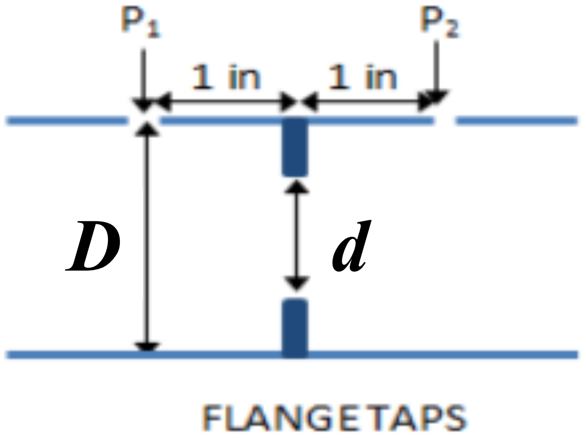
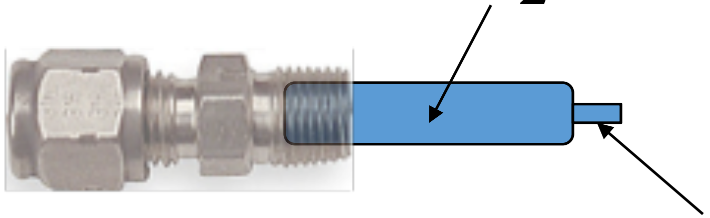
Example Incompressible Injector Calculation

Injector Geometry Model



D= 0.5 cm

$$p_2 = 1000_{kPa}$$

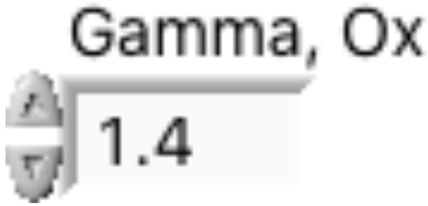


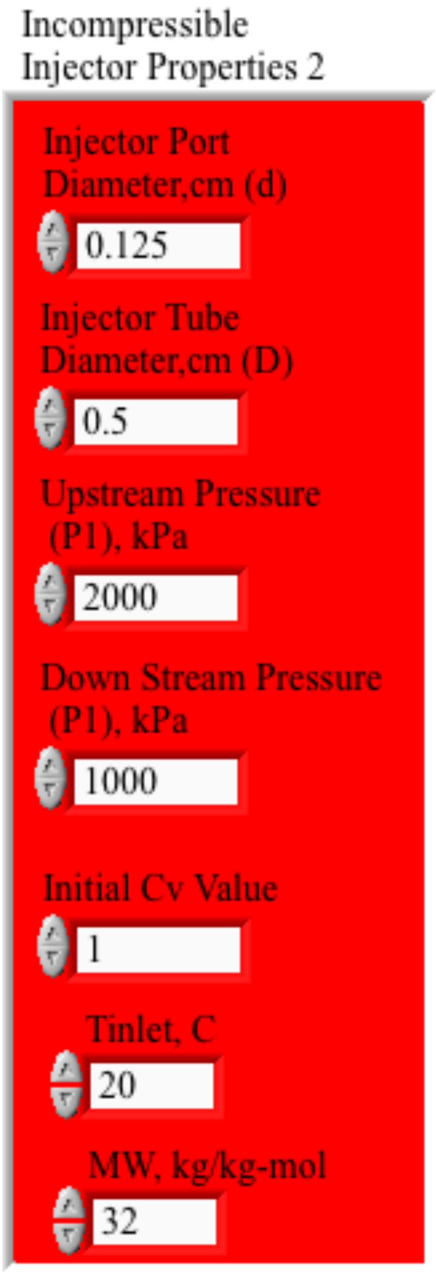
$$p_1 = 2000_{kPa}$$

d= 0.125 cm

$$\tilde{P}_{0_c} = p_2$$

Initial Chamber Pressure Guess Downstream of Injector



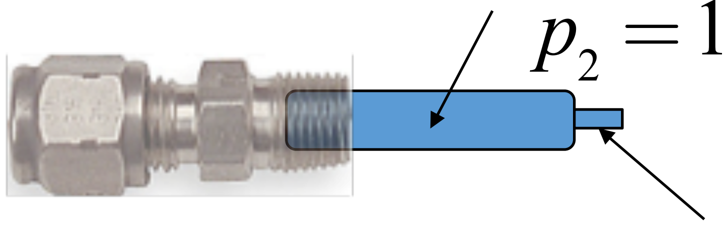


Incompressible Massflow Calculation

Injector Geometry Model

D= 0.5 cm

$p_2 = 1000_{kPa}$



$p_1 = 2000_{kPa}$

d= 0.125 cm

$$\rho_{inc} = \frac{P_1}{R_g \cdot T_1} = 2000 \frac{1000}{\frac{8314.4612}{32}(20+273.15)} = 26.2577 \; kg/m^3$$

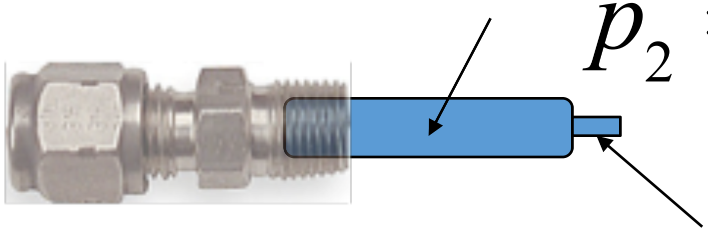
- **Assume Initial Cv=1**

$$\dot{m} = \left(\frac{Cd \cdot A_2}{\sqrt{1-\left(\frac{A_2}{A_1}\right)^2}} \right) \sqrt{2 \cdot \rho \cdot (p_1 - p_2)} = \frac{\left(\frac{0.125^2 \frac{\pi}{4}}{10000} \right) (2 \cdot 26.2577 \, (2000-1000) \, 1000)^{0.5} \, 1000}{\left(1 - \left(\frac{0.125}{0.5}\right)^4\right)^{0.5}} = 8.91052 \; g/sec$$

Calculate GOX Viscosity

Southerland's Law for GOX VISCOSITY

Injector Geometry Model



$$p_2 = 1000_{kPa}$$

$$p_1 = 2000_{kPa}$$

mu0, Nt-sec/m^2	Cs, deg K	T0, deg K
2.018E-5	127	292.25

$$\mu(T_0) = \mu_s \left(\frac{T_0}{T_s}\right)^{3/2} \left(\frac{T_s + C_s}{T_0 + C_s}\right) =$$

$$2.018 \cdot 10^{-5} \left(\frac{20 + 273.15}{292.25}\right)^{3/2} \left(\frac{292.25 + 127}{20 + 273.15 + 127}\right) = 2.02299\text{e-}05$$

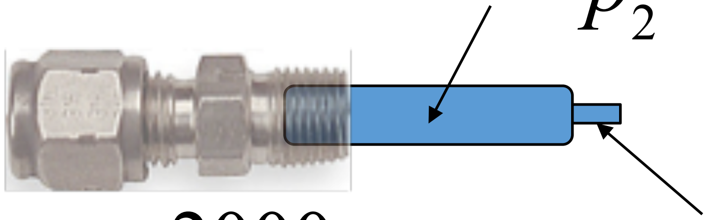
Pa-sec

Calculate Injector Inlet Flow Reynolds Number

Injector Geometry Model

D= 0.5 cm

$$p_2 = 1000_{kPa}$$



$$p_1 = 2000_{kPa}$$

d= 0.125 cm

$\rightarrow$ *Calculate Reynolds number*

$$\mathrm{R}_{e_{D_1}} = \frac{\rho_1 \cdot V_1 \cdot D_1}{\mu_1} = \frac{\frac{\dot{m}}{A_1} \cdot D_1}{\mu_1} = \frac{\frac{\dot{m}}{\frac{\pi}{4} D_1^2} \cdot D_1}{\mu_1} = \frac{4 \cdot \dot{m}}{\pi \cdot D_1 \cdot \mu_1} =$$

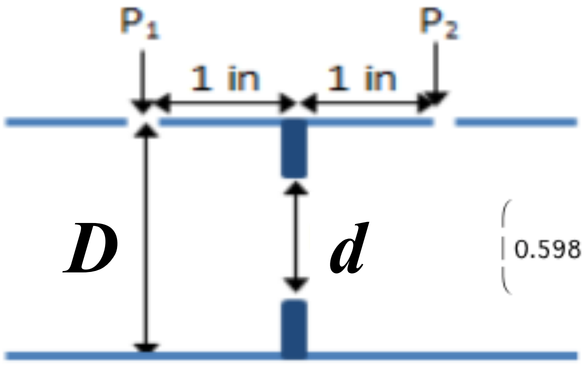
$$\frac{4 \, \frac{8.91051}{1000}}{\pi \, \frac{0.5}{100} \, 2.02299 \cdot 10^{-5}} = 112163$$

Calculate Injector Incompressible Discharge Coefficient

$R_{e_{D_1}}$ = 112163

Flange Tap: (*D*, *d* --> *inches*)

$$C_v i = \left[0.598 + 0.468 \cdot \left(\frac{d}{D}\right)^4 \left(1 + 10 \cdot \left(\frac{d}{D}\right)^8\right)\right] \cdot \sqrt{1 - \left(\frac{d}{D}\right)^4} + \left(0.87 + 8.1 \cdot \left(\frac{d}{D}\right)^4\right) \cdot \sqrt{\left(1 - \left(\frac{d}{D}\right)^4\right) \frac{1}{R_{e_D}}} =$$



FLANGE TAPS

$$\left(0.598 + 0.468 \left(\frac{0.125}{0.5}\right)^4 \left(1 + 10 \left(\left(\frac{0.125}{0.5}\right)^8\right)\right)\right) \left(1 - \left(\frac{0.125}{0.5}\right)^4\right)^{0.5} + \left(0.871 + 8.1 \left(\left(\frac{0.125}{0.5}\right)^4\right)\right) \left(\left(1 - \left(\frac{0.125}{0.5}\right)^4\right) \frac{1}{112163}\right)^{0.5} = 0.601346$$

D= 0.5 cm
0.19685 in

d= 0.125 cm
=0.0492126 in

$$\rightarrow \text{Calculate } C_d i \rightarrow \beta = \sqrt{1 - \left(\frac{D_2}{D_1}\right)^4} \rightarrow C_d i = \frac{C_v i}{\sqrt{1 - \left(\frac{D_2}{D_1}\right)^4}} =$$

$$\frac{0.601346}{\left(1 - \left(\frac{0.125}{0.5}\right)^4\right)^{0.5}}$$

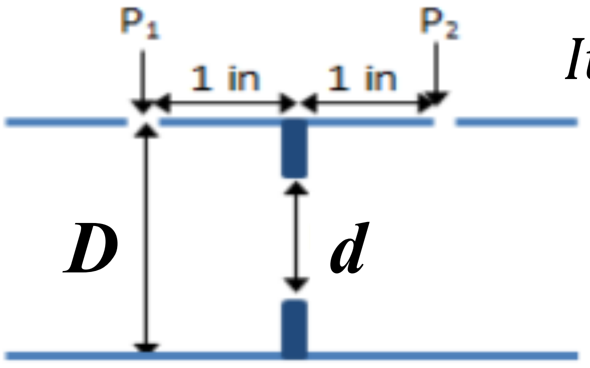
=0.60251

Iterate Injector Incompressible Discharge Coefficient

$R_{e_{D_1}}$ = 112163

Return to Slide 2 with New value for Cv_i

$$\text{Iterate Until} \left| \frac{\left(C_d i\right)_{j+1} - \left(C_d i\right)_j}{\left(C_d i\right)_j} \right| < \varepsilon$$

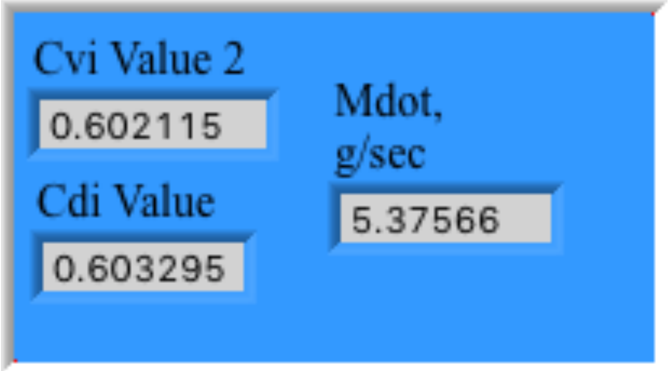


FLANGETAPS

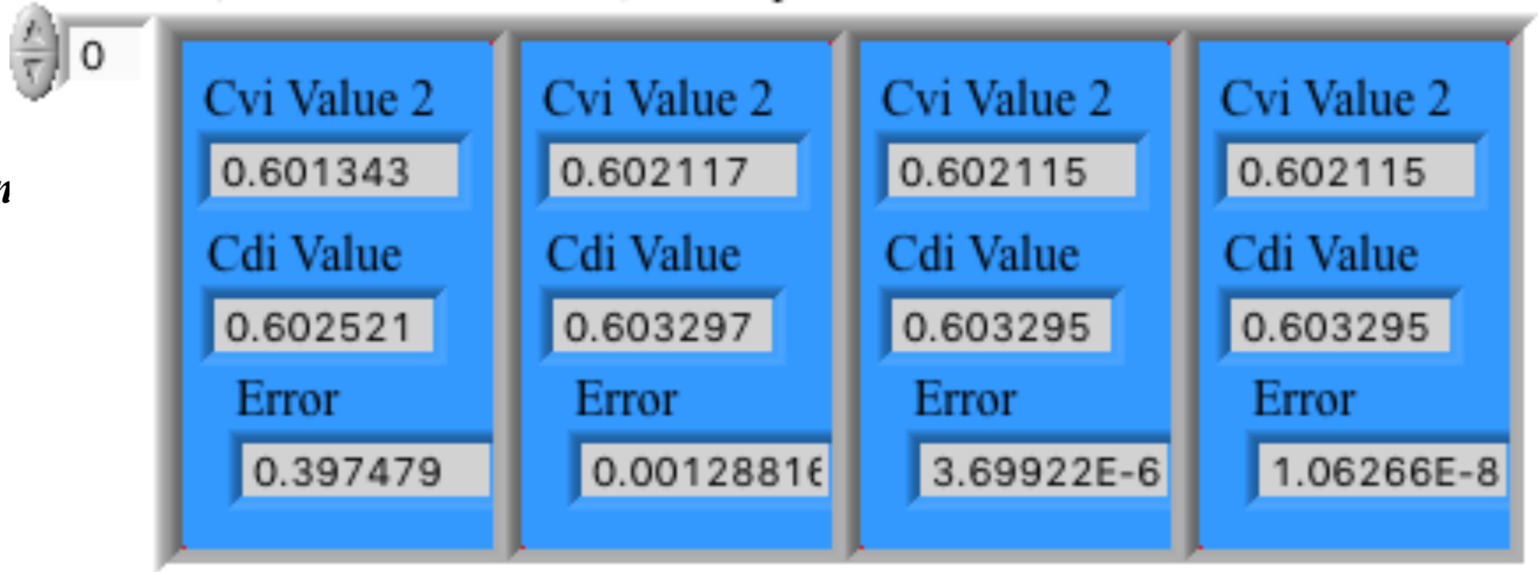
D= 0.5 cm
0.19685 in

d= 0.125 cm
=0.0492126 in

ASME
CV, CD FINAL
VALUES



Cv, Cd Iteration Values, Incompressible



Now Correct for Compressibility

$$P_0 = \left[\frac{\left(\frac{A_1}{A_2}\right)^2 \cdot (p_1)^{\frac{\gamma+1}{\gamma}} - (p_2)^{\frac{\gamma+1}{\gamma}}}{\left(\frac{A_1}{A_2}\right)^2 \cdot (p_1)^{\frac{2}{\gamma}} - (p_2)^{\frac{2}{\gamma}}}\right]^{\frac{\gamma}{\gamma-1}} = \left[\frac{\left(\frac{D_1}{D_2}\right)^4 \cdot (p_1)^{\frac{\gamma+1}{\gamma}} - (p_2)^{\frac{\gamma+1}{\gamma}}}{\left(\frac{D_1}{D_2}\right)^4 \cdot (p_1)^{\frac{2}{\gamma}} - (p_2)^{\frac{2}{\gamma}}}\right]^{\frac{\gamma}{\gamma-1}} = \left(\frac{\left(\frac{0.5}{0.125}\right)^4 2000^{\frac{1.4+1}{1.4}} - 1000^{\frac{1.4+1}{1.4}}}{\left(\frac{0.5}{0.125}\right)^4 2000^{2/1.4} - 1000^{2/1.4}}\right)^{\frac{1.4}{1.4-1}}$$

$= 2001.83$ kPa

$$\frac{\tilde{P}_{0_c}}{P_0} = \frac{p_2}{P_0} = \frac{1000}{2001.83} = 0.499543$$

$$r = \frac{\tilde{P}_{0_c}}{P_0} = \frac{1000}{2001.83} = 0.499543 < r^* = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} = 0.528282$$

Example Compressible Injector Calculation

Proceedings of the Institute of Mechanical Engineers
On the Flow of a Compressible Fluid through Orifices
D. A. Jobson,
First Published June 1, 1955 Research Article
https://doi.org/10.1243/PIME_PROC_1955_169_077_02

Subcritical Flow: $\left(\frac{p}{P_0}\right) > \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}$

$$r = \frac{p}{P_0}$$

$$f = \frac{1}{\left(C_d\right)_{inc}} - \frac{1}{2\left(C_d\right)^2_{inc}}$$

$$K_n = \sqrt{\frac{2\gamma}{\gamma-1} \cdot r^{\frac{2}{\gamma}}\left[1 - r^{\frac{\gamma-1}{\gamma}}\right]}$$

$$C_d = \frac{1 - \sqrt{1 - f \cdot \left(2 \cdot r^{\frac{1}{\gamma}}\right)^2 \left(\frac{1-r}{K_n^{\,2}}\right)}}{2 \cdot f \cdot r^{\frac{1}{\gamma}}}$$

$$\rightarrow \dot{m} = K_n \cdot C_d \cdot A \cdot \sqrt{P_0 \cdot \rho_0}$$

Supercritical Flow:

$$\left(\frac{p}{P_0}\right) \le \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} \qquad r^* = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}$$

$$f = \frac{1}{\left(C_d\right)_{inc}} - \frac{1}{2\left(C_d\right)^2_{inc}}$$

$$K_n = \sqrt{\frac{2\gamma}{\gamma-1} \cdot {r^*}^{\frac{2}{\gamma}}\left[1 - {r^*}^{\frac{\gamma-1}{\gamma}}\right]} = \sqrt{\gamma \cdot \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}}$$

$$C_d = \left(\frac{1}{2 \cdot f \cdot {r^*}^{\frac{1}{\gamma}}}\right) \cdot \left[\left\{1 + \frac{\left(r^* - \left(\frac{p}{P_0}\right)\right) \cdot {r^*}^{\frac{1}{\gamma}}}{K_n^2}\right\} - \sqrt{\left(1 + \frac{\left(r^* - \left(\frac{p}{P_0}\right)\right) \cdot {r^*}^{\frac{1}{\gamma}}}{K_n^2}\right)^2 - \frac{\left(2 \cdot {r^*}^{\frac{1}{\gamma}}\right)^2 \cdot \left(1 - \left(\frac{p}{P_0}\right)\right) \cdot f}{K_n^2}}\right]$$

$$\rightarrow \dot{m} = K_n \cdot C_d \cdot A \cdot \sqrt{P_0 \cdot \rho_0}$$

Now Correct for Compressibility

Supercritical Flow

$$r = \frac{\tilde{P}_{0_c}}{P_0} = \frac{1000}{2001.83} = 0.499543 < r^* = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} = 0.528282$$

$$f = \frac{1}{\left(C_d\right)_{inc}} - \frac{1}{2\left(C_d\right)^2_{inc}} = \frac{1}{0.603295} - \frac{1}{2\,(0.603295)^2} = 0.283805$$

$$K_n = \sqrt{\frac{2\gamma}{\gamma-1} \cdot r^{*\frac{2}{\gamma}}\left[1 - r^{*\frac{\gamma-1}{\gamma}}\right]} = \sqrt{\gamma \cdot \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}} = \left(1.4\left(\frac{2}{1.4+1}\right)^{\frac{1.4+1}{1.4-1}}\right)^{0.5} = 0.684731$$

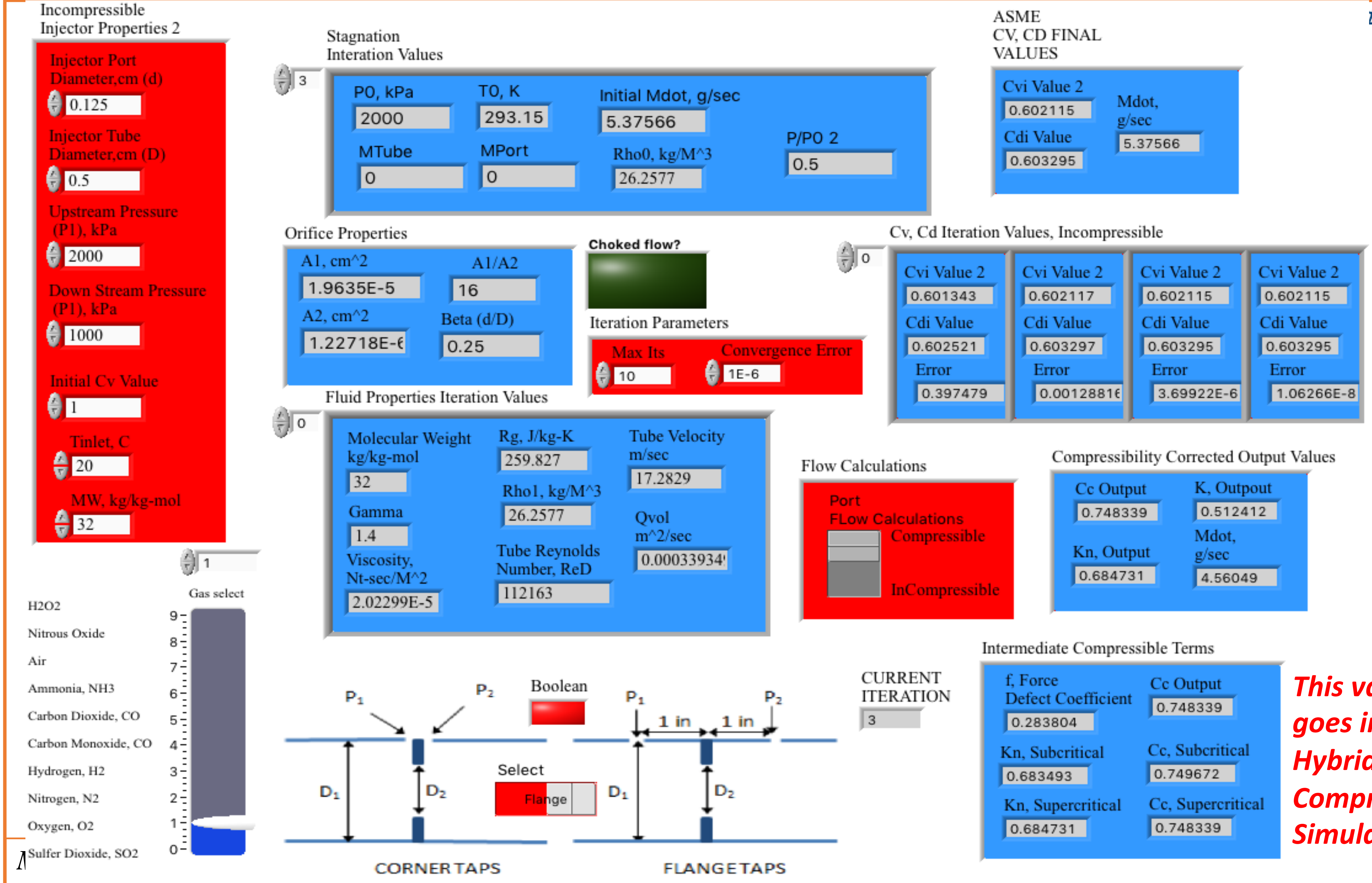
Correct for Compressibility (2)

$$C_d = \left(\frac{1}{2 \cdot f \cdot r^{*\frac{1}{\gamma}}}\right) \cdot \left[\left\{ 1 + \frac{\left(r^* - \left(\frac{p}{P_0}\right)\right) \cdot r^{*\frac{1}{\gamma}}}{K_n^2} \right\} - \sqrt{\left(1 + \frac{\left(r^* - \left(\frac{p}{P_0}\right)\right) \cdot r^{*\frac{1}{\gamma}}}{K_n^2}\right)^2 - \frac{\left(2 \cdot r^{*\frac{1}{\gamma}}\right)^2 \cdot \left(1 - \left(\frac{p}{P_0}\right)\right) \cdot f}{K_n^2}} \right] =$$

$$\frac{\left(1 + \left(\frac{(0.528282 - 0.499543)}{0.684731^2}\right) 0.528282^{1/1.4}\right) - \left(\left(\left(1 + \left(\frac{(0.528282 - 0.499543)}{0.684731^2}\right) 0.528282^{1/1.4}\right)^2\right) - 4 \cdot 0.283804 \left(0.528282^{2/1.4}\right) \frac{(1 - 0.499543)}{0.6847312^2}\right)^{0.5}}{2 \cdot 0.283804 \left(0.528282^{1/1.4}\right)}$$

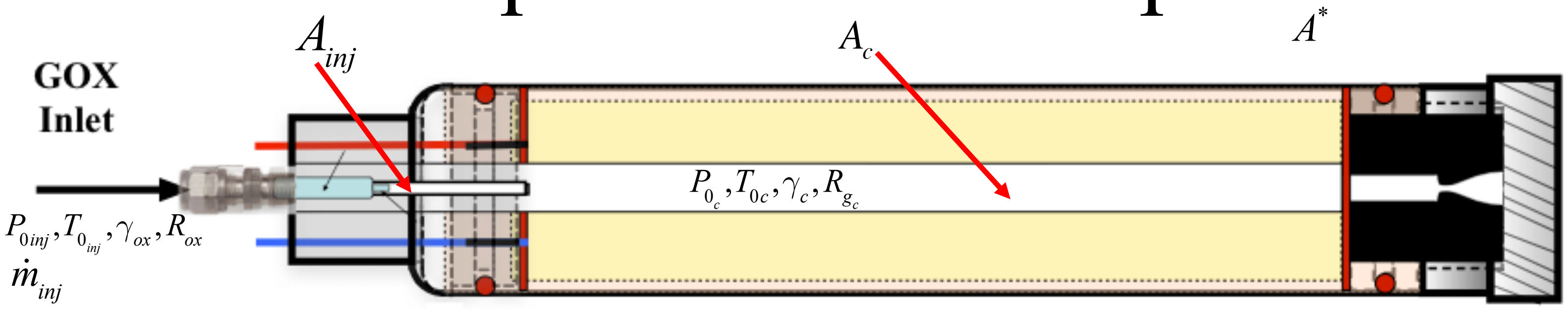
$$=0.74834$$

This value goes into your Hybrid Compressible Simulation



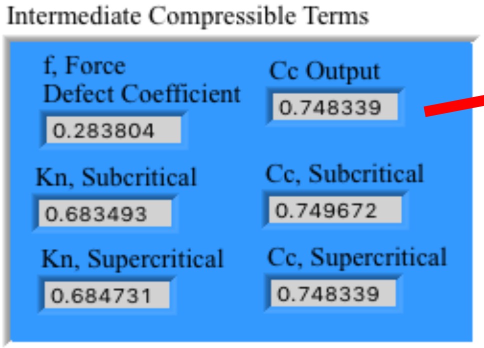
This value goes into your Hybrid Compressible Simulation

Hybrid Ballistic Equations for Compressible Oxidizer



$$Supercritical: \left(\frac{P_{0inj}}{P_{0_c}}\right) \geq \left(\frac{\gamma_{ox}+1}{2}\right)^{\frac{\gamma_{ox}}{\gamma_{px}-1}}$$

$$\rightarrow Injector\ Choked \rightarrow r_c = \left(\frac{2}{\gamma_{0x}+1}\right)^{\frac{\gamma_{ox}}{\gamma_{ox}-1}}$$



$$K_n = \sqrt{\gamma_{ox} \cdot \left(\frac{2}{\gamma_{ox}+1}\right)^{\frac{\gamma_{ox}+1}{\gamma_{px}-1}}} \rightarrow \dot{m}_{ox} = \left(K_n \cdot C_d \cdot A\right)_{inj} \cdot \frac{P_{0inj}}{\sqrt{R_{g_{ox}} T_{0_{inj}}}}$$

$$C_d = \left(\frac{1}{2 \cdot f \cdot r_c^{\frac{1}{\gamma_{ox}}}}\right) \cdot \left[\left\{1 + \frac{\left(r_c - \left(\frac{P_{0_c}}{P_{0_{inj}}}\right)\right) \cdot r_c^{\frac{1}{\gamma_{ox}}}}{K_n^2}\right\} - \sqrt{1 + \frac{\left(r_c - \left(\frac{P_{0_c}}{P_{0_{inj}}}\right)\right) \cdot r_c^{\frac{1}{\gamma_{ox}}}}{K_n^2} - \frac{\left(2 \cdot r_c^{\frac{1}{\gamma_{ox}}}\right)^2 \cdot \left(1 - \left(\frac{P_{0_c}}{P_{0_{inj}}}\right)\right) \cdot f}{K_n^2}}\right]$$

Chamber Pressure:

$$\frac{\partial P_{0c}}{\partial t} = \frac{A_{burn} \dot{r}_{fuel}}{V_c}\left[\rho_{fuel} R_{g_c} T_{0c} - P_{0c}\right] - P_{0c}\left[\frac{A^*}{V_c}\sqrt{\gamma_c R_{g_c} T_0 \left(\frac{2}{\gamma_c+1}\right)^{\frac{\gamma_c+1}{(\gamma_c-1)}}}\right] + \frac{R_{g_c} T_{0_c}}{V_c} \cdot \left\{\left(K_n \cdot C_d \cdot A_{inj}\right)_{inj} \cdot \frac{P_{0inj}}{\sqrt{R_{g_{ox}} \cdot T_{0_{inj}}}}\right\}$$

Regression:

$$\dot{r}_{fuel} = \left(\frac{0.047}{\rho_{fuel} \cdot \left(P_{r_c}\right)^{2/3}}\right) \cdot \left(\frac{C_{P_c} \cdot \left(T_{0_c} - T_{fuel_{surf}}\right)}{h_{v_{fuel}}}\right) \cdot \left(\frac{\dot{m}_{ox}}{A_c}\right)^{4/5} \cdot \left(\frac{\mu_c}{L}\right)^{1/5}$$

$$\dot{m}_{fuel} = \rho_{fuel} \cdot A_{burn} \cdot \dot{r}_{fuel} = \rho_{fuel} \cdot \pi \cdot (D \cdot L)_{port} \cdot \dot{r}_{fuel} \rightarrow O/F = \frac{\dot{m}_{ox}}{\dot{m}_{fuel}} = \frac{\left(K_n \cdot C_d \cdot A\right)_{inj} \cdot \frac{P_{0inj}}{\sqrt{R_{g_{ox}} T_{0_{inj}}}}}{\rho_{fuel} \cdot \pi \cdot (D \cdot L)_{port} \cdot \dot{r}_{fuel}}$$