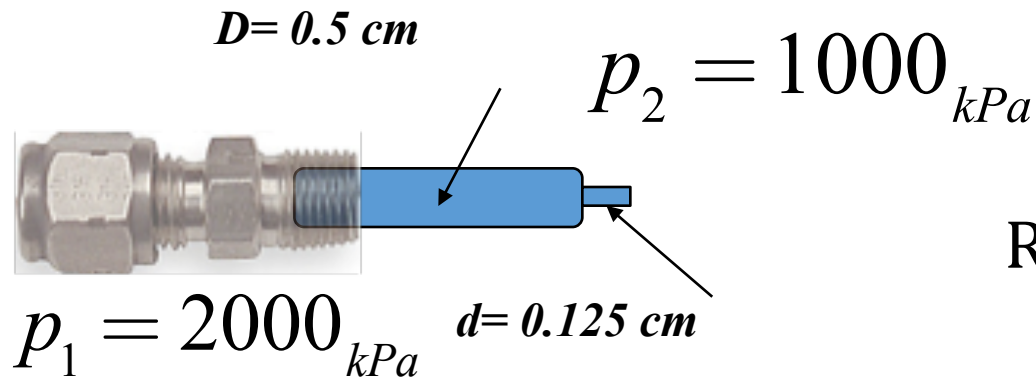
292.25

$$\mu(T_0) = \mu_s \left(\frac{T_0}{T_s} \right)^{3/2} \left(\frac{T_s + C_s}{T_0 + C_s} \right) =$$

$$2.018 \cdot 10^{-5} \left(\frac{20 + 273.15}{292.25} \right)^{3/2} \left(\frac{292.25 + 127}{20 + 273.15 + 127} \right) = 2.02299\text{e-}05 \text{ Pa-sec}$$

Calculate Injector Inlet Flow Reynolds Number

Injector Geometry Model



→ Calculate Reynolds number

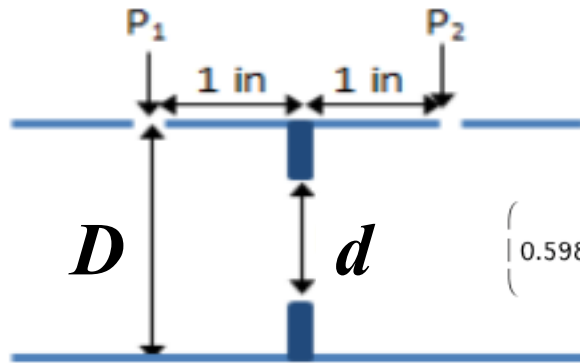
$$\begin{aligned}
 R_{e_{D_1}} &= \frac{\rho_1 \cdot V_1 \cdot D_1}{\mu_1} = \frac{\dot{m} \cdot D_1}{A_1 \cdot \mu_1} = \frac{\dot{m} \cdot D_1}{\frac{\pi}{4} D_1^2 \cdot \mu_1} = \frac{4 \cdot \dot{m}}{\pi \cdot D_1 \cdot \mu_1} = \\
 &= \frac{4 \cdot 8.91051}{\pi \cdot 0.5 \cdot 2.02299 \cdot 10^{-5}} = 112163
 \end{aligned}$$

Calculate Injector Incompressible Discharge Coefficient

$$R_{e_{D_1}} = 112163$$

Flange Tap: $(D, d \rightarrow \text{inches})$

$$C_{v_i} = \left[0.598 + 0.468 \cdot \left(\frac{d}{D}\right)^4 \left(1 + 10 \cdot \left(\frac{d}{D}\right)^8 \right) \right] \cdot \sqrt{1 - \left(\frac{d}{D}\right)^4} + \left(0.87 + 8.1 \cdot \left(\frac{d}{D}\right)^4 \right) \cdot \sqrt{\left(1 - \left(\frac{d}{D}\right)^4\right) \frac{1}{R_{e_d}}} =$$



$$\left(0.598 + 0.468 \left(\frac{0.125}{0.5}\right)^4 \left(1 + 10 \left(\frac{0.125}{0.5}\right)^8 \right) \right) \left(1 - \left(\frac{0.125}{0.5}\right)^4 \right)^{0.5} + \left(0.871 + 8.1 \left(\frac{0.125}{0.5}\right)^4 \right) \left(\left(1 - \left(\frac{0.125}{0.5}\right)^4 \right) \frac{1}{112163} \right)^{0.5} = 0.601346$$

FLANGE TAPS

$D = 0.5 \text{ cm}$
 0.19685 in

$d = 0.125 \text{ cm}$
 $= 0.0492126 \text{ in}$

$$\rightarrow \text{Calculate } C_{d_i} \rightarrow \beta = \sqrt{1 - \left(\frac{D_2}{D_1}\right)^4} \rightarrow C_{d_i} = \frac{C_{v_i}}{\sqrt{1 - \left(\frac{D_2}{D_1}\right)^4}} =$$

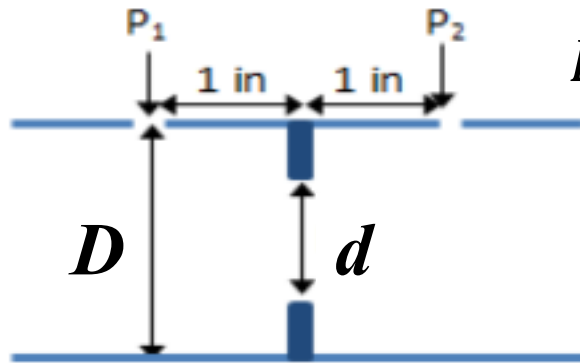
$$\frac{0.601346}{\left(1 - \left(\frac{0.125}{0.5}\right)^4 \right)^{0.5}} = 0.60251$$

Iterate Injector Incompressible Discharge Coefficient

$$R_{e_{D_1}} = 112163$$

Return to Slide 2 with New value for Cv_i

Iterate Until $\left| \frac{(C_{di})_{j+1} - (C_{di})_j}{(C_{di})_j} \right| < \epsilon$



FLANGETAPS

$D = 0.5 \text{ cm}$
 0.19685 in

$d = 0.125 \text{ cm}$
 $= 0.0492126 \text{ in}$

ASME
CV, CD FINAL
VALUES

Cvi Value 2	Mdot, g/sec
0.602115	
Cdi Value	5.37566
0.603295	

Cv, Cd Iteration Values, Incompressible

0			
Cvi Value 2	Cvi Value 2	Cvi Value 2	Cvi Value 2
0.601343	0.602117	0.602115	0.602115
Cdi Value	Cdi Value	Cdi Value	Cdi Value
0.602521	0.603297	0.603295	0.603295
Error	Error	Error	Error
0.397479	0.00128816	3.69922E-6	1.06266E-8

Now Correct for Compressibility

$$P_0 = \frac{\left[\left(\frac{A_1}{A_2} \right)^2 \cdot (p_1)^{\frac{\gamma+1}{\gamma}} - (p_2)^{\frac{\gamma+1}{\gamma}} \right]^{\frac{\gamma}{\gamma-1}}}{\left[\left(\frac{A_1}{A_2} \right)^2 \cdot (p_1)^{\frac{2}{\gamma}} - (p_2)^{\frac{2}{\gamma}} \right]^{\frac{\gamma}{\gamma-1}}} = \frac{\left[\left(\frac{D_1}{D_2} \right)^4 \cdot (p_1)^{\frac{\gamma+1}{\gamma}} - (p_2)^{\frac{\gamma+1}{\gamma}} \right]^{\frac{\gamma}{\gamma-1}}}{\left[\left(\frac{D_1}{D_2} \right)^4 \cdot (p_1)^{\frac{2}{\gamma}} - (p_2)^{\frac{2}{\gamma}} \right]^{\frac{\gamma}{\gamma-1}}} = \frac{\left[\left(\frac{0.5}{0.125} \right)^4 \cdot 2000^{\frac{1.4+1}{1.4}} - 1000^{\frac{1.4+1}{1.4}} \right]^{\frac{1.4}{1.4-1}}}{\left[\left(\frac{0.5}{0.125} \right)^4 \cdot 2000^{2/1.4} - 1000^{2/1.4} \right]^{\frac{1.4}{1.4-1}}}$$

$$= 2001.83 \text{ kPa} \quad \frac{\tilde{P}_{0c}}{P_0} = \frac{p_2}{P_0} = \frac{1000}{2001.83} = 0.499543$$

$$r = \frac{\tilde{P}_{0c}}{P_0} = \frac{1000}{2001.83} = 0.499543 < r^* = \left(\frac{2}{\gamma+1} \right)^{\frac{\gamma}{\gamma-1}} = 0.528282$$

Example Compressible Injector Calculation

Proceedings of the Institute of Mechanical Engineers
On the Flow of a Compressible Fluid through Orifices
D. A. Jobson,
First Published June 1, 1955 Research Article
https://doi.org/10.1243/PIME_PROC_1955_169_077_02

$$\text{Subcritical Flow: } \left(\frac{p}{P_0}\right) > \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}$$

$$r = \frac{p}{P_0}$$

$$f = \frac{1}{(C_d)_{inc}} - \frac{1}{2(C_d)_{inc}^2}$$

$$K_n = \sqrt{\frac{2\gamma}{\gamma-1} \cdot r^{\frac{2}{\gamma}} \left[1 - r^{\frac{\gamma-1}{\gamma}}\right]}$$

$$C_d = \frac{1 - \sqrt{1 - f \cdot \left(2 \cdot r^{\frac{1}{\gamma}}\right)^2 \left(\frac{1-r}{K_n^2}\right)}}{2 \cdot f \cdot r^{\frac{1}{\gamma}}}$$

$$\rightarrow \dot{m} = K_n \cdot C_d \cdot A \cdot \sqrt{P_0 \cdot \rho_0}$$

Supercritical Flow:

$$\left(\frac{p}{P_0}\right) \leq \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} \quad r^* = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}$$

$$f = \frac{1}{(C_d)_{inc}} - \frac{1}{2(C_d)_{inc}^2}$$

$$K_n = \sqrt{\frac{2\gamma}{\gamma-1} \cdot r^{*\frac{2}{\gamma}} \left[1 - r^{*\frac{\gamma-1}{\gamma}}\right]} = \sqrt{\gamma \cdot \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}}$$

$$C_d = \left(\frac{1}{2 \cdot f \cdot r^{*\frac{1}{\gamma}}}\right) \cdot \left[\left(1 + \frac{\left(r^* - \left(\frac{p}{P_0}\right)\right) \cdot r^{*\frac{1}{\gamma}}}{K_n^2}\right) - \sqrt{\left(1 + \frac{\left(r^* - \left(\frac{p}{P_0}\right)\right) \cdot r^{*\frac{1}{\gamma}}}{K_n^2}\right)^2 - \frac{\left(2 \cdot r^{*\frac{1}{\gamma}}\right)^2 \cdot \left(1 - \left(\frac{p}{P_0}\right)\right) \cdot f}{K_n^2}} \right]$$

$$\rightarrow \dot{m} = K_n \cdot C_d \cdot A \cdot \sqrt{P_0 \cdot \rho_0}$$

Now Correct for Compressibility

Supercritical Flow

$$r = \frac{\tilde{P}_{0_c}}{P_0} = \frac{1000}{2001.83} = 0.499543 < r^* = \left(\frac{2}{\gamma + 1} \right)^{\frac{\gamma}{\gamma - 1}} = 0.528282$$

$$f = \frac{1}{(C_d)_{inc}} - \frac{1}{2(C_d)_{inc}^2} = \frac{1}{0.603295} - \frac{1}{2(0.603295)^2} = 0.283805$$

$$K_n = \sqrt{\frac{2\gamma}{\gamma - 1} \cdot r^{*\frac{2}{\gamma}} \left[1 - r^{*\frac{\gamma - 1}{\gamma}} \right]} = \sqrt{\gamma \cdot \left(\frac{2}{\gamma + 1} \right)^{\frac{\gamma + 1}{\gamma - 1}}} = \left(1.4 \left(\frac{2}{1.4 + 1} \right)^{\frac{1.4 + 1}{1.4 - 1}} \right)^{0.5} = 0.684731$$

Correct for Compressibility (2)

$$C_d = \left(\frac{1}{2 \cdot f \cdot r^{*\frac{1}{\gamma}}} \right) \cdot \left[\left\{ 1 + \frac{\left(r^* - \left(\frac{p}{P_0} \right) \right) \cdot r^{*\frac{1}{\gamma}}}{K_n^2} \right\} - \sqrt{ \left\{ 1 + \frac{\left(r^* - \left(\frac{p}{P_0} \right) \right) \cdot r^{*\frac{1}{\gamma}}}{K_n^2} \right\}^2 - \frac{\left(2 \cdot r^{*\frac{1}{\gamma}} \right)^2 \cdot \left(1 - \left(\frac{p}{P_0} \right) \right) \cdot f}{K_n^2} } \right] =$$

$$\frac{\left(1 + \left(\frac{(0.528282 - 0.499543)}{0.684731^2} \right) 0.528282^{1/1.4} \right) - \left(\left(1 + \left(\frac{(0.528282 - 0.499543)}{0.684731^2} \right) 0.528282^{1/1.4} \right)^2 - 4 \cdot 0.283804 (0.528282^{2/1.4}) \frac{(1 - 0.499543)}{0.684731^2} \right)^{0.5}}{2 \cdot 0.283804 (0.528282^{1/1.4})}$$

=0.74834

This value goes into your Hybrid Compressible Simulation

Incompressible
Injector Properties 2

Injector Port Diameter, cm (d)
0.125

Injector Tube Diameter, cm (D)
0.5

Upstream Pressure (P1), kPa
2000

Down Stream Pressure (P1), kPa
1000

Initial Cv Value
1

Tinlet, C
20

MW, kg/kg-mol
32

Stagnation Iteration Values

P0, kPa	T0, K	Initial Mdot, g/sec	
2000	293.15	5.37566	
MTube	MPort	Rho0, kg/M ³	P/PO 2
0	0	26.2577	0.5

ASME
CV, CD FINAL
VALUES

Cvi Value 2	Mdot, g/sec
0.602115	5.37566
Cdi Value	
0.603295	

Orifice Properties

A1, cm ²	A1/A2
1.9635E-5	16
A2, cm ²	Beta (d/D)
1.22718E-6	0.25

Choked flow?

Iteration Parameters

Max Its	Convergence Error
10	1E-6

Cv, Cd Iteration Values, Incompressible

Cvi Value 2	Cvi Value 2	Cvi Value 2	Cvi Value 2
0.601343	0.602117	0.602115	0.602115
Cdi Value	Cdi Value	Cdi Value	Cdi Value
0.602521	0.603297	0.603295	0.603295
Error	Error	Error	Error
0.397479	0.0012881E	3.69922E-6	1.06266E-8

Fluid Properties Iteration Values

Molecular Weight kg/kg-mol	Rg, J/kg-K	Tube Velocity m/sec
32	259.827	17.2829
Gamma	Rho1, kg/M ³	Qvol m ² /sec
1.4	26.2577	0.00033934
Viscosity, Nt-sec/M ²	Tube Reynolds Number, ReD	
2.02299E-5	112163	

Flow Calculations

Port

Flow Calculations

Compressible

InCompressible

Compressibility Corrected Output Values

Cc Output	K, Output
0.748339	0.512412
Kn, Output	Mdot, g/sec
0.684731	4.56049

Gas select

H2O2

Nitrous Oxide

Air

Ammonia, NH3

Carbon Dioxide, CO

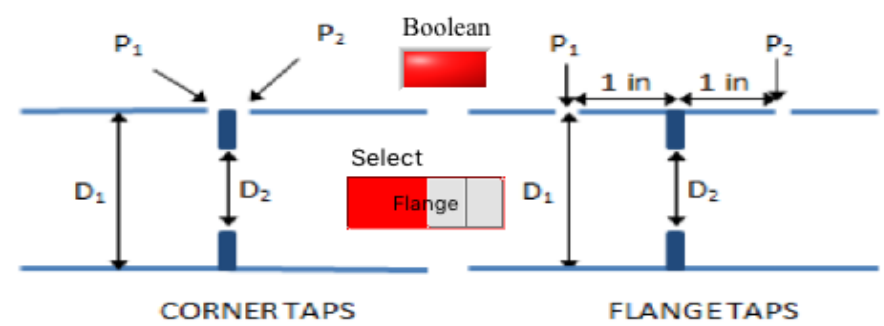
Carbon Monoxide, CO

Hydrogen, H2

Nitrogen, N2

Oxygen, O2

Sulfur Dioxide, SO2



CURRENT
ITERATION

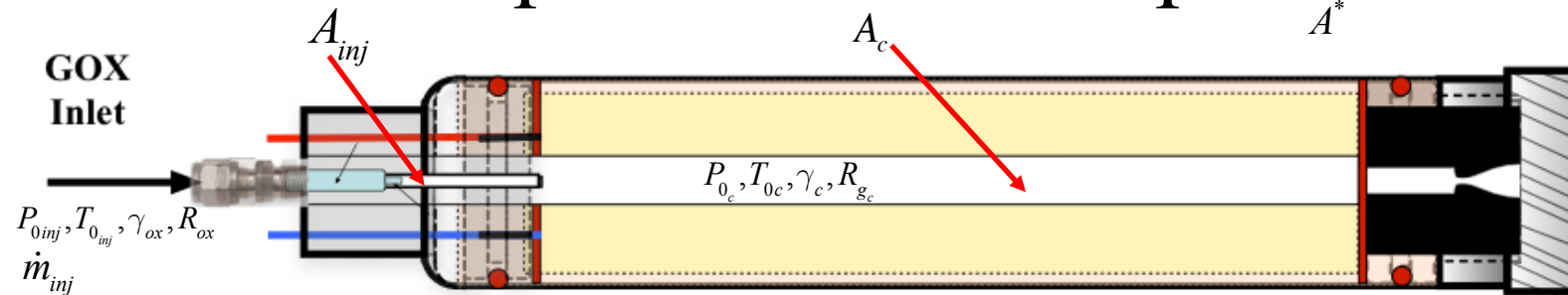
3

Intermediate Compressible Terms

f, Force Defect Coefficient	Cc Output
0.283804	0.748339
Kn, Subcritical	Cc, Subcritical
0.683493	0.749672
Kn, Supercritical	Cc, Supercritical
0.684731	0.748339

This value goes into your Hybrid Compressible Simulation

Hybrid Ballistic Equations for Compressible Oxidizer



Supercritical: $\left(\frac{P_{0inj}}{P_{0c}}\right) \geq \left(\frac{\gamma_{ox} + 1}{2}\right)^{\frac{\gamma_{ox}}{\gamma_{ox} - 1}}$

→ Injector Choked → $r_c = \left(\frac{2}{\gamma_{ox} + 1}\right)^{\frac{\gamma_{ox}}{\gamma_{ox} - 1}}$

$$K_n = \sqrt{\gamma_{ox} \cdot \left(\frac{2}{\gamma_{ox} + 1}\right)^{\frac{\gamma_{ox} + 1}{\gamma_{ox} - 1}}} \rightarrow \dot{m}_{ox} = (K_n \cdot C_d \cdot A)_{inj} \cdot \frac{P_{0inj}}{\sqrt{R_{g_{ox}} T_{0inj}}}$$

$$C_d = \left(\frac{1}{2 \cdot f \cdot r_c^{\frac{1}{\gamma_{ox}}}}\right) \cdot \left[1 + \frac{\left(r_c - \left(\frac{P_{0c}}{P_{0inj}}\right)\right) \cdot r_c^{\frac{1}{\gamma_{ox}}}}{K_n^2} \right] \cdot \sqrt{1 + \frac{\left(r_c - \left(\frac{P_{0c}}{P_{0inj}}\right)\right) \cdot r_c^{\frac{1}{\gamma_{ox}}}}{K_n^2} - \frac{\left(2 \cdot r_c^{\frac{1}{\gamma_{ox}}}\right)^2 \cdot \left(1 - \left(\frac{P_{0c}}{P_{0inj}}\right)\right)}{K_n^2}} \cdot f}$$

Intermediate Compressible Terms

f, Force Defect Coefficient	Cc Output
0.283804	0.748339
Kn, Subcritical	Cc, Subcritical
0.683493	0.749672
Kn, Supercritical	Cc, Supercritical
0.684731	0.748339

Chamber Pressure:

$$\frac{\partial P_{0c}}{\partial t} = \frac{A_{burn} \cdot \dot{r}_{fuel}}{V_c} \left[\rho_{fuel} R_{gc} T_{0c} - P_{0c} \right] - P_{0c} \left[\frac{A^*}{V_c} \sqrt{\gamma_c R_{gc} T_{0c} \left(\frac{2}{\gamma_c + 1}\right)^{\frac{\gamma_c + 1}{\gamma_c - 1}}} + \frac{R_{gc} T_{0c}}{V_c} \cdot \left\{ (K_n \cdot C_d \cdot A)_{inj} \cdot \frac{P_{0inj}}{\sqrt{R_{g_{ox}} T_{0inj}}} \right\} \right]$$

Regression:

$$\dot{r}_{fuel} = \left(\frac{0.047}{\rho_{fuel} \cdot (P_r)^{2/3}}\right) \cdot \left(\frac{C_{P_c} \cdot (T_{0c} - T_{fuel,surf})}{h_{v,fuel}}\right) \cdot \left(\frac{\dot{m}_{ox}}{A_c}\right)^{4/5} \cdot \left(\frac{\mu_c}{L}\right)^{1/5}$$

$$\dot{m}_{fuel} = \rho_{fuel} \cdot A_{burn} \cdot \dot{r}_{fuel} = \rho_{fuel} \cdot \pi \cdot (D \cdot L)_{port} \cdot \dot{r}_{fuel} \rightarrow O/F = \frac{\dot{m}_{ox}}{\dot{m}_{fuel}} = \frac{(K_n \cdot C_d \cdot A)_{inj} \cdot \frac{P_{0inj}}{\sqrt{R_{g_{ox}} T_{0inj}}}}{\rho_{fuel} \cdot \pi \cdot (D \cdot L)_{port} \cdot \dot{r}_{fuel}}$$